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USING LOTTERIES TO FINANCE PUBLIC GOODS: THEORY AND EXPERIMENTAL EVIDENCE*

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This study explores the economics of charitable fund-raising. We begin by developing theory that examines the optimal lottery design while explicitly relaxing both risk-neutrality and preference homogeneity assumptions. We test our theory using a battery of experimental treatments and find that our theoretical predictions are largely confirmed. Specifically, we find that single- and multiple-prize lotteries dominate the voluntary contribution mechanism both in total dollars raised and the number of contributors attracted. Moreover, we find that the optimal fund-raising mechanism depends critically on the risk postures of potential contributors and preference heterogeneity.

1. INTRODUCTION

A rich literature has developed in the past several decades that systematically examines the supply side of public goods provisioning. Leaders in the field have grappled with the importance of altruism, fairness, reciprocity, inequity aversion, and the like in explaining the behavior of agents in such environments.[2] Yet as Andreoni (1998) aptly points out, the demand side of charitable fund-raising remains largely unexplored and many critical issues remain unresolved. One line of research that has begun to fill these gaps is the exploration of charitable lotteries as a means to finance public goods. In an important study, Morgan (2000) shows that lotteries obtain higher levels of public goods provision than a voluntary contributions mechanism because the lottery rules introduce additional private benefits from contributing. This serves to reduce the gap between private and social marginal benefits, mitigating the tendency for agents to free ride.[3]

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[2] For general models of reciprocity see Rabin (1993), Dufwenberg and Kirchsteiger (2004), Falk and Fischbacher (2006), and Charness and Rabin (2002); for models of inequity aversion, see Fehr and Schmidt (1999) and Bolton and Ockenfels (2000); and on altruism, see Andreoni and Miller (2002).

[3] In a related line of research, scholars have explored the use of charitable auctions as a means to finance public goods. For example, Goeree et al. (2005) compare the performance of the single-prize lottery with both winner-pays and all-pay auctions.

The goals of this article are to make both theoretical and empirical advances on the demand side of charitable fund-raising by developing and testing theory on the optimal lottery design. An important feature of our theoretical model is that the introduction of a distribution over prizes enables a charitable fund-raiser to exploit individual risk preferences and potential asymmetries in underlying marginal valuations for the public good. Our theory provides several testable predictions: (1) when agents are risk neutral and have symmetric marginal valuations for the public good, contributions in the single-prize lottery are greater than those in an equivalent-valued multiple-prize lottery; (2) when agents have symmetric marginal valuations for the public good, there exists a level of individual risk aversion above which contributions in the multiple-prize lottery are greater than those in an equivalent-valued single-prize lottery; and (3) with sufficient asymmetry in the marginal valuations for the public good, risk-neutral agents contribute more to the public good under the multiple-prize lottery than an identical set of agents contribute under an equivalent-valued single-prize lottery. Furthermore, we find that equilibria exist whereby lotteries induce greater participation rates than the voluntary contribution mechanism (VCM).

We evaluate our theoretical conjectures via a series of experimental treatments that examines the contribution decisions of agents across a number of settings. The first set of treatments compares the outcomes of the VCM, the single-prize lottery, and the multiple-prize lottery for agents who have symmetric valuations for the public good, but who may differ in revealed risk preference. A second set of treatments introduces individual heterogeneities in the marginal valuations for the public good.

We find results that are generally consistent with our theory. First, we find that average contributions and participation rates under both the single- and multiple-prize lotteries are larger than under the VCM. Second, we find that the optimal fund-raising mechanism depends crucially on individual risk posture. For example, an interesting data pattern not anticipated by extant theory is that the decline in contributions to the public good with respect to increases in risk posture is greater in the single-prize lottery than in an equivalently valued multiple-prize lottery. Yet, these tendencies are consistent with our theory of behavior for agents with preferences that exhibit constant absolute risk aversion (CARA). The data also draw attention to the fact that the optimal lottery mechanism relies on the degree of asymmetry across individual marginal valuations for the public good. Finally, we find that total provision of the public good is largest under the single-prize lottery.

We view the contribution of our study as threefold. First, our analysis has implications for practitioners in the design of fund-raising campaigns. Second, our theory and empirical tests highlight avenues for future empirical and theoretical work on charitable giving. Finally, our insights underscore the theoretical properties and viability of a mechanism that can finance public goods without too much government direction.

The remainder of the article is crafted as follows. Section 2 presents a model of investment decisions for individuals with CARA preferences. Section 3 outlines our experimental design. Section 4 describes our results. Section 5 concludes.

2. LOTTERY THEORY

Consider an economy that is populated by agents, $i = 1, \ldots, n$, with quasi-linear utility functions of the form

$$u_i = y_i + h_i(G), \tag{1}$$

where y_i is a numeraire and G represents the provision level of the public good. Each agent is endowed with initial income w, which can be converted into the public good G on a one-for-one basis.[4] We assume $h_i(\cdot)$ to be increasing and concave ($h_i'(\cdot) > 0, h_i''(\cdot) \leq 0$), and make the standard assumption that it is socially desirable to provide a positive amount of the public good, that is, $\sum_i h_i'(0) > 1$. Assuming an interior solution, the socially optimal contribution level for each agent is given by

$$\sum_i h_i'(G^*) = 1. \tag{2}$$

2.1. *Voluntary Contributions.* Consider an organization that relies on voluntary contributions for public good provisioning. Denoting individual contributions by b_i and the total contribution by B, agent i would maximize her utility $w - b_i + h_i(B)$ by choosing the contribution level b_i according to

$$h_i'(B) \leq 1 \tag{3}$$

(with equality if $b_i > 0$), which determines the Nash equilibrium provision level G^N. From concavity of $h_i(\cdot)$, we immediately obtain the following result:

PROPOSITION 1. *With quasi-linear preferences, voluntary contributions underprovide the public good relative to first-best levels:* $G^N < G^*$.

In voluntary settings, agents fail to internalize the benefits conferred on all other agents when investing in the public good. Thus, each agent tends to contribute less than is socially optimal. In the extreme, that is, if $h_i'(0) \leq 1$ for all i, each individual contributes zero to the public good.[5]

2.2. *Lotteries.* To alleviate free riding, a charitable fund-raiser can link contributions to the public good with the chance of winning a prize in a lottery. Generalizing Morgan (2000) to allow for multiple prizes, we consider a lottery

[4] We focus on private provisioning of public goods. Alternatively, there is an interesting line of work that explores dual (public and private) provisioning of public goods (see, e.g., Epple and Romano, 2003).

[5] In our case of quasi-linear utility, all agents who contribute under the VCM have identical marginal valuation of the public good. Similarly, Andreoni (1988) and Fries et al. (1991) show for more general utility functions that for large (replicated) economies only one type of agent contributes. To reverse this standard result of free riding, motivations beyond the consumption value of the public good could be introduced. Sugden (1982, 1984) and Andreoni (1990) are among the many interesting studies that relax the link between utility and the level of public good provided and define utility as a function of both the level of the public good provided and own contributions.

that pays $k \leq n$ prizes, ordered by $P_1 \geq \cdots \geq P_k$. For ease of notation, we define $P_{k+1} = \cdots = P_n = 0$. We assume that the total prize level distributed by the charity is exogenously given by P and is taken out of the contributions. In contrast to Morgan (2000), however, we assume that the lottery will be carried out regardless of whether total contributions (ticket sales) are sufficient to cover the prizes. Thus, if contributions are less than the prize level, the charity runs a deficit.[6] Total provision of the public good is hence given by

$$G(B) = B - P.$$

Consider first the case of symmetric agents with identical valuations of the public good, $h(\cdot) := h_i(\cdot)$. Each player i contributes to the public good by purchasing b_i lottery tickets. The probability player i wins prize s is denoted by π_{si} and depends on contribution levels. We assume that each agent can only win one prize. The expected utility of agent i is therefore given by

$$EU_i = \sum_{s=1}^{n} \pi_{si} \rho(P_s - b_i + h(B - P)),$$

where $\rho(\cdot)$ is a Bernoulli utility function. We assume that players have CARA preferences represented as $\rho(z) = -\exp(-\sigma z)/\sigma$, where σ denotes the level of risk aversion. Therefore,

(4)

$$\begin{aligned} EU_i &= \sum_{s=1}^{n} \pi_{si} \rho(P_s - b_i + h(B - P)) = -\frac{1}{\sigma} \sum_{s=1}^{n} \pi_{si} \exp[-\sigma(P_s - b_i + h(B - P))] \\ &= -\frac{1}{\sigma} \exp[-\sigma(P_n - b_i + h(B - P))] \left[1 + \sum_{s=1}^{n-1} \pi_{si} (\exp[-\sigma(P_s - P_n)] - 1) \right], \end{aligned}$$

where $\pi_{ni} = 1 - \sum_{s=1}^{n-1} \pi_{si}$.

