Ratio-of-Mediator-Probability Weighting for Causal Mediation Analysis in the Presence of Treatment-by-Mediator Interaction

Online Supplement of Simulation Results

Table S1

Misspecified Nonlinear Propensity Score Models: Low-Degree Nonlinearity

	Model	N = 5,000			N = 800		
		RMPW	*NRMPW 3×3*	*NRMPW 4×4*	*RMPW*	*NRMPW 3×3*	*NRMPW 4×4*
Direct Effect Estimate							
% Bias removal	(a)	0.9106	0.8525	0.9001	0.8918	0.8352	0.8645
	(b)	0.8842	0.8671	0.9151	0.8617	0.8468	0.8619
	(c)	0.9160	0.8525	0.8984	0.9046	0.8192	0.7656
Relative efficiency	(a)	0.9335	0.9529	0.9612	0.9294	0.9145	0.8775
	(b)	1.0293	1.0695	1.0738	0.9973	1.0100	0.9560
	(c)	0.8780	0.9628	0.9475	0.8161	0.7880	0.8028
MSE	(a)	0.0023	0.0030	0.0024	0.0113	0.0123	0.0123
	(b)	0.0042	0.0042	0.0037	0.0214	0.0213	0.0223
	(c)	0.0067	0.0099	0.0071	0.0324	0.0394	0.0444
Indirect Effect Estimate							
% Bias removal	(a)	0.9092	0.8511	0.8986	0.9059	0.8484	0.8782
	(b)	0.8731	0.8562	0.9036	0.8706	0.8555	0.8708
	(c)	0.9104	0.8472	0.8929	0.9085	0.8228	0.7689
Relative efficiency	(a)	1.5800	1.8114	1.7021	1.1918	1.3194	1.0190
	(b)	0.6741	0.7422	0.7239	0.6171	0.6996	0.6488
	(c)	0.7978	0.9910	0.9386	0.7011	0.7370	0.7441
MSE	(a)	0.0006	0.0013	0.0007	0.0021	0.0027	0.0027
	(b)	0.0026	0.0026	0.0021	0.0122	0.0111	0.0116
	(c)	0.0045	0.0079	0.0050	0.0186	0.0235	0.0289

Table S2

Misspecified Nonlinear Propensity Score Models: Moderate-Degree Nonlinearity

	Model	*N* = 5,000			*N* = 800		
		RMPW	*NRMPW* 3×3	*NRMPW* 4×4	*RMPW*	*NRMPW* 3×3	*NRMPW* 4×4
Direct Effect Estimate							
% Bias removal	(a)	0.8412	0.8404	0.8839	0.8187	0.8249	0.8527
	(b)	0.8029	0.8608	0.9054	0.7768	0.8421	0.8598
	(c)	0.8703	0.8400	0.8802	0.8494	0.7960	0.7366
Relative efficiency	(a)	0.9688	0.9674	0.9738	0.9704	0.9354	0.9019
	(b)	1.0423	1.0708	1.0808	1.0167	1.0163	0.9722
	(c)	0.9459	0.9987	0.9675	0.8951	0.8717	0.8469
MSE	(a)	0.0028	0.0028	0.0024	0.0113	0.0116	0.0117
	(b)	0.0049	0.0041	0.0037	0.0213	0.0204	0.0210
	(c)	0.0082	0.0097	0.0075	0.0341	0.0389	0.0456
Indirect Effect Estimate							
% Bias removal	(a)	0.8397	0.8389	0.8823	0.8333	0.8396	0.8679
	(b)	0.7900	0.8470	0.8909	0.7815	0.8471	0.8650
	(c)	0.8656	0.8355	0.8755	0.8547	0.8010	0.7412
Relative efficiency	(a)	1.5569	1.7352	1.7215	1.1579	1.2258	1.0218
	(b)	0.6859	0.7706	0.7651	0.6129	0.6988	0.6650
	(c)	0.9208	1.0569	0.9657	0.8197	0.8282	0.7968
MSE	(a)	0.0012	0.0012	0.0007	0.0026	0.0025	0.0025
	(b)	0.0035	0.0026	0.0022	0.0137	0.0114	0.0117
	(c)	0.0060	0.0076	0.0054	0.0201	0.0238	0.0300

