Subjective Beliefs in Asset Pricing

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Subjective beliefs in asset pricing

- Asset prices are forward looking: Expectations about distant future payoffs and their determinants (GDP, Inflation, etc.) play a big role in valuation for many assets
- An outsider to the field of asset pricing might think that the empirical study of actual investors' formation of beliefs about future payoffs, macro variables, risk exposures, would be at the core of asset pricing research
- This is not (yet?) the case — but it should be.

Subjective beliefs in asset pricing

- Consider one-period economy with constant risk-free rate R_F and single risky asset that pays off random X next date.
- Risky asset price P reflects investors' subjective probabilities $\tilde{\pi}$, with expectation $\tilde{\mathbb{E}}[.]$,

$$P = \tilde{\mathbb{E}}\left[\tilde{M}X\right]$$

  where $\tilde{M}$ is subjective SDF reflecting investor preferences.
- Suppose an econometrician studies data from this economy under objective distribution (= empirical distribution)

$$P = \mathbb{E}\left[MX\right]$$

  How do subjective beliefs of investors affect the objective SDF M that econometrician would back out from the data?

Subjective beliefs in asset pricing: Log-normal example

- Let $x = \log X$ and assume that $x \sim N(\mathbb{E}[x], \sigma_x^2)$, which investors perceive as $x \sim N(\tilde{\mathbb{E}}[x], \sigma_x^2)$
- Let $\tilde{m} = \log \tilde{M}$, $r_f = \log R_F$, and suppose investors price asset with a CAPM-style SDF that is log-linear in x.
- Imposing $\log \tilde{\mathbb{E}}[\tilde{M}] = -r_f$, we get

$$\tilde{m} = -r_f - \frac{1}{2}\tilde{\phi}^2\sigma_x^2 - \tilde{\phi}(x - \tilde{\mathbb{E}}[x]),$$

  where $\tilde{\phi}$ is the subjective price of risk.
- Then we can express $p = \log P$ as

$$p = \log \tilde{\mathbb{E}}\left[\tilde{M}X\right] = -r_f + \frac{1}{2}\sigma_x^2 + \tilde{\mathbb{E}}[x] - \tilde{\phi}\sigma_x^2 \qquad (1)$$

Subjective beliefs in asset pricing: Econometrician's view

- Consider this one-period setting repeated many times observed by an outside econometrician with $T \to \infty$: effectively, the econometrician knows the objective probabilities π (= empirical distribution)
- The econometrician wants to fit a CAPM-style SDF

$$m = -r_f - \frac{1}{2}\phi^2\sigma_x^2 - \phi(x - \mathbb{E}[x]),$$

  where $\log \mathbb{E}[M] = -r_f$, and ϕ is the objective price of risk.
- From econometrician's viewpoint

$$p = \log \mathbb{E}\left[MX\right] = -r_f + \frac{1}{2}\sigma_x^2 + \mathbb{E}[x] - \phi\sigma_x^2 \qquad (2)$$

  where $\mathbb{E}[x]$ and σ_x^2 can be estimated with sample moments.

Subjective beliefs in asset pricing: Effect of imposing RE

- Taking differences of subjective and objective pricing equations (1) and (2)

$$\tilde{\mathbb{E}}[x] - \mathbb{E}[x] = (\tilde{\phi} - \phi)\sigma_x^2$$

- So far, just a reduced-form statistical exercise: Econometrician looks for an SDF to describe observed prices
- Conventionally, adding rational expectations (RE) assumption,

$$\mathbb{E}[x] = \text{expectation of investors},$$
$$\phi = \text{price of risk demanded by investors}$$

  brings strong economic content: if correct, assumption allows econometrician to back investors' SDF and preferences
- But wrong economic conclusions if $\tilde{\mathbb{E}}[x] \neq \mathbb{E}[x]$.
  - Example: If $\tilde{\mathbb{E}}[x] < \mathbb{E}[x] \Rightarrow \phi > \tilde{\phi}$ and econometrician would misinterpret pessimism as high price of risk

Subjective beliefs in asset pricing: Effect of imposing RE

- RE implicitly assumes investors know the true model generating X, including parameter values, i.e., much stronger than just "investors are rational"
- RE assumption was an attempt to sidestep the problem of having to specify and study $\tilde{\pi}$ by fixing $\tilde{\pi} = \pi$, but it's becoming increasingly clear that $\tilde{\pi}$ could be the key to understanding asset prices
- Tweaking SDF (here: ϕ) is only game in town for explaining asset prices under RE
  - exception: modifications to stochastic process generating payoffs that have suitable asset price implications, but are hard to reject with conventional sample sizes (e.g., Bansal-Yaron long-run risk model).