Under the lottery rules, the probabilities of agent i winning prize s depend on the contributions of all agents. Assuming that the other (symmetric) players each contribute b_{-i}, these probabilities are given by

$$\begin{aligned} \pi_{1i} &= \frac{b_i}{b_i + (n-1)b_{-i}} \\ \pi_{2i} &= \frac{(n-1)b_{-i}}{b_i + (n-1)b_{-i}} \frac{b_i}{b_i + (n-2)b_{-i}} \\ &\cdots \\ \pi_{si} &= \frac{(n-1)b_{-i}}{b_i + (n-1)b_{-i}} \frac{(n-2)b_{-i}}{b_i + (n-2)b_{-i}} \cdots \frac{b_i}{b_i + (n-s)b_{-i}}. \end{aligned} \tag{5}$$

[6] Canceling the lottery and returning the money paid for lottery tickets to all contributors is often infeasible for real-world charities (List and Lucking-Reiley, 2002). In addition, a charity might decide to run a loss from a charity event if by doing so it can generate a warm list of potential donors that reduces the solicitation costs in subsequent fund-raising drives.

For example, π_{2i} is the probability that some other agent wins the first prize multiplied by the probability of agent i winning the second prize (conditional on all the first-prize winner's tickets taken out). For a symmetric equilibrium ($b_i = b_{-i} = b$), these probabilities reduce to

$$\pi_{si} = \frac{1}{n} \quad \text{for all } s = 1, \ldots, n. \tag{6}$$

The marginal effect of b_i on the probabilities, π_{si}, is given by

$$\frac{\partial \pi_{si}}{\partial b_i} = \frac{\pi_{si}}{b_i} - \sum_{j=1}^{s} \frac{\pi_{si}}{b_i + (n-j)b_{-i}}, \tag{7}$$

which gives the following representation of the symmetric equilibrium:

$$\frac{\partial \pi_{si}}{\partial b_i} = \frac{1}{nb}\left(1 - \sum_{j=0}^{s-1} \frac{1}{n-j}\right) =: \frac{1}{nb} H(s). \tag{8}$$

Using these preliminaries, we now discuss the equilibrium contribution levels of agents in a multiple-prize lottery. For an interior solution, the first-order condition for agent i who maximizes her expected utility (4) is given by

$$\begin{aligned} 0 &= \sigma(1 - h'(B-P)) \\ &\quad + \sum_{s=1}^{n-1}\left(\sigma(1 - h'(B-P))\frac{1}{n} + \frac{1}{nb}H(s)\right)(\exp[-\sigma(P_s - P_n)] - 1). \end{aligned}$$

Rearranging this equation provides the following expression for the optimal contribution level of an agent in the multiple-prize lottery:[7]

$$B(1 - h'(B-P)) = R := \frac{n}{\sigma} \frac{\sum_{s=1}^{n-1} H(s)(1 - \exp[-\sigma(P_s - P_n)])}{1 + \sum_{s=1}^{n-1} \exp[-\sigma(P_s - P_n)]}. \tag{9}$$

Because the left-hand side of (9) is increasing in B, a symmetric equilibrium in which contributions cover prize payments exists only if for $B = P$ we have $P(1 - h'(0)) < R$.

Interestingly, equilibrium contributions depend solely on prize differences $P_s - P_n$. The intuition is that any arbitrary small contribution guarantees the agent the

[7] Note that the right-hand side R of (9) does not depend on the valuation $h(\cdot)$ of the public good. For all $h(\cdot)$, equilibrium contributions are increasing in R. Thus, the discussion of optimal lottery design is independent of the specifics of the underlying public good.

minimum prize P_n, which is equivalent to a certain payment and does not change the incentives to contribute to the public good. Hence, it is always optimal for the fund-raiser to set the lowest prize value equal to 0 and award fewer prizes than there are agents competing for them.

2.3. *Optimal Lottery—Symmetric Risk-Neutral Players.* For risk-neutral agents, applying l'Hospital's rule for $\sigma \to 0$ to condition (9) leads to the following condition defining an optimal contribution level:

$$B(1 - h'(B - P)) = \sum_{s=1}^{n-1} H(s)(P_s - P_n). \tag{10}$$

Since $H(s)$ (defined in (8)) is decreasing in s and $\sum_t P_t = P$, condition (10) implies that optimally only one prize $P_1 = P > 0$ should be provided.[8] Equilibrium contribution levels exceed total prize payments and are given by

$$B(1 - h'(B - P)) = \frac{n-1}{n} P, \tag{11}$$

as our assumption that the public good is socially desirable, that is, $nh'(0) > 1$, this immediately implies that $P(1 - h'(0)) < \frac{n-1}{n} P$. Similar to Morgan (2000), we are thus able to show the existence of a symmetric equilibrium in which total contributions to the public good are sufficient to cover the prize payments. Because condition (11) can only be satisfied if $h'(B - P) < 1$ and $B > 0$, this lottery leads to a net provision level larger than that obtained by a VCM.

PROPOSITION 2 (Optimal Lottery—Risk-Neutral Players). *If a lottery is used to finance a public good and agents are risk neutral, contributions are maximized by providing only one prize whenever the overall prize budget is fixed. The lottery yields a provision level of the public good in excess of the VCM level G^N.*

2.4. *Charitable Lotteries—Symmetric, Risk-Averse Agents.* Having established that the single-prize lottery is indeed the optimal lottery for symmetric, risk-neutral players, we turn to an examination of relaxing risk neutrality and symmetry assumptions. If agents are risk averse and (9) holds, agents receive less utility from the chance of winning one big prize. Their expected utility can be increased by flattening the payoffs, that is, by splitting the single prize into two or several smaller prizes. Hence, it may be optimal for the fundraiser to provide more than one prize. Indeed, it is possible that the introduction of multiple prizes is necessary to cover the costs of the lottery. For example, this would be the case if

[8] An interesting parallel to the optimality of providing a single prize was pointed out by a referee: Moldovanu and Sela (2001) consider a contest setting (or equivalently an all-pay auction setting) with incomplete information on effort costs. They show that the provision of a single prize maximizes the average effort level with a linear effort–cost relationship, whereas multiple prizes might be optimal for nonlinear effort–costs.

the right-hand side of condition (9) satisfies $P(1 - h'(0)) > R$ for the single-prize lottery, whereas for an appropriately designed multiprize lottery, the inequality is reversed.

Note, however, that for any given distribution of prizes the right-hand side of (9) approaches 0 as risk aversion goes to infinity. Therefore, the net public good provision converges toward that obtained under a VCM, G^N. Yet, in such instances, lottery contributions will always cover the prize payments provided that $h'(0) > 1$. In contrast, aggregate contributions will not cover the prizes if $h'(0) < 1$, and agents are sufficiently risk averse. In this latter case, the fund-raiser will run a deficit and run the lottery.

PROPOSITION 3 (Multiple-Prize Lotteries—Symmetric Risk-Averse Players). *If the total prize budget is fixed at P and the number of (potential) participants is given by n, more than one prize should be provided if*

$$\sigma > \sigma* = \frac{1}{P} \log\left[\frac{n^2 - 2n + 2}{n^2 - 3n + 2}\right].$$

If the level of risk aversion is large enough, the optimal lottery provides n – 1 prizes. Provided that $h'(0) > 1$, *the provision from any lottery converges toward the VCM level* G^N *from above if risk aversion gets infinitely large. The lottery contributions fall short of the prize level if* $h'(0) < 1$ *and agents are sufficiently risk averse.*

PROOF OF PROPOSITION 3. See the Appendix.

Risk aversion therefore provides one impetus for the use of a multiple-prize lottery. As shown in Proposition 3, the critical level of risk aversion necessary to induce a charitable fund-raiser to optimally introduce (at least) a second prize is decreasing in both the prize budget P and the number of participants n. Similar arguments hold for the introduction of up to $n - 1$ prizes. Consequently, for a charity to maximize the provision level of the public good, it should consider the risk posture of potential contributors, particularly when designing high-valued lotteries with a large number of participants. However, it should be noted that charitable lotteries may fail to generate a net increase in public good provision relative to a VCM when potential donors are sufficiently risk averse.

