



**Repeated Games
and Reputations**

Long-Run Relationships

George J. Mailath and Larry Samuelson

OXFORD
UNIVERSITY PRESS
2006

Contents

1 Introduction	1
1.1 Intertemporal Incentives	1
1.2 The Prisoners' Dilemma	3
1.3 Oligopoly	4
1.4 The Prisoner's Dilemma under Imperfect Monitoring	5
1.5 The Product-Choice Game	7
1.6 Discussion	8
1.7 A Reader's Guide	10
1.8 The Scope of the Book	10

Part I **Games with Perfect Monitoring**

2 The Basic Structure of Repeated Games with Perfect Monitoring	15
2.1 The Canonical Repeated Game	15
2.1.1 <i>The Stage Game</i>	15
2.1.2 <i>Public Correlation</i>	17
2.1.3 <i>The Repeated Game</i>	19
2.1.4 <i>Subgame-Perfect Equilibrium of the Repeated Game</i>	22
2.2 The One-Shot Deviation Principle	24
2.3 Automaton Representations of Strategy Profiles	29
2.4 Credible Continuation Promises	32
2.5 Generating Equilibria	37
2.5.1 <i>Constructing Equilibria: Self-Generation</i>	37
2.5.2 <i>Example: Mutual Effort</i>	40
2.5.3 <i>Example: The Folk Theorem</i>	41
2.5.4 <i>Example: Constructing Equilibria for Low δ</i>	44
2.5.5 <i>Example: Failure of Monotonicity</i>	46
2.5.6 <i>Example: Public Correlation</i>	49
2.6 Constructing Equilibria: Simple Strategies and Penal Codes	51
2.6.1 <i>Simple Strategies and Penal Codes</i>	51
2.6.2 <i>Example: Oligopoly</i>	54
2.7 Long-Lived and Short-Lived Players	61
2.7.1 <i>Minmax Payoffs</i>	63
2.7.2 <i>Constraints on Payoffs</i>	66

3 The Folk Theorem with Perfect Monitoring	69
3.1 Examples	70
3.2 Interpreting the Folk Theorem	72
3.2.1 <i>Implications</i>	72
3.2.2 <i>Patient Players</i>	73
3.2.3 <i>Patience and Incentives</i>	75
3.2.4 <i>Observable Mixtures</i>	76
3.3 The Pure-Action Folk Theorem for Two Players	76
3.4 The Folk Theorem with More than Two Players	80
3.4.1 <i>A Counterexample</i>	80
3.4.2 <i>Player-Specific Punishments</i>	82
3.5 Non-Equivalent Utilities	87
3.6 Long-Lived and Short-Lived Players	91
3.7 Convexifying the Equilibrium Payoff Set Without Public Correlation	96
3.8 Mixed-Action Individual Rationality	101
4 How Long Is Forever?	105
4.1 Is the Horizon Ever Infinite?	105
4.2 Uncertain Horizons	106
4.3 Declining Discount Factors	107
4.4 Finitely Repeated Games	112
4.5 Approximate Equilibria	118
4.6 Renegotiation	120
4.6.1 <i>Finitely Repeated Games</i>	122
4.6.2 <i>Infinitely Repeated Games</i>	134
5 Variations on the Game	145
5.1 Random Matching	145
5.1.1 <i>Public Histories</i>	146
5.1.2 <i>Personal Histories</i>	147
5.2 Relationships in Context	152
5.2.1 <i>A Frictionless Market</i>	153
5.2.2 <i>Future Benefits</i>	154
5.2.3 <i>Adverse Selection</i>	155
5.2.4 <i>Starting Small</i>	158
5.3 Multimarket Interactions	161
5.4 Repeated Extensive Forms	162
5.4.1 <i>Repeated Extensive-Form Games Have More Subgames</i>	163
5.4.2 <i>Player-Specific Punishments in Repeated Extensive-Form Games</i>	165
5.4.3 <i>Extensive-Form Games and Imperfect Monitoring</i>	167
5.4.4 <i>Extensive-Form Games and Weak Individual Rationality</i>	168
5.4.5 <i>Asynchronous Moves</i>	169
5.4.6 <i>Simple Strategies</i>	172

