Premuneration Values, Investments, and Pricing in Matching Markets

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Introduction

- Many interesting economic problems involve matching of economic agents with complementary attributes.
- The output generated in a matched pair depends on the attributes of the members of the match.
- These attributes are affected by investments, which are typically made prior to matching.
- When ex ante contracting is impossible, inefficiencies may arise:
  - holdup problem, and
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  - coordination failure.

Does competition among potential partners ameliorate or exacerbate the inefficiencies?

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  - prices are personalized—prices can be written as a function of the agents' attributes.
- Equilibria may display
  - underinvestment, or
  - overinvestment.
- Moreover, the ex ante efficient outcome is an equilibrium.

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Part II: Uniform Pricing and Premuneration Values

- What if prices cannot be personalized, but instead must be uniform, so that the price depends only on one side's attribute? This might reflect
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  - prohibitive personalization costs, or
  - legal or institutional prohibitions.

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  - prohibitive personalization costs, or
  - legal or institutional prohibitions.
- Premuneration values now matter.
- A matched agent's premuneration value is her value from the match in the absence of transfers (i.e., pre-transfer).
- Premuneration values are irrelevant if prices are personalized, but they are key to understanding the nature of the inefficiencies under uniform pricing.

The Papers

- Part I:
  - Cole, Mailath, Postlewaite (2001a, b): Infinite and finite population assignment games with prematch investments.
- Part II:
  - Mailath, Postlewaite, and Samuelson (2013a): Introduces premuneration values and a general one sided incomplete info model of prematch investments.
  - Mailath, Postlewaite, and Samuelson (2013b): A parameterized version of MPS (2013a) to do comparative statics.
- Related:
  - Liu, Mailath, Postlewaite, and Samuelson (2014): A general notion of incomplete information stability.

The Model

(CMP 2001a, as reinterpreted by MPS 2013a)

- Researchers $\rho \in [0,1]$ and laboratories $\lambda \in [0,1]$.
- Researcher ρ chooses attribute $r \in \mathbb{R}_+$, at cost $c(r,\rho)$, where c satisfies the usual monotonicity and single crossing conditions.
- Laboratory λ chooses attribute $\ell \in \mathbb{R}_+$, at cost $\psi(\ell,\lambda)$.
- Researchers and laboratories match, with a match between researcher r and laboratory ℓ creating surplus $v(\ell,r)$, with v supermodular.
- Premuneration values:

$$v(\ell,r) = h_R(\ell,r) + h_L(\ell,r).$$

Feasible Outcomes

Suppose researcher and laboratory attribute choice functions $\mathfrak{r} : [0,1] \to \mathbb{R}_+$ and $\mathfrak{l} : [0,1] \to \mathbb{R}_+$ are strictly increasing when positive, and set

$$\mathcal{R} := \text{cl}(\mathfrak{r}[0,1]) \quad \text{and} \quad \mathcal{L} := \text{cl}(\mathfrak{l}[0,1]).$$

A feasible matching is $\tilde{r} : \mathcal{L} \to \mathcal{R}$ and $\tilde{\ell} : \mathcal{R} \to \mathcal{L}$ such that

$$\tilde{r}(\mathfrak{l}(\lambda)) = \mathfrak{r}(\lambda) \quad \text{and} \quad \tilde{\ell}(\mathfrak{r}(\rho)) = \mathfrak{l}(\rho),$$

so that

$$\tilde{r}(\tilde{\ell}(r)) = r \quad \text{for all} \quad r \in \mathcal{R}.$$

A feasible outcome is $((\mathfrak{r}, \mathfrak{l}), (\tilde{r}, \tilde{\ell}))$.

Personalized Price Equilibrium

A personalized price is $p_P : \mathcal{R} \times \mathcal{L} \to \mathbb{R}_+$.