Relevance of subjective beliefs for asset pricing

- Motivation for considering models in which $\tilde{\pi} \neq \pi$
  - How could investors possibly know the true DGP parameter values, the true model? Even with rational (Bayesian) learning $\tilde{\pi} \neq \pi$
  - Evidence from survey data on investor return expectations, inflation expectations, professional forecasts, etc. indicates $\tilde{\pi} \neq \pi$
  - Evidence on behavioral biases in updating from observed information
- We therefore need
  - models of subjective belief dynamics
  - microdata evidence on subjective belief dynamics

Payoff expectations and return expectations

- Many surveys collect expectations about future asset returns rather than future payoffs. Log returns in our one-period setting:

$$r = x - p$$

- For purpose of empirical measurement, simple equivalence: Given p,

$$\tilde{\mathbb{E}}r = \tilde{\mathbb{E}}x - p \tag{3}$$

- However, for equilibrium modeling of asset prices we can't think of p as given: expectations about (exogenous) payoffs are different in important ways from expectations about (endogenous) returns.

Payoff expectations and return expectations

- Substituting for p in (3) using subjective pricing equation (1), we obtain

$$\tilde{\mathbb{E}}r - r_f + \frac{1}{2}\sigma_x^2 = \tilde{\phi}\sigma_x^2$$

  i.e., return expectation does not depend on $\tilde{\mathbb{E}}[x]$.
  - In equilibrium, optimistic beliefs about $\mathbb{E}[x]$ generate high prices, not high expected returns, unless subjective price of risk ($\tilde{\phi}$) goes up at the same time
- Important for interpretation of empirical analysis of subjective return expectations data.
  - Example: Hypothesis that a variable z_t drives time-variation in $\tilde{\mathbb{E}}_t[x_{t+1}]$ and asset prices does not imply that we should find $\mathrm{cov}\,(\tilde{\mathbb{E}}_t[r_{t+1}], z_t) > 0$

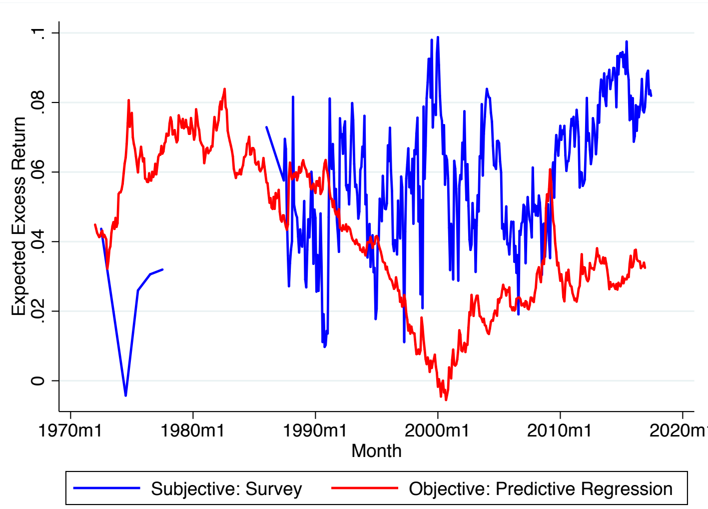
Subjective return expectations: Individual investors