5.5	Dynamic Games: Introduction	174
5.5.1	<i>The Game</i>	175
5.5.2	<i>Markov Equilibrium</i>	177
5.5.3	<i>Examples</i>	178
5.6	Dynamic Games: Foundations	186
5.6.1	<i>Consistent Partitions</i>	187
5.6.2	<i>Coherent Consistency</i>	188
5.6.3	<i>Markov Equilibrium</i>	190
5.7	Dynamic Games: Equilibrium	192
5.7.1	<i>The Structure of Equilibria</i>	192
5.7.2	<i>A Folk Theorem</i>	195
6	Applications	201
6.1	Price Wars	201
6.1.1	<i>Independent Price Shocks</i>	201
6.1.2	<i>Correlated Price Shocks</i>	203
6.2	Time Consistency	204
6.2.1	<i>The Stage Game</i>	204
6.2.2	<i>Equilibrium, Commitment, and Time Consistency</i>	206
6.2.3	<i>The Infinitely Repeated Game</i>	207
6.3	Risk Sharing	208
6.3.1	<i>The Economy</i>	209
6.3.2	<i>Full Insurance Allocations</i>	210
6.3.3	<i>Partial Insurance</i>	212
6.3.4	<i>Consumption Dynamics</i>	213
6.3.5	<i>Intertemporal Consumption Sensitivity</i>	219

Part II **Games with (Imperfect) Public Monitoring**

7	The Basic Structure of Repeated Games with Imperfect Public Monitoring	225
7.1	The Canonical Repeated Game	225
7.1.1	<i>The Stage Game</i>	225
7.1.2	<i>The Repeated Game</i>	226
7.1.3	<i>Recovering a Recursive Structure: Public Strategies and Perfect Public Equilibria</i>	228
7.2	A Repeated Prisoners' Dilemma Example	232
7.2.1	<i>Punishments Happen</i>	233
7.2.2	<i>Forgiving Strategies</i>	235
7.2.3	<i>Strongly Symmetric Behavior Implies Inefficiency</i>	239
7.3	Decomposability and Self-Generation	241
7.4	The Impact of Increased Precision	249
7.5	The Bang-Bang Result	251

7.6	An Example with Short-Lived Players	255
7.6.1	<i>Perfect Monitoring</i>	256
7.6.2	<i>Imperfect Public Monitoring of the Long-Lived Player</i>	260
7.7	The Repeated Prisoners' Dilemma Redux	264
7.7.1	<i>Symmetric Inefficiency Revisited</i>	264
7.7.2	<i>Enforcing a Mixed-Action Profile</i>	267
7.8	Anonymous Players	269
8	Bounding Perfect Public Equilibrium Payoffs	273
8.1	Decomposing on Half-Spaces	273
8.2	The Inefficiency of Strongly Symmetric Equilibria	278
8.3	Short-Lived Players	280
8.3.1	<i>The Upper Bound on Payoffs</i>	280
8.3.2	<i>Binding Moral Hazard</i>	281
8.4	The Prisoners' Dilemma	282
8.4.1	<i>Bounds on Efficiency: Pure Actions</i>	282
8.4.2	<i>Bounds on Efficiency: Mixed Actions</i>	284
8.4.3	<i>A Characterization with Two Signals</i>	287
8.4.4	<i>Efficiency with Three Signals</i>	289
8.4.5	<i>Efficient Asymmetry</i>	291
9	The Folk Theorem with Imperfect Public Monitoring	293
9.1	Characterizing the Limit Set of PPE Payoffs	293
9.2	The Rank Conditions and a Public Monitoring Folk Theorem	298
9.3	Perfect Monitoring Characterizations	303
9.3.1	<i>The Folk Theorem with Long-Lived Players</i>	303
9.3.2	<i>Long-Lived and Short-Lived Players</i>	303
9.4	Enforceability and Identifiability	305
9.5	Games with a Product Structure	309
9.6	Repeated Extensive-Form Games	311
9.7	Games of Symmetric Incomplete Information	316
9.7.1	<i>Equilibrium</i>	318
9.7.2	<i>A Folk Theorem</i>	320
9.8	Short Period Length	326
10	Private Strategies in Games with Imperfect Public Monitoring	329
10.1	Sequential Equilibrium	329
10.2	A Reduced-Form Example	331
10.2.1	<i>Pure Strategies</i>	331
10.2.2	<i>Public Correlation</i>	332
10.2.3	<i>Mixed Public Strategies</i>	332
10.2.4	<i>Private Strategies</i>	333
10.3	Two-Period Examples	334
10.3.1	<i>Equilibrium Punishments Need Not Be Equilibria</i>	334
10.3.2	<i>Payoffs by Correlation</i>	337
10.3.3	<i>Inconsistent Beliefs</i>	338