Definition

Given $((\mathfrak{r}, \mathfrak{l}), (\tilde{r}, \tilde{\ell}), p_P)$, researcher ρ is optimizing if

$$(\mathfrak{r}(\rho), \tilde{\ell}(\mathfrak{r}(\rho))) \in \arg\max_{(r,\ell)\in\mathcal{R}\times\mathcal{L}} h_R(r, \ell) - p_P(r, \ell) - c(r, \rho),$$

Laboratory λ is optimizing if

$$(\tilde{r}(\mathfrak{l}(\lambda)), \mathfrak{l}(\lambda)) \in \arg\max_{(r,\ell)\in\mathcal{R}\times\mathcal{L}} h_L(r, \ell) + p_P(r, \ell) - \psi(\ell, \lambda).$$

A personalized price equilibrium is an outcome at which all agents are optimizing and no agent finds it profitable to deviate outside the set of chosen attributes.

Equilibrium and Stability

A personalized price equilibrium is the equilibrium of a two stage game:

1. in the first stage, agents simultaneously choose attributes, and
2. in the second stage, as a function of the chosen attributes, agents are matched (and compensated) in a pairwise stable matching.

The second stage outcome is a stable outcome of the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$.

Ex Post Stability

Matching is stable in the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$ if, for all pairs of attributes $(r, \ell) \in \mathcal{R} \times \mathcal{L}$,

$$\underbrace{h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))}_{\text{researcher payoff}} + \underbrace{h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell)}_{\text{laboratory payoff}} \geq v(r, \ell).$$

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Remark

With personalized prices, the division of surplus into premuneration values is irrelevant. Any reassignment of premuneration values can be undone by prices.

Ex Ante Stability

Given $((\mathfrak{r}, \mathfrak{l}), (\tilde{r}, \tilde{\ell}), p_P)$,

$$u_R(\rho) := h_R(\mathfrak{r}(\rho), \tilde{\ell}(\mathfrak{r}(\rho))) - p_P(\mathfrak{r}(\rho), \tilde{\ell}(\mathfrak{r}(\rho))) - c(\mathfrak{r}(\rho), \rho)$$

and

$$u_L(\lambda) := h_L(\tilde{r}(\mathfrak{l}(\lambda)), \mathfrak{l}(\lambda)) + p_P(\tilde{r}(\mathfrak{l}(\lambda)), \mathfrak{l}(\lambda)) - \psi(\mathfrak{l}(\lambda), \lambda).$$

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The matching is stable in the ex ante assignment game if, for all researcher-laboratory pairs $(\rho, \lambda) \in [0, 1]^2$,

$$u_R(\rho) + u_L(\lambda) \geq \max_{r,\ell} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda).$$

Stability Comparison

A matching is stable in the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$ if, for all pairs of attributes, $(r, \ell) \in \mathcal{R} \times \mathcal{L}$,

$$
\begin{aligned}
h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r)) \\
+ h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell) \\
\geq v(r, \ell).
\end{aligned}
$$

Stability Comparison

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A matching is stable in the ex ante assignment game if, for all researcher-laboratory pairs $(\rho, \lambda) \in [0, 1]^2$, with $r = \mathfrak{r}(\rho)$ and $\ell = \mathfrak{l}(\lambda)$,

$$h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r)) - c(r, \rho) + h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell) - \psi(\ell, \lambda) \geq \max_{r', \ell'} v(r', \ell') - c(r', \rho) - \psi(\ell', \lambda).$$

Example

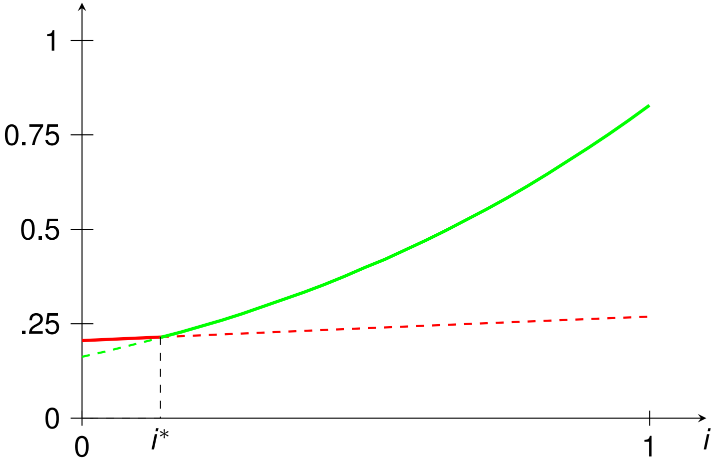
- $v(r,\ell) = \begin{cases} r\ell & \text{if } r\ell \le \frac{1}{2}, \\ 2(r\ell)^2 & \text{if } r\ell > \frac{1}{2}. \end{cases}$.
- $c(r,\rho) = 2r^5/(\rho+2)$.
- $\psi(\ell,\lambda) = 2\ell^5/(\lambda+2)$.
- ex ante efficient attribute choices:

