# Repeated Games and Reputations

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June 26, 2014

9th Tinbergen Institute Conference:
70 Years of
Theory of Games and Economic Behaviour

The slides and associated bibliography
are on my webpage
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# Introduction

- Games and Economic Behavior does not mention repeated games.
- Yet we find in Luce and Raiffa (1957, page 457): “The relevant literature which dates back, even if we are generous; only to 1949, is already extensive.”
- Two basic themes emerged early:
  - intertemporal incentives discourage opportunistic behavior, and
  - actions can convey information.
- Work originally focused on repeated zero sum games, and their applications (though the repeated prisoners’ dilemma makes a significant appearance in L&R).
- Fruitful applications to economics and other social sciences required understanding repeated non-zero sum games and discounting.

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  - In particular, the tools typically focus on continuation values and patient players.
  - They do not focus on behavior directly.

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- The current set of tools are well suited to proving folk theorems, but do not to provide a positive theory of behavior in long run relationships.
  - In particular, the tools typically focus on continuation values and patient players.
  - They do not focus on behavior directly.
- There are some suggestive results with bounded recall.
- The reputations literature has a larger positive content, but has serious limitations.

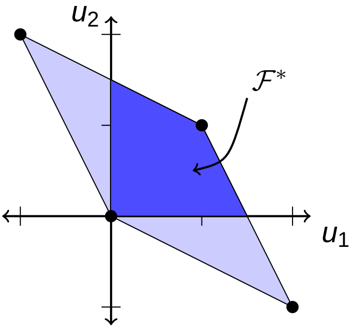
# Examples of Long-Run Relationships

- Buyer-seller.
- Traders selling goods on consignment.
- Employer and employees.
- Partners in a firm.
- A government/leader of a community interacting with its members.
- Firm selling to a population of “small” customers.

# Games with Perfect Monitoring

## The prisoners' dilemma

| | $E$ | $S$ |
|---|---|---|
| $E$ | $1, 1$ | $-1, 2$ |
| $S$ | $2, -1$ | $0, 0$ |




- each player has minmax payoff of 0.
- $\mathcal{F}^*$ is the set of feasible and individually rational payoffs.
- action spaces $A_i := \{E, S\}$ and $A := A_1 \times A_2$.

# The Repeated Game $\Gamma(\delta)$

- set of histories, $H := \cup_{t \geq 0} A^t$.
- pure strategy for player $i$, $\sigma_i : H \to A_i$.
- outcome induced by $\sigma = (\sigma_1, \sigma_2)$,

$$a(\sigma) := (a^0(\sigma), a^1(\sigma), a^2(\sigma), \ldots),$$

  where $a^0(\sigma) := \sigma(\varnothing)$ and $a^t(\sigma) := \sigma(a^{t-1}(\sigma))$.
- payoffs in the repeated game,

$$U_i(\sigma) = \sum\nolimits_{t \geq 0} (1 - \delta)\delta^t u_i(a^t(\sigma))$$

### Definition

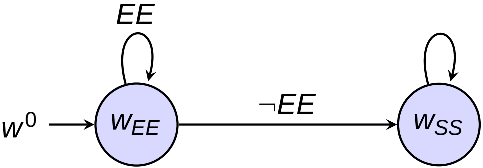
The profile $\sigma^*$ is a Nash eq of $\Gamma(\delta)$ if $U_i(\sigma^*) \geq U_i(\sigma_i, \sigma^*_{-i})\ \forall \sigma_i$.
The profile $\sigma^*$ is a subgame perfect eq of $\Gamma(\delta)$ if $\sigma^*|_{h^t}$ is a Nash eq for all histories $h^t \in H$, where $\sigma^*|_{h^t}$ is the continuation profile induced by $h^t$.

# Characterizing Equilibria

- Let $\mathcal{E}^p(\delta) \subset \mathcal{F}^*$ be the set of (pure strategy) subgame perfect equilibria of $\Gamma(\delta)$.
- Difficult problem: many possible deviations in many possible subgames.
- But repeated games are recursive, and the one shot deviation principle (from dynamic programming) holds.
- Simple penal codes (Abreu, 1988): use $i$'s worst eq to punish any (and all) deviation by $i$.

