

Quine and Bolzano

(1) *Though formulated with reference to language, the above clarification [the definition of logical truth] does not of itself hint that logical truths owe their truth to language. What we have thus far is only a delimitation of the class, per accidens if you please. Afterwards the linguistic doctrine of logical truth, which is an epistemological doctrine, goes on to say that logical truths are true by virtue purely of the intended meanings, or intended usage, of the logical words.*" (1966, 110)

(2) *"[M]y much cited definition of logical truth was meant only as an improved exposition of a long-current idea. So I was not taken aback at Bar-Hillel's finding the idea in Bolzano [...]"* (Quine 1960, 65; see also 1966, 110)

Bolzano's Syntax

- There are simple and complex object-ideas. Simple ideas are treated as unstructured. (Cf. 1837, §56, 243ff.)
- A complex object-idea [A] is typically attributive i.e. of the form [Something which has a] where [which has] is an idea-forming operator (cf. 1837, §60, 259ff.).
- All propositions have the form 'A has b' (Cf. 1837, §127, 9f) If the subject-idea [A] of a proposition [A has b] is complex, its analysis is: [Something which has a, has b]. (Cf. 1837, §§58–59, 251ff.)

Logical constants: Certain ideas (such as [has] and [which has]) connect other types of ideas (such as [human], [creature] and [mortality]) to compose more complex ideas (e.g. [Creatures, which have humanity]) or propositions (e.g. [Humans have mortality])

Logical connectives: To say that Pierre is tall and Marie is hungry is to say that the collection of the two propositions: {[Pierre is tall], [Marie is hungry]} is a collection of truths. (1837, §192, 300, 301). To say that either it is raining or grass is green is to say that the collection of the two propositions: {[It is raining], [Grass is green]} contains at least one truth. (1837, §166, 205) Add to this Bolzano's systematic treatment of sentential negation (1837, §141, 63), conditionality (1837, §164, 199, 200), universal quantification (1837, §147, 77ff), existential quantification (1837, §172, 215) and relational predicates; introduce minimal notational devices and what one gets is a language whose expressive power is at least as significant as that of first order predicate logic.

Logical concepts: Unlike Quine's list of logical words, Bolzano's list of logical concepts is not restricted to logical constants, that is, to "particles" whose role is to determine logical form. In addition to including (his version of the set of) logical constants, Bolzano's list also includes categorial determinations and metalogical concepts. Formal ontological categories such as [something], [collection] and [object], on the one hand, and metalogical notions such as [propositions], [ideas], [objectuality], [analyticity], on the other all "pertain to logic" on Bolzano's account.

Form

(3) *One can distinguish two kinds of properties in objects that are composed of several parts: those such that by stating them one specifies which are the parts of which it is composed without determining the manner of its composition; and those that deal with the latter. But since we usually call the parts of which an object is constituted, taken together, its matter and the manner of their connection its form, one can call a property-idea which only states the parts of an object an idea of its matter, by contrast, one that describes the manner of the connection between these parts an idea of its form.* (1837, §81, 389f)

(4) *[...] we consider the form of propositions and ideas when we keep an eye only on what they have in common with many others, that is, when we speak of entire species or genera of the latter. [...] one calls a species or genus of proposition formal if in order to determine it one only needs to specify certain parts that appear in these ideas or propositions while the rest of the parts which one calls the stuff or matter remain arbitrary.*" (1837 §12, 51)

'X, who is A is B' is a "determinate connection of words or signs" through which the class to which [Caius, who is a man is mortal] belongs can be "represented". (See 1837, §81, 393)

A schematic expression always represents a structural property and the only way to have cognitive access to the property is through the schematic expression. The idea that the form of [Caius, who is a man is mortal] is 'X, who is A is B' implies both that the expression 'X, who is A is B' represents the set of all propositions that have the same structure as [Caius, who is a man is mortal] and that we can generate that set by considering the propositions that are expressed when we arbitrarily substitute the components designated by the schematic letters.