$$\mathfrak{r}^*(i) = \mathfrak{l}^*(i) = \begin{cases} \sqrt[3]{(i+2)/10}, & \text{if } i \le i^*, \\ (2i+4)/5, & \text{if } i > i^*, \end{cases}$$

where $i^* \approx .1$.

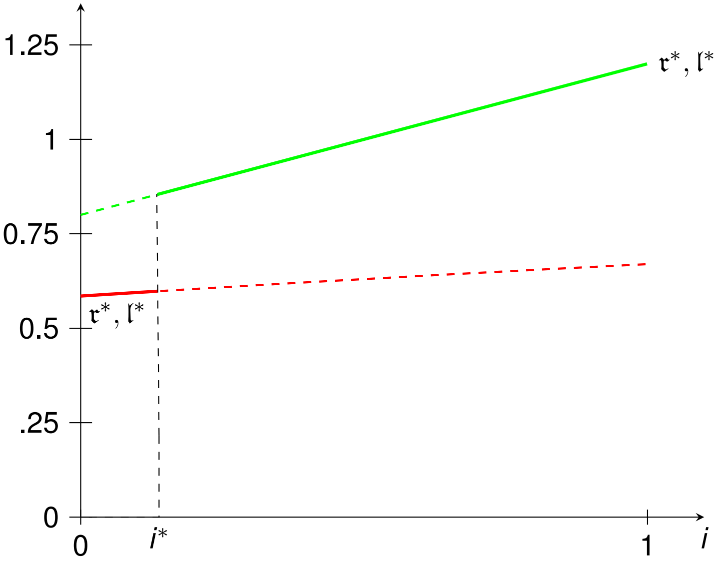
Example

net surplus



Example

attribute choices



Efficiency

Theorem

Suppose $((\mathfrak{r}, \mathfrak{l}), (\tilde{r}, \tilde{\ell}), p_P)$ *is a personalized price equilibrium. Then,* $h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))$ *is differentiable where both* $\mathfrak{r}$ *and* $\mathfrak{l}$ *are continuous, and*

$$\frac{d}{dr}[h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))] = \frac{\partial v(r, \tilde{\ell}(r))}{\partial r}.$$

A similar statement holds for laboratories.

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Theorem

There exists a personalized price p_P *such the ex ante efficient allocation is part of a personalized price equilibrium.*

Constrained Efficiency

Definition

The allocation $((\mathfrak{r}, \mathfrak{l}), (\tilde{r}, \tilde{\ell}), p_P)$, is constrained efficient if

$$u_R(\rho) + u_L(\lambda) \geq \sup_{r \in \mathbb{R}_+, \ell \in \mathcal{L}} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda)$$

and

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Theorem

Every personalized price equilibrium outcome is constrained efficient.

Uniform Pricing

- Suppose researcher attributes are not public information (prices cannot condition on r).
- A laboratory deviating to an unpriced attribute now needs beliefs over the researcher attributes it attracts.
- Mailath, Postlewaite and Samuelson (2013a) introduces a general model of uniform pricing, establishing
  - sufficient conditions for existence of equilibrium, and
  - necessary and sufficient conditions for equilibria to be efficient.
- Mailath, Postlewaite and Samuelson (2013b) imposes more structure. In particular:
  - only one side makes investment decisions, and
  - the cost function and premuneration values have a simple functional form.