2.5. *Lottery—Asymmetric Risk-Neutral Agents.* Thus far we have assumed that agents have identical preferences for the public good and have thus focused on symmetric equilibria. In this section, we explore behavior among risk-neutral agents who are *asymmetric* in their valuations $h_i(B)$ of the public good B. The following proposition derives conditions under which a fund-raiser should introduce at least a second prize (i.e., under which a shift from $P_1 = P$ and $P_2 = 0$ to $P_1 = P - \varepsilon$ and $P_2 = \varepsilon$ for small $\varepsilon > 0$ leads to increased contributions).

PROPOSITION 4. *For asymmetric risk-neutral agents, the lottery contributions are sufficient to cover the prizes. If under a single-prize lottery a set S of k agents* *contribute a total of B^1 to the public good and their marginal valuations of the public good are given by $h_i'(B^1 - P)$, it is optimal to introduce (at least) a second prize if*

$$(12) \qquad \left[\frac{1}{k}\sum_{i\in S} 1 - h_i'(B^1 - P)\right]\left[\frac{1}{k}\sum_{i\in S}\frac{1}{1 - h_i'(B^1 - P)}\right] > \frac{(k-1)^2}{k(k-2)}(>1).$$

The lottery leads to a provision level in excess of the VCM level G^N.

PROOF OF PROPOSITION 4. See the Appendix.

For symmetric valuations of the public good, the left-hand side of (12) equals 1. Hence, the inequality cannot hold and contribution levels will decrease if one shifts weight to the second prize. If, however, agents are heterogeneous with respect to their marginal valuations of the public good, the left-hand side of the equation is larger than 1 and the introduction of a second prize may increase aggregate contributions.

The intuition for this result is straightforward: compared to high marginal valuation types, contributions to the public good and lottery are expensive for low marginal valuation agents. Therefore, given the contributions of high marginal value types, the low types place only a small probability on winning the prize and may decide not to contribute. If multiple prizes are provided, however, the chances of winning one of these prizes increase, leading to positive contributions. Given a large number of such low-valuation types, the total contributions to the public good may increase.

EXAMPLE 1. Assume two types of agents with linear marginal valuation of the public good: $h_i'(G) = \bar{h}_i G$, $\bar{h}_1 = 0.75$, and $\bar{h}_2 = 0.5$. The number of type 1 agents is given by $n_1 = 2$, the total number is given by $n = n_1 + n_2$. One can easily show that only type 1 agents contribute if one prize is provided: $b_1 = P$ and $b_2 = 0$.

Using Proposition 4, we obtain that the fund-raiser can improve on the performance of the single-prize lottery if

$$\begin{aligned}(0.25n_1 + 0.5n_2)(4n_1 + 2n_2) &> \frac{(n-1)^2 n}{n-2}\\ \Leftrightarrow (n-1)(2+n) &> \frac{(n-1)^2 n}{n-2}\\ \Leftrightarrow n &> 4\end{aligned}$$

By providing two prizes, the fund-raiser provides higher incentives for type 2 agents to contribute and thereby increase the aggregate contribution level if more than two low-type agents exist: $n_2 = n - n_1 > 2$.

Example 1 highlights a potential "double-dividend" of the multiprize lottery: Under certain parameter values, both aggregate contributions and participation rates are increased. Indeed, this represents a general attractiveness of lotteries versus a VCM. Fund-raising strategists around the globe understand the importance of building a "donor development pyramid," which includes as its base first-time donors. The base is commonly understood to be the most difficult, yet most important, stage in building a successful long-term fund-raising effort.

3. EXPERIMENTAL DESIGN

In summary, our theory provides several testable predictions: (1) When agents are risk neutral and have symmetric marginal per capita returns (MPCRs) for the public good, contributions in the single-prize lottery are greater than those in an equivalently valued multiple-prize lottery; (2) when agents have symmetric MPCRs, there exists a level of risk aversion, σ, above which contributions in the multiple-prize lottery are greater than those generated in an equivalently valued single-prize lottery; and (3) there exists a level of individual MPCR heterogeneity for which risk-neutral agents contribute more to the public good under the multiple-prize lottery than the same set of agents contribute under an equivalently valued single-prize lottery.

We design an experiment that follows our theory to examine these conjectures. Table 1 provides a design summary. We begin with the traditional control treatment that induces symmetric MPCRs, denoted VCM-Symmetric. Our other VCM treatment introduces heterogeneous valuations for the public good, but holds constant the average MPCR: denoted VCM-Asymmetric. We cross both of these treatments with comparable single-prize (denoted SPL-Symmetric and SPL-Asymmetric) and multiple-prize treatments (denoted NPL-Symmetric and NPL-Asymmetric), leading to six treatments.

All treatments were conducted at the University of Maryland–College Park. The symmetric treatments consisted of five sessions (one for the VCM and two each for the NPL and SPL) held on separate days with different subjects. The asymmetric treatments consisted of six sessions (two sessions for each treatment) held on separate days with different subjects. Each session consisted of two parts, the first to gather information on contribution levels across the various treatments. The second part was included to gather data on individual risk postures.

3.1. *Part 1.*

The first part of each session was designed to compare contribution levels across the SPL, the NPL, and the VCM. The VCM treatment and the SPL follow the instructions from Morgan and Sefton (2000) to enable direct comparison. Table 1 summarizes the key features of our experimental design and the number of participants in each session. Subjects were recruited on campus using posters and e-mails that advertised subjects could "earn extra cash by participating in an experiment in economic decision-making." The message stated that students would be paid in cash at the end of the session and that sessions generally take less than an hour and a half. The same protocol was used to ensure that each session was run identically.

Each subject was seated at linked computer terminals that were used to transmit all decision and payoff information. The sessions each consisted of 12 rounds, the first 2 being practice. The subjects were instructed that the practice rounds would

TABLE 1

EXPERIMENTAL DESIGN

	Session 1	Session 2
VCM-Symmetric		
MPCR = 0.30	$N = 20$ subjects	
Endowment = 100	10 rounds	
	200 observations	
SPL-Symmetric		
MPCR = 0.30	$N = 20$ subjects	$N = 16$ subjects
Endowment = 100	10 rounds	10 rounds
Prize = 80	200 observations	160 observations
NPL-Symmetric		
MPCR = 0.30	$N = 20$ subjects	$N = 16$ subjects
Endowment = 100	10 rounds	10 rounds
Prizes = {50, 20, 10}	200 observations	160 observations
VCM-Asymmetric		
MPCR = {0.9, 0.1, 0.1, 0.1}	$N = 20$ subjects	$N = 12$ subjects
Endowment = 100	10 rounds	10 rounds
	200 observations	120 observations
SPL-Asymmetric		
MPCR = {0.9, 0.1, 0.1, 0.1}	$N = 20$ subjects	$N = 16$ subjects
Endowment = 100	10 rounds	10 rounds
Prize = 80	200 observations	160 observations
NPL-Asymmetric		
MPCR = {0.9, 0.1, 0.1, 0.1}	$N = 20$ subjects	$N = 16$ subjects
Endowment = 100	10 rounds	10 rounds
Prize = {50, 20, 10}	200 observations	160 observations

NOTES: Cell entries provide the experimental design and parameters for each treatment. For example, in the VCM-Symmetric treatment, the MPCR = 0.30 and the subjects were endowed with 100 tokens. In this treatment, there was one session of 20 subjects that lasted for 10 rounds. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: in the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

not affect earnings. Once the individuals were seated and logged into the terminals, a set of instructions and a record sheet were handed out. The subjects were asked to follow along as the instructions, which are available on request, were read aloud. After the instructions were read and any questions were answered, the first practice round began.

At the beginning of each round, subjects were randomly assigned to groups of four. The subjects were not aware of whom they were grouped with, but they did know that the groups changed every round. Each round the subjects were endowed with 100 tokens. Their task was simple: decide how many tokens to place in the group account and how many to place in their private account. After all subjects made their choice, the computer informed them of the total number of tokens placed in their group account, the number of points from the group account and the private account, as well as any bonus points that were earned. The payoff for the round was determined by summing the points from the group account, points from the private account, and any bonus points received.

