Partial Equilibrium Thinking, Extrapolation, and Bubbles*

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Abstract

We develop a dynamic theory of "Partial Equilibrium Thinking" (PET), and provide a micro-foundation for time-varying price extrapolation. The two-way feedback between prices and beliefs is present at all times, but only sometimes manifests itself in explosive ways. In normal times, PET generates constant price extrapolation and momentum. By contrast, following a "displacement shock" that increases uncertainty, PET leads to stronger and time-varying extrapolation, triggering bubbles and endogenous crashes. Our theory sheds light on both normal times market dynamics and the Kindleberger (1978) narrative of bubbles within a unified framework.

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Sustained periods of over-optimism that eventually end in a crash are at the heart of many macro-economic events, such as stock market bubbles, house price bubbles, investment booms, credit cycles, or financial crises (Mackay 1841, Bagehot 1873, Galbraith 1954, Kindleberger 1978, Shiller 2000, Jordà et al. 2015, Greenwood et al. 2021). Given the real consequences of bubbles and crashes, there has been widespread interest in understanding their anatomy and the beliefs that support them.

In terms of anatomy, Kindleberger (1978)'s historical narrative of bubbles provides us with some guidance, by identifying three key stages of bubbles and crashes. The first stage is characterized by what Kindleberger refers to as a *displacement*, "some outside event that changes horizons, expectations, anticipated profit opportunities, behavior." Examples include technological revolutions, such as the railroads in the 1840s, the radio and automobiles in the 1920s, and the internet in the 1990s, or financial innovations such as securitization prior to the 2008 financial crisis. The second stage is characterized by *euphoria* and acceleration. As investors respond to such shocks, the good news leads to a wave of optimism and rising prices. This in turn encourages further buying in a self-sustaining feedback between prices and beliefs that decouples prices from fundamentals. More recent empirical evidence has also shown that this stage is also associated with destabilizing speculation (De Long et al. 1990, Brunnermeier and Nagel 2004), accelerating and convex price paths (Greenwood et al. 2019), and heavy trading (Ofek and Richardson 2003, Hong and Stein 2007, Barberis 2018, DeFusco et al. 2020). Eventually, in the third stage of the bubble, agents who rode the bubble exit, leading to a *crash*.

Turning to beliefs, early theories of bubbles maintain the assumption of rational expectations (Blanchard and Watson 1982, Tirole 1985, Martin and Ventura 2012). However, as well as being at odds with empirical evidence on prices (Giglio et al. 2016), these theories are also unable to speak to the pervasive empirical and experimental evidence on extrapolative beliefs (Smith et al. 1988, Haruvy et al. 2007, Case et al. 2012, Greenwood and Shleifer 2014). Behavioral theories have instead turned to over-confidence and short-sale constraints (Harrison and Kreps 1978, Scheinkman and Xiong

2003),[1] and more recently to modeling extrapolative expectations themselves (Cutler et al. 1990, De Long et al. 1990, Hong and Stein 1999, Glaeser and Nathanson 2017, Barberis et al. 2018, Bordalo et al. 2021, Liao et al. 2021, Chodorow-Reich et al. 2021).[2] Following a sequence of positive news, investors extrapolate recent price rises, and become more optimistic. This then translates into even higher prices, and even more optimistic future beliefs. By directly modeling the self-sustaining feedback between outcomes and beliefs that is characteristic of bubbles, these models generate many features of the Kindleberger (1978) narrative.[3]

At the same time, the reduced form nature of extrapolation considered in these theories leaves several questions open. First, what are the foundations of extrapolative expectations, and what determines how strongly traders extrapolate price changes in updating their future beliefs? Second, why is it that "[b]y no means does every upswing in business excess lead inevitably to mania and panic" (Kindleberger 1978)? In other words, why is it that the same type of extrapolative beliefs sometimes leads prices and beliefs to become extreme and decoupled from fundamentals, while in normal times we don't observe such extreme responses to shocks?

To answer these questions we first provide a micro-foundation for the degree of price extrapolation with a theory of "Partial Equilibrium Thinking" (PET) (Bastianello and Fontanier 2021) in which traders fail to realize the general equilibrium consequences of their actions when learning information from prices. Second, consistent with the Kindleberger narrative, we draw a distinction between normal times shocks and displacement shocks, and show that while partial equilibrium thinking leads to constant price extrapolation in normal times, it leads to stronger and time-varying extrapolation following a displacement.

Micro-founding the degree of extrapolation in this way provides a unifying theory

[1]There is also a literature on coarse reasoning in financial markets (e.g. Bianchi and Jehiel 2010, Eyster and Piccione 2013). Unlike these papers, our mechanism can deliver sustained periods of overpricing without relying on short-sale constraints.

[2]See also Barberis et al. (2015), Hirshleifer et al. (2015), Jin and Sui (2022), and Nagel and Xu (2022) for models of extrapolative expectations that address other asset pricing anomalies.

[3]See Brunnermeier and Oehmke (2013), Xiong (2013) and Barberis (2018) for exhaustive surveys on bubbles and crashes, and Hirshleifer (2015) for a broader survey on behavioral finance.

in which the two-way feedback between prices and beliefs is present at all times, but only manifests itself in explosive ways under very specific circumstances. According to Soros (2015): "[...] in most situations [the two-way feedback] is so feeble that it can safely be ignored. We may distinguish between near-equilibrium conditions where certain corrective mechanisms prevent perceptions and reality from drifting too far apart, and far-from equilibrium conditions where a reflexive double-feedback mechanism is at work and there is no tendency for perceptions and reality to come closer together [...]." We formalize this notion of "near-equilibrium" and "far-from equilibrium" conditions by modeling the distinction between normal times shocks which do not generate large changes to the environment, and Kindleberger-type displacements which instead do.

To illustrate our notion of partial equilibrium thinking, consider some investors who see the price of a stock rise, but do not know what caused this. They may think that some other more informed investors in the market received positive news about this stock and decided to buy, pushing up its price. Given this thought process, they infer positive news about it, and also buy, leading to a further price increase. At this point, rational agents perfectly understand that this additional price rise is not due to further good news, but simply reflects the lagged response of uninformed agents who are thinking and behaving just like them. As a result, they no longer update their beliefs in response to this second price rise, and the two-way feedback between prices and beliefs fails to materialize, as shown in the top panel of Figure 1.

However, for uninformed agents to learn the right information from prices, they must perfectly understand what generates the price changes they observe at each point in time, which in turn requires them to perfectly understand all other agents' actions and beliefs. Theories of rational expectations model this level of understanding by assuming common knowledge of rationality, which has been widely rejected by experimental evidence (Crawford et al. 2013, Kübler and Weizsäcker 2004, Penczynski 2017, Eyster et al. 2018). We relax this assumption by instead assuming that agents think in partial equilibrium, whereby "otherwise rational expectations fail to take into account the strength of similar responses by others" (Kindleberger 1978). PET agents neglect that all other uninformed agents are thinking and behaving just like them, and attribute any price change they observe to new information alone. Following the second price rise in the example in Figure 1, PET agents attribute it to further good news, encouraging further buying and inducing further price rises in a self-sustaining feedback between prices and beliefs. In this paper we formalize the intuition behind this example and show how, depending on the information structure, the strength of this feedback effect may be time-varying.

Figure 1: The Feedback-Loop Theory of Bubbles. Changes in prices and beliefs after a one-off shock to fundamentals, under rational expectations (top panel) and under partial equilibrium thinking (bottom panel).

REE: $\Delta\mathbb{E}_{I,1} \implies \Delta P_1 \implies \Delta\mathbb{E}_{U,2} \implies \Delta P_2$

PET: $\Delta\mathbb{E}_{I,1} \implies \Delta P_1 \implies \Delta\mathbb{E}_{U,2} \implies \Delta P_2 \implies \Delta\mathbb{E}_{U,t+1} \quad \Delta P_{t+1}$

This notion of partial equilibrium thinking builds on a vast literature in social learning that has documented agents' tendency to neglect the extent to which other agents infer information from aggregate outcomes (Kübler and Weizsäcker 2004, Penczynski 2017, Eyster et al. 2018, Enke and Zimmermann 2019),[4] and has studied this type of bias in the form of correlation neglect, naïve herding, cursedness, and k-level thinking (DeMarzo et al. 2003, Eyster and Rabin 2005, Eyster and Rabin 2010).[5,6] We contribute to this literature in two ways. First, we introduce this type of bias in a general equilibrium environment, where prices don't only have a purely informational role, but they also have a market feedback effect role, as they act as a measure of scarcity. Second, by

[4]More recently, Liu et al. (2021) find that a perceived information advantage is one of the dominant trading motives among retail traders in China.

[5]See also Bohren (2016), Esponda and Pouzo (2016), Gagnon-Bartsch and Rabin (2016), Fudenberg et al. (2017), Bohren and Hauser (2021), Frick et al. (2020), Fudenberg et al. (2021), Gagnon-Bartsch et al. (2021) among others for theoretical studies of misinference in social learning contexts.

[6]While one can think of partial equilibrium thinking as being an example of level-2 thinking, we differ from models of $K-$level thinking that do not involve an inference problem from equilibrium outcomes (e.g. Angeletos and Lian (2017), Farhi and Werning (2019), among others), an ingredient which instead lies at the heart of our two-way feedback effect between outcomes and beliefs. We also differ from models that study the implications of higher order beliefs in a fully rational model (e.g. Allen et al. 2006).

drawing a distinction between normal times and displacement shocks, we study how the latter introduce time-variation in the relative strength of the informational and scarcity roles of prices, and show how this allows for reversals even after periods where outcomes and beliefs have become extreme and decoupled from fundamentals.

We begin by introducing partial equilibrium thinking into a standard infinite horizon model of a financial market where each period a continuum of investors solve a portfolio choice problem between a risky and a riskless asset. Our agents differ in their ability to observe fundamental news: a fraction of agents are informed and observe fundamental shocks, and the remaining fraction of agents are uninformed and instead infer information from prices. Motivated by empirical and experimental evidence that traders extrapolate trends as opposed to instantaneous price movements (Andreassen and Kraus 1990, Case et al. 2012), we assume that traders learn information from past as opposed to current prices as in De Long et al. (1990), Hong and Stein (2007) and Barberis et al. (2018).[7]

Given this information structure, in each period price changes reflect both the contemporaneous response of informed agents to news, and the lagged response of uninformed agents who learn from past prices. However, when uninformed agents think in partial equilibrium, they neglect the second source of variation and attribute any price change to new information alone, leading to a simple type of price extrapolation.

The key prediction of the model which leads to different dynamics in response to different types of shocks is that the degree of extrapolation and the bias that partial equilibrium thinking generates are decreasing in informed traders' informational edge. This edge is simply defined as the aggregate confidence of informed traders relative to the aggregate confidence of uninformed traders, and is higher when there are more informed traders in the market, and when the precision of the additional information informed traders hold is higher. When this informational edge is high, informed traders trade more aggressively, and the influence on prices of uninformed traders' beliefs is lower. This leads partial equilibrium thinkers to neglect a smaller source of price variation,

[7]This assumption allows us to model the evolution of the two-way feedback between outcomes and beliefs dynamically. Bastianello and Fontanier (2021) explores the implications of partial equilibrium thinking and more general types of model misspecification in a static framework.

therefore leading to a smaller bias and a smaller strength of the feedback between prices and beliefs. Conversely, when informed traders' edge is low, partial equilibrium thinkers neglect a bigger source of price variation, leading to a larger bias and a stronger feedback effect. By understanding how this edge varies in response to different types of shocks, we can then understand how partial equilibrium thinking generates different dynamics in normal times, and following a displacement.

We show that in normal times informed agents' edge is constant over time. For example, normal times shocks may come in the form of earnings announcements: sophisticated traders are better able to understand the long run implications of such shocks, and uninformed retail traders can learn about them more slowly by observing how the market reacts to such news. When this is the case, informed traders are always one step ahead of uninformed traders, and their edge is high and constant, meaning that partial equilibrium thinkers neglect a small source of price variation, thus leading to weak departures from rationality, as when Soros' notion of "near equilibrium" conditions are satisfied.

This is no longer true following a Kindleberger-type displacement, when the informational edge becomes time-varying. Specifically, displacements are "something new under the sun," and the implications of such shocks for long term outcomes can be learnt only gradually over time. These shocks wipe out much of informed agents' edge as not even the most informed of informed agents are able to immediately grasp the full long-term implications of such events. This leads informed agents to trade less aggressively, and to a rise in the influence on prices of uninformed traders' beliefs. Partial equilibrium thinkers then neglect a greater source of price variation, leading to a stronger bias. This fuels the strength of the feedback between prices and beliefs, allowing both to accelerate away from fundamentals, as "far-from equilibrium" conditions take over in determining equilibrium dynamics. As informed traders learn more about the displacement over time, they regain their edge, leading to a gradual fall in the degree of extrapolation, and in the strength of the feedback effect. When the feedback effect runs out of steam, the bubble bursts, and prices and beliefs converge back towards fundamentals. The exact shape of the bubble then depends on the speed with which informed traders learn more about the displacement over time.

Relative to earlier micro-foundations of price extrapolation (Hong and Stein 1999, Malmendier and Nagel 2011, Fuster et al. 2012, Glaeser and Nathanson 2017, Greenwood and Hanson 2015), this paper draws a distinction between normal times shocks and displacement shocks, and focuses on the endogenous time-variation in extrapolative beliefs. Unlike previous papers, we are then able to exploit the properties of unstable and non-stationary regions, as displacements make the transition to such regions only temporary. This allows us to offer an explanation for why not *every* large positive shock leads to bubbles and crashes, in a way that is consistent with both historical narratives and more recent empirical evidence (Kindleberger 1978, Greenwood et al. 2019).

Finally, we study how our bias interacts with speculative motives, and show that whether speculators amplify bubbles or arbitrage them away depends on their beliefs of whether mispricing is temporary or predictable. If they think that mispricing is temporary, they arbitrage it away immediately, and bubbles and crashes do not arise. If instead they realize that future mispricing is predictable and that they will be able to sell the asset to "a greater fool" at a higher price in the future, they increase their position in the asset, thus pushing prices up further, and amplifying the bubble. These predictions are consistent with bubbles being associated with the type of destabilizing speculation described in the latter case (Keynes 1936), and with more sophisticated traders initially riding the bubble (Brunnermeier and Nagel 2004, Temin and Voth 2004, Griffin et al. 2011, An et al. 2022).

This paper proceeds as follows. In Section 1 we introduce our notion of partial equilibrium thinking and study it in the context of normal times shocks. Section 2 models displacements and shows how these shocks interact with partial equilibrium thinking in generating bubbles and crashes. In Section 3 we add speculative motives. Section 4 concludes and discusses some directions of future research. While prices are a very natural equilibrium outcome agents may learn from, partial equilibrium thinking can be applied more broadly to any setup where agents learn information from a general equilibrium variable, thus lending itself to a variety of other macro and finance applications, such as credit cycles and investment booms.

1 Normal Times

In this section we introduce our notion of partial equilibrium thinking (PET) in normal times, which we think of as periods where shocks come in the form of regular earning announcements that do not cause significant changes in the composition of traders in the market, or in the relative confidence of traders.

1.1 Setup

Agents solve a portfolio choice problem between a risk-free and a risky asset. The risk-free asset is in zero net supply and we normalize its price and its risk-free rate to one. The risky asset is in fixed net supply Z and pays a liquidating dividend when it dies at an uncertain terminal date.[8] In each period, with probability β the asset remains alive and produces $u_t \overset{iid}{\sim} N(0, \sigma_u^2)$ worth of terminal dividends, and with probability $(1-\beta)$ the asset dies, and all accumulated dividends are paid out (Blanchard 1985). From the point of view of period t, the asset is still alive in period $t+h$ with probability β^h. Taking expectations over all possible terminal dates, the present value of the terminal dividend in period t, conditional on realized future shocks $\{u_{t+h}\}_{h=1}^{\infty}$, can be written as:[9]

$$D_T = \bar{D} + \sum_{j=0}^{t} u_j + \sum_{h=1}^{\infty} \beta^h u_{t+h} \tag{2}$$

[8]Having the net supply of the risky asset being *fixed*, instead of stochastic, ensures that prices are fully revealing (Grossman 1976). In Appendix D.2 we relax this assumption and allow for the supply of the risky asset to be stochastic, so that prices are only partially revealing (Diamond and Verrecchia 1981). The key intuitions at the heart of partial equilibrium thinking remain unchanged.

[9]Notice that conditional on dying in period $t+h$, the realized terminal dividend evolves as a random walk:

$$D_{t+h} = \bar{D} + \sum_{j=0}^{t+h} u_j \tag{1}$$

where $\bar{D} > 0$ is constant and is common knowledge. Moreover, this expression simply reflects that from the point of view of period t, the asset has produced $\sum_{j=0}^{t} u_j$ worth of terminal dividends while alive in these first t periods, and with probability β^h the asset is still alive in period $t+h$, and if so it will produce an amount u_{t+h}.

This death probability β then acts as a very natural discount rate such that dividends paid further into the future receive a lower weight today. Introducing this uncertain terminal date is a simple and effective modeling device that increases tractability by serving two key purposes: it avoids horizon effects from approaching a fixed terminal date, and it keeps variances bounded even as we allow the terminal date to be arbitrarily far into the future.

Our economy is populated by a continuum of measure one of fundamental traders, who have CARA utility over terminal wealth and trade as if they were going to hold the asset until its death, even though they rebalance their portfolio every period.[10] In each period t all agents then solve the following portfolio choice problem:

$$\max_{X_{i,t}} \quad \left\{ X_{i,t} \left(\mathbb{E}_{i,t}[D_T] - P_t \right) - \frac{1}{2} \mathcal{A} X_{i,t}^2 \mathbb{V}_{i,t}[D_T] \right\} \tag{3}$$

where $X_{i,t}$ is the dollar amount that agent i invests in the risky asset in period t, $\mathcal{A}$ is the coefficient of absolute risk aversion, and $\mathbb{E}_{i,t}[D_T]$ and $\mathbb{V}_{i,t}[D_T]$ refer to agent i's posterior mean and variance beliefs about the fundamental value of the asset conditional on their information set in period t. The corresponding first order condition yields the following standard demand function for the risky asset:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[D_T] - P_t}{\mathcal{A} \mathbb{V}_{i,t}[D_T]} \tag{4}$$

which is increasing in agent i's expected payoff, and decreasing in the risk they associate

[10] Our fundamental traders behave as if they were going to hold their position forever, even though they rebalance every period. This ensures tractability, and allows us to explore the mechanism at the core of partial equilibrium thinking in the simplest and most transparent way. In Section 3 we relax this assumption and model traders who have CARA utility over next period wealth, and forecast next period prices as opposed to long-term fundamentals. The main intuitions are unchanged.

with holding the asset.

Turning to the information structure, we assume that a fraction ϕ of agents are informed, and observe the fundamental shock u_t, in every period. The remaining fraction $(1-\phi)$ of agents are uninformed and do not observe any of the fundamental shocks, but can learn information from prices. Given experimental evidence by Andreassen and Kraus (1990), we then assume that traders learn information from past as opposed to current prices, in the spirit of the positive feedback traders in De Long et al. (1990), Hong and Stein (1999), and Barberis et al. (2018).[11]

To solve the model, we proceed in three steps. First, we solve for the true price function which generates the outcomes that agents observe. Second, we turn to PET agents' beliefs of what generates the prices they observe, which allows us to pin down the mapping that PET agents use to learn information from prices. Finally, we solve the equilibrium recursively, and study the properties of equilibrium outcomes.