The Extended Example

- Researchers choose attributes r, at cost

$$c(r,\rho) = \frac{r^{2+k}}{(2+k)\rho^k}.$$

- Laboratories have attributes $\ell = \lambda$,
- Researchers and laboratories match, with a match between researcher r and laboratory ℓ creating surplus ℓr.
- Premuneration values are given by
  - for the researcher

$$h_R(r,\ell) = \theta \ell r, \text{ and}$$

  - for the laboratory

$$h_L(r,\ell) = (1-\theta)\ell r.$$

Equilibrium

Definition

A price function p and researcher choices (ℓ_R, r_R) constitute a matching equilibrium if,

1. for every $\rho \in [0, 1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves the researcher optimization problem,

$$\max_{\ell, r} \; \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}.$$

2. every researcher and laboratory earns nonnegative payoffs, and
3. ℓ_R is market-clearing (i.e., ℓ_R is $1:1$, onto and measure preserving).

Some Equilibrium Properties

Lemma

Every equilibrium price function p is strictly increasing and continuous.

Lemma

The equilibrium researcher attribute-choice function r_R is strictly increasing.

Lemma

The equilibrium researcher laboratory-choice function ℓ_R is given by

$$\ell_R(\rho) = \rho.$$

The Efficient Outcome

Efficiency requires that each pair $\rho = \lambda$ match and maximize their surplus:

$$\max_r \ \rho r - \frac{r^{2+k}}{(2+k)\rho^k}.$$

The first-order condition is

$$\rho = \frac{r^{1+k}}{\rho^k},$$

immediately implying

$$r = \rho.$$

The Equilibrium Outcome

Researcher ρ's problem is to choose ℓ and r to maximize

$$\theta \ell r - p(\ell) - c(r, \rho) \; = \; \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}.$$

The first order conditions are

$$\theta \ell = \frac{r^{1+k}}{\rho^k}$$

and

$$\theta r = p'(\ell).$$

Theorem

The unique matching equilibrium is given by

$$r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}.$$

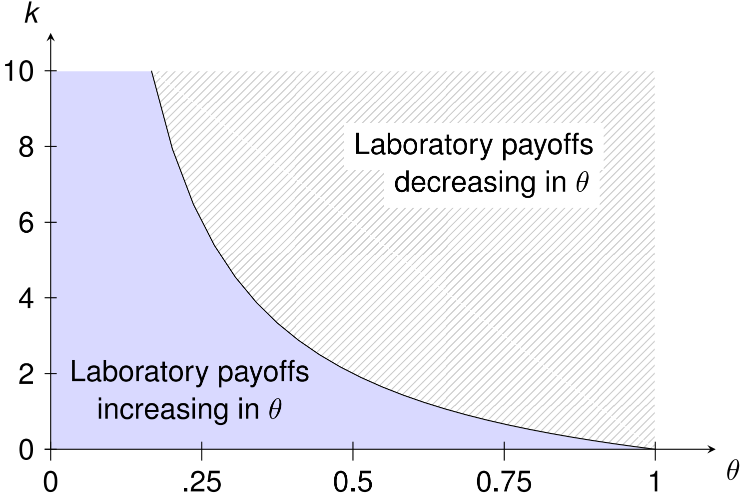
and

$$p(\ell) = \frac{1}{2}\ell^2 \cdot \theta^{\frac{2+k}{1+k}}.$$

If $\theta < 1$, so that the laboratory premuneration value share is positive, then in equilibrium, researchers invest less than the efficient level.

Equilibrium Payoffs

$$u_L(\theta, k, \lambda) = \frac{1}{2}\theta^{\frac{1}{1+k}}(2-\theta)\lambda^2.$$



Endogenous Information

Information Acquisition

Suppose laboratories could, at fixed cost κ, acquire a technology allowing them to observe researchers' types.

- Some laboratories would choose to do so, for sufficiently small κ.
- Some laboratories would not do so.
- We consider an equilibrium in which laboratories $\lambda > \tilde{\lambda}$ acquire information, laboratories $\lambda < \tilde{\lambda}$ do not.

Informed Laboratories ($\lambda \geq \tilde{\lambda}$)

Equilibrium prices

- Informed laboratories can set different prices for different researchers.
- For appropriate values of ϕ, the personalized price function

$$\hat{p}(\ell, r) = \phi + \frac{\ell^2}{2} - (1-\theta)\ell r$$

  implies efficient investments (because one-sided).

