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RESEARCH ARTICLE

Toward an efficiency rationale for the public provision of private goods

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Abstract Public provision of a private goods is justified on efficiency grounds in a model with no redistributive preferences. A government's involvement in the provision of a private good generates information about preferences that facilitates more efficient revenue extraction for the provision of public goods. Public provision of the private good improves economic efficiency under a condition that is always fulfilled under independence and satisfied for an open set of joint distributions. The efficiency gains require that consumers cannot arbitrage the publicly provided private good, so our analysis applies to private goods where it is easy to keep track of the ultimate user, such as schooling and health care, but not to easily tradable consumer goods.

Keywords Publicly provided private goods · Constrained efficiency · Optimal taxation

JEL Classification D61 · D82 · H42

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1 Introduction

Governments at all levels not only provide public goods, but also devote considerable resources to the provision of private goods and services such as health care, insurance, child care, housing and education. This is puzzling in light of the standard public finance justifications for government intervention in the marketplace.

With a few exceptions, the existing literature takes for granted that public provision of private goods should be understood as an in-kind transfer.[1] The puzzle is then to explain the reasons for why governments choose to redistribute in-kind rather than to rely on cash transfers. Many possible justifications have been put forward in the literature, such as paternalistic preferences, self-targeting in environments with incomplete information, second best arguments to reduce inefficiencies in the tax code, commitment problems for altruistic donors, credit constraints and pecuniary effects. However, they all ultimately explain public provision of private goods as an *instrument to redistribute,* either from rich to poor or toward those who need a particular good or service.

The contribution of this paper is to provide a simple and plausible argument showing that public provision of a private good can be welfare improving also in an environment where there are no preferences for redistribution. Since some publicly provided private goods, such as publicly provided higher education, have neutral or regressive distributional effects, we believe that our explanation complements the existing literature in a highly relevant dimension.

We consider a model where consumers have preferences defined over a private good and an excludable public good. Preferences are private information to the individuals, agreements must be voluntary, and there are no outside resources available to finance the provision of the public good.

As a benchmark, we consider what we refer to as the *separate provision problem,* where *by assumption* the public good and the private good are provided separately. More exactly, we assume that information about the willingness to pay for the private good cannot affect the probability of access or the access fee for the public good and, symmetrically, that the willingness to pay for the public good cannot affect the price an individual pays for the private good. This can be interpreted as a problem facing two independent government agencies sharing the same objective function and operating under an integrated budget constraint, who cannot share information about individual consumers with each other. Apart from the separability assumption, we impose no other restrictions on the feasible mechanisms other than incentive compatibility, feasibility and individual rationality.

Under a mild technical assumption, the best separate provision mechanism is very simple and can be implemented by setting an access fee for the public good, a provision probability of the public good and charging a uniform sales tax on the private good. Hence, there is no need to centralize the provision of the private good in this case.

[1] We are aware of two exceptions. Manski (2010) who provides an argument based on the mathematics (Jensen's inequality) of aggregating individual payoffs in an environment of private decision making under uncertainty. Garratt and Marshall (1994) argue that public financing of college education creates a welfare-enhancing lottery because of indivisibilities.

We contrast the best separate provision mechanism with the case where the goods are jointly provided. By "joint provision," we mean that *all information* can be used for decisions on pricing and provision of *both* goods. This leads to a setup similar to commodity bundling problems in the industrial organization literature, with the difference that the objective is to maximize consumer welfare under a break-even constraint as opposed to maximize profits.

As types are multi-dimensional, we cannot derive the optimal joint provision mechanism analytically. However, our primary interest is the qualitative question of *when public provision of the private good is welfare improving*. For this, it is sufficient to demonstrate that there is *some joint provision mechanism that outperforms the best separate provision mechanism*. We therefore consider small perturbations from the best separate provision mechanism. Under a condition, which is satisfied by a large set of joint distributions, including the case when valuations are independent, there are perturbations that improve upon the best separate provision mechanism.

A constrained optimal mechanism must weakly outperform the perturbations we consider. Hence, any optimal joint provision mechanism must condition prices for private good consumption on valuations for the public good. This cannot be implemented using an anonymous market mechanism for the private good, which is why we interpret the non-separability between the markets as public provision of the private good. An alternative interpretation is as a non-linear optimal commodity tax, but it is crucial that consumption of both goods must be monitored.

The intuition for the result is best explained in terms of the benefits of bundling for a multi-product monopolist. It is generally desirable for the monopolist to sell the goods both as stand-alone products and in bundles containing more than one good. This is because the additional pricing instruments can be used so as to relax incentive constraints, which allows the provider to extract more revenue. Because the public good is under-provided (even when cross subsidies are allowed), the shadow price of revenue exceeds unity, implying that bundling of the public and the private good is welfare enhancing. Our model makes many simplifying assumptions: unit demands; transferable utility; a single public good; a single private good; and no outside revenue source. Extending the result to many public and private goods is easy, but most other simplifications are imposed to keep the separate provision benchmark simple. However, as long as raising revenue is distortionary on the margin and some socially valuable projects are under-funded, we believe that the logic should generalize.

While the model is stylized, there are many potential applications, some that may generate interesting extensions. Quantitatively, the most important examples of publicly provided private goods are health care and education. Our model complements the existing literature by offering an additional reason why it may be beneficial to let these arguably pure private goods be managed by some government institution.

Publicly provided or subsidized health care is provided because it allows for investments in infrastructure and other goods and services that generate positive externalities without making too many taxpayers worse off. Public schools are there as a part of a bundle set up to make local residents able to better provide adequate policing, fire services and other local public goods. While there are a number of plausible alternative explanations, it should be noted that a crucial part of our explanation is that the same government body provides both the private good as well as seeking to overcome a market failure.[2] In contrast, most existing explanations would suggest that the boundaries of the jurisdiction for each government provided good would be independent: an institution for smog abatement would ideally consist of the relevant "smogisphere," whereas fire and police districts would be set up based on other considerations. As seemingly independent goods and services tend to be provided by the same jurisdiction, we think that some form of bundling may be an important part of the puzzle.

Health care and education are also examples of services that often can be purchased either as part of a government bundle or as a stand-alone good. Many countries provide heavily subsidized health care to its citizens, but also allow foreign nationals to purchase care at substantially higher prices. For example, in the USA, Medicare eligibility requires that an individual pay into Medicare/Social Security taxes, and thus participate in both the Federal and State income tax system, for at least ten years prior to retirement. These income taxes of course fund most of the public goods such as infrastructure and national parks etc. Since foreign elderly did not pay the income taxes, they would have to pay for the health-care services at a much higher price if they were to come to the USA for their medical needs. Similarly, Canadian public schools are free for tax-paying residents, but foreigners can purchase access to K-12 education provided that capacity constraints do not bind.

We also note that if health care, schooling, elderly care, social security, daycare, etc. are nothing but mechanisms to transfer resources from wealthy to poor individuals, as much of the existing literature on publicly provided private goods assumes, it is unclear why such programs are open also to the wealthy. Means testing would be easy in most developed countries, but is often not used. The economics in our model provides an explanation for why means testing may be a bad idea: it could erode popular support for the programs and therefore reduce the tax revenue that can be collected to mitigate market failures.

As pointed out already, an unusual feature of our model is that public provision of private good is justified without any desire to redistribute resources. Not only does this imply that it may make sense to allow the wealthy to get access, it also explains how provision of goods with neutral or regressive distributional consequences can be desirable. An obvious example of this is public universities. All public universities in the USA charge higher tuition for out-of-state students than in-state students. In light of our model, we can interpret this is a scheme where higher education, a private good, is used to provide incentives for the citizens (i.e., parents of current and future college students) to remain in the state and pay taxes to finance necessary infrastructure, local public goods and other expenditures. Notice that this may be a good policy also if real-world policy makers have preferences for redistribution as budget constraints can be relaxed. By subsidizing in-state tuition, extra revenue can be generated that could be used also for redistributive purposes.

The model could, with a few adjustment, also be taken to other problems in economics. For example, provided that there is a market failure in the provision of a good, the screening benefits of bundling may provide a reason to subsidize that good

[2] See Sect. 2 for a detailed discussion of the existing literature.

as part of a compensation package to a worker. That is, one could think about employer provided health insurance and pension benefits as a way for firms to reduce labor costs.

Our analysis also illustrates a more general point. Since Ramsey (1927), it has been standard in the public finance literature to ask how to best raise a given target revenue without considering what the tax revenue is intended for. Similarly, public good provision and other public expenditure problems always take the size of any outside funds as given. This paper provides an example where the taxation and expenditure problems are non-separable: the optimal commodity tax to finance the public good should depend on whether the consumer gets access to the public good.[3] Hence, the analytically convenient dichotomy between government expenditures and revenue is not without loss of generality.

Just like in the literature that explains public provision of private goods as an in-kind transfer, it is crucial for our analysis that consumers cannot trade the private good *ex post*. Such arbitrage would restore a uniform price for the private good and therefore make bundling irrelevant. While this property is not unique to the setup in this paper, we nevertheless consider it a desirable property. The asymmetric information setup creates a distinction between anonymous commodity-like goods—which should be left to the market—and services where the identity of the final consumer is observable—which could be useful screening devices. The no arbitrage restriction also seems realistic for many goods that are publicly provided in the real world. For example, publicly provided education, health insurance, child care and health care are all difficult to resell. Hence, neither our theory nor complementary explanations that justify public provision as an in-kind transfer suggest that all private goods should be publicly provided.

The remainder of the paper is structured as follows. In Sect. 2, we review the related literature in greater details and explain our contribution relative existing models. Section 3 contains a motivating example, and Sect. 4 sets up a more general model and characterizes the best separate provision mechanism. In Sect. 5, we present the main result, which establishes that there is public provision of private goods in the constrained optimal mechanism. Finally, we conclude in Sect. 6.

2 Related literature

The novelty of our approach is that public provision of private goods is justified in an environment in which there are no preferences for redistribution. In contrast, the previous literature has focused almost exclusively on distributional effects. The two exceptions we are aware of are Garratt and Marshall (1994) and Manski (2010), but the arguments in these two papers are qualitatively very different from the considerations that make public provision desirable in this paper. In Garratt and Marshall (1994), the role of public provision is to convexify payoffs by providing lotteries to government-financed higher education, and in Manski (2010), public provision reduces uncertainty in payoffs because the consensus choice is closer to an underlying state of nature that is unknown by the agents.

[3] Related points are made in Boadway and Keen (1993), Boadway et al. (1998) and Blomquist et al. (2010).