Table S3

Misspecified Non-additive Propensity Score Models: Low-Degree Non-additivity

	Model	N = 5,000			N = 800		
		RMPW	*NRMPW 3×3*	*NRMPW 4×4*	*RMPW*	*NRMPW 3×3*	*NRMPW 4×4*
Direct Effect Estimate							
% Bias removal	(a)	0.9393	0.8594	0.9070	0.9196	0.8435	0.8663
	(b)	0.9249	0.8732	0.9208	0.8994	0.8508	0.8504
	(c)	0.9263	0.8567	0.9029	0.9136	0.8432	0.8378
Relative efficiency	(a)	0.9394	0.9608	0.9716	0.9061	0.9040	0.8968
	(b)	1.0228	1.0544	1.0628	0.9876	0.9933	0.9554
	(c)	0.8877	0.9639	0.9701	0.8116	0.8586	0.8228
MSE	(a)	0.0021	0.0029	0.0023	0.0114	0.0123	0.0121
	(b)	0.0038	0.0042	0.0037	0.0214	0.0219	0.0227
	(c)	0.0051	0.0081	0.0057	0.0255	0.0280	0.0293
Indirect Effect Estimate							
% Bias removal	(a)	0.9378	0.8580	0.9055	0.9339	0.8567	0.8799
	(b)	0.9156	0.8644	0.9115	0.9132	0.8638	0.8634
	(c)	0.9231	0.8537	0.8997	0.9178	0.8471	0.8417
Relative efficiency	(a)	1.4852	1.8095	1.6943	1.2177	1.3721	1.1092
	(b)	0.6531	0.7102	0.7063	0.6320	0.6825	0.6343
	(c)	0.7640	0.9555	0.9258	0.6711	0.7531	0.6913
MSE	(a)	0.0005	0.0012	0.0007	0.0020	0.0026	0.0027
	(b)	0.0022	0.0025	0.0020	0.0115	0.0112	0.0120
	(c)	0.0029	0.0060	0.0036	0.0127	0.0151	0.0163

Table S4

Misspecified Non-additive Propensity Score Models: Moderate-Degree Non-additivity

	Model	*N* = 5,000			*N* = 800		
		RMPW	*NRMPW 3×3*	*NRMPW 4×4*	*RMPW*	*NRMPW 3×3*	*NRMPW 4×4*
Direct Effect Estimate							
% Bias removal	(a)	0.8757	0.8618	0.9079	0.8576	0.8471	0.8583
	(b)	0.8439	0.8816	0.9280	0.8221	0.8617	0.8447
	(c)	0.8500	0.8588	0.9020	0.8357	0.8388	0.8209
Relative efficiency	(a)	1.2890	0.9565	0.9600	0.9291	0.9051	0.8900
	(b)	0.6931	1.0681	1.0810	1.0309	0.9939	0.9773
	(c)	0.8917	0.9646	0.9464	0.9439	0.8747	0.8455
MSE	(a)	0.0026	0.0027	0.0023	0.0116	0.0120	0.0120
	(b)	0.0045	0.0041	0.0037	0.0212	0.0214	0.0220
	(c)	0.0077	0.0073	0.0055	0.0258	0.0273	0.0292
Indirect Effect Estimate							
% Bias removal	(a)	0.8742	0.8604	0.9064	0.8718	0.8611	0.8725
	(b)	0.8292	0.8662	0.9118	0.8262	0.8661	0.8489
	(c)	0.8479	0.8566	0.8997	0.8419	0.8450	0.8269
Relative efficiency	(a)	1.5349	1.7647	1.6173	1.2890	1.2282	1.0362
	(b)	0.7229	0.7570	0.7467	0.6931	0.6949	0.6848
	(c)	0.9438	0.9704	0.9262	0.8917	0.7942	0.7436
MSE	(a)	0.0009	0.0011	0.0006	0.0024	0.0026	0.0028
	(b)	0.0030	0.0025	0.0021	0.0117	0.0112	0.0116
	(c)	0.0056	0.0051	0.0032	0.0132	0.0140	0.0158

Online Supplement of Stata code for RMPW Analyses

I. Parametric RMPW analysis for a binary mediator using Generalized Method of Moments (GMM) to account for estimation error in the weights