  Researcher ρ's payoff from ℓ and r is

$$\theta \ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k} = \ell r - \phi - \frac{1}{2}\ell^2 - \frac{r^{2+k}}{(2+k)\rho^k}.$$

  Maximizing the payoff yields $\ell = \rho$ and $r = \rho$.
- The value of ϕ will be determined by the laboratory incentives to become informed.

Informed Laboratories

Exogenously fixed $\tilde{\lambda}$

- If $\widetilde{\lambda} = 0$, then individual rationality implies $\phi = 0$, and the equilibrium is unique.
- If $\widetilde{\lambda} > 0$, there is a one parameter family of equilibrium price functions, indexed by

$$\phi \in [-\hat{u}_L(\theta, k, \widetilde{\lambda}),\ \hat{u}_R(\theta, k, \widetilde{\lambda})] = \left[\frac{-\widetilde{\lambda}^2}{2},\ \frac{k\widetilde{\lambda}^2}{2(2+k)} \right];$$

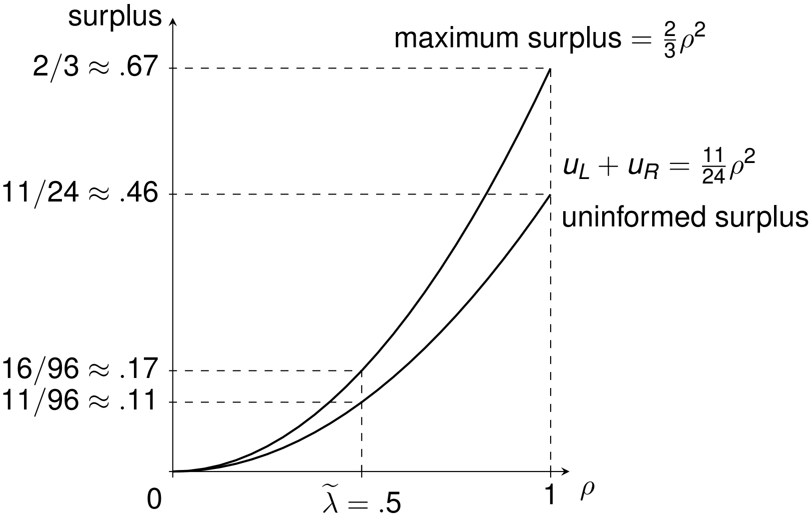
  the bounds on ϕ come from the participation constraint.
- The total net surplus of the pair with index ρ is

$$\frac{k}{2(2+k)}\rho^2 + \frac{1}{2}\rho^2 = \frac{(1+k)}{(2+k)}\rho^2,$$

  which is the maximum (i.e., efficient) value of the ρ-match.

