Discussion of
Portfolio Choice with Model Misspecification:
A Foundation for Alpha and Beta Portfolios
by R. Uppal and P. Zaffaroni

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This paper: No light reading...
Focus of this paper: Robust portfolio optimization

- With estimated moments, portfolio optimization is difficult in practice
  - Sample covariance matrix (close to) singular, badly estimated
  - Big estimation error in mean returns
- Michaud (1989): Portfolio optimization is “estimation-error maximization”
- Existing methods to achieve robustness
  - Portfolio constraints
    - Jagannathan and Ma (2003): no short sales
  - Shrink covariance matrix:
    - Ledoit and Wolf (2003): towards single factor model
    - Ledoit and Wolf (2004): towards constant pairwise correlation
  - Shrink expected returns:
    - Jorion (1986): towards average return of global min. var. portfolio
    - Pastor (2000): towards zero factor model alphas

Approach in this paper: Sharpe Ratio bound

- Here: Jointly tilt covariance matrix and expected return estimates by imposing Sharpe Ratio bounds
- Basic idea: Extremely high Sharpe Ratios are implausible
  - Sources of high in-sample SR = likely spurious
  - Make optimizer disregard features of the data that cause the SR to be so high
- SR constraint incorporated into ML estimation of mean returns and covariances
- Sensible economically motivated approach
Non-robustness of high in-sample Sharpe Ratios

Figure 4: In-sample and out-of-sample maximum squared Sharpe Ratios (annualized) of first $K$ principal components (incl. level factor). In panels (a) and (b) the sample period is split into two halves. We extract PCs in the first sub-period and calculate SR-maximizing combination of the first $K$ PCs. We then apply the portfolio weights implied by this combination in the out-of-sample period (second sub-period). In panels (c) and (d) we randomly sample (without replacement) half of the returns to extract PCs and calculate SR-maximizing combination of first $K$ PCs in the subsample. We then apply the portfolio weights implied by this combination in the out-of-sample period (remainder of the data). The procedure is repeated 1,000 times; average squared SRs are shown.

From Kozak, Nagel, and Santosh (2016)

Approach in this paper: Outline of method

- Simplified example: Suppose

\[ R_t = a + \beta_f \lambda + \beta_f (f_t + \eta - \lambda) + \beta_g \gamma + \beta_g (g_t - \gamma) + \epsilon_t \]

where the covariance matrix of $\epsilon$ is $\sigma^2 I$ and $f$ and $g$ are uncorrelated.

- If $f$ pre-specified as factor, and $g$ unobserved, the alpha is

\[ \alpha = a + \beta_g \gamma + \beta_f E[\eta] \]

- Proposed approach:
  1. To remove $\beta_f E[\eta]$: EIV correction
  2. To remove $\beta_g \gamma$: Remove major principal components of factor model ($f$) residuals
  3. To prevent mistaking $\bar{\epsilon}$ for $\alpha$: SR bound
Approach in this paper: Results

- Simulations
  - Estimate portfolio optimization inputs on a rolling basis with data up to $t$
  - Evaluate Sharpe Ratio from out-of-sample returns of optimized portfolio in period $t+1$.
- Results: constrained-ML optimal portfolios outperform out-of-sample compared with
  - equal-weighted portfolio
  - global MV portfolio
  - MVE portfolio based on sample means and covariances
  - MVE portfolio based on single-factor model

Comment: Motivation of the SR constraint

- SR constraint imposed in estimation
  \[ \alpha_N \Sigma_N^{-1} \alpha_N < \delta < \infty \]
- Much of first part of the paper: APT to motivate SR bound
  - Absence of asymptotic arbitrage as $N \to \infty$
- But: Absence of asymptotic arbitrage in APT does not yield restriction for finite $N$
  - APT consistent with any finite $\delta$
  - Recognized by authors in fn. 28
- Also recognized by Ross (1976, p. 353): In empirical example he suggests (ad-hoc) to set $\delta$ to twice squared SR of market portfolio
- Why not start directly with an SR bound?
  - Economically plausible in wide variety of models (incl. with mispricing, many irrational investors)
  - Some share of rational SR-max. investors $\Rightarrow$ SR bounded
Comment: Motivation of focus on alpha component of portfolio weights

- Optimal portfolio weights can be decomposed into two components
  1. Positions in (observed) factors $f$: exposed to factor risk
  2. Positions to exploit $\alpha$: exposed to idiosyncratic risk

- Authors use asymptotic argument to motivate SR bound on $\alpha$, but not factor premia
  - Under SR bound, as $N \to \infty$, $\alpha$ must shrink along with risk $\Rightarrow$ requires higher leverage to reach expected return target.
  - As $N \to \infty$, bets on $\alpha$ dominate bets on factors

- Asymptotic argument can mislead for practically relevant case of finite $N$
  - With badly estimated factor premium and covariances, (erroneous) factor bets could well be important
  - Especially when number of factors is high
  - Especially when alphas are relatively small

Comment: Motivation of focus on alpha component of portfolio weights

- Example: A factor may appear to have small risk and be uncorrelated with other (high-variance) factors in sample
- Out of sample, such factors often turn out to have substantial correlation with (high-variance) factors
- E.g., factor (hedge fund) that appears to be market neutral in sample but not out-of-sample
- Bound on factor SR would help guard against taking big bets on such factors
Comment: Comparison with alternative approaches

- Comparison with
  - MVA based on sample mean and covariances
  - equal-weighted portfolio
  
  may be setting the bar a little too low.

- Would be interesting to compare instead with alternative robust methods in the literature
  - Shrinkage of covariance matrix (Ledoit and Wolf)
  - Shrinkage of expected returns or alphas (Jorion; Pastor)

- More generally, relation to Bayesian approaches to portfolio optimization?
  - Informative prior on max. SR?

Comment: Apply to empirical data

- ... would be a useful extension
Concluding remarks

- Promising approach to portfolio optimization
- SR bound economically sensible
- Not quite clear yet how much of an edge the proposed method has over other sophisticated approaches
- Value added of lengthy asymptotic analysis until p. 30 not clear. Potential gains from greater focus on core innovation.