# Example I

## Grim Trigger




This is an eq if

$$
\begin{aligned}
1 \geq & (1-\delta) \times 2 + \delta \times 0 \\
& \Rightarrow \delta \geq \frac{1}{2}.
\end{aligned}
$$

[Automata representation of strategy profiles is due to Osborne and Rubinstein 1994.]

# A Folk Theorem

**A Folk Theorem (Fudenberg and Maskin 1986)**

For every feasible and strictly individually rational $v \in \mathcal{F}^*$, with $\mathcal{F}^*$ having full dimension, there exists $\underline{\delta} \in (0, 1)$ such that $v \in \mathcal{E}^p(\delta)$ for all $\delta \in (\underline{\delta}, 1)$.
For our prisoners' dilemma, for $\delta \geq 3/4$,

$$\mathcal{E}^p = \mathcal{F}^*.$$

- In general, order of quantifiers matters, and there are games with weakly IR payoffs that are not in $\mathcal{E}^p(\delta)$ for any $\delta$.
- Restriction to pure minmax is not necessary (though then need to use mixed strategies).
- Original proof used simple penal codes to explicitly construct eq.

# Interpretation

- While efficient payoffs are consistent with eq, so are many other payoffs, and associated eq behaviors. (Consistent with experimental evidence: Dal Bo and Fréchette 2011, Dreber, Fudenberg, and Rand 2014.)
- Moreover, multiple eq are consistent with the same payoff.
- The theorem does not justify restricting attention to efficient payoffs.

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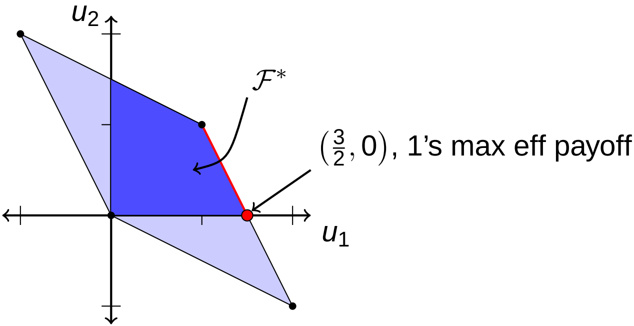
- Moreover, multiple eq are consistent with the same payoff.
- The theorem does not justify restricting attention to efficient payoffs.

Nontheless:

- In many situations, understanding the potential scope of equilibrium incentives helps us to understand possible plausible behaviors.
- Understanding what it takes to achieve efficiency gives us important insights into the nature of equilibrium incentives.

# Impatient Players

- What if $\delta < 3/4$?
- $\delta < 1/2$, only subgame perfect equilibrium payoff is $(0, 0)$.
- Difficult to characterize the equilibrium set in general for intermediate $\delta$. Mailath, Obara, and Sekiguchi (2002) ask: "What is player 1's maximum efficient equilibrium payoff?"



- If $\delta \in \left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, then 1's maximum efficient equilibrium payoff is

$$2 - \delta.$$

  This is decreasing in the discount factor (same eq payoff: $SE, EE^{\infty}$).
- If $\delta = \frac{1}{\sqrt{2}}$, then $SE, SE, EE^{\infty}$ is an eq outcome, yielding a payoff of $\frac{3}{2}$ to player 1.
- Let $\mathcal{W} \subset \left(\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$ be the set of discount factors for which 1's max eff eq payoff is $\frac{3}{2}$. Then,
    - $\mathcal{W}$ is uncountable, but
    - its complement is open and dense in $\left(\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$. Moreover, for some $\delta$, there is an inefficient eq payoff yielding 1 a higher payoff.
- Thus, the associated eq behaviors are not robust to small changes in $\delta$.

