Extracting returns from cash flows

- Paper deals with fundamental question: Can we extract a series of “realized returns” from realized cash flows of illiquid investments?
- Wide applicability, if method works:
  - Private equity
  - Real estate
  - Infrastructure investments
- Focus of my discussion (entirely) on basic conceptual questions:
  - How does ACGP’s method work?
  - What are the assumptions necessary for it to work?
  - Are there (better?) alternative methods?
Extracting returns from cash flows: Accounting identity

- Accounting identity from definition of (gross) returns $R$
  \[ P_0 = \frac{D_1 + P_1}{R_1} = \frac{D_1}{R_1} + \frac{P_2 + D_2}{R_1 R_2} = \frac{D_1}{R_1} + \frac{D_2}{R_1 R_2} + \frac{D_3}{R_1 R_2 R_3} + \ldots = D_1 w_1 + D_2 w_2 + D_3 w_3 + \ldots = D w \]

- Note on terminology
  - $R_t$ are not discount rates, they are realized returns
  - $w_t$ are not discount factors, they are just inverse of compounded realized returns.

- Given $w$, one can back out the returns $R_1 = w_1^{-1}$, $R_2 = w_2^{-1}/R_1$, ...

- With one cash flow stream over $T$ time periods, there are $T$ unknowns in $w$, but only one equation

Extracting returns from cash flows: ACGP’s method

- ACGP: Identification by using multiple cash-flow streams
  - Example: Consider 2 funds, each with initial investment of $\$1$
    \[ 1 = D^a_1 w^a_1 + D^a_2 w^a_2 = D^a w^a \]
    \[ 1 = D^b_1 w^b_1 + D^b_2 w^b_2 = D^b w^b \]

- Fund-specific returns undetermined: 2 eqs, 4 unknowns
  - ACGP: Among the many solutions, pick the unique one where $w^a = w^b = w$

- Let $P_0 \equiv (1,1)'$ and $D = (D^a', D^b')'$. Then,
  \[ P_0 = D w \]

- which we can solve for
  \[ w = D^{-1} P_0 \]

  and from $w$ we can back out $R_1$ and $R_2$. 

Interpreting the results from ACGP’s method

- What’s the interpretation of the $R_1$ and $R_2$ backed out under the assumption that $w^a = w^b = w$?
- Let’s call $R_1$ and $R_2$ **quasi-returns**.
- Quasi-returns $\neq$ true fund-level realized returns
- Quasi-returns = mean return of the two funds? Typically, not:
  - Mean return typically satisfies neither one of the two fund-level accounting identities w/o error
  - Easy to construct examples where quasi-returns are completely unrelated to true mean return each period

Quasi-returns are typically not equal to mean returns

- Suppose average true value is $V_1$, $V_2$ and funds pay out a fraction $s$ of the true value in first period:

$$D = \begin{pmatrix}
    s(V_1 + x) & (1 - s)(V_2 + y) \\
    s(V_1 - x) & (1 - s)(V_2 - y)
\end{pmatrix}$$

- True mean returns

$$\mu_1 = V_1$$
$$\mu_2 = \frac{V_2}{V_1}$$

- Extracted quasi-returns are equal to true mean returns iff

$$\frac{V_2}{V_1} = -\frac{(1 - s) y}{s x}$$

i.e., only for specific realizations of idiosyncratic shocks and payout ratios
Quasi-returns compared with mean returns: Simulation

- Simulate $N$ overlapping series of two-period funds with $1$ initial investment.
  - True log value follows Brownian motion. Shocks independent across funds (adding factor shocks does not change results).
  - Payout of fraction $s = 1/2$ of true value in first period of fund’s life, remainder in second period.

- Example ($N = 5$):

  \[
  D = \begin{pmatrix}
  D_1^a & D_2^a & 0 & 0 & 0 \\
  -1 & D_2^b & D_3^b & 0 & 0 \\
  0 & -1 & D_3^c & D_4^c & 0 \\
  0 & 0 & -1 & D_4^d & D_5^d \\
  0 & 0 & -1 & D_4^e & D_5^e \\
  \end{pmatrix}
  \]

  \[
  P_0 = \begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0 \\
  0 \\
  \end{pmatrix}
  \]

- $N$ equations and $N$ unknown quasi-returns: $w = D^{-1}P_0$

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Results from 10,000 simulations ($N = 100$)

<table>
<thead>
<tr>
<th></th>
<th>quasi-returns</th>
<th>true returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>std.dev.</td>
<td>2.13</td>
<td>0.07</td>
</tr>
<tr>
<td>corr. w/ true returns</td>
<td>0.18</td>
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</tr>
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Bottom line: Not clear how to interpret quasi-returns. They can be very different from the true realized returns of the average fund.