Note: the procedure below does not exclude observations for which there is no common support.

```
*** Generate a constant to be used in the GMM command

generate cons = 1

*** Specify moment equations, storing them in "locals"
*** deltaC is the control group mean outcome; deltaE the experimental group mean outcome
*** deltaStar1 is the counterfactual mean outcome of the experimental group
*** deltaStart0 is the counterfactual mean outcome of the control group

local equation1 ( Z - ( ( 1 / ( 1 + exp(-{xb1: X1 X2 X3 X4 X5 X6 X7 X8 X9 cons}) ) ) ) ) * A
local equation2 ( Z - ( ( 1 / ( 1 + exp(-{xb2: X1 X2 X3 X4 X5 X6 X7 X8 X9 cons}) ) ) ) ) * (1 - A)
local equation3 ( Y - {deltaC} ) * ( 1 - A)
local equation4 ( Y - {deltaE} ) * A
local equation5 ( Y - {deltaStar1} ) * ///
            ( ( ( Z * ( ( 1 / ( 1 + exp(-{xb2:}) ) )/( 1 / ( 1 + exp(-{xb1:}) ) ) ) ) ) + ( (1-Z) * ///
            ( exp(-{xb2:}) / ( 1 + (exp(-{xb2:})) ) ) / ( exp(-{xb1:}) / ( 1 + (exp(-{xb1:})))) ) ) ) * A
local equation6 ( Y - {deltaStar0} ) * ///
  ( ( ( Z * ( ( 1 / ( 1 + exp(-{xb1:}) ) )/( 1 / ( 1 + exp(-{xb2:}) ) ) ) ) ) + ( (1-Z) * ///
  ( exp(-{xb1:}) / ( 1 + (exp(-{xb1:})) ) ) / ( exp(-{xb2:}) / ( 1 + (exp(-{xb2:})))) ) ) ) * (1 – A)

*** Specify "instruments," storing them in locals

local equation1inst X1 X2 X3 X4 X5 X6 X7 X8 X9
local equation2inst X1 X2 X3 X4 X5 X6 X7 X8 X9

*** Execute GMM command

gmm (eq1: `equation1') (eq2: `equation2') (eq3: `equation3') (eq4: `equation4') (eq5: `equation5') ///
(eq6: `equation6'), instruments(eq1: `equation1inst') instruments(eq2: `equation2inst')
instruments(eq3: ) instruments(eq4: ) instruments(eq5: ) instruments(eq6: ) winitial(identity) onestep

*** Estimate Natural Indirect Effect

lincom _b[/deltaE] - _b[/deltaStar1]

*** Estimate Natural Direct Effect

lincom _b[/deltaStar1] - _b[/deltaC]
```

***Estimate Pure Indirect Effect

```
lincom _b[/deltaStar0] - _b[/deltaC]
```

*** Estimate Treatment-by-Mediator Interaction Effect

```
lincom (_b[/deltaE] - _b[/deltaStar1]) - (_b[/deltaStar0] - _b[/deltaC])
```

II. Parametric RMPW analysis for a binary mediator

Note: this code does not account for estimation error in the RMPW weights, but does exclude observations due to lack of common support.

```
***  Run binary logit for the control group

logit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==0

*** Generate predicted probability, that is pr(Z=1|X, A=0), for both experimental and control groups.

* Note that this will be an in-sample prediction for those in the control group
* and an out-of-sample prediction for those in the experimental group.

predict p0, pr

predict xb0, xb

*** Run binary logit for the experimental group

logit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==1

*** Generate predicted probability, that is pr(Z=1|X, A=1).  Note that this will be an
* in-sample prediction for those in the experimental group
* and an out-of-sample prediction for those in the control group.

predict p1, pr

predict xb1, xb

*** Variables xb0 and xb1 are the logit scores of the respective propensity models for
* the experimental and control groups.

* Loop over logit scores (a= 0, 1), treatment groups (i= 0, 1), and mediator values (j=0, 1)

forvalues a=0(1)1  {

* Calculate the standard deviation of each logit score, to be used below.

qui sum xb`a'

sca sd`a'=r(sd)

forvalues i=0(1)1  {

forvalues j=0(1)2  {
```

```
qui sum xb`a' if A==`i' & Z==`j'

* Calculate the "minimum" and “maximum” of each logit score for each treatment-by-mediator group.
* Where the "maximum"("minimum") is actually 20% of a standard deviation of the logit score
* above (below) the actual maximum (minimum).

sca max`a'`i'`j'=r(max) + .2*sd`a'

sca min`a'`i'`j'=r(min) - .2*sd`a'

}

}

}

*** Generate an "exclude" indicator

* Loop over each logit score.

gen exclude=0

forvalues a=0(1)1  {

replace exclude=1 if xb`a'<max(min`a'00, min`a'01, min`a'10, min`a'11)

replace exclude=1 if xb`a'>min(max`a'00, max`a'01, max`a'10, max`a'11)

}

*** Generate parametric RMPW

gen rmpw=1 if exclude==0

replace rmpw=p0/p1 if A==1 & Z==1 & exclude==0

replace rmpw=(1-p0)/(1-p1) if A==1 & Z==0 & exclude==0

*** Generate a unique identifier, called "obs", for each person.  This will allow
* duplicates to have the same identifier, which will be necessary for obtaining the correct
* standard errors.

gen obs=_n

*** Generate duplicate observations for the experimental group, where D1 is the indicator
* for duplicate. D1=0 for all control group observations and original experimental group
```

* observations.

expand 2 if A==1, gen(D1)

*** Make sure duplicates get a weight=1. Note that duplicate observations receive a different
* weight than their original.

replace rmpw=1 if D1==1 & exclude==0

*** Outcome model.

* This command weights each observation and clusters standard errors at the person level,
* adjusting for correlation in errors within each set of duplicates.