# Reputations

## The product-choice game

| | $c$ | $s$ |
|---|---|---|
| $H$ | $2, 3$ | $0, 2$ |
| $L$ | $3, 0$ | $1, 1$ |

- Row player is a long-lived firm choosing *H*igh or *L*ow quality.
- Column player is a short-lived customer choosing the *c*ustomized or *s*tandard product.
- In the game with perfect monitoring, grim trigger (play $Hc$ till 1 plays $L$, then revert to perpetual $Ls$) is an eq if $\delta \geq \frac{1}{2}$.
- The set of eq payoffs for Row player: $[1, 2]$.

# Reputations

- Suppose the customers are uncertain about the characteristics of the firm. In particular, suppose they assign prior prob $\rho > 0$ that the firm is a behavioral type that chooses $H$ after any history.
- Then, for sufficiently patient firms, every Nash eq must give the firm a payoff close to 2 (Fudenberg and Levine, 1989).
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- Then, for sufficiently patient firms, every Nash eq must give the firm a payoff close to 2 (Fudenberg and Levine, 1989).
- This is a natural way to reduce the set of eq payoffs and behaviors in some settings.
- But is it always natural? Tit-for-tat in finitely repeated PD or overly aggressive firms (as done in the original reputation papers of KWMR)? Or mixed Stackelberg types?
- There are no good general results for two long-lived players (and counterexamples indicating that under perfect monitoring, there can't be).

# What do we learn from perfect monitoring

- Multiplicity of equilibria.
  - This is necessary for repeated game to serve as a building block for any theory of institutions.
  - Selection of eq can (should) be part of modelling.
- It is sometimes argued that the punishments imposed are too severe. But often weaker punishments will work.
- Histories coordinate behavior.
- But intertermporal incentives require deviations to be punished, which requires monitoring, and a future (so end game effects can be important).
  - Noisy monitoring.
  - Overlapping generations.
  - Random matching (assumptions on observability of histories).

# Games with Public Monitoring

Continuing with prisoners' dilemma example

- Actions are not observed. There is a noisy signal of actions (output), $y \in \{\underline{y}, \bar{y}\} =: Y$,

$$\Pr(\bar{y} \mid a) := \rho(\bar{y} \mid a) = \begin{cases} p, & \text{if } a = EE, \\ q, & \text{if } a = SE \text{ or } SE, \text{ and} \\ r, & \text{if } a = SS. \end{cases}$$

- Realized payoffs, $u_i^*(a_i, y)$, and ex ante payoffs,

$$u_i(a_i, y) := \sum_y u_i^*(a_i, y)\rho(y \mid a).$$

- Player $i$'s histories, $H_i := \cup_{t \geq 0}(A_i \times Y)^t$.
- Public histories, $H := \cup_{t \geq 0} Y^t$.
- Pure strategy for player $i$, $\sigma_i : H_i \to A_i$.

# Equilibrium Notion

- Game has no proper subgames, so how to usefully capture sequential rationality?
- Public strategy for player $i$, $\sigma_i : H \to A_i$.

**Definition**

The profile of public strategies $\sigma^*$ is a perfect public eq if $\sigma^*|_{h^t}$ is a Nash eq for all histories $h^t \in H$, where $\sigma^*|_{h^t}$ is the continuation public profile induced by $h^t$.

# Characterizing PPE

## The Role of Continuation Values

- Let $\mathcal{E}^p(\delta) \subset \mathcal{F}^*$ be the set of (pure strategy) PPE.
- If $v \in \mathcal{E}^p(\delta)$, then there exists $a' \in A$ and $\gamma : Y \to \mathcal{E}^p(\delta)$ so that, for all $i$,

$$\begin{aligned} v_i &= (1-\delta)u_i(a') + \delta \sum\nolimits_y \gamma_i(y)\rho(y \mid a') \\ &\geq (1-\delta)u_i(a_i, a'_{-i}) + \delta \sum\nolimits_y \gamma_i(y)\rho(y \mid a_i, a'_{-i}) \quad \forall a_i \in A_i. \end{aligned}$$