In contrast, the literature on in-kind redistribution is huge. We will mainly discuss the literature based on informational asymmetries, which has some similarities with our model, but the reader may consult Currie and Firouz (2007) for a more complete discussion. However, many of the most well-known papers explain public provision of private goods as a political economy phenomenon. Influential examples are Fernandez and Rogerson (1995) and Epple and Romano (1996), and more recent contributions in this strand include Levy (2005), Kimura and Yasui (2009) and Borck and Wrohlich (2011). There is also a long tradition, going back to Musgrave, Richard (1959), of interpreting public provision of certain goods as paternalism.[4]

More closely related to our approach, there is also a large literature that takes a normative perspective. Some of these papers assume that the transfers must be provided in-kind for reasons exogenous to the model. For example, Besley and Coate (1991) consider a model where households may opt out from public provision and purchase a higher-quality version of the good if they are dissatisfied with the quality of the publicly provided good. If the willingness to pay for quality is increasing in wealth, mainly rich households opt out from the publicly provided private goods, implying that the system of public provision of private goods can serve as a transfer toward poor individuals.[5]

However, it has long been recognized that there are circumstances where in-kind transfers make it easier to target the intended recipients than cash transfers. This is formalized by Nichols and Zeckhauser (1982) and Blackorby and Donaldson (1988), who construct models where in-kind transfers are superior to cash transfers because of its screening role. This argument may be particularly important in the case of health care, as it seems that the incentive to lie about health status to obtain cash is a much more severe problem than the incentive to lie to obtain a medical procedure.

In-kind transfers have also been studied in the literature on optimal taxation. As in the political economy literature, the underlying driving force is a desire to redistribute from the rich to the poor, but this literature allows for a richer set of transfer instruments. Most papers in this literature, such as Blomquist and Christiansen (1995), Cremer and Gahvari (1997) and Blomquist et al. (2010), assume that preferences over leisure and the publicly provided good are non-separable. The role of public provision of private goods may therefore be interpreted as a tool to reduce the inefficiencies associated with income taxation. Note that the logic is similar to that in our paper in that the underlying problem is an inefficiency unrelated to the private good (taxes distorting labor supply decisions or under-provision of a public good) and that public provision is an "indirect" way to mitigate the fundamental inefficiency.

The need for preferences to be non-separable in these models is due to the famous Atkinson and Stiglitz (1976) result that commodity taxation is useless when preferences are separable. However, if types are multi-dimensional, it has been showed by Cremer et al. (2002) that this result does not generalize, so public provision could be useful also in this case. Again, it is interesting to note that there is a parallel with our

[4] Paternalism is again being considered due to the recent interest in irrational behavior. For example, Pirttilä and Tenhunen (2008) consider a behavioral model where public provision is efficiency enhancing because of bounded rationality.

[5] Gahvari and Mattos (2007) extends Besley and Coate (1991). See also Blomquist and Christiansen (1999).

model, as ultimately it is the multi-dimensional type space that makes it desirable to bundle the public and private goods. Limited commitment by altruistic donors can also justify public provision of private goods. Coate (1995) analyzed an environment where the rich has altruistic preferences toward the poor and would like to insure the poor's income risks. If the poor are given cash, they may still opt not to purchase insurance to exploit the Samaritan's Dilemma. As a result, the rich may prefer to give the poor an in-kind transfer of insurance to overcome the difficulty of committing not to provide *ex post* assistance. A very similar argument is considered by Bruce and Waldman (1991), with the difference that the efficiency problem is that *ex post* transfers harm incentives to accumulate human capital.

Finally, our paper is related to a large literature on commodity bundling. Of particular relevance is McAfee et al. (1989), where a condition for when mixed bundling improves the profit for a multi-product monopolist. Our condition shares some terms with this condition, but is different for two reasons. First of all, we are trying to find ways to use bundling to increase consumer welfare rather than profits, which generates a constrained optimization problem with a zero profit constraint. Secondly, our non-bundling benchmark is also a constrained optimization problem, considerably more complicated than a standard monopoly problem. This requires us to link multiplier values across optimization problems, which is one of the main analytical difficulties relative (McAfee et al. 1989) and also makes it a non-trivial exercise to demonstrate that our condition holds when valuations are independent.

3 An example

The simplest example to illustrate the benefits of bundling is when the valuations are perfectly negatively correlated.[6] Take the extreme case with the valuation of a public good, θ_G, and the willingness to pay for a private good, θ_P, are both uniformly distributed, but where $\theta_P = 1 - \theta_G$. If goods are sold separately, it is necessary to exclude some customers from either the public or the private good as otherwise no revenue can be raised. In contrast, if the planner taxes all consumers any sum less than unity and gives away both goods to anyone willing to pay the tax there are no longer any consumers excluded from usage. Hence, public provision can achieve first best in this case.

For a somewhat less extreme example assume that θ_G and θ_P are independent and uniformly distributed on $[0, 1]$ and that utility is quasi-linear in money, with no credit constraints for the consumers. In this case, it will no longer be possible to achieve first best, but it will nevertheless be the case that public provision may outperform the Ramsey taxation benchmark for a range of cost parameters. Later, we will consider arbitrary smooth joint densities.

The private good is produced at constant unit cost c. In contrast, the public good can be provided to everyone in the economy by incurring a fixed cost K, but there is a technology in place making use exclusions feasible at no cost. There are sev-

[6] Indeed, this is the case considered by Adams and Yellen (1976) in their seminal paper on commodity bundling.

eral possible interpretations. One is that the good is a local public good. If the local government can enforce tax collection within its jurisdiction, but where citizens can move elsewhere if they want, then the exclusion technology is that those who move elsewhere to avoid the taxes also loose access to the local public goods. Of course, the use exclusions can also be explicit restrictions on access to the government provided non-rival good.

3.1 Self financing mechanisms

As a first step, imagine that the public good must be self-financing. Let π be the probability that the good is provided and f be the user fee.[7] The best self-financing mechanism $\langle \pi, f \rangle$ is thus the solution to the problem

$$\max_{\{\pi, f\}} \pi \int_f^1 \theta_G \mathrm{d}\theta_G \tag{1}$$

$$\text{s.t.} \quad f(1-f) \geq \pi K, \tag{2}$$

which has solution

$$\pi^* = \begin{cases} 1 & \text{if } K \leq \frac{1}{4} \\ 0 & \text{if } K > \frac{1}{4}, \end{cases} \qquad f^* = \begin{cases} \frac{1}{2} - \sqrt{\frac{1}{4} - K} & \text{if } K \leq \frac{1}{4} \\ 0 & \text{if } K > \frac{1}{4}. \end{cases} \tag{3}$$

To understand (3), note that a profit maximizer would set access fee $f = \frac{1}{2}$, resulting in maximized revenue of $\frac{1}{4}$. Hence, it is possible to provide the good and break even if and only if $K \leq \frac{1}{4}$, and, from a social standpoint, it is always desirable to provide the public good if K is in this range. The constrained optimal fee is then obtained by solving the quadratic equation one obtains by setting $\pi = 1$ and requiring that constraint (2) is satisfied with equality.

3.2 Ramsey taxation

Next, suppose that, in the spirit of Ramsey taxation, a government uses revenue from a sales tax on the private good to subsidize access to the public good. Assuming that T dollars are collected from the private good sales tax and that randomizations are not allowed, the public good provision problem is just like (1) with fixed cost $K - T$.[8] Hence, the best pricing and provision scheme is

[7] In our general analysis, we allow for arbitrary incentive feasible mechanisms, but linear pricing is optimal when goods are provided separately given that "virtual valuations" are monotone, which is the case in this example (see Proposition 2 below).

[8] When $K > \frac{1}{4} + T$, randomizations will be part of an optimal mechanism.

$$\pi(T) = \begin{cases} 1 & \text{if } K \leq \frac{1}{4} + T \\ 0 & \text{if } K > \frac{1}{4} + T, \end{cases} \quad f(T) = \begin{cases} \frac{1}{2} - \sqrt{\frac{1}{4} + T - K} & \text{if } K \leq \frac{1}{4} + T \\ 0 & \text{if } K > \frac{1}{4} + T. \end{cases} \tag{4}$$

In order to raise T, we have to set a unit sales tax t on the private good satisfying $T = t(1-c-t)$. Solving the quadratic for its smaller root, we find that the relationship between the tax revenue and the associated tax rate is

$$t = \frac{1-c}{2} - \sqrt{\left(\frac{1-c}{2}\right)^2 - T}. \tag{5}$$

This is a valid solution for our problem if and only if T is less than $\left(\frac{1-c}{2}\right)^2$, which is the maximized monopoly profit from the private good. It is obviously impossible to raise more revenue.

In order to set the sales tax on the private good optimally, the benevolent planner trades off the distortion created in the market for the private good with the efficiency gains from being able to lower the fee for access to the public good. This optimization problem can be formulated as

$$\max_{0 \leq T \leq \left(\frac{1-c}{2}\right)^2} \int_{\frac{1}{2}-\sqrt{\frac{1}{4}+T-K}}^{1} \theta_G \mathrm{d}\theta_G + \int_{c+\left[\frac{1-c}{2}-\sqrt{\left(\frac{1-c}{2}\right)^2-T}\right]}^{1} (\theta_P - c)\,\mathrm{d}\theta_P. \tag{6}$$

It is easy to verify from the first-order condition that the unique solution to (6) is to set the subsidy to the public good as

$$T^* = \frac{(1-c)^2 K}{1+(1-c)^2}. \tag{7}$$

Note that T^*—the share of the cost for the public good that is financed by taxing the private good—is decreasing in the unit cost of the private good, which simply reflects that fewer consumers will purchase the good when the price is higher, so it becomes harder to generate tax revenue.

From now on, we will specialize the example further to simplify the algebra by assuming that $c = 0$, so that $T^* = \frac{K}{2}$. The maximized welfare in (6) can then be directly calculated as

$$\frac{1}{2}\left[\theta_G^2\right]\Big|_{\frac{1}{2}-\sqrt{\frac{1}{4}+T^*-K}}^{1} + \frac{1}{2}\left[\theta_P^2\right]\Big|_{\frac{1}{2}-\sqrt{\frac{1}{4}-T^*}}^{1} - K = \frac{1}{2}\left(1+\sqrt{1-2K}+K\right) - K. \tag{8}$$

3.3 Pure public provision of the private good

While T^* corresponds to an optimal commodity tax under the restriction that the only link between the public and the private good is the government budget constraint, we will now show that the government may do better.[9] For simplicity, we will consider the case with "socialized provision" of the private good, so only citizens that agree to pay the taxes of the jurisdiction can get access to the private good. This "pure bundling mechanism" is clearly very crude and only dominates the Ramsey solution in particular parameter regions. However, our general analysis will show that an element of public provision of the private good is desirable generically.

