

To a large extent, this is due to the focus on individual states. If we instead consider sets of states (strategy profiles), as I discussed at the end of Section 5.1, there is hope for more positive results.<sup>37</sup>

The lack of support for the standard refinement of backward induction is in some ways a success. Backward induction has always been a problematic principle, with some examples (like the centipede game) casting doubt on its universal applicability. The reasons for the lack of support improve our understanding of when backward induction is an appropriate principle to apply.

The ability to discriminate between different strict equilibria and provide a formalization of the intuition of strategic uncertainty is also a major contribution of the area.

I suspect that the current evolutionary modeling is still too stylized to be used directly in applications. Rather, applied researchers need to be aware of what they are implicitly assuming when they do equilibrium analysis.

In many ways, there is an important parallel with the refinements literature. Originally, this literature was driven by the hope that theorists could identify the unique “right” equilibrium. If that original hope had been met, applied researchers need never worry about a multiplicity problem. Of course, that

hope was not met, and we now understand that that hope, in principle, could never be met. The refinements literature still serves the useful role of providing a language to describe the properties of different equilibria. Applied researchers find the refinements literature of value for this reason, even though they cannot rely on it mechanically to eliminate “uninteresting” equilibria. The refinements literature is currently out of fashion because there were too many papers in which one example suggested a minor modification of an existing refinement and no persuasive general refinement theory emerged.

There is a danger that evolutionary game theory could end up like refinements. It is similar in that there was a lot of early hope and enthusiasm. And, again, there have been many perturbations of models and dynamic processes, not always well motivated. As yet, the overall picture is still somewhat unclear.

However, on the positive side, important insights are still emerging from evolutionary game theory (for example, the improving understanding of when backward induction is appropriate and the formalization of strategic uncertainty). Interesting games have many equilibria, and evolutionary game theory is an important tool in understanding which equilibria are particularly relevant in different environments.

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- <sup>37</sup> Sets of strategy profiles that are asymptotically stable under plausible deterministic dynamics turn out also to have strong Elon Kohlberg and Jean-Francois Mertens (1986) type stability properties (Swinkels (1993)), in particular, the property of robustness to deletion of never weak best replies. This latter property implies many of the refinements that have played an important role in the refinements literature and signaling games, such as the intuitive criterion, the test of equilibrium domination, and D1 (In-Koo Cho and Kreps 1987). A similar result under different conditions was subsequently proved by Ritzberger and Weibull (1995), who also characterize the sets of profiles that can be asymptotically stable under certain conditions.
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