1.2 True Price Function in Normal Times

To solve for the true market clearing price function, we need to specify agents' posterior beliefs, compute agents' asset demand functions, and impose market clearing. Starting from agents' beliefs, we know that in period t all informed agents trade on the information they receive, and update their beliefs accordingly:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t-1}[D_T] + u_t \tag{5}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty}\beta^h u_{t+h}\right] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_I \tag{6}$$

Instead, all uninformed agents learn information from past prices. Let $\tilde{u}_{t-1}$ be the fundamental shock which uninformed traders learn from the past price they observe,

[11] Appendix D.1 shows that the main intuitions of the model still go through if we assume that uninformed traders submit market orders that do not condition on the current price level.

P_{t-1}. We can then write uninformed traders' posterior beliefs as:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} \tag{7}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{I,t}\left[u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_U \tag{8}$$

More generally, throughout the paper we denote with a $\tilde{\cdot}$ uninformed traders' beliefs about a variable. In this case, since prices are fully revealing, uninformed traders believe they are extracting from P_{t-1} the exact fundamental shock that informed traders observe in $t-1$, so $\tilde{u}_{t-1}$ is uninformed agents' beliefs of the $t-1$ fundamental shock, u_{t-1}.

Whether $\tilde{u}_{t-1} = u_{t-1}$ or $\tilde{u}_{t-1} \neq u_{t-1}$ depends on the mapping uninformed traders use to extract information from prices. In Sections 1.3 and 1.4 we show that if traders have rational expectations, then $\tilde{u}_{t-1} = u_{t-1}$, but if instead they use a misspecified mapping, as with partial equilibrium thinking, they extract biased information from prices and $\tilde{u}_{t-1} \neq u_{t-1}$. For now, treat $\tilde{u}_{t-1}$ as a generic signal uninformed traders learn from past prices, and we derive this as an equilibrium object in the next section.

The last equality in both (8) and (6) shows that both informed and uninformed agents face *constant* uncertainty over time: informed traders always face uncertainty over all future fundamental shocks, while uninformed traders additionally face uncertainty over the current fundamental shock, as they only learn information from past prices. Since informed traders are always one step ahead of uninformed traders, we can define ζ_t to be the aggregate informational edge of informed agents relative to uninformed agents in period t as follows.

Definition 1 (Aggregate Informational Edge)**.** *The aggregate informational edge of informed traders in period t is defined as the aggregate confidence of informed traders relative to the aggregate confidence of uninformed traders:*

$$\zeta_t \equiv \frac{\phi}{(1-\phi)} \frac{\tau_{I,t}}{\tau_{U,t}} \tag{9}$$

where $\tau_{i,t} = (\mathbb{V}_{i,t})^{-1}$ is the confidence of agent $i \in \{I, U\}$ in period t.

This edge is increasing in the fraction of informed traders in the market (ϕ), and in the relative individual level confidence of informed and uninformed traders (${}^{\tau_{I,t}}/_{\tau_{U,t}}$). Importantly, since in normal times ϕ and ${}^{\tau_{I,t}}/_{\tau_{U,t}}$ are constant, the informational edge is also constant.

Given these posterior beliefs, we can compute agents' asset demand functions and impose market clearing by simply equating the aggregate demand for the risky asset to the fixed supply Z:

$$\phi \underbrace{\left(\frac{\mathbb{E}_{I,t-1}[D_T] + u_t - P_t}{\mathcal{A}\mathbb{V}_I} \right)}_{X_{I,t}} + (1-\phi) \underbrace{\left(\frac{\mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} - P_t}{\mathcal{A}\mathbb{V}_U} \right)}_{X_{U,t}} = Z \tag{10}$$

The true market clearing price function is then given by:

$$P_t = a\left(\mathbb{E}_{I,t-1}[D_T] + u_t\right) + b\left(\mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1}\right) - c \tag{11}$$

where:[12]

$$a \equiv \frac{\phi\tau_I}{\phi\tau_I + (1-\phi)\tau_U} = \frac{\zeta}{1+\zeta} \tag{12}$$

$$b \equiv \frac{(1-\phi)\tau_U}{\phi\tau_I + (1-\phi)\tau_U} = \frac{1}{1+\zeta} \tag{13}$$

$$c \equiv \frac{\mathcal{A}Z}{\phi\tau_I + (1-\phi)\tau_U} \tag{14}$$

so that prices reflect a weighted average of agents' beliefs minus a risk-premium component which compensates agents for bearing risk. The last equality in (12) and (13) then shows that the weight on informed agents' beliefs is increasing in their informational edge, and the opposite comparative static holds for the weight on uninformed agents' beliefs.

Taking first differences of the price function in (11) and of agents' beliefs in (5) and

[12]The last equality in (12) and (13) is obtained by dividing the numerator and denominator by $\phi\tau_I$ and $(1-\phi)\tau_U$ respectively, and using the definition of ζ in (9).

(9), we find that:

$$\Delta P_t = \underbrace{a u_t}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{b \tilde{u}_{t-1}}_{\substack{\text{lagged response of } U \\ \text{from learning from past prices}}} \tag{15}$$

which shows that price changes reflect both the instantaneous response to shocks of informed agents, and the lagged response of uninformed agents who learn information from past prices.

To specify what information uninformed agents extract from past prices we need to understand what uninformed agents think is generating the price changes that they observe. In what follows we first explore the inference problem under rational expectations, and then turn to the inference problem under partial equilibrium thinking.

1.3 Rational Expectations Benchmark

If uninformed traders have rational expectations, they perfectly understand that (15) generates the price changes they observe.[13] As discussed further in Appendix B.1, they then invert the following mapping to learn information from past price changes:[14]

$$\Delta P_{t-1} = \underbrace{a \tilde{u}_{t-1}}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{b \tilde{u}_{t-2}}_{\substack{\text{lagged response of } U \\ \text{from learning from past prices}}} \tag{16}$$

where $\tilde{u}_{t-1}$ is uninformed traders' belief of the shock that hit the economy in period $t-1$ (and which they wish to infer from past prices), and $\tilde{u}_{t-2}$ is the signal uninformed traders already learnt in period $t-1$ (and is already in their information set).[15]

Inverting (16), we find that uninformed rational traders use the following mapping

[13]To keep this rational benchmark as close as possible to our notion of partial equilibrium thinking, we restrict uninformed rational traders to also learn information from *past* prices.

[14]The lag on the price change reflects the fact that uninformed traders are learning information from past as opposed to current prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

[15]Moreover, because of common knowledge of rationality, rational uninformed traders are correctly specified about $\tilde{u}_{t-2}$ (the signal other traders extracted from past prices in the previous period).

to infer information from prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{a}\right) \Delta P_{t-1} - \frac{b}{a}\tilde{u}_{t-2} \tag{17}$$

Lagging the true price function in (15) and substituting it into the above expression, we see that since rational traders understand what generates the price changes they observe, they are able to extract the right information from past prices:

$$\tilde{u}_{t-1} = u_{t-1} \tag{18}$$

However, for uninformed agents to learn the right information from prices, they must perfectly understand what generates every single price change they observe, which in turn requires them to perfectly understand other agents' actions and beliefs. In what follows, we relax this assumption.

1.4 Partial Equilibrium Thinking

When agents think in partial equilibrium, they misunderstand what generates the price changes that they observe because they fail to realize the general equilibrium consequences of their actions (Bastianello and Fontanier 2021). The way that PET manifests itself in this setup is that all agents learn information from prices, but they fail to realize that other agents do too. In other words, PET agents think that they are the only ones inferring information from prices, and that all other agents trade on their unconditional priors.[16]

Formally, PET agents think that in period $t-1$ informed agents update their beliefs

[16] Our notion of partial equilibrium thinking captures how traders misunderstand the endogenous part of the price change that comes from learning from *equilibrium* outcomes, even though they may correctly understand the part of the price change that comes from informed traders directly responding to *exogenous* signals.

with the new fundamental information they receive, $\tilde{u}_{t-1}$:[17]

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} \tag{19}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_I \tag{20}$$

On the other hand, they think that all other uninformed agents do not learn information from prices, and instead trade on the same unconditional prior beliefs they held in period $t = 0$:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D] = \bar{D} \tag{21}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_U \tag{22}$$

where the equivalences in (20) and (22) highlight that in normal times, PET agents understand that all agents face constant uncertainty over time. Moreover, since $\tilde{\mathbb{V}}_I = \mathbb{V}_I < \tilde{\mathbb{V}}_U = V_U$, we see that PET agents are not misspecified about other agents' second moment beliefs, and they understand that informed agents have an informational edge.

Importantly, all agents are atomistic and do not consider the effect of their own asset demand on prices. PET agents then think that the equilibrium price in period $t-1$ is generated by the following market clearing condition:

$$\phi \underbrace{\left(\frac{\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-1} - P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_I}\right)}_{\tilde{X}_{I,t-1}} + (1-\phi)\underbrace{\left(\frac{\bar{D} - P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_U}\right)}_{\tilde{X}_{U,t-1}} = Z \tag{23}$$

which leads to the following price function:

$$P_{t-1} = \tilde{a}\left(\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\bar{D} - \tilde{c} \tag{24}$$

[17]The use of $t-1$ subscripts instead of t is to highlight that uninformed agents learn information from past prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

where:

$$\tilde{a} \equiv \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{\tilde{\zeta}}{1+\tilde{\zeta}} \tag{25}$$

$$\tilde{b} \equiv \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{1}{1+\tilde{\zeta}} \tag{26}$$

$$\tilde{c} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} \tag{27}$$

Since the only source of price variation perceived by PET agents is given by changes in informed agents' beliefs, we can take first differences of (24) and rewrite this as:

$$\Delta P_{t-1} = \underbrace{\tilde{a}\tilde{u}_{t-1}}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} \tag{28}$$

which shows that when agents think in partial equilibrium they attribute any price change they observe to new information alone. They instead neglect the second source of price variation in (16), which is due to the lagged response of all other uninformed traders. PET agents then invert the mapping in (28) to extract $\tilde{u}_{t-1}$ from prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{\tilde{a}}\right)\Delta P_{t-1} \tag{29}$$

Therefore, PET provides a micro-foundation for extrapolative expectations as uninformed traders extract a positive signal and become more optimistic whenever they see a price rise, and extract a negative signal and become more pessimistic whenever they see a price fall. This is unlike the rational expectations benchmark in (17), where uninformed traders become more optimistic (pessimistic) following a price rise (fall) *only* if that price change is due to new information. If the price change they observe is instead due to the lagged response of uninformed traders who are learning information from past prices, rational traders do not update their beliefs.

The bias inherent in partial equilibrium thinking is then increasing in the source of price variation they neglect, which, in turn, is decreasing in informed traders' informational edge. Intuitively, a lower edge (from a smaller fraction of informed traders in the market, or from a lower confidence of informed relative to uninformed traders) increases the influence on prices of uninformed agents' beliefs, leading PET agents to omit a greater source of price variation.

Proposition 1 (Micro-foundation of Price Extrapolation)**.** *Partial equilibrium thinking provides a micro-foundation for extrapolative expectations:*

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{30}$$

where $\frac{1}{\tilde{a}} = 1 + \frac{1}{\tilde{\zeta}}$*. Moreover, given a one-off shock to fundamentals, the bias is decreasing in the true and perceived informational edge of informed traders:*

$$\tilde{u}_{t-1} - u_{t-1} = \left(\frac{b}{\tilde{a}}\right) \tilde{u}_{t-2} \tag{31}$$

where $\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right)$*.*

Proof. All proofs are in Appendix A. □

1.5 The Feedback-Loop Theory of Bubbles

Combining the expressions of the true price function in (15) and of the extracted signal in (29), we find that when traders think in partial equilibrium changes in prices and in beliefs evolve as an AR(1):

$$\tilde{u}_{t-1} = u_{t-1} + \left(\frac{b}{\tilde{a}}\right) \tilde{u}_{t-2} \tag{32}$$

$$\Delta P_t = a u_t + \left(\frac{b}{\tilde{a}}\right) \Delta P_{t-1} \tag{33}$$

This is in contrast to the rational benchmark where, combining (15) and (17), we find that when traders are rational price changes evolve as an MA(1):

$$\tilde{u}_{t-1} = u_{t-1} \tag{34}$$

$$\Delta P_t = a u_t + b u_{t-1} \tag{35}$$

Intuitively, partial equilibrium thinkers mistakenly infer a sequence of shocks from a one-off shock, and this leads to over-reaction, as is clear from the second term in (32) relative to its rational counterpart in (34). Following a one-off shock, PET agents fail to realize that the second price rise is due to the buying pressure of all other uninformed agents, and instead attribute it to further good news, which in turn fuels even higher prices and more optimistic beliefs, in a self-sustaining feedback loop, just as we saw in the example in Figure 1 in the introduction.

1.5.1 Strength of the Feedback Effect

As with all AR(1) processes, the AR(1) coefficient in the processes that describe equilibrium changes in prices and in beliefs in (32) and (33) is key to determining the properties of equilibrium outcomes. In our case, this quantity also has a special meaning, in that it captures the strength of the feedback effect between prices and beliefs, and it is decreasing both in the true informational edge (ζ), and in uninformed agents' perception of it ($\tilde{\zeta}$):

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) \tag{36}$$

Intuitively, when uninformed agents' perception of the informational edge is low, they neglect a greater source of price variation, leading to a greater bias. Moreover, when the true informational edge of informed agents is low, the influence on prices of uninformed traders' biased beliefs is higher. Both these forces contribute to fuelling the feedback between outcomes and beliefs. We summarize these results in the following proposition.

Proposition 2 (Strength of the Feedback Effect)**.** *When agents think in partial equilibrium, the strength of the feedback between outcomes and beliefs is decreasing both in the true informational edge (ζ), and in uninformed agents' perception of it ($\tilde{\zeta}$). Specifically, environments with a smaller fraction of informed traders (ϕ), and with a lower true and perceived confidence of informed agents relative to uninformed agents (τ_I/τ_U, $\tilde{\tau}_I/\tilde{\tau}_U$) are* *characterized by a stronger feedback between prices and beliefs.*

Equation (32) then shows that deviations from rationality are increasing in the strength of the feedback effect, leading to the following empirical prediction both in the cross-section, and over time.

Proposition 3 (Deviations from Rationality)**.** *Deviations from rationality in both prices and beliefs are decreasing in the true and perceived informational edges (ζ, $\tilde{\zeta}$). Specifically, following a one-off shock to fundamentals, environments with a smaller fraction of informed agents (ϕ), and with a lower true and perceived confidence of informed agents relative to uninformed agents ($^{\tau_I}/_{\tau_U}, ^{\tilde{\tau}_I}/_{\tilde{\tau}_U}$) exhibit greater departures from rationality.*

1.5.2 Stable and Unstable Regions

Another feature of the AR(1) processes in (32) and (33) is that the system can be stationary or non-stationary, depending on whether $^b/_{\tilde{a}} < 1$ or $^b/_{\tilde{a}} > 1$.

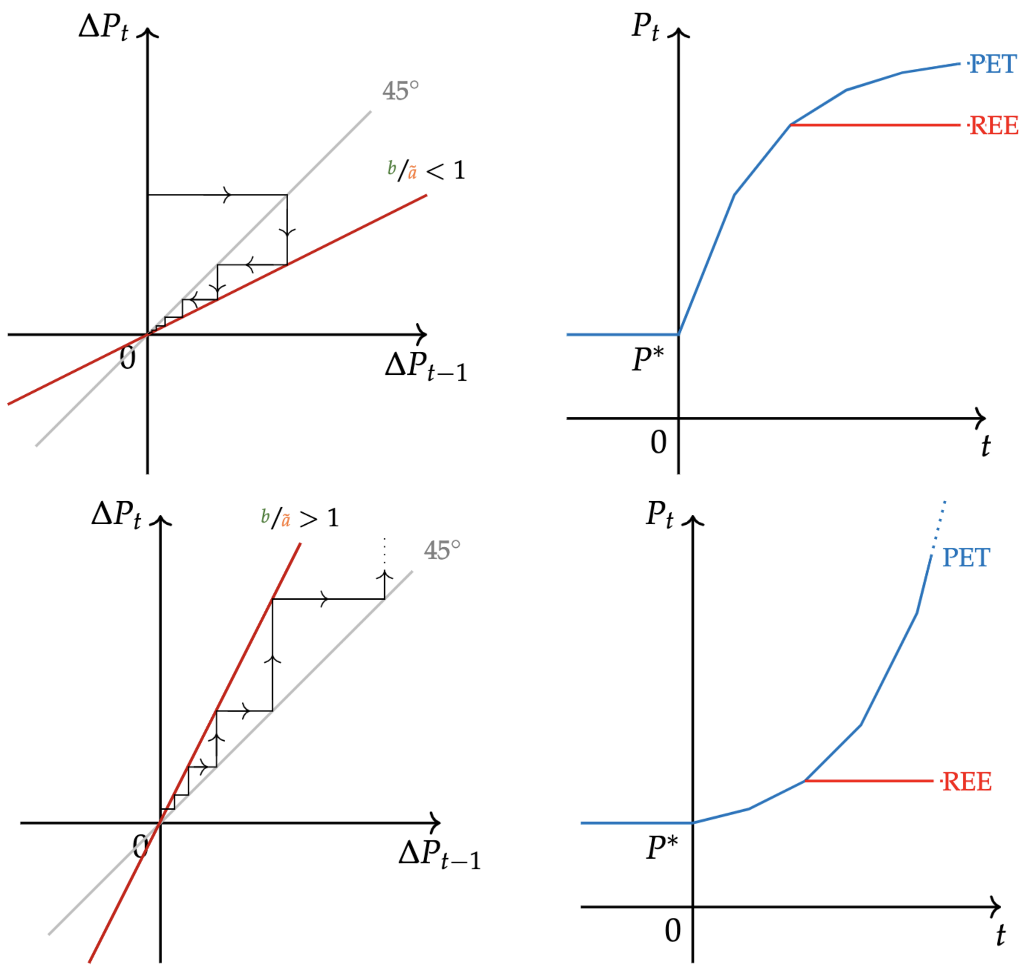
When $^b/_{\tilde{a}} < 1$, changes in prices and in beliefs in (32) and (33) are stationary, and shocks eventually die out, so that prices and beliefs exhibit momentum and converge to a new steady state (as shown in the left panel of Figure 2).[18] On the other hand, when $^b/_{\tilde{a}} > 1$ the system is non-stationary and the influence of the feedback effect is explosive: consecutive changes in prices and beliefs get larger and larger, and prices and beliefs accelerate in a convex way, becoming extreme and decoupled from fundamentals (as shown in the right panel of Figure 2).

Definition 2 (Stable and Unstable Regions.)**.** *We refer to stationary regions with $^b/_{\tilde{a}} < 1$ as stable regions, and non-stationary regions with $^b/_{\tilde{a}} > 1$ as unstable regions.*

Since we do not observe unbounded prices and beliefs in response to normal times shocks (e.g. following earnings announcements), it is plausible to assume that in normal times changes in prices and beliefs are stationary.

[18]Notice that PET outcomes do not converge to the rational expectations equilibrium as $t \to \infty$. Conditional on not observing the liquidating dividend, PET agents never unlearn their misinferred information, as in Gagnon-Bartsch and Rabin (2016). In this respect, PET is attentionally stable in the sense of Gagnon-Bartsch et al. (2021).