Surpluses

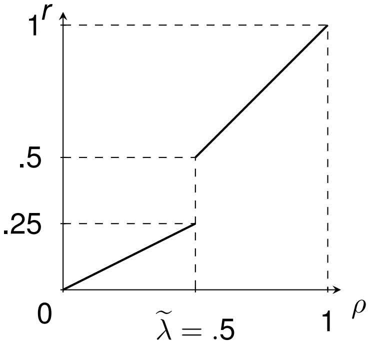
$\theta = \frac{1}{4}$ and $k = 1$



Endogenously Informed Laboratories

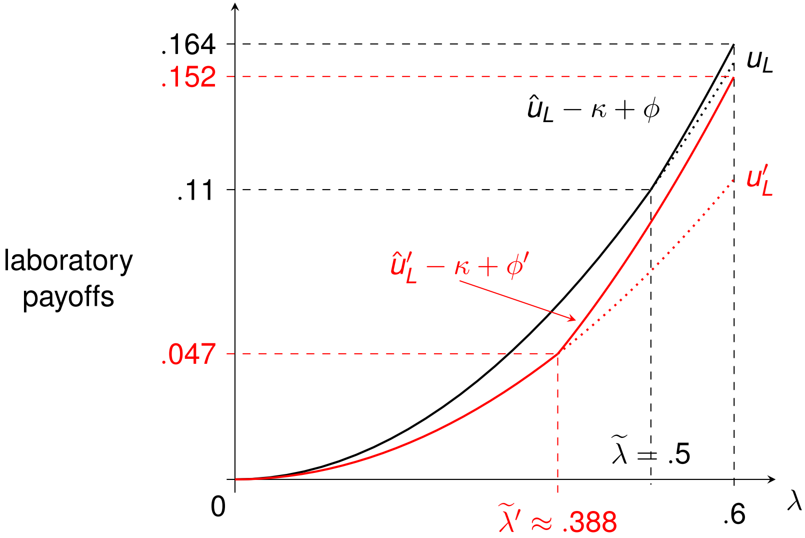
Researcher attribute choices

- $\theta = \frac{1}{4}$, $k = 1$ and $\kappa = \frac{5}{96}$, so $\tilde{\lambda} = \frac{1}{2}$.
- Laboratories with $\lambda \geq \frac{1}{2}$ informed and $\lambda < \frac{1}{2}$ uninformed.



Comparative Statics w.r.t. θ

Black is $\theta = \frac{1}{4}$, red is $\theta = \frac{1}{9}$



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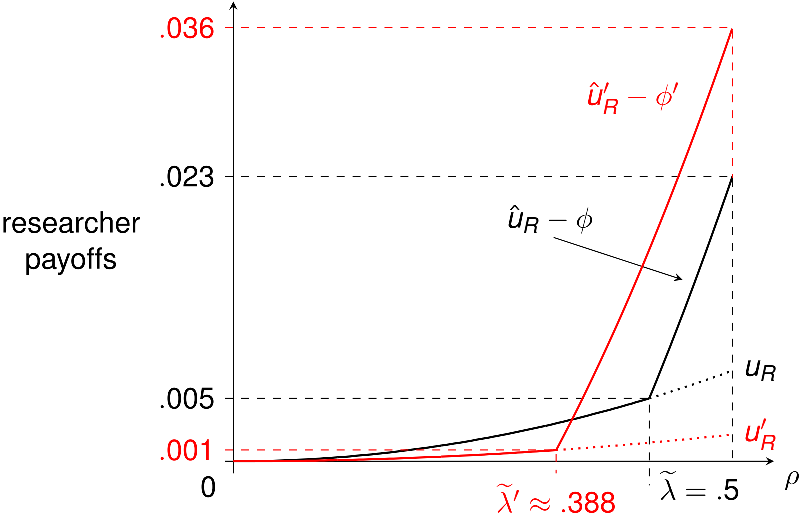


Laboratories Invest

- The Researchers' Problem:
  - Researcher ρ has exogenously determined attribute $r = \rho$.
  - Given the price function p^*, researcher ρ chooses ℓ to maximize

$$\theta \ell \rho - p^*(\ell).$$

- The Laboratories' Problem:
  - The cost of attribute $\ell \in \mathbb{R}_+$ to laboratory λ is

$$\psi(\ell, \lambda) = \frac{\ell^{2+k}}{(2+k)\lambda^k}, \qquad k \in \mathbb{R}_+.$$

  - Laboratories choose attributes to maximize

$$(1-\theta)\ell r_L^*(\ell) + p^*(\ell) - \psi(\ell, \lambda).$$

Equilibrium

Definition

A price function p, matching function r_L^*, and strictly increasing laboratory attribute choices (ℓ_L^*, ℓ_R^*) constitute a matching equilibrium if

1. $\ell_R^*(\rho)$ is an optimal laboratory attribute for researcher ρ, for all $\rho \in [0, 1]$,
2. $\ell_L^*(\lambda)$ is an optimal laboratory attribute for laboratory λ, for all $\lambda \in [0, 1]$,
3. every researcher and laboratory earns nonnegative payoffs, and
4. markets clear: $r_L^*(\ell_R^*(\rho)) = \rho$ for all $\rho \in [0, 1]$ and $\ell_R^*(\lambda) = \ell_L^*(\lambda)$ for all $\lambda \in [0, 1]$.