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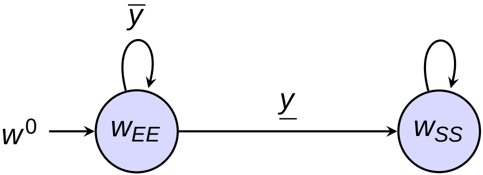
- (Self-generation, Abreu, Pearce and Stacchetti 1990) $B \subset \mathcal{E}^p(\delta)$ if and only if for all $v \in B$, there exists $a' \in A$ and $\gamma : Y \to B$ so that, for all $i$,

$$\begin{aligned} v_i &= (1-\delta)u_i(a') + \delta \sum\nolimits_y \gamma_i(y)\rho(y \mid a') \\ &\geq (1-\delta)u_i(a_i, a'_{-i}) + \delta \sum\nolimits_y \gamma_i(y)\rho(y \mid a_i, a'_{-i}) \quad \forall a_i \in A_i. \end{aligned}$$

# Example II

## Grim Trigger



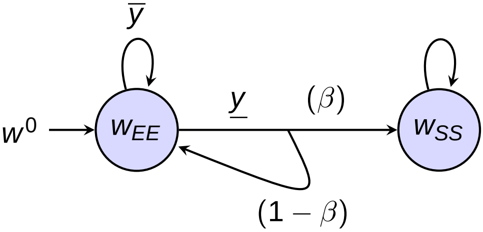

This is an eq if

$$\begin{aligned} V &= (1-\delta) + \delta[pV + (1-p)\times 0] \\ &\geq (1-\delta)2 + \delta[qV + (1-q)\times 0] \\ &\Rightarrow \frac{\delta(p-q)}{(1-\delta p)} \geq 1 \quad \iff \quad \delta \geq \frac{1}{2p-q}. \end{aligned}$$

Note that $V = \frac{(1-\delta)}{(1-\delta p)}$.

# Example III

## The value of “forgiveness”




This is an eq if

$$\begin{aligned} V &= (1-\delta) + \delta(p + (1-p)\beta)V \\ &\geq (1-\delta)2 + \delta(q + (1-q)\beta)V \end{aligned}$$

In the most efficient eq (requires $p > q$ and $\delta(2p - q) > 1$),

$$\beta = \frac{\delta(2p-q)1}{\delta(2p-q-1)} \quad \text{and} \quad V = 1 - \frac{1-p}{p-q} < 1.$$

# Implications

- Providing intertemporal incentives requires imposing punishments on the equilibrium path.
- These punishments may generate inefficiencies, and the greater the noise, the greater the inefficiency.
- How to impose punishments without creating inefficiencies: transfer value rather than destroying it.
- In PD example, impossible to distinguish $ES$ from $SE$. Similar problem arises in Green and Porter (1984).
- Other examples reveal the need for asymmetric/ nonstationary behavior in symmetric stationary environments.

# Another Folk Theorem

**"Theorem" (Fudenberg, Levine, and Maskin, 1994)**

Suppose the set of feasible and individually rational payoffs has nonempty interior, and that all action profiles are statistically distinguishable (pairwise full rank). Every strictly individually rational and feasible payoff is a PPE payoff, provided players are patient enough.

- Note that the monitoring can be arbitrarily noisy, as long as it remains statistically informative.
- Proof relies on a clever linear programming approach to bounding eq payoffs, due to Fudenberg and Levine (1994).

# Reputations I

- Noisy monitoring does not, in general, restrict eq payoffs.
- Reputation arguments continue to hold (Fudenberg and Levine, 1992; there is a beautiful proof by Gossner, 2011).
- But:

**“Theorem” (Cripps, Mailath, and Samuelson, 2004, 2007)**

Let $p_t$ denote the posterior prob assigned by the uninformed players to the behavioral type. Suppose the behavioral type’s action is not part of a static Nash eq, and monitoring has full support. Then, conditional on informed player not being a behavioral type, with probability one,

$$p_t \longrightarrow 0.$$

# Reputations II

## On the plus side

- The temporary reputations result does not hold when the uncertainty is continually refreshed, either because the types are themselves changing (Ekmekci, Gossner, and Wilson, 2012), or the uninformed agents have bounded memory (Liu, 2011).
- There is a glimmer of positive reputation results for two long-lived players (Cripps and Faingold, 2014).