Figure 2: Impulse response functions following a normal times shock. This Figure compares the path of equilibrium price changes (left panel) and the corresponding path of equilibrium prices (right panel) following a one-off fundamental shock $u_1 > 0$ under rational expectations (REE) and under partial equilibrium thinking (PET). The top panel plots the impulse response function when the economy is in a stable region, with $^b/_{\tilde{a}} < 1$, and shows that price changes dies out, and prices gradually converge to a new steady state level. The bottom panel plots the impulse response function when the economy is in an unstable region, with $^b/_{\tilde{a}} > 1$, and shows that price changes become larger and larger over time, and prices accelerate away from fundamentals in a convex way, and are unbounded.



Moreover, since in (20) and (22) we showed that in normal times $\tau_i = \tilde{\tau}_i$ for $i \in \{I, U\}$, it follows that $\tilde{\zeta} = \zeta$, and the strength of the feedback effect reduces to:

$$\frac{b}{\tilde{a}} = \frac{1}{\zeta} \tag{37}$$

so that for the response of the economy to normal times shocks not to be explosive it must be that the aggregate confidence of informed agents is greater than the aggregate confidence of uninformed agents.

$$\frac{b}{\tilde{a}} < 1 \iff \zeta > 1 \iff \phi\tau_I > (1-\phi)\tau_U \tag{38}$$

As long as the feedback between outcomes and beliefs is *constant*, the economy either responds to shocks by monotonically converging to a new state-dependent steady state, or it accelerates away from fundamentals, leading prices and beliefs to become extreme. While the acceleration characteristic of the unstable regions of this theory may seem well-suited to model the formation of bubbles (Greenwood et al. 2019), it leaves no room for endogenous reversals and crashes.

In the next section, we show how displacements can generate *time-variation* in the strength of the feedback effect, and shift the economy across stable and unstable regions. By bringing the explosive properties of unstable regions into play before the convergent properties of stable regions take over again, displacements can lead to the formation of bubbles and endogenous crashes.

2 Displacements

"Displacement is some outside event that changes horizons, expectations, profit opportunities, behavior – some sudden advice many times unexpected. Each day's events produce some changes in outlook, but few significant enough to qualify as displacements" (Kindleberger 1978). Examples include the widespread adoption of a ground-breaking discovery, such as railroads in the 1840s, the radio and automobiles in the 1920s, and the internet in the 1990s; financial liberalization in Japan in the 1980s; or financial innovations such as securitization prior to the 2008 financial crisis (Aliber and Kindleberger 2015).

While the exact nature of the displacement varies from one bubble episode to another, what these shocks have in common is that they represent "something new under the sun," and their full implications for long term outcomes can only be understood gradually over time, as more information becomes available (Pástor and Veronesi 2006, Pástor and Veronesi 2009). When the internet was first made available to the public in 1993, investors were aware of this new technology, but at the time nobody knew the full potential of this invention. The development of blockchains as decentralized ledgers has paved the way for cryptocurrencies. However, we are yet to learn about the full implications of this technology, and assets that are associated with them have indeed been prone to bubbly behavior.

This is in stark contrast to normal times shocks, which may come in the form of regular earnings announcements. Following these news events, sophisticated traders are well trained to immediately process and understand the content of such news (e.g. the implications of same store sales on long term outcomes), while uninformed traders can learn about their implications more slowly, by seeing how the market reacts to them. As we saw in Section 1, in normal times informed traders are always one step ahead of uninformed traders, and their informational edge is constant.

In this section we show how displacement shocks generate time-variation in informed agents' edge, which in turn leads to time-varying extrapolation, and a time-varying strength of the feedback between prices and beliefs. This can shift the economy between stable and unstable regions. Specifically, when the displacement first materializes, informed agents' edge is wiped out, thus increasing the influence on prices of uninformed agents' beliefs and the strength with which they extrapolate. Both of these forces fuel the feedback between prices and beliefs. If the uncertainty associated with the displacement is high and persistent enough, the economy can enter the unstable region, leading prices and beliefs to accelerate away from fundamentals. Then, as informed agents learn about the new technology and regain their edge, the feedback effect weakens, and the economy re-enters the stable region. This leads the bubble to burst and prices and beliefs to return back towards fundamentals.

We conclude this section by discussing how the speed of information arrival shapes the duration and amplitude of bubbles, as well as alternative ways of modeling a displacement.

2.1 Displacement Shocks

We model displacements as an uncertain positive shock to long-term outcomes that agents can learn about only gradually over time. Starting from a normal-times steady state where uninformed agents' beliefs are consistent with the price they observe, in period $t = 0$ both informed and uninformed traders learn that there is "something new under the sun," but do not know the exact implications of such shock for long-term outcomes. Specifically, in period $t = 0$, all agents learn that the terminal dividend changes by an uncertain amount $\omega \sim N(\mu_0, \tau_0^{-1})$, where $\mu_0 > 0$:[19]

$$D_T = \bar{D} + \sum_{j=0}^{\infty} \beta^j u_j + \omega \tag{39}$$

Initially, all agents share the same unconditional prior over ω. Starting in period $t = 1$, each period informed agents observe a common signal that is informative about the displacement, $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim^{iid} N(0, \tau_s^{-1})$. Uninformed agents do not observe these signals but still learn information from past prices.

We solve the model using the same three steps we used in normal times: first, we specify what truly generates price changes agents observe. Second, we specify what uninformed agents think is generating these price changes, and find the mapping PET agents use to extract information from prices. Third, we solve the model recursively, and discuss the properties of equilibrium outcomes.

[19] In Appendix C.1 we also consider the case where $\mu_0 < 0$, and show how with partial equilibrium thinking negative bubbles are dampened relative to positive ones.

2.2 True Price Function following a Displacement

Following a displacement, informed agents observe new signals u_t and s_t in each period, and they revise their beliefs accordingly, via standard Bayesian updating:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t-1}[D_T] + u_t + w_t \tag{40}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{V}_I + (t\tau_s + \tau_0)^{-1} \tag{41}$$

where $w_t \equiv \mathbb{E}_{I,t}[\omega] - \mathbb{E}_{I,t-1}[\omega] = \frac{\tau_s}{t\tau_s+\tau_0}(s_t - \mathbb{E}_{I,t-1}[\omega])$ is informed agents' revision of their beliefs about the displacement ω in light of the new signal s_t. Equation (41) shows that when the displacement is announced, informed agents face greater uncertainty than before, but their confidence gradually rises back towards its steady state level as they receive more signals about ω and learn more about the displacement over time.

On the other hand, in each period t, uninformed agents are learning $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ from the price change they observe in period $t-1$, and their posterior beliefs are given by:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1} \tag{42}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{U,t}\left[u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1} \tag{43}$$

where (41) shows that uninformed agents also face greater uncertainty when the displacement is announced, but their confidence also rises back towards its steady state level as they learn about ω from past prices over time. Specifically, after t periods, PET agents have learnt about the displacement from $(t-1)$ price changes.

Combining the information in (41) and (43), informed agents' edge is initially diluted by the increase in aggregate uncertainty, but then gradually rises back to its steady state level:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}}\right) \tag{44}$$

Given these beliefs, we find that price changes capture both changes in mean beliefs, and changes in confidence levels:[20]

$$\Delta P_t = \underbrace{a_t\left(u_t + w_t\right)}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{b_t\left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right)}_{\substack{\text{lagged response of } U \\ \text{from learning from past prices}}} + \underbrace{\left(P_{t|t-1} - P_{t-1}\right)}_{\text{changes in confidence}} \tag{46}$$

where the influence on prices of informed and uninformed traders' beliefs are given by:

$$a_t \equiv \frac{\zeta_t}{1+\zeta_t} = 1 - b_t \qquad b_t \equiv \frac{1}{1+\zeta_t} \tag{47}$$

Moreover, the change in price that is due to changes in agents' levels of confidence is:

$$(P_{t|t-1} - P_{t-1}) \equiv \Delta a_t \mathbb{E}_{I,t-1}[D_T] + \Delta b_t \mathbb{E}_{U,t-1}[D_T] - \Delta c_t \tag{48}$$

where $c_t \equiv \frac{\mathcal{A}Z}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}$ is the risk-premium component.

There are two important points to notice from these expressions. First, time-variation in informed agents' edge generates variation in the relative influence on prices of informed and uninformed agents' beliefs ($\tilde{a}_t$ and $\tilde{b}_t$, respectively). As informed traders initially lose their edge and then regain it, the influence on prices of informed traders' beliefs initially drops and then gradually rises again. By symmetry, the influence on prices of uninformed traders' biased beliefs initially rises, and then gradually falls.

Second, (46) shows that price changes now reflect three components. The first two components are due to changes in mean beliefs of both informed and uninformed traders, just as in normal times. However, displacements now bring into play a third source of price variation, which is due to changes in informed and uninformed traders' relative confidence levels. As shown in the definition of $(P_{t|t-1} - P_{t-1})$ above, changes in relative confidence levels manifest themselves in two ways. First, holding individual level beliefs fixed, changes in relative confidence levels lead to a change in the weighted av-

[20]Market clearing yields:

$$P_t = a_t \mathbb{E}_{i,t}[D_T] + b_t \mathbb{E}_{U,t}[D_T] - c_t \tag{45}$$

where a_t, b_t, and c_t are defined in the main text. Taking first differences of this expression, using agents' posterior beliefs in (40) and (42), and rearranging yields the expression in (46).

erage of beliefs, by changing the relative weights on informed and uninformed traders' beliefs ($\Delta a_t \mathbb{E}_{I,t-1}[D_T] + \Delta b_t \mathbb{E}_{U,t-1}[D_T]$). Second, changes in confidence levels also lead to changes in the aggregate risk-bearing capacity, therefore adding an additional source of price variation via changes in the risk premium component (Δc_t).

Both these features contribute to the dynamics of bubbles of and crashes, as will become clear when discussing equilibrium dynamics in Section 2.4. Before that, we now turn to specifying the mapping uninformed traders use to extract information from prices.

2.3 Micro-founding Time-varying Price Extrapolation

Just as we did in Section 1, to understand what information uninformed agents extract from past prices, we start by specifying what uninformed agents think is generating the price changes they observe. This, in turn, requires us to work out PET agents' beliefs about other agents' actions and beliefs. Following a displacement, PET agents think that in period $t-1$ informed agents trade on all signals they have received up until period $t-1$, $\{\tilde{u}_j\}_{j=0}^{t-1}$ and $\{\tilde{s}_j\}_{j=1}^{t-1}$:

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1} \tag{49}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1} \tag{50}$$

where $\tilde{w}_t \equiv \tilde{\mathbb{E}}_{I,t}[\omega] - \tilde{\mathbb{E}}_{I,t-1}[\omega] = \frac{\tau_s}{t\tau_s+\tau_0}\left(\tilde{s}_t - \tilde{\mathbb{E}}_{I,t-1}[\omega]\right)$. Moreover, (50) reflects that after $(t-1)$ periods informed agents have observed $(t-1)$ price changes which incorporate $(t-1)$ signals about the displacement. Notice that $\tilde{\mathbb{V}}_{I,t-1}[D_T]$ it time-varying as uninformed agents recognize that informed agents' confidence decreases when the displacement is announced, and then increases over time as they learn more about it.

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on their fixed prior beliefs:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D_T] = \bar{D} + \mu_0 \tag{51}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \mathbb{V}_U + (\tau_0)^{-1} \tag{52}$$

where (52) shows that following a displacement PET agents believe that other uninformed agents face greater and constant uncertainty as they do not learn new information after the displacement is announced.

Combining the information in (50) and (52), PET agents' perception of informed agents' edge ($\tilde{\zeta}_{t-1}$) is initially diluted by the rise in aggregate uncertainty due to the displacement, and then gradually rises over time as informed agents learn more about it:

$$\tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}\right) \tag{53}$$

While these dynamics mirror those in the true informational edge in (44), notice that $\tilde{\zeta}_t$ rises at a faster rate than ζ_t, as depicted in panels (a) and (b) of Figure 3. Intuitively, since PET agents think that uninformed agents are not learning more information over time, they think that informed agents regain their edge over uninformed agents at a faster rate than they do in reality: in period t PET agents think informed agents know t more signals than uninformed agents, when in reality all uninformed agents learn from past prices and informed agents are only one period ahead of uninformed agents.

Given these beliefs, PET agents think that price changes now reflect two components:[21]

$$\Delta P_{t-1} = \underbrace{\tilde{a}_{t-1}\left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right)}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)}_{\text{changes in confidence}} \tag{55}$$

where:

$$\tilde{a}_{t-1} \equiv \frac{\tilde{\zeta}_{t-1}}{1+\tilde{\zeta}_{t-1}} \qquad \tilde{b}_{t-1} \equiv \frac{1}{1+\tilde{\zeta}_{t-1}} \tag{56}$$

[21] The perceived market clearing condition yields:

$$P_t = \tilde{a}_t \tilde{\mathbb{E}}_{i,t}[D_T] + \tilde{b}_t \tilde{\mathbb{E}}_{U,t}[D_T] - \tilde{c}_t \tag{54}$$

where $\tilde{a}_t$, $\tilde{b}_t$, and $\tilde{c}_t$ are defined in the main text. Taking first differences of this expression, using agents' posterior beliefs in (49) and (51), and rearranging yields the expression in (46). Notice in particular that uninformed traders think other uninformed traders never update their beliefs, so this term does not show up in (55).

and $\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)$ captures changes in prices due to increases in confidence levels:

$$\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right) = \left(\Delta\tilde{a}_{t-1}\tilde{E}_{I,t-2}[D_T] + \Delta\tilde{b}_{t-1}\tilde{\mathbb{E}}_{U,t-2}[D_T]\right) - \Delta\tilde{c}_{t-1} \tag{57}$$

where $\tilde{c}_{t-1} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_{I,t-1}+(1-\phi)\tilde{\tau}_{U'}}$. Fixing individual level beliefs, this term reflects both perceived changes in weights on informed and uninformed traders' beliefs, and perceived changes in risk-premia. In what follows we argue that at the peak of the bubble partial equilibrium thinkers over-estimate changes in prices due to changes in confidence levels. This leads them to expect greater price changes than the ones they observe. As their beliefs are disappointed, they attribute this discrepancy to negative fundamental news, leading the bubble to burst.

PET agents then invert the mapping in (55), and attribute the unexpected part of the price change they observe to new information $(\tilde{u}_{t-1} + \tilde{w}_{t-1})$, leading to *time-varying extrapolation*.

Proposition 4 (Time-varying Extrapolation)**.** *Following a displacement shock, partial equilibrium thinking leads to time-varying price extrapolation, with traders extrapolating the unexpected part of the price change they observe. Posterior beliefs are given by:*

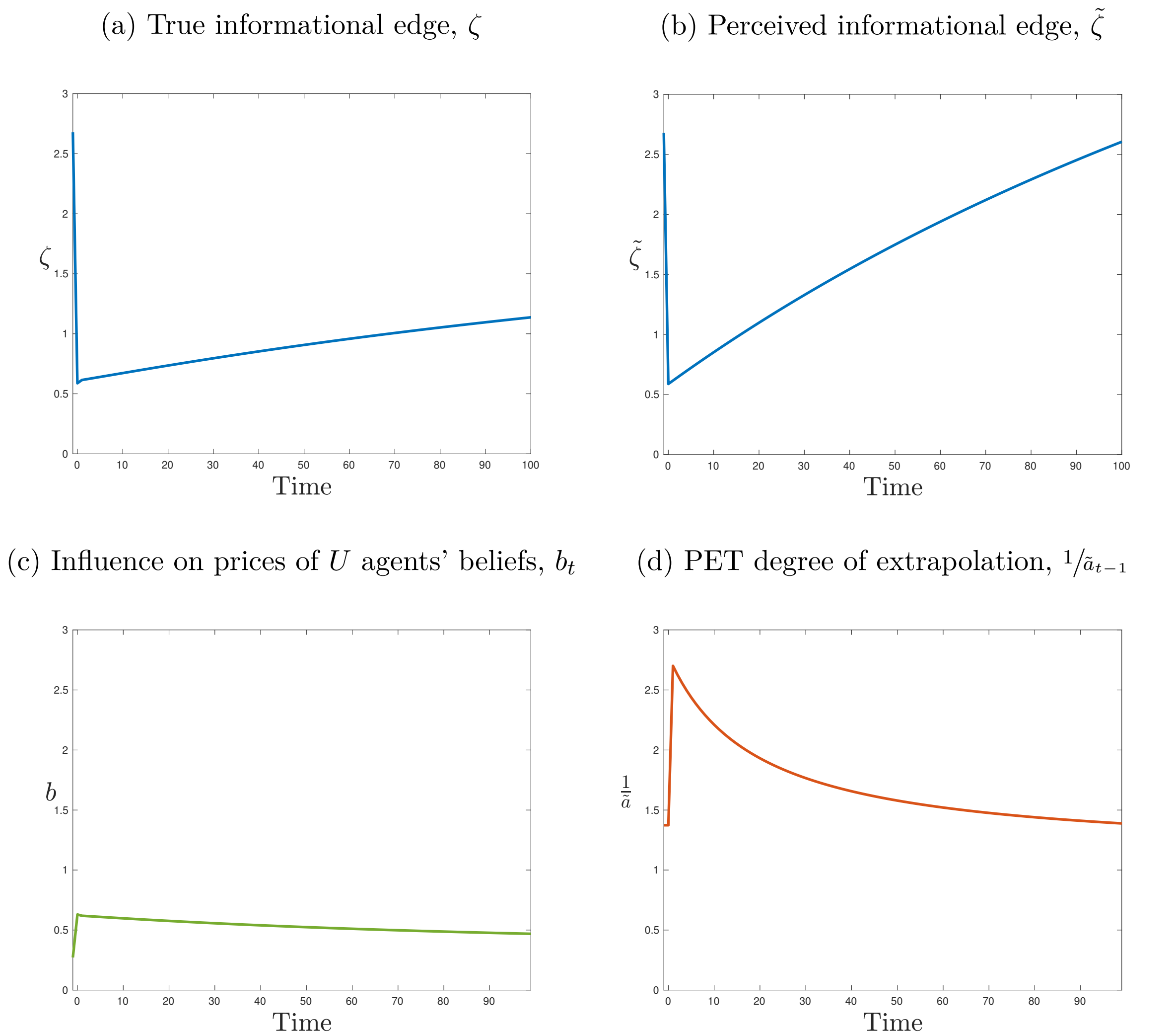
$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \frac{1}{\tilde{a}_{t-1}}\left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{58}$$

where $\frac{1}{\tilde{a}_{t-1}} = 1 + \frac{1}{\tilde{\zeta}_{t-1}}$.

The time-varying extrapolation parameter is also shown in panel (d) of Figure 3. As well as being consistent with empirical evidence that documents a time-varying extrapolation parameter (Cassella and Gulen 2018), micro-founding the extrapolation parameter in this way allows us to understand the assumptions implicit in models of constant price extrapolation. Specifically, they assume that following a large structural break in prices, agents still forecast prices in exactly the same way as they did before the structural break, which is counterfactual.

