► , *The Logic of Impredicativity.*

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E-mail: .

A definition is said to be impredicative if it quantifies over a totality to which the *definiendum* belongs. In this talk I will examine the phenomenon of impredicativity in the light of different accounts of quantification, in order to test the hypothesis that the circularity usually attributed to it is not problematic *per se*, but only in virtue of the meaning of the quantification in classical logic. Thus the aim of the talk is to relativise Russell's Vicious Circle Principle (VCP) and predicativism ([8], [6], [1], [4]) to classical logic and to compare three non-classical treatments of impredicativity.

Classical rules support an explanation of quantification as the exhaustive conjunction or disjunction of its instances. In this context, the circularity arises because the *definiendum* of an impredicative definition is inevitably one of these instances, namely one of the possible values of the variable bound by the quantifier.

Alternative approaches to impredicativity are based on different interpretations of the quantification. I will explore three non-classical settings, respectively formalised in constructive type theory ([5], [2]), semi-intuitionistic logic ([3] and substructural logic ([7], [9]).

All these approaches share the idea that the account of predicativism depends strictly on the notion of generality involved in the quantification (on which the impredicativity depends). However, on the semi-intuitonistic route, we still presuppose a notion of *proper* totality, namely the kind of generality inherent in the infinite nature of every infinite collection (e.g. integers, real numbers...); on the other hand, on the type-theoretic and substructural routes, we presuppose what we can call *schematic* totality, namely the generality of the syntactic rules governing substitution, instantiation and elimination processes.

These approaches will be discussed and applied, as a case study, to the abstractionist programs in philosophy of mathematics, which are typically based on impredicative abstraction principles, understood as implicit definitions of the primitive terms of the theory.

[1] Dummett, M., ***Frege: Philosophy of Mathematics*** Cambridge University Press 1991.

[2] Fruchart, T., Longo, G. *Carnap's remarks on impredicative definitions and the genericity theorem.* ***Logic and Foundations of Mathematics: Selected Contributed Papers of the Tenth International Congress of Logic, Methodology and Philosophy of Science,*** Florence, 1999 September Dordrecht: Springer Netherlands.

[3] Linnebo, Ø *Generality explained.* ***Journal of Philosophy***, 119(7), 349-379, 2022.

[4] Linnebo, Ø., Shapiro, S.. *Predicativism as a form of potentialism.* ***The Review of Symbolic Logic***, 16(1), 1-32, 2023.

[5] Pistone, P. *Polymorphism and the Obstinate Circularity of Second-Order Logic: a Victim's Tale,* ***Bulletin of Symbolic Logic***, 24(1), 1-52, 2018.

[6] Poincaré, H. *Les mathématiques et la logique,* ***Revue de Métaphysique et de Morale***, 14, 294–317, 1906.

[7] Paoli, F. *The ambiguity of quantifiers,* ***Philosophical Studies***, 124, 313-330, 2005.

[8] Russell, B. *Mathematical logic as based on a theory of types,* ***American Journal of Mathematics***, 30, 222–262, 1908.

[9] Elia Zardini ∀ *and* ω, Alessandro Torza, editor, ***Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language***, 489–526.

Springer, Dordrecht, 2015.