# Games with Private Monitoring

- Intertemporal incentives arise when public histories coordinate continuation play.
- Can intertemporal incentives be provided when the monitoring is private?
- Stigler (1964) suggested that that answer is often NO, and so collusion is not likely to be a problem when monitoring problems are severe.
- While intuitive, we need to evaluate this argument within a formal model.

# The Problem

- Fix a strategy profile $\sigma$. Player $i$'s strategy $\sigma_i$ is sequentially rational if, after all private histories, $h_i^t$, the continuation strategy $\sigma_i \mid_{h_i^t}$ is a best reply to the other players' continuation strategies $(\sigma_j \mid_{h_j^t})_{j \neq i}$ (which depend on their private histories).
- That is, player $i$ is best responding to the other players' behavior, given his beliefs over the private histories of the other players.
- While player $i$ knows his beliefs, we typically do not.
- Most researchers thought this problem was intractable.

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- Most researchers thought this problem was intractable.

**"Theorem" (Sekiguchi, 1997)**

There exists an almost efficient eq for the PD with conditionally-independent almost-perfect private monitoring.

# Automaton Representation

- "Recall" that any public strategy profile can be represented by an automaton, $(\mathcal{W}, w^0, f, \tau)$, where
  - $\mathcal{W}$ is set of states,
  - $w^0 \in \mathcal{W}$ is the initial state,
  - $f : \mathcal{W} \to A$ is output function (decision rule), and
  - $\tau : \mathcal{W} \times Y \to \mathcal{W}$ is transition function.
- Similarly, any strategy can be represented by an automaton, $(\mathcal{W}_i, w_i^0, f_i, \tau_i)$, where
  - $\mathcal{W}_i$ is set of states,
  - $w_i^0 \in \mathcal{W}_i$ is $i$'s initial state,
  - $f_i : \mathcal{W}_i \to A_i$ is output function (decision rule), and
  - $\tau_i : \mathcal{W}_i \times A_i \times Y_i \to \mathcal{W}_i$ is transition function.

# A Helpful Observation

- Fix an automaton representation for player $j$. Consider the problem for player $i$ of predicting player $j$'s continuation play.
- In terms of continuation play, it is $i$'s belief over player $j$'s private state that is important.
- Since the set of states $\mathcal{W}_j$ partitions the set of histories, differences in beliefs over histories are only relevant in so far as they imply differences in beliefs over private states.

# Almost Public Monitoring

- How robust are PPE in the game with public monitoring to the introduction of private monitoring?
- Perturb the public signal, so that player $i$ observes the conditionally (on $y$) independent signal $y_i \in \{\underline{y}, \overline{y}\}$, with probabilities given by

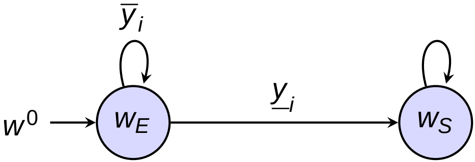
$$\pi(y_1, y_2 \mid y) = \pi_1(y_1 \mid y)\pi_2(y_2 \mid y),$$

and

$$\pi_i(y_i \mid y) = \begin{cases} 1 - \varepsilon, & \text{if } y_i = y, \\ \varepsilon, & \text{if } y_i \neq y. \end{cases}$$

# Grim Trigger

- Suppose $\frac{1}{2p-q} < \delta < 1$, so grim trigger is a strict PPE.
- Strategy in game with private monitoring is




- If $1 > p > q > r > 0$, profile is not a Nash eq (for any $\varepsilon > 0$).
  Consider private history $E\underline{y}_1, S\bar{y}_1, S\bar{y}_1, S\bar{y}_1, \cdots$
- If $1 > p > r > q > 0$, profile is a Nash eq (but not sequential).

# Behavioral Robustness

An eq is behaviorally robust if the same profile is an eq in near-by games.

