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ABSTRACT

Two potentially asymmetric players compete for a prize of common value, which is initially unknown, by exerting efforts. A designer has two instruments for contest design. First, she decides whether and how to disclose an informative signal of the prize value to players. Second, she sets the scoring rule of the contest, which varies the relative competitiveness of the players. We show that the optimum depends on the designer's objective. A bilateral symmetric contest—in which information is symmetrically distributed and the scoring bias is set to offset the initial asymmetry between players—always maximizes the expected total effort. However, the optimal contest may deliberately create bilateral asymmetry—which discloses the signal privately to one player, while favoring the other in terms of the scoring rule—when the designer is concerned about the expected winner's effort. The two