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**ABSTRACT**

We study the effect of the life settlement market on the structure of long term contracts offered by the primary market for life insurance, as well as the effect on consumer welfare, using a dynamic model of life insurance with one sided commitment and bequest-driven lapsation. We show that the presence of life settlement affects the extent as well as the form of dynamic reclassification risk insurance in the equilibrium of the primary insurance market, and that the settlement market generally leads to lower consumer welfare. We also examine the primary insurers' response to the settlement market when they can offer enriched contracts by specifying optimally chosen cash surrender values (CSVs).

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# 1 Introduction

A life settlement is a financial transaction in which a policyholder sells his/her life insurance policy to a third party – the life settlement firm – for more than the cash value offered by the policy itself. The life settlement firm subsequently assumes responsibility for all future premium payments to the life insurance company, and becomes the new beneficiary of the life insurance policy if the original policyholder dies within the coverage period.[1] The life settlement industry is quite recent, growing from just a few billion dollars in the late 1990s to about $12-$15 billion in 2007, and according to some projections, is expected to grow to more than $150 billion in the next decade (see Chandik, 2008).[2]

To provide some background information on the life insurance market, the main categories of life insurance products are Term Life Insurance and Whole Life Insurance.[3] A term life insurance policy covers a person for a specific duration at a fixed or variable premium for each year. If the person dies during the coverage period, the life insurance company pays the face amount of the policy to his/her beneficiaries, provided that the premium payment has never lapsed. The most popular type of term life insurance has a fixed premium during the coverage period and is called Level Term Life Insurance. A whole life insurance policy, on the other hand, covers a person's entire life, usually at a fixed premium. Besides the difference in the period of coverage, term and whole life insurance policies also differ in the amount of cash surrender value (CSV) received if the policyholder surrenders the policy to the insurance company before the end of the coverage period. For term life insurance, the CSV is zero; for whole life insurance, the CSV is typically positive and pre-specified to depend on the length of time that the policyholder has owned the policy. Importantly, the CSV on whole life policies does *not* depend on the health status of the policyholder when surrendering the policy.[4]

The opportunity for the life settlement market results from two main features of life insurance contracts. First, most life insurance policies purchased by consumers, either term or whole life, have the feature that the insurance premium stays fixed over the course of the policy. Because pol-

[1]The legal basis for the life settlement market seems to be the Supreme Court ruling in *Grigsby v. Russell* [222 U.S. 149, 1911], which upheld that for life insurance an "insurable interest" only needs to be established at the time the policy becomes effective, but does not have to exist at the time the loss occurs. The life insurance industry has typically included a two-year contestability period during which transfer of the life insurance policy will void the insurance.

[2]The life settlement industry actively targets wealthy seniors 65 years of age and older with life expectancies from 2 to up to 12-15 years. This differs from the earlier viatical settlement market developed during the 1980s in response to the AIDS crisis, which targeted persons in the 25-44 age band diagnosed with AIDS with life expectancy of 24 months or less. The viatical market largely evaporated after medical advances dramatically prolonged the life expectancy of an AIDS diagnosis.

[3]There are other variations such as Universal Life Insurance and Variable Life Insurance that combine some features of both Term and Whole Life Insurances (see Gilbert and Schultz, 1994).

[4]The life insurance industry typically thinks of the CSV from the whole life insurance as a form of tax-advantaged investment instrument (see Gilbert and Schultz, 1994).

icyholders' health typically deteriorates over time, the fixed premium implies that policyholders initially pay a premium that is higher than actuarially fair, but in later years the same premium is typically actuarially favorable. This phenomenon is known as *front-loading*. Front-loading implies that policyholders of long-term life insurance policies, especially those with impaired health, often have locked in premiums that are much more favorable than what they could obtain in the spot market. This generates what has been known as the *actuarial value* of the life insurance policy (see the Deloitte Report, 2005). Second, as we mentioned earlier, the cash surrender value for life insurance policies is either zero for term life insurance, or at a level that does not depend on the health status of the policyholder. Because the actuarial value of a life insurance policy is much higher for individuals with impaired health, the fact that the CSV does not respond to health status provides an opening for the gains of trade between policyholders with impaired health and the life settlement companies.[5] Life settlement firms operate by offering policyholders, who are intending to either lapse or surrender their life insurance policies, more cash than the cash surrender value offered by the insurers.

The emerging life settlement market has triggered controversies between life insurance companies who oppose it, and the life settlement industry who supports it. The views from the two opposing camps are represented by Doherty and Singer (2002) and Singer and Stallard (2005) on the proponent side, and the Deloitte Report (2005) on the opponent side. Doherty and Singer (2002) argued that a secondary market for life insurance enhances the liquidity to life insurance policyholders by eroding the monopsony power of the carrier. This will increase the surplus of policyholders and in the long run will lead to a larger primary insurance market. On the other side, life insurance companies, as represented by the Deloitte Report (2005), claim that the life settlement market, by denying them the return on lapsing or surrendered policies, increases the costs of providing policies in the primary market. They allege that these costs will have be passed on to consumers, which would ultimately make the consumers worse off.

A key issue in the contention between the opposing sides is the role of lapsing or surrendering in the pricing of life insurance in the primary market (see Daily, 2004). There are a variety of situations in which policyholders may choose to lapse or surrender. First, the beneficiary for whom the policy was originally purchased could be deceased or no longer need the policy; second, the policyholder may experience a negative income shock (or a large expense shock) that leads

[5]The Deloitte Report (2005, p. 3) states that the CSVs of whole life insurance policies are, by regulation, not allowed to be conditioned on health impairments of the policyholder who surrenders the policy. Doherty and Singer (2002, p. 18) also argue that regulatory constraints faced by life insurance carriers deter life insurance companies from offering health dependent cash surrender values: "Such an offering of explicit health-dependent surrender values by a life insurance carrier, however, would be fraught with regulatory, actuarial, and administrative difficulties. Life insurance carriers do not offer health-adjusted surrender values, which suggests that these difficulties outweigh the benefits that carriers would obtain by offering health-dependent surrender values to consumers." Life settlement firms so far are not yet regulated in their pricing of life insurance policies.

him to favor more cash now than to leave a bequest.[6] In the absence of the life settlement market, when a health-impaired policyholder chooses to lapse or surrender its policy, the life insurance company pockets the intrinsic economic value of these policies, which potentially allows the life insurance company to offer insurance at a lower premium. In the presence of the life settlement market, these policies will be purchased by the life settlement firms as assets, thus the primary insurance company will always have to pay their face amount if the original policyholder dies within the coverage period.

In this paper, we analyze the effect of life settlement on the primary life insurance market. Using a dynamic equilibrium model of life insurance similar to Hendel and Lizzeri (2003, HL henceforth) and Daily, Hendel and Lizzeri (2008, DHL henceforth), we study how equilibrium contracts and consumer welfare are affected by the presence of a life settlements market. We focus on how the equilibrium properties of the life insurance contracts and the consumer welfare are affected by life settlement firms in a dynamic equilibrium model of life insurance. Hendel and Lizzeri (2003) studied a model where life insurance companies are risk neutral and can commit to contractual terms (including future premiums and face amounts), and the consumers are risk averse and cannot commit to remain in the contract they earlier have chosen. Consumers are subject to mortality risk, which is assumed to be symmetrically observed by the consumers and the life insurance company. Because consumers' mortality risks may change over time, they will face *reclassification risk* (i.e., changes in insurance premiums) if they have to purchase one-period contracts from the spot market. HL showed that the competitive life insurance market will in equilibrium offer long-term life insurance policies that at least partially insure the policyholders against the reclassification risk, via front-loading of insurance premiums. In a recent paper, Daily, Hendel and Lizzeri (2008) further assumed that consumers in the second period may lose bequest motive, and introduced the life settlement market. They compared the consumer welfare with and without the life settlement market.

In this paper, we first fully characterize the equilibrium life insurance contract in the presence of a settlement market, assuming that the primary insurers cannot enrich their contract space to set optimally chosen cash surrender values. We show that the life settlement market affects the equilibrium life insurance contracts in a qualitatively important manner: with the settlement market, risk reclassification insurance will be offered in the form of *premium discounts*, rather than in the form of flat premiums, as is the case without a settlement market. This may lead to a *smaller* degree of front-loading in the first period. We also show a general welfare result that the presence of the settlement market always leads to a decrease of consumer welfare relative to what could be achieved in the absence of the settlement market. Moreover, we provide conditions under which

[6] For example, *Wall Street Journal* reports that older adults are turning to the "life settlement" industry to help them through tough times in an article titled "Source of Cash for Seniors are Drying Up" (November 13, 2008).

the life settlement market could lead to a complete collapse of reclassification risk insurance as a result of unraveling. We then relax the assumption that prohibits endogenously chosen CSVs, and find that whether or not CSVs can be made health-contingent has crucial implications. If the cash surrender values are restricted to be non-health-contingent, we show that endogenous CSV is an ineffective tool for the primary insurance companies to counter the threat of the life settlement industry.

Our paper is also related to a large industrial organization literature on secondary markets for durable goods (see, e.g., Hendel and Lizzeri, 1999; Stolyarov, 2002; House and Leahy, 2004). The key difference between the life settlement market and the secondary market for durable goods is as follows. In the durable good case, once the transaction between the primary seller and the buyer is consummated, the seller's payoff is not directly affected by whether the buyer sells the used durable in the secondary market. In contrast, for the case of life settlements, the primary insurer's payoff is directly impacted by whether the policyholder chooses to lapse, surrender for cash value, or to sell the contract to a settlement firm.[7]

The remainder of the paper is structured as follows. In Section 2 we present a baseline model without the life settlement market. In Section 3 we extend the baseline model to include the life settlement market and analyze its effect on primary market contracts and on consumer welfare. In Section 4 we consider how the welfare results are affected if the life insurance companies can respond to life settlements by specifying endogenous cash surrender values in the life insurance contract; in particular we show that whether health-contingent CSVs are allowed has a crucial effect. Finally, Section 5 summarizes our findings and discusses directions for future research. All proofs are collected in the Appendix.

## 2 The Baseline Model of Life Insurance without Settlement Market

In this section, we present and analyze a model of dynamic life insurance slightly modified from Hendel and Lizzeri (2003) and Daily, Hendel and Lizzeri (2008).[8]

[7]Other secondary markets in financial services, such as the home mortgages resale market and the catastrophic risk reinsurance, operates more like the secondary market for durable goods (see Doherty and Singer, 2002 for some description of these markets).

[8]The two main modifications are as follows. First, we consider a continuous distribution for second-period health states while HL and DHL considered discrete health states. Second, we slightly change the timing of events: we assume that income realization and premium payments occur before the resolution of death uncertainty, while HL and DHL assumed that income and consumption are realized after the death uncertainty is resolved. Neither modification makes any qualitative difference for this baseline model without the life settlement market; but they simplify some of our arguments when we introduce the life settlement market.

## 2.1 The Model

**Health, Income and Bequest Motives.** Consider a perfectly competitive primary market for life insurance that includes individuals (policyholders) and life insurance companies. There are two periods. In the first period, the policyholder has a probability of death $p_1 \in (0,1)$ known to both himself and the insurance companies. In the second period, the policyholder has a new probability of death $p_2 \in [0,1]$ which is randomly drawn from a continuous and differential cumulative distribution function $\Phi(\cdot)$ with a corresponding density $\phi(\cdot)$. A consumer's period 2 health state $p_2$ is not known in period 1, but $p_2$ is symmetrically learned by the insurance company and the consumer, and thus common knowledge, at the start of period 2.

The policyholder's income stream is $y - g$ in period 1 and $y + g$ in period 2, where $y$ is interpreted as the mean life-cycle income and $g \in (0, \bar{g}]$ with $\bar{g} < y$ captures the income growth over the periods. Both $y$ and $g$ are assumed to be common knowledge.

The policyholder has two sources of utility: his own consumption should he live, and his dependents' consumption should he die. If the policyholder lives, he derives utility $u(c)$ if he consumes $c \geq 0$; if he dies, then he has a utility $v(c)$ if his dependents consume $c \geq 0$. $u(\cdot)$ and $v(\cdot)$ are both strictly concave and twice differentiable.

However, in period 2, there is a chance that the policyholder no longer has a bequest motive. We denote by $q \in (0,1)$ the probability that the policyholder loses his bequest motive.[9] The bequest motive is realized at the same time as the period 2 health state; however, we assume that it is private information to the policyholder and cannot be contracted upon. If the policyholder retains his bequest motive, his utility in period 2 is again $u(\cdot)$ if he is alive and $v(\cdot)$ if he dies; if the policyholder loses bequest motive, then his utility is $u(\cdot)$ if he stays alive, and some constant which is normalized to zero if he dies.

We assume that there are no capital markets, thus the consumer cannot transfer income from period 1 to period 2. The only way for the consumer to ensure a stream of income for his dependents is to purchase life insurance.[10]

**Timing, Commitment, and Contracts.** Now we provide more details about the timing of events. At the beginning period 1, after learning the period-1 health state $p_1$, the consumer may purchase a long-term contract from an insurance company. A *long-term contract* specifies a premium and face value for period 1, $\langle Q_1, F_1 \rangle$, and a menu of health-contingent premiums and face values $\langle Q_2(p_2), F_2(p_2) \rangle$ for each period-2 health state $p_2 \in [0,1]$. In contrast, a spot contract is simply a

[9] A loss of bequest motive could result from divorce, or from changes in the circumstances of the intended beneficiaries of the life insurance policy.

[10] Studying how access to capital markets might affect the demand for life insurance and the welfare effect of life settlement is an important area for future research.

premium and a face value $\langle Q, F \rangle$ which earns zero expected profit for a given coverage period.

The key assumption is that the insurance companies can commit to these terms in period 2, but that the policyholders cannot. The *one-sided commitment* assumption has two important implications. First, it implies that the period-2 terms of the long-term insurance contract must be at least as desirable to the policyholder as what he could obtain in the period-2 spot market; otherwise, the policyholder will lapse the long-term contract into a new spot contract. This imposes a constraint on the set of feasible long term contracts that consumers will demand in period 1. Second, if a policyholder suddenly finds himself without a bequest motive, he could lapse his policy by refusing to pay the second period premium.

In period 2, after learning the period 2 health state $p_2$, the policyholder has three options. He can either continue with his long-term contract purchased in period 1, or he can let the long-term policy lapse and buy a period-2 spot contract, or he can let the long-term policy lapse and simply remain uninsured.