- Direct measures of one-year expected returns in %:
  - UBS/Gallup survey, 1998-2007, monthly
  - Vanguard Research Initiative survey of Vanguard customers Ameriks et al. (2016), one survey in 2014
  - Surveys of Lease et al. (1974) and Lewellen et al. (1977), annual, 1972 and 1973
- Coarse measures: Impute percentage expectations through projection
  - Michigan Survey of Consumers, monthly 2002-2017: prob(stock market up)
  - Conference Board Survey, monthly 1987-2017: likely up/down/same
  - Roper Center Surveys, annual, 1974-1977: likely up/down/same

Objective return expectations

- As econometricians, we don't know the true model of the world. How to specify π to evaluate RE benchmark?
- Within a RE model, we know it: e.g. Campbell and Cochrane (1999): P/D should capture objective time-varying expected returns.
- Empirically, existing evidence that P/D or similar valuation ratios predict returns
- Here: use fitted value ($\times$ 4) in regression of next-quarter CRSP value-weighted index returns returns in excess of T-bill yield on the log P/D ratio as objective expected excess returns.

Disconnect between objective and subjective expected stock market excess returns



Source: Subjective = one-year expected stock market returns in excess of one-year Treasury yield from various individual investor surveys in Nagel and Xu (2018). Objective = Fitted value from predictive regression of stock market excess returns on log price-dividend ratio estimated on quarterly data 1927-2017.

Subjective expectations and bond pricing

- Over the past 30+ years, long-term (LT) Treasury bonds have earned high returns in excess of T-bills
- Reason: multi-decade trend towards lower interest rates from $> 10\%$ in early 1980s to close to zero today.
- To an econometrician working under the RE assumption, this looks like a big risk premium for LT bonds
- But did investors in early 1980s really price in such a high risk premium, expecting to earn a high excess return from LT bonds?
- Or did the extremely persistent downward trend in interest rates surprise them?

Persistent subjective expectations errors: Blue Chip fed funds rate forecasts

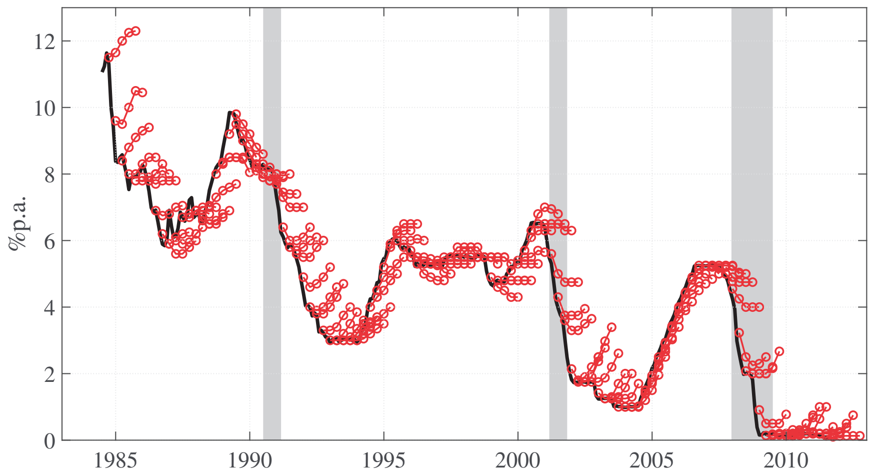


Figure 3
Conditional term-structures of FFR survey forecasts
The figure plots the term structures of FFR forecasts in the BCFF survey. The forecasts are for the current quarter up to four quarters ahead. The plot shows forecasts made in the middle month of each quarter. The shaded areas are NBER-dated recessions.

Cieslak (2018, RFS)

Criticisms of survey expectations as measures of subjective beliefs

Survey expectations $\neq$ objective expectations

- Do RE asset pricing models get something fundamentally wrong?
- Or is the problem that preferences/risk-adjustments distort the expectations reported in surveys?
- Risk-neutral expectations hypothesis (RNE)

If people report the risk-neutral expectation, then many surveys make sence [sic]. — Cochrane (2011)

Risk-neutral expectations hypothesis

- RNE hypothesis asserts that survey respondents report marginal-utility (SDF) weighted expectation of returns

$$\mathcal{E}_{i,t}[R_{t+1}] = \mathbb{E}_{i,t}\left[\frac{M_{i,t+1}}{\mathbb{E}_{i,t}\left[M_{i,t+1}\right]}R_{t+1}\right],$$