The points for each round were determined as follows. For all sessions, subjects received 1 point for each token placed in their private account. In the five sessions with symmetric valuations for the public good, they were awarded 0.3 points for each token placed in the group account by themselves and the other members of their group. In the six sessions with asymmetric valuations for the public good, subjects were awarded either 0.9 or 0.1 points for each token placed in the group account by themselves and the other members of their group.[9] In addition, each session had a different method for earning bonus points.

We follow Morgan and Sefton (2000) by adding the cost of the prize (80 tokens) to the group account in the VCM, which makes the VCM treatment comparable to the lottery treatments. In the symmetric VCM session, all subjects, regardless of their contributions to the group account, therefore earned 24 bonus points; in the asymmetric VCM session, subjects received either 72 or 8 bonus points. These bonus points represent the value of 80 tokens placed in the group accounts.

In the SPL sessions, group members competed for a lottery prize of 80 points. Each subject's chance of winning the prize was based on his or her contribution to the group account compared with the aggregate number of tokens placed in the group account. For the NPL sessions, group members competed for one of three lottery prizes of values 50, 20, and 10 points, respectively. As in the SPL sessions, subjects' chances of winning the first prize were based on his or her share of group contributions. The three prizes were awarded in order of value, and without replacement, meaning that in each round, three of the four group members would receive some bonus points.

At the end of the experiment, one of nonpractice rounds was chosen at random as the round that would determine earnings.[10] Subjects were paid $1.00 for every 15 points earned. They recorded their earnings for Part 1 of the session and added those to their earnings for Part 2 of the experiment to determine total earnings for the session.

3.2. *Part 2.* The second part of the session was designed to lend insights into subjects' risk postures and link those preferences to behavior in the public goods game described earlier. Attempting to measure risk postures in one game and applying them to more closely explore behavior in another is not novel to this study (see, e.g., Eckel and Wilson, 2004). There are issues, however, with such an approach, including whether risk preferences are stable across games, over

[9] In the asymmetric sessions, there was one agent in each group of four who had a valuation for the public account of 0.9 and three agents who had valuations of 0.1 for tokens placed in the group account. Individual valuations were held constant throughout the session, and each group of four had exactly one member with the high valuation and three members with the low valuation.

[10] This practice has become increasingly common in economics experiments. With a fixed budget, this approach permits us to observe a large number of individual decisions over (perhaps) higher payoffs for each decision since only one decision is used for payment. Laury (2005) reports an experiment that tests this approach and finds evidence in favor of its effectiveness (i.e., subjects do not appear to scale-down payoffs to account for the random selection that is made).

time, etc. and whether individual unobservables that influence risk posture are correlated with behavior in the public goods game. Clearly, because risk posture is not exogenously imposed on players (such as MPCRs are induced in the public goods game) an important caveat must be placed on the results from such an exercise.

In this part of the session, the low payoff treatment of Holt and Laury (2002) was replicated with all values multiplied by a factor of 4 (the instructions are available on request).[11] In each of the 11 sessions, this part was conducted in an identical manner. The treatment is based on ten choices between paired lotteries (see the Appendix). The payoff possibilities for Option A, $8.00 or $6.40, are much less variable than those for Option B, $15.40 or $0.40, which was considered the risky option. The odds of winning the higher payoff for each of the options increase with each decision. The paired choices are designed such that a risk-neutral individual should choose Option A for the first four decisions and then switch to Option B for the remaining six decisions. The paired choices are also designed to determine degrees of risk aversion. The implied CARA risk preferences for our experimental parameters are summarized in the Appendix.

On completion of Part 1 of the session, instructions and a decision sheet were handed out. After the directions were read and questions were answered, the subjects were asked to complete their decision sheets by choosing either A or B for each of the 10 decisions. The subjects were instructed that one of the decisions would be randomly selected ex post and used to determine their payoffs. Part of a deck of cards was used to determine payoffs, cards 2–10 and the Ace to represent 1. After each subject completed his or her decision sheet, a monitor would approach the desk and randomly draw a card twice, once to select which of the 10 decisions to use, and a second time to determine what the payoff was for the option chosen, A or B, for the particular decision selected. After the first card was selected, it was placed back in the pile, the deck was reshuffled, and the second card was drawn.

After all the subjects' payoffs were determined, they combined their payoff from Part 1 with that of Part 2 to compute their final earnings. The final payoffs were then verified against the computer records, and subjects were paid privately in cash for their earnings. Each of the sessions took approximately 75 minutes and subjects earned an average of $18.79 for participating.

4. RESULTS

Our experimental design enables us to test a number of theoretical predictions regarding contribution levels across our various treatments. We craft the results summary by first pooling the data across subjects of all risk postures, but later explore the effects of risk preference on contribution schedules. This approach permits a comparison of our results with the voluminous public goods literature, which implicitly assumes risk neutrality, and therefore represents joint hypothesis testing in some cases.

[11] The payoffs for the Holt and Laury experiment were multiplied by a factor of 4 so that the domain of earnings from this experiment ($0.40, $15.40) would correspond with the domain of potential earnings from the public goods game ($1.20, $29.33).

TABLE 2

EXPERIMENTAL RESULT—MEAN CONTRIBUTION LEVELS BY TREATMENT

	Mean Donation		
	All Agents	High-Valuation Agents Only	Low-Valuation Agents Only
VCM-Symmetric	22.845 tokens (17.336 tokens)		
SPL-Symmetric	42.647 tokens (18.531 tokens)		
NPL-Symmetric	32.830 tokens (20.998 tokens)		
VCM-Asymmetric	25.425 tokens (24.269 tokens)	50.85 tokens (32.774 tokens)	16.917 tokens (23.436 tokens)
SPL-Asymmetric	47.228 tokens (27.271 tokens)	74.833 tokens (22.287 tokens)	38.026 tokens (22.284 tokens)
NPL-Asymmetric	40.95 tokens (28.913 tokens)	63.9 tokens (35.707 tokens)	33.3 tokens (22.135 tokens)

NOTES: Cell entries provide the mean and standard deviation for each treatment. For example, in the VCM-Symmetric treatment, the average token contribution was 22.845 with a standard deviation of 17.336 tokens. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

Our first hypothesis concerns whether lotteries introduce a compensating externality that serves to attenuate the tendency for agents to free ride in every treatment. This hypothesis is directly testable using our experimental data and implies that mean contributions in our symmetric (asymmetric) lottery sessions should be greater than mean contributions in our symmetric (asymmetric) VCM sessions.[12] Table 2 provides mean contribution levels for each of our treatments.

As can be seen from Table 2, contribution levels in the two lottery treatments (SPL and NPL) are greater than those in the VCM when marginal valuations are either symmetric or asymmetric: mean contribution levels were 42.7 (32.8) tokens in the symmetric SPL (NPL) treatment and were 47.2 (41.0) tokens for the asymmetric SPL (NPL) treatment, significantly larger than contributions under the VCM: 22.9 (25.4) tokens in the symmetric (asymmetric) treatments.

Statistical support of these results can be found in Table 3, which provides differences in mean contribution levels across treatment. Entries in the table provide the difference between the mean contributions in the column treatment with the corresponding mean contribution level in the row treatment. For example, the entry in row 1 column 1 indicates that mean contribution levels in the symmetric SPL treatment were 19.8 tokens greater than contributions in the symmetric

[12] Of course, under our design our model predicts that VCM contributions should be 0, whereas contributing all tokens would be efficient.

TABLE 3

DIFFERENCE IN MEAN CONTRIBUTION LEVELS

	SPL-Symmetric	NPL-Symmetric	VCM-Asymmetric	SPL-Asymmetric	NPL-Asymmetric
VCM-Symmetric	19.802**	9.986*	2.58	24.383**	18.105**
SPL-Symmetric		−9.817**	−17.222**	4.581	−1.697
NPL-Symmetric			−7.405*	14.397**	8.12*
VCM-Asymmetric				21.803**	15.525**
SPL-Asymmetric					−6.277

**Denotes statistically significant at the $p < 0.05$ level.
*Denotes statistically significant at the $p < 0.10$ level.
NOTES: Cell entries provide the difference in mean contribution levels between the column and row treatments. For example, the difference in mean contributions between the symmetric SPL sessions and the symmetric VCM treatment is 19.802 tokens with this difference being statistically significant at the $p < 0.05$ level using a Mann–Whitney test. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case each player has an MPCR of 0.30; in the asymmetric case the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

VCM treatment, a difference that is statistically significant at the $p < 0.05$ level using the Mann–Whitney test.[13] In the symmetric (asymmetric) NPL treatment, mean contribution levels were 10 tokens (15.5 tokens) greater than those in the symmetric (asymmetric) VCM treatment, and both of these differences are statistically significant at conventional levels. Figure 1 shows that these differences are robust to period, as the treatment effect exists in all 10 periods, but is the largest in magnitude in latter periods, when contribution rates decay in the VCM treatments.