If the only way to get access to the private good is to opt in to the government bundle, the relevant willingness to pay is that for joint consumption of the two goods. This joint willingness to pay, denoted by $\theta = \theta_G + \theta_P$, is distributed on $[0, 2]$ in accordance with the symmetric triangular density function

$$h(\theta) = \begin{cases} \theta & \theta \in [0, 1] \\ 2 - \theta & \theta \in [1, 2]. \end{cases}, \tag{9}$$

and we denote by $H(\theta)$ the associated cumulative density. Let b denote the access fee to the private–public good bundle. Because the maximal revenue under the density (9) is attained at $b < 1$, we only consider K that are small enough so that the best bundle price solves the equation

$$[1 - H(b)]\, b = \left(1 - \frac{b^2}{2}\right) b = K. \tag{10}$$

For concreteness, let $K = 7/16$, which is convenient because it translates into an optimal bundle price of $1/2$. This implies that only $1/8$ of the consumers are excluded from consumption of the bundle. Recall that for $K = 7/16$ in the separate provision taxes-and-subsidies regime, the use fee for the public good and the sales tax of the private good as given by (4) and (5) are (by setting $c = 0$, and $T^* = K/2 = 7/32$):

$$t^* = f^* = \frac{1}{2} - \sqrt{\frac{1}{32}} \approx 0.323 \tag{11}$$

Hence, the prices under separate provision are higher than the price for the bundle on a per good basis. This implies that there are fewer exclusions from usage under bundling than under the benchmark tax and subsidy regime. The reason for this is that the tails of the distribution of willingness to pay are fatter when goods are provided separately than with bundling.[10] Clearly, fewer use exclusions is a beneficial effect from providing the private good jointly with the public good.

However, there is also a negative effect that is created by the cruder pricing scheme when goods are jointly provided. Some agents with a high valuation for the private good may decide to opt out of the bundle because they have a low valuation of the public good and vice versa. Obviously, this generates a misallocation that tends to

[9] The reader may worry that this is because we only consider a linear tax. However, our general analysis establishes that a linear tax is without loss of generality in the "Ramsey tax regime" (again, see Proposition 2 below).

[10] See Fang and Norman (2006) for details.

make separate provision more desirable. In general, either effect can dominate, but for the example with $K = 7/16$, the welfare gain from the reduction in use exclusions is large enough to make the regime with public provision of the private good more desirable.

To see this, we calculate the welfare in the regime where the government jointly provides the two goods at price $b = 1/2$ as

$$\int_{\frac{1}{2}}^{1} \theta^2 \mathrm{d}\theta + \int_{1}^{2} \theta(2-\theta)\,\mathrm{d}\theta - K = \frac{23}{24} - \frac{7}{16} \approx 0.9583 - \frac{7}{16}, \tag{12}$$

which is to be compared with the welfare under the regime with taxes and subsidies (from (8) by setting $K = 7/16$),

$$\frac{1}{2}\left(1 + \sqrt{1 - 2\frac{7}{16} + \frac{7}{16}}\right) - K = \frac{23 + 4\sqrt{2}}{32} - \frac{7}{16} \approx 0.89 - \frac{7}{16}. \tag{13}$$

Hence, the pure bundling mechanism outperforms the best Ramsey commodity taxation.

4 The general case: separate provision mechanisms

Just like in the example, an economy is populated by a continuum of *ex ante* identical agents with preferences over a binary and excludable public good and a binary private good. Utility is transferable and the public good can be produced at a per capita cost $K > 0$, whereas the private good is produced at unit cost $c \geq 0$.

An agent is characterized by her type $\boldsymbol{\theta} \equiv (\theta_G, \theta_P) \in \boldsymbol{\Theta} \equiv \Theta_G \times \Theta_P = [\underline{\theta}_G, \overline{\theta}_G] \times [\underline{\theta}_P, \overline{\theta}_P]$ where θ_G is her valuation for the public good and θ_P is her valuation for the private good, and these valuations are her private information. To avoid trivialities, we assume that $\underline{\theta}_P \leq c < \overline{\theta}_P$ and $0 < K < \overline{\theta}_G$. We assume that the probability measure over $\boldsymbol{\Theta}$ can be represented by a smooth cumulative distribution $H : \boldsymbol{\Theta} \to [0, 1]$, and write $H_G : \Theta_G \to [0, 1]$ and $H_P : \Theta_P \to [0, 1]$ for the respective marginal cumulative distributions. Probability density functions exist and are denoted by h, h_G and h_P, respectively.

Agents are risk neutral with payoffs given by

$$I_G \theta_G + I_P \theta_P - m, \tag{14}$$

where I_G and I_P are dummy variables indicating wether the goods are consumed or not and m is a transfer from the consumer to the provider..

A *separate provision mechanism* is a mechanism $(\pi, \phi_G, t_G, \phi_P, t_P)$, where:

- $\pi \in [0, 1]$ is the probability that the public good is provided;
- $\phi_G : \Theta_G \to [0, 1]$ is the probability of access to the public good;
- $t_G : \Theta_G \to \mathrm{R}$ is the fee for consuming the public good;

- $\phi_P : \Theta_P \to [0, 1]$ is the probability of consuming the private good;
- $t_P : \Theta_P \to R$ is the fee for consuming the private good.

Allowing π to be a non-degenerate randomization may seem like spurious generality, but an optimal mechanism may provide with a probability strictly in between zero and one. This is because of the discreteness of the public project, which implies that a lottery may help to convexify the problem. Concretely, it may be that the tax distortion in the private market that is needed in order to provide the good for sure may be too big to justify provision. Using a lottery, one can get a desirable outcome at least with positive probability.

It is important to note that we have assumed here that (ϕ_G, t_G) are functions of θ_G only and *cannot depend on* θ_P, and that (ϕ_P, t_P) are functions of θ_P only and *cannot depend on* θ_G. This independence across goods is the defining property of a separate provision mechanism and may be justified if markets are physically separated in space and if the government agencies cannot track the behavior of individual agents across markets.

Note that the provision probability π is a scalar and that $(\phi_G, t_G, \phi_P, t_P)$ are all independent of the distribution of realized types. This specification is a direct result of modeling the set of agents as a continuum, which simplifies the analysis tremendously relative to the corresponding finite agent model, where all decision rules depend on the full type profile. However, in closely related models, it has been shown that the continuum specification is a good approximation of the model with a large finite agent economy because of the interaction between participation and self-financing constraints (see Schmitz 1997; Hellwig 2003; Norman 2004; Fang and Norman 2010). The reason is that most agents must have a negligible influence on the probability to provide a public good in a large economy, thus creating a situation similar to the "Paradox of Voting."

4.1 The problem

The best separate provision mechanism maximizes the *ex ante* average utility of the consumers subject to incentive feasibility. Because utility is transferable, the associated allocation is also the *ex ante* Pareto frontier. Formally, the problem may be written as,

$$\max_{\{\pi,(\phi_J,t_J)_{J\in\{P,G\}}\}} \sum_{J\in\{G,P\}} \int_{\Theta_J} [\phi_J(\theta_J)\theta_J - t_J(\theta_J)]\,\mathrm{d}H_J(\theta_J) \tag{15}$$

$$\text{s.t.} \sum_{J\in\{G,P\}} [\phi_J(\theta_J)\theta_J - t_J(\theta_J)] \geq \sum_{J\in\{G,P\}} \left[\phi_J(\widehat{\theta}_J)\theta_J - t_J(\widehat{\theta}_J)\right] \quad \forall \boldsymbol{\theta}, \widehat{\boldsymbol{\theta}} \in \boldsymbol{\Theta} \tag{16}$$

$$\sum_{J\in\{G,P\}} [\phi_J(\theta_J)\theta_J - t_J(\theta_J)] \geq 0 \qquad \forall \boldsymbol{\theta} \in \boldsymbol{\Theta} \tag{17}$$

$$K\pi + \int_{\Theta_P} c\phi_P(\theta)\,\mathrm{d}H_P(\theta_P) \leq \sum_{J\in\{G,P\}} \int_{\Theta_J} t_J(\theta_J)\,\mathrm{d}H_J(\theta_J) \tag{18}$$

$$0 \leq \phi_G(\theta_G) \leq \pi \qquad \forall \theta_G \in \Theta_G \tag{19}$$

where:

- (15) is the *ex ante* expected utility of a representative citizen;
- (16) are the (Bayesian) incentive compatibility constraints;
- (17) are the participation constraints, which ensure that no agent is made worse off than from the status quo. This constraint implies that the lowest type must be provided with a minimum utility level, which makes lump sum taxes infeasible;
- The resource constraint (18) guarantees that the total costs for the production of the public and the private goods do not exceed the total revenue collected from the agents. Note that the cost to provide the public good depends on the probability of provision π, but not on the probability of consumption $\phi_G(\cdot)$.[11]
  Clearly, it is usually unrealistic to assume that commodity taxation and user fees are the only sources of revenue. However, as long as any other source of revenue would involve distortions, the qualitative results will be unchanged. Indeed, a particularly tractable extension is to assume that each dollar raised from outside of the model costs a constant $\eta > 1$ in reduced surplus. This would put an upper bound on the multiplier on the resource constraint with no other change of the results other than implying that financing may now come from a mix of commodity taxes, user fees and this alternative source.
- (19) restricts the probability of accessing the public good to be no more than the probability that the public good is provided.

While the other constraints are non-controversial, the use of participation constraints in public finance is sometimes debated. It is sometimes argued that the power of taxation implies that participation constraints are irrelevant. However, in the context of local governments, " opting out" can be interpreted as a shorthand for moving to a different jurisdiction. Additionally, in the case of a federal government with no migration, the participation constraint could be interpreted as a (very special) form of inequality aversion or as a minimum sustenance level. Finally, if interpreting the problem as representing the "constitutional stage," participation constraints are again appropriate.

4.2 The characterization

To avoid randomizations in the constrained optimum, we will only consider the regular case where the *virtual valuations*,

$$x_J(\theta_J) \equiv \theta_J - \frac{1 - H_J(\theta_J)}{h_J(\theta_J)}, \tag{20}$$

are strictly increasing in both the public and the private good.[12] This is a standard regularity condition in the auction literature, and it allows us to obtain the following characterization result:

[11] While (18) only requires that the resources be balanced in expectation, this is without loss of generality because one can adjust transfers without changing the interim expected payoffs in such a way as to balance the budget *ex post* for sure (see Borgers and Norman 2009).

[12] The non-regular case can be dealt with using the methods in Myerson (1981).