Theorem

A *Matching Equilibrium* *is a vector* $(p^*, r_L^*, \ell_R^*, \ell_L^*)$*, where*

$$\begin{aligned} p^*(\ell) &= \frac{\theta\ell^2}{2\alpha}, & \ell &\in \mathbb{R}_+, \\ r_L^*(\ell) &= \ell/\alpha, & \ell &\in [0, \alpha], \\ \ell_L^*(\lambda) &= \ell_R^*(\lambda) = \alpha\lambda, & \lambda &\in [0, 1], \end{aligned}$$

for $\alpha = (2-\theta)^{\frac{1}{k+1}}$*. Laboratory payoffs are given by*

$$u_L^*(\theta, k, \lambda) = \frac{k}{2(k+2)}(2-\theta)^{(k+2)/(k+1)}\lambda^2.$$

Researcher payoffs are given by

$$u_R^*(\theta, k, \rho) = \frac{1}{2}\theta(2-\theta)^{1/(k+1)}\rho^2.$$

Properties of Equilibrium

- If $\theta = 1$, the outcome is efficient.
- If $\theta < 1$, laboratories overinvest.
- Researchers' payoffs increase in θ; laboratories' payoffs decrease in θ.

Discussion

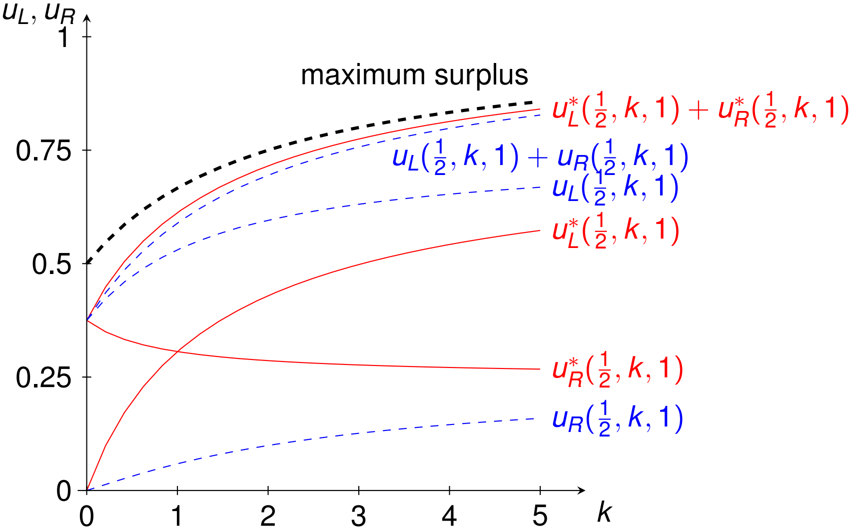
Premuneration Values

- Premuneration values affect investment incentives.
- When researchers' attributes cannot be observed and $\theta < 1$, researchers underinvest.
- Increasing θ can mitigate this inefficiency: Labs' eq payoffs increase in θ if for small θ, and decrease in θ otherwise. Researchers' payoffs increase in θ.
- When lab's can become informed (at some cost), θ determines which lab's choose to become informed and the payoffs of all lab's (both informed and uninformed) and researchers. Lab's may gain by having θ increase. The impact on researchers is even more ambiguous.
- When laboratories invest, laboratories overinvest in order to more effectively compete for researchers.

Discussion

Competition

- $k = 0$ researchers are homogeneous, and so competitive.
- As $\rightarrow \infty$, researchers become more heterogenous.
- When researchers invest, as k increases, there are enhanced investment incentives arising from reduced competition and reduced investment costs.
- When laboratories invest, reduced researcher competition leads to smaller laboratory investments.
- Lab payoffs under either scenario increase with k.
  - As $k \rightarrow \infty$, enhanced researcher investment incentives (when researchers make investments), and the reduced cost of investments (when laboratories make investments) dominate the impact of reduced competition for laboratories.


u_L, u_R
1
0.75
0.5
0.25
0
0
1
2
3
4
5
k
maximum surplus
$u_L^*(\frac{1}{2}, k, 1) + u_R^*(\frac{1}{2}, k, 1)$
$u_L(\frac{1}{2}, k, 1) + u_R(\frac{1}{2}, k, 1)$
$u_L(\frac{1}{2}, k, 1)$
$u_L^*(\frac{1}{2}, k, 1)$
$u_R^*(\frac{1}{2}, k, 1)$
$u_R(\frac{1}{2}, k, 1)$

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