## 2.2 Equilibrium Contracts

To characterize the equilibrium set of contracts, we first consider the actions of a policyholder in the second period who no longer has a bequest motive. Given the absence of secondary market, and we have not yet allowed the insurance companies to buy back contracts through CSVs, the best course of action for those who no longer have a bequest motive is to simply let the long-term policy lapse and become uninsured.[11]

Competition among primary insurance companies ensures that the equilibrium contract is a long-term contract $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0,1] \rangle$ that solves:

$$\max \left[ u\left(y-g-Q_1\right) + p_1 v\left(F_1\right) \right] + \left(1-p_1\right) \int \left\{ \left(1-q\right) \left[ \begin{array}{c} u\left(y+g-Q_2\left(p_2\right)\right) \\ +p_2 v\left(F_2\left(p_2\right)\right) \end{array} \right] + qu\left(y+g\right) \right\} d\Phi(p_2) \tag{1}$$

$$\text{s.t.} \quad Q_1 - p_1 F_1 + \left(1-p_1\right)\left(1-q\right) \int \left[Q_2(p_2) - p_2 F_2(p_2)\right] d\Phi(p_2) = 0, \tag{2}$$

$$Q_2(p_2) - p_2 F_2(p_2) \leq 0, \text{ for all } p_2 \in [0,1], \tag{3}$$

where (1) is the expected utility the policyholders receive from the contract, (2) is the *zero-profit* constraint that reflects perfect competition in the primary market, and constraints (3) guarantee that there will not be lapsation among policyholders with a bequest motive in the second period.[12]

[11]We introduce "cash surrender value" in Section 4 when we consider the primary life insurers' response to the settlement market.

[12]See Hendel and Lizzeri (2003, Appendix) for a formal argument for why (3) guarantee "no lapsation" for those

The first order conditions for problem (1) with respect to $Q_1, F_1, Q_2(p_2)$ and $F_2(p_2)$ are, respectively:

$$u'(y-g-Q_1) = \mu, \tag{4a}$$

$$v'(F_1) = \mu, \tag{4b}$$

$$u'(y+g-Q_2(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)(1-q)\,\phi(p_2)}, \tag{4c}$$

$$v'(F_2(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)(1-q)\,\phi(p_2)}, \tag{4d}$$

where $\mu$ and $\lambda(p_2)$ are respectively the Lagrange multipliers for constraints (2) and (3), and $\mu > 0$ and $\lambda(p_2) \leq 0$ must also satisfy the complementary slackness conditions:

$$\lambda(p_2)\left[Q_2(p_2) - p_2 F_2(p_2)\right] = 0. \tag{5}$$

Notice that the first order conditions (4) imply that:

$$u'(y-g-Q_1) = v'(F_1), \tag{6a}$$

$$u'(y+g-Q_2(p_2)) = v'(F_2(p_2)) \text{ for all } p_2 \in [0,1]. \tag{6b}$$

Thus in equilibrium a policyholder obtains *full-event insurance* in every state in both periods. (6a)-(6b) also imply that there is a one-to-one relationship between the face amounts ($F_1$ and $F_2(p_2)$, respectively) and premiums ($Q_1$ and $Q_2(p_2)$, respectively) policyholders will obtain in equilibrium; thus it suffices to characterize the equilibrium premiums $Q_1$ and $Q_2(p_2)$ for all $p_2 \in [0,1]$. Moreover, because $u'$ and $v'$ are decreasing, the face amounts must *decrease* with the premium in every state in both periods.

To characterize the equilibrium premiums, it is useful to divide the support of the second-period health states $p_2$ into two subsets $\mathcal{B}$ and $\mathcal{NB}$ depending on whether the no-lapsation constraint (3) binds. We have the following lemma:

**Lemma 1.** *If $p_2 \in \mathcal{B}$ and $p_2' \in \mathcal{NB}$ then $p_2 < p_2'$ and $Q_2(p_2) \leq Q_2(p_2')$.*

Lemma 1 implies the existence of a threshold $p_2^*$ such that $p_2 \in \mathcal{B}$ if $p_2 < p_2^*$ and $p_2 \in \mathcal{NB}$ if $p_2 > p_2^*$. To characterize the equilibrium premiums $Q_2(p_2)$, it is useful to define the *fair premium and face amount* for full-event insurance under health state $p_2$ in the second period, denoted by

with bequest motives in the second period.

$Q_2^{FI}(p_2)$ and $F_2^{FI}(p_2)$, which uniquely solve the following pair of equations:

$$\begin{aligned} Q_2^{FI}(p_2) - p_2 F_2^{FI}(p_2) &= 0, \quad (7) \\ u'\left(y + g - Q_2^{FI}(p_2)\right) &= v'\left(F_2^{FI}(p_2)\right), \quad (8) \end{aligned}$$

where (7) ensures that the premium is actuarially fair, and (8) ensures full-event insurance as defined earlier.[13]

From Lemma 1, we have that for all $p_2 < p_2^*$, (3) is binding; together with the full-event insurance conditions (6b), we must have $Q_2(p_2) = Q_2^{FI}(p_2)$ for all $p_2 < p_2^*$. If $p_2 > p_2^*$, then we know from Lemma 1 that $p_2 \in \mathcal{NB}$, thus $\lambda(p_2) = 0$. Therefore the first order conditions (4a) and (4c) imply that:

$$u'(y - g - Q_1) = u'\left(y + g - Q_2(p_2)\right). \quad (9)$$

Equation (9) implies that if $p_2 > p_2^*$, the premium $Q_2(p_2)$ must be *independent of* $p_2$; moreover, it must satisfy

$$Q_2(p_2) = 2g + Q_1. \quad (10)$$

Finally, the following lemma characterizes the equilibrium premium for $p_2 = p_2^*$ if $p_2^* < 1$ :

**Lemma 2.** *If $p_2^* < 1$, then the equilibrium contract satisfies the following at $p_2 = p_2^*$ :*

$$\begin{aligned} Q_2(p_2^*) &= Q_2^{FI}(p_2^*), \quad (11) \\ u'\left(y + g - Q_2^{FI}(p_2^*)\right) &= u'(y - g - Q_1). \quad (12) \end{aligned}$$

Lemma 2 implies that the threshold $p_2^*$ is uniquely determined by the following equation:

$$Q_2^{FI}(p_2^*) = 2g + Q_1. \quad (13)$$

Summarizing the above discussions, we have shown that in the baseline model, the equilibrium premium profile over period-2 health states follows the increasing actuarially fair premium profile $Q_2^{FI}(\cdot)$ as $p_2$ approaches $p_2^* = \left[Q_2^{FI}\right]^{-1}(2g + Q_1)$, but remains constant at $Q_1 + 2g$ for $p_2 > p_2^*$. Figure 1 depicts the equilibrium premiums in the second period as a function of the period-2 health state $p_2$.[14]

The remaining element of the equilibrium long-term contract, i.e. the first period premium $Q_1$ (and thus also $F_1$), is determined from the zero profit condition (2). Since we have shown that for

[13] The equation system (7) and (8) has a unique solution because $u'$ and $v'$ are both decreasing in their arguments.

[14] Hendel and Lizzeri (2003) made the ingeneous observation that excatly the same outcome for the consumers would obtain if the insurance company offers a contract that guarantees the second period premium to be $Q_1 + 2g$ for *all* health states.

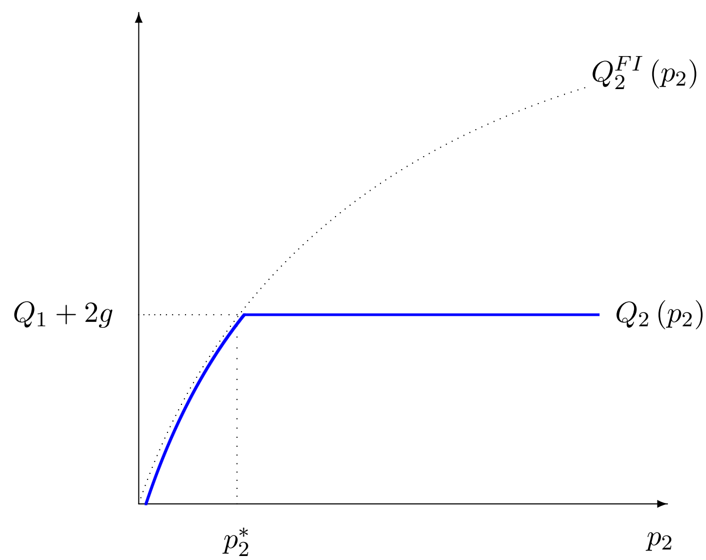


Figure 1: Equilibrium Period-2 Premium Profile $Q_2(p_2)$: The No Life Settlement Market Case.

all $p_2 \le p_2^*$, $Q_2(p_2) = p_2 F_2(p_2)$, (2) can now be rewritten as:

$$(Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_{p_2^*}^{1} [Q_2(p_2) - p_2 F_2(p_2)]\, d\Phi(p_2) = 0. \tag{14}$$

Because $Q_2(p_2) < p_2 F_2(p_2)$ for all $p_2 > p_2^*$, the second term in the left hand side of (14) is negative, i.e., the insurance company in equilibrium will for sure suffer a loss in the second period; thus, the zero-profit condition (14) requires that $Q_1 - p_1 F_1 > 0$. In other words, the insurance company demands in the first period a premium $Q_1$ that is higher than the actuarially fair premium $p_1 F_1$ for the period-1 coverage alone. This is exactly the phenomenon of *front-loading*. In equilibrium, consumers accept a front loaded premium in exchange for *reclassification risk insurance*. This reclassification risk insurance takes the form of flat premiums in the second period for all health states $p_2 > p_2^*$.

Two useful observations are in order. First, one can show that $p_2^*$ will always be greater than $p_1$, the period-1 death probability. To see this, suppose to the contrary that $p_2^* < p_1$, hence $p_1 \in \mathcal{NB}$. From (10), we have $Q_2(p_1) = Q_1 + 2g$. The first order conditions (6a) and (6b) then imply that if $Q_2(p_1) = Q_1 + 2g$, it must be the case that $F_2(p_1) = F_1$. Since by assumption $p_1 \in \mathcal{NB}$, we have $Q_2(p_1) - p_1 F_2(p_1) = (Q_1 + 2g) - p_1 F_1 < 0$; hence $Q_1 - p_1 F_1 < -2g < 0$. This contradicts the zero-profit constraint, which requires that $Q_1 - p_1 F_1$ is positive whenever $p_2^* < 1$.

One can also show that when $g$ is sufficiently small, then $p_2^* < 1$; i.e., there must be some degree of reclassification risk insurance and front-loading in equilibrium. To see this, suppose that $p_2^* = 1$, that is, suppose that no-lapsation constraint (3) binds for all $p_2$, then first order conditions (4) imply that $u'(y + g - Q_2(p_2)) \leq u'(y - g - Q_1)$ for all $p_2 \in [0, 1)$. Since $u$ is concave, this in turn implies that $Q_2(p_2) \leq Q_1 + 2g$ for all $p_2$. However, for any $p > p_1$, it must be the case that $Q_2(p_2) > Q_1$ if $p_2 \in \mathcal{B}$.[15] Thus when $g$ is sufficiently small, it is impossible to have both $Q_2(p_2) \leq Q_1 + 2g$ for all $p_2$ and $Q_2(p_2) > Q_1$ for all $p_2 > p_1$.

The above discussions are summarized below, which replicates Parts 1-3 of Proposition 1 in Hendel and Lizzeri (2003) with some slight modifications.

**Proposition 1.** ***(Hendel and Lizzeri 2003)*** *The equilibrium set of contracts satisfies the following:*

1. *All policyholders obtain full event insurance in period 1, and in all period-2 health states as defined by (6);*

2. *There is a period-2 threshold health state $p_2^*$ (which is higher than the period 1 death probability $p_1$) such that for all $p_2 \leq p_2^*$ the period-2 premiums are actuarially fair, and for all $p_2 > p_2^*$ the period-2 premiums are constant, actuarially favorable and given by $Q_2(p_2) = Q_2(p_2^*) = Q_1 + 2g$;*

3. *When the income growth parameter $g$ is sufficiently small, $p_2^*$ is strictly less than 1, i.e. reclassification risk insurance is provided for policyholders with low income growth.*

The following proposition provides a comparative statics result about how the probability of bequest motive loss $q$ affects the contract profile, front-loading and reclassification risk insurance:

**Proposition 2.** *Let $\hat{q} > q$. Let unhatted and hatted variables denote equilibrium for $q$ and $\hat{q}$ respectively. Suppose that $p_2^* < 1$. Then in equilibrium $\hat{Q}_1 < Q_1$ and $\hat{p}_2^* < p_2^*$.*

Proposition 2 states that as the probability of losing bequest motive increases from $q$ to $\hat{q}$, the first period premium will be lower, which implies that there will be less front-loading in the first period in equilibrium.[16] At the same time, however, a higher degree of reclassification risk is offered in the second period, i.e., more states are offered actuarially favorable contract terms and premiums are lower across the board. The intuition is quite simple. In the current setting without a settlement market, policyholders who lose bequest motives in period 2 will lapse their policy,

[15] To see this, note that under the hypothesis that the set $\mathcal{NB}$ is empty, it must be the case that $(Q_1, F_1)$ is actuarially fair, i.e. $Q_1 - p_1 F_1 = 0$. Because $(Q_1, F_1)$ and $(Q_2(p_2), F_2(p_2))$ must both provide full-event insurance as defined by (6a) and (6b), if $Q_2(p_2) \leq Q_1$, it must be that $F_2(p_2) > F_1$ for any $g > 0$. But then for all $p_2 > p_1$,

$$Q_2(p_2) - p_2 F_2(p_2) < Q_1 - p_2 F_1 < Q_1 - p_1 F_1 = 0,$$

contradicting the assumption $p_2 \in \mathcal{B}$.

[16] Because period-one premium $Q_1$ is lower and period-one face amount $F_1$ (from full-event insurance condition) is higher, the amount of front-loading, namely $Q_1 - p_1 F_1$, must be lower.

which results in a net profit for the life insurance companies; because the life insurance companies are competitive, they are able to pass these profits onto consumers in the form of lower first period premiums and a higher degree of reclassification risk insurance. We call this phenomenon *lapsation based pricing*.

## 3 Introducing the Life Settlement Market

We now introduce the life settlement market. Policyholders who lose bequest motive in period 2 can now sell their contracts to life settlement firms. If a policyholder with period-2 death probability $p_2$ sells her policy to a life settlement firm, the life settlement firm will pay her premium to the life insurance company and collect the death benefit if the policyholder dies.

We now specify the amount that the policyholder receives from the settlement firm when she sells her policy. To this end, we introduce the concept of the *actuarial value* of a life insurance policy. Suppose that a policyholder purchased a long-term life insurance policy in period 1 that specifies a premium $Q_2(p_2)$ for a death benefit of $F_2(p_2)$ if her period-2 health state realization is $p_2$. Then the actuarial value of the contract at health state $p_2$ is $V_2(p_2) \equiv p_2 F_2(p_2) - Q_2(p_2)$. As an example, recall from Proposition 1 that in the equilibrium without a secondary market, the long-term policies are such that in period-2, the actuarial values $V_2(p_2) \equiv p_2 F_2(p_2) - Q_2(p_2)$ is strictly positive for $p_2 > p_2^*$. If policyholders with period-2 health states $p_2 > p_2^*$ lose bequest motive, they will prefer to capture some of the positive actuarial value rather than let the policy lapse. This, as we described in the introduction, is the source of surplus for the life settlement market. We assume that a policyholder will receive a fraction $\beta \in (0,1)$ of the actuarial value of the policy if she sells her life insurance policy to the settlement firm, where $\beta < 1$ represents either the degree of competition in the secondary market or the amount of fees/commissions/profits etc. that are spent by settlement firms.[17]

We assume that the secondary market only operates in period 2.[18] We also assume that policyholders make their decisions about whether to sell a contract on the secondary market after learning the realization of period-2 health state and bequest motive, but before the realization of the death uncertainty. Finally, we make the natural assumption that the resale price of the contract is paid to the policyholder upon transfer of the policy to the settlement company and before the death uncertainty is realized.