Overall, these data generate our first set of results:

RESULT 1a. Mean contribution levels in the single-prize lottery (SPL) are greater than mean contribution levels in the VCM for both the symmetric and asymmetric MPCR sessions.

RESULT 1b. Mean contribution levels in the multiple-prize lottery (NPL) are greater than mean contribution levels in the VCM for both the symmetric and asymmetric MPCR sessions.

The first part of Result 1a replicates the findings of Morgan and Sefton (2000), who used a higher MPCR of 0.75. The second part of Result 1a and Result 1b are novel to the literature.

[13] In deriving the Mann–Whitney test statistic, we use as the unit of observation the mean contribution levels for each agent in the session. Thus, we are basing the test statistic on a comparison of 36 observations for the symmetric and asymmetric lottery sessions versus 20 (32) observations in the symmetric (asymmetric) VCM. Yet, it might be the case that data within sessions are not statistically independent. To attenuate these concerns, we have run regression models that include controls for individual and session-specific random effects. These results, which are available on request, are consonant with the results presented in the text.

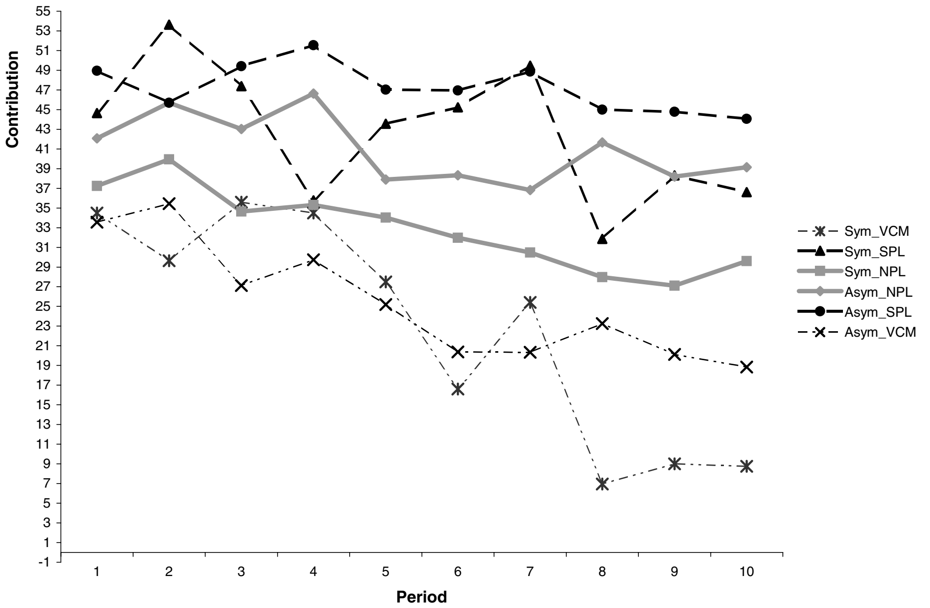

FIGURE 1

MEAN CONTRIBUTIONS PER PERIOD—BY TREATMENT

4.1. *Single- Versus Multiple-Prize Lottery.* Our theory provides a number of testable hypotheses regarding the performance of the SPL versus the NPL conditioned on the underlying distribution of marginal valuations for the public good. For symmetric, risk-neutral agents, contributions in a lottery that provides a single prize should dominate those obtained from an equivalently valued NPL. This insight is contained in Table 4, which provides Nash equilibrium predictions for contribution levels across our four lottery treatments. As rows 1 and 2 of Table 4 illustrate, in our environment, total contribution levels in the symmetric one-prize lottery should exceed those from the symmetric NPL by approximately 21 tokens.

This leads to our second testable hypothesis: contributions in the symmetric SPL should be greater than those in the symmetric NPL if risk neutrality represents a reasonable approximation of the risk posture of our subjects. Table 2 indicates that mean contribution levels for agents in the SPL treatment was 42.7 tokens, whereas mean contribution levels in the NPL treatment was 32.8 tokens.

Further statistical support of this insight is provided in Table 3 and Figure 1. As indicated in the table, the difference in contribution levels between these two treatments of 9.8 tokens is statistically significant at the $p < 0.05$ level using a Mann–Whitney test. This difference is roughly one-half as large as our theoretical prediction, yet if our sample includes agents of varying risk postures this result might follow (we return to this possibility below). These data lead to our next result:

TABLE 4

NASH EQUILIBRIUM PREDICTIONS (RISK NEUTRAL AGENTS)—LOTTERY TREATMENTS

	Total Group Contributions	Individual Donation	High-Valuation Agents	Low-Valuation Agents
SPL-Symmetric	85.7 tokens	21.425 tokens		
NPL-Symmetric	64.4 tokens	16.1 tokens		
SPL-Asymmetric	85.7 tokens		76.5 tokens	3.1 tokens
NPL-Asymmetric	94.06 tokens		68.7 tokens	8.5 tokens

NOTES: Cell entries provide the Nash equilibrium predictions for risk-neutral agents in our four lottery treatments. For example, in the symmetric single-prize lottery (SPL-Symmetric), each agent is predicted to contribute 21.425 tokens to the public account. In the asymmetric single-prize lottery, the high-valuation agent is predicted to contribute 76.5 tokens to the public good and the three low-valuation agents are each predicted to contribute 3.1 tokens. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

RESULT 2. Agents in the symmetric SPL treatments contribute more to the public good than do agents in the symmetric NPL treatment.

We have found evidence suggesting that, in general, agents with symmetric preferences for the public good prefer the SPL. Our theory also provides several testable predictions regarding asymmetries in marginal valuations of the public good. Indeed, when asymmetries are introduced, a series of conjectures results for risk-neutral agents. First, contributions in the symmetric SPL are statistically indistinguishable from contributions in the asymmetric SPL. Second, contributions in the asymmetric NPL are greater than contributions in the asymmetric SPL. Third, contributions in the asymmetric NPL are greater than contributions in the symmetric NPL.

A test of these theoretical conjectures can be carried out by examining data across the symmetric and asymmetric lotteries. Table 4 provides a summary of the Nash equilibrium predictions for contribution levels in all cases. The table provides the basis for the various hypotheses. For example, total contributions in the asymmetric NPL should exceed those from the asymmetric SPL by approximately 8 tokens. This increase is composed of two parts: although the high-valuation agent (MPCR $= 0.9$) decreases contributions by 7.8 tokens, this decrease is offset by an increase in contributions of 16.2 tokens by the three low-valuation agents (MPCR $= 0.1$).

Table 3 provides statistical evidence concerning this set of results. The data show several interesting patterns. For example, we find that average contributions in the symmetric SPL of 42.6 tokens are not significantly different from the 47.2 tokens contributed in the asymmetric SPL. This null result is consonant with our theoretical predictions. However, our finding that contributions in the asymmetric NPL are statistically indistinguishable from contributions in the two SPL treatments (41 tokens vs. 47.2 and 42.6 tokens, respectively) is at odds with our theory. This result is surprising given that in pilot experiments using an MPCR averaging 0.75, we found statistically significant results suggesting that contributions in the NPL dominate contributions in the SPL. Indeed, if we pooled those data with the data herein, statistical significance is achieved as well.[14]

A further insight is that contributions in the asymmetric NPL weakly dominate contributions in the symmetric NPL: Table 3 shows that average contributions in the asymmetric NPL sessions are 8.12 tokens greater than those in the symmetric NPL sessions, with this difference significant at the $p < 0.10$ level. This result is consistent with our theory. In sum, these insights provide three new results:

RESULT 3a. Contributions in the symmetric single-prize lottery are statistically indistinguishable from contributions in the asymmetric single-prize lottery.

RESULT 3b. Contributions in the asymmetric multiple-prize lottery are statistically indistinguishable from contributions in the single-prize lottery.

RESULT 3c. Contributions in the asymmetric multiple-prize lottery weakly dominate contributions in the symmetric multiple-prize lottery.

