Discussion of

Estimating Private Equity Returns from Limited Partner Cash Flows

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Extracting returns from cash flows

- Paper deals with fundamental question: Can we extract a series of "realized returns" from realized cash flows of illiquid investments?
- Wide applicability, if method works:
  - Private equity
  - Real estate
  - Infrastructure investments
- Focus of my discussion (entirely) on basic conceptual questions:
  - How does ACGP's method work?
  - What are the assumptions necessary for it to work?
  - Are there (better?) alternative methods?

Extracting returns from cash flows: Accounting identity

- Accounting identity from definition of (gross) returns R

$$P_0 = \frac{D_1 + P_1}{R_1} = \frac{D_1}{R_1} + \frac{P_2 + D_2}{R_1 R_2} = \frac{D_1}{R_1} + \frac{D_2}{R_1 R_2} + \frac{D_3}{R_1 R_2 R_3} + \dots$$
$$= D_1 w_1 + D_2 w_2 + D_3 w_3 + \dots$$
$$= Dw$$

- Note on terminology
  - R_t are not discount rates, they are realized returns
  - w_t are not discount factors, they are just inverse of compounded realized returns.
- Given w, one can back out the returns $R_1 = w_1^{-1}$, $R_2 = w_2^{-1}/R_1$, ...
- With one cash flow stream over T time periods, there are T unknowns in w, but only one equation

Extracting returns from cash flows: ACGP's method

- ACGP: Identification by using multiple cash-flow streams
- Example: Consider 2 funds, each with initial investment of \$1

$$1 = D_1^a w_1^a + D_2^a w_2^a = D^a w^a$$
$$1 = D_1^b w_1^b + D_2^b w_2^b = D^b w^b$$

- Fund-specific returns undetermined: 2 eqs, 4 unknowns
- ACGP: Among the many solutions, pick the unique one where

$$w^a = w^b = w$$

- Let $P_0 \equiv (1,1)'$ and $D = (D^{a'}, D^{b'})'$. Then,

$$P_0 = Dw$$

- which we can solve for

$$w = D^{-1} P_0$$

and from w we can back out R_1 and R_2.

Interpreting the results from ACGP's method

- What's the interpretation of the R_1 and R_2 backed out under the assumption that $w^a = w^b = w$?
- Let's call R_1 and R_2 quasi-returns.
- Quasi-returns $\neq$ true fund-level realized returns
- Quasi-returns $=$ mean return of the two funds? Typically, not:
  - Mean return typically satisfies neither one of the two fund-level accounting identities w/o error
  - Easy to construct examples where quasi-returns are completely unrelated to true mean return each period

Quasi-returns are typically not equal to mean returns

- Suppose average true value is V_1, V_2 and funds pay out a fraction s of the true value in first period:

$$D = \begin{pmatrix} s(V_1 + x) & (1-s)(V_2 + y) \\ s(V_1 - x) & (1-s)(V_2 - y) \end{pmatrix}$$

- True mean returns

$$\mu_1 = V_1$$
$$\mu_2 = \frac{V_2}{V_1}$$

- Extracted quasi-returns are equal to true mean returns iff

$$\frac{V_2}{V_1} = -\frac{(1-s)}{s}\frac{y}{x}$$

i.e., only for specific realizations of idiosyncratic shocks and payout ratios

Quasi-returns compared with mean returns: Simulation

- Simulate N overlapping series of two-period funds with \$1 initial investment.
  - True log value follows Brownian motion. Shocks independent across funds (adding factor shocks does not change results).
  - Payout of fraction $s = 1/2$ of true value in first period of fund's life, remainder in second period.
- Example ($N = 5$):

$$D = \begin{pmatrix} D_1^a & D_2^a & 0 & 0 & 0 \\ -1 & D_2^b & D_3^b & 0 & 0 \\ 0 & -1 & D_3^c & D_4^c & 0 \\ 0 & 0 & -1 & D_4^d & D_5^d \\ 0 & 0 & -1 & D_4^e & D_5^e \end{pmatrix} \quad P_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- N equations and N unknown quasi-returns: $w = D^{-1} P_0$

Quasi-returns compared with mean returns: Simulation

- Results from 10,000 simulations ($N = 100$)

	quasi-returns	true returns
mean	0.90	1.00
std.dev.	2.13	0.07
corr. w/ true returns	0.18	

- Bottom line: Not clear how to interpret quasi-returns. They can be very different from the true realized returns of the average fund.
- Lots of open questions: Under which assumptions about
  - true value process
  - cash-flow generating payout policy

  does the method work/fail?

Why do quasi returns in ACGP look more sensible?

- ACGP make three additional tweaks to the method that result in plausible (but not necessarily informative) quasi-returns
- #1: Allow for fund-specific error in accounting identity

$$Dw = P_0 + e$$

- #2: A factor model for returns

$$R_t = R_t^f + \beta' F_t + f_t$$

where the PE-specific part f_t has autocorrelation ϕ.
- #3: Imposition of prior information in Bayesian setting, e.g., priors on
  - β
  - ϕ
  - $\mathrm{Var}(e)$.