[17] Currently the life settlement industry typically offers about 20% of the *death benefits* to sellers after commissions and fees. Since $\beta$ is relative to the actuarial value, the plausible range of $\beta$ is 0.4 to 0.6 (see Life Insurance Settlement Association (2006)).

[18] This is an innocuous assumption because the zero-profit condition on the primary market ensures that all period 1 contracts are actuarially fair, and thus there is no surplus to be recovered on a secondary market for period 1 contracts.

### 3.1 Equilibrium Contracts With Settlement Market

Now we characterize the equilibrium contract in the presence of the secondary market. To start, we note that in period 2 a policyholder will sell her contract to the settlement firm if and only if she no longer has a bequest motive.[19] In the presence of settlement firms, the insurance companies in the primary market will choose a long-term contract $\langle (Q_1^s, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\}\rangle$ to solve:[20]

$$\max\left[u(y-g-Q_1^s)+p_1 v(F_1^s)\right] \tag{15}$$
$$+(1-p_1)\int\left\{(1-q)\left[\begin{array}{c} u\left(y+g-Q_2^s(p_2)\right) \\ +p_2 v(F_2^s(p_2)) \end{array}\right]+qu\left(y+g+\beta V_2^s(p_2)\right)\right\}d\Phi(p_2)$$

$$\text{s.t.}\quad Q_1^s - p_1 F_1^s + (1-p_1)\int\left[Q_2^s(p_2)-p_2 F_2^s(p_2)\right]d\Phi(p_2) = 0 \tag{16}$$
$$Q_2^s(p_2) - p_2 F_2^s(p_2) \le 0 \text{ for all } p_2 \in [0,1], \tag{17}$$

where $V_2^s(p_2) \equiv p_2 F_2^s(p_2) - Q_2(p_2)$ is the actuarial value of the contract in period 2 for a policyholder with health status $p_2$. Note that there are *two* key differences between the problems with and without the settlement market. First, the consumer's expected utility functions (1) and (15) differ in that, for the case with the secondary market, the term $\beta V_2^s(p_2)$ enters in (15), reflecting the added amount of consumption from selling the policy to the settlement firm when the policyholder no longer has bequest motive. Second, the zero-profit condition for the insurance company now does not have the $1-q$ term multiplying $\int[Q_2^s(p_2)-p_2F_2^s(p_2)]\,d\Phi(p_2)$ in (16). The $1-q$ term is no longer in the zero profit condition because when there is a secondary market, no policyholders with a positive actuarial value will let their contracts lapse, and the insurance companies are liable for paying the death benefits for *all* policies in the second period and of course also collect all contracted second-period premiums.

It is useful to emphasize that the problem without settlement market *does not* correspond to a special case of the problem with settlement firms by setting $\beta = 0$. Setting $\beta = 0$ would have restored the objective function in problem (15) to be identical to that in (1), but the zero profit condition (16) would still be different from that of (2). Put it differently, even if policyholders are selling their policies for free, the primary insurer is still liable for every policy sold to the settlement market, thus $1-q$ does not enter the zero profit constraint.

Similar to the case without the secondary market, the first order conditions for an optimum

[19] If a policyholder still has bequest motive, she will never sell her original policy and repurchase on the spot market because the best she could do if she sells is to get the full actuarial value of her original contract, but she will subsequently repurchase a spot contract with the same face amount.

[20] We use superscript $s$ to denote the contract terms for the secondary market case.

with respect to $Q_1^s, F_1^s, Q_2^s(p_2)$ and $F_2^s(p_2)$ are respectively as follows:

$$u'(y - g - Q_1^s) = \mu \tag{18a}$$

$$v'(F_1^s) = \mu \tag{18b}$$

$$(1-q)\, u'(y + g - Q_2^s(p_2)) + \beta q u'(y + g + \beta V_2^s(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)\phi(p_2)} \tag{18c}$$

$$(1-q)\, v'(F_2^s(p_2)) + \beta q u'(y + g + \beta V_2^s(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)\phi(p_2)} \tag{18d}$$

where, again, $\mu \geq 0$ is the Lagrange multipliers for (16) and $\lambda(p_2) \leq 0$ is the Lagrange multiplier for the no-lapsation constraint (18d) for period-2 health state $p_2$, with the complementarity slackness condition:

$$\lambda(p_2)\left[Q_2^s(p_2) - p_2 F_2^s(p_2)\right] = 0. \tag{19}$$

From the first order conditions (18), we immediately have:

$$u'(y - g - Q_1^s) = v'(F_1^s) \tag{20}$$

$$u'(y + g - Q_2^s(p_2)) = v'(F_2^s(p_2)) \text{ for all } p_2 \in [0,1]\,. \tag{21}$$

Thus, as in the case without settlement firms, we again see that the equilibrium terms of the contract $\langle (Q_1^s, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\}\rangle$ must provide full-event insurance in both periods and in all health states. Denote the set of states for which the no-lapsation constraints bind and do not bind respectively as $\mathcal{B}^s$ and $\mathcal{NB}^s$. Lemma 3 is an analog of Lemma 1 for the case with secondary market:

**Lemma 3.** *If $p_2 \in \mathcal{B}^s$ and $p_2' \in \mathcal{NB}^s$, then $p_2 < p_2'$ and $Q_2^s(p_2) < Q_2^s(p_2')$.*

As in the case without a secondary market, Lemma 3 below shows that there exists a threshold period-2 health state $p_2^{s*} \in [0,1]$ such that $p_2 \in \mathcal{B}^s$ if $p_2 < p_2^{s*}$, and $p_2 \in \mathcal{NB}^s$ if $p_2 > p_2^{s*}$. The threshold period-2 health state $p_2^{s*}$ is characterized by Lemma 4:

**Lemma 4.** *If $p_2^{s*} < 1$, then the equilibrium contract satisfies the following at $p_2 = p_2^{s*}$ :*

$$Q_2^s(p_2^{s*}) = Q_2^{FI}(p_2^{s*}) \tag{22}$$

$$(1-q)\, u'(y + g - Q_2^{FI}(p_2^{s*})) + \beta q u'(y+g) = u'(y - g - Q_1^s). \tag{23}$$

Recall from Lemma 2 that, in the case without a settlement market, the threshold $p_2^*$ does not depend on q, the probability of bequest motive loss. In contrast, Lemma 4 shows when there is a settlement market, $p_2^{s*}$ is related to both both $\beta$ and $q$. This fact plays an important role for the unraveling result reported in Proposition 6 below.

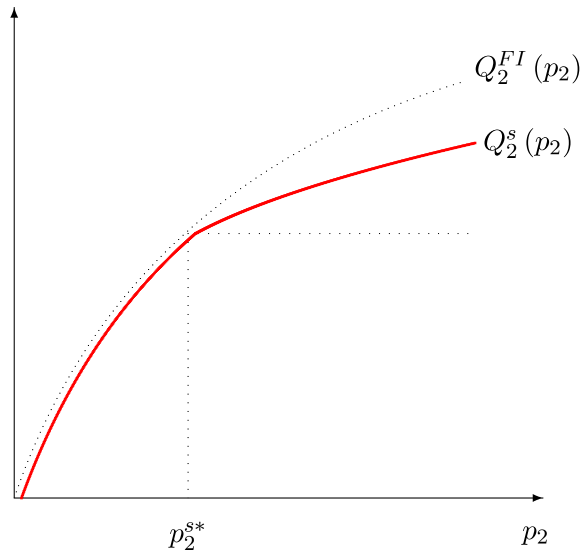


Figure 2: Equilibrium Period-2 Premium Profile $Q_2^s(p_2)$: The Life Settlement Market Case.

Lemmas 3 and 4 establish that the equilibrium contract with a secondary market has some resemblance to that without the secondary market. Proposition 3 shows, however, that the secondary market also leads to an important qualitative difference in the way the primary insurance market provides reclassification risk insurance to the consumers.

**Proposition 3.** ***(Period-2 Premium at Non-binding Health States)*** *In the presence of the life settlement market, for $p_2 > p_2^{s*}$, the second-period premium $Q_2^s(p_2)$ is such that:*

1. $Q_2^s(p_2) < Q_2^{FI}(p_2)$;
2. $Q_2^s(p_2)$ *is strictly increasing in $p_2$ when $q > 0$.*

Proposition 3 shows that with the life settlement market, reclassification risk insurance will no longer take the form of guaranteed flat premiums in the second period, as is the case when the settlement market does not exist; instead, the long-term contract now offers partial reclassification risk insurance in the form of *premium discounts* relative to the spot market premium. Figure 2 depicts the qualitative features of the second-period premium file $Q_2^s(p_2)$. Of course, the health states for which such partial reclassification risk insurance will be provided, i.e., the set $\mathcal{NB}^s$, may be different from the set $\mathcal{NB}$ that is relevant for the case without secondary market.

We now provide some comparative statics results with the settlement market.

**Proposition 4.** *If $q > \hat{q}$ then $Q_1^s \leq \hat{Q}_1^s$ where $Q_1^s$ and $\hat{Q}_1^s$ are respectively the equilibrium period-one premium for $q$ and $\hat{q}$.*

Proposition 4 tells us that the higher the probability that the consumers may lose bequest motives, the lower is the first-period premium. At a first glance, this seems to be just an analog of Proposition 2 for the case without the settlement market, but this analogy is misleading. When there is no settlement market, a higher $q$ directly increases the probability of lapsation and thus lowers the probability that the insurance company has to pay the death benefits in period 2. The primary insurance companies under competition will thus lower first period premiums (i.e., lower front-loading) and lower $p_2^*$ (i.e. offer more dynamic insurance), as shown in Proposition 2. Notice that lower front-loading can be compatible with more dynamic insurance when there is no settlement market because offering more dynamic insurance (i.e. lowering $p_2^*$) can still be less costly if more of these contracts will lapse (and hence no death benefit payments are necessary) as a result of a higher $q$.

The presence of the settlement market, however, ensures that the primary insurer will not be able avoid paying death benefits even when the policyholder loses her bequest motive. Thus the comparative statics result reported in Proposition 4 does not result from the direct effect that a higher $q$ reduces the probability of paying out death benefits. It instead arises because the *nature* of the dynamic insurance in equilibrium is fundamentally changed: it is now in the form of premium discounts instead of flat premiums,as proved in proposition 3. Offering premium discounts is a less costly way of providing dynamic insurance than offering flat premiums, which allows the primary insurers to lower the first period premium.

Another useful piece of intuition to explain both Propositions 2 and 4 is that the demand for consumption smoothing increases when $q$ gets higher. Specifically, as $q$ gets higher, it becomes more likely for the policyholder to be in period 2 with high income and no bequest motive. As such, they would prefer to transfer income from this period-2 state to the first period, where income is lower, if they could. This transfer occurs indirectly through lower first period premiums when $q$ is higher.

An important difference in terms of comparative statics results with respect to $q$ between the cases with and without the settlement market is in how $q$ affects $p_2^*$ and $p_2^{s*}$. Proposition 2 tells us that when there is no settlement market, the insurance companies will respond to a higher $q$ by lowering the first period premium *and* increasing dynamic insurance (i.e. a lower $p_2^*$). With the settlement market, we already showed that the primary insurers will also respond to a higher $q$ by lowering the first period premium, but it is no longer clear that $p_2^{s*}$ is also lowered. Indeed, a more plausible conjecture would be that $p_2^{s*}$ increases with $q$. The intuition is that when the period-one premium is lowered, the zero-profit condition would require that the primary insurers offer less insurance (here, again, note importantly that a higher $q$ does not allow the primary insurers to pay less death benefits when there is settlement market). This intuition is complicated, however, by the fact that the shape of $Q_2^s(\cdot)$ itself may be affected by $q$, as shown in Figure 2, and the first order conditions (18a)-(18d). At a *global* level, we must expect to see a decline of $p_2^{s*}$ as $q$ decreases from very large values to small values.[21] However, at a *local* level, without further assumptions, we are unable to prove that an increase in $p_2^{s*}$ lowers the second-period loss for the primary insurers.

We also state a simple comparative statics result in terms of consumer welfare with respect to $\beta$, the parameter that measures the competitiveness or the efficiency (i.e. loadings) of the settlement market. The result follows from the envelope theorem after recognizing that, in the optimization problem given by (15)-(17), the parameter $\beta$ only appears in the objective function.

**Proposition 5.** *An increase in $\beta$ increases consumer welfare.*

The effect of $\beta$ on the structure of equilibrium contracts is much harder to establish. The complication is similar to what we described above related to the effect of $q$, in that a change in $\beta$ can potentially affect the shape of $Q_2^s(\cdot)$.

### 3.2 Welfare Effects of the Settlement Market

In this section, we describe the effects of the settlement market on consumer welfare. We first consider a limiting result to demonstrate a potentially stark effect of the secondary market on the extent to which reclassification risk insurance can be achieved by the primary insurers.

**Proposition 6.** ***(Potential for Unraveling)*** *Fix $u(\cdot,), v(\cdot), y$ and $\Phi(\cdot)$. There is a threshold $\hat{q} > 0$ such that if $q > \hat{q}$, then $\mathcal{NB}^s = \emptyset$ for* any *$g$, that is, the equilibrium contract is the set of spot market contracts for all period-2 health states.*

Proposition 6 shows that in the presence of the life settlement market, the primary life insurance market can no longer offer any dynamic reclassification risk insurance for *any* level of $g$, when $q$ is sufficiently large. This result provides a stark contrast to Claim (3) in Proposition 1 which states that, without life settlement, the equilibrium contract offered by life insurance companies must involve some degree of reclassification risk insurance *when $g$ is sufficiently small for any $q < 1$*. Proposition 6 thus tells us that settlement market may lead to the unraveling of the capacity of primary life insurance market to offer dynamic reclassification risk insurance.

Note that Proposition 6 provides a clear welfare ranking of the equilibria with and without the settlement market for environments with small $g$ and large $q$ : without the settlement market,

[21] For sufficiently large values of $q$, Proposition 6 below shows that the settlement market unravels reclassification risk insurance, and thus $p_2^{s*} = 1$. But for $q = 0$, there is no scope for a settlement market and Proposition 1 shows $p_2^{s*} = p_2^* < 1$.

when $g$ is small and $q$ is large, the equilibrium contracts must offer dynamic reclassification risk insurance; with the settlement market, the equilibrium insurance contracts must be spot contracts. Thus the settlement market reduces consumer welfare when $g$ is small and $q$ is large.

Our next proposition shows that the same welfare ranking between the cases with and without a settlement market holds more generally, even when reclassification risk insurance is offered in both environments.

**Proposition 7.** ***(Welfare Effects of the Secondary Market)*** *Consumer welfare is reduced by the presence of a life settlement market.*

The argument we use in proving Proposition 7 is as follows: for any contract that is feasible – including the equilibrium contract – with a secondary market, we show that there exist a feasible contract without the secondary market. The constructed contract for the no settlement market case offers identical coverage as the original contract for the settlement market case, except at a lower first-period premium, which is made possible by lapsation pricing. We show that consumers are weakly better off under the constructed contract.