Lots of open questions: Under which assumptions about
  - true value process
  - cash-flow generating payout policy
does the method work/fail?
Why do quasi returns in ACGP look more sensible?

- ACGP make three additional tweaks to the method that result in plausible (but not necessarily informative) quasi-returns
- #1: Allow for fund-specific error in accounting identity
  \[ Dw = P_0 + e \]
- #2: A factor model for returns
  \[ R_t = R_f^t + \beta^t F_t + f_t \]
  where the PE-specific part \( f_t \) has autocorrelation \( \phi \).
- #3: Imposition of prior information in Bayesian setting, e.g., priors on
  - \( \beta \)
  - \( \phi \)
  - \( \text{Var}(e) \).

Parameter posterior means remain close to prior means

- Parameter estimates from ACGP method

<table>
<thead>
<tr>
<th>Model</th>
<th>( \beta_{\text{market}} )</th>
<th>( \beta_{\text{size}} )</th>
<th>( \beta_{\text{value}} )</th>
<th>( \beta_{\text{illiquidity}} )</th>
<th>In-sample Alpha</th>
<th>Persistence of Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>1.41(^a) 0.24</td>
<td></td>
<td></td>
<td></td>
<td>0.05(^a) 0.01</td>
<td>0.40 0.19</td>
</tr>
<tr>
<td>3 factors (FF)</td>
<td>1.49(^a) 0.41 0.09</td>
<td>0.23</td>
<td>0.31</td>
<td>0.27</td>
<td>0.04(^a) 0.01</td>
<td>0.43 0.19</td>
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<tr>
<td>4 factors (PS)</td>
<td>1.41(^a) 0.41 0.03 0.36</td>
<td>0.21</td>
<td>0.26</td>
<td>0.23</td>
<td>0.00 0.02</td>
<td>0.48 0.19</td>
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- Prior means: 1.3 0.55 0.05 0.50 0.50
- Appendix C: increasing prior mean of beta to 1.8 \( \Rightarrow \) posterior mean beta rises to 1.65
- Concern: PE cash flows do not appear to provide much incremental information to move parameters away from priors.
Can we find a method that works reasonably well using only PE cash flow data (no prior information on factor loadings, persistence, ...)?

Observation #1: All information about mean returns should be in total (or average) cash flow each period. In overlapping funds simulation, $1 invested initially generates cash flows

$$\bar{D} = (D_1^a - 1 \ D_2^a + D_2^b - 1 \ D_3^b + D_3^c - 1 \ ... ) \quad (1)$$

For this total cash flow series we have

$$\bar{D}_w = 1,$$

i.e., one equation with $N$ unknown elements of $w$: Many solutions
Observation #2: Cash flows $\bar{D}$ from $1$ investment are akin to a yield and cumulative returns should be positively related to cumulative “yield”

Let $x_t$ be the cumulative "yield" from 1 to $t$

\[
x_1 = \log(1.5 + \bar{D}_1) \\
x_2 = x_1 + \log(1 + \bar{D}_2) \\
x_3 = x_2 + \log(1 + \bar{D}_3) \\
... ... \\
\]

Let $y$ be a vector with elements

\[
y_t = \frac{1}{\exp(x_t)} \\
\]

Now let’s look for the unique $w = yb$, i.e., that is spanned by $y$, and solves

\[
1 = \bar{D}w \\
\]

Solution

\[
w = \frac{1}{\bar{D}y}y \\
\]

from which we can back out the period-by-period returns.

Some analogy to projecting stochastic discount factors (SDF) on the payoff space (Hansen and Richard 1987; Hansen and Jagannathan 1991), but here with returns, not SDF
Sketch of an alternative method

- Results from 10,000 simulations

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- Still lots of open questions for this method, too: How does performance vary with nature of underlying value process and payout policy?
- But general idea of projection of all possible returns (that solve accounting identity) on subspace spanned by cash-flows seems reasonable.

Conclusion

- Paper attacks a fundamental problem: How to estimate realized period-by-period returns on illiquid investments
- Difficult task: Some very basic conceptual issues are still unresolved
  - Assumptions under which proposed method works/does not work
  - Role of priors
- A whole paper could/should be written just on these basic questions, before even going to applications
- Alternative methods like the one I proposed here may also merit further investigation