*** Optional adjustment for covariate X1, centered at its sample mean, for improving precision.

reg Y A D1 X1 [pweight=rmpw], vce(cluster obs)

*** To decompose natural indirect effect into the pure indirect effect and the natural
*treatment-by-mediator interaction effect

*Create a duplicate set of the control group, which will be weighted

expand 2 if A==0, gen(D0)

* Generate a new set of weights for the duplicate control group

replace rmpw = p1/p0   if A==0 & Z==1 & D0==1 & exclude==0

replace rmpw = (1-p1)/(1-p0)   if A==0 & Z==0 & D0==1 & exclude==0

*** Outcome model to estimate the pure indirect effect and the natural treatment-by-mediator
* interaction effect

*** Optional adjustment for covariate X1, centered at its sample mean, for improving precision.

reg Y A D1 D0 X1 [pweight=rmpw], vce(cluster obs)

lincom D1 – D0

*** The coefficient for D0 represents the pure indirect effect. The coefficient for D1 represents
* the total indirect effect.

*** The post-estimation command estimates, and does a significance test on, the natural
* treatment-by-mediator interaction effect, which is the total indirect effect less the pure indirect *
effect.

III. Nonparametric RMPW analysis for a binary mediator

Note: this code does not account for estimation error in the RMPW weights, but does exclude observations due to lack of common support.

```
*** Run binary logit for the control group.

logit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==0

*** Generate logit score (not probability), which will be used later to create a categorical
* variable used in creating nonparametric weights.

predict xb0, xb

*** Run binary logit for the experimental  group.

logit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==1

***  Generate logit score, which will be used later to create a categorical variable used in

* creating nonparametric weights.

predict xb1, xb

*** Identify common support: the code is the same as that under parametric analysis

*** Generate 3×3 nonparametric RMPW

*** Place all observations into three equal-sized categories based on their logit score
* from the experimental group model.

* Generate categorical variable h1 = (0, 1, 2) based on terciles in xb1.

egen h1=cut(xb1), group(3)

*** Within each category of h1, generate categorical variables (h00, h01, h02) = (0, 1, 2)
* based on terciles in xb0.

* Loop through each category of h1.

forvalues j=0(1)2  {

egen h0`j'=cut(xb0) if h1==`j', group(3)
```

```
}

***  Generate a strata variable to place each observation into one of 9 strata,
* based on joint distribution of xb0 and xb1.

gen strata=.

replace strata=0 if h1==0 & h00==0

replace strata=1 if h1==0 & h00==1

replace strata=2 if h1==0 & h00==2

replace strata=3 if h1==1 & h01==0

replace strata=4 if h1==1 & h01==1

replace strata=5 if h1==1 & h01==2

replace strata=6 if h1==2 & h02==0

replace strata=7 if h1==2 & h02==1

replace strata=8 if h1==2 & h02==2

*** Calculate probabilities P(Z=1| A, strata). “Prij” is the probability that Z=1 in treatment
* group i and strata j. Loop over treatment groups (i = 0, 1) and strata (j = 0, 1, . . . , 8).

forvalues i=0(1)1  {

forvalues j=0(1)8  {

qui sum Z if A==`i' & strata==`j' & exclude==0

sca pr`i'`j'=r(mean)

}

}

***  Generate nonparametric RMPW weights based on these calculated probabilities,
* treatment group membership, strata membership, and Z.

gen nrmpw=1 if exclude==0

* Loop over strata categories (j= 0, 1, . . . , 8).
```

```
forvalues j=0(1)8  {

replace nrmpw = pr0`j'/pr1`j' if A==1 & strata==`j' & Z==1 & exclude==0

replace nrmpw = (1-pr0`j')/(1-pr1`j') if A==1 & strata==`j' & Z==0 & exclude==0

}

***  Use the same process here as in the parametric case to create a person-specific identifier
* and generate duplicate observations.

gen obs=_n

expand 2 if A==1, gen(D1)

* Ensure duplicates receive a weight equal to 1.

replace nrmpw=1 if D1==1 & exclude==0

*** Outcome model.

*Command weights each observation and clusters standard errors at the person level,
* adjusting for correlation in errors within each set of duplicates.

*** Optional adjustment for covariate X1, centered at its sample mean, for improving precision.

reg Y A D1 X1 [weight=nrmpw], vce(cluster obs)

*** To decompose the indirect effect into the pure indirect effect and the natural
*treatment-by-mediator interaction effect

*Create a duplicate set of the control group, which will be weighted

expand 2 if A==0, gen(D0)

* Generate new set of weights for the duplicate control group

* Loop over strata categories (j= 0, 1, . . . , 8).

forvalues j=0(1)8  {

replace nrmpw = pr1`j'/pr0`j' if A==0 & strata==`j' & Z==1 & D0==1 & exclude==0

replace nrmpw = (1-pr1`j')/(1-pr0`j') if A==0 & strata==`j' & Z==0 & D0==1 & exclude==0

}
```

```
*** Outcome model to estimate the pure indirect effect and the natural treatment-by-mediator
* interaction effect

*** Option adjustment for covariate X1, centered at its sample mean, for improving precision.

reg Y A D1 D0 X1 [pweight=nrmpw], vce(cluster obs)

lincom D1 – D0
```