Proposition 7 formalizes an intuitive argument provided in Claim 3 of Proposition 2 in Daily, Hendel and Lizzeri (2008). They argued that the settlement market effectively transfers resources from period 1 when income is relatively low to period 2 when income is relatively high in the event of losing bequest motive. Such transfers, due to concavity of the utility function, are welfare reducing. The informal argument provided in their paper hinges on the hypothesis that the first-period *equilibrium* premium in the primary market is higher with the settlement market than without. This hypothesis does not hold in general. An extreme example of this is provided already in Proposition 6. When $q$ is sufficiently large, Proposition 6 establishes that the primary insurance market can not offer any dynamic insurance, which implies that the first period premium is $Q_1^s = Q_1^{FI}$, the actuarially fair premium. In contrast, when $g$ is small, Proposition 1 tells us that without the settlement market there will be dynamic insurance, implying that the first period premium $Q_1 > Q_1^{FI}$ because of front-loading. Thus, for sufficiently large $q$ and small $g$, the first-period equilibrium premium in the primary market is lower with the settlement market than without. Our proof does not rely on such cross-regime comparisons of first-period *equilibrium* premiums. Even though the first period equilibrium premium may be lower with the settlement market than without the settlement market, the reduction in first period premium occurs only by forgoing potentially welfare enhancing reclassification risk insurance.

Many would consider the emergence of the settlement firms as a form of market completion (e.g., Doherty and Singer (2002)). After all, consumers who lose their bequest motives in period 2 can share the surplus in the actuarial value of their policy with the settlement firm, something they could not do when there is no settlement market. So on a first glance, the welfare result in Proposition 7 is somewhat counter-intuitive. However, from the classical Lipsey and Lancaster (1956), we know that, loosely, once we depart from complete markets (or *all* the conditions required for the optimality of equilibrium), it is possible that the next-best solution may not be the one with the least degree of market incompleteness. Because the market incompleteness due to the lack of commitment is present with or without the settlement firms, we are in a second-best world.[22]

Another intuition that may be useful for understanding the welfare result is the following. The settlement market allows policyholders access to the actuarial value in their policies, and thus may be interpreted as contributing to market completeness. At the same time, however, the settlement market *weakens* the consumer's ability to commit to not asking for a return of their front loaded premiums in the event that they lose their bequest motive. This weakening of the consumers' commitment power can be interpreted as contributing further to market incompleteness.

# 4 Endogenous Cash Surrender Values

So far, we have assumed that the response of the primary market to settlement firms is restricted to choosing contracts of the form $\langle (Q_1^s, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\} \rangle$ . It is possible that, facing the threat from the life settlement market, the primary insurers may start to enrich their contracts by specifying *endogenously chosen cash surrender values* (CSV). We distinguish two possibilities of cash surrender values depending on whether they could be contingent on the health of the policyholder at the time she surrenders the policy to the primary insurer. The categories of market regimes we consider in this section, defined by the combination of whether there is a settlement market and what type of CSVs are allowed in the contract, are summarized in Table 1. The case of non-health contingent CSVs is interesting for at least two reasons. First and foremost, current U.S. regulations allows life insurance companies to offer CSVs that depend on the tenure of the policy, but prohibits them from offering health-contingent CSVs (see, e.g., Gilbert and Schultz (1994, Chapter 6)). In fact, currently almost all term life insurance policies have zero CSVs and most whole life insurance policies have very low and non-health-contingent CSVs. Second, health-contingent CSVs may not be easy to enforce.

In this section, we first study the case in which the primary insurers can specify health-contingent CSVs in the life insurance contracts of the form $\langle (Q_1, F_1), \{(Q_2(p_2), F_2(p_2), S(p_2)) : p_2 \in [0,1]\} \rangle$ where the additional term $S(p_2)$ specifies the cash surrender value a policyholder can receive from the primary insurer if she surrenders the contract when her period-2 health status is $p_2$. Notice that by paying $S(p_2)$ , the primary insurer can *avoid* paying the death benefit, but does not receive the premium payment in period-2. This is significantly different from when the policyholder sells the

[22] For example, Levin (2001) showed that in an Akerlof lemons model, greater information asymmetries between the buyers and the sellers do not necessarily reduce the equilibrium gains from trade.

| | | Cash Surrender Value | | |
|---|---|---|---|---|
| | | None | Non-Health Contingent | Health Contingent |
| Settlement Market | No | A | A′ | A″ |
| | Yes | B | C | D |

Table 1: Categorization of Market Regimes.

contract to the settlement market, in which case the primary insurer still receives payment of the period-2 premium, but remains responsible for paying the death benefits. We show in Section 4.1 that in the absence of the settlement market, the option of specifying health-contingent CSVs will not be exercised in equilibrium by the life insurance companies. We thus immediately know that the equilibrium contracts and equilibrium consumer welfare under regimes A, A′ and A″ will be identical. We then show in Section **??** that under regime D, when there is a settlement market and when the primary insurers are allowed to enrich their contracts by offering health-contingent CSVs, the consumer welfare will be improved relative to that under regime B, but still lower than that under regime A. In Section 4.3, we study regime C where there is a settlement market and the primary insurers are restricted to specify only non-health contingent CSVs. We prove a surprising result that, if restricted to non-health contingent CSVs, the primary insurers will offer zero CSV in equilibrium, and thus the consumer welfare and equilibrium contract are identical to those under regime B.

### 4.1 Endogenous Cash Surrender Values Without a Secondary Market

As a benchmark, we here show that, in the absence of a life settlement market, the consumers will optimally choose to set $S(p_2) = 0$ when the feasible contract space is enriched to include health contingent CSVs (regime A′).

The competitive insurance companies under this regime will choose a long-term contract in the form of $\langle (Q_1, F_1), \{(Q_2(p_2), F_2(p_2), S(p_2)) : p_2 \in [0,1]\}\rangle$ to solve:

$$\max u(y-g-Q_1)+p_1 v(F_1) \tag{24}$$
$$+(1-p_1)\int\left\{(1-q)\left[\begin{array}{c} u(y+g-Q_2(p_2)) \\ +p_2 v(F_2(p_2)) \end{array}\right]+qu(y+g+S(p_2))\right\}d\Phi(p_2)$$
$$\text{s.t. } Q_1-p_1F_1+(1-p_1)\int\{(1-q)\left[Q_2(p_2)-p_2F_2(p_2)\right]-qS(p_2)\}\,d\Phi(p_2)=0, \tag{25}$$
$$Q_2(p_2)-p_2F_2(p_2)\leq 0 \text{ for all } p_2, \tag{26}$$
$$S(p_2)\geq 0 \text{ for all } p_2, \tag{27}$$

where (25) is the zero-profit constraint reflecting the competitive nature of the primary insurance market, (26) is again the no-lapsation constraint, and (27) is simply the constraint that the CSV can not be negative because consumers can not commit to contracts requiring them to pay the insurance company at termination. The first order conditions for the optimum with respect to $Q_1, F_1, Q_2(p_2), F_2(p_2)$ and $S(p_2)$ are respectively:

$$u'(y-g-Q_1)=\mu \tag{28a}$$
$$v'(F_1)=\mu \tag{28b}$$
$$u'(y+g-Q_2(p_2))=\mu+\frac{\lambda(p_2)}{(1-p_1)(1-q)\,\phi(p_2)} \tag{28c}$$
$$v'(F_2(p_2))=\mu+\frac{\lambda(p_2)}{(1-p_1)(1-q)\,\phi(p_2)} \tag{28d}$$
$$u'(y+g+S(p_2))=\mu+\frac{\gamma(p_2)}{(1-p_1)q\phi(p_2)}, \tag{28e}$$

where, as in Section 2, $\mu$ and $\lambda(p_2)$ are respectively the Lagrange multipliers for constraints (25) and (26), and $\gamma(p_2)$ is the Lagrange multiplier for constraint (27).

It is easy to see that (27) must bind, i.e. $S(p_2)=0$, for all $p_2$ If it were slack for some $p_2\in[0,1]$, then for such $p_2$, we will have $\gamma(p_2)=0$; then the first order conditions (28a) and (28e) would immediately imply that $u'(y+g+S(p_2))=u'(y-g-Q_1)$, which is impossible. We can thus conclude that, without a secondary market, the primary insurance companies will choose in equilibrium not to include a health-contingent CSV option in their long-term contract. The proposition below summarizes this discussion:

**Proposition 8.** *In the absence of a settlement market, the option to include surrender values in long-term contracts will not be used by the life insurance companies. Thus equilibrium contracts and outcomes for the consumers will be identical regardless of whether health-contingent CSVs are allowed.*

Proposition 8 is consistent with the observation that term life insurance products indeed do not carry any cash surrender value. Whole Life insurance policies, as we mentioned in the introduction, often do specify a low amount of cash surrender value if the policyholder cancels the policy prior to death. The industry has typically advertised the cash surrender value option as a redemption of front-loaded premium payments, even though many industry analysts disagree and think that it should be better interpreted as a saving instrument that exploits the tax advantages of life insurance payouts (see, e.g., Gilbert and Schultz (1994, Chapter 6)). Proposition 8 suggests the latter interpretation is more appropriate; in the absence of threats from secondary market, it would have been efficient not to specify any cash surrender value in a pure life insurance contract.

It is interesting to note that Proposition 7 can be seen as a corollary of Proposition 8 and Proposition 5. To see this, note that when $\beta = 1$, problem (15) is a special case of problem (24) by restricting $S(p_2) = V_2^s(p_2)$ for all $p_2$. Thus the consumer welfare at the optimum of problem (24), and thus by Proposition 8 the consumer welfare under regime A as well, is at least as high as that of problem (15), even when $\beta = 1$. But from Proposition 5, the consumer welfare is actually highest when $\beta = 1$ for regime B. Thus consumer welfare is always higher under regime A than under regime B for all $\beta \in (0, 1]$, which is precisely what Proposition 7 states.

### 4.2 Endogenous Health-Contingent Cash Surrender Values with a Settlement Market

Now we consider equilibrium contracts with health-contingent surrender values in the presence of a settlement market. The key difference from problem (24) above is that the cash surrender value now must be no less than what the policyholder could obtain from the settlement market, while in problem (24) $S(p_2)$ needs to be no less than $0$. Denote the equilibrium contract with a settlement market and endogenous CSVs with the superscript $ss$. The competitive insurance companies offer a long-term contract $\langle (Q_1^{ss}, F_1^{ss}), \{(Q_2^{ss}(p_2), F_2^{ss}(p_2), S^{ss}(p_2)) : p_2 \in [0,1]\}\rangle$ to solve:

$$\max u(y - g - Q_1^{ss}) + p_1 v(F_1^{ss}) \tag{29}$$

$$+ (1-p_1) \int \left\{ (1-q) \left[ \begin{array}{c} u(y + g - Q_2^{ss}(p_2)) \\ +p_2 v(F_2^{ss}(p_2)) \end{array} \right] + q u(y + g + S^{ss}(p_2)) \right\} d\Phi(p_2)$$

$$\text{s.t.} Q_1^{ss} - p_1 F_1^{ss} + (1-p_1) \int \{(1-q)\left[Q_2^{ss}(p_2) - p_2 F_2^{ss}(p_2)\right] - q S^{ss}(p_2)\} d\Phi(p_2) = 0 \tag{30}$$

$$Q_2^{ss}(p_2) - p_2 F_2^{ss}(p_2) \leq 0 \text{ for all } p_2, \tag{31}$$

$$\beta \left[p_2 F_2^{ss}(p_2) - Q_2^{ss}(p_2)\right] - S^{ss}(p_2) \leq 0 \text{ for all } p_2, \tag{32}$$

where constraint (32) reflects the requirement that the endogenous CSV must be at least as high as what the policyholder can obtain from the settlement firms, i.e., $\beta \left[p_2 F_2^{ss}(p_2) - Q_2^{ss}(p_2)\right]$.[23] The insurance company will never set the cash surrender value to be lower than what could be obtained on the secondary market because by offering just an $\varepsilon$ more, the insurance company can repurchase the policy for $\beta V_2^{ss}(p_2) + \varepsilon$. This is preferred to letting the policy sold on the settlement market, in which case the insurance compnay is liable for $V_2^{ss}(p_2)$.

Arguments similar to those used in the proof of Proposition 8 above show that:

**Lemma 5.** *In the presence of a secondary market, health-contingent cash surrender values $S^{ss}(p_2)$ will be optimally chosen to be equal to the amount that can be obtained from the secondary market.*

Arguments analogous to those used in the proof of Lemmas 3 and 4 can be easily adapted to show that there exists a threshold period-2 health state $p_2^{ss*}$ above which second period premiums are actuarially favorable; moreover at $p_2^{ss*}$, the equilibrium contract must satisfy conditions analogous to those of Lemma 4.[24] Thus, the equilibrium contracts with endogenous health-contingent CSVs and the settlement market (i.e. regime D) are *qualitatively* similar to the case of settlement market without CSVs (i.e. regime B). However, as we emphasized earlier, even though the primary insurance companies would have to offer cash surrender values that exactly match the secondary market, their payoffs are fundamentally different depending on whether the primary insurers are offering endogenously chosen CSVs. When $S^{ss}(p_2)$ is endogenously chosen, once a consumer with health state $p_2$ loses bequest motive, she will surrender her policy to the primary insurance company in exchange for CSV $S^{ss}(p_2)$, but the insurance company does not receive further premium payment $Q_2^{ss}(p_2)$ and would not have to pay out death benefits $F^{ss}(p_2)$. When there are no CSVs, the primary insurer will continue to receive premium payments from the settlement firm, but will have to pay the death benefit. It is a priori not clear how the endogenous CSV will affect the quantitative features of the contracts. The following proposition, however, shows that in the presence of the settlement market, the option of endogenously choosing health-contingent cash surrender values will lower the period-2 health threshold above which partial reclassification risk insurance is provided by the dynamic contract:

**Proposition 9.** *In the presence of a settlement market, $p_2^{ss*} < p_2^{s*}$ where $p_2^{ss*}$ and $p_2^{s*}$ are respectively the period-2 health state thresholds above which reclassification risk insurance is provided under regimes D and B respectively.*

The intuition for Proposition 9 is quite simple. When primary insurers are allowed to offer health-contingent CSVs, they will choose the CSVs $S^{ss}(p_2)$ to preempt the settlement firms, which

[23] We omit another constraint for no-lapsation: $S^{ss}(p_2) \leq V_2^{ss}(p_2) \equiv p_2 F_2^{ss}(p_2) - Q_2^{ss}(p_2)$ in the formulation of the problem. Lemma 5 below ensures that this constraint never binds and thus the solution to the problem is not affected by the omission.

[24] The formal statement of this result and its proof are omitted for brevity.

prevents the settlement firms from receiving part of the actuarial value $V_2^{ss}(p_2)$. This in turns allows the primary insurers to offer reclassification risk insurance for a wider range of period-2 types.