Proposition 1 *Suppose that $x_J(\theta_J)$ defined in (20) is weakly increasing in θ_J for $J \in \{G, P\}$ and that $\left(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*\right)$ solves the planning problem (15). Then:*

1. *there exists some f such that*

$$\phi_G^*(\theta_G) = \begin{cases} 0 & \text{if } \pi^*\theta_G < f \\ \pi^* & \text{if } \pi^*\theta_G \geq f; \end{cases} \quad \text{and} \quad t_G^*(\theta_G) = \begin{cases} 0 & \text{if } \pi^*\theta_G < f \\ f & \text{if } \pi^*\theta_G \geq f; \end{cases} \tag{21}$$

2. *there exists p such that*

$$\phi_P^*(\theta_P) = \begin{cases} 0 & \text{if } \theta_P < p \\ 1 & \text{if } \theta_P \geq p. \end{cases} \quad \text{and} \quad t_P^*(\theta_P) = \begin{cases} 0 & \text{if } \theta_P < p \\ 1 & \text{if } \theta_P \geq p. \end{cases} \tag{22}$$

In words, Proposition 1 states that it is without loss of generality to restrict attention to single price mechanisms fully characterized by a probability of public good provision π^*, a price on the private good p, and a fixed user fee for the public good f.[13] In particular, the sole government intervention in the private good market under the best separate provision mechanism can be interpreted as a "unit sales tax" in the amount of $p^* - c$. Otherwise, the operation of the private good market can be left completely to the private sector, which provides the private good via a competitive market subject to a sales tax.

Proposition 1 implies that a solution to (15) may be obtained by solving the following simplified planning problem:

$$\max_{\{\pi, f, p\}} \pi \left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} \theta_G \mathrm{d}H_G(\theta_G) - K \right] + \int_{p}^{\overline{\theta}_P} (\theta_P - c)\, \mathrm{d}H_P(\theta_P) \tag{23}$$

$$\text{s.t } 0 \leq f\left[1 - H_G(f\pi)\right] - \pi K + (p - c)\left[1 - H_P(p)\right], \tag{24}$$

$$0 \leq \pi \leq 1, \tag{25}$$

and by inspecting the optimality conditions of this simplified planning problem we obtain::

Proposition 2 *Suppose that* $\mathrm{E}(\theta_G | \theta_G \geq 0) > K$ *and that* $\underline{\theta}_G < K$*. Then,* $p^* > c$ *and* $\pi^* > 0$ *in any constrained optimal mechanism. If, in addition,* $\underline{\theta}_G \leq 0$*, then* $f^* > 0$.

To summarize, there is a positive sales tax, a positive probability of provision and a strictly positive user fee whenever the public good is socially desirable. The revenue for the sales tax is used to partially fund the public good, which is not be surprising as the welfare loss from a small tax on the private good is of second order, whereas increasing the probability of public good provision or reducing access fees for the

[13] In the case of a non-trivial randomization $\frac{f}{\pi^*}$ would be the appropriate user fee charged conditional on completion.

public good leads to a first-order welfare gain because there is always under-provision of the public good at the constrained optimum. The intuition for the strictly positive public good user fee f^* is similar: excluding consumers with valuations just above zero leads to only a second-order welfare loss, whereas the associated revenue from charging a positive f^* generates a first-order welfare gain. Finally, the reason for the provision probability to be strictly positive whenever providing the public good is that even a small tax can fund the provision with a small probability.

Departing from separability in preferences or going beyond unit demands would make the model significantly more complex. However, one extension is very easy: adding more public and/or private goods leads to a characterization just like Propositions 1 and 2, except that now typically some, but not necessarily all, socially beneficial public goods should be provided. See Hellwig (2007) for a related model with excludable public goods only.

4.3 Proofs of Propositions 1 and 2

Because neither access probabilities or fees can depend on reports about the willingness to pay for the other good in a separate provision mechanism, the incentive compatibility constraints (16) hold if and only if,

$$\phi_J(\theta_J)\,\theta_J - t_J(\theta_J) \geq \phi_J(\widehat{\theta}_J)\theta_J - t_J(\widehat{\theta}_J), \quad \forall \theta_J, \widehat{\theta}_J \in \Theta_J. \tag{26}$$

for each good $J \in \{G, P\}$. In words, "public good incentive compatibility" of a mechanism separates from "private good incentive compatibility." This simple observation allows us to decompose the maximization problem (15) into two separate mechanism design problems. The key property of this decomposition is that it delivers a pair of unidimensional problems, which can be analyzed using standard techniques.

Specifically, let $\left(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*\right)$ be a solution to problem (15). Then we define the *public good provision problem* as

$$\max_{\{\pi, \phi_G\}} \int_{\Theta_G} \left[\phi_G(\theta_G)\,\theta_G\right] \mathrm{d}H_G(\theta_G) - K\pi \tag{27}$$

$$\text{s.t. } 0 \leq \int_{\Theta_G} \phi_G(\theta_G)\,x_G(\theta_G)\,\mathrm{d}H_G(\theta_G) - K\pi + \int_{\Theta_P} \left[t_P^*(\theta_P) - c\phi_P^*(\theta_P)\right] \mathrm{d}H_P(\theta_P) \tag{28}$$

$$0 \leq \phi_G(\theta_G) \leq \pi \text{ for all } \theta_G \tag{29}$$

$$\phi_G(\cdot) \text{ is weakly increasing,} \tag{30}$$

where the optimal private good provisions and transfers $\left(\phi_P^*, t_P^*\right)$ are taken as given. Symmetrically, the *private good provision problem*, which takes $\left(\pi^*, \phi_G^*, t_G^*\right)$ as given, is

$$\max_{\{\phi_P\}} \int_{\Theta_P} [\phi_P(\theta_P)(\theta_P - c)]\,\mathrm{d}H_P(\theta_P) \tag{31}$$

$$\text{s.t. } 0 \leq \int_{\Theta_P} \phi_P(\theta_P)\, x_P(\theta_P)\,\mathrm{d}H_P(\theta_P) + \int_{\Theta_G} t_G^*(\theta_G)\,\mathrm{d}H_G(\theta_G) - K\pi^* \tag{32}$$

$$0 \leq \phi_P(\theta_P) \leq 1 \text{ for all } \theta_P \tag{33}$$

$$\phi_P(\cdot) \text{ is weakly increasing.} \tag{34}$$

The following lemma, whose detailed derivation is in the Appendix A, provides the details on the relationship between the original problem (15) and problems (27) and (31):

Lemma 1 *If $(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*)$ solves the planning problem (15), then (π^*, ϕ_G^*) solves (27) and ϕ_P^* solves (31); moreover, for $J \in \{G, P\}$,*

$$t_J^*(\theta_J) = \theta_J \phi_J^*(\theta_J) - \int_{\underline{\theta}_J}^{\theta_J} \phi_J^*(x)\,\mathrm{d}x \tag{35}$$

for all $\theta_J \in \Theta_J$.

That is, if we knew the size of the cross-subsidy $\int_{\Theta_P}\left[t_P^*(\theta_P) - c\phi_P^*(\theta_P)\right]\mathrm{d}H_P(\theta_P)$, then all we need to do to find the optimal public good provision probability and transfer is to solve (27) and then use formula (35) to find he transfer. To interpret, notice that in any incentive compatible mechanism that provides only the public good, it must be that $t_G(\cdot)$ is increasing and that[14]

$$\begin{aligned}\int_{\Theta_G} t_J(\theta_J)\,\mathrm{d}H_G(\theta_G) &= \int_{\Theta_G}\left[\theta_G\phi_G(\theta_G) - \int_{\underline{\theta}_G}^{\theta_G}\phi_G(x)\,\mathrm{d}x\right]\mathrm{d}H_G(\theta_G) - U(\underline{\theta}_G)\\ &= \int_{\Theta_G}\phi_G(\theta_G)\,x_G(\theta_G)\,\mathrm{d}H_G(\theta_G) - U(\underline{\theta}_G),\end{aligned} \tag{36}$$

where we recall that the virtual valuation for the public good $x_G(\theta_G)$ is defined in (20). Hence, with participation constraints given by $U(\theta_G) \geq 0$ it follows that a *profit maximizing* provider of the public good would seek to maximize

$$\int_{\Theta_G}\phi_G(\theta_G)\,x_G(\theta_G)\,\mathrm{d}H_G(\theta_G) - K\pi \tag{37}$$

subject only to the boundary and monotonicity constraints (29) and (30). This problem can be solved in two steps. First, assume that the good is provided for sure and ask

[14] (36) follows from the now standard calculations that appear for example in Myerson (1981).

how much revenue the monopolist can extract. This problem is solved by a fixed price mechanism, even when the virtual valuation is non-monotonic in the valuation. Then, as a second step, one simply checks whether the maximized revenue exceeds costs, in which case, the profit maximizing provision probability is unity. Willingness to pay is proportional to the probability of access, so the probability of provision will be zero whenever the monopolist would make a loss when providing for sure.

In our problem, we do not have profits in the objective function, but since there is a zero profit constraint, there is some shadow price on the profit motive, and the optimization problem boils down to maximizing a weighted average of consumer welfare and profits. Unlike the case with a for-profit monopolist, it is then necessary to restrict attention to the regular case with monotonic virtual valuations to avoid the possibility of randomizations at the constrained optimum. However, given that the virtual valuation is increasing, the revenue extraction and surplus maximization are aligned in the sense that the average fee that can be charged by giving access to all types above a threshold is increasing in the threshold type.

4.3.1 Proof of Proposition 1

To prove Proposition 1 from Lemma 1 one relies on the fact that $\phi_J(\theta_J)$ appear linearly in both the objective functions and the constraints. As the argument is quite standard, we only provide a sketch, and we take as granted that the problem can be solved by Lagrangian techniques (see Hellwig (2005) for justification).