10.4	An Infinitely Repeated Prisoner's Dilemma	340
10.4.1	<i>Public Transitions</i>	340
10.4.2	<i>An Infinitely Repeated Prisoners' Dilemma: Indifference</i>	343
11	Applications	347
11.1	Oligopoly with Imperfect Monitoring	347
11.1.1	<i>The Game</i>	347
11.1.2	<i>Optimal Collusion</i>	348
11.1.3	<i>Which News Is Bad News?</i>	350
11.1.4	<i>Imperfect Collusion</i>	352
11.2	Repeated Adverse Selection	354
11.2.1	<i>General Structure</i>	354
11.2.2	<i>An Oligopoly with Private Costs: The Game</i>	355
11.2.3	<i>A Uniform-Price Equilibrium</i>	356
11.2.4	<i>A Stationary-Outcome Separating Equilibrium</i>	357
11.2.5	<i>Efficiency</i>	359
11.2.6	<i>Nonstationary-Outcome Equilibria</i>	360
11.3	Risk Sharing	365
11.4	Principal-Agent Problems	370
11.4.1	<i>Hidden Actions</i>	370
11.4.2	<i>Incomplete Contracts: The Stage Game</i>	371
11.4.3	<i>Incomplete Contracts: The Repeated Game</i>	372
11.4.4	<i>Risk Aversion: The Stage Game</i>	374
11.4.5	<i>Risk Aversion: Review Strategies in the Repeated Game</i>	375

Part III **Games with Private Monitoring**

12	Private Monitoring	385
12.1	A Two-Period Example	385
12.1.1	<i>Almost Public Monitoring</i>	387
12.1.2	<i>Conditionally Independent Monitoring</i>	389
12.1.3	<i>Intertemporal Incentives from Second-Period Randomization</i>	392
12.2	Private Monitoring Games: Basic Structure	394
12.3	Almost Public Monitoring: Robustness in the Infinitely Repeated Prisoner's Dilemma	397
12.3.1	<i>The Forgiving Profile</i>	398
12.3.2	<i>Grim Trigger</i>	400
12.4	Independent Monitoring: A Belief-Based Equilibrium for the Infinitely Repeated Prisoner's Dilemma	404
12.5	A Belief-Free Example	410
13	Almost Public Monitoring Games	415
13.1	When Is Monitoring Almost Public?	415
13.2	Nearby Games with Almost Public Monitoring	418

13.2.1	<i>Payoffs</i>	418
13.2.2	<i>Continuation Values</i>	419
13.2.3	<i>Best Responses</i>	421
13.2.4	<i>Equilibrium</i>	421
13.3	Public Profiles with Bounded Recall	423
13.4	Failure of Coordination under Unbounded Recall	425
13.4.1	<i>Examples</i>	425
13.4.2	<i>Incentives to Deviate</i>	427
13.4.3	<i>Separating Profiles</i>	428
13.4.4	<i>Rich Monitoring</i>	432
13.4.5	<i>Coordination Failure</i>	434
13.5	Patient Players	434
13.5.1	<i>Patient Strictness</i>	435
13.5.2	<i>Equilibria in Nearby Games</i>	437
13.6	A Folk Theorem	441
14	Belief-Free Equilibria in Private Monitoring Games	445
14.1	Definition and Examples	445
14.1.1	<i>Repeated Prisoners' Dilemma with Perfect Monitoring</i>	447
14.1.2	<i>Repeated Prisoners' Dilemma with Private Monitoring</i>	451
14.2	Strong Self-Generation	453