In terms of equilibrium consumer welfare, we have the following unambiguous ranking across regimes:

**Proposition 10.** *When there is the settlement market, equilibrium consumer welfare is higher (strictly higher if $\beta < 1$) when life insurance companies can offer health-contingent CSVs (regime D) than when they are not allowed to offer CSVs (regime B).*

**Proposition 11.** *Equilibrium consumer welfare is lower when there is a settlement market than when there is no settlement market (regime A, A′ or A″) even if endogenous health-contingent CSVs are allowed (regime D).*

The arguments used in the proofs of both Propositions 10 and 11 are similar to that for Proposition 7: for any contract that is feasible under the "dominated" regime (regime B in Proposition 10 and regime D in Proposition 11), we construct a feasible contract under the "dominant" regime (regime D in Proposition 10 and regime A in Proposition 11) which offers identical coverage as the original contract for the "dominated" regime, except for a lower first-period premium. We show that consumers are weakly better off under the constructed contract.

### 4.3 Non-Health Contingent CSV with the Settlement Market

Finally, we consider regime C where primary life insurers are only allowed to offer non-health contingent CSVs in the presence of life settlement firms. That is, we consider a contract space in the form of $\langle (Q_1, F_1), \{(Q_2(p_2), F_2(p_2), S) : p_2 \in [0,1]\}\rangle$ where $S$ denotes a non-health-contingent CSV. As we mentioned earlier, the restrictions on CSV to be non-health contingent could result from explicit government regulations or from the difficulties in enforcing contracts with health-contingent CSVs. We first state an immediate corollary of Proposition 8:

**Corollary 1** (of Proposition 8)**.** *In the absence of the settlement market, the primary insurance companies will set $S = 0$ when they are restricted to offer only non-heath contingent CSV.*

When the primary insurance companies face the threat from the settlement market, but are restricted to react to the threat by offering a non-health contingent CSV, they will choose a contract in the form of $\langle (Q_1, F_1), \{(Q_2(p_2), F_2(p_2), S) : p_2 \in [0,1]\}\rangle$ to solve:

$$\max u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)(1-q)\int_0^1 [u(y+g-Q_2(p_2)) + p_2 v(F_2(p_2))]\, d\Phi(p_2)$$
$$+ (1-p_1)q\int_{S\geq \beta V_2(p_2)} u(y+g+S) d\Phi(p_2) \tag{33}$$
$$+ (1-p_1)q\int_{S< \beta V_2(p_2)} u(y+g+\beta V_2(p_2)) d\Phi(p_2)$$

$$\text{s.t. } V_2(p_2) \geq S, \text{ for all } p_2, \tag{34}$$

$$S \geq 0, \tag{35}$$

$$Q_1 - p_1 F_1 = (1-p_1)(1-q)\int_0^1 V_2(p_2) d\Phi(p_2) + (1-p_1)q\int_{S\geq \beta V_2(p_2)} S d\Phi(p_2) \tag{36}$$
$$+ (1-p_1)q\int_{S< \beta V_2(p_2)} V_2(p_2) d\Phi(p_2)$$

where as before $V_2(p_2) \equiv p_2 F_2(p_2) - Q_2(p_2)$ denotes the actuarial value of the policy at period-2 heath state $p_2$. To understand the above problem, let us first explain the constraints. Constraint (34) is the analog of the no-lapsation constraint in this setting, which requires that the actuarial value of the contract terms for any period-2 health state be at least equal to the CSV. As before, this requirement reflects the consumer's inability to commit: if the actuarial value of the contract was less than the CSV, the consumer would simply surrender the contract and repurchase better insurance on the spot market. Constraint (35) requires that the CSV $S$ be non-negative to reflect the consumer's inability to commit to a negative payout in any state. Constraint (36) is the zero-profit condition reflecting competitiveness of the primary market. The first integral in the right hand side of (36) is the expected loss the insurance company suffers from consumers who retain their bequest motive. Constraint (34) implies that for these consumers, the insurance company's expected period-2 loss is always equal to $V_2(p_2)$. The second integral in the right hand of (36) is the expected loss the insurance company suffers from consumers who lose their bequest motive and find it optimal to surrender the policy back to the original insurer (for any of such consumers the insurance company's period-2 loss is $S$). The third integral is the expected period-2 loss the insurer suffers from consumers who lose their bequest motive but find it optimal to sell the policy on the secondary market (for a consumer in this category with health state $p_2$, the expected period-2 loss is $V_2(p_2)$ for the insurance company).

Now let us explain the objective function. The first integral in (33) is the expected second period utility to consumers with a bequest motive, for whom constraint (34) ensures that they remain with the original contract terms; the second integral is the expected second period utility for consumers who lose their bequest motive, and find it optimal to surrender their contract back

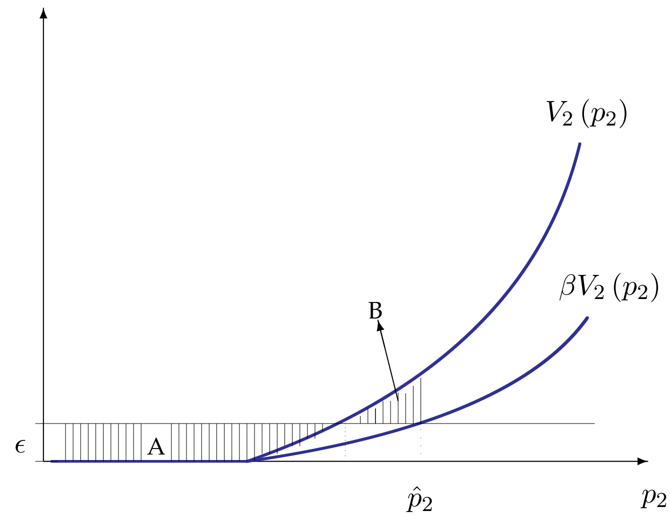


Figure 3: The Effect of Increasing $S$ by $\epsilon > 0$ on Primary Insurer's Period-2 Profit.

to the insurance company for CSV $S$; the third integral is the expected second period utility for consumers without bequest motive who find it optimal to sell their contract on the settlement market for payment of $\beta V_2(p_2)$.

Note that problem (33) is substantially more complicated than problem (29) because now the policyholders who lose bequest motives need to choose whether to sell the policy to the primary insurer or the settlement firms. However, using a rather intuitive perturbation argument we can prove the following somewhat surprising result:

**Proposition 12.** *In the presence of a settlement market, if the primary insurers are restricted to offer only non-health contingent CSVs, they will choose $S^* = 0$ in equilibrium.*

To understand the intuition for Proposition 12, it is useful to consider the effect of raising $S$ from $0$ to $\varepsilon$ on the firm's second period profits. In Figure 3, the curve labelled $V_2(p_2)$ depicts the period-2 actuarial value of the primary insurer's long-term policy at health state $p_2$, and the curve labelled $\beta V_2(p_2)$ is the settlement firm's payment for such policies. If the primary insurer raises the non-health contingent CSV $S$ from $0$ to $\varepsilon$, policyholders with period 2 health in region A who no longer have a bequest motive will surrender their policies to the primary insurer for a payment of $\varepsilon > \beta V_2(p_2)$. The area labeled A captures the loss in profit from such a change in the sense that the firm will be paying these consumers $\varepsilon$ under such a change, whereas they would have would have cost the primary insurer $V_2(p_2)$, which is less than $\varepsilon$, before the change. Policyholders who no longer have bequest motives but with period-2 health state in the region B will also decide to surrender their policy to the primary insurer instead of selling them to the life settlement firms because $\beta V_2(p_2) < \varepsilon$. For these policyholders the primary insurers were losing

$V_2(p_2)$ before the change, and now are losing only $\varepsilon$. Since $V_2(p_2) > \varepsilon$ for these policyholders, area B then represents the gain for the primary insurer's profit from increasing $S$ from $0$ to $\varepsilon$.[25] As is clear from the graph, area A is first-order proportional to $\varepsilon$, while Area B is second-order proportional to $\varepsilon$. When $\varepsilon$ is small, the firm's second period losses increase as a result of increasing $S$ from $0$ to $\varepsilon$. As a result, the insurance company has to increase the first period premium $Q_1$ to maintain zero profit. It is easy to see that the utility cost of increasing the first period premium is exactly $\mu \equiv u'(y - g - Q_1^*)$. It turns out that the utility gain for the consumer when $S$ increases from 0 to $\varepsilon$ is captured by $(1 - p_1)qu'(y + g)\Phi(\hat{p}_2)$ where $\hat{p}_2$ is defined by $V_2(\hat{p}_2) = \varepsilon$. The marginal utility gain in the second period is thus smaller than the marginal loss from the increase in the first period premium, and so this tradeoff is welfare reducing. Similar perturbation argument can be used to show that marginally decreasing $S$ from any positive level is always welfare improving. Thus the optimal $S^* = 0$.

Proposition 12 tells us that when primary insurance companies are restricted to respond to the threat of the settlement market by optimally choosing non-health contingent CSVs, such an option is essentially useless. Thus the consumer welfare in regime C is exactly the same as in regime B.

**Corollary 2** (to Proposition 12)**.** *Equilibrium contracts and consumer welfare are identical under regimes B and C.*

### 4.4 Summary

Table 2 summarizes our results for consumer welfare ranking under the various market regimes categorized in Table 1. Proposition 8 and its Corollary 2 imply that when there is no settlement market, the equilibrium consumer welfare does not depend on whether endogenous cash surrender values are available to the primary insurer. Thus consumer welfare is the same under regimes A, A′ and A″. Proposition 11 shows that consumer welfare is lower under regime D (with settlement market and endogenous health-contingent CSVs) than regimes A (and A′, A″). Proposition 12 and its Corollary 2 show that consumer welfare is the same under regimes B and C. Finally, Proposition 10 shows that consumer welfare is higher under regime D than regime B (and thus C). Table 2 thus summarizes our overall finding that the presence of settlement market unambiguously lowers the equilibrium consumer welfare in our environment, irrespective of what the primary insurers are allowed to do in response to the settlement firms. In particular, restricting the primary insurers' CSVs to be non-health contingent is undesirable for consumers.

[25] Areas A and B in Figure 3 respectively correspond to Terms A and B in expression (A16) in the proof (see the Appendix).

| | | Cash Surrender Value | | | | |
|---|---|---|---|---|---|---|
| | | None | | Non-Health Contingent | | Health Contingent |
| Settlement Market | No | A | $=$ | A′ | $=$ | A″ |
| | | | | | | $\veebar$ |
| | Yes | B | $=$ | C | $\leq$ | D |

Table 2: Comparison of Consumer Welfare Across Market Regimes: A Summary.

Note: The two weak inequalities are strict when $\beta$ is strictly less than $1$.

## 5 Conclusion and Future Research

In this paper, we have examined in detail the effect of the life settlement market on the structure of the long term contracts offered by the primary insurance market, as well as the effect of the life settlement market on consumer welfare, using a dynamic model of life insurance with one sided commitment and bequest-driven lapsation ( à la Hendel and Lizzeri (2003) and Daily, Hendel and Lizzeri (2008)). We show that the presence of the life settlement market affects the extent as well as the form of dynamic reclassification risk insurance in the equilibrium of the primary insurance market. In the absence of a life settlement market, reclassification risk insurance is provided through actuarially favorable level premiums for individuals with second period health state worse than a threshold $p_2^*$. In contrast, when there is a secondary market, reclassification risk is provided through premium discounts (relative to the actuarially fair premium) for individuals whose health is worse than a threshold $p_2^{s*}$. Moreover, $p_2^{s*}$ may be different from $p_2^*$, so reclassification risk insurance may be provided for a smaller set of health realizations when there is a secondary market. We show that in general, the settlement market always leads to worse consumer welfare than when there is no secondary market (Proposition 7). In the most extreme form, the presence of the settlement market can completely unravel the dynamic contracts to a sequence of short-term spot contracts with no dynamic risk classification risk insurance at all (Proposition 6).

We also examine the primary insurers' response to the settlement market when they can offer enriched contracts by specifying optimally chosen cash surrender values. We show that when there are no settlement firms, the primary insurers will not exercise the option of specifying CSVs; but when there is a threat from settlement firms, primary insurers will choose CSVs to preempt the settlement market. Allowing for optimally chosen CSVs improves consumer welfare, but consumers are still worse off than if there was no secondary market. We also showed that the option of primary insurers to endogenously choose the CSV is useless if the CSVs are restricted to be non-health contingent as required by the current regulation. However, if CSVs can be health-contingent, then the primary insurance companies can partially mitigate the welfare losses induced by the emergence of the settlement market (see Table 2).

**Directions for Future Research.** There are several important venues for further research. First, this paper, as well as Daily, Hendel and Lizzeri (2008), studies the effects of life settlement markets when life insurance policy lapsation is driven only by the loss of bequest motives. Selling of life insurance policies could, however, be a result of large income losses (or equivalently expense increase), as is the case for the viatical market for AIDS patients, as well as the story reported in the *Wall Street Journal*. In a companion paper, Fang and Kung (2010a), we consider a model of life insurance market that explicitly features both income and mortality risks and examine the effects of life settlement market on consumer welfare when policyholders' lapsation could be driven by income shocks. The life settlement market allows life insurance policies to be used as an instrument for consumption smoothing when the policyholder experiences a large negative income shock. Because payments received from life settlement firms (or from cash surrender values of the primary insurer) in such low income states have a large marginal value, the life settlement market can indeed make consumers better off. We also find that, when lapsations are driven by income shocks, the welfare effects of the settlement market depend on what other consumption smoothing instruments are available to the consumers, and whether they are allowed to hold multiple policies.

The theoretical analysis thus establishes that the welfare effects of life settlement market depends on why policyholders lapse. If policyholders lapse only because of their loss of bequest motives, then we have shown in this paper that the settlement market is bad for consumers. However, if lapsations are driven by income shocks, then our companion paper Fang and Kung (2010a) shows that settlement market may improve consumer welfare. Therefore, it is crucially important to empirically understand why policyholders lapse their policies. Surprisingly, to the best of our knowledge, there has been no formal empirical analysis of this issue in the literature. In Fang and Kung (2010b), we use data from the HRS to estimate a dynamic structural model of life insurance purchase, renewal, and lapsation. We then use these estimates to disentangle the contributions of health shocks, income shocks and bequest motive shocks to the observed lapsation of life insurance policies.

It is also interesting to empirically test the models' predictions of how the primary insurance market responds to the threat from the settlement market. For example, Proposition 3 showed that level term life insurance policies are no longer optimal. Propositions 10 and 12 showed that primary insurers should have incentives to offer health-contingent CSVs in response to the settlement market, but a non-health contingent CSV is of no use. Do we see these developments in the primary market? It is also interesting to examine the model's prediction of who will sell life insurance to the settlement firms. For example, in the current model, only those with no bequest motives but with bad health (whose original life insurance policy has strictly positive actuarial value) will sell to settlement firms. Does the evidence support this prediction?

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# Appendix.