This also highlights another important point. We model partial equilibrium thinking

Figure 3: Time-variation in the true and perceived informational edges, and in b_t and $1/\tilde{a}_{t-1}$ following a displacement shock. Panels (a) and (b) plot the true and perceived informational edges after a displacement shock, respectively. Following a displacement shock in period $t = 0$, both ζ and $\tilde{\zeta}$ are initially wiped out, and then gradually rise over time as traders learn more about the displacement over time. Comparing panels (a) and (b) shows that the true informational edge rises at a faster rate than the perceived informational edge. Panel (c) plots how the influence on prices of uninformed agents' beliefs (b_t) varies over time following a displacement: b_t initially rises and then gradually declines. Panel (d) plots how the strength with which PET agents extrapolate past prices ($1/\tilde{a}_t$) varies over time following a displacement: when the displacement is announced, PET agents initially extrapolate past prices more aggressively, and then the degree with which they extrapolate declines over time.



by staying as close as possible to the rational expectations benchmark. While the inference problem is much simpler than the rational counterpart (since PET agents do not have to think about higher-order beliefs) it still requires some degree of sophistication on the part of uninformed traders. On the one hand, this is inherent in the nature of our bias, where traders think they are the only ones learning information from prices, and think they have an edge relative to their peers.[22] On the other hand, the reduced form nature of our bias translates into a very simple strategy and heuristic, which does not require much sophistication. If traders think about what generates the price changes they are learning from, it is natural for them to engage in constant price extrapolation when the properties of the environment they are learning from are stable, and to adjust the degree of extrapolation in response to a structural break. In other words, our theory can be understood as explaining when and why agents change heuristics: they do so in response to different type of shocks that change the properties of the environment.

2.4 Displacement, Bubbles and Crashes

By combining the results from Sections 2.2 and 2.3, we find that following a displacement PET agents' prices and beliefs evolve as follows:

$$\Delta P_t = a_t(u_t + w_t) + \left(\frac{b_t}{\tilde{a}_{t-1}}\right)\Delta P_{t-1} - \left(\left(\frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\tilde{P}_{t-1|t-2} - P_{t-2}) - (P_{t|t-1} - P_{t-1})\right) \quad (59)$$

$$(\tilde{u}_{t-1} + \tilde{w}_{t-1}) = \left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(u_{t-1} + w_{t-1}) + \left(\frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\tilde{u}_{t-2} + \tilde{w}_{t-2}) - \frac{1}{\tilde{a}_{t-1}}\left(\tilde{P}_{t-1|t-2} - P_{t-1|t-2}\right) \quad (60)$$

These expressions are reminiscent of the AR(1) processes in (32) and (33), with two key differences, which together allow for the formation of bubbles and crashes following a displacement shock, as shown in Figure 5. First, the strength of the feedback between prices and beliefs is now time-varying, so that equilibrium dynamics can now shift across stable and unstable regions. When the equilibrium dynamics shift to a non-stationary

[22]Partial equilibrium thinking can either be seen as an example of the Lake-Wobegan (or better-than-average) effect (Svenson 1981, Maxwell and Lopus 1994), or as agents paying limited attention to others' informational inferences, rather than having false beliefs about others' inference (Eyster and Rabin 2010).

region, prices and beliefs accelerate away from fundamentals leading to the build up of the bubble. Second, the last term in both (59) and (60) acts as a pull-back force, that dampens increases in prices and beliefs during the formation of the bubble. It is this term that ultimately allows uninformed agents' beliefs to be disappointed at the peak of the bubble, leading to reversals and a crash. We now discuss both of these differences in detail.

Substituting (47) and (56) into the pseudo-AR(1) coefficient in the evolution of beliefs in (60), we find that the strength of the feedback effect now takes the following form:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) \tag{61}$$

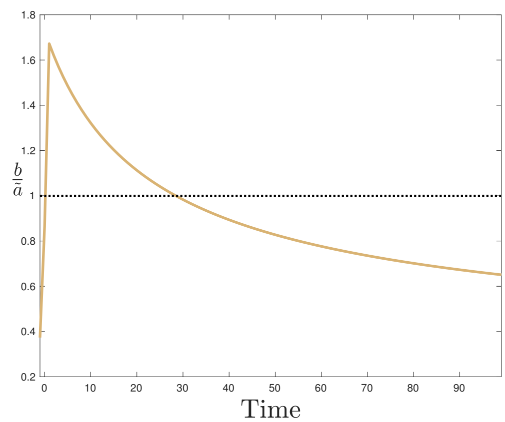
Figure 4 shows that when the displacement materializes in period $t = 0$, the strength of the feedback effect initially increases as the economy is flooded with uncertainty, and both the true and the perceived informational edges are diluted. However, as agents start learning about the displacement, the strength of the feedback effect gradually declines.

Starting from a stable region in normal times, Appendix B.2 shows that if the increase in uncertainty generated by the displacement is large enough, the economy enters an unstable region (${}^{b_t}/_{\tilde{a}_t} > 1$), allowing prices and beliefs to accelerate away from fundamentals. In the long run the economy always returns to a stable region, as $\lim_{t\to\infty} {}^{b_t}/_{\tilde{a}_t} < {}^{b}/_{\tilde{a}} < 1$ since $\lim_{t\to\infty}(b_t - b) = 0$ and $\lim_{t\to\infty}(\tilde{a}_t - \tilde{a}) > 0$, with prices and beliefs converging to a new steady state.

Proposition 5 (Time-varying Strength of the Feedback Effect)**.** *When agents think in partial equilibrium, the strength of the feedback effect between prices and beliefs becomes time varying in response to a displacement shock. In each period t, it is decreasing both in the true informational edge (ζ_t), and in uninformed agents' perception of it ($\tilde{\zeta}_t$). In the long-run, the feedback effect converges to a steady-state value strictly lower than 1.*

However, a time-varying feedback effect in and of itself is not enough to lead to the bursting of the bubble. Indeed, we need uninformed agents to infer *negative* information from prices ($\tilde{u}_{t-1} + \tilde{w}_{t-1} < 0$) and price changes to become negative ($\Delta P_t < 0$) for prices

Figure 4: Time variation in the strength of the feedback effect following a displacement. This figure shows how the strength of the feedback between outcomes and beliefs varies over time following a displacement. The dotted line at $b/\tilde{a} = 1$ separates the stable region ($b/\tilde{a} < 1$) from the unstable region ($b/\tilde{a} > 1$). Starting from a normal times steady state where the strength of the feedback effect is less than one, a displacement is announced in period $t = 0$, and this leads the strength of the feedback effect to initially rise and then gradually decline over time. The initial increase in $b/\tilde{a}$ is increasing in the uncertainty associated with the displacement $(\tau_0)^{-1}$, and this figure depicts a scenario where $(\tau_0)^{-1}$ is large enough to initially shift the economy to an unstable region. Eventually, as informed agents learn more about the displacement, the strength of the feedback effect weakens and the economy returns to a stable region.



and beliefs to revert back towards fundamentals and for the bubble to burst. Moving from an unstable to a stable region simply ensures that price changes go from being positive and increasing over time to positive and decreasing over time, but does not deliver *negative* price changes on its own.[23] Instead, to achieve the reversal, we need stability together with the presence of the last correction term in (59), which allows price changes to become negative.

To gain further intuition as to why PET traders' beliefs are eventually disappointed, notice that the intercept term in (59) is coming from uninformed traders' misunderstanding of the part of the price change due to changes in confidence alone. Following a positive displacement shock, PET agents mistakenly think that informed traders are

[23]In other words, a time-varying $b_t/\tilde{a}_{t-1}$ would not be enough to get a reversal if equilibrium price changes evolved as follows:

$$\Delta P_t = a_t(u_t + w_t) + \left(\frac{b_t}{\tilde{a}_{t-1}}\right)\Delta P_{t-1} \tag{62}$$

Following a one-off positive shock to fundamentals ($u_t + w_t > 0$ for $t = 0$ and $u_t + w_t = 0$ for $t > 0$), there would be no term that allows for ΔP_t to become negative, unlike the additional term in (59).

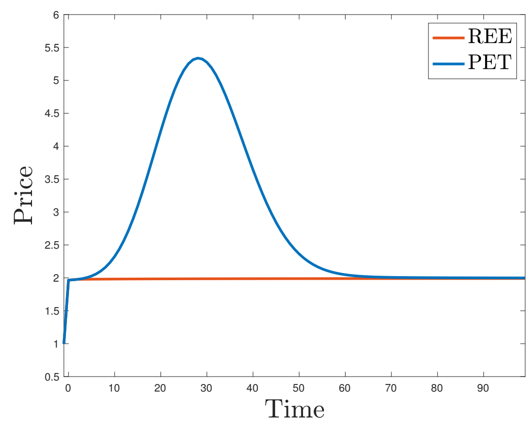
more optimistic than uninformed traders. Fixing individual beliefs, as informed traders regain their edge over time, PET traders think that the average belief becomes more optimistic ($\Delta\tilde{a}_t\tilde{\mathbb{E}}_{I,t}[D_T] + \Delta\tilde{b}_t\bar{D} > 0$), and that this pushes prices up further. In reality informed traders are *less optimistic* than uninformed traders, so that, as informed traders regain their edge, the average belief actually becomes less optimistic over time and closer to the rational benchmark ($\Delta a_t\mathbb{E}_{I,t}[D_T] + \Delta b_t\mathbb{E}_{U,t} < 0$). This puts a negative (corrective) pressure on prices. By over-estimating the part of the price change due to changes in confidence levels, partial equilibrium thinkers eventually expect price rises that are higher than the price changes that they observe. When this occurs, their beliefs are disappointed, which leads them to become more pessimistic, and the bubble to burst. In Appendix B.3 we show more formally how these forces can only induce a reversal once the economy has returned to a stable region.

Figure 5 shows the path of equilibrium outcomes following a displacement shock. Initially, as the economy enters the unstable region, prices and beliefs accelerate away from fundamentals in a convex way, and reach levels several multiples of the fundamental value of the asset (Greenwood et al. 2019). As the strength of the feedback effect weakens, and the economy re-enters the stable region, PET agents' expectations are disappointed, leading the bubble to burst, and prices and beliefs to converge back towards fundamentals. Partial equilibrium thinking naturally delivers these key characteristics of bubbles by exploiting the properties of unstable regions. The duration of the bubble is then longer and its amplitude greater when the uncertainty associated with the displacement is higher, and it takes longer to resolve over time, as in these cases equilibrium dynamics spend longer in the non-stationary region.

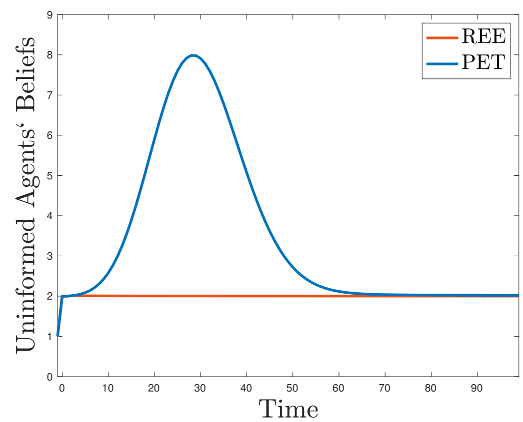
Moreover, notice that while the initial stage of the bubble is associated with high trading volume (Barberis 2018, Hong and Stein 2007), our model is also consistent with empirical evidence in DeFusco et al. (2020) that documents a quiet period before the bust, during which trading volume is falling while prices are still rising. Intuitively, partial equilibrium thinking leads to endogenously heterogeneous beliefs, and during the formation of the bubble disagreement initially increases at an increasing rate, and then increases at a decreasing rate. Similar dynamics occur during the bust, this time in reverse.

Figure 5: Bubbles and crashes following a displacement. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$, and we let its realized value be $\omega = \mu_0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim N(0, \tau_s^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. This figure compares the path of equilibrium prices, uninformed agents' beliefs, trading volume and agents' positions in the risky asset following a displacement which temporarily shifts the economy into an unstable region, under rational expectations and under partial equilibrium thinking. As the economy shifts into an unstable region when the displacement is announced, prices and beliefs accelerate away from fundamentals. This phase of the bubble is also associated with high trading volume, and PET agents being long the asset. Eventually, as the strength of the feedback effect weakens, the economy returns to a stable region and uninformed agents' beliefs are disappointed, leading to a crash.

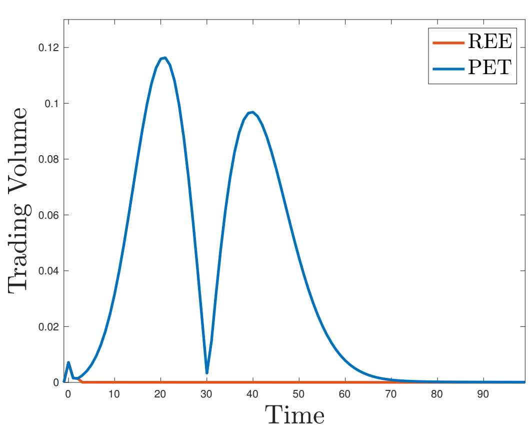
(a) Price



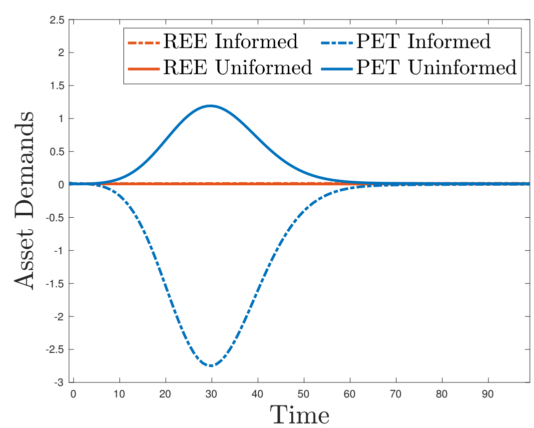
(b) Uninformed agents' beliefs



(c) Trading volume



(d) Asset demands



2.5 Other Features of Bubbles

Appendix C.1 considers a negative displacement shock, with $\mu_0 < 0$. Interestingly, negative bubbles are not merely symmetric, and instead are dampened relative to positive bubbles. To understand why this is the case, we ought to focus on the true and perceived risk-premium components. Regardless of the sign of the displacement shock, the gradual resolution of uncertainty over time exerts an upward force on prices, as the increased risk-bearing capacity reduces the risk-premium component. However, PET agents under-estimate this upward force, as they believe that other uninformed traders are not learning and becoming more confident over time. By under-estimating the increase in risk-bearing capacity, they then under-estimate the upward force on prices coming from changes in risk-premia, and instead attribute part of this to better fundamentals. This force is as at play both when the cash flow shock of the displacement is positive, and when it is negative, therefore amplifying positive bubbles and dampening negative ones. This in contrast to equilibrium dynamics with constant price extrapolation, where this dampening channel is absent, and where negative bubbles would actually be more pronounced than positive ones following a displacement shock.[24]

Appendix C.2 then shows how the exact shape of the bubble depends on the speed with which information about the displacement becomes available over time. For example, Figure 9 shows how the model can deliver a slower boom and a faster crash if information about the displacement is revealed slowly at first, and at a faster rate once the bubble bursts (see Veldkamp (2005) and Ordonez (2013) for models with endogenous information flows during booms and busts).

Finally Appendix C.3 considers the case where uninformed traders are misspecified about the frequency of information arrival. When this is the case, time-variation in the degree of extrapolation ensures that bubbles can burst even as the aggregate risk

[24]Intuitively, the initial increase in uncertainty associated with the displacement exerts a downward pressure on prices, which dampens positive cash flow shocks, and amplifies negative cash flow shocks. Fixing the size of the cash flow shock in absolute value, this asymmetry then leads to a greater initial price change following a negative shock relative to the same size positive shock. Extrapolating a greater initial price change with constant price extrapolation then leads to more amplified dynamics in response to negative shocks.

bearing capacity of informed traders is decreasing relative to that of uninformed traders, something which would not be possible if we had constant price extrapolation.

2.6 Other Types of Displacements

A key lesson from our analysis so far is that shocks that generate bubbles and crashes must have two properties: they must shift the economy to an unstable region, and such a shift must be temporary. So far, we have considered one possible way to achieve this via a positive shock that creates uncertainty, which gradually resolves over time. However, the sources of variation in $\frac{b_t}{\tilde{a}_t}$ discussed in Proposition 2 are informative about other types of shocks which may contribute to the formation of bubbles and crashes.

Specifically, we can write the strength of the feedback effect as follows:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) < 1 \iff \left(\frac{\phi_t}{1-\phi_t}\frac{\tau_{I,t}}{\tau_{U,t}}\right)\left(\frac{\tilde{\phi}_t}{1-\tilde{\phi}_t}\frac{\tilde{\tau}_{I,t}}{\tilde{\tau}_{U,t}}\right) > 1 \tag{63}$$

where the second inequality simply follows from re-arranging the first one, and using the definition of the true and perceived informational edges.[25] Moreover, (63) generalizes our earlier expressions by allowing the fraction of informed agents in the market to be time-varying, and by allowing uninformed agents to be misspecified about this quantity ($\tilde{\phi}_t \neq \phi_t$). There are four components of the information structure that can then lead to time-variation in the strength of the feedback effect: the true and the perceived confidence of informed agents relative to uninformed agents, and the true and the perceived composition of agents in the market. Temporary shocks to these quantities can also contribute to the time-varying strength of the feedback effect.

For example, Greenwood and Nagel (2009) find that young inexperienced investors increased exposure to technology stocks during the dot.com bubble, and decreased it during the crash. More generally, historical narratives associate displacements with large changes in the composition of agents in the market (Mackay 1841, Brooks 1999, Brennan 2004, Aliber and Kindleberger 2015). This paper highlights how changes in

[25]Re-arranging the first inequality, we get: $(1+\zeta_t) > \left(\frac{1+\tilde{\zeta}_t}{\tilde{\zeta}_t}\right) \iff \zeta_t\tilde{\zeta}_t > 1$.

the composition of traders constitute another source of time-variation in the strength of the feedback effect, and hints to how the timing of these changes can play an important role in determining the shape and amplitude of bubbles.