Analyticity

(5) *If however there is even only one single idea in a proposition that may be exchanged arbitrarily without altering its*
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truth or falsity; i.e. if all propositions which turn up through the exchange of this idea with any other are all true or all false provided only that they have objectuality, then this property of the proposition is remarkable enough to be distinguished from all others for which this is not the case. (1837, §148, 83)

Take: [Caius, who is a bachelor, is unmarried]. This proposition cannot be Bolzano-analytic (with respect to [Caius]) unless we reject from the range of acceptable substitution instances, variants thereof such as: [Triangle, who is a bachelor, is unmarried] whose subject-idea [triangle, who is a bachelor] is empty and therefore make the proposition vacuously false.

To say that: 'Caius, who is a man, is mortal' is analytic with respect to 'Caius' and to say that all interpretations of: 'X, who is a man, is mortal' are true amounts to the same. But if this is the case, 'is analytic with respect to ...' is, like the universal quantifier of first order predicate logic, an operator that binds a variable to express generality.

Logical Analyticity

[Caius, who is a man is mortal] vs [Caius, who is a man is a man]

(6) In order to appraise the analytic nature of the [latter kind of] propositions no other than logical knowledge is necessary, since the concepts which form the invariable part of these propositions all belong to logic. On the other hand, for the appraisal of the truth and falsity of propositions like [the first one] a wholly different kind of knowledge is required, since concepts alien to logic intrude. This distinction, I admit, is rather unstable, as the whole domain of concepts belonging to logic is not circumscribed to the extent that controversies could not arise at times. Nevertheless, it might be profitable to keep this distinction in mind. Hence propositions like those [...] may be called logically analytic, or analytic in the narrower sense. (Bolzano 1837 §148, 84)

(i) In logically analytic propositions, all non logical concepts are considered to be arbitrarily variable, that is, only logical concepts occur in them "essentially".

(ii) We can know that logically analytic propositions are analytic by virtue of mere "logical knowledge".

(iii) We can know that logically analytic propositions are true or false on the basis of logical cognitions alone since they contain only logical concepts essentially.

(iv) The distinction between logically analytic and analytic proposition rests on the distinction between logical and non-logical components.

On Bolzano's account, only logical concepts occur essentially in both: (1) [Caius who is a man, is a man] and (2) [[Something] is objectual]. But only (1) is logically analytic. (2) is not.

(7) In my opinion not even one principle in logic, or in any other science, should be a merely analytic truth. For I look upon merely analytic propositions as much too unimportant to be laid down in any science as proper theorems of it. Who would want to fill up geometry, for example, with propositions like: an equilateral triangle is a triangle, or is an equilateral figure, etc.? (1837, §12, 51, 52)

(8) [...] we find judgments of this sort not only in mathematics, in the pure natural sciences and in metaphysics, as Kant proves it incontestably, but they are also to be found in logic, namely not merely among the theorems that belong to this discipline if we understand it, with Bolzano, according to a wider concept, but in the very part of it which one calls analytic and which has been worked on since Aristotle. (Příhonský 1850, 42, 43)

Epistemic Necessity

(9) If a given proposition consists of mere concepts, such as, for instance, the proposition that virtue deserves respect [...]; then the truth or falsity of the latter depends only on the properties of these concepts; and, at least in many cases, nothing else will be required in order to convince yourself of its truth that you examine attentively the concepts themselves of which it is composed. Thus, it will be possible for you to recognise the truth that virtue deserves respect from the mere fact that you have the concepts virtue, to deserve and respect. (Bolzano 1837, §42, 180, 181)

The property b can be inferred from [A] iff (i) [A has b] is true; and (ii) [A] is a pure concept (i.e. does not contain an intuition)

Note: In a conceptual order, if [A has b] is a proposition, then both (i) and (ii) are systematically fulfilled and b can be inferred from [A]. Since [A has b] will be either an axiom or deducible from an axiom then in order to know a concept [A], i.e. in order to know which properties I can infer from [A] I need to know the relevant part of the deductive order in which [A] is embedded and be in a position to draw the relevant consequences.