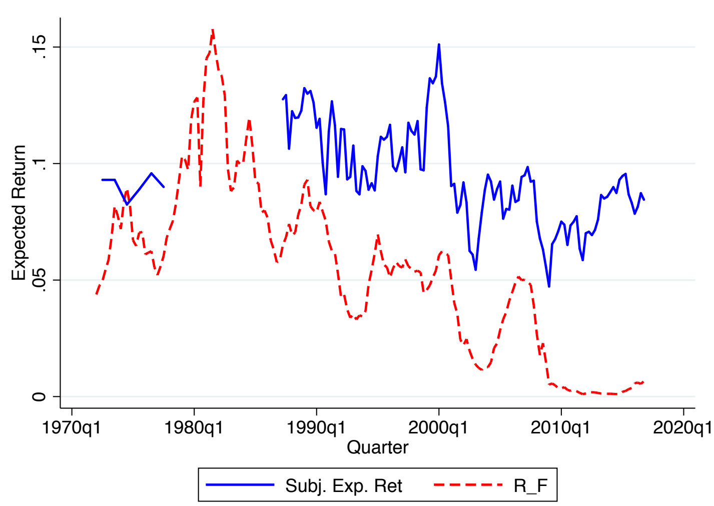
  - Idea: Good times $\Rightarrow$ lower risk aversion $\Rightarrow$ lower weight on bad outcomes $\Rightarrow$ Optimism
- But, in the absence of trading frictions and risk-free asset in existence

$$\begin{aligned} 1 &= \mathbb{E}_{i,t}[M_{i,t+1}R_{t+1}] \\ 1 &= \mathbb{E}_{i,t}[M_{i,t+1}]R_{f,t}, \end{aligned}$$

- And so RNE hypothesis implies

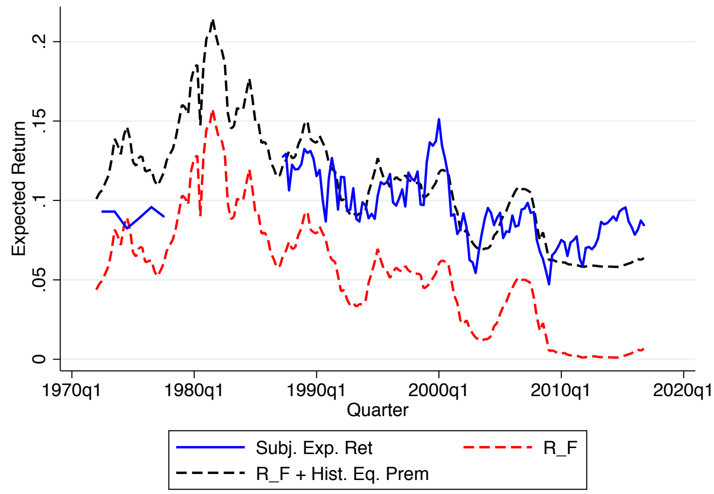
$$\mathcal{E}_{i,t}[R_{t+1}] = R_{f,t},$$

Empirical evidence not consistent with risk-neutral expectations hypothesis



Source: Based on Adam et al. (2018). Subjective expected return = one-year expected stock market returns from various individual investor surveys in Nagel and Xu (2018). R_F = one-year Treasury yield

Empirical evidence not consistent with unconditional pessimism bias in survey expectations



Source: Based on Adam et al. (2018). Subjective expected return = one-year expected stock market returns from various individual investor surveys in Nagel and Xu (2018). R_F = one-year Treasury yield. Historical equity premium: Average return of CRSP value-weighted index in excess of one-year Treasury yield 1926-2016.