3 Speculative Motives

A noted feature of bubbles neglected so far is the role of destabilizing speculation. When explaining the stage of 'euphoria' characteristic of bubbles, Kindleberger (1978) describes how "[i]nvestors buy goods and securities to profit from the capital gains associated with the anticipated increases in the prices of these goods and securities."[26]

To model speculative motives, we change agents' objective function. Specifically, instead of having agents who are only concerned with forecasting the terminal dividend as in (4), we now assume that agents have CARA utility over next period wealth, and forecast next period's payoff:

$$\Pi_{t+1} = \beta P_{t+1} + (1-\beta) D_t \tag{64}$$

which simply reflects traders' beliefs that with probability β the asset is alive next period, and is worth P_{t+1}, and with probability $(1-\beta)$ the asset dies, and pays out a terminal dividend $D_t = \bar{D} + \sum_{j=0}^{t} u_j$ in normal times and $D_t = \bar{D} + \sum_{j=0}^{t} u_j + \omega$ following a displacement. Taking first order conditions, we have that agents now trade according to the following asset demand function, given their beliefs:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[\Pi_{t+1}]} \tag{65}$$

In Appendix B.4 we solve the model with speculative motives using the same three steps as in Section 2, and show that the true price function is linear in agents' beliefs, and that partial equilibrium thinking still provides a micro-foundation for price

[26]Using survey data in the Michigan Survey of Consumers, Piazzesi and Schneider (2009) find that during the recent US housing boom there was a growing cluster of households who thought it was a good time to buy because they believed house prices would rise further.

extrapolation:

$$P_t = a_t \mathbb{E}_{I,t}[\Pi_{t+1}] + b_t \mathbb{E}_{I,t}[\Pi_{t+1}] - c_t \tag{66}$$

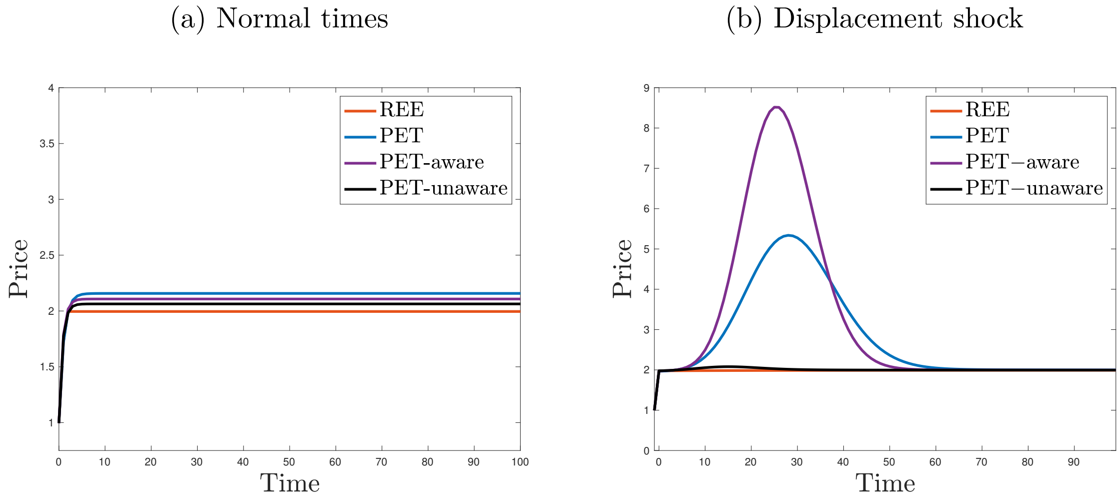
$$\mathbb{E}_{U,t}[\Pi_{t+1}] = \mathbb{E}_{U,t-1}[\Pi_{t+1}] + \theta_t \left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{67}$$

where a_t, b_t, c_t and θ_t are once again constant in normal times, but become time-varying following a displacement. While these coefficients still depend on the properties of the environment, their functional form depends on agents' higher order beliefs. Specifically, since agents are forecasting future *endogenous* outcomes, they need to forecast other agents' future beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their own private information and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

In this section, we consider two cases. First, we let informed agents understand uninformed agents' biased beliefs, which in turn implies that they understand that mispricing is predictable. Second, we consider the case where informed agents mistakenly believe that all other agents are rational and extract the right information from prices. We refer to the first type of speculators as being "PET-aware," and to the second type as being "PET-unaware." This lines up with the distinction in practical asset management between investors who think about behavioral biases in the market, and those who only concentrate on the gap between market prices and their estimates of fundamentals.

Figure 6 contrasts the dynamics of equilibrium outcomes in normal times and following a displacement, with and without speculative motives. As in the case without speculation, panel (a) shows that normal times dynamics only exhibit a small degree of momentum and speculative motives keep prices closer to fundamentals. After a displacement shock, however, panel (b) of Figure 6 makes clear that the dynamics heavily depend on the behavior of informed speculators. When informed agents understand other agents' biases, they engage in destabilizing speculation and amplify the bubble. Intuitively, when informed agents realize that mispricing is predictable, they understand that higher prices today translate into more optimistic beliefs by uninformed agents and higher prices tomorrow. This increases informed agents' expected capital gains and induces them to demand more of the asset today, inflating prices further (as in De Long et al. 1990).

Figure 6: Normal Times and Bubbles and crashes with speculators. Panel (a) compares the path of equilibrium prices under rational expectations, partial equilibrium thinking, "PET-aware" speculation, and "PET-unaware" speculation in normal times. Starting from a normal times steady state, Panel (b) considers a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ in each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \ \forall t > 1$. "PET-aware" speculation amplifies the bubble relative to the case with no speculative motives, while "PET-unaware" speculation arbitrages the bubble away.



To take advantage of predictable mispricing, "PET-aware" speculators require a high level of understanding of other agents' actions and beliefs. Alternatively, we consider the case where informed agents mistakenly believe that they live in a rational world and think that uninformed agents are able to recover the right information from past prices. In this case, informed agents believe that any current mispricing will be corrected next period. This leads them to trade more aggressively on their own information, thus keeping prices closer to fundamentals, and effectively arbitraging the bubble away.

This analysis highlights the importance of higher order beliefs in the formation of bubbles: only if investors think that mispricing is likely to persist do they engage in destabilizing speculation. If instead they think mispricing is temporary, they engage in fundamental speculation and arbitrage it away (Abreu and Brunnermeier 2002, Abreu and Brunnermeier 2003).

4 Conclusion

In this paper we provide a micro-foundation for the degree of price extrapolation with a theory of "Partial Equilibrium Thinking" (PET), in which uninformed agents mistakenly attribute any price change they observe to new information alone, when in reality part of the price change is due to other agents' buying/selling pressure. We show that when agents think in partial equilibrium the degree of extrapolation varies with the information structure, and is decreasing in informed agents' informational edge.

This micro-foundation provides a unifying theory of both weak departures from rationality in normal times, and extreme bubbles and crashes following a displacement. These are simply different manifestations of the same two-way feedback between prices and beliefs. In normal times, informed agents' edge is constant, and PET delivers constant price extrapolation. By contrast, following a displacement, informed agents' edge is temporarily wiped out, and PET agents' degree of extrapolation is stronger at first, but then gradually dies down, leading to bubbles and endogenous crashes.

While this paper provides a first step in micro-founding the degree of price extrapolation, our analysis leaves several open avenues for future work. First, a quantitative assessment of our theory would shed light on the extent of amplification that time-varying extrapolation can provide in explaining departures from rationality, and would clarify the importance of this channel. Second, by looking at the variation in the degree of price extrapolation and in individual level forecasts, our model offers two predictions that distinguish it from models of constant price extrapolation, and of fundamental extrapolation: i) unlike models of constant price extrapolation, when agents think in partial equilibrium the degree of price extrapolation is stronger when there are fewer informed agents in the market, and when informed agents' edge is greater; ii) unlike models of fundamental extrapolation, when agents think in partial equilibrium the bias in individual level forecasts depends on the composition of agents in the market, as this affects the extent of misspecification. These predictions can be tested both in the cross-section and over time. As the literature moves to incorporating non rational expectations into macro and finance models, and to studying their quantitative and policy implications, distinguishing between different sources of irrationality is increasingly important, and evidence that sheds light on these issues is a fruitful avenue for future research.

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Appendices

A Proofs

A.1 Proof of Proposition 1: Micro-foundation of Price Extrapolation

In equation (29) we showed that in period t partial equilibrium thinkers extract the following fundamental shock from past prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{A.1}$$

Combining this with the expression for uninformed traders' posterior beliefs in (7), we find that:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{A.2}$$

which provides a micro-foundation for extrapolative beliefs where uninformed traders extrapolate recent price changes in forecasting future fundamentals.

To understand how the size of the bias inherent in partial equilibrium thinking varies with informed traders' edge, start from the expression in (32) for the equilibrium evolution of signals uninformed traders extract from prices:

$$\tilde{u}_{t-1} = u_{t-1} + \left(\frac{b}{\tilde{a}}\right) \tilde{u}_{t-2} \tag{A.3}$$

If we consider the impulse response function to a one-off shock to fundamentals in period $t = 1$, so that $u_t \neq 0$ for $t = 1$ and $u_t = 0$ for $t > 1$, we can iterate the above expression backwards, and find that:

$$\tilde{u}_t = \left(\frac{b}{\tilde{a}}\right)^{t-1} u_1 \tag{A.4}$$

which shows that while uninformed traders extract the right signal in the first period after the shock, they extract a biased signal in each period thereafter. Specifically, since

$u_t = 0$ for $t > 1$, we have that:

$$\tilde{u}_t - u_t = \left(\frac{b}{\tilde{a}}\right)^{t-1} u_1 \tag{A.5}$$

so that for a given fundamental shock u_1 the bias is increasing in the strength of the feedback effect $b/\tilde{a}$ in (36):

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) \tag{A.6}$$

Since the strength of the feedback effect is decreasing in the true and perceived informational edges, it follows that the bias in uninformed traders' beliefs is also decreasing in both these terms. □

A.2 Proof of Proposition 2: Strength of the Feedback Effect

Equation (36) shows that the strength of the feedback effect is given by:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) \tag{A.7}$$

Combining this with the definitions of the informed agents' edge ζ in (9), and of uninformed agents' perception of it, $\tilde{\zeta}$, we find that:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) = \left(\frac{1}{1+\frac{1}{\frac{1}{\phi}-1}\frac{\tau_I}{\tau_U}}\right)\left(1+\left(\frac{1}{\phi}-1\right)\frac{1}{\frac{\tilde{\tau}_I}{\tilde{\tau}_U}}\right) \tag{A.8}$$

The first equality shows that the strength of the feedback effect is decreasing in both the true informational edge, ζ, and in uninformed agents' perception of it, $\tilde{\zeta}$. The second equality shows that the strength of the feedback effect is decreasing in the fraction of informed agents in the market, ϕ (by inspection, both $\frac{1}{1+\zeta}$ and $\frac{1}{\tilde{\zeta}}$ are decreasing in ϕ) and in the true and perceived confidence of informed agents relative to uninformed agents τ_I/τ_U, $\tilde{\tau}_I/\tilde{\tau}_U$. □

A.3 Proof of Proposition 3: Deviations from Rationality

When traders have rational expectations, they infer the right information from prices at each point in time. Following, a one-off shock in period 0, uninformed traders learn the following information from price:

$$\tilde{u}_0^{REE} = u_0 \neq 0 \tag{A.9}$$

$$\tilde{u}_t^{REE} = u_t = 0 \quad \forall t > 0 \tag{A.10}$$

It follows that under rational expectations, uninformed traders' beliefs are given by:

$$\mathbb{E}_{U,0}[D_T]^{REE} = \bar{D} \tag{A.11}$$

$$\mathbb{E}_{U,t}[D_T]^{REE} = \bar{D} + u_0 \quad \forall t > 0 \tag{A.12}$$

This reflects that rational uninformed traders understand that there is no new information after period 0, and that all other price changes they observe are due to the lagged response of all uninformed traders who are also learning information from prices. Therefore, they no longer update their beliefs following the second price rise, as in the example in Figure 1. The corresponding equilibrium prices are then given by:

$$P_0^{REE} = \bar{P} + \Delta P_0 = \bar{P} + au_0 \tag{A.13}$$

$$P_t^{REE} = \bar{P} + \Delta P_0 + \Delta P_1 + \underbrace{\sum_{j=2}^{t} \Delta P_t}_{=0} = \bar{P} + au_0 + bu_0 \quad \forall t > 0 \tag{A.14}$$

where $\sum_{j=2}^{t} \Delta P_t = 0$ as neither informed nor uninformed agents update their beliefs after period $t = 1$, and in normal times the risk-premium component $\left(\frac{\mathcal{A}Z}{\phi\tau_I + (1-\phi)\tau_U}\right)$ is also constant over time.

On the other hand, from (15) and (A.2) together with the fact that in normal times $a = \tilde{a}$, we know that when uninformed traders think in partial equilibrium, equilibrium beliefs and prices are given by:

$$\mathbb{E}_{U,0}[D_T] = \bar{D} \tag{A.15}$$

$$\mathbb{E}_{U,1}[D_T] = \bar{D} + u_0 \tag{A.16}$$

$$\mathbb{E}_{U,t}[D_T] = \bar{D} + u_0 + \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \tag{A.17}$$

and:

$$P_0 = \bar{P} + au_0 \tag{A.18}$$

$$P_1 = \bar{P} + au_0 + bu_0 \tag{A.19}$$

$$P_t = \bar{P} + au_0 + bu_0 + \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) \tag{A.20}$$

Comparing PET to REE outcomes, we see that when traders think in partial equilibrium, deviations from rational outcomes are given by:

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{U,t}^{REE}[D_T] = 0 \qquad \text{for } t = 0, 1 \tag{A.21}$$

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{U,t}^{REE}[D_T] = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \quad \forall t > 1 \tag{A.22}$$

and:

$$P_t - P_t^{REE} = 0 \qquad \text{for } t = 0, 1 \tag{A.23}$$

$$P_t - P_t^{REE} = \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j (bu_0) \quad \forall t > 1 \tag{A.24}$$

where the last equality uses the fact that in normal times $\tilde{a} = a$.

From Proposition 2, we know that $\frac{b}{\tilde{a}}$ is decreasing in ζ, $\tilde{\zeta}$, ϕ, $\frac{\tau_I}{\tau_U}$ and $\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$. Moreover, from (13) we know that b is also decreasing in ζ, which is itself increasing in ϕ and $\frac{\tau_I}{\tau_U}$. Combining these results with (A.22) and (A.24), we obtain the comparative statics in Proposition 3 $\forall t > 1$. In particular, when the equilibrium is stable these comparative statics also hold in the limit as $t \to \infty$, as the economy approaches the new steady state. $\square$

A.4 Proof of Proposition 4: Time-varying Extrapolation

Before the displacement is announced, the degree of extrapolation in normal times is simply given by:

$$\theta = 1 + \frac{1}{\tilde{\zeta}} = 1 + \left(\frac{1}{\phi} - 1\right)\frac{\mathbb{V}_I}{\mathbb{V}_U} \tag{A.25}$$

Following a displacement, inverting equation (55) yields:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \frac{1}{\tilde{a}_{t-1}}\left(\Delta P_{t-1} - \left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)\right) \tag{A.26}$$

Using the fact that $\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_t + \tilde{w}_t$, and also that $\Delta P_{t-1} - \tilde{P}_{t-1|t-2} + P_{t-2} = P_{t-1} - \tilde{P}_{t-1|t-2}$, we get:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \frac{1}{\tilde{a}_{t-1}}\left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{A.27}$$

where $\frac{1}{\tilde{a}_{t-1}} = 1 + \frac{1}{\tilde{\zeta}_{t-1}}$ is time-varying (as discussed in the main text), and captures the time-varying strength with which partial equilibrium thinkers extrapolate recent price changes. $\square$

A.5 Proof of Proposition 5: Time-varying Strength of the Feedback Effect

In (61), we showed that, following a displacement, the strength of the feedback effect takes the following form:

$$\frac{b_{t-1}}{\tilde{a}_{t-1}} = \left(\frac{1}{1 + \zeta_{t-1}}\right)\left(1 + \frac{1}{\tilde{\zeta}_{t-1}}\right) \tag{A.28}$$

which directly shows that the feedback effect is decreasing in both the true and perceived informational edges. The true and perceived informational edges were derived in (44) and (53) as follows:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}}\right) \tag{A.29}$$

$$\tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}\right) \tag{A.30}$$

Since both these quantities are time-varying, it follows that (A.28) is also time-varying. Taking the limit of this expression, we find that:

$$\lim_{t\to\infty} \zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_I}\right) \tag{A.31}$$

$$\lim_{t\to\infty} \tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I}\right) \tag{A.32}$$

and hence that $\lim_{t\to\infty} \zeta_t < \lim_{t\to\infty} \tilde{\zeta}_{t-1}$ which directly implies $\lim_{t\to\infty} \frac{b_{t-1}}{\tilde{a}_{t-1}} < 1$

B Derivations

B.1 Rational Expectations

When uninformed traders have rational expectations, they perfectly understand what generates price changes they observe. In turn, this requires them to understand other traders' beliefs, and actions.

Formally, rational agents think that in period $t-1$ informed agents update their beliefs with the new fundamental information they receive, $\tilde{u}_{t-1}$:[27,28]

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} \tag{B.1}$$

[27]Here we still denote with a ˜ uninformed traders' beliefs about a variable, and in this case $\tilde{u}_{t-1}$ is uninformed traders' beliefs about the fundamental shock that hit the economy in period $t-1$, and which uninformed traders wish to extract from past prices.

[28]The use of $t-1$ subscripts instead of t is to highlight that uninformed agents learn information from past prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_I = \mathbb{V}_I \tag{B.2}$$

where the last equality highlights how uninformed traders are correctly specified about informed traders' posterior variance, as can be seen from comparing (B.2) to (20).

Moreover, they also understand that all other uninformed agents learn information from past prices. Specifically, they know that in period $t-1$ uninformed traders update their beliefs by $\tilde{u}_{t-2}$, which is the same signal that they extract from the past price they observe in that period, P_{t-2}:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D_T] + \tilde{u}_{t-2} \tag{B.3}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_U = \mathbb{V}_U \tag{B.4}$$

where the last equality highlights how uninformed traders are correctly specified about other uninformed traders' posterior variance, as can be seen from comparing (B.4) to (22).

To be clear on notation, notice that, while $\tilde{u}_{t-2}$ is in uninformed traders' information set starting in period $t-1$, $\tilde{u}_{t-1}$ is the signal that uninformed traders are extracting from prices in period t.

Rational agents then think that the equilibrium price in period $t-1$ is generated by the following market clearing condition:

$$\phi\underbrace{\left(\frac{\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-1} - P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_I}\right)}_{\tilde{X}_{I,t-1}} + (1-\phi)\underbrace{\left(\frac{\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-2} - P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_U}\right)}_{\tilde{X}_{U,t-1}} = Z \tag{B.5}$$

which leads to the following price function:

$$P_{t-1} = \tilde{a}\left(\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\left(\mathbb{E}_{U,t-2}[D_T] + \tilde{u}_{t-2}\right) - \tilde{c} \tag{B.6}$$

where: $\tilde{a} \equiv \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{\tilde{\zeta}}{1+\tilde{\zeta}}$, $\tilde{b} \equiv \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U} = \frac{1}{1+\tilde{\zeta}}$ and $\tilde{c} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$. Since we saw in (B.2) and (B.4) that uninformed traders are have correct beliefs about the posterior variances of both informed and uninformed traders, it follows that $\tilde{a} = a$, $\tilde{b} = b$ and $\tilde{c} = c$, where a, b and c are the coefficients in the true price function in (11).

Taking first differences of (B.2) and (B.4), substituting them into the first difference of (B.6), and using the fact that $\tilde{a} = a$, $\tilde{b} = b$ and $\tilde{c} = c$, we find that rational traders understand that price changes reflect two sources of price variation, which capture changes in beliefs of both informed and uninformed traders:

$$\Delta P_{t-1} = \underbrace{a\, \tilde{u}_{t-1}}_{\text{instantaneous response}} + \underbrace{b\, \tilde{u}_{t-2}}_{\text{lagged response}} \tag{B.7}$$

They then invert this mapping to extract the following signal from past prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{a}\right) \Delta P_{t-1} - \left(\frac{b}{a}\right) \tilde{u}_{t-2} \tag{B.8}$$

Lagging the true price function (15), and substituting it into (B.8), we then find that uninformed traders are able to extract the right information from past prices:

$$\tilde{u}_{t-1} = u_{t-1} \tag{B.9}$$

B.2 Displacements, Bubbles and Crashes

In normal times, the strength of the feedback effect is given by:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right) = \frac{1}{\zeta} < 1 \tag{B.10}$$

where the second equality follows from the fact that in normal times:

$$\zeta = \tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I} \tag{B.11}$$

and the last inequality in (B.10) follows from the fact that the economy must be in a stable region in normal times.

Following a displacement, the strength of the feedback effect is given by:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) \tag{B.12}$$

where in $t = 0$:

$$\zeta_0 = \tilde{\zeta}_0 = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (\tau_0)^{-1}} \tag{B.13}$$

and in $t > 0$:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{B.14}$$

$$\tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{B.15}$$

Combining (B.12) and (B.13), we find that in period $t = 0$ the strength of the feedback effect is given by:

$$\frac{b_0}{\tilde{a}_0} = \frac{1}{\zeta_0} \tag{B.16}$$

$$= \frac{1}{\zeta} + \left(\frac{1}{\zeta_0} - \frac{1}{\zeta}\right) \tag{B.17}$$

$$= \frac{b}{\tilde{a}} + \left(\frac{1-\phi}{\phi}\right)\left(\frac{\mathbb{V}_U - \mathbb{V}_I}{\mathbb{V}_U}\right)\frac{(\tau_0)^{-1}}{\mathbb{V}_U + (\tau_0)^{-1}} \tag{B.18}$$

where the second equality simply adds and subtracts the strength of the feedback effect in normal times $\frac{b}{\tilde{a}} = \frac{1}{\zeta}$, and the last equality uses the expressions for ζ and ζ_0 in (B.11) and (B.13) above, and rearranges.