4.2. *Lottery Incentives and the Tendency to Free Ride.* One theoretical prediction that necessarily falls out of our model is that charitable lotteries attenuate the tendency for strong free riding. Theoretically, charitable lotteries induce positive donations from agents that would not otherwise contribute to the public good/charity under a VCM. Furthermore, our theory predicts that under our parameter values, there is a greater incentive for low-value agents to give in the NPL than in the SPL. Given that most charitable fund-raising guides highlight the value of securing a "warm list" of donors who have previously given, charitable lotteries might provide a double dividend for fund-raisers.

To examine whether the tendency to strongly free ride is attenuated by the lottery incentives, we estimate a random effects probit model. In estimating the model, we make use of the random effects probit specification of Butler and Moffitt (1982),

$$T_{it} = \beta' X_{it} + e_{it} \quad e_{it} \sim N[0, 1],$$

where T_{it} equals unity if agent i donated 0 in period t, and equals 0 otherwise, and X_{it} are model covariates. The vector X_{it} includes treatment dummies and a one-period-lagged value of the total group donations for agent i.

Table 5 provides empirical results for our estimated random effects probit model. Testing the hypothesis that individuals are less likely to strongly free ride in our lottery treatments is equivalent to testing whether the estimated coefficients on the indicators for our four lottery sessions are negative and statistically significant. Results from our model support this hypothesis. As indicated in column 2, the estimated coefficients on all four lottery indicators are negative and statistically significant at the $p < 0.01$ level. These coefficient estimates suggest that conditioned on underlying model covariates, agents in the lottery treatments are less likely to contribute 0 than in an equivalent VCM.

[14] These results, and results from our pilot data, are available on request.

TABLE 5

RANDOM EFFECTS PROBIT OF FREE-RIDING BEHAVIOR

	$T_{it} = 1$ If Agent i Free Rides in Period t
Constant	−0.5258
	(0.273)
NPL-Symmetric	−1.224**
	(0.308)
SPL-Symmetric	−1.345**
	(0.206)
VCM-Asymmetric	−0.747*
	(0.325)
SPL-Asymmetric	−1.318**
	(0.378)
NPL-Asymmetric	−2.289**
	(0.421)
One-period-lagged group donations	−0.002**
	(0.0009)
Total number of observations	1620
Total number of agents	180

**Denotes statistically significant at the $p < 0.01$ level.
*Denotes statistically significant at the $p < 0.05$ level.
NOTES: Cell entries provide parameter estimates from a random effects probit model where $T_{it} = 1$ if agent i contributed 0 to the public good in period t. For example, the negative and significant coefficient on the symmetric N-prize lottery treatment dummy variable suggests that, relative to the symmetric VCM, agents in this treatment are less likely to free ride in any given period. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

Further support for this insight is provided by t-tests comparing the estimated parameter values for the various lottery treatments with the parameter estimate for the VCM treatment. For the symmetric lottery sessions, the estimated coefficients on both the SPL and NPL are smaller than the associated parameter for the VCM (the constant term in the regression) at the $p < 0.05$ level of significance. For the asymmetric lottery treatments, the coefficient estimate of the SPL (NPL) is less than the estimated coefficient on the VCM at the $p < 0.10$ ($p < 0.05$) level of significance.[15] We conclude that

[15] Furthermore, parameter estimates are consistent with our theoretical prediction that strong free-riding incentives are lower in the asymmetric NPL than the asymmetric SPL. Agents in the asymmetric NPL are approximately 1.2% less likely to free ride than in the asymmetric SPL, with this difference significant at the $p < 0.05$ level. Estimated probabilities are evaluated at the mean value for one-period-lagged group donations in the respective SPL (188.8 tokens) and NPL (168.3 tokens) treatments.

RESULT 4. The introduction of a charitable lottery attenuates the tendency of agents to "strongly free ride," that is, increases the number of contributing agents.

4.3. *Lottery Incentives and the Provision of the Public Good.* Thus far we have focused on contributions of individuals in the various treatments. A further prediction of our theory is that lotteries can increase the total provision of the public good. Recall that in the lotteries the exogenous prize amount was 80 tokens. Accordingly, we must account for these prizes by subtracting 20 tokens from each individual's contribution in the lottery treatments. Following Morgan and Sefton (2000), we provide Table 6, which summarizes the total provision of the public good across the various treatments for the final round of play. We also provide results from a Wilcoxon Rank-Sum test on whether the total public good provision is larger in the lottery treatments than the VCM treatments.

The data highlight the power of the lottery mechanism. In all four comparisons, the data indicate that lotteries provide greater levels of the public good than the comparable VCM. Yet, this enthusiasm should be tempered, as the noisiness of the data renders all statistical tests insignificant. These results are in line with those in Morgan and Sefton (2000), although they do find some marginal significance.

TABLE 6
DIFFERENCE IN PUBLIC GOOD PROVISION—FINAL ROUND ONLY

	Net Provision	Multiple-Prize Lottery	Single-Prize Lottery
Symmetric sessions			
VCM	35.0	3.44	31.44
	(35.7)	(0.45)	(0.15)
NPL	38.4		28.0
	(65.5)		(0.20)
SPL	66.4		
	(74.6)		
Asymmetric sessions			
VCM	75.4	1.74	20.96
	(48.9)	(0.47)	(0.23)
NPL	77.1		19.22
	(42.2)		(0.24)
SPL	96.3		
	(66.6)		

NOTES: Cell entries in column 1 provide the average net provision level of the public good in round 10 for each of our experimental treatments. The associated standard deviations are in parentheses. Columns 2 and 3 provide the difference in average provision levels between the associated column and row treatments. The p-value for a one-sided Wilcoxon Rank-Sum test that the column value is greater than the associated row value is given in parentheses. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

TABLE 7

HOLT–LAURY RISK EXPERIMENT—BY TREATMENT AND MPCR

	Not Risk Averse	Risk Averse
VCM-Symmetric	$N = 11$ subjects (11 subjects)	$N = 9$ subjects (9 subjects)
VCM-Asymmetric	$N = 15$ subjects (14 subjects)	$N = 17$ subjects (18 subjects)
SPL-Symmetric	$N = 13$ subjects (11 subjects)	$N = 23$ subjects (25 subjects)
SPL-Asymmetric	$N = 18$ subjects (14 subjects)	$N = 18$ subjects (22 subjects)
NPL-Symmetric	$N = 22$ subjects (23 subjects)	$N = 14$ subjects (13 subjects)
NPL-Asymmetric	$N = 20$ subjects (17 subjects)	$N = 16$ subjects (19 subjects)

NOTES: Entries provide the number of agents who reveal a given risk posture. Agents classified as risk averse select more than five of the "safe" alternative As. Numbers in parentheses recalculate implied risk aversion as the midpoint of any interval around which the subject is revealed indifferent between lottery A and lottery B by switching responses between these alternatives. Cell entries can be read as follows: in the VCM-Symmetric treatment there were nine subjects who are classified as risk averse and nine subjects who are classified as risk averse using the revised procedure. VCM, SPL, and NPL denote voluntary contributions mechanism, single-prize lottery, and multiple-prize lottery. Symmetric and Asymmetric denote induced preferences for the public good: In the symmetric case, each player has an MPCR of 0.30; in the asymmetric case, the MPCRs are 0.9, 0.1, 0.1, and 0.1 for the four players.

4.4. *Risk Aversion and Lottery Contributions.* We can examine our data at a level deeper based on our theoretical predictions and subjects' revealed risk preferences in Part 2 of our experiment. Risk preferences, summarized in Table 7, were assigned on the basis of the observed choices in the Holt and Laury (2002) experimental design.[16] Cell entries provide the number of subjects in each treatment that revealed a given risk posture. For example, 69.45% (25 out of 36) of the subjects in our symmetric SPL treatment were classified as risk averse. In the symmetric NPL, 38.89% (14 out of 36) of the subjects were classified as risk averse.

Our theory provides two testable implications of risk aversion on contributions in our symmetric lottery sessions: (1) In both lottery treatments, contributions should be a decreasing function in risk aversion and (2) contributions should decline more rapidly in the SPL than in the NPL treatments. Table 8 summarizes mean contribution levels (by revealed risk posture) for these two lottery treatments. Perusal of the data presented highlights an important difference in the behavior of risk-averse agents across these two treatments: contributions in the SPL decline in risk aversion, whereas no such decline occurs in the NPL. Although

[16] Agents classified as risk averse select the safe alternative A for the first six or more choice alternatives. The corresponding CARA values for such agents are greater than 0.08 (see the Appendix). Our theory predicts that under this definition of risk aversion, contributions of risk-averse agents should be less than contributions of agents classified as non-risk-averse.