Professional forecasters: Livingston/Philadelphia Fed survey of professional forecasters

- Sample periods: 1952-2017, semiannual
- We use mean of individual forecasts ($N \approx 50$) each period
- Forecasters forecast level of S&P500 index (or pre-cursors of the index) 6- and 12-months ahead
- Problem: Not known exactly on which day forecasters fill out survey (and what level of S&P500 is on that day)
- Solution (Gultekin 1983): Construct one-year expected return from forecasts F_{t+6}, F_{t+12} for 6- and 12-months ahead

$$\tilde{\mathbb{E}}_t[R_{t\to t+12}] = 2 \times \left(\frac{F_{t+12}}{F_{t+6}} - 1\right) + \frac{D_t}{P_t}$$

Empirical dynamics of subjective return expectations and expectations errors

- How do subjective return expectations relate to cycles in asset prices?

$$\tilde{\mathbb{E}}_t R_{t+1} - R_{f,t} = a_0 + a_1 \log(P/D) + e_t$$

  - Benchmark: RE models predict strongly negative a_1

- How do subjective return expectations relate to expectations errors

$$R_{t+1} - \tilde{\mathbb{E}}_t R_{t+1} = b_0 + b_1 \log(P/D) + e_t$$

  - Benchmark: RE models predict strongly negative $b_1 = 0$

Empirical dynamics of subjective return expectations and expectations errors

	(1)	(2)	(3)	(4)
	Subjective eq. prem.		Subjective exp. err.	
	Indiv.	Prof.	Indiv.	Prof.
Log P/D	0.02	0.05	-0.14	-0.15
(s.e.)	(0.01)	(0.02)	(0.05)	(0.04)
Lagged one-year return	0.02	-0.14	-0.06	-0.05
(s.e.)	(0.01)	(0.03)	(0.09)	(0.09)
Constant	-0.03	-0.10	0.58	0.58
(s.e.)	(0.02)	(0.08)	(0.19)	(0.15)
Observations	126	128	126	128
Adjusted R^2	0.199	0.201	0.058	0.088

Source: Subjective expected stock market returns in excess of one-year Treasury yield from various individual investor surveys in Nagel and Xu (2018). Professional forecasts from Livingston/Philadelphia Fed survey.

Empirical dynamics of subjective return expectations and expectations errors: Summary

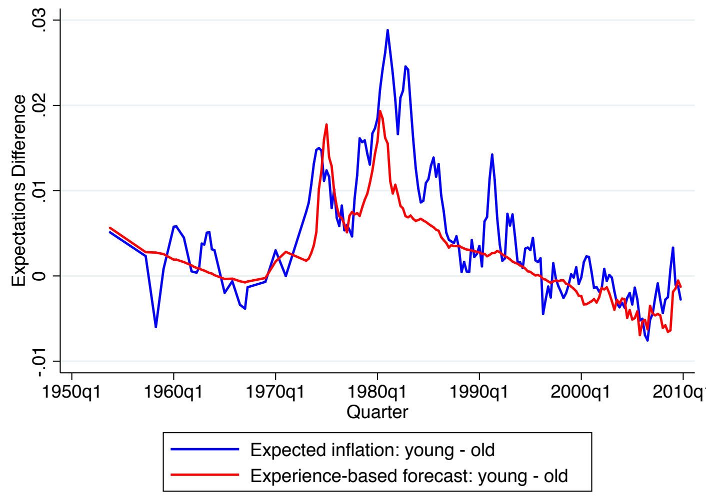
1. Subjective expected excess returns are not counter-cyclical w.r.t. P/D (a_1 is not negative)
2. Subjective expectations errors are strongly counter-cyclical w.r.t. P/D ($b_1 < 0$)
3. Both facts are true for individual investor and professional forecaster expectations
4. Individuals and professionals disagree based past one-year returns
   - Individuals: "momentum" beliefs
   - Professionals: "contrarian" beliefs

   but in their respective sample periods, they are both not quite contrarian enough (similar negative coefficients in expectations error regression)

Modeling subjective belief dynamics

- Required: Asset pricing model that
  - produces counter-cyclical variation in objective expected excess returns
  - and is also consistent with these facts about subjective return expectations (I will focus on facts 1-3 here)
- Ideally, belief formation mechanism in AP model should also be consistent with cross-sectional micro-evidence
  - Malmendier and Nagel (2011, 2016): Evidence that individuals extrapolate from life-time experiences when forming macroeconomic expectations, return expectations, and making portfolio choices