**Proof of Lemma 1:**

*Proof.* If $p_2 \in \mathcal{B}$ and $p_2' \in \mathcal{NB}$, the complementary slackness conditions (5) implies that $\lambda(p_2) \leq 0$ and $\lambda(p_2') = 0$. First order conditions (4c) for $Q_2(p_2)$ and $Q_2(p_2')$ and $p'$ then imply:

$$u'(y + g - Q_2(p_2)) = \mu + \frac{\lambda(p_2)}{(1 - p_1)q\phi(p_2)} \leq u'\left(y + g - Q_2(p_2')\right) = \mu.$$

Since $u'$ is decreasing, it must be that $Q_2(p_2) \leq Q_2(p_2')$. The full event insurance conditions (6a) and (6b) then imply that $F_2(p_2) \geq F_2(p_2')$.

To prove that $p_2 < p_2'$, suppose to the contrary. Since $p_2' \in \mathcal{NB}$ implies that $Q_2(p_2') < p_2' F_2(p_2')$, we have

$$Q_2(p_2) \leq Q_2(p_2') < p_2' F_2(p_2') \leq p_2 F_2(p_2)$$

where the last inequality follows from postulated $p_2 \geq p_2'$, and the fact that $F_2(p_2) \geq F_2(p_2')$ established above. Thus, $Q_2(p_2) < p_2 F_2(p_2)$, a contradiction to $p_2 \in \mathcal{B}$. □

**Proof of Lemma 2:**

*Proof.* Suppose $Q_2(p_2^*) < Q_2^{FI}(p_2^*)$. Since $\langle Q_2(p_2^*), F_2(p_2^*)\rangle$ must satisfy the full-event insurance condition (6b), we have $F_2(p_2^*) > F_2^{FI}(p_2^*)$. Thus, $Q_2(p_2^*) - p_2^* F_2(p_2^*) < 0$, hence $p_2^* \in \mathcal{NB}$. Therefore, $\lambda(p_2^*) = 0$ and thus the first order conditions (4) imply that

$$u'(y + g - Q_2(p_2^*)) = u'(y - g - Q_1). \tag{A1}$$

But for all $p_2 < p_2^*$, we already established that $Q_2(p_2) = Q_2^{FI}(p_2)$ and $\lambda(p_2) \le 0$, thus

$$\lim_{p_2 \to p_{2-}^*} u'(y + g - Q_2(p_2)) = u'(y + g - Q_2^{FI}(p_2^*)) \le u'(y - g - Q_1). \tag{A2}$$

(A1) and (A2) imply that $u'(y + g - Q_2^{FI}(p_2^*)) \le u'(y + g - Q_2(p_2^*))$, hence $Q_2(p_2^*) \ge Q_2^{FI}(p_2^*)$, a contradiction.

To prove (12), suppose instead $u'(y + g - Q_2^{FI}(p_2^*)) < u'(y - g - Q_1)$. Because $Q_2^{FI}(\cdot)$ is continuous, there must exist $\hat{p}_2 > p_2^*$ such that $u'(y + g - Q_2^{FI}(\hat{p}_2)) < u'(y - g - Q_1)$. Since by Lemma 1, $\hat{p}_2 \in \mathcal{NB}$, it must be that $Q_2(\hat{p}_2) < Q_2^{FI}(\hat{p}_2)$. Thus $u'(y + g - Q_2(\hat{p}_2)) < u'(y + g - Q_2^{FI}(\hat{p}_2)) < u'(y - g - Q_1)$, a contradiction to $\hat{p}_2 \in \mathcal{NB}$. □

**Proof of Proposition 2:**

*Proof.* Let $\hat{q} > q$ and suppose $\hat{Q}_1 \ge Q_1$. Then the concavity of $u$ implies that $u'(y - g - \hat{Q}_1) \ge u'(y - g - Q_1)$. Lemma 2 then implies that:[26]

$$u'(y + g - Q_2^{FI}(\hat{p}_2^*)) \ge u'(y + g - Q_2^{FI}(p_2^*)).$$

Since $Q_2^{FI}(\cdot)$ is increasing in its argument, we thus have $\hat{p}_2^* \ge p^*$. This in turn implies that $\hat{Q}_2(p_2) \ge Q_2(p_2)$ and $\hat{F}_2(p_2) \le F_2(p_2)$ for all $p_2$. Hence $0 \ge \hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2) \ge Q_2(p_2) - p_2 F_2(p_2)$ for all $p_2 \in [0, 1]$. Hence, if $p_2^* < 1$, we must have:

$$(1 - \hat{q}) \int \left[\hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2)\right] d\Phi(p_2) > (1 - q) \int [p_2 F_2(p_2) - Q_2(p_2)] \, d\Phi(p_2).$$

The above inequality, together with the postulated $\hat{Q}_1 \ge Q_1$, contradicts the zero profit condition for both $q$ and $\hat{q}$. □

[26]Note that $Q_2^{FI}(\cdot)$ as defined by (7) and (8) does not depend on $q$.

**Proof of Lemma 3:**

*Proof.* If $p_2 \in \mathcal{B}^s$ and $p_2' \in \mathcal{NB}^s$, then $\lambda(p_2) \leq 0$ and $\lambda(p_2') = 0$, and $V_2^s(p_2) = 0$ and $V_2^s(p_2') > 0$. The first order conditions (18c) corresponding to heath states $p_2$ and $p_2'$ imply that

$$(1-q)\,u'(y+g-Q_2^s(p_2)) + \beta q u'(y+g) \leq (1-q)\,u'(y+g-Q_2^s(p_2')) + \beta q u'(y+g+\beta V_2^s(p_2')).$$

Since $u'(y+g) > u'(y+g+\beta V_2^s(p_2'))$, the above inequality can hold only if $Q_2^s(p_2) < Q_2^s(p_2')$.

To prove that $p_2 < p_2'$, suppose to the contrary that $p_2 \geq p_2'$. Then note that $p_2' \in \mathcal{NB}^s$ implies that $Q_2^s(p_2') < p_2' F_2^s(p_2')$; hence

$$Q_2^s(p_2) < Q_2^s(p_2') < p_2' F_2^s(p_2') \leq p_2 F_2^s(p_2),$$

where the last inequality because $F_2^s(p_2) \geq F_2^s(p_2')$ which follows from $Q_2^s(p_2) < Q_2^s(p_2')$ and the fact that full-event insurance requires that face amount decreases with premium. Hence $Q_2^s(p_2) < p_2 F_2^s(p_2)$, contradicting the assumption that $p_2 \in \mathcal{B}^s$. □

**Proof of Lemma 4:**

*Proof.* The structure of the proof is similar to that for Proposition 2. Suppose that $Q_2^s(p_2^{s*}) < Q_2^{FI}(p_2^{s*})$. Since $\langle Q_2^s(p_2^{s*}), F_2^s(p_2^{s*})\rangle$ must satisfy the full-event insurance condition (21), we have $F_2^s(p_2^{s*}) > F_2^{FI}(p_2^{s*})$. Thus, $Q_2^s(p_2^{s*}) - p_2^{s*} F_2^s(p_2^{s*}) < 0$, hence $p_2^{s*} \in \mathcal{NB}^s$. Therefore $\lambda(p_2^{s*}) = 0$. Thus the first order conditions imply:

$$(1-q)\,u'(y+g-Q_2^s(p_2^{s*})) + \beta q u'(y+g+\beta V_2^s(p_2^{s*})) = u'(y-g-Q_1^s). \tag{A3}$$

Since $Q_2^s(p_2) = Q_2^{FI}(p_2)$ for all $p_2 < p_2^{s*}$, we have

$$\lim_{p_2 \to p_{2-}^{s*}} (1-q)\,u'(y+g-Q_2^s(p_2)) + \beta q u'(y+g) = (1-q)\,u'(y+g-Q_2^{FI}(p_2^{s*})) + \beta q u'(y+g) \leq u'(y-g-Q_1^s). \tag{A4}$$

(A3) and (A4) together imply:

$$(1-q)\,u'(y+g-Q_2^{FI}(p_2^{s*})) + \beta q u'(y+g) \leq (1-q)\,u'(y+g-Q_2^s(p_2^{s*})) + \beta q u'(y+g+\beta V_2^s(p_2^{s*})),$$

but this is impossible because we postulated that $Q_2^{FI}(p_2^{s*}) > Q_2^s(p_2^{s*})$ and hence $V_2(p_2^{s*}) > 0$.

To prove (23), we suppose instead that $(1-q)\,u'(y+g-Q_2^{FI}(p_2^{s*})) + \beta q u'(y+g) < u'(y-g-Q_1^s)$.

Then there must exist $p_2 > p_2^{s*}$ but sufficiently close to $p_2^{s*}$ such that:

$$(1-q)\, u'(y+g-Q_2^s(p_2)) + \beta q u'(y+g+\beta V_2^s(p_2)) < u'(y-g-Q_1^s),$$

contradicting that $p_2 \in \mathcal{NB}$ for all $p_2 > p_2^{s*}$. □

**Proof of Proposition 3:**

*Proof.* The first assertion directly follows from the fact that $p_2 \in \mathcal{NB}^s$ if $p_2 > p_2^{s*}$. To show that $Q_2^s(p_2)$ increases in $p_2$ for $p_2 > p_2^{s*}$, note that from the first order conditions (18), $F_2^s(p_2)$ and $Q_2^s(p_2)$ must satisfy, for all $p_2 > p_2^*$ the following system:

$$\begin{aligned}
&(1-q)\, u'(y+g-Q_2^s(p_2)) + \beta q u'(y+g+\beta V_2^s(p_2)) = u'(y-g-Q_1^s),\\
&v'(F_2^s(p_2)) = u'(y+g-Q_2^s(p_2)),\\
&V_2^s(p_2) = p_2 F_2^s(p_2) - Q_2^s(p_2).
\end{aligned}$$

Taking derivatives with respect to $p_2$ for each equation, we obtain:

$$\begin{aligned}
&(1-q)\, u''(y+g-Q_2^s(p_2))\frac{dQ_2^s}{dp_2} = \beta^2 q u''(y+g+\beta V_2^s(p_2))\frac{dV_2^s}{dp_2}\\
&v''(F_2^s(p_2))\frac{dF_2^s}{dp_2} = -u''(y+g-Q_2^s(p_2))\frac{dQ_2^s}{dp_2}\\
&\frac{dV_2^s}{dp_2} = F_2^s(p_2) + p_2\frac{dF_2^s}{dp_2} - \frac{dQ_2^s}{dp_2}
\end{aligned}$$

Solving for $dQ_2^s/dp_2$, we obtain:

$$\frac{dQ_2^s}{dp_2} = \frac{F_2^s(p_2)}{\frac{(1-q)u''(y+g-Q_2^s(p_2))}{\beta^2 q u''(y+g+\beta V_2^s(p_2))} + \left[1 + p_2\frac{u''(y+g-Q_2^s(p_2))}{v''(F_2^s(p_2))}\right]}, \tag{A5}$$

which is strictly positive if $q > 0$. □

**Proof of Proposition 4:**

*Proof.* Suppose to the contrary that $Q_1^s > \hat{Q}_1^s$. Then, $u'(y-g-Q_1^s) > u'(y-g-\hat{Q}_1^s)$. Using (23) in Lemma 4, we have:

$$(1-q)\, u'\left(y+g-Q_2^{FI}(p_2^{s*})\right) + \beta q u'(y+g) > (1-\hat{q})\, u'(y+g-Q_2^{FI}(\hat{p}_2^{s*})) + \beta\hat{q}u'(y+g).$$

Rearranging the above inequality yields:

$$\beta(q-\hat{q})u'(y+g) > (1-\hat{q})\,u'\left(y+g-Q_2^{FI}(\hat{p}_2^{s*})\right) - (1-q)\,u'\left(y+g-Q_2^{FI}(p_2^{s*})\right). \tag{A6}$$

Suppose, for the first case, that $\hat{p}_2^{s*} \geq p_2^{s*}$. Then $Q_2^{FI}\left(\hat{p}_2^{s*}\right) \geq Q_2^{FI}\left(p_2^{s*}\right)$, thus $u'\left(y+g-Q_2^{FI}(\hat{p}_2^{s*})\right) \geq u'\left(y+g-Q_2^{FI}(p_2^{s*})\right)$. Thus, (A6) implies that:

$$\beta\,(q-\hat{q})\,u'(y+g) > (q-\hat{q})\,u'(y+g-Q_2^{FI}(p_2^{s*})),$$

which is impossible when $q > \hat{q}$.

Now, suppose, for the second case, that $\hat{p}_2^{s*} < p_2^{s*}$. Then there must exist $\tilde{p}_2 > p_2^{s*}$ such that $V_2^s(\tilde{p}_2) = \hat{V}_2^s(\tilde{p}_2)$. Such $\tilde{p}_2$ must exist for the following reasons. If $\hat{p}_2^{s*} < p_2^{s*}$, we know that $\hat{V}_2^s(p_2) > V_2^s(p_2)$ for all $\hat{p}_2^{s*} < p_2 < p_2^{s*}$. The zero-profit conditions together with the postulated $Q_1^s > \hat{Q}_1^s$ then imply that $Q_1^s - p_1 F_1^s = \int V_2^s(p_2)d\Phi(p_2) > \hat{Q}_1^s - p_1\hat{F}_1^s = \int \hat{V}_2^s(p_2)d\Phi(p_2)$, hence $V_2^s(\cdot)$ must cross $\hat{V}_2^s(\cdot)$ at some point $\tilde{p}_2 > p_2^{s*}$. We now argue that $V_2^s(\tilde{p}_2) = \hat{V}_2^s(\tilde{p}_2)$ must imply that $Q_2^s(\tilde{p}_2) = \hat{Q}_2^s(\tilde{p}_2)$. To see this, note that both $\langle Q_2^s(\tilde{p}_2), F_2^s(\tilde{p}_2)\rangle$ and $\left\langle \hat{Q}_2^s(\tilde{p}_2), \hat{F}_2^s(\tilde{p}_2)\right\rangle$ must both provide full-event insurance as defined by (21). That is,

$$\begin{aligned} u'\left(y+g-Q_2^s(\tilde{p}_2)\right) &= v'\left(F_2^s(\tilde{p}_2)\right) \\ u'\left(y+g-\hat{Q}_2^s(\tilde{p}_2)\right) &= v'\left(\hat{F}_2^s(\tilde{p}_2)\right). \end{aligned}$$