Let $\lambda \geq 0$ be the multiplier on the integral constraint (28), $\gamma(\theta_G)$ be the multiplier on constraint $\phi_G(\theta_G) \geq 0$, and $\omega(\theta_G)$ be the multiplier on constraint $\pi - \phi_G(\theta_G) \geq 0$. The optimality condition for $\phi_G(\theta_G)$ then reads:

$$\theta_G + \lambda x_G(\theta_G) + \frac{\gamma(\theta_G) - \omega(\theta_G)}{h_G(\theta_G)} = 0, \tag{38}$$

together with the appropriate complementary slackness conditions for the non-negativity constraints. As $\theta_G + \lambda x_G(\theta_G)$ is continuous and strictly increasing under the regularity condition that $x_G(\cdot)$ is weakly increasing in θ_G, it follows from (38) that the solution has a threshold property, i.e., there exists some θ_G^* such that $\phi_G^*(\theta_G) = 0$ if $\theta_G < \theta_G^*$, and $\phi_G^*(\theta_G) = \pi^*$ if $\theta_G \geq \theta_G^*$, exactly as described by (21) if we let $f = \theta_G^*/\pi^*$. Moreover, inserting the threshold rule into the condition (35) we obtain,

$$t_G^*(\theta_G) = \theta_G \phi_G^*(\theta_G) - \int_{\underline{\theta}_G}^{\theta_G} \phi_G^*(x)\,\mathrm{d}x = \begin{cases} 0 & \text{if } \pi^*\theta_G < f \\ f & \text{if } \pi^*\theta_G \geq f. \end{cases}.$$

We may thus interpret f as an access fee for a lottery where consumers who pay f get access to the public good if it is provided, which happens with probability π^*. Alternatively, the fee could be charged only conditional on the good being provided, in which case the relevant access price would be $\frac{f}{\pi^*} = \theta_G^*$.

4.3.2 Proof of Proposition 2

Here, we show that Proposition 2 follows directly from inspection of the optimality conditions associated with problem (23).

First, write $\chi \equiv \frac{f}{\pi}$ and let λ, μ and γ, respectively, be the Lagrangian multiplier for the constraint (24) and the boundary constraints $\pi \geq 0$ and $\pi \leq 1$. The Kuhn–Tucker necessary conditions for an optimum are:

$$0 = \int_{\chi}^{\overline{\theta}_G} \theta_G \mathrm{d}H_G(\theta_G) + (1+\lambda)\left[h_G(\chi)\chi^2 - K\right] + \mu - \gamma \tag{39}$$

$$0 = -\chi h_G(\chi) + \lambda\left[1 - H_G(\chi) - h_G(\chi)\chi\right] \tag{40}$$

$$0 = -(p-c)h_P(p) + \lambda\left[(1 - H_P(p)) - (p-c)h_P(p)\right] \tag{41}$$

$$0 = \lambda\left\{\pi\chi\left[1 - H_G(\chi)\right] + (p-c)\left[1 - H_P(p)\right] - K\pi\right\}, \lambda \geq 0 \tag{42}$$

$$\mu\pi = 0, \gamma(1-\pi) = 0, \mu \geq 0, \gamma \geq 0 \tag{43}$$

(Part 1): If $p^* < c$, then the first term on the right-hand side in (41) is strictly positive and the second is weakly positive; thus, the condition cannot hold. Suppose that $p^* = c$. Then, from (41) either $\lambda = 0$ or $1 - H_P(c) = 0$. Since the second possibility is ruled out by the assumption that $c < \overline{\theta}_P$, the only possibility that remains is that $\lambda = 0$. But if $\lambda = 0$ at the optimal solution, then constraint (24) is not binding, which implies that χ^*, π^* must solve the following problem:

$$\max_{\{\chi,\pi\}} \pi\left[\int_{\chi}^{\overline{\theta}_G} \theta_G \mathrm{d}H_G(\theta_G) - K\right]$$
$$\text{s.t.} \quad 0 \leq \pi \leq 1.$$

If $\pi^* > 0$ in the solution to the above problem, then the objective function is monotonically decreasing in χ over $\left[\max\left\{\underline{\theta}_G, 0\right\}, \overline{\theta}_G\right]$; thus it must be that $\chi^* = \max\left\{\underline{\theta}_G, 0\right\}$. Thus, if $\pi^* > 0$, it must maximize

$$\pi\left[\int_{\max\{\underline{\theta}_G,0\}}^{\overline{\theta}_G} \theta_G \mathrm{d}H_G(\theta_G) - K\right] = \pi\left[\mathrm{E}(\theta_G|\theta_G \geq 0) - K\right].$$

By assumption $\mathrm{E}(\theta_G|\theta_G \geq 0) > K$, so the solution must be $\pi^* = 1$, and thus $\chi^* = \max\left\{\underline{\theta}_G, 0\right\}$. But, at $p^* = c$, $\chi^* = \max\left\{\underline{\theta}_G, 0\right\}$ and $\pi^* = 1$, the right hand side of the budget constraint (24) reads

$$\pi^*\left[\chi^*\left(1 - H_G(\chi^*)\right) - K\right] + \left(p^* - c\right)\left[1 - H_P(p^*)\right]$$
$$= \max\left\{\underline{\theta}_G, 0\right\}\left(1 - H_G\left(\max\left\{\underline{\theta}_G, 0\right\}\right)\right) - K,$$

which is strictly negative, thus the budget constraint (24) is violated, a contradiction. Hence, $p^* > c$ in any solution to (23).

(Part 2): As Part 1 establishes that $p^* > c$ in any optimum, it follows that there is a strict budget surplus if $\pi^* = 0$ (the tax collected from the private goods due to $p^* > c$ is unspent). Instead, consider a positive public good provision probability π' as given by

$$\pi' = \frac{(p^* - c)\left[1 - H_P(p^*)\right]}{K} > 0.$$

By construction, constraint (24) is satisfied by the alternative simple mechanism $\left(\pi', \chi = 0, p^*\right)$. Clearly $\left(\pi', \chi = 0, p^*\right)$ improves the objective of (23) upon $(\pi^* = 0, \chi = 0, , p^*)$. A contradiction.

(Part 3): Suppose $\chi^* = 0$ and $\underline{\theta}_G \leq 0$. From condition (40), it follows that $\lambda [1 - H_G(0)] = 0$, which can only hold if $\lambda = 0$. But, from the proof of Part 1, if $\lambda = 0$, then $p^* = c$, which contradicts our conclusion in Part 1.

5 Publicly provided private goods

In this section, we introduce an additional policy instrument that allows the government to condition the provision probability and price for each of the two goods on the reported valuations of *both goods*. We interpret this as *public provision of both goods* since such joint provision mechanisms is not implementable if the private good is traded anonymously in the private sector.

For tractability, we will study small perturbations of the best separate provision mechanism characterized in Proposition 2. Specifically, we will add a price τ, which is the fee charged to a consumer who consumes the bundle consisting of both the public and the private goods. Hence, the interpretation of f is now as the user fee for the consumption of the *public good only* and p is the price for the private good charged to consumers who *do not get access to the public good*. If $\tau \neq f + p$, this requires that the government is actively involved in provision of the private good, because such a scheme is feasible for the government only if it could monitor the consumers' personal consumption of the private goods. We study perturbations where the provision probability is fixed at the same level as in the optimal separate provision mechanism and (f, p, τ) is in a neighborhood of $(f^*, p^*, f^* + p^*)$, where (f^*, p^*) are the fees in the optimal solution to the separate provision benchmark. The main result is that, generally, charging $\tau \neq f + p$ improves welfare.

Our local argument is silent on what the optimal joint provision mechanism is. However, we are mainly interested in the *qualitative* question of whether public provision of a private good can be efficiency enhancing. As we show below, the perturbation outperforms the best separate provision mechanism we identified in Sect. 4, implying that the constrained optimal joint provision mechanism must be one in which the government takes an active part in the provision of the private good.

5.1 Demands, budget surplus and social surplus

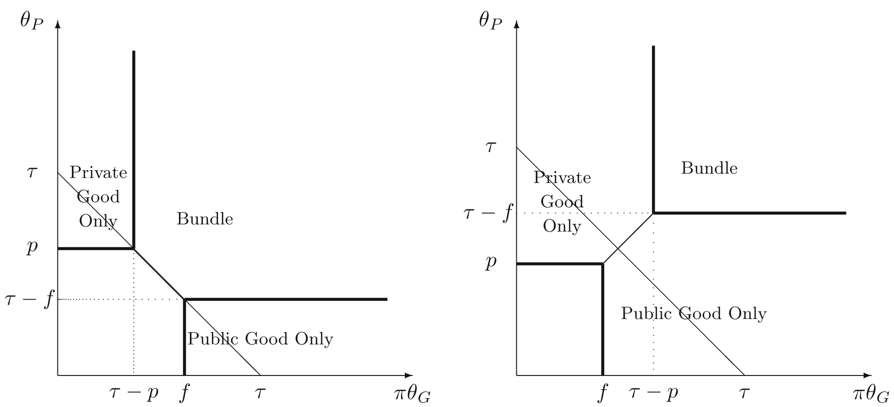
As a first step, we need to calculate the social surplus as well as the budget surplus under simple pricing mechanisms of the form (π, f, p, τ). To do this, we first derive the demands for the two goods and the bundle for a given allocation rule (π, f, p, τ).

Given (π, f, p, τ), a type-(θ_G, θ_P) consumer's demand is easy to characterize:

- she will demand *only the public good* if $\pi\theta_G - f \geq 0$, $\pi\theta_G - f \geq \theta_P - p$, and $\pi\theta_G - f \geq \pi\theta_G + \theta_P - \tau$;
- she will demand *only the private good* if $\theta_P - p \geq 0$, $\theta_P - p \geq \pi\theta_G - f$, and $\theta_P - p \geq \pi\theta_G + \theta_P - \tau$;
- she will demand the bundle if $\pi\theta_G + \theta_P - \tau \geq 0$, $\pi\theta_G + \theta_P - \tau \geq \pi\theta_G - f$, and $\pi\theta_G + \theta_P - \tau \geq \theta_P - p$.

In each of the above three cases, which of inequalities are relevant depends on whether the bundle is cheaper or more expensive than the components. The two graphs in Fig. 1 plot the consumer's demand for the private good and public goods as a function of her valuations (θ_G, θ_P) for the case $\tau < f + p$ and the case $\tau > f + p$, respectively. Notice that we have scaled down the valuation for the public good by the provision probability π in the graphs. Calculation based on Fig. 1 immediately yields that the relevant demands can be summarized as in Table 1.

For notational simplicity, let $z = (\pi, f, p, \tau)$. Define by $G_1(z)$ the *budget surplus* (if positive) or budget deficit (if negative) given prices $z = (\pi, f, p, \tau)$ for the case $\tau \leq f + p$. Using the demands described in Table 1, we can write this as:



(a) Case $\tau \leq f + p$ **(b)** Case $\tau \geq f + p$

Fig. 1 Consumer's demand for private good only, public good only and the bundle

Table 1 Summary of demands

	Case 1: $\tau \le f + p$	Case 2: $\tau \ge f + p$
Public good only	$\int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$	$\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G+p-f} h(\boldsymbol{\theta})\,\mathrm{d}\theta + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$
Private good only	$\int_{p}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$	$\int_{p}^{\tau-f} \int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} + \int_{\tau-f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$
Bundle	$\int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau-\pi\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$	$\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$

$$G_1(z) = f\left[\int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\right] + (p-c)\left[\int_{p}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\right] + (\tau-c)\left[\int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau-\pi\theta_G}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,d\boldsymbol{\theta}\right] - K\pi, \tag{44}$$

Symmetrically, we let $G_2(z)$ denote the *budget surplus/deficit* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \ge f + p$:

$$G_2(z) = f\left[\int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G+p-f} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\right] + (p-c)\left[\int_{p}^{\tau-f} \int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} h(\boldsymbol{\theta})\,d\boldsymbol{\theta} + \int_{\tau-f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\right] + (\tau-c)\left[\int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau-f}^{\overline{\theta}_P} h(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\right] - K\pi. \tag{45}$$

Note that $G_1(z) = G_2(z)$ when $\tau = f + p$ which can be seen by substituting $\tau = f + p$ into (44) and (45).