Ceteris paribus, for the strength of the feedback effect to enter the unstable region we need the uncertainty associated with the displacement $(\tau_0)^{-1}$ to be high enough:

$$\frac{b_0}{\tilde{a}_0} > 1 \iff (\tau_0)^{-1} > \frac{\left(1-\frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)\mathbb{V}_U}{1-\left(1-\frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)} \tag{B.19}$$

where$(1 - {}^{b}/_{\tilde{a}}) > 0$ from (B.10). In the long run, as uncertainty about the displacement is resolved, we have that:

$$\zeta_\infty \equiv \lim_{t\to\infty} \zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I} = \zeta \tag{B.20}$$

$$\tilde{\zeta}_\infty \equiv \lim_{t\to\infty} \tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I} > \tilde{\zeta} \tag{B.21}$$

Combining these expressions:

$$\lim_{t\to\infty} \frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_\infty}\right)\left(1+\frac{1}{\tilde{\zeta}_\infty}\right) < \frac{b}{\tilde{a}} < 1 \tag{B.22}$$

which shows that in the long run the economy always returns to a stable region, with a steady state feedback effect that is weaker than the original normal times feedback effect. In the main text we show that when the strength of the feedback effect evolves in this way, prices and beliefs are initially non-stationary and accelerate away from fundamentals in a convex way. As the feedback effect then weakens towards its new steady state level, it eventually returns into a stable region, leading uninformed agents' beliefs to be disappointed, the bubble to burst, and prices and beliefs to converge back towards fundamentals.

B.3 Bursting the Bubble

To see how these forces play a joint role in bursting the bubble, and how the reversal can only occur once the economy returns to a stable region, we can substitute the definitions of $(P_{t-1} - P_{t-1|t-2})$ and $(P_{t-1} - \tilde{P}_{t-1|t-2})$ into (60), to find that beliefs evolve as follows:

$$\begin{aligned}\tilde{u}_{t-1} + \tilde{w}_{t-1} = &\left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{I,t-1}[D_T] - \mathbb{E}_0[D_T]) \\ &- \left(1 - \frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T]) + \frac{1}{\tilde{a}_{t-1}}(\tilde{c}_{t-1} - c_{t-1})\end{aligned} \tag{B.23}$$

where $\mathbb{E}_0[D_T] = \bar{D} + \mu_0$ is agents' unconditional prior belief when the displacement is announced. For the bubble to burst, we need $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ to eventually turn negative.

If we consider a one-off positive shock, such that $\mathbb{E}_{I,t-1}[D_T] = \mathbb{E}_{I,1}[D_T] > \mathbb{E}_0[D_T]$ for all $t \geq 1$, equation (B.23) makes clear that as long as the economy is in a unstable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} > 1$, PET agents continue to extract positive information from prices, and therefore become increasingly optimistic.[29] In other words, when the economy is in an unstable region, the lagged response of uninformed agents always raises prices by more than what uninformed agents would expect from changes in confidence alone. On the other hand, this is no longer the case once the economy returns to a stable region and the feedback between outcomes and beliefs runs out of steam. At the peak of the bubble uninformed agents' beliefs vastly exceed fundamentals, and the term in $(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T])$ dominates in determining the sign of the news that uninformed agents extract from past prices in (B.23). Once the economy returns into a stable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} < 1$, PET agents expect higher price rises than the ones they observe. As their beliefs are disappointed, they become more pessimistic ($\tilde{u}_{t-1} + \tilde{w}_{t-1} < 0$) and the bubble bursts.

B.4 Adding Speculative Motives

To model speculative motives, we let agents have Constant Absolute Risk Aversion (CARA) utility over *next* period wealth.

When traders have these preferences, their asset demand function conditional on their beliefs is given by:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[\Pi_{t+1}]} \tag{B.24}$$

where the expected next period payoff is given by:

$$\Pi_{t+1} \equiv \beta P_{t+1} + (1-\beta) D_t \tag{B.25}$$

and simply reflects that with probability β the asset is alive next period and worth P_{t+1}, and with probability $(1-\beta)$ the asset dies and pays out its terminal dividend D_t.

[29] Notice that the last term in $\tilde{c}_{t-1} - c_{t-1} > 0$, as uninformed traders under-estimate the aggregate risk bearing capacity following a displacement.

Since agents are forecasting prices, which are *endogenous* outcomes, they now need to forecast other agents' future beliefs. Therefore, in solving the model with speculative motives, we need to specify agents' higher order beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their private information alone, and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

We consider two cases. In Section B.4.1 we let informed agents be "PET-aware," so that they perfectly understand uninformed agents' biased beliefs. In Section B.4.2, we consider a case where informed agents are "PET-unaware" and mistakenly believe that all other agents are rational, and that uninformed agents extract the right information from prices. This lines up with the distinction in practical asset management between investors who concentrate on the gap between market prices and their estimates of fundamentals, and those who also think about the behavioral biases in the market.

B.4.1 "PET−aware" Speculation

In solving the model, we proceed in the same three steps we used in the baseline model. First, we solve for the true price function which generates the prices agents observe. Second, we specify the mapping that uninformed agents use to extract information from prices. Third, we solve the model forward, starting from the steady state in normal times. The one key difference to our baseline setup is that since all agents are now forecasting an endogenous outcome, we now need to solve for the first two steps by backwards induction. To do so, we use the new steady state after the uncertainty surrounding the displacement has been resolved as our terminal point.

Step 1: True Market Clearing Price Function. To determine the true market clearing condition which determines the prices agents observe, we know that in period t all informed agent trade on the whole history of signals they have received up until that date ($\{u_j\}_{j=0}^{t}$, $\{s_j\}_{j=1}^{t}$) and all uninformed agents trade on the information they have learnt from past prices.

We define $\mathcal{D}_t \equiv \bar{D} + \sum_{j=1}^{t} u_t$ and $\mathcal{W}_t \equiv \frac{\tau_0}{t\tau_s+\tau_0}\mu_0 + \frac{\tau_s}{t\tau_s+\tau_0}\sum_{j=1}^{t}\tilde{s}_t$ to be informed agents' period t belief of normal times shocks and of the displacement respectively, and $\tilde{\mathcal{D}}_t$ and $\tilde{\mathcal{W}}_t$ are uninformed agents' beliefs about these quantities.

We can then guess that the true price function takes the following form:

$$P_t = A_t(\mathcal{D}_t + \mathcal{W}_t) + B_t(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}) - K_t \tag{B.26}$$

where $\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}$ is the information that uninformed agents extract from past prices, and A_t, B_t and K_t are time-varying and deterministic coefficients that depend on the properties of the environment.

To verify our guess, notice that if informed agents are aware of uninformed agents' bias, their beliefs about next period payoff are given by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] = (1-\beta+\beta A_{t+1}) \underbrace{(\mathcal{D}_t + \mathcal{W}_t)}_{\mathbb{E}_{I,t}[\tilde{\mathcal{D}}_{t+1}+\tilde{\mathcal{W}}_{t+1}]} + \beta B_{t+1} \underbrace{\left(\frac{P_t - \tilde{B}_t(\bar{D}+\mu_0) + \tilde{K}_t}{\tilde{A}_t}\right)}_{\mathbb{E}_{I,t}[\tilde{\mathcal{D}}_t+\tilde{\mathcal{W}}_t]} - \beta K_{t+1} \tag{B.27}$$

$$\mathbb{V}_{I,t}[\Pi_{t+1}] = \mathbb{V}_{I,t}\left[\beta A_{t+1}u_{t+1} + \beta A_{t+1}\frac{\tau_s}{(t+1)\tau_s+\tau_0}(\omega+\epsilon_{t+1}) + (1-\beta)\omega\right] \tag{B.28}$$

$$= (\beta A_{t+1})^2\sigma_u^2 + \left(\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1} + \left(1-\beta+\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(t\tau_s+\tau_0)^{-1} = \mathbb{V}_{I,t} \tag{B.29}$$

where the variance term captures how the uncertain components of expected profits in equation B.25 are (i) the future dividend component u_{t+1} ; (ii) the signal informed agents receive in period $t+1$, $s_{t+1} = \omega + \epsilon_{t+1}$; and (iii) the displacement shock ω.

Turning to uninformed agents' beliefs:

$$\mathbb{E}_{U,t}[\Pi_{t+1}] = (1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1}) + \beta\tilde{B}_{t+1}(\bar{D}+\mu_0) - \beta\tilde{K}_{t+1} \tag{B.30}$$

$$
\begin{aligned}
\mathbb{V}_{U,t}[\Pi_{t+1}] =& \mathbb{V}_{U,t}\left[\beta \tilde{A}_{t+1}\left(u_{t+1}+u_t+\frac{2\tau_s}{(t+1)\tau_s+\tau_0}\omega+\frac{\tau_s}{(t+1)\tau_s+\tau_0}(\epsilon_{t+1}+\epsilon_t)\right)+(1-\beta)(u_t+\omega)\right] \\
=&(\beta \tilde{A}_{t+1})^2\sigma_u^2+(1-\beta+\beta\tilde{A}_{t+1})^2\sigma_u^2 \\
&+\left(1-\beta+\beta\tilde{A}_{t+1}\frac{2\tau_s}{(t+1)\tau_s}\right)^2((t-1)\tau_s+\tau_0)^{-1} \\
&+2\left(\frac{\tau_s\beta\tilde{A}_{t+1}}{(t+1)\tau_s+\tau_0}\right)^2(\tau_s)^{-1}=\mathbb{V}_{U,t}
\end{aligned}
\tag{B.31}
$$

where the first equality captures that in period t uninformed traders are uncertain about ut, u_{t+1}, ϵ_t and ϵ_{t+1} and ω, and the last equality simply simplifies notation to highlight that $\mathbb{V}_{U,t}$ is deterministic and time-varying.

Given these beliefs, the true market clearing condition which generates the prices agents observe is given by:

$$
\phi\left(\frac{\mathbb{E}_{I,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\mathbb{V}_{I,t}[\Pi_{t+1}]}\right)+(1-\phi)\left(\frac{\mathbb{E}_{U,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\mathbb{V}_{U,t}[\Pi_{t+1}]}\right)=Z \tag{B.32}
$$

and the resulting market clearing price function is given by:

$$
\begin{aligned}
P_t =& \left(\frac{\phi\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{I,t}[\Pi_{t+1}] \\
&+\left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{U,t}[\Pi_{t+1}] \\
&-\frac{\mathcal{A}Z\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi\mathbb{V}_{U,t}+(1-\phi)\mathbb{V}_{I,t}}
\end{aligned}
\tag{B.33}
$$

Since (B.27), (B.29), (B.30) and (B.31) show that $\mathbb{E}_{I,t}[\Pi_{t+1}]$ is linear in $(\mathcal{D}_t+\mathcal{W}_t)$ and $(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1})$, $\mathbb{E}_{U,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1})$, and that $\mathbb{V}[\Pi_{t+1}]$ and $\mathbb{V}[\Pi_{t+1}]$ are deterministic, we see that the true price function does indeed take the form in (B.26). Substituting (B.27), (B.29), (B.30) and (B.31) into (B.33), and matching coefficients, yields:

$$A_t = \left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \right)(1-\beta+\beta A_{t+1}) \tag{B.34}$$

$$B_t = \left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \right)(1-\beta+\beta \tilde{A}_{t+1}) \tag{B.35}$$

$$\begin{aligned} K_t = &\left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \right)\left(\beta K_{t+1}+\beta\frac{B_{t+1}}{\tilde{A}_t}\left(-\tilde{B}_t(\bar{D}+\mu_0)+\tilde{K}_t\right)\right) \\ &+\left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \right)\left(-\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)+\beta\tilde{K}_{t+1}\right) \\ &+\frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \end{aligned} \tag{B.36}$$

These expressions give recursive equations for the coefficients which determine equilibrium prices at each point in time. To solve for this mapping, we then need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved as the end point. Specifically, the new steady state is given by:

$$A' = \left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}} \right)(1-\beta+\beta A') \tag{B.37}$$

$$B' = \left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}} \right)(1-\beta+\beta \tilde{A}') \tag{B.38}$$

$$K' = \left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}} \right)\left(\beta K'+\beta\frac{B'}{\tilde{A}'}\left(-\tilde{B}'(\bar{D}+\mu_0)+\tilde{K}'\right)\right)$$

$$+\left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\right)\left(-\beta\tilde{B}'(\bar{D}+\mu_0)+\beta\tilde{K}'\right)$$

$$+\frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}} \tag{B.39}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are the coefficients of the mapping PET agents use to extract information from prices in the new steady state, and which we solve for in (B.52), (B.53) and (B.54) in the next section respectively. Moreover, $\mathbb{V}'_I$ and $\mathbb{V}'_U$ are the variances of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\mathbb{V}'_I=\lim_{t\to\infty}\mathbb{V}_{I,t}=(\beta A')^2\sigma_u^2 \tag{B.40}$$

$$\mathbb{V}'_U=\lim_{t\to\infty}\mathbb{V}_{U,t}=(\beta\tilde{A}')^2\sigma_u^2+(1-\beta+\beta\tilde{A}')^2\sigma_u^2 \tag{B.41}$$

Using this steady state as our end point, we can then solve for the true price function which generates the prices agents observe by backward induction.

Step 2: Mapping to Infer Information from Prices. Just as in the baseline model without speculation, PET agents think that in period t informed agents trade on the information they have received so far, $\{u_j\}_{j=1}^t$, $\{s_j\}_{j=1}^t$, and that uninformed agents only trade on their prior beliefs. Therefore, we can guess as before their beliefs about the equilibrium price function takes the following form:

$$P_t=\tilde{A}_t(\tilde{\mathcal{D}}_t+\tilde{\mathcal{W}}_t)+\tilde{B}_t(\bar{D}+\mu_0)-\tilde{K}_t \tag{B.42}$$

where $\tilde{A}_t$, $\tilde{B}_t$ and $\tilde{K}_t$ are time-varying and deterministic coefficients.

To verify that this is the price function which would arise in equilibrium if agents traded on their own private information alone, notice that, given this price function, informed agents' beliefs would take the following form:

$$\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}]=\tilde{\mathbb{E}}_{I,t}[\beta\left(\tilde{A}_{t+1}(\tilde{\mathcal{D}}_{t+1}+\tilde{\mathcal{W}}_{t+1})+\tilde{B}_{t+1}(\bar{D}+\mu_0)-\tilde{K}_{t+1}\right)+(1-\beta)(\tilde{\mathcal{D}}_t+\tilde{\omega})]$$

$$=(1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_t+\tilde{\mathcal{W}}_t)+\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)-\beta\tilde{K}_{t+1} \tag{B.43}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{I,t}[\Pi_{t+1}] =&\tilde{\mathbb{V}}_{I,t}\left[\beta\tilde{A}_{t+1}\tilde{u}_{t+1}+\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)(\tilde{\omega}+\tilde{\epsilon}_{t+1})+(1-\beta)\tilde{\omega}\right]\\ =&\left(\beta\tilde{A}_{t+1}\right)^2\sigma_u^2+\left(\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1}\\ &+\left(1-\beta+\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(t\tau_s+\tau_0)^{-1}=\tilde{\mathbb{V}}_{I,t}\end{aligned} \tag{B.44}$$

where $\tilde{\mathbb{V}}_{I,t}$ is time-varying and deterministic. Turning to PET agents' beliefs of other uninformed agents' beliefs:

$$\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}]=(1-\beta+\beta\tilde{A}_{t+1}+\beta\tilde{B}_{t+1})(\bar{D}+\mu_0)-\beta\tilde{K}_{t+1} \tag{B.45}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{U,t}[\Pi_{t+1}] =&\tilde{\mathbb{V}}_{I,t}\left[\beta\tilde{A}_{t+1}(\tilde{u}_{t+1}+\tilde{u}_t)+\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)(2\tilde{\omega}+\tilde{\epsilon}_t+\tilde{\epsilon}_{t+1})+(1-\beta)(\tilde{u}_t+\tilde{\omega})\right]\\ =&\left(\beta\tilde{A}_{t+1}\right)^2\sigma_u^2+\left(1-\beta+\beta\tilde{A}_{t+1}\right)^2\sigma_u^2+2\left(\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1}\\ &+\left(1-\beta+2\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_0)^{-1}=\tilde{\mathbb{V}}_{U,t}\end{aligned} \tag{B.46}$$

where $\mathbb{V}_{U,t}$ is time-varying and deterministic.[30]

Given these beliefs, the market clearing condition which PET agents think is generating the price that they observe is given by:

$$\phi\left(\frac{\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\tilde{\mathbb{V}}_{I,t}[\Pi_{t+1}]}\right)+(1-\phi)\left(\frac{\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}]-P_t}{\mathcal{A}\tilde{\mathbb{V}}_{U,t}[\Pi_{t+1}]}\right)=Z \tag{B.47}$$

[30] In solving the model we assume that partial equilibrium thinkers believe other uninformed traders think past fundamental shocks simply did not realize - since they did not receive private information about them, they think they did not happen. Our results are robust to alternative assumptions about traders' higher order beliefs. For example, we could just as easily have assumed that PET traders believe that other uninformed traders think no news ever arrives, and having them trade on fixed prior beliefs even following a displacement.

and the resulting market clearing price function is given by:

$$P_t = \left(\frac{\phi \tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \right) \tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] + \left(\frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \right) \tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] - \frac{\mathcal{A}Z\tilde{\mathbb{V}}_{I,t}\tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}} \tag{B.48}$$

Since (B.43), (B.44), (B.45) and (B.46) show that $\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t)$ and $(\bar{D} + \mu_0)$, that $\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}]$ is linear in $(\bar{D} + \mu_0)$ and that $\tilde{\mathbb{V}}_{I,t+1}[\Pi_{t+1}]$ and $\tilde{\mathbb{V}}_{U,t+1}[\Pi_{t+1}]$ are deterministic, we see that given PET agents' beliefs about other agents, the price function which generates the prices they observe does indeed take the form in (B.42). Substituting (B.43), (B.44), (B.45) and (B.46) into (B.48), and matching coefficients yields:

$$\tilde{A}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1}) \tag{B.49}$$

$$\tilde{B}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{B}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1} + \beta \tilde{B}_{t+1}) \tag{B.50}$$

$$\tilde{K}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} - \frac{\mathcal{A}Z}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \tag{B.51}$$

These expressions give recursive equations for the coefficients with determine equilibrium prices at each point in time. Therefore, to solve for this mapping, we need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved. Specifically, uninformed agents think that the new steady state is given by:

$$\tilde{A}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) (1 - \beta + \beta \tilde{A}') \tag{B.52}$$

$$\tilde{B}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}}\right)\beta\tilde{B}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}}\right)(1-\beta+\beta\tilde{A}'+\beta\tilde{B}') \tag{B.53}$$

$$\tilde{K}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}}\right)\beta\tilde{K}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}}\right)\beta\tilde{K} - \frac{\mathcal{A}Z}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \tag{B.54}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are PET agents' beliefs of the coefficients of the price function in the new steady state after the uncertainty associated with the displacement is resolved, and $\tilde{\mathbb{V}}'_I$ and $\tilde{\mathbb{V}}'_U$ are PET agents' beliefs of the variance of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\tilde{V}'_I = \lim_{t\to\infty} \tilde{\mathbb{V}}_{I,t} = (\beta\tilde{A})^2\sigma_u^2 \tag{B.55}$$

$$\tilde{V}'_U = \lim_{t\to\infty} \tilde{\mathbb{V}}_{U,t} = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 + (1-\beta)^2(\tau_0)^{-1} \tag{B.56}$$

Using this steady state as our end point, we can then solve for the mapping uninformed agents use to extract information from prices by backward induction.