If, moreover, $V_2^s(\tilde{p}_2) = \tilde{p}_2 F_2^s(\tilde{p}_2) - Q_2^s(\tilde{p}_2) = \tilde{p}_2\hat{F}_2^s(\tilde{p}_2) - \hat{Q}_2^s(\tilde{p}_2) = \hat{V}_2^s(\tilde{p}_2)$, then it must be the case that $Q_2^s(\tilde{p}_2) = \hat{Q}_2^s(\tilde{p}_2)$ and $F_2^s(\tilde{p}_2) = \hat{F}_2^s(\tilde{p}_2)$. Thus we have established that there exists $\tilde{p}_2 > p_2^{s*} > \hat{p}_2^{s*}$ such that $V_2^s(\tilde{p}_2) = \hat{V}_2^s(\tilde{p}_2)$ and $Q_2^s(\tilde{p}_2) = \hat{Q}_2^s(\tilde{p}_2)$. Now, from the first order conditions (18), we have that at $p_2 = \tilde{p}_2$,

$$\begin{aligned} & (1-q)\,u'(y+g-Q_2^s(\tilde{p}_2)) + \beta q u'(y+g+\beta V_2^s(\tilde{p}_2)) = u'\left(y-g-Q_1^s\right) \\ > \; & u'\left(y-g-\hat{Q}_1^s\right) \\ = \; & (1-\hat{q})\,u'(y+g-\hat{Q}_2^s(\tilde{p}_2)) + \beta\hat{q}u'(y+g+\beta\hat{V}_2^s(\tilde{p}_2)) \\ = \; & (1-\hat{q})\,u'(y+g-Q_2^s(\tilde{p}_2)) + \beta\hat{q}u'(y+g+\beta V_2^s(\tilde{p}_2)), \end{aligned}$$

which could not hold if $q > \hat{q}$. □

**Proof of Proposition 5:**

*Proof.* See discussion in the main text. □

**Proof of Proposition 6:**

*Proof.* If $\mathcal{NB}^s$ is not empty, then for any $p_2 \in \mathcal{NB}^s$, from first order conditions (18) the contract terms must satisfy:

$$(1-q)\, u'(y+g-Q_2^s(p_2)) + \beta q u'(y+g+\beta V_2^s(p_2)) = u'(y-g-Q_1^s), \tag{A7}$$

which can be rewritten as:

$$(1-q)\left[u'(y+g-Q_2^s(p_2)) - \beta u'(y+g+\beta V_2^s(p_2))\right] = u'(y-g-Q_1^s) - \beta u'(y+g+\beta V_2^s(p_2)). \tag{A8}$$

First note that the zero-profit condition (16) implies that if $\mathcal{NB}^s$ is not empty, then it must be the case that $Q_1^s \geq Q_1^{FI} > 0$, where $Q_1^{FI}$ denotes the actuarially fair premium for period-1 full event insurance. Specifically, $\left\langle Q_1^{FI}, F_1^{FI}\right\rangle$ are implicitly defined by the unique solution to the following system of equations:

$$\begin{aligned} u'(y-g-Q_1^{FI}) &= v'\left(F_1^{FI}\right) \\ Q_1^{FI} &= p_1 F_1^{FI}. \end{aligned}$$

Notice that $Q_1^{FI}$ does *not* depend on $q$, but it is decreasing in $g$. Let $\bar{g}$ be the upper-bound of the values that $g$ can take, and let $\underline{Q}_1^{FI} > 0$ denotes the actuarially fair full-insurance premium at $g = \bar{g}$. Thus $Q_1^{FI} \geq \underline{Q}_1^{FI}$ for all $g$. Therefore the right hand side (RHS) of (A8) is bounded below, *for any* $g > 0$, by:

$$RHS > u'(y-\underline{Q}_1^{FI}) - \beta u'(y).$$

Now examine the left hand side (LHS) of (A8). We will consider two cases. For the first case, suppose that $\lim_{x\to 0} u'(x) \equiv u'(0) < \infty$. Because $Q_2^s(p_2)$ is always smaller than $y+g$ in equilibrium, we have that

$$LHS = u'(y+g-Q_2^s(p)) - \beta u'(y+g+\beta V_2^s(p)) < u'(0).$$

Thus if

$$q < \hat{q} = \frac{u'(y-\underline{Q}_1^{FI}) - \beta u'(y)}{u'(0)},$$

then the LHS of (A8) will always be smaller than its RHS; i.e., equation (A7) can never be satisfied for any $p_2$. Thus, $\mathcal{NB}^s$ must be empty.

For the second case, suppose that $\lim_{x\to 0} u'(x) = \infty$. Since $p_2 \in \mathcal{NB}^s$, we have $p_2 F_2^s(p_2) -$

$Q_2^s(p_2) > 0$. Plugging (21) into the above inequality, we obtain:

$$p_2 v'^{-1}(u'(y+g-Q_2^s(p_2))) > Q_2^s(p_2), \tag{A9}$$

or equivalently,

$$u'(y+g-Q_2^s(p_2)) < v'\left(\frac{Q_2^s(p_2)}{p_2}\right). \tag{A10}$$

Notice that the LHS of (A10) is increasing as $Q_2^s(p_2)$ varies from 0 to $y+g$, and that its RHS is decreasing in $Q_2^s(p_2)$ over the same interval. If $u'(y+g) \geq v'(0)$ then (A10) cannot be satisfied for any value of $Q_2^s(p)$ and hence$\mathcal{NB}^s$ must be empty. Thus we can without loss of generality consider the case that $u'(y+g) < v'(0)$. Since we are now considering the case in which $u'(0) = \infty$, we know that at $Q_2^s(p) = y+g$, LHS of (A10) is $u'(0) > v'((y+g)/p_2)$ for all $p_2$. Because LHS of (A10) is continuous and monotonically increasing in $Q_2^s(p_2)$, while the RHS of (A10) is continuous and monotonically decreasing in $Q_2^s(p_2)$, there must exist, for each $p_2 \in \mathcal{NB}^s$ some $x(p_2;g) < y+g$ such that $u'(y+g-x(p_2;g)) = v'(x(p_2;g)/p)$, and hence $Q_2^s(p)$ must be bounded above by $x(p_2;g)$. Moreover, note that, for all $g$, it can be easily shown that $x(p_2;g)$ is increasing in $p_2$. Thus we can write $\sup_{p_2 \in \mathcal{NB}^s} x(p_2;g) = x(1;g) \equiv x(g) < y+g$, for all $g$. Now denote $\bar{u}' \equiv \max_g u'(y+g-x(g)) < \infty$. We hence have

$$\begin{aligned} u'(y+g-Q_2^s(p_2)) - \beta u'(y+g+\beta V_2^s(p_2)) &< u'(y+g-Q_2^s(p_2)) < u'(y+g-x(p_2;g)) \\ &\leq u'(y+g-x(g)) \leq \bar{u}', \end{aligned}$$

where the second inequality follows from $Q_2^s(p_2) < x(p_2;g)$; the third inequality follows from $x(p_2,g) \leq x(g)$, and the last inequality follows from $\bar{u}' \equiv \max_g u'(y+g-x(g))$. Thus, if

$$q < \hat{q} \equiv \frac{u'(y-\underline{Q}_1^{FI}) - \beta u'(y)}{\bar{u}'},$$

then the LHS of (A8) will always be smaller than its RHS; i.e., equation (A7) can never be satisfied for any $p_2$. Thus, $\mathcal{NB}^s$ must be empty. □

**Proof of Proposition 7:**

*Proof.* We will show that for feasible contract for problem (15), we can construct a feasible contract for problem (1) that which makes the consumers weakly better off.

Let $C^s = \langle (Q_1^s, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\}\rangle$ be a feasible contract for problem (15) when there is a settlement market. Thus, $Q_1^s - p_1 F_1^s = (1-p_1)\int V_2^s(p_2)d\Phi(p_2)$, where $V_2^s(p_2) \equiv p_2 F_2^s(p_2) - Q_2^s(p_2)$.

Now consider a contract $\hat{C} \equiv \left\langle (\hat{Q}_1, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\} \right\rangle$ where $\hat{Q}_1$ is given by:

$$\hat{Q}_1 - p_1 F_1^s = (1-p_1)(1-q) \int V_2^s(p_2) d\Phi(p_2).$$

Since $q \in (0,1)$, we know that $\hat{Q}_1 < Q_1^s$. That is, $\hat{C}$ is exactly the same contract as $C^s$ except that the first period premium is decreased from $Q_1^s$ until the zero profit condition for the no-settlement-market case (2) holds. It is easy to show that $\hat{C}$ is a feasible contract for problem (1).

We will now show that $\hat{C}$ in a world without settlement market is better than $C^s$ in a world with settlement market. To see this, let

$$\begin{aligned} W^s(C^s) = {} & p_1 v(F_1^s) + u(y - g - Q_1^s) + \\ & (1-p_1) \int \left\{ \begin{array}{c} (1-q)\left[p_2 v(F_2^s(p_2)) + u(y + g - Q_2^s(p_2))\right] \\ + q u(y + g + \beta V_2^s(p_2)) \end{array} \right\} d\Phi(p_2) \end{aligned}$$

denote the expected consumer welfare associated with contract $C^s$ in a world with the settlement market. Let

$$\begin{aligned} W\left(\hat{C}\right) = {} & p_1 v(F_1^s) + u(y - g - \hat{Q}_1) + \\ & (1-p_1) \int \left\{ \begin{array}{c} (1-q)\left[p_2 v(F_2^s(p_2)) + u(y + g - Q_2^s(p_2))\right] \\ + q u(y + g) \end{array} \right\} d\Phi(p_2) \end{aligned}$$

denote the expected consumer welfare associated with contract $\hat{C}$ in a world without the settlement market. Note that

$$\begin{aligned} W\left(\hat{C}\right) - W^s(C^s) &= u(y - g - \hat{Q}_1) - u(y - g - Q_1^s) \\ &\quad - (1-p_1) q \int \left[u(y + g + \beta V_2^s(p_2)) - u(y+g)\right] d\Phi(p_2) \\ &\geq u(y - g - \hat{Q}_1) - u(y - g - Q_1^s) \\ &\quad - (1-p_1) q \left[u\left(y + g + \beta \int V_2^s(p_2) d\Phi(p_2)\right) - u(y+g)\right] \end{aligned}$$

where the inequality follows from Jensen's inequality due to the concavity of $u(\cdot)$. Further note that:

$$
\begin{aligned}
& q\left[u\left(y+g+\beta \int V_2^s(p_2)d\Phi\left(p_2\right)\right)-u\left(y+g\right)\right] \\
& = qu\left(y+g+\beta \int V_2^s(p_2)d\Phi\left(p_2\right)\right)+\left(1-q\right)u(y+g)-u(y+g) \\
& \leq u\left(y+g+\beta q \int V_2^s(p)d\Phi\left(p\right)\right)-u(y+g),
\end{aligned}
$$

where again the inequality follows from Jensen's inequality. Thus,

$$
\begin{aligned}
W\left(\hat{C}\right)-W^s\left(C^s\right) \geq\ & u(y-g-\hat{Q}_1)-u(y-g-Q_1^s) \\
& -\left(1-p_1\right)\left[u\left(y+g+\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right)-u(y+g)\right]
\end{aligned}
$$

First note that $Q_1^s-\hat{Q}_1=(1-p_1)q\int V_2^s(p_2)d\Phi\left(p_2\right)$. By the continuous function theorem, we know that there exists $\delta_1 \in (0,1)$ such that

$$
\begin{aligned}
& u(y-g-\hat{Q}_1)-u(y-g-Q_1^s) \\
= \ & u'\left(y-g-Q_1^s+\delta'\left(Q_1^s-\hat{Q}_1\right)\right)\left(Q_1^s-\hat{Q}_1\right).
\end{aligned}
$$

Similarly, there exists $\delta_2 \in (0,1)$ such that

$$
\begin{aligned}
& \left(1-p_1\right)\left[u\left(y+g+\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right)-u(y+g)\right] \\
= \ & \left(1-p_1\right)\left[u'\left(y+g+\delta_2\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right)\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right] \\
= \ & u'\left(y+g+\delta_2\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right)\beta\left(Q_1^s-\hat{Q}_1\right).
\end{aligned}
$$

Hence

$$
\begin{aligned}
& W\left(\hat{C}\right)-W^s\left(C^s\right) \\
\geq \ & \left[u'\left(y-g-Q_1^s+\delta'\left(Q_1^s-\hat{Q}_1\right)\right)-\beta u'\left(y+g+\delta_2\beta q \int V_2^s(p_2)d\Phi\left(p_2\right)\right)\right]\left(Q_1^s-\hat{Q}_1\right) \geq 0
\end{aligned}
$$

where the last inequality will be strict if $Q_1^s-\hat{Q}_1$ is strictly positive, i.e., if there is dynamic reclassification risk insurance under contract $C^s$.

Now let $C^s$ be the *equilibrium* contract in the presence of the settlement market. The above argument shows that the contract $\hat{C}$ constructed through a simple reduction of first period premium is feasible for the problem without the settlement market; and $\hat{C}$ provides weakly (or strictly, if

$C^s$ offers some dynamic insurance) higher expected utility to the consumers for the case without settlement market than $C^s$ would provide for consumers with settlement market. Because $\hat{C}$ is only a candidate contract for the case without settlement market, the equilibrium contract in that case must provide no lower expected consumer welfare than $\hat{C}$. □

**Proof of Proposition 8:**

*Proof.* See discussion in the main text. □

**Proof of Lemma 5:**

*Proof.* The first order conditions for the solution to problem (29) are:

$$u'(y-g-Q_1^{ss}) = \mu \tag{A11a}$$

$$v'(F_1^{ss}) = \mu \tag{A11b}$$

$$u'(y+g-Q_2^{ss}(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)(1-q)\phi(p_2)} - \frac{\beta\gamma(p_2)}{(1-p_1)(1-q)\phi(p_2)} \tag{A11c}$$

$$v'(F_2^{ss}(p_2)) = \mu + \frac{\lambda(p_2)}{(1-p_1)(1-q)\phi(p_2)} - \frac{\beta\gamma(p_2)}{(1-p_1)(1-q)\phi(p_2)} \tag{A11d}$$

$$u'(y+g+S^{ss}(p_2)) = \mu + \frac{\gamma(p_2)}{(1-p_1)q\phi(p_2)} \tag{A11e}$$

where $\gamma(p_2)$ is the Lagrange multiplier for constraint (32).

From these conditions, we see that constraint (32) must bind for all $p_2$ because otherwise, $\gamma(p_2)$ must be equal to $0$, and then (A11a) and (A11e) together would have implied that $u'(y+g+S^{ss}(p_2)) = u'(y-g-Q_1^{ss})$, which cannot hold. □

**Proof of Proposition 9:**

*Proof.* We consider two cases. For the first case, suppose that $Q_1^s \leq Q_1^{ss}$. Then we have, from the full-event insurance condition, $Q_1^s - p_1F_1^s \leq Q_1^{ss} - p_1F_1^{ss}$. From Lemma 5, we know that in equilibrium $S^{ss}(p_2) = V_2^{ss}(p_2)$. Thus the zero profit conditions imply that:

$$\int V_2^s(p_2)d\Phi(p_2) \leq \int (1-q)\,V_2^{ss}(p_2) + \beta q V_2^{ss}(p_2)d\Phi(p_2) \leq \int V_2^{ss}(p_2)d\Phi(p_2).$$

So there must exist $\tilde{p}_2$ such that $V_2^s(\tilde{p}_2) \leq V_2^{ss}(\tilde{p}_2)$ and $Q_2^s(\tilde{p}_2) \geq Q_2^{ss}(\tilde{p}_2)$. At such a $\tilde{p}_2$, the following must hold:

$$\begin{aligned}&(1-q)\,u'(y+g-Q_2(\tilde{p}_2)) + \beta q u'(y+g+\beta V_2^s(\tilde{p}_2))\\ &\geq (1-q)\,u'(y+g-Q_2^{ss}(\tilde{p}_2)) + \beta q u'(y+g+\beta V_2^{ss}(\tilde{p}_2))\end{aligned}$$

Now from the first order conditions for problem (15) detailed in (18), the left hand side of the above inequality is equal to $u'(y-g-Q_1^s)$; and the right hand side, from the first order conditions for problem (29), is large than $[(1-q)+\beta q]\,u'(y-g-Q_1^{ss})$. That is,

$$u'(y-g-Q_1^s) \geq [(1-q)+\beta q]\,u'(y-g-Q_1^{ss}).$$

Now, Lemma 4 for $p_2^{s*}$ and the analogous lemma for $p_2^{ss*}$ imply that:[27]

$$(1-q)\,u'(y+g-Q_2^{FI}(p_2^{s*})) + \beta q u'(y+g) \geq (1-q)\,u'(y+g-Q_2^{FI}(p_2^{ss*})) + \beta q u'(y+g).$$

Hence, $p_2^{s*} \geq p_2^{ss*}$.