IV. Parametric RMPW analysis for a three-category mediator

Note: this code does not account for estimation error in the RMPW weights, but does exclude observations due to lack of common support.

```
*****  Run ordered logit  ***

*** Same as binary parametric analysis except that we have an ordered logit, where Z= 0, 1, 2
* with three predicted probabilities under each treatment.

ologit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==0

predict p00 p01 p02, pr

ologit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==1

predict p10 p11 p12, pr

*** Run ordered logit for each treatment group and generate logit scores.

ologit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==0

predict xbo0, xb

ologit Z X1 X2 X3 X4 X5 X6 X7 X8 X9 if A==1

predict xbo1, xb

* Loop over logit scores (a=0, 1), treatment groups “i” and mediator values “j”.

forvalues a=0(1)1  {

* Calculate the standard deviation of each logit score.

qui sum xbo`a'

sca sdo`a'=r(sd)

forvalues i=0(1)1  {

forvalues j=0(1)2  {

* Calculate "minimums" and "maximums" for each treatment-by-mediator group as above.

qui sum xbo`a' if A==`i' & Z==`j'
```

```
sca max`a'`i'`j'=r(max) + .2*sdo`a'

sca min`a'`i'`j'=r(min) - .2*sdo`a'

}

}

}

*** Identify common support; generate an "exclude" indicator

* Loop over each logit score xbo “a”.

gen exclude3=0

forvalues a=0(1)1  {

replace exclude3=1 if xbo`a'>min(max`a'00, max`a'01, max`a'02, max`a'10, max`a'11, max`a'12)

replace exclude3=1 if xbo`a'<max(min`a'00, min`a'01, min`a'02, min`a'10, min`a'11, min`a'12)

}

*** Generate weight

gen rmpw3=1 if exclude3==0

forvalues j=0(1)2  {

replace rmpw3=p0`j'/p1`j' if A==1 & Z==`j' & exclude3==0

}

***  Create a person-specific identifier,generate duplicate LFA observations, and give duplicates a weight equal to 1.

gen obs=_n

expand 2 if A==1, gen(D1)

replace rmpw3=1 if D1==1 & exclude3==0

*** Outcome model

*** Optional adjustment for covariate X1, centered at its sample mean, for improving precision.
```

```
reg Y A D X1 [weight=rmpw3], vce(cluster obs)

*** To decompose natural indirect effect into the pure indirect effect and the natural
*treatment-by-mediator interaction effect

*Create a duplicate set of the control group, which will be weighted

expand 2 if A==0, gen(D0)

* Generate a new set of weights for the duplicate control group

forvalues j=0(1)2  {

replace rmpw3=p1`j'/p0`j' if A==0 & Z==`j' & exclude3==0

}

*** Outcome model to estimate the pure indirect effect and the natural treatment-by-mediator
* interaction effect

*** Optional adjustment for covariate X1, centered at its sample mean, for improving precision.

reg Y A D1 D0 X1 [pweight=rmpw], vce(cluster obs)

lincom D1 – D0

*** The coefficient for D0 represents the pure indirect effect. The coefficient for D1 represents
* the total indirect effect.

*** The post-estimation command estimates, and does a significance test on, the natural
* treatment-by-mediator interaction effect, which is the total indirect effect less the pure indirect *
effect.
```