Given this mapping, uninformed agents extract the following information from prices:

$$\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1} = \frac{P_{t-1} - \tilde{B}_{t-1}(\bar{D}+\mu_0) + \tilde{K}_{t-1}}{\tilde{A}_{t-1}} \tag{B.57}$$

Or, given their information set in period t, they extract the following *new information* from the unexpected price change they observe in period $t-1$:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \frac{1}{\tilde{A}_{t-1}}(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]) \tag{B.58}$$

where $\tilde{w}_{t-1} = \tilde{\mathcal{W}}_{t-1} - \tilde{\mathcal{W}}_{t-2}$. This verifies our claim in the text that PET agents extrapolate unexpected price changes even when we allow for speculative motives.

Step 3: Solving the Model Recursively. We solve for the normal times steady state before the displacement is announced by solving the system of equations in (B.52), (B.53), (B.54) and (B.37), (B.38), (B.39), using the following normal times variances:

$$\tilde{\mathbb{V}}_I = (\beta\tilde{A})^2\sigma_u^2 \tag{B.59}$$

$$\tilde{\mathbb{V}}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.60}$$

$$\mathbb{V}_I = (\beta A)^2\sigma_u^2 \tag{B.61}$$

$$\mathbb{V}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.62}$$

Starting from the normal times steady state, we can then simulate the equilibrium path of our economy forward for a given set of signals.

B.4.2 "PET−unaware" Speculation - Mistakenly Rational

If informed agents are not omniscient, and instead mistakenly believe that the world is rational, and that uninformed agents are able to recover the correct information form prices, then their posterior beliefs in (B.27) should be replaced by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] = (1-\beta+\beta A_{t+1})(\mathcal{D}_t+\mathcal{W}_t) + \beta B_{t+1}(\mathcal{D}_t+\mathcal{W}_t) - \beta K_{t+1} \tag{B.63}$$

The posterior variance is identical since, as in the "PET−aware" case, Informed agents are certain about the beliefs that Uninformed agents will have next period.

Following the same steps as in Section B.4.1 above, it follows that the equilibrium price becomes:

$$P_t = A_t(\mathcal{D}_t+\mathcal{W}_t) + B_t(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1}) - K_t \tag{B.64}$$

where:

$$A_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta A_{t+1}+\beta B_{t+1}) \tag{B.65}$$

$$B_t = \left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta\tilde{A}_{t+1}) \tag{B.66}$$

$$K_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}} + \frac{1-\phi}{\mathbb{V}_{U,t}}} \right) \beta K_{t+1} + \left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}} + \frac{1-\phi}{\mathbb{V}_{U,t}}} \right) \left(-\beta \tilde{B}_{t+1}(\bar{D} + \mu_0) + \tilde{K}_{t+1} \right) + \frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}} + \frac{1-\phi}{\mathbb{V}_{U,t}}} \tag{B.67}$$

Since the mapping used by PET agents to extract information from prices is unchanged relative to the one in Section B.4.1, we can use this alternative price function to simulate the path of equilibrium prices and beliefs by following the same steps as in Section B.4.1. The results of these simulation for prices, beliefs, trading volume and asset demand are presented in Figure 7.

C Additional Results

C.1 Negative Bubbles

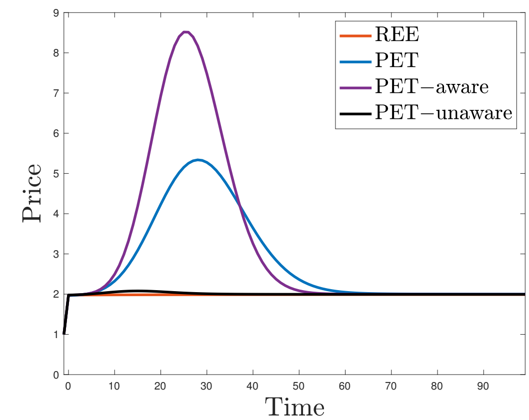
Negative bubbles, defined as episodes of substantial under-valuation, are far less common than positive bubbles (Barberis 2018). To achieve this asymmetry between positive and negative bubbles, models of bubbles generally rely on short-sale constraints (Scheinkman and Xiong 2003, Harrison and Kreps 1978): when the asset becomes too under-valued, over-pessimistic agents cannot take on extreme short positions, thus limiting the extent to which their beliefs get incorporated into prices and dampening the extent of under-valuation (Daniel et al. 2021).

In our model we abstract from short-sale constraints for simplicity, but negative bubbles are still dampened relative to positive bubbles because of the asymmetry inherent in displacement shocks. Specifically, our notion of displacement leads to changes in agents' expectations of both the mean of the fundamental value of the asset, and of the uncertainty associated with it.

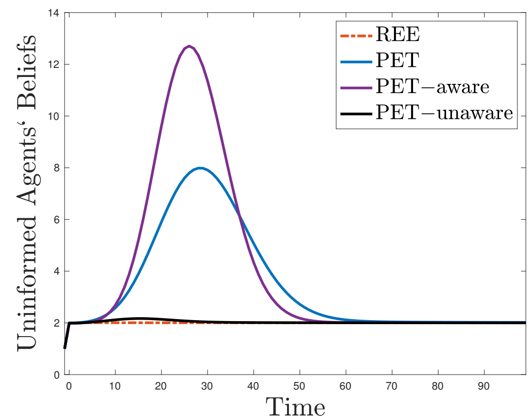
Changes in uncertainty then lead to changes in the risk-premium component, which initially rises and then gradually decreases as traders start learning about the new shock. In turn, the declining risk-premium exerts an upwards force on prices. However, partial equilibrium thinkers *under-estimate this upward force*, because they over-estimate the

Figure 7: Bubbles and crashes with "PET-aware" and "PET-unaware" speculators. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ in each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \; \forall t > 1$. This figure compares the path of equilibrium prices, uninformed agents' beliefs, trading volume and agents' positions in the risky asset under rational expectations, partial equilibrium thinking, "PET-aware" speculation, and "PET-unaware" speculation. "PET-aware" speculation amplifies the bubble relative to the case with no speculative motives, while "PET-unaware" speculation arbitrages the bubble away.

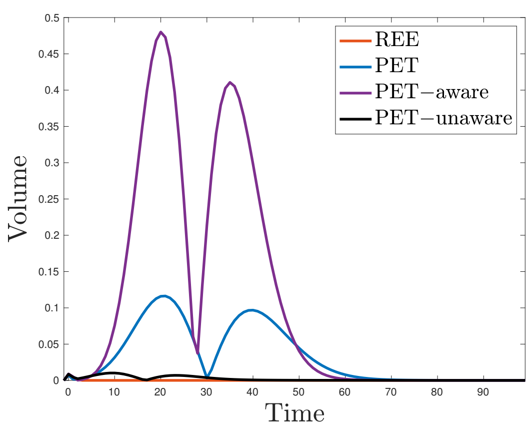
(a) Price



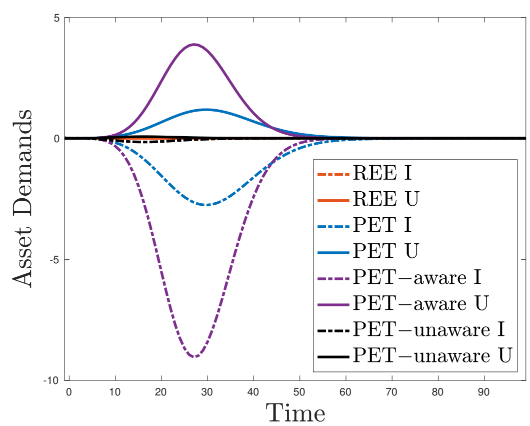
(b) Uninformed agents' beliefs



(c) Trading volume



(d) Asset demands



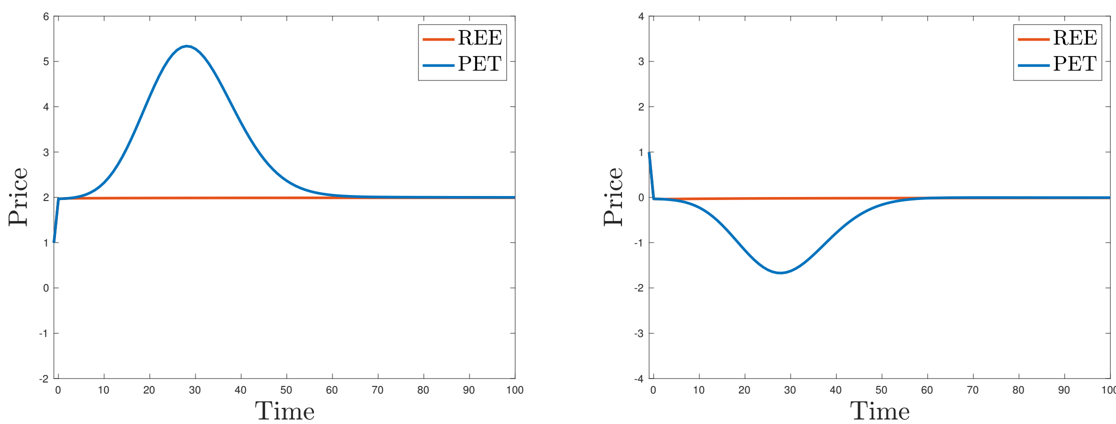
risk-premium component.[31]

The discrepancy between the true and the perceived risk premium is then attributed to good news, regardless of the sign of the shock. This amplifies positive shocks, and in-

[31]Partial equilibrium thinkers think that other uninformed traders are not learning over time. So they effectively think that uninformed traders face greater uncertainty than they do. This greater uncertainty then translates into a higher perceived aggregate risk-premium.

stead dampens or can even reverse the negative cash-flow shock associated with negative bubbles.[32] Figure 8 shows this asymmetry by simulating the impulse response function following a positive and negative cash flow shock of the same absolute value.

Figure 8: Asymmetry between Positive and Negative Bubbles. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim N(0, \tau_s^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. This figure compares the path of equilibrium prices for positive bubbles ($\mu_0 > 0$, left panel) and negative bubbles ($\mu_0 < 0$, right panel). For a given size shock in absolute value, negative bubbles are dampened relative to positive bubbles.



C.2 Slow Boom, and Faster Crash

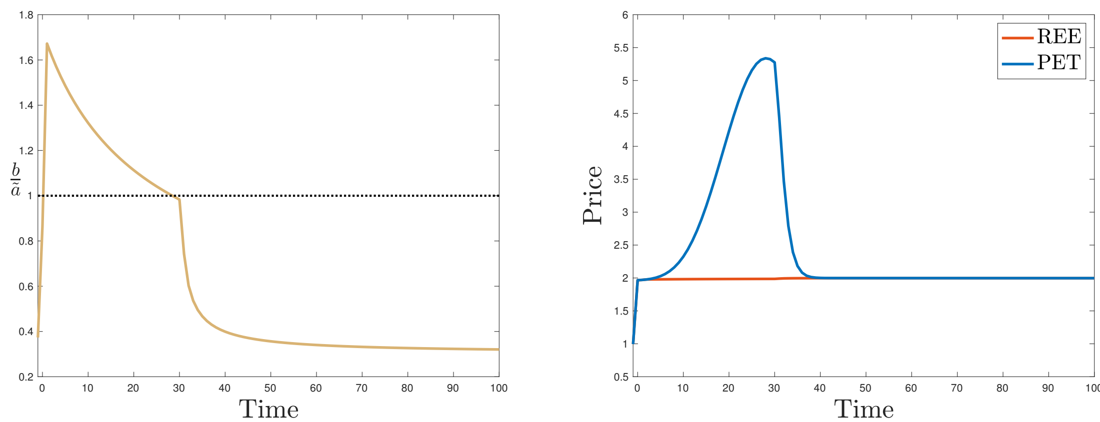
In the baseline model presented in the core of the paper, we assumed that the precision of each incremental piece of news was constant. Here, we check the robustness of our results if we instead assume that signals are very noisy at first, but become more precise after a certain amount of time. Specifically, we simulate a situation where for the first 30 periods, signals are of precision τ_s, and are of precision $\tau_s' > \tau_s$ afterwards. Figure 9 shows how a bubble and a crash still take place, but the crash is accelerated by the increased precision of signals. Intuitively, this is simply because a high τ_s' makes the feedback effect decrease more rapidly with time.

[32] If we mute this risk-premium component, for example by setting the supply of the risky asset to zero, the bubble goes back to being symmetric.

Figure 9: Asymmetric bubbles and crashes. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon \sim N(0, \tau_{s,t}^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \; \forall t > 1$. Moreover, $\tau_{s,t} = \tau_s$ for $t \leq 30$ and $\tau_{s,t} = \tau_s' > \tau_s$ for $t > 30$, which reflects that information is revealed at a faster rate once the bubble bursts. The left panel illustrates the evolution of the strength of the feedback effect. The right panel illustrates the evolution of equilibrium prices, which now exhibit a slower boom and a faster crash.

(a) Strength of the feedback effect

(b) Price



C.3 Misunderstanding the Frequency of Information Arrival

By assuming that informed agents receive new information in each period following a displacement, we are implicitly assuming that uninformed agents understand the frequency with which informed agents receive new information. However, if we change the frequency of information arrival, the true confidence of informed agents becomes decoupled from uninformed agents' perception of it.

In our model, following a displacement, uninformed agents observe a price change in each period, and they attribute each price change to new information. Regardless of the frequency of information arrival, having observed t price changes after t periods, uninformed agents' perception of informed agents' confidence is given by:

$$\tilde{\tau}_{I,t} = \left(\mathbb{V}_{I,0} + (t\tau_s + \tau_0)^{-1}\right)^{-1} \tag{C.1}$$

If informed agents receive news in each period, then $\tilde{\tau}_{I,t} = \tau_{I,t}$. Suppose instead that after t period, informed agents have received only $n_t < t$ signals. Their true confidence is now given by:

$$\tau_{I,t} = \left(\mathbb{V}_{I,0} + (n_t\tau_s + \tau_0)^{-1}\right)^{-1} < \tilde{\tau}_{I,t} \tag{C.2}$$

With this information structure, informed agents need to receive only a finite number of signals for the bubble to burst. Let n_∞ be the total number of signals informed agents receive about the displacement over the whole lifetime of the asset. Long run stability then requires:

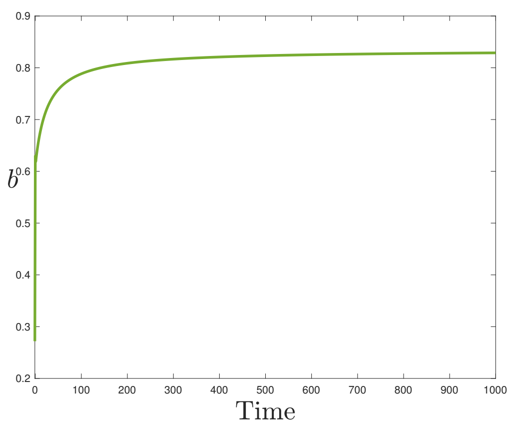
$$n_\infty > \bar{n} \tag{C.3}$$

where $\bar{n} = \frac{1}{\tau_s}\left(\frac{1}{\mathbb{V}_{I,0}(\tilde{\zeta}_\infty\zeta_0 - 1)} - \tau_0\right)$, and $\tilde{\zeta}_\infty = \lim_{t\to\infty}\tilde{\zeta}_t$. This implies that bubbles may burst even if the true confidence of informed agents is lower than the true confidence of uninformed agents. This is not the case with models of constant price extrapolation, which instead rely on changes in the true relative confidence of informed and uninformed agents in order to generate bubbles and crashes.

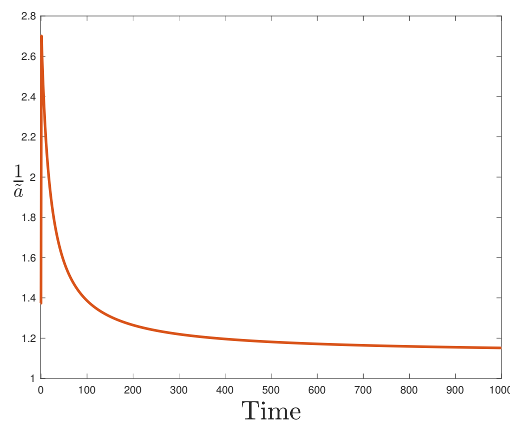
To illustrate this point, Figure 10 shows the response of the economy if informed agents receive a single signal in period $t = 1$, and then receive no further information about the displacement thereafter, so that $n_\infty = 1$. When this is the case, the confidence of uninformed agents rises relative to the confidence of uninformed agents, as shown in the top left panel of Figure 10. However, even though the influence on prices of uninformed agents' biased beliefs rises over time, the economy can still return to a stable region because the strength with which PET agents extrapolate past prices falls over time. Intuitively, PET agents still attribute any price change they observe to additional news about the displacement, and thus think that informed agents' edge is rising over time. Comparing the path of equilibrium prices in the bottom right panel of Figure 10 to the one in Figure 5 we see that when informed agents receive a single shock, the bubble is much more accentuated and takes much longer to die out as the market spends more time in the unstable region. However, the key take-away is that a time-varying extrapolation coefficient allows for bubbles and endogenous crashes that are not driven by changes in agents' relative confidence levels, which would instead be necessary with constant price-extrapolation.

Figure 10: Response of the economy when informed agents receive a single signal in period $t = 1$, and no further information thereafter. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$, and then informed agents receive a *single* signal $s_1 = \omega + \epsilon_1$ with $\epsilon_1 > 0$ and no more signals thereafter. Panels (a) and (b) show how the components of the feedback effect vary over time given this information structure, and Panels (c) and (d) show the evolution of the strength of the feedback effect and of equilibrium prices. Even though b rises over time, the degree of extrapolation still falls after its initial rise, thus allowing the strength of the feedback effect to return to a stable region ($b/\tilde{a} < 1$). Panel (d) shows that the bubble is much more accentuated than the one in Figure 5, as the economy spends longer in the unstable region.