For the second case, suppose $Q_1^s > Q_1^{ss}$. Then:

$$u'(y-g-Q_1) > u'(y-g-Q_1^{ss}) \geq [(1-q)+\beta q]\,u'(y-g-Q_1^{ss}).$$

Then identical argument as the step immediately above implies that $p_2^{s*} > p_2^{ss*}$. □

**Proof Proposition 10:**

*Proof.* Let the equilibrium in regime B be denoted by $C^s = \langle (Q_1^s, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2)) : p_2 \in [0,1]\}\rangle$. Consider a contract $\hat{C}^{ss}$ that is feasible in regime D constructed as follows:

$$\hat{C}^{ss} = \left\langle (Q_1^{ss} = \hat{Q}_1, F_1^s), \{(Q_2^s(p_2), F_2^s(p_2), S^{ss}(p_2) = \beta\,[p_2 F_2^s(p_2) - Q_2^s(p_2)]) : p_2 \in [0,1]\}\right\rangle,$$

where $\hat{Q}_1$ is chosen to satisfy the zero-profit condition (30), i.e.,

$$\hat{Q}_1 = p_1 F_1^s - (1-p_1)\int \{[(1-q)+q\beta]\,[Q_2^s(p_2) - p_2 F_2^s(p_2)]\}\,d\Phi(p_2).$$

[27] The analogous lemma for $p_2^{ss*}$ is omitted from the text for brevity. It states that under regime D, the equilibrium contract at $p_2 = p_2^{ss*}$ satisfies:

$$\begin{aligned}Q_2^{ss}(p_2^{ss*}) &= Q_2^{FI}(p_2^{ss*})\\ (1-q)\,u'(y+g-Q_2^{FI}(p_2^{ss*})) + \beta q u'(y+g) &= [(1-q)+\beta q]\,u'(y-g-Q_1^{ss}).\end{aligned}$$

In contrast, $Q_1^s$ in $C^s$ must satisfy the zero-profit condition (16), which implies that:

$$Q_1^s = p_1 F_1^s + (1 - p_1) \int \{Q_2^s(p_2) - p_2 F_2^s(p_2)\} \, d\Phi(p_2).$$

For any $\beta \in (0,1)$, $\hat{Q}_1 < Q_1^s$. Thus the consumer is strictly better off in regime D with contract $\hat{C}^{ss}$ than in regime B with the optimal contract $C^s$. □

**Proof Proposition 11:**

*Proof.* The proof is similar to that of Proposition 7. Let

$$C^{ss} = \langle (Q_1^{ss}, F_1^{ss}), \{(Q_2^{ss}(p_2), F_2^{ss}(p_2), S^{ss}(p_2)) : p_2 \in [0,1]\} \rangle$$

be the optimal contract with endogenous health-contingent CSVs in the presence of a settlement market. As Lemma 5 shows, $S^{ss}(p_2) = \beta V_2^{ss}(p_2) \equiv p_2 F_2^{ss}(p_2) - Q_2^{ss}(p_2)$ for all $p_2$. Thus, the zero-profit condition implies:

$$Q_1^{ss} = p_1 F_1^{ss} + (1 - p_1) \left[(1 - q) + \beta q\right] \int V_2^{ss}(p_2) d\Phi(p_2).$$

Consider the contract $\hat{C} = \left\langle (\hat{Q}_1, F_1^{ss}), \{(Q_2^{ss}(p_2), F_2^{ss}(p_2)) : p_2 \in [0,1]\} \right\rangle$ where $\hat{Q}_1$ is given by:

$$\hat{Q}_1 = p_1 F_1^{ss} + (1 - p_1)(1 - q) \int V_2^{ss}(p_2) d\Phi(p_2).$$

Since $q \in (0,1)$ and $\beta > 0$, we know that $\hat{Q}_1 < Q_1^{ss}$. That is, $\hat{C}$ offers exactly the same coverage as $C^{ss}$, except at a lower first period premium. It is easy to see that $\hat{C}$ is a feasible contract for regime A (the case without a secondary market), but outside the feasible set for regime D.

We will now show that $\hat{C}$ in a world without secondary market provides consumers with higher welfare than $C^{ss}$ does in a world with secondary market. To see this, let

$$\begin{aligned} W^{ss}(C^{ss}) &= p_1 v(F_1^{ss}) + u(y - g - Q_1^{ss}) \\ &\quad + (1 - p_1) \int \{(1 - q)\left[p_2 v(F_2^{ss}(p_2)) + u(y + g - Q_2^{ss}(p_2))\right] + q u(y + g + S^{ss}(p_2))\} \, d\Phi(p_2) \end{aligned}$$

denote the expected consumer welfare associated with contract $C^{ss}$ in regime D; and let

$$\begin{aligned} W\left(\hat{C}\right) &= p_1 v(F_1^{ss}) + u(y - g - \hat{Q}_1) \\ &\quad + (1 - p_1) \int \{(1 - q)\left[p_2 v(F_2^{ss}(p_2)) + u(y + g - Q_2^{ss}(p_2))\right] + q u(y + g)\} \, d\Phi(p_2) \end{aligned}$$

denote the expected consumer welfare associated with contract $\hat{C}$ in regime A. Note that since $S^{ss}(p_2) = \beta V_2^{ss}(p_2)$ for all $p_2$, all of the equations in the proof of Proposition 7 hold until we reach the inequality:

$$W\left(\hat{C}\right) - W^{ss}\left(C^{s}\right) \geq u(y - g - \hat{Q}_1) - u(y - g - Q_1^{ss})$$
$$- (1 - p_1)\left[u\left(y + g + \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right) - u(y + g)\right].$$

By the continuous function theorem, we know that there exists $\delta_1 \in (0, 1)$ such that

$$\begin{aligned} & u(y - g - \hat{Q}_1) - u(y - g - Q_1^{ss}) \\ = \ & u'\left(y - g - Q_1^{ss} + \delta_1\left(Q_1^{ss} - \hat{Q}_1\right)\right)\left(Q_1^{ss} - \hat{Q}_1\right). \end{aligned}$$

Similarly, there exists $\delta_2 \in (0, 1)$ such that

$$\begin{aligned} & (1 - p_1)\left[u\left(y + g + \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right) - u(y + g)\right] \\ = \ & (1 - p_1)\left[u'\left(y + g + \delta_2 \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right) \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right] \\ = \ & u'\left(y + g + \delta_2 \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right)\left(Q_1^{ss} - \hat{Q}_1\right), \end{aligned}$$

where the last equality follows from $Q_1^{ss} - \hat{Q}_1 = \beta(1 - p_1) q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)$. Hence

$$\begin{aligned} & W\left(\hat{C}\right) - W^{ss}\left(C^{ss}\right) \\ \geq \ & \left[u'\left(y - g - Q_1^{ss} + \delta_1\left(Q_1^{ss} - \hat{Q}_1\right)\right) - u'\left(y + g + \delta_2 \beta q \int V_2^{ss}(p_2) d\Phi\left(p_2\right)\right)\right]\left(Q_1^{ss} - \hat{Q}_1\right) \geq 0 \end{aligned}$$

where the last inequality will be strict if $Q_1^{ss} - \hat{Q}_1$ is strictly positive, i.e., if there is dynamic reclassification risk insurance under contract $C^{ss}$. Since $C^{ss}$ is the optimal contract under regime D and since $\hat{C}$ is only a feasible contract under regime A, we conclude that equilibrium consumer welfare must be no lower under regime A than under regime D. □

**Proof of Proposition 12:**

*Proof.* The Lagrangian for problem (33) is:

$$\begin{aligned}\mathcal{L} &= u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)(1-q)\int_0^1 \left[u(y+g-Q_2(p_2)) + pv(F_2(p_2))\right] d\Phi(p_2) \\ &+ (1-p_1)q\int_{S\geq\beta V_2(p_2)} u(y+g+S)d\Phi(p_2) + (1-p_1)q\int_{S<\beta V_2(p_2)} u(y+g+\beta V_2(p_2))d\Phi(p_2) \\ &+ \int_0^1 \lambda(p)\left[Q_2(p_2) - p_2F_2(p_2) + S\right] d\Phi(p_2) + \gamma S \\ &+ \mu\left[\begin{array}{c}(Q_1 - p_1F_1) + (1-p_1)(1-q)\int_0^1 \left[Q_2(p_2) - p_2F_2(p_2)\right] d\Phi(p_2) \\ -(1-p_1)q\int_{S\geq\beta V_2(p_2)} Sd\Phi(p_2) + (1-p_1)(1-q)\int_{S<\beta V_2(p)}\left[Q_2(p) - p_2F_2(p_2)\right] d\Phi(p_2)\end{array}\right]\end{aligned} \tag{A12}$$

where $\{\lambda(p_2) \leq 0 : p_2 \in [0,1]\}, \gamma \geq 0, \mu \geq 0$ are respectively the Lagrange multiplier for constraints (34), (35), and (36).

Using standard arguments, we can show that under the optimum, $V_2(\cdot)$ must be continuous and monotonically increasing in $p_2$, with $V_2(p_2) > 0$ for some $p_2$ if there is some dynamic reclassification risk insurance in equilibrium. Thus we know that for every $S \geq 0$ with $S$ sufficiently small, there exists a $\hat{p}_2$ such that $\beta V_2(p_2) \geq S$ if and only if $p_2 \geq \hat{p}_2$ where $\beta V_2(\hat{p}_2) = S$. Thus from the Implicit Function Theorem, we have:

$$\frac{d\hat{p}_2}{dS} = \frac{1}{\beta V_2'(\hat{p}_2)}. \tag{A13}$$

Therefore, the Lagrangian (A12) can be rewritten as:

$$\begin{aligned}\mathcal{L} &= u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)(1-q)\int_0^1 \left[u(y+g-Q_2(p_2)) + pv(F_2(p_2))\right] d\Phi(p_2) \\ &+ (1-p_1)q\int_0^{\hat{p}_2} u(y+g+S)d\Phi(p_2) + (1-p_1)q\int_{\hat{p}_2}^1 u(y+g+\beta V_2(p_2))d\Phi(p_2) \\ &+ \int_0^1 \lambda(p)\left[Q_2(p_2) - p_2F_2(p_2) + S\right] d\Phi(p_2) + \gamma S \\ &+ \mu\left[\begin{array}{c}(Q_1 - p_1F_1) + (1-p_1)(1-q)\int_0^1 \left[Q_2(p_2) - p_2F_2(p_2)\right] d\Phi(p_2) \\ -(1-p_1)q\int_0^{\hat{p}_2} Sd\Phi(p_2) + (1-p_1)(1-q)\int_{\hat{p}_2}^1\left[Q_2(p) - p_2F_2(p_2)\right] d\Phi(p_2)\end{array}\right].\end{aligned} \tag{A14}$$

Applying the Leibniz rule and (A13), we have that the derivative of the Lagrangian (A14) with respect to $S$, evaluated at the optimum (superscripted by $*$), is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial S} &= (1-p_1)q\int_0^{\hat{p}_2^*} u'(y+g+S^*)d\Phi(p_2) + (1-p_1)qu(y+g+S^*)\frac{\phi(\hat{p}_2^*)}{\beta V_2^{*\prime}(\hat{p}_2^*)} \\ &- (1-p_1)qu(y+g+\beta V_2^*(\hat{p}_2^*))\frac{\phi(\hat{p}_2^*)}{\beta V_2^{*\prime}(\hat{p}_2^*)} + \int_0^1 \lambda(p_2)d\Phi\left(p_2\right) + \gamma \\ &-\mu(1-p_1)q\int_0^{\hat{p}_2^*} d\Phi(p_2) - \mu(1-p_1)qS^*\frac{\phi(\hat{p}_2^*)}{\beta V_2^{*\prime}(\hat{p}_2^*)} \\ &-\mu(1-p_1)q\left[Q_2^*(\hat{p}_2^*) - \hat{p}_2^* F_2^*(\hat{p}_2^*)\right]\frac{\phi(\hat{p}_2^*)}{\beta V_2^{*\prime}(\hat{p}_2^*)}. \end{aligned} \tag{A15}$$

Since by definition, $\beta V_2^*(\hat{p}_2^*) = S^*$, (A15) simplifies to:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial S} &= (1-p_1)qu'(y+g+S^*)\Phi(\hat{p}_2^*) + \int_0^1 \lambda(p_2)d\Phi\left(p_2\right) + \gamma \\ &- \overbrace{\mu(1-p_1)q\Phi(\hat{p}_2^*)}^{\text{Term A}} + \overbrace{\mu(1-p_1)q(1-\beta)V_2^*(\hat{p}_2^*)\frac{\phi(\hat{p}_2^*)}{\beta V_2^{*\prime}(\hat{p}_2^*)}}^{\text{Term B}}. \end{aligned} \tag{A16}$$

We now argue that $\frac{\partial \mathcal{L}}{\partial S}$ is strictly negative when $S$ deviates from $0$ to a small $\varepsilon > 0$. To see this, note that in the $\varepsilon$-neighborhood of $S = 0$, we have $\gamma = 0$, $\lim_{s\to\varepsilon=0^+} V_2^*\left(\hat{p}_2^*\right) = \varepsilon$, thus

$$\lim_{s\to\varepsilon=0^+} \frac{\partial \mathcal{L}}{\partial S} = (1-p_1)q\left[u'(y+g) - \mu\right]\Phi(\hat{p}_2^*\left(0\right)) + \int_0^1 \lambda(p_2)d\Phi\left(p_2\right),$$

where $\hat{p}_2^*\left(0\right) = \lim_{\omega\to 0^+} \hat{p}_2\left(\varepsilon\right)$ and $\hat{p}_2\left(\varepsilon\right)$ solves $\beta V_2\left(\hat{p}_2\left(\varepsilon\right)\right) = \varepsilon$. Note that the first order condition with respect to $Q_1$ implies that $u'\left(y-g-Q_1^*\right) = \mu > u'(y+g)$ and that $\lambda\left(p_2\right) \le 0$ for all $p_2$, we have:

$$\lim_{s\to\varepsilon=0^+} \frac{\partial \mathcal{L}}{\partial S} < 0.$$

The same argument can be used to show that if the optimal $S^*$ was strictly positive, a deviation of $S$ from $S^*$ to $S^* - \varepsilon$ will be strictly preferred. Thus the optimal $S^*$ must be equal to $0$. □