TABLE 8
MEAN CONTRIBUTION LEVELS—SYMMETRIC LOTTERIES (BY RISK PREFERENCE)

	Non-Risk-Averse Agents	Risk-Averse Agents
SPL-Symmetric	48.109 tokens	40.24 tokens
NPL-Symmetric	32.759 tokens	32.943 tokens

NOTES: Entries provide the average contribution levels by revealed risk posture for our symmetric lottery treatments. For example, risk-averse agents in the single-prize lottery contribute on average 40.24 tokens to the public good. Non-risk-averse agents in this treatment contribute an average of 48.104 tokens. Agents are assigned to risk class based on the midpoint of any interval of indifference between Option A and Option B in the Holt and Laury experimental design. There are a total of 25 agents classified as risk averse in the symmetric SPL and 14 agents classified as risk averse in the symmetric NPL sessions. SPL and NPL denote single-prize lottery and multiple-prize lottery. Symmetric denotes that each player has an MPCR of 0.30.

these data patterns are consonant with our theory, the 7.9 token difference in the SPL is not statistically significant at conventional levels.

5. CONCLUSIONS

Numerous mechanisms have been designed to elicit socially optimal levels of public goods contributions. Theoretically, complex taxation/allocation schemes have been designed that solve the free-rider problem. In practice, however, these schemes have generally failed to achieve socially optimal contribution levels or require a degree of coercion that exceeds acceptable levels. Perhaps using this as an impetus, scholars have recently begun to explore the effectiveness of less coercive mechanisms, including auctions and lotteries. Although this literature is nascent, preliminary findings suggest that these mechanisms have an ability to diminish free riding, and contributions levels can, in theory, approach first best.

Our goal in this article is to provide theoretical and empirical evidence on an alternate mechanism for the financing of public goods: the multiple-prize lottery. Although symmetric risk-neutral agents strictly prefer single-prize lotteries, we show that plausible levels of risk aversion and asymmetries in preferences for the public good can generate an optimal lottery that includes more than one prize. We test our theory using a series of laboratory treatments and find evidence in favor of many of our theoretical predictions. Perhaps most importantly, contribution levels under both the multiple-prize and single-prize lottery dominate those of the VCM. Moreover, we find that risk posture and asymmetries in underlying marginal valuations for the public good are critical components determining the optimal lottery. For example, the Golden Rule in fund-raising guides reminds us that generating a warm list of givers is central to any fund-raiser interested in long-term viability. In this spirit, our results suggest that lotteries induce greater levels of participation than VCMs.

Before we can begin to make strong arguments that behavior observed in the laboratory is a good indicator of behavior in the field, we must explore whether similar findings are observed in the field. The next step in our research agenda is to examine our theory outside of the laboratory in a real-world fund-raiser in the spirit of List and Lucking-Reiley (2002). Although field experiments may not be as "clean" as laboratory experiments, where researchers have more control by inducing preferences to accord with theoretical assumptions, and excluding other complicating factors, such an approach has the virtue of resembling natural economic phenomena as closely as possible. In addition, field experiments provide a robustness check of the laboratory results in a natural setting, where the mathematical assumptions of the theory cannot necessarily be guaranteed to hold. Discussion of these results will be reserved for another occasion.

APPENDIX

A.1. *Proofs*

PROOF OF PROPOSITION 3. Consider first, the single-prize lottery. Here, $P_1 = P$ and $P_t = 0$ for all $t > 1$, $H(1) = \frac{n-1}{n}$, and $H(t) = H(1) - \sum_{s=1}^{t-1} \frac{1}{n-s}$. From (9), we know that a lottery prize distribution maximizes contributions if

$$\hat{R} := \frac{\sum_{s=1}^{n-1} H(s)[1 - \exp(-\sigma P_s)]}{1 + \sum_{s=1}^{n-1} \exp(-\sigma P_s)} \tag{A.1}$$

is maximized under the condition $\sum_{s=1}^{n-1} P_s = P(P_s \geq 0)$. From (A.1), we immediately obtain

$$\frac{\partial \hat{R}}{\partial P_t} \bigg/ \sigma = \frac{(H(t) + \hat{R}) \exp(-\sigma P_t)}{1 + \sum_{s=1}^{n-1} \exp(-\sigma P_s)}. \tag{A.2}$$

We will show that it is optimal to have only one prize if and only if $\frac{\partial \hat{R}}{\partial P_1} \geq \frac{\partial \hat{R}}{\partial P_2}$ at $P_1 = P$:

(i) It is obvious that one can improve on the single-prize lottery by introducing a second prize, if $\frac{\partial \hat{R}}{\partial P_1} < \frac{\partial \hat{R}}{\partial P_2}$.

(ii) Assume now that it is optimal to introduce $k > 1$ prizes $P_s^{\text{opt}} > 0$ for all $s = 1, \ldots, k$. First note that for the interior optimum, it is necessary that $\frac{\partial \hat{R}}{\partial P_t} = \frac{\partial \hat{R}}{\partial P_r}$ for all $r, t = 1, \ldots, k$. Then (A.2) implies that $P_1^{\text{opt}} > P_2^{\text{opt}} > \cdots > P_k^{\text{opt}}$ as $H(t)$ is decreasing in t.

Now consider the lotteries $P(\lambda)[\lambda \in (0, 1)]$ given by $P_s(\lambda) = \lambda P_s^{\text{opt}}$ for $s = 2, \ldots, k$ and $P_1(\lambda) = \lambda P_1^{\text{opt}} + (1 - \lambda)P$. Obviously this lottery satisfies the budget constraint $P = \sum_{s=1}^{k} P_s(\lambda)$. Furthermore, we obtain for the derivative

$$\frac{d\hat{R}(P(\lambda))}{d\lambda}\Big/\sigma = \frac{\sum_{s=1}^{k}[H(s)+\hat{R}(P(\lambda))]\exp[-\sigma P_s(\lambda)](P_s(1)-P_s(0))}{1+\sum_{s=1}^{n-1}\exp[-\sigma P_s(\lambda)]}$$

$$\frac{d^2\hat{R}(P(\lambda))}{d\lambda^2}\Big/\sigma = \frac{d\hat{R}(P(\lambda))}{d\lambda}\frac{2\sum_{s=1}^{n-1}\exp[-\sigma P_s(\lambda)](P_s(1)-P_s(0))}{1+\sum_{s=1}^{n-1}\exp[-\sigma P_s(\lambda)]} - \frac{\sum_{s=1}^{k}[H(s)+\hat{R}(P(\lambda))]\exp[-\sigma P_s(\lambda)](P_s(1)-P_s(0))^2}{1+\sum_{s=1}^{n-1}\exp[-\sigma P_s(\lambda)]}.$$

Therefore, if $\frac{d\hat{R}(P(\lambda))}{d\lambda}=0$, we obtain local concavity ($\frac{d^2\hat{R}(P(\lambda))}{d\lambda^2}<0$). That is, no local minimum exists. Since the assumed optimality of $P(1)$ implies $\frac{d\hat{R}(P(1))}{d\lambda}=0$, it follows that $0<\frac{d\hat{R}(P(0))}{d\lambda}$, or equivalently,

$$\begin{aligned}
0 &< \sum_{s=1}^{k}[H(s)+\hat{R}(P(0))]\exp[-\sigma P_s(0)](P_s(1)-P_s(0)) \\
&= [H(1)+\hat{R}(P(0))]\exp(-\sigma P)(P_s(1)-P)+\sum_{s=2}^{k}[H(s)+\hat{R}(P(0))]P_s(1) \\
&< [H(1)+\hat{R}(P(0)]\exp(-\sigma P)(P_s(1)-P)+[H(2)+\hat{R}(P(0))]\sum_{s=2}^{k}P_s(1) \\
&= \{[H(1)+\hat{R}(P(0)]\exp(-\sigma P)-[H(2)+\hat{R}(P(0))]\}(P_s(1)-P).
\end{aligned}$$

Therefore, we obtain $H(1)+\hat{R}(P(0))\exp(-\sigma P) < H(2)+\hat{R}(P(0))$ which is equivalent to $\frac{\partial\hat{R}}{\partial P_1}<\frac{\partial\hat{R}}{\partial P_2}$ at $P_1=P$.