Next, let $S_1(z)$ denote the *social surplus* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \leq f + p$,

$$S_1(z) = \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} \pi\theta_G h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} + \int_{p}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$
$$+ \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau - \pi\theta_G}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$
$$+ \int_{\frac{f}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} - K\pi. \tag{46}$$

Symmetrically, let $S_2(z)$ be the *social surplus* given prices $z = (\pi, f, p, \tau)$ for the case $\tau \geq f + p$:

$$S_2(z) = \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G + p - f} \pi\theta_G h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\underline{\theta}_P}^{\tau - f} \pi\theta_G h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$
$$+ \int_{p}^{\tau - f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} + \int_{\tau - f}^{\overline{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$
$$+ \int_{\frac{\tau-p}{\pi}}^{\overline{\theta}_G} \int_{\tau - f}^{\overline{\theta}_P} (\pi\theta_G + \theta_P - c)\, h(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} - K\pi. \tag{47}$$

For the same reasons as that for the budget surplus, $S_1(z) = S_2(z)$ when $\tau = f + p$.

5.2 The main result

Our main result provides a sufficient condition under which public provision of private goods will improve welfare over the best separate provision mechanism. To state the result, we let $DS_i(z)$ and $DG_i(z)$ for $i = 1, 2$ denote the gradients with respect to the simplified policy vector z of the social surplus and budget surplus in the two different cases. Explicit expressions of the corresponding partial derivatives can be found below in the proof.

Proposition 3 *Let λ^* be the multiplier on constraint (24) corresponding to a solution (π^*, f^*, p^*) of problem (23). Then, there exists a feasible simple pricing policy of* *the form* (f, p, τ) *that generates a higher social surplus than an optimal separate provision mechanism whenever*

$$DS_1\left(z^*\right) + \lambda^* DG_1\left(z^*\right) \neq 0. \tag{48}$$

We have expressed the sufficient condition (48) in terms of the gradient vectors for the case with bundling discounts. This is because, *when evaluated at the optimal separate provision benchmark*, the gradients in the two cases coincide (see Lemma 2 below).

Proposition 3 does not give us a concrete characterization of the optimal joint provision mechanism. However, since any joint provision mechanism must collect and use "private goods data" when allocating the public good, it implies that the optimal mechanism has some public provision of the private good whenever condition (48) is satisfied.

Condition (48) is ultimately a condition on the joint distribution over (θ_G, θ_P) and the cost parameters (c, K) as z^* is uniquely determined from these exogenous variables. We conjecture that (48) is satisfied for almost all distributions over (θ_G, θ_P) given any pair of cost parameters. Unfortunately, we have not been able to prove this. One difficulty is that the multiplier λ^* depends on the marginal distributions of willingness to pay, so the weight on revenue changes with perturbations of the joint density. As there are many joint distributions that generate the same marginals, we believe that (48) is generic in the sense, of being satisfied almost always, but as we were not able to prove this, we will instead provide a weaker genericity claim.

5.3 Independence

Now, we use Proposition 3 above to examine the case where θ_G and θ_P are independent. In this case, there is indeed always an improvement over the best separate provision policy.

Proposition 4 *Suppose that* θ_G *and* θ_P *are independent. Then*

$$\frac{\partial S_1\left(z^*\right)}{\partial p} + \lambda^* \frac{\partial G_1\left(z^*\right)}{\partial p} > 0.$$

Proof When $h_P\left(\theta_P|\theta_G\right) = h_P\left(\theta_P\right)$ for all θ_P, we have that

$$\begin{aligned}\frac{\partial G_1\left(z^*\right)}{\partial p} &= \left[1 - H_P\left(p^*\right) - \left(p^* - c\right) h_P\left(p^*\right)\right] H_G\left(\frac{f^*}{\pi^*}\right) \\ &\quad + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \frac{\left(p^* - c\right) h_P\left(p^*\right)}{\lambda^*} H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right)\end{aligned}$$

where the second equality uses (41), the first-order condition for p^* in the separate provision case. Next,

$$\frac{\partial S_1(z^*)}{\partial p} = -\left(p^* - c\right) h_P\left(p^*\right) H_G\left(\frac{f^*}{\pi^*}\right) + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right)$$

Hence,

$$\begin{aligned}\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} &= -\left(p^* - c\right) h_P\left(p^*\right) H_G\left(\frac{f^*}{\pi^*}\right) \\ &\quad + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \\ &\quad + \lambda^* \left\{ \frac{\left(p^* - c\right) h_P\left(p^*\right)}{\lambda^*} H_G\left(\frac{f^*}{\pi^*}\right) \right. \\ &\quad \left. + \frac{f^*}{\phi_G^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \right\} \\ &= \left(1 + \lambda^*\right) \frac{f^*}{\pi^*}\left[1 - H_P\left(p^*\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) > 0.\end{aligned}$$

□

Proposition 4 establishes that public provision of private goods is welfare improving in the important special case of independent valuations. By continuity, it also implies that there exists an *open set* of joint distribution functions for θ_G and θ_P for which public provision of private goods is efficiency enhancing.

5.4 What if valuations are strongly correlated?

We conjecture that the condition (48) is satisfied for virtually any joint density, but have not been able to prove that almost all joint densities satisfy the condition. However, we have studied parametric classes of densities failures of our condition are non-generic in a strong sense.

A simple example is a family of distributions corresponding to joint densities $h_\alpha(\theta_G, \theta_P) = \alpha h_1(\theta_G, \theta_P) + (1 - \alpha) h_2(\theta_G, \theta_P)$ for $\alpha \in [0, 1]$, where

$$h_1(\theta_G, \theta_P) = \begin{cases} 1 & \text{if } 0 \le \theta_G \le 1 \text{ and } \theta_G \le \theta_P \le \theta_G + 1 \\ 0 & \text{otherwise} \end{cases}. \quad (49)$$

$$h_2(\theta_G, \theta_P) = \begin{cases} 1 & \text{if } 0 \le \theta_G \le 1 \text{ and } 1 - \theta_G \le \theta_P \le 2 - \theta_G \\ 0 & \text{otherwise} \end{cases}$$

Valuations are positively correlated when drawn from the first distribution above, and negatively correlated when drawn from the second. However, their marginal distributions are identical, so by creating mixtures over the two underlying distributions, one obtains a family of distributions with the same marginal distributions, but with the correlation coefficient being a strictly increasing function of α. As the marginal distributions are constant in α, the separate provision problem has the same solution irrespective of α. However, by looking at the partial derivative with respect to p, it is easy to show that condition (48) can fail at most for a single value of α. That is, private provision is useful not only for nearly independent valuations, but also if valuations are strongly positively or negatively correlated.

5.5 What if there are many public and private goods?

While many other extensions would be difficult, it is easy to extend our result to the case with an arbitrary number of public and private goods. When goods are provided separately, we now obtain a characterization where at least one public good is provided with some probability and where all private goods are sold at prices above marginal cost. As the problem simplifies to a version of Ramsey optimal taxation, we would also get familiar predictions on relative taxes having to do with demand elasticities.

However, from our point of view, the most important observation is that Propositions 3 and 4 extend immediately to this setup. This follows by observing that it is enough to pick one of the public goods that are provided with positive probability and an arbitrary private good and redefine H as the "marginal joint distribution" over these two goods. The generalization is then immediate.

5.6 The Proof of Proposition 3

In order to provide conditions for when public provision of the private good outperforms the best decentralized outcome, we construct two *auxiliary* optimization problems. The reason is that perturbations with $\tau < f + p$ and $\tau > f + p$ result in different demands, implying that it is non-obvious that the demands are differentiable at points where $\tau = f + p$.

To address this issue, we consider two problems which define the optimal price vectors (π, f, p, τ) under the restrictions that $\tau \leq f + p$ and $\tau \geq f + p$, respectively. Specifically, let (π^*, f^*, p^*) be the best separate provision mechanism characterized in Proposition 2. Then, we let the first problem be to maximize welfare under the break-even constraint and the constraint that the bundle is cheaper than the components,

$$\begin{aligned} &\max_{(f,p,\tau)} S_1\left(\pi^*, f, p, \tau\right) \\ \text{s.t.} \quad &G_1\left(\pi^*, f, p, \tau\right) \geq 0 \\ &f + p - \tau \geq 0. \end{aligned} \tag{50}$$

Notice the restriction that $\pi = \pi^*$, that is, the public good provision probability is fixed at the same level as in the best separate provision mechanism characterized in Proposition 2. Also note, which is the point with the construction, that S_1 and G_1 are smooth functions.

Similarly, the second problem is:

$$\begin{aligned}&\max_{(f,p,\tau)} S_2\left(\pi^*, f, p, \tau\right)\\ \text{s.t.}\quad & G_2\left(\pi^*, f, p, \tau\right) \geq 0\\ & \tau - f - p \geq 0.\end{aligned} \tag{51}$$

Problem (51) defines the best simple pricing policy in the form of (π, f, p, τ) under the restriction that the public good provision probability is fixed at the level as in the best separate provision mechanism characterized in Proposition 2, and the bundle is restricted to be no cheaper than its components.

As $(f^*, p^*, f^* + p^*)$ is a feasible solution to both (50) and (51), it follows that *a necessary condition for the best separate provision mechanism to be optimal when joint provision is feasible is that* $(f^*, p^*, f^* + p^*)$ *solves both* (50) *and* (51). Our strategy in proving that public provision of the private good is desirable is therefore to find a condition [condition (48)] under which $(f^*, p^*, f^* + p^*)$ cannot solve both problems (50) and (51).