Parameter posterior means remain close to prior means

- Parameter estimates from ACGP method

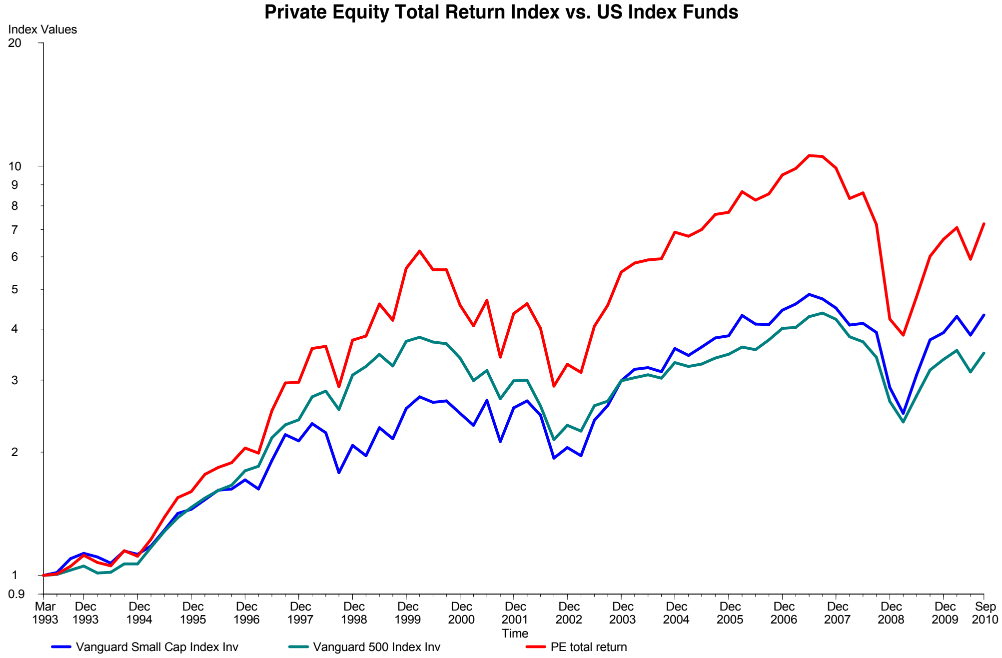
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	In-sample Alpha	Persistence of Alpha
CAPM	1.41[a]				0.05[a]	0.40
	0.24				*0.01*	*0.19*
3 factors (FF)	1.49[a]	0.41	0.09		0.04[a]	0.43
	0.23	*0.31*	*0.27*		*0.01*	*0.19*
4 factors (PS)	1.41[a]	0.41	0.03	0.36	0.00	0.48
	0.21	*0.26*	*0.23*	*0.27*	*0.02*	*0.19*

- Prior means: 1.3 0.55 0.05 0.50 0.50
- Appendix C: increasing prior mean of beta to 1.8 $\Rightarrow$ posterior mean beta rises to 1.65
- Concern: PE cash flows do not appear to provide much incremental information to move parameters away from priors.

Extracted PE Return ≈ Prior beta × Market Return?

Figure 1

Private Equity Total Return Index vs. US Index Funds



Sketch of an alternative method

- Can we find a method that works reasonably well using only PE cash flow data (no prior information on factor loadings, persistence, ...)?
- Observation #1: All information about mean returns should be in total (or average) cash flow each period. In overlapping funds simulation, \$1 invested initially generates cash flows

$$\bar{D} = \begin{pmatrix} D_1^a - 1 & D_2^a + D_2^b - 1 & D_3^b + D_3^c - 1 & \dots \end{pmatrix} \qquad (1)$$

- For this total cash flow series we have

$$\bar{D}w = 1,$$

  i.e., one equation with N unknown elements of w: Many solutions

Sketch of an alternative method

- Observation #2: Cash flows $\bar{D}$ from \$1 investment are akin to a yield and cumulative returns should be positively related to cumulative "yield"
- Let x_t be the cumulative "yield" from 1 to t

$$\begin{aligned} x_1 &= \log(1.5 + \bar{D}_1) \\ x_2 &= x_1 + \log(1 + \bar{D}_2) \\ x_3 &= x_2 + \log(1 + \bar{D}_3) \\ \dots & \quad \dots \end{aligned}$$

- Let y be a vector with elements

$$y_t = \frac{1}{\exp(x_t)}$$

Sketch of an alternative method

- Now let's look for the unique $w = yb$, i.e., that is spanned by y, and solves

$$1 = \bar{D}w$$

- Solution

$$w = \frac{1}{\bar{D}y} y$$

from which we can back out the period-by-period returns.

- Some analogy to projecting stochastic discount factors (SDF) on the payoff space (Hansen and Richard 1987; Hansen and Jagannathan 1991), but here with returns, not SDF

Sketch of an alternative method

- Results from 10,000 simulations

	quasi-returns	alt. method	true returns
mean	0.90	1.01	1.00
std.dev.	2.13	0.16	0.07
corr. w/ true returns	0.18	0.63	

- Still lots of open questions for this method, too: How does performance vary with nature of underlying value process and payout policy?
- But general idea of projection of all possible returns (that solve accounting identity) on subspace spanned by cash-flows seems reasonable.

Conclusion

- Paper attacks a fundamental problem: How to estimate realized period-by-period returns on illiquid investments
- Difficult task: Some very basic conceptual issues are still unresolved
  - Assumptions under which proposed method works/does not work
  - Role of priors
- A whole paper could/should be written just on these basic questions, before even going to applications
- Alternative methods like the one I proposed here may also merit further investigation