It is therefore optimal to provide only one prize if and only if

$$\begin{aligned}
\exp(-\sigma P) \geq \frac{H(2)+\hat{R}}{H(1)+\hat{R}} &= \frac{H(1)-\frac{1}{n-1}+\frac{H(1)[1-\exp(-\sigma P)]}{n-1+\exp(-\sigma P)}}{H(1)+\frac{H(1)[1-\exp(-\sigma P)]}{n-1+\exp(-\sigma P)}} \\
&= \frac{n-1-\frac{n-1+\exp(-\sigma P)}{n-1}}{n-1}
\end{aligned}$$

$$\Leftrightarrow \exp(-\sigma P)((n-1)^2+1) \geq (n-1)^2-(n-1)$$

$$\Leftrightarrow \exp(-\sigma P) \geq \frac{n^2-3n+2}{n^2-2n+2}$$

$$\Leftrightarrow \sigma \leq \sigma^* = \frac{1}{P}\log\left[\frac{n^2-2n+2}{n^2-3n+2}\right].$$

Note that the critical CARA level σ^* is decreasing in the prize budget P and the number of participants n.

We finally show that for sufficiently large CARA level, the optimal lottery pays $n-1$ prizes. Assume to the contrary that the optimal lottery pays $k<n-1$ prizes, $P_1 \geq \cdots \geq P_k > P_{k+1} = \cdots = P_n = 0$. Then, $\frac{\partial R}{\partial P_t} = \frac{\partial \hat{R}}{\partial P_t}/\sigma$ as defined in (A.2) converges for increasing risk-aversion, $\sigma \to \infty$, to

$$\frac{\partial R}{\partial P_t} \to \begin{cases} 0 & \text{for } t \leq k, \\ \dfrac{H(t)(n-k) + \sum_{s=1}^{k} H(s)}{(n-k)^2} & \text{for } t > k. \end{cases}$$

From the definition of $H(s)$ it follows that

$$\begin{aligned} & H(k+1)(n-k) + \sum_{s=1}^{k} H(s) \\ & = n - \sum_{j=0}^{k} \frac{n-k}{n-j} - \sum_{s=1}^{k} \sum_{j=0}^{s-1} \frac{1}{n-j} = n - \sum_{j=0}^{k} \frac{n-k}{n-j} - \sum_{j=0}^{k-1} \frac{k-j}{n-j} \\ & = n-k-1 \geq 0, \end{aligned}$$

from which we obtain

$$\frac{\partial R}{\partial P_{k+1}} \to \frac{H(k+1)(n-k) + \sum_{s=1}^{k} H(s)}{(n-k)^2} > 0.$$

It is clear that under these conditions, the $k<n-1$ prize lottery is not optimal and (at least) one additional prize should be introduced. Iterating this argument, it follows immediately that a $n-1$ prize lottery is optimal if agents' CARA level exceeds a certain threshold. ■

PROOF OF PROPOSITION 4. Each contributing player maximizes expected utility

$$\mathrm{EU}_i = w - b_i + h_i(B-P) + P_1 \frac{b_i}{B} + P_2 \frac{b_i}{B} \sum_{j \neq i} \frac{b_j}{B - b_j}$$

by choosing contribution b_i according to

(A.3)

$$0 \geq -1 + h_i'(B-P) + P_1 \frac{B-b_i}{B^2} + P_2 \frac{B-b_i}{B^2} \sum_{j \neq i} \frac{b_j}{B-b_j} - P_2 \frac{b_i}{B} \sum_{j \neq i} \frac{b_j}{(B-b_j)^2}$$

with equality if $b_i > 0$, which leads to

$$0 = -k + \sum_{j \in S} h'_j(B-P) + P_1 \frac{k-1}{B} + P_2 \frac{k-2}{B} \sum_j \frac{b_j}{B-b_j} - \frac{P_2}{B}, \tag{A.4}$$

where S is the set and k the number of agents with positive contribution level (or first-order condition holding with equality). To explore the set of contributing agents S, consider the one-prize lottery ($P_1 = P$, $P_2 = 0$). Here, let us first order the agents according to the maximum of contribution levels, B_i, of opponents for which an agent i still would contribute ($0 = -1 + h'_i(B_i - P) + P/B_i$):$B_1 \geq B_2 \geq \cdots \geq B_n$. Now consider $S_k = \{1, \ldots, k\}$ and the resulting the total contribution level $B(S_k)$ determined by (A.4). As $B(S_k)$ is increasing in k and $B(S_1) \leq B_1$, there exists a maximal k for which $B(S_{k^*}) \leq B_{k^*}$. It only remains to show that $B_{k^*+1} \leq B(S_{k^*})$: Assuming the contrary, we would obtain $0 < -(k^*+1) + \sum_{j \in S_{k^*+1}} h'_j(B(S_{k^*})) + P\frac{k^*}{B(S_{k^*})}$ and, hence, we could increase $B(S_{k^*})$ to $B(S_{k^*+1})$, whereas still $B(S_{k^*+1}) \leq B_{k^*+1}$ would hold. This, however, contradicts the maximality assumption on k^*.

Hence, S_{k^*} forms the set of agents who participate in the one-prize lottery in equilibrium. From (A.3) we immediately obtain $1 - h'_1(B-P) > 0$, which implies that the public good provision level exceeds the VCM level G^N.

To see, when contributions increase if one shifts prizes from $P_1 = P$ and $P_2 = 0$ to $P_1 = P - \varepsilon$ and $P_2 = \varepsilon$, implicitly differentiate (A.4) by ε at $\varepsilon = 0$:

$$B'(\varepsilon = 0) = B \frac{-k^* + (k^*-2) \sum_{j \in S_{k^*}} \frac{b_j}{B - b_j}}{(k^*-1)P - B^2 \sum_{j \in S_{k^*}} h''_j(B-P)},$$

where from the first-order condition we have

$$b_i = -(1 - h'_i(B-P)) \frac{B^2}{P} + B,$$

$$B = \frac{k^*-1}{\sum_{i \in S_{k^*}} 1 - h'_i(B-P)} P,$$

$$\frac{b_i}{B-b_i} = -1 + \frac{1}{1 - h'_i(B-P)} \frac{P}{B},$$

$$\sum_{i \in S_{k^*}} \frac{b_i}{B-b_i} = -k^* + \frac{P}{B} \sum_{i \in S_{k^*}} \frac{1}{1 - h'_i(B-P)},$$

and hence,

$$B' > 0 \Leftrightarrow -k^* + (k^*-2) \sum_{i \in S_{k^*}} \frac{b_i}{B-b_i} > 0$$

$$\Leftrightarrow \left[\frac{1}{k^*} \sum_{i \in S_{k^*}} 1 - h'_i(B-P) \right] \left[\frac{1}{k^*} \sum_{i \in S_{k^*}} \frac{1}{1 - h'_i(B-P)} \right] > \frac{(k^*-1)^2}{k^*(k^*-2)} (> 1).$$

■

A.2. *Risk Aversion Decision Sheet and Implied Cara Risk Preference*

Decision Sheet		
OPTION A	OPTION B	DECISION
1/10 of \$8.00, 9/10 of \$6.40	1/10 of \$15.40, 9/10 of \$0.40	
2/10 of \$8.00, 8/10 of \$6.40	2/10 of \$15.40, 8/10 of \$0.40	
3/10 of \$8.00, 7/10 of \$6.40	3/10 of \$15.40, 7/10 of \$0.40	
4/10 of \$8.00, 6/10 of \$6.40	4/10 of \$15.40, 6/10 of \$0.40	
5/10 of \$8.00, 5/10 of \$6.40	5/10 of \$15.40, 5/10 of \$0.40	
6/10 of \$8.00, 4/10 of \$6.40	6/10 of \$15.40, 4/10 of \$0.40	
7/10 of \$8.00, 3/10 of \$6.40	7/10 of \$15.40, 3/10 of \$0.40	
8/10 of \$8.00, 2/10 of \$6.40	8/10 of \$15.40, 2/10 of \$0.40	
9/10 of \$8.00, 1/10 of \$6.40	9/10 of \$15.40, 1/10 of \$0.40	
10/10 of \$8.00, 0/10 of \$6.40	10/10 of \$15.40, 0/10 of \$0.40	

CARA Risk Preference		
Number of Safe Choices	Implied CARA Risk Preference	Classification of Risk Posture
0–3	CARA < 0.00	Not risk averse
4	CARA = 0.00	Not risk averse
5	CARA = 0.03	Not risk averse
6	CARA = 0.08	Risk averse
7	CARA = 0.13	Risk averse
8	CARA = 0.20	Risk averse
9–10	CARA = 0.30	Risk averse

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