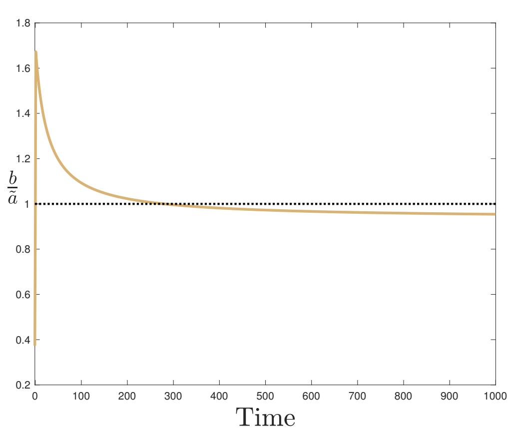
(a) Influence on prices of U agents' beliefs



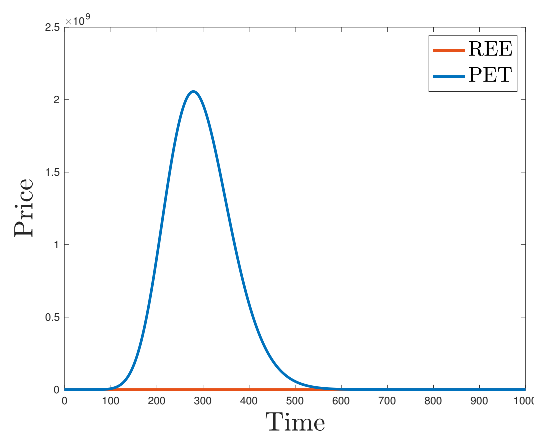
(b) PET degree of extrapolation



(c) Strength of the feedback effect



(d) Price



D Extensions

D.1 Market Orders

In this section we consider the case where uninformed traders submit market orders, so that they do not condition on current prices in computing their demand for the risky asset. When this is the case, uninformed traders effectively end up changing the net supply of the risky asset available to the informed traders. Partial equilibrium thinkers then think that other uninformed traders hold a constant amount in the risky asset and that the net supply available to informed traders is fixed, when in reality it is time-varying as uninformed traders update their demand based on information they learn from past prices.

While the exact functional form of the results changes, the key intuitions and results from the baseline model still go through. Specifically, partial equilibrium thinkers still generate a bias that is decreasing in the perceived informational edge of informed traders, and it still leads to constant price extrapolation in normal times, and time-varying extrapolation following a displacement.

D.1.1 Setup

We maintain the same assumptions about the setup and information structure as in the baseline model. Specifically, in each period t, informed traders receive signals about the terminal dividend, and uninformed traders can learn information from past prices.

The only difference to our baseline model is that we now assume that uninformed traders do not condition on current prices, and instead submit market orders, and submit the following demand for the risky asset (Kyle 1985, Campbell and Kyle 1993, Campbell 2017):

$$X_{U,t} = \frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_{U,t}[D_T]} \tag{D.1}$$

To solve the model we then take similar steps as in the main text. First, we compute the true price function, conditional on traders' posterior beliefs. Second, we compute the price function which uninformed traders think is generating the price changes they observe, and which they use to infer information from prices. Third, we combine these two mappings and consider the properties of equilibrium outcomes.

We first solve the model in normal times, and then add displacement shocks.

D.1.2 Normal Times

In this section we show that, even when uninformed traders submit market orders, in normal times: i) partial equilibrium thinkers still extrapolate recent price changes they observe, ii) the bias is still decreasing in informed traders' informational edge, and iii) stationarity still requires the aggregate confidence of informed traders to be greater than the aggregate confidence of uninformed traders.

Step 1: True Market Clearing Price Function. The market clearing condition which equates the aggregate demand for the risky asset to the fixed supply is given by:

$$\phi\left(\frac{\mathbb{E}_{I,t}[D_T]-P_t}{\mathcal{A}\mathbb{V}_I}\right)+(1-\phi)\left(\frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_U}\right)=Z \tag{D.2}$$

where $\mathbb{V}_I = \mathbb{V}_{I,t}[D_T]$ and $\mathbb{V}_U = \mathbb{V}_{U,t}[D_T]$ are constant and equal to the normal time variances we had in the baseline model in (6) and (8), respectively. Solving for P_t, and using the definition of the aggregate informational edge of informed traders relative to uninformed traders, $\zeta = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_I}\right)$, we find that the true price function, conditional on agents' posterior beliefs, is given by the following expression:

$$P_t = \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\mathbb{E}_{U,t}[D_T] - \frac{\mathcal{A}\mathbb{V}_I}{\phi}Z \tag{D.3}$$

Taking first differences, and using the fact that $\Delta\mathbb{E}_{I,t}[D_T] = u_t$ and $\Delta\mathbb{E}_{U,t}[D_T] = \tilde{u}_{t-1}$, we find that price changes reflect changes in beliefs of both informed and uninformed traders, just as in the baseline model:

$$\Delta P_t = u_t + \frac{1}{\zeta}\tilde{u}_{t-1} \tag{D.4}$$

Step 2: Partial Equilibrium Thinking Mapping. Partial equilibrium thinkers think that other uninformed traders do not learn information from prices, and trade on their unconditional prior beliefs. Since uninformed traders learn information from past prices, we consider the market clearing condition for period $t-1$, as this provides us with an expression for P_{t-1}, the price they are learning from in period t:

$$\phi\left(\frac{\tilde{\mathbb{E}}_{I,t-1}[D_T]-P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_I}\right)+(1-\phi)\left(\frac{\bar{D}}{\mathcal{A}\tilde{\mathbb{V}}_U}\right)=Z \tag{D.5}$$

Solving for P_{t-1}, and using the definition of the perceived informational edge as in our main setup, $\tilde{\zeta}\equiv\left(\frac{\phi}{1-\phi}\right)\left(\frac{\tilde{\mathbb{V}}_U}{\tilde{\mathbb{V}}_I}\right)$, we obtain the following perceived price function:

$$P_t=\tilde{\mathbb{E}}_{I,t-1}[D_T]+\frac{1}{\tilde{\zeta}}\bar{D}-\frac{\mathcal{A}\mathbb{V}_I}{\phi}Z \tag{D.6}$$

Taking first differences, and using the fact that $\Delta\tilde{\mathbb{E}}_{I,t-1}=\tilde{u}_{t-1}$, we see that partial equilibrium thinker still attribute every price change to new information alone, as in the baseline model:

$$\Delta P_t=\tilde{u}_{t-1} \tag{D.7}$$

Partial equilibrium thinkers then trivially invert this mapping to extract the following signal from past price changes they observe:

$$\tilde{u}_{t-1}=\Delta P_{t-1} \tag{D.8}$$

so that they still extrapolate price changes they observe, and the fact that they extrapolate one-to-one simply reflects that informed traders' beliefs are now incorporated into prices one-to-one.[33]

[33] We can compare this to the rational benchmark where uninformed traders understand what generates the price changes they observe, and use the following mapping in their inference:

$$\tilde{u}^{REE}_{t-1}[D_T]=\Delta P_{t-1}-\frac{1}{\zeta}\tilde{u}_{t-2} \tag{D.9}$$

Comparing (D.8) to (D.9), we notice that, just as in the baseline model, the bias inherent in partial equilibrium thinking doesn't come directly from the weight that uninformed traders put on past price

Step 3: Properties of Equilibrium Outcomes. Combining the results in (D.4) and (D.8), we find that changes in prices and in beliefs evolve as follows:

$$\Delta P_t = u_t + \frac{1}{\zeta}\Delta P_{t-1} \tag{D.10}$$

$$\tilde{u}_{t-1} = u_{t-1} + \frac{1}{\zeta}\tilde{u}_{t-2} \tag{D.11}$$

which closely mirrors the expressions in (32) and (33) in the baseline model. Specifically, (D.11) shows that the bias in the signal uninformed traders extract from past prices $\tilde{u}_{t-1} - u_{t-1}$ is still decreasing in informed traders' informational edge, and the AR(1) coefficient in (D.10) and (D.11) shows that in normal times stationarity still requires that the $\zeta < 1$, or that the aggregate confidence of informed traders be greater than the aggregate confidence of uninformed traders, as in the baseline model.

D.1.3 Displacements

In this section, we introduce displacement shocks as in (39), and show that, even when uninformed traders can only submit market orders, i) partial equilibrium thinking still leads to time-varying price extrapolation, and that ii) local stationarity depends on the true informational edge.

Step 1: True Market Clearing Price Function. The market clearing condition which equates the aggregate demand for the risky asset to the fixed supply is now given by:

$$\phi\left(\frac{\mathbb{E}_{I,t}[D_T] - P_t}{\mathcal{A}\mathbb{V}_{I,t}[D_T]}\right) + (1-\phi)\left(\frac{\mathbb{E}_{U,t}[D_T]}{\mathcal{A}\mathbb{V}_{U,t}[D_T]}\right) = Z \tag{D.12}$$

Solving for P_t, and using the the definition of the aggregate informational edge of informed traders relative to uninformed traders: $\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_{U,t}[D_T]}{\mathbb{V}_{I,t}[D_T]}\right)$, we obtain the true

changes (in this case 1), but rather it comes from the part of the price variation they neglect. Specifically, rational uninformed traders do condition on past price changes, but the also have a correction term to account for the fact that part of the price change they observe comes from the lagged response of all other uninformed traders who are also learning information from prices with a lag, as shown in the second term in (D.9), which is instead missing in the PET mapping in (D.8).

price function, conditional on agents' posterior beliefs:

$$P_t = \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\mathbb{E}_{U,t}[D_T] - \frac{\mathcal{A}\mathbb{V}_{I,t}[D_T]}{\phi}Z \tag{D.13}$$

Step 2: Partial Equilibrium Thinking Mapping. Partial equilibrium thinkers think that other uninformed traders do not learn information from prices, and trade on their unconditional prior beliefs. Therefore, they think that P_{t-1} (the price they are learning from in period t) is determined from the following market clearing condition:

$$\phi\left(\frac{\tilde{\mathbb{E}}_{I,t-1}[D_T] - P_{t-1}}{\mathcal{A}\tilde{\mathbb{V}}_{I,t-1}[D_T]}\right) + (1-\phi)\left(\frac{\bar{D} + \mu_0}{\mathcal{A}\tilde{\mathbb{V}}_{U,t-1}[D_T]}\right) = Z \tag{D.14}$$

Solving for P_{t-1}, and using the definition of the perceived informational edge as in our main setup, $\tilde{\zeta}_t \equiv \left(\frac{\phi}{1-\phi}\right)\left(\frac{\tilde{\mathbb{V}}_{U,t}[D_T]}{\tilde{\mathbb{V}}_{I,t}[D_T]}\right)$, we obtain the following perceived price function:

$$P_{t-1} = \tilde{\mathbb{E}}_{I,t-1}[D_T] + \frac{1}{\tilde{\zeta}_{t-1}}\left(\bar{D} + \mu_0\right) - \frac{\mathcal{A}Z}{\phi}\mathbb{V}_{I,t-1} \tag{D.15}$$

where we also define $\mathbb{V}_{i,t-1} \equiv \mathbb{V}_{i,t-1}[D_T]$ for $i \in \{I, U\}$, for ease of notation. Taking first differences, and rearranging, we find that partial equilibrium thinkers still extrapolate unexpected price changes:

$$\Delta\mathbb{E}_{U,t}[D_T] = \Delta P_{t-1} + \left(\frac{\Delta\tilde{\zeta}_{t-1}}{\tilde{\zeta}_{t-1}\tilde{\zeta}_{t-2}}\right)\left(\bar{D} + \mu_0\right) + \frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1} \tag{D.16}$$

Notice that while the degree of price extrapolation is still 1, this is still not the same as constant price extrapolation, since the second and third terms in the above expressions are still time-varying (which wouldn't be the case with constant price extrapolation).[34]

[34] We can once again compare this to the rational benchmark, where uninformed traders take into account that other uninformed traders are also learning information from past prices. In this case, uninformed traders' changes in beliefs would evolve as follows:

$$\Delta\mathbb{E}_{U,t}[D_T] = \Delta P_{t-1} + \left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\mathbb{E}_{U,t-1}[D_T] - \frac{1}{\zeta_{t-2}}\Delta\mathbb{E}_{U,t-1}[D_T] + \frac{\mathcal{A}Z}{\phi}\Delta\mathbb{V}_{I,t-1} \tag{D.17}$$

Step 3: Properties of Equilibrium Outcomes. Whether the price function is in a stationary or non-stationary region now purely depends on the true informational edge:

$$\Delta P_t = \Delta \mathbb{E}_{I,t}[D_T] + \frac{1}{\zeta_t}\Delta \mathbb{E}_{U,t}[D_T] + \Delta\left(\frac{1}{\zeta_t}\right)\mathbb{E}_{U,t-1}[D_T] - \frac{\Delta \mathbb{V}_{I,t}}{\phi}\mathcal{A}Z \tag{D.19}$$

which we can re-write as:

$$\begin{aligned}\Delta P_t = \Delta \mathbb{E}_{I,t}[D_T] &+ \frac{1}{\zeta_t}\Delta P_{t-1} \\ &+ \frac{1}{\zeta_t}\left(\frac{\Delta\tilde{\zeta}_{t-1}}{\tilde{\zeta}_{t-1}\tilde{\zeta}_{t-2}}\right)\left(\bar{D}+\mu_0\right) + \Delta\left(\frac{1}{\zeta_t}\right)\mathbb{E}_{U,t-1}[D_T] \\ &- \frac{\mathcal{A}Z}{\phi}\Delta \mathbb{V}_{I,t} + \frac{1}{\zeta_t}\frac{\mathcal{A}Z}{\phi}\Delta \mathbb{V}_{I,t-1}\end{aligned} \tag{D.20}$$

However, deviations from rationality still depend both on the true and the perceived informational edges. An intuitive way to see this is to express the difference between uninformed traders' beliefs at t and informed traders' beliefs at $t-1$. In the rational benchmark, that difference is simply 0. Instead, when traders think in partial equilibrium, this difference is given by:

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{I,t-1}[D_T] = \frac{\mathbb{E}_{U,t-1}[D_T]}{\zeta_{t-1}} - \frac{\bar{D}+\mu_0}{\tilde{\zeta}_{t-1}} \tag{D.21}$$

which depends both on the true and perceived informational edges, as well as past PET beliefs.

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and we can re-write this expression in a way that highlights the source of price variation that partial equilibrium thinkers neglect:

$$\Delta \mathbb{E}_{U,t}[D_T] = \Delta P_{t-1} + \left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\bar{D} + \underbrace{\left(\frac{\Delta\zeta_{t-1}}{\zeta_{t-1}\zeta_{t-2}}\right)\left(\mathbb{E}_{U,t-1}[D_T] - \left(\bar{D}+\mu_0\right)\right) - \frac{1}{\zeta_{t-2}}\Delta \mathbb{E}_{U,t-1}}_{\text{source of price variation PET traders neglect}} + \frac{\mathcal{A}Z}{\phi}\Delta \mathbb{V}_{I,t-1} \tag{D.18}$$

As in the baseline framework, this bias is time-varying following a displacement.

D.2 Partially Revealing Prices

When prices are fully revealing, the extrapolation parameter used by PET agents is decreasing in informed agents' informational edge. In this section, we study how the extrapolation parameter changes if we allow for noise, so that prices are no longer fully revealing.

D.2.1 Stochastic Supply and Information Structure

To consider the effect of noise on PET agents' inference problem, we assume that the supply of the risky asset is stochastic, and given by $z_t \overset{iid}{\sim} N(Z, \sigma_z^2)$.

To illustrate the effect of noise in the simplest possible way, we assume that agents learn about the realization of the supply of the risky asset after two periods. In each period t, all agents are uncertain about $z_{t-j} \overset{iid}{\sim} N(Z, \sigma_z^2)$ for $j \leq 1$ and they know the realization of z_{t-h} for $h \geq 2$. Even though one period lagged prices are partially revealing, this assumption makes prices fully revealing at further lags, thus simplifying PET agents' inference problem.

D.2.2 Inference Problem with Noise

When prices are fully revealing, uninformed agents think they can extract from prices the exact information that informed agents received in the previous period. This is no longer true when prices are partially revealing. When this is the case uninformed agents can only infer a noisy signal of fundamentals from prices.

Specifically, in normal times, uninformed agents think that prices take the following form:

$$P_{t-1} = \tilde{a}\left(\tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\bar{D} - \tilde{c}z_{t-1} \tag{D.22}$$

where $\tilde{a} = \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$, $\tilde{b} = \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$ and $\tilde{c} = \frac{\mathcal{A}}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$. Since prices are fully revealing in period $t-2$, but they are partially revealing in period $t-1$, uninformed agents extract the following noisy signal from prices:[35]

$$\frac{P_{t-1} - \tilde{a}\tilde{D}_{t-2} - \tilde{b}\bar{D} + \tilde{c}Z}{\tilde{a}} = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{D.23}$$

and we can re-write this more simply as:

$$\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{t-1}[P_{t-1}]\right) = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{D.24}$$

This shows that uninformed agents are now uncertain as to whether the unexpected price change they observe is due to new information, or to changes in the stochastic supply of the risky asset. Either way, PET agents still extrapolate past prices to recover a (noisy) signal from them.

Given the noisy information that uninformed agents extract from prices, their beliefs in period t are given by:

$$\mathbb{E}_{U,t}[D_T] = \tilde{D}_{t-2} + \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2}\right)\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{D.25}$$

$$= \tilde{D}_{t-2} + \frac{\kappa}{\tilde{a}}\left(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]\right) \tag{D.26}$$

where $\kappa = \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2}\right) \leq 1$ is the weight that PET agents put on the noisy signal they extract from past prices. This shows that the extrapolation parameter θ now depends on two components:

$$\theta \equiv \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{1}{\phi\tilde{\tau}_I}\right)^2 \sigma_z^2}\right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right)\frac{\tilde{\tau}_U}{\tilde{\tau}_I}\right)}_{\text{inference}} \tag{D.27}$$

where $(\tilde{\tau}_U)^{-1} = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 = (\tilde{\tau}_I)^{-1} + \sigma_u^2$ and $(\tilde{\tau}_I)^{-1} = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2$. Starting from the second component in (D.27), $1/\tilde{a}$ is the extrapolation parameter that would prevail if

[35] The assumption that prices are fully revealing in period $t-2$ means that uninformed agents think they know the exact value of $\tilde{\mathbb{E}}_{I,t-2}[D_T] = \tilde{D}_{t-2}$, as opposed to being uncertain about it.

$\sigma_z^2 = 0$ and prices were fully revealing: the more sensitive prices are to shocks, the less strongly do PET agents need to extrapolate unexpected price changes to recover the (in their mind unbiased) noisy signal $\tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}(z_{t-1} - Z)$ from prices. Turning to the first component in (D.27), $\kappa \leq 1$ is the weight that PET agents put on the information they extract from prices when forming their posterior beliefs. Whenever $\sigma_z^2 > 0$, $\kappa < 1$, and PET agents extrapolate prices less strongly than when prices are fully revealing, and this simply reflects the noisy nature of the signal they are able to infer from prices.

To draw comparative statics, we can substitute the expressions for $\tilde{\tau}_I$ and $\tilde{\tau}_U$ into (D.27), and re-write the extrapolation parameter in terms of the primitives of the model:

$$\theta = \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{1}{1 + \left(\frac{1}{\phi}\right)^2 \left(\frac{\beta^2}{1-\beta^2}\right)^2 \sigma_u^2 \sigma_z^2} \right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right) \beta^2 \right)}_{\text{inference}} \tag{D.28}$$

From this expression, we see that the extrapolation parameter is decreasing in all sources of noise (σ_u^2 and σ_z^2), as this reduces the informativeness of the signal uninformed agents extract from prices.

On the other hand, increasing the perceived information advantage ($1/\beta^2$) and the fraction of informed agents in the market (ϕ) both have two competing roles. Increasing $1/\beta^2$ (or ϕ) decreases the fully revealing extrapolation parameter $1/\tilde{a}$ as prices are more sensitive to news, but it also increases the weight κ, as prices are a more informative signal. For small enough noise, the first effect dominates, and the extrapolation parameter is decreasing in the informational edge, and in the fraction of informed agents in the market. On the other hand, if there is too much noise in prices, the second effect dominates and the comparative statics are reversed.[36]

[36] Notice that it is a more general property of learning models that the effects of learning are dampened when noise is greater. Therefore, in this section we see that in circumstances where learning is relevant, the comparative statics described in the main text still hold.