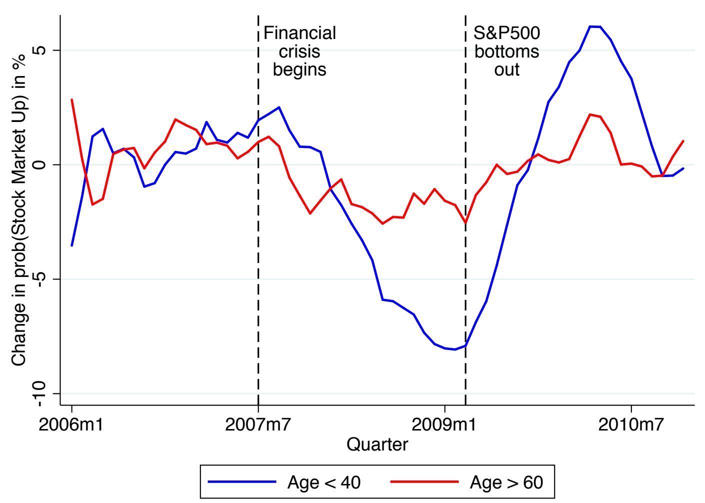
Learning from experience: Inflation



Based on Malmendier and Nagel (2016). **Inflation Expectations**: Michigan Survey of Consumers, one-year expected inflation rate. **Experience-based forecast**: AR(1) model forecast estimated based on weighted life-time inflation data for each survey respondent. Figure shows **differences**: average for individuals of age $<$ 40 minus average for individuals of age $>$ 60.

Update on MN (2011): Heterogeneity in belief updating during financial crisis

Six-month change in survey respondents' subjective probability of a rise in the stock market



Source: Michigan Survey of Consumers, panel component. Respondents' stated probability that stock market index will rise over the next 12 months compared to their response to the same question in the survey six months earlier. Averaged within age groups ($<$ 40 and $>$ 60) and over 12-month moving windows.

Learning from experience and asset pricing

- Most natural approach to bring learning from experience into an asset pricing model: empirical findings are about learning from cohort-level experiences $\Rightarrow$ overlapping generations (OLG) asset pricing model
- Several existing papers use this OLG approach. But unwieldy, difficult to solve, and to make quantitatively realistic.
- Essence of learning from experience (for asset pricing) is the gradual fading of memory for the average investor, not so much the cross-sectional heterogeneity in experiences
- Our approach: Representative agent with fading memory
  - MN (2016): dynamics of average (across cohorts) belief is well-approximated by constant-gain learning

Bayesian updating

- Representative agent learns about μ in endowment growth

$$\Delta c_t = \mu + \sigma \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0,1)$$

  where σ is known to the agent.
- Uses history $H_t \equiv \{\Delta c_1, \Delta c_2, \ldots, \Delta c_t\}$, to estimate μ.
- Bayesian (flat prior): Posterior mean is equal-weighted average

$$\hat{\mu}_t = \frac{1}{t} \sum_{s=1}^{t} \Delta c_s$$

- Recursive representation with decreasing gain

$$\hat{\mu}_t = \hat{\mu}_{t-1} + \frac{1}{t}(\Delta c_t - \hat{\mu}_{t-1})$$

  - gain $1/t = 1/$size of data set

Modification of Bayesian updating: Learning with fading memory (Nagel and Xu 2018)

- Representative agent is Bayesian in all respects except fading memory: Posterior is formed with weighted likelihood

$$p(\mu|H_t) \propto p(\mu) \prod_{j=0}^{\infty} \left[\exp\left(-\frac{(\Delta c_{t-j} - \mu)^2}{2\sigma^2} \right) \right]^{(1-\nu)^j},$$