A public profile has bounded recall if there exists $L$ such that after any history of length at least $L$, continuation play only depends on the last $L$ periods of the public history.

**"Theorem" (Mailath and Morris, 2002)**

A strict PPE with bounded recall is behaviorally robust to private monitoring that is almost public.

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**“Theorem” (Mailath and Morris, 2006)**

If the private monitoring is sufficiently rich, a strict PPE is behaviorally robust to private monitoring that is almost public if and only if it has bounded recall.

# Bounded Recall I

Because of these results, it is tempting to think that bounded recall provides an attractive restriction on behavior. Bounded recall may capture some notion of simplicity and bounded rationality.

- The only strongly symmetric bounded recall equilibrium in some parameterizations of the public monitoring repeated PD is perpetual *SS* (Cole and Kocherlakota, 2005). For the same parameterizations, the set of strongly symmetric infinite recall PPE is strictly larger.

# Bounded Recall II

But:

**“Theorem” (Hörner and Olszewski, 2009)**

The public monitoring folk theorem holds using bounded recall strategies. The folk theorem also holds using bounded recall strategies for games with almost-public monitoring.

- Eq use “communication phases” to encode histories before the recall dates (actions convey information). If cheap talk is possible, public monitoring result is immediate (the private monitoring result requires more work, Compte 1998 and Kandori & Matsushima 1998).
- This private monitoring folk theorem is not behaviorally robust. Eq incentives rely on indifferences (and a public correlating device), with behavior depending upon the fine details of the monitoring.

# Bounded Recall III

**“Theorem” (Mailath and Olszewski, 2011)**

The perfect monitoring folk theorem holds using bounded recall strategies with uniformly strict incentives. Moreover, the resulting equilibrium is behaviorally robust to almost-perfect almost-public monitoring.

- These strategies also use “communication phases” to encode histories before the recall dates.
- While behaviorally robust, the constructed profiles are not simple!

# Belief-Free Equilibria

Another approach is to specify behavior in such a way that the beliefs are irrelevant.

**Definition**

The profile $((\mathcal{W}_1, w_1^0, f_1, \tau_1), (\mathcal{W}_2, w_2^0, f_2, \tau_2))$ is a belief-free eq if for all $(w_1, w_2) \in \mathcal{W}_1 \times \mathcal{W}_1$, $(\mathcal{W}_i, w_i, f_i, \tau_i)$ is a best reply to $(\mathcal{W}_j, w_j, f_j, \tau_j)$, all $i \neq j$.

This approach is due to Piccione (2002), with a refinement by Ely and Valimaki (2002). Belief-free eq are characterized by Ely, Hörner, and Olszewski (2005).

# Illustration of Belief Free Eq

## The product-choice game redux

| | $c$ | $s$ |
|---|---|---|
| $H$ | $2, 3$ | $0, 2$ |
| $L$ | $3, 0$ | $1, 1$ |

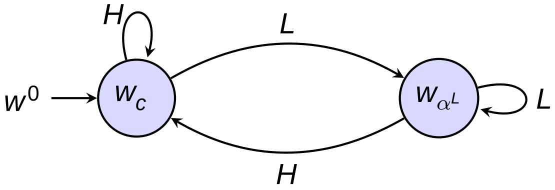
- Row player is a firm choosing $H$igh or $L$ow quality.
- Column player is a short-lived customer choosing the $c$ustomized or $s$tandard product.
- In the game with perfect monitoring, grim trigger (play $Hc$ till 1 plays $L$, then revert to perpetual $Ls$) is an eq if $\delta \geq \frac{1}{2}$.

The belief-free eq that achieves a payoff of 2 for the row player:

- Row player always plays $\frac{1}{2} \circ H + \frac{1}{2} \circ L$. (Trivial automaton)
- Column player's strategy has one period memory. Play $c$ for sure after $H$ in the previous period, and play

$$\alpha^L := \left(1 - \frac{1}{2\delta}\right) \circ c + \frac{1}{2\delta} \circ s$$

after $L$ in the previous period. Player 2's automaton:



- Histories are not being used to coordinate continuation play!
- This is to be contrasted with strict PPE.
- Rather, lump sum taxes are being imposed after “deviant” behavior is “suggested.”
- This is essentially what Ely Valimaki (2002) do in the repeated prisoners’ dilemma.

# Imperfect Monitoring

- This works for public and private monitoring.
- No hope for behavioral robustness.

**"Theorem" (Hörner and Olszewski, 2006)**

The folk theorem holds for games with private almost-perfect monitoring.

- Result uses belief-free ideas in a central way, but the equilibria constructed are not belief free.

# Purifiability

- Belief-free equilibria typically have the property that players randomize the same way after different histories (and so with different beliefs over the private states of the other player(s)).
- Harsanyi (1973) purification is perhaps the best rationale for randomizing behavior in finite normal form games.
- Can we purify belief-free equilibria (Bhaskar, Mailath, and Morris, 2008)?
  - The one period memory belief free equilibria of Ely and Valimaki (2002), as exemplified above, is not purifiable using one period memory strategies.
  - They are purifiable using unbounded memory strategies.
  - Open question: can they be purified using bounded memory strategies? (It turns out that for sequential games, only Markov equilibria can be purified using bounded memory strategies, Bhaskar, Mailath, and Morris 2013).

# What about noisy monitoring?

Current best result is Sugaya (2013):

**"Theorem"**

The folk theorem generically holds for the repeated two-player prisoners' dilemma with private monitoring if the support of each player's signal distribution is sufficiently large. Neither cheap talk communication nor public randomization is necessary, and the monitoring can be very noisy.

# Ex Post Equilibria

- The belief-free idea is very powerful.
- Suppose there is an unknown state determining payoffs and monitoring.

| $\omega_E$ | $E$ | $S$ |
|---|---|---|
| $E$ | $1, 1$ | $-1, 2$ |
| $S$ | $2, -1$ | $0, 0$ |

| $\omega_S$ | $E$ | $S$ |
|---|---|---|
| $E$ | $0, 0$ | $2, -1$ |
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- Let $\Gamma(\delta; \omega)$ denote the complete-information repeated game when state $\omega$ is common knowledge. The monitoring may be perfect or imperfect public.

# Perfect Public Ex Post Equilibria

$\Gamma(\delta; \omega)$ is complete-information repeated game at $\omega$.

**Definition**

The profile of public strategies $\sigma^*$ is a perfect public ex post eq if $\sigma^*|_{h^t}$ is a Nash eq of $\Gamma(\delta; \omega)$ for all histories $h^t \in H$, where $\sigma^*|_{h^t}$ is the continuation public profile induced by $h^t$.

- These equilibria can be strict; histories do coordinate play.
- But the eq are belief free.

# Perfect Public Ex Post Equilibria

$\Gamma(\delta;\omega)$ is complete-information repeated game at $\omega$.

**Definition**

The profile of public strategies $\sigma^*$ is a perfect public ex post eq if $\sigma^*|_{h^t}$ is a Nash eq of $\Gamma(\delta;\omega)$ for all histories $h^t \in H$, where $\sigma^*|_{h^t}$ is the continuation public profile induced by $h^t$.

- These equilibria can be strict; histories do coordinate play.
- But the eq are belief free.

**"Theorem" (Fudenberg and Yamamoto 2010)**

Suppose the signals are statistically informative (about actions and states). The folk theorem holds state-by-state.

These ideas also can be used in some classes of reputation games (Hörner and Lovo, 2009) and in games with private monitoring (Yamamoto, 2014).

# Conclusion

- The current theory of repeated games shows that the long relationships can discourage opportunistic behavior, it does not show that long run relationships will discourage opportunistic behavior.
- The current set of tools are well suited to proving folk theorems, but do not to provide a positive theory of behavior in long run relationships.
  - In particular, the tools typically focus on continuation values and patient players.
  - They do not focus on behavior directly.
- There are some suggestive results with bounded recall or memory.
- The reputations literature has a larger positive content, but has serious limitations.
- We need a better positive theory of behavior in repeated games.