Differentiating (44) and evaluating at $z^* = (\pi^*, f^*, p^*, f^* + p^*)$, we have that the components of the gradient to the budget surplus function in the case with a bundling discount are[15]

$$\frac{\partial G_1(z^*)}{\partial f} = \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} \left\{H_P\left(p^* \middle| \theta_G\right) + \left(p^* - c\right) h_P\left(p^* \middle| \theta_G\right)\right\} h_G(\theta_G)\, \mathrm{d}\theta_G - \frac{f^*}{\pi^*} H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \tag{52a}$$

$$\frac{\partial G_1(z^*)}{\partial p} = \int_{\underline{\theta}_G}^{\frac{f^*}{\phi_G^*}} \left[\left\{\left(1 - H_P\left(p^* \middle| \theta_G\right)\right) - \left(p^* - c\right) h_P\left(p^* \middle| \theta_G\right)\right\}\right] h_G(\theta_G)\, \mathrm{d}\theta_G + \frac{f^*}{\phi_G^*}\left[1 - H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \tag{52b}$$

$$\frac{\partial G_1(z^*)}{\partial \tau} = \int_{\frac{f^*}{\phi_G^*}}^{\overline{\theta}_G} \left\{\left(1 - H_P\left(p^* \middle| \theta_G\right)\right) - \left(p^* - c\right) h_P\left(p^* \middle| \theta_G\right)\right\} h_G(\theta_G)\, \mathrm{d}\theta_G - \frac{f^*}{\phi_G^*}\left[1 - H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \tag{52c}$$

These expressions inform us about the effect on the budget when one slightly perturbs the relevant prices f, p and τ from $z^* = (\pi^*, f^*, p^*, f^* + p^*)$. Likewise, if we differentiate the welfare function in (46) and evaluate at $z = z^*$, we obtain

[15] The details of the derivations for (52) and (53) are available as an external appendix.

$$\frac{\partial S_1(z^*)}{\partial f} = \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c)\, h_P\left(p^* \middle| \theta_G\right) h_G(\theta_G)\, \mathrm{d}\theta_G - \frac{f^*}{\pi^*} H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \tag{53a}$$

$$\frac{\partial S_1(z^*)}{\partial p} = -\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c)\, h_P\left(p^* \middle| \theta_G\right) h_G(\theta_G)\, \mathrm{d}\theta_G + \frac{f^*}{\pi^*}\left[1 - H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right) \tag{53b}$$

$$\frac{\partial S_1(z^*)}{\partial \tau} = -\int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} (p^* - c)\, h_P\left(p^* \middle| \theta_G\right) h_G(\theta_G)\, \mathrm{d}\theta_G - \frac{f^*}{\pi^*}\left[1 - H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right). \tag{53c}$$

As a consequence of the Lemma below, we omit explicit expressions for the gradients to S_2 and G_2. These expressions are available in the external appendix which proves Lemma 2. This result establishes that the gradients of G_1 and G_2 coincide and that the gradients of S_1 and S_2 coincide when evaluated at $z^* = (\pi^*, f^*, p^*, f^* + p^*)$. Letting $DG_i(z)$ and $DS_i(z)$ denote the gradient vectors for $i = 1, 2$, we thus have that:

Lemma 2 *$DG_1(z^*) = DG_2(z^*)$ and $DS_1(z^*) = DS_2(z^*)$*

Lemma 2 is analogous to the well-known "smooth pasting" condition in optimal control problem with switching points (see, e.g., Dixit 1993). Here, the relevant budget surplus function switch from G_1 to G_2 at z^*, and we have the analog of "value matching" of G_1 and G_2 at z^*, i.e., $G_1(z^*) = G_2(z^*)$. Lemma 2 simply states that the two functions are smoothly pasted at the switch point z^*. The same is true for the social surplus functions S_1 and S_2 at the switch point z^*.

Next, we establish that if the optimal separate provision mechanism solves either of the auxiliary problems, then it must be that the multiplier on the constraint in that problem coincides with the multiplier from the separate provision problem. While this requires a proof, it should be intuitively rather plausible as the additional dimension is useless in this case, so the shadow price on the break-even constraint should be just like in the simpler problem.

Lemma 3 *Let λ^* be the multiplier on constraint (24) corresponding to the solution (π^*, f^*, p^*) of problem (23). Also, let λ_i be the multiplier on the resource constraint $G_i(f, p, \tau; \pi^*)$ for $i = 1, 2$ in problem (50) and (51). Then,*

1. $\lambda_1 = \lambda^*$ if z^* solves problem (50);

2. *$\lambda_2 = \lambda^*$ if z^* solves problem (51).*

Proof First consider (50). If z^* solves the problem, the Kuhn–Tucker necessary conditions for a solution must be fulfilled at z^*. Hence, there exists $\lambda_1 > 0$ and $\mu_1 \geq 0$ such that

$$\frac{\partial S_1(z^*)}{\partial f} + \lambda_1 \frac{\partial G_1(z^*)}{\partial f} + \mu_1 = 0 \tag{54a}$$

$$\frac{\partial S_1(z^*)}{\partial p} + \lambda_1 \frac{\partial G_1(z^*)}{\partial p} + \mu_1 = 0 \tag{54b}$$

$$\frac{\partial S_1(z^*)}{\partial \tau} + \lambda_1 \frac{\partial G_1(z^*)}{\partial \tau} - \mu_1 = 0 \tag{54c}$$

$$\mu_1 (f + p - \tau) = 0, \quad \mu_1 \geq 0 \tag{54d}$$

Using the expressions for the partial derivatives in (52) and (53), it is easy to check that:

$$\frac{\partial S_1(z^*)}{\partial f} + \frac{\partial S_1(z^*)}{\partial \tau} = -\frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) \tag{55a}$$

$$\begin{aligned}\frac{\partial G_1(z^*)}{\partial f} + \frac{\partial G_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\overline{\theta}_G} h_G(\theta_G)\, \mathrm{d}\theta_G - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \left[1 - H_G\left(\frac{f^*}{\pi^*}\right)\right] - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right).\end{aligned} \tag{55b}$$

Combining (54a) and (54c), and using (55), we have that

$$-\frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right) + \lambda_1 \left\{\left[1 - H_G\left(\frac{f^*}{\pi^*}\right)\right] - \frac{f^*}{\phi_G^*} h_G\left(\frac{f^*}{\pi^*}\right)\right\} = 0. \tag{56}$$

This condition is the same as (40), which is the first-order conditions to separate provision problem (23) with respect to the user fee f. It follows that $\lambda_1 = \lambda^*$, since otherwise (56) will be violated. This proves the first part.

For the second part, we note that the Kuhn–Tucker conditions for Problem (51) are

$$\begin{aligned}\frac{\partial S_2(z^*)}{\partial f} + \lambda_2 \frac{\partial G_2(z^*)}{\partial f} - \mu_2 &= \frac{\partial S_1(z^*)}{\partial f} + \lambda_2 \frac{\partial G_1(z^*)}{\partial f} - \mu_2 = 0 \\ \frac{\partial S_2(z^*)}{\partial p} + \lambda_2 \frac{\partial G_2(z^*)}{\partial p} - \mu_2 &= \frac{\partial S_1(z^*)}{\partial p} + \lambda_2 \frac{\partial G_1(z^*)}{\partial p} - \mu_2 = 0 \\ \frac{\partial S_2(z^*)}{\partial \tau} + \lambda_2 \frac{\partial G_2(z^*)}{\partial \tau} + \mu_2 &= \frac{\partial S_1(z^*)}{\partial \tau} + \lambda_2 \frac{\partial G_1(z^*)}{\partial \tau} + \mu_2 = 0 \\ \mu_2 (f + p - \tau) = 0&, \mu_2 \geq 0\end{aligned}$$

where the first equality in each line follows from Lemma 2. The same argument applies. □

Together, Lemmas 2 and 3 imply that the optimality conditions for problem (50) identical with those for problem (51) in case the optimal solution coincides with the separate provision benchmark.

5.6.1 Proof of Proposition 3

Armed with Lemmas 2 and 3 we are now in a position to prove the main result.

Proof Suppose to the contrary, $(f^*, p^*, f^* + p^*)$ solves both problems (50) and (51). Lemma 3 then implies that the multiplier in each problem must be given by λ^*. Thus if z^* is the best simple pricing policy for problem (50), then

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} + \mu_1 = 0 \\
\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} + \mu_1 = 0 \\
\frac{\partial S_1(z^*)}{\partial \tau} + \lambda^* \frac{\partial G_1(z^*)}{\partial \tau} - \mu_1 = 0 \\
\mu_1 (f + p - \tau) = 0, \quad \mu_1 \geq 0.
\end{aligned} \tag{57}$$

Similarly if z^* is the best simple pricing policy for problem (51), then by using Lemma 2, we have

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} - \mu_2 = 0 \\
\frac{\partial S_1(z^*)}{\partial p} + \lambda^* \frac{\partial G_1(z^*)}{\partial p} - \mu_2 = 0 \\
\frac{\partial S_1(z^*)}{\partial \tau} + \lambda^* \frac{\partial G_1(z^*)}{\partial \tau} + \mu_2 = 0 \\
\mu_2 (f + p - \tau) = 0, \quad \mu_2 \geq 0
\end{aligned} \tag{58}$$

Assume that $\mu_1 > 0$. Then, (57) implies that $\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} < 0$, which makes it impossible to find $\mu_2 \geq 0$ such that (58) holds. Symmetrically, if $\mu_2 > 0$, then $\frac{\partial S_1(z^*)}{\partial f} + \lambda^* \frac{\partial G_1(z^*)}{\partial f} > 0$, which makes it impossible to find $\mu_1 \geq 0$ such that (57) holds. Since z^* must solve both (50) and (51) for there to be no improvement we conclude that $\mu_1 = \mu_2 = 0$, or else there is some z better than z^*. The claim follows. □

How our problem differs from McAfee et al. (1989)? Our expression $\partial G_1(z^*) / \partial p$ in (52b) is identical to the condition in Proposition 1 of McAfee et al. (1989). Expression (52a) can also be written in that form by reversing the roles of θ_G and θ_P. This is not a coincidence. The derivatives in (52a), (52b) and (52c) are the effects on profits given a marginal increase in f, p and τ, respectively. In the case of McAfee et al. (1989), going from (52b) to their main result is relatively straightforward since they demonstrated that *profits* may be increased relative to separate pricing, by adding the bundle. Since they study profit maximization their analogue of (f^*, p^*) are chosen to solve a monopolist profit maximization problem under separate pricing. Thus, their p^*, for example, must satisfy the first-order optimality condition $\left[1 - H_P(p^*)\right] - (p^* - c)\, h_P(p^*) = 0$. If θ_G and θ_P are independent, the first term in (52b) becomes:

$$\int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \left\{\left[1 - H_P\left(p^* \middle| \theta_G\right)\right] - \left(p^* - c\right) h_P\left(p^* \middle| \theta_G\right)\right\} h_G(\theta_G)\, \mathrm{d}\theta_G$$
$$= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \left\{\left[1 - H_P\left(p^*\right)\right] - \left(p^* - c\right) h_P\left(p^*\right)\right\} h_G(\theta_G), \tag{59}$$

which is equal to zero from the optimality condition of p^* in their problem. Thus, it follows immediately from (52b) that a small increase in the price of the private good (or a small decrease in the price from the bundle) would increase the profits in the case of independence. It can also be verified that a small increase in the price of the public good also increases profits if θ_G and θ_P are stochastically independent.