Part IV Reputations

15	Reputations with Short-Lived Players	459
15.1	The Adverse Selection Approach to Reputations	459
15.2	Commitment Types	463
15.3	Perfect Monitoring Games	466
15.3.1	<i>Building a Reputation</i>	470
15.3.2	<i>The Reputation Bound</i>	474
15.3.3	<i>An Example: Time Consistency</i>	477
15.4	Imperfect Monitoring Games	478
15.4.1	<i>Stackelberg Payoffs</i>	480
15.4.2	<i>The Reputation Bound</i>	484
15.4.3	<i>Small Players with Idiosyncratic Signals</i>	492
15.5	Temporary Reputations	493
15.5.1	<i>Asymptotic Beliefs</i>	494
15.5.2	<i>Uniformly Disappearing Reputations</i>	496
15.5.3	<i>Asymptotic Equilibrium Play</i>	497
15.6	Temporary Reputations: The Proof of Proposition 15.5.1	500
15.6.1	<i>Player 2's Posterior Beliefs</i>	500
15.6.2	<i>Player 2's Beliefs about Her Future Behavior</i>	502

15.6.3	<i>Player 1's Beliefs about Player 2's Future Behavior</i>	503
15.6.4	<i>Proof of Proposition 15.5.1</i>	509
16	Reputations with Long-Lived Players	511
16.1	The Basic Issue	511
16.2	Perfect Monitoring and Minmax-Action Reputations	515
16.2.1	<i>Minmax-Action Types and Conflicting Interests</i>	515
16.2.2	<i>Examples</i>	518
16.2.3	<i>Two-Sided Incomplete Information</i>	520
16.3	Weaker Reputations for Any Action	521
16.4	Imperfect Public Monitoring	524
16.5	Commitment Types Who Punish	531
16.6	Equal Discount Factors	533
16.6.1	<i>Example 1: Common Interests</i>	534
16.6.2	<i>Example 2: Conflicting Interests</i>	537
16.6.3	<i>Example 3: Strictly Dominant Action Games</i>	540
16.6.4	<i>Example 4: Strictly Conflicting Interests</i>	541
16.6.5	<i>Bounded Recall</i>	544
16.6.6	<i>Reputations and Bargaining</i>	546
16.7	Temporary Reputations	547
17	Finitely Repeated Games	549
17.1	The Chain Store Game	550
17.2	The Prisoners' Dilemma	554
17.3	The Product-Choice Game	560
17.3.1	<i>The Last Period</i>	562
17.3.2	<i>The First Period, Player 1</i>	562
17.3.3	<i>The First Period, Player 2</i>	565
18	Modeling Reputations	567
18.1	An Alternative Model of Reputations	568
18.1.1	<i>Modeling Reputations</i>	568
18.1.2	<i>The Market</i>	570
18.1.3	<i>Reputation with Replacements</i>	573
18.1.4	<i>How Different Is It?</i>	576
18.2	The Role of Replacements	576
18.3	Good Types and Bad Types	580
18.3.1	<i>Bad Types</i>	580
18.3.2	<i>Good Types</i>	581
18.4	Reputations with Common Consumers	584
18.4.1	<i>Belief-Free Equilibria with Idiosyncratic Consumers</i>	585
18.4.2	<i>Common Consumers</i>	586
18.4.3	<i>Reputations</i>	587
18.4.4	<i>Replacements</i>	588
18.4.5	<i>Continuity at the Boundary and Markov Equilibria</i>	590
18.4.6	<i>Competitive Markets</i>	594

18.5 Discrete Choices	596
18.6 Lost Consumers	599
18.6.1 <i>The Purchase Game</i>	599
18.6.2 <i>Bad Reputations: The Stage Game</i>	600
18.6.3 <i>The Repeated Game</i>	601
18.6.4 <i>Incomplete Information</i>	603
18.6.5 <i>Good Firms</i>	607
18.6.6 <i>Captive Consumers</i>	608
18.7 Markets for Reputations	610
18.7.1 <i>Reputations Have Value</i>	610
18.7.2 <i>Buying Reputations</i>	613
Bibliography	619
Symbols	629
Index	631