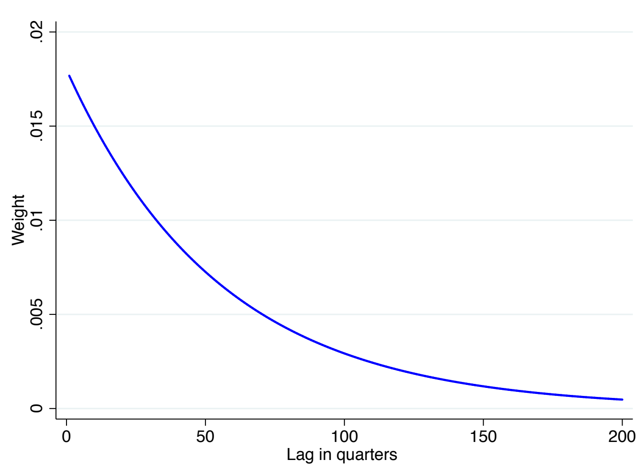
  where $\nu > 0$ induces fading memory.
- Yields posterior mean dynamics

$$\tilde{\mu}_t = \tilde{\mu}_{t-1} + \nu(\Delta c_t - \tilde{\mu}_{t-1}).$$

  i.e., updating with constant gain ν: perpetual learning.
- $S = 1/\nu$ is effective sample size

Pinning down the gain with micro-evidence on weighting of past data

- MN (2011, 2016) show that macro expectations & stock return beliefs are both fit well with $\nu \approx 0.018$ (quarterly): $S = 1/\nu \approx 56$ quarters.
- Weights on past data implied by $\nu = 0.018$



Predictive distribution

- Predictive distribution

$$\Delta c_{t+j} \sim \mathcal{N}\left(\tilde{\mu}_t, (1+\nu)\sigma^2\right), \qquad j = 1, 2, ...,$$

- Unlike in full-memory Bayesian learning, here no gain in precision over time
- Subjective long-run risk

$$\tilde{\mu}_{t+1} = \tilde{\mu}_t + \nu\sigma\sqrt{1+\nu}\tilde{\varepsilon}_{t+1},$$

where

$$\tilde{\varepsilon}_{t+1} = \frac{\Delta c_{t+1} - \tilde{\mu}_t}{\sigma\sqrt{1+\nu}} \sim \mathcal{N}(0, 1)$$

under time-t predictive distribution,

Pricing consumption claim

- Baseline: Epstein-Zin preferences with EIS $\psi = 1$ and flat prior $p(\mu)$.
- Subjective log SDF

$$m^1_{t+1|t} = \tilde{\mu}_m - \tilde{\mu}_t - \xi\sigma\tilde{\varepsilon}_{t+1},$$

- Consumption-wealth ratio constant

$$\zeta = \frac{1}{1-\delta}$$

Pricing dividend claim

- Levered dividends

$$\Delta d_{t+1} = \lambda \Delta c_{t+1} - \alpha(d_t - c_t - \mu_{dc}) + \sigma_d \eta_{t+1}$$

  where
  - μ_{dc}, α, λ are positive constants known to agent.
  - Cointegration ensures finite price of dividend claim.
- Solve for prices and returns of dividend strips
- Equity claim is a portfolio of these strips

Subjective and objective risk premia

- Subjective risk premium on "infinite-horizon" dividend strip

$$\log \tilde{\mathbb{E}}_t[R^{\infty}_{t+1}] - r_{f,t} = \left[1 + \nu \frac{\lambda - 1}{\alpha}\right] \xi \sqrt{1+\nu}\sigma^2,$$

  i.e., constant.
  - Equity claim: Subjective premium time-varying as higher $\tilde{\mu}_t \Rightarrow$ Higher weight on (risky) long-term strips
- Objective risk premium on "infinite-horizon" dividend strip

$$\log \mathbb{E}_t[R^{\infty}_{t+1}] - r_{f,t} = \text{subj. prem.} - \frac{1}{2}\nu \left(1 + \nu \frac{\lambda - 1}{\alpha}\right)^2 \sigma^2 + \left(1 + \nu \frac{\lambda - 1}{\alpha}\right)(\mu - \tilde{\mu}_t),$$

  i.e., time-varying.

Baseline calibration

Parameter	Symbol	Value	Source
Belief updating			
Gain	ν	0.018	MN (2016) (survey data)
Endowment process			
Leverage ratio	λ	3	CJL (2017)
Dividend cointegration parameter	α	0.001	
Mean consumption growth	μ	0.45%	CJL (2017)
Consumption growth volatility	σ	1.35%	CJL (2017)
Dividend growth volatility	σ_d	1%	
Preferences			
Risk aversion	γ	4	
EIS	ψ	1	
Time discount factor	δ	0.9967	BKY (2012)

Unconditional moments

	Data 1927-2016	Model $\psi = 1$, $\phi = 1$	Model $\psi = 1.5$, $\phi = 0.99$
$\mathbb{E}(\Delta c)$	1.84	1.80	1.80
$\sigma(\Delta c)$	2.72	2.70	2.70
$\mathbb{E}(\Delta d)$	2.38	1.80	1.80
$\sigma(\Delta d)$	13.31	8.35	8.35
$\sigma(\tilde{\mu})$	-	0.51	0.51
$\rho(\tilde{\mu})$	-	0.98	0.98
$\sigma(\tilde{\mu}_d)$	1.32	1.55	1.55
$\rho(\tilde{\mu}_d)$	0.97	0.98	0.98
$\mathbb{E}(R_m - R_f)$	8.11	7.16	7.65
$\sigma(R_m - R_f)$	22.41	13.31	16.35
$SR(R_m - R_f)$	0.36	0.54	0.47
$\mathbb{E}(p - d)$	3.40	2.81	2.98
$\sigma(p - d)$	0.44	0.14	0.22
$\rho(p - d)$	0.97	0.91	0.94
$\mathbb{E}(r_f)$	0.67	1.64	0.61
$\sigma(r_f)$	2.47	0.51	0.34

Subjective and objective risk premia dynamics in model and data

- Predicting quarter-ahead excess returns with $\tilde{\mu}_d$
  - Model: coeff. -2.40, $R^2 = 0.01$
  - Data (1927-2016): -5.79, $R^2 = 0.03$
- Regression of $\tilde{\mathbb{E}}_t[R_{t+1}] - R_{f,t}$ on $\tilde{\mu}_d$
  - Model: coeff. 0.83 (Economically, ≈ 0)
  - Data: coeff. 0.34 (Economically, ≈ 0)
- Regression of $R_{t+1} - \tilde{\mathbb{E}}_t[R_{t+1}]$ on $\tilde{\mu}_d$
  - Model: coeff. -10.75
  - Data: coeff. -12.31

Conditional return volatility de-linked from movements in objective risk premium

- Many RE models have almost perfectly correlated time-variation in equity premium and conditional market return volatility
  - e.g., Bansal and Yaron (2004); Campbell and Cochrane (1999)

  but literature has not paid much attention to this prediction.
- Fading memory model: Objective risk premium approx. unrelated to volatility
- NB: Neither model (ours, BY, CC) captures high short-run volatility following market crashes

Conditional return volatility de-linked from movements in objective risk premium

Predicting next quarter volatility (square root of sum of daily squared returns within quarter) in the data

	(1) 1927-2016	(2) 1927-2016	(3) 1946-2016	(4) 1946-2016
Experienced payout growth	1.58		1.39	
	(1.65)		(1.41)	
$(p-d)/100$		-0.67		2.20
		(1.56)		(0.66)
Constant	0.06	0.10	0.05	-0.01
	(0.01)	(0.06)	(0.01)	(0.02)
Observations	360	360	284	284
Adjusted R^2	0.010	0.001	0.010	0.068

Conclusion

- Survey expectations data informative about investor expectations
  - No marginal-utility weighting / risk-adjustment
  - No asymmetric loss in forecast construction
- Subjective beliefs data useful to inform about and constrain belief dynamics in asset pricing models
- Learning with fading memory can reconcile survey expectations and explain asset pricing puzzles
  - with simple IID DGP and constant risk aversion (unlike CC and BY models)
  - in highly tractable framework (unlike OLG models)
  - with rate of memory loss calibrated to microdata
- Potential further applications: Production economy; multi-country asset pricing

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