Our problem differs from McAfee et al. (1989) in two respects. First of all, our goal is to demonstrate that bundling can increase *social welfare* rather than profits. This is why we get additional terms from the joint distribution in our condition that characterizes when bundling is desirable. Secondly, because (f^*, p^*) in our problem, as characterized in Proposition 2, are *not chosen to maximize profits*, we cannot use the first-order conditions from a standard monopoly problem in the same way as McAfee et al. (1989). Instead, both the separate provision problem and the maximization problem with the added bundle price are constrained optimization problems. This requires us to link multiplier values across optimization problems, which is one of the main analytical difficulties relative McAfee et al. (1989). In addition, whereas the condition in McAfee et al. (1989) is a condition purely on the joint density, we have a condition that depends on the joint density and a Lagrange multiplier, which in turn depends on other the parameters of the separate provision problem. This also implies that proving that our condition is satisfied for the case with independence now is non-trivial.

6 Concluding remarks

It has been shown that public provision of private goods may be justified on pure efficiency grounds, even if the local government does not seek to redistribute from rich to poor citizens. This is important for two reasons. Firstly, while the literature has identified situations where in-kind redistribution leads to improvements in how well targeted the transfers are, cash transfers are still superior to in-kind redistribution in many cases since the latter misallocates resources. Secondly, there are large-scale programs that publicly provide private goods for which the redistributive effects are neutral or regressive. In principle, this could be explained from an optimal taxation model with non-separable leisure, but our model offers a more direct rationale.

The theory developed in this paper is based on the premise that governments also provide non-rival goods. In such an environment, we show that public provision of a private good generates information that facilitates more efficient revenue extraction, which helps overcome inefficiencies in public good provision. Our main result establishes that public provision of the private good improves economic efficiency under a condition that is always fulfilled under independence and satisfied for a large set of joint distributions. Indeed, in an example, we even showed that complete socialization of a private good may be better than market provision and optimal taxes. Unfortunately, we cannot say anything analytical about the optimal mechanism without making simplifying assumptions on either the set of admissible mechanisms or looking at a very sparse set of types. Hence, our analysis is silent on identifying circumstances when the benefits of public provision are large. We expect that this will depend on similar factors as the desirability of bundling for a multi-product monopoly provider, as well as the severity of the public good provision problem. However, as one would take a completely different approach to make progress on these questions, we leave this for future research.

Finally, our analysis also exemplifies a more general point about the optimal taxation literature. Our model combines optimal commodity taxation with a decision on how to provide an excludable public good. We found that the marginal price for access to the public good for consumers that purchase the private good should be different from those who do not. Hence, our model is a stylized example where provision and user fees for the public good and taxes on the private good must be jointly determined in order to achieve economic efficiency. Our paper thus illustrates that the common practice of separating the question of how a given budget should be spent from the question of how a given tax revenue should be raised generates efficiency losses.

We have illustrated our point in a very stylized model with a single public good and a single private good, but, as we have already noted, the argument extends to a model with an arbitrary number of goods. There are other extensions that would lead to a more complex separate provision benchmark, but it seems unlikely that this would affect the basic logic of our argument as long as there are inefficiencies when the private good is traded separately and if raising additional revenue is distortionary.

At a more general level, it has been known since Guesnerie and Roberts (1984) that asymmetric information can justify using quantity controls and prices simultaneously to improve sorting. The literature in commodity bundling considers very particular quantity controls where the marginal price for a good is made contingent on what other goods are consumed. Our paper shows that these well-known screening advantages of quantity controls/bundling can be translated into a novel theory for why private goods are publicly provided. We have chosen to illustrate this point in the simplest possible setup with unit demands, transferable utility and linear preferences in order to be able to obtain a clean characterization of the non-bundling benchmark. However, we see no particular reason why these simplifying assumptions should have any qualitative importance.

7 Appendix A: Details about Lemma 1

Let $\left(\pi^*, \phi_G^*, t_G^*, \phi_P^*, t_P^*\right)$ solve the planning problem (15). Because, as argued in (26), the incentive compatibility constraints (16) can be separated for the private good and the public good, we know that, fix $\left(\phi_P^*, t_P^*\right)$, $\left(\pi^*, \phi_G^*, t_G^*\right)$ must solve:

$$\max_{(\pi,\phi_G,t_G)} \int_{\Theta} \left[\phi_G(\theta_G)\theta_G - t_G(\theta_G)\right] \mathrm{d}H_G(\theta_G) \tag{60}$$

$$\text{s.t.} \quad 0 \le \phi_G(\theta_G)\theta_G - t_G(\theta_G) - \phi_G(\widehat{\theta}_G)\theta_G + t_G(\widehat{\theta}_G), \quad \forall \theta_G, \widehat{\theta}_G \in \Theta_G,$$

$$0 \le \phi_G(\theta_G)\theta_G - t_G(\theta_G) + \phi_P^*(\theta_P)\theta_P - t_P^*(\theta_P), \quad \forall (\theta_G, \theta_P) \in \Theta_G \times \Theta_P, \tag{61}$$

$$0 \le \int_{\Theta_G} t_G(\theta_G)\,\mathrm{d}H_G(\theta_G) - K\pi + \underbrace{\int_{\Theta_P} t_P^*(\theta_P)\,\mathrm{d}H_P(\theta_P) - \int_{\Theta_P} c\phi_P^*(\theta_P)\,\mathrm{d}H_P(\theta_P)}_{\text{constant}}$$

$$0 \le \phi_G(\theta_G) \le \pi. \tag{62}$$

The optimization problem (60) takes the private good allocation rule $\left(\phi_P^*, t_P^*\right)$ from an optimal mechanism as given and solves for an optimal allocation of the public good conditional on the transfer between markets (the constant in 62) and the reservation utilities implied by $\left(\phi_P^*, t_P^*\right)$.

Symmetrically, fix $\left(\pi^*, \phi_G^*, t_G^*\right)$, $\left(\phi_P^*, t_P^*\right)$ must solve:

$$\max_{(\phi_P,t_P)} \int_{\Theta_P} \left[\phi_P(\theta_P)\theta_P - t_P(\theta_P)\right] \mathrm{d}H_P(\theta_P) \tag{63}$$

$$\text{s.t.} \quad 0 \le \phi_P(\theta_P)\theta_P - t_P(\theta_P) - \phi_P(\widehat{\theta}_P)\theta_P + t_P(\widehat{\theta}_P) \quad \forall \theta_P, \widehat{\theta}_P \in \Theta_P$$

$$0 \le \phi_G^*(\theta_G)\theta_G - t_G^*(\theta_G) + \phi_P(\theta_P)\theta_P - t_P(\theta_P) \quad \forall (\theta_G, \theta_P) \in \Theta_G \times \Theta_P$$

$$0 \le \underbrace{\int_{\Theta_G} t_G^*(\theta_G)\,\mathrm{d}H_G(\theta_G) - K\pi^*}_{\text{constant}}$$

$$+ \int_{\Theta_P} t_P(\theta_P)\,\mathrm{d}H_P(\theta_P) - \int_{\Theta_P} c\phi_P(\theta_P)\,\mathrm{d}H_P(\theta_P). \tag{64}$$

That is, taking the public good provision as given, $\left(\phi_P^*, t_P^*\right)$ solves for the least distorted allocation of private goods conditional on the transfer to the other market and reservation utilities implied.

Define the "indirect utility functions"

$$U_J(\theta_J) \equiv \theta_J \phi_J(\theta_J) - t_J(\theta_J), \tag{65}$$

for $J = G, P$. A routine argument based on reasoning akin to the envelope theorem (see, e.g., Myerson 1981 or Mas-Colell et al. 1995, p. 888) can be used to establish:

Lemma 4 *Suppose that the marginal density $h_J(\theta_J)$ is strictly positive on its support $\Theta_J = \left[\underline{\theta_J}, \overline{\theta_J}\right]$. Then, (ϕ_J, t_J) satisfies the incentive compatibility constraints in (61) and (64, respectively, if and only if ϕ_J is weakly increasing in θ_J and*

$$U_J(\theta_J) = U_J(\widehat{\theta}_J) + \int_{\widehat{\theta}_J}^{\theta_J} \phi_J(x)\,\mathrm{d}x \quad \forall \theta_J, \widehat{\theta}_J \in \Theta_J.$$

Equally routine procedures using the characterization in Lemma 4 show that the aggregate transfer revenues from the public goods fees and the private goods fees, respectively, can be determined uniquely from the utility of the lowest type and the provision rules.

Lemma 5 *Suppose that* (π, ϕ_G, t_G) *and* (ϕ_P, t_P) *satisfy the incentive compatibility constraints in (61) and (64) receptively if and only* ϕ_J *is weakly increasing in* θ_J *and*

$$\int_{\Theta_J} t_J(\theta_J)\,\mathrm{d}H_J(\theta_J) = \int_{\Theta_J} \phi_J(\theta_J)\,x_J(\theta_J)\,\mathrm{d}H_J(\theta_J) - U_J(\underline{\theta_J}).$$

Furthermore, it is without loss of generality to assume that the participation constraint of type $(\underline{\theta}_G, \underline{\theta}_P)$ binds, so that

$$\phi^*(\underline{\theta}_G)\,\underline{\theta}_G - t_G^*(\underline{\theta}_G) + \phi_P^*(\underline{\theta}_P)\,\underline{\theta}_P - t_P^*(\underline{\theta}_P) = 0,$$

at a solution to problem (15). Higher types can mimic $(\underline{\theta}_G, \underline{\theta}_P)$, so, just like in unidimensional problems, incentive compatibility automatically implies that the participation constraints hold for higher types, provided that it is satisfied for the lowest type.

Lemma 5 together with the binding participation constraint for $(\underline{\theta}_G, \underline{\theta}_P)$ suggests a reformulation of the public good problem (60) as Problem (27). Specifically, Problem (27) is derived from problem (60) as follows: (1) the transfers are eliminated from the objective function by substitution from the feasibility constraint (62), which is assumed to bind (all constants have been eliminated); (2) All redundant participation constraints have been eliminated.

Problem (31) is derived from problem (63) analogously.

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