Time Consistency, Reputations, and the Importance of Perpetual Uncertainty: Implications for Macroeconomics

June 23, 2003

Introduction

“Reputations” play a central role in the study of time consistency in policy, and more generally, moral hazard.

Reputation has two meanings in the literature, one motivated by repeated game considerations, and the other by incomplete information.

I will argue that the repeated-game view of reputation is not particularly helpful, and that (based on joint work Martin Cripps and Larry Samuelson) the approach based on incomplete information often requires some mechanism to maintain asymptotically uncertainty about an actor’s characteristics.

We begin with a one-period economy consisting of competitive agents (modelled as a continuum of identical agents) and a government.

From an exogenous endowment of 2, each agent decides on a level of investment (low, 1 or high, 2). Output $= 2\times$investment.

The government then chooses the tax rate, τ, on output. Tax receipts are used to provide a public good, g per \$ tax receipt.

If θ agents choose high investment, then total output is

$$\theta \cdot 4 + (1-\theta) \cdot 2 = 2 + 2\theta.$$

Public good provision is $\qquad g\tau(2+2\theta).$

An agent's utility from high investment is

$$(1-\tau)\cdot 4 + g\tau(2+2\theta),$$

and from low investment,

$$1 + (1-\tau)\cdot 2 + g\tau(2+2\theta).$$

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Average utility is

$$3 + \theta + \tau(g-1)(2+2\theta).$$

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If $g < 2$ and the government can commit ex ante to a tax regime, then the unique outcome is all agents choosing high investment under the low tax regime.

This is the Ramsey (or Stackelberg) outcome. The agents are, of course, better off in this outcome than in the unique equilibrium outcome.

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On the other hand, if society is infinitely lived (and agents are sufficiently patient and/or g is not too much larger than 1, there are many equilibria:

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But should one period of high taxes result in a dramatic loss of reputation?

More plausible to allow for the possibility of imperfect monitoring of the government's intended actions.

Suppose the implemented tax regime is the government's choice with probability $1 - \epsilon > \frac{1}{2}$, and the other regime with probability $\epsilon > 0$.

Model abstracts from uncertainty. Whether some level of taxes is high or low will typically depend on the realization of the state of the world, about which actors are asymmetrically informed.

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A version of the folk theorem still holds under imperfect monitoring, but the greatest payoff (independent of discounting) to the government in any equilibrium is

$$3 + g - \frac{2\epsilon(g-1)}{1-2\epsilon}.$$

(Recall that the payoff to the government, under perfect monitoring, in the high investment/low tax regime is $3 + g$.)

However (since this is an example of the folk theorem):

This explanation is too powerful. It can support outcomes such as low investment/high taxes on prime dates, and high investment/low taxes at all other times.

Moreover, in richer environments, it can support outcomes with very low payoffs of the government and agents.

Repeated game-type arguments do not provide a theory of reputations.

Incomplete information

Can uncertainty about the characteristics of the government allow the government to develop a reputation for low taxes?

The gov't as previously described is the normal type.

Two possibilities of interest:

Commitment types and inept (opportunistic, corrupt) types.

A commitment type necessarily chooses low taxes in every period.

An inept type necessarily chooses high taxes in every period.

Reputation as separation

The gov't is either inept or normal.

Ex ante, agents do not know type of the gov't, with prior $0 < \phi_0 < 1$ on normal.

There are many equilibria.

Consider Markov perfect equilibria—behavior of all actors depends only on the prior of the uninformed competitive agents.

Can the desire to separate from inept types provide a rationale for a low tax reputation?

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In any Markov perfect eq in which the gov't chooses low taxes for some belief ϕ, there exist two sequences of beliefs $\{\phi_k\}$ and $\{\phi_k'\}$ with

$$\lim_{k\to\infty} \phi_k = \lim_{k\to\infty} \phi_k' = 1$$

such that

$$\lim_{k\to\infty} \tau\left(\phi_k\right) > \lim_{k\to\infty} \tau(\phi_k'),$$

where $\tau(\phi)$ is the probability of low taxes at belief ϕ.

Intuition:

Candidate eq where normal gov't always chooses low taxes.

Eventually, if the gov't is indeed normal, uninformed agents assign a probability very close to 1 they face a normal gov't.

Any one signal of high taxes (indeed, any finite sequence of high taxes for sufficiently high beliefs) does not change the belief of the agents enough to lead to low investment.

But then, the normal gov't has no incentive not to choose high taxes.

If the imperfections in monitoring are asymmetric, there is an eq in which the normal gov't chooses low taxes until the first high taxes is observed, after which behavior reverts to the low investment/high tax regime.

This is a Markovian implementation of trigger behavior.

Perpetual uncertainty

Problem is uninformed agents become too confident about the type of the gov't.

Suppose the type of the gov't is subject to unobserved shocks. In particular, in each period, with probability θ, the gov't's type switches from normal to inept (and conversely, although this is less important).

Uninformed agents do not observe a switch, if it occurs.

Now, the uninformed agents can never be too confident about the type of the gov't.

Thm (Mailath/Samuelson '01) Fix the switching probability $\theta \in (0,1)$ and the discount factor $\delta \in (0,1)$. If public good production is not too attractive (i.e., there exists $\bar{g} > 1$ such that $g \in [1, \bar{g})$), there is a Markov perfect eq in which the normal type of gov't always chooses low taxes.

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In this eq, when beliefs are pessimistic, agents choose low investment since they assign high probability to the gov't being inept. Nonetheless, the normal gov't chooses low taxes in an attempt to signal to investors (recall monitoring is imperfect) that the gov't is not inept.

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The critical observation is that, with replacements, the expected time to an observation of low taxes rather than high taxes changing behavior of investors is uniformly bounded (for all eq beliefs).

Reputation as pooling

The gov't is either normal or the commitment type.

For $\varepsilon > 0$ small, if agents are patient, then the normal gov't's ex ante eq payoff is at least

$$3 + g - \varepsilon > 3 + g - \frac{2\epsilon(g-1)}{1-2\epsilon},$$

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Note that, for patient agents, the outcome in which the normal type of gov't always chooses high tax, and agents low investment is inconsistent with eq.

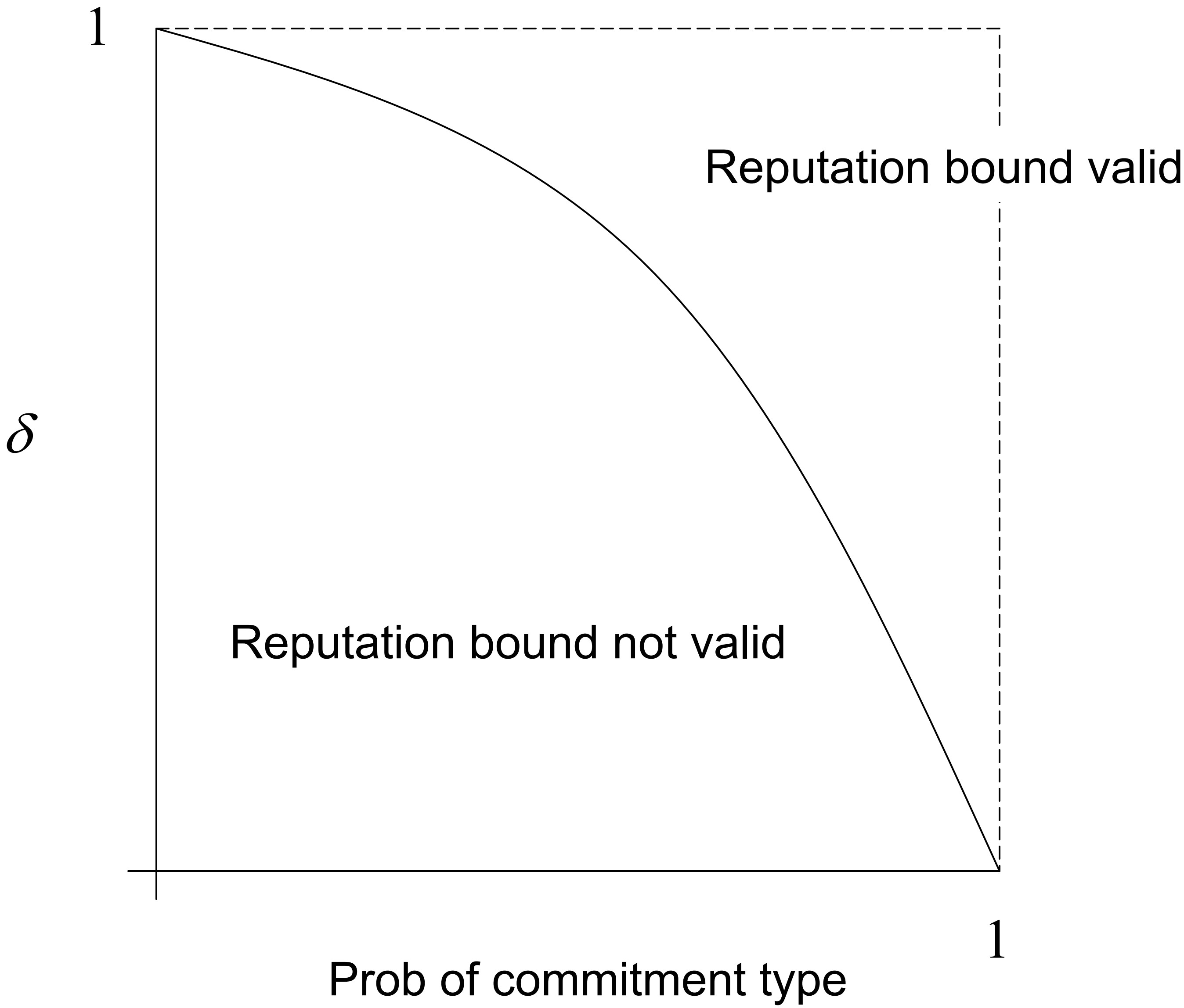
Intuition: Fix an equilibrium.

A lower bound on the normal type's eq payoff is given by the payoff from always choosing low taxes.

Eventually, the competitive agents must assign a high probability to low taxes in the next period, and so choose high investment.

Eventually is a function of the monitoring and the prior, but not of discounting.

Then, if the gov't is sufficiently patient, the contribution to payoffs from periods before eventually is small.


1
δ
Reputation bound valid
Reputation bound not valid
1
Prob of commitment type

Asymptotic reputations

Consider Markov equilibria.

State is the uninformed agents' posterior belief (the gov't's reputation).

In equilibrium, in every period normal type must play high taxes with positive probability.

$\Longrightarrow$ eventually uninformed agents learn that the gov't is the normal type.

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What about other equilibria?

(Markov perfect in complete information game $\Longrightarrow$
(low investment, high tax) always).

Some intuitions

First suppose reputation does not disappear.
Then $\exists$ an eq with both types given positive probability in the limit.

$\Rightarrow$ agents cannot distinguish between signals received from two types

$\Rightarrow$ he believes normal and commitment types playing same strategies.

$\Rightarrow$ he should be choosing high investment.

The normal type can then benefit by choosing high taxes for 1 period.

Contradiction of Equilibrium

Mixed commitment types

The commitment type may be mixed. For example, the action type's payoffs are subject to shocks. In some periods, the commitment type just likes to tax, but not in others.

Suppose the commitment type is the simple type that chooses low tax with probability $\frac{2}{3}$ and high tax with probability $\frac{1}{3}$. The best response of the agents is still to choose high investment. The payoff from this strategy exceeds the payoff from choosing low tax, and provides a higher FL bound.

The possibility of mixed commitment types complicates the inference problem for agents.

Impermanent reputations

At any Nash eq, the normal type's reputation in period t is the uninformed agents' posterior belief that the gov't is the commitment type, denoted p_t.

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Asymptotic equilibrium play

Thm: In any Nash eq, if the gov't is normal, with probability one, behavior converges to a Nash eq of the complete information game.

Asymptotic restrictions on behavior

Moreover, under a continuity assumption, we have:

Given any prior p_0 and any δ, for all $\varepsilon > 0$, there exists a Nash equilibrium of the incomplete information game, such that, if the gov't is normal, the probability of the event that eventually low investment/high taxes occurs in every period is at least $1 - \varepsilon$.

This is true even if the prior and discount factor pair, (p_0, δ), is in the parameter region where the reputation bound of Fudenberg and Levine is valid.

The results are significantly more general than the example:

- Many commitment types.
- More complicated commitment types (including payoff types).
- Long-lived competitive agents, as long as the commitment type is a finite-state public automaton and the competitive agents' behavior is "well-behaved."
- Game has perfect monitoring, but commitment type of long-lived player is a behavior strategy that has full support.

- The public signal of the gov't's action depends on the agents' actions through an aggregate statistic.

- Private beliefs. In the game-theoretic context, with a finite number of uninformed players whose actions (and so beliefs) are private.

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If reputation is due to a desire to separate from inept types, then need perpetual uncertainty about types to have any eq reputation.

If reputation arises from the possible existence of commitment types, then reputation is transitory in the absence of perpetual uncertainty. (In some situations, this is enough—the establishment of a new institution.)

However, uncertainty about types is continually refreshed (Holmstrom('82/99), Cole, Dow, and English ('95), Mailath and Samuelson ('01), Phelan ('01)).