Discussion of

Investor Beliefs and Asset Prices Under Selective Memory

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June 2024

Big picture: Asset pricing with subjective beliefs

- Campbell-Shiller PV identity under objective probabilities (of econometrician studying large sample)

$$p_t - d_t = \text{const.} + \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}}_{\text{approx. constant}} - \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}$$

- Campbell-Shiller PV identity under subjective probabilities (of representative investor)

$$p_t - d_t = \text{const.} + \widetilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - \underbrace{\widetilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}}_{\text{approx. constant}}$$

- Needed: Models of $\widetilde{\mathbb{E}}_t \Delta d_{t+1+j}$.

- This paper: Selective memory.

Selective memory model: EZ log-normal case

- Log endowment growth

$$g_t = \mu + \sigma \epsilon_t, \qquad \epsilon_t \overset{\text{IID}}{\sim} \mathcal{N}(0,1)$$

- Representative agent learns about distribution of g from recalled history of g

- Recall governed by memory function that gives higher recall probability

$$m(g_\tau) = \exp\left(-\frac{(g_\tau - g_t)^2}{2\kappa}\right)$$

  to past observations g_τ close to current g_t

- Consequence: Posterior mean tilted towards g_t

$$\hat{\mu}_t = \frac{\kappa}{\kappa + \sigma^2}\mu + \frac{\sigma^2}{\kappa + \sigma^2} g_t$$

Selective memory model: EZ log-normal case

- Asset pricing ($\rho = 0.95$)

$$d_t - p_t = \text{const.} + \left(\frac{1}{\psi} - \lambda\right) \hat{\mu}_t$$

$$\log \widetilde{\mathbb{E}}_t r_{t+1} - r_{f,t} = \text{const.}$$

$$\log \mathbb{E}_t r_{t+1} - r_{f,t} = \text{const.} - \left(\frac{1}{1-\rho}\lambda - \frac{\rho}{1-\rho}\frac{1}{\psi}\right) \hat{\mu}_t$$

- High $g_t \Rightarrow$ high $\hat{\mu}_t \Rightarrow$ low $d_t - p_t$, low $\log \mathbb{E}_t r_{t+1} - r_{f,t}$

- $\sigma(\hat{\mu}_t) \approx 0.02$, $\left(\frac{1}{1-\rho}\lambda - \frac{\rho}{1-\rho}\frac{1}{\psi}\right) \approx 47$, so in predictive regression with standardized $\hat{\mu}_t$ or $d_t - p_t$, coefficient $0.02 \times 47 = 0.94$. Table 3:

	No parameter uncertainty		
	RP_{Subj}	RP_{Obj}	$\hat{b}_{CG}$
dp_t	0.002	1.119	

Comment #1: Lack of persistence

- In data, $d - p$ highly persistent: Autocorrelation ≈ 0.97 quarterly
- In infinite-history version of this model:

$$AC(d_t - p_t) = AC(\hat{\mu}_t) = AC(g_t) \approx 0$$

- Objective risk premium mean-reverts fast, unlike in data
  - $d_t - p_t$ predicts r_{t+1}, but not r_{t+2}, r_{t+3}, ..., unlike in data
- Subjective cash flow beliefs mean-revert fast, unlike in data
  - E.g., analyst long-term growth expectations are persistent
- Higher persistence in finite-history version of model, but this persistence fades away over time

Comment #2: Definition of context and evolution of context

- When agent assesses similarity between past and present context, what is the definition of context?
  - Here g_t quarterly, but why not daily, or annual, or ... ?
- This has asset pricing consequences
  - Example: if context measured in terms of annual g_t, $d_t - p_t$ and objective risk premium will be more persistent than if measured in terms of daily g_t.
- More generally, question of context evolution: In retrieved-context theory (Howard and Kahana 1999, 2002), context drifts with persistence

$$c_t = \phi c_{t-1} + (1 - \phi) g_t$$

Could this be a potential mechanism to obtain more persistence in beliefs, $d_t - p_t$, expected returns, ... ?

Comment #3: More focus on unique predictions of similarity-weighting

- Paper would benefit from focusing more developing predictions unique to the similarity-weighted memory model
  - e.g., currently emphasized distinctions such as subjective risk premium U-shape in g_t are very subtle

- Allow for more role of higher moments, or perhaps even non-parametric learning of distribution?
  - e.g., high recent g_t should make similarity-weighted recalled history more positively skewed

- Predictions about higher moments could be studied empirically with
  - Option prices
  - Perceived crash probabilities (e.g., ICF Yale survey)

Summary

- Similarity-weighted memory is an interesting framework for belief formation in asset pricing

- Still some challenges in matching asset price properties

- Potential improvements from
  - Modifying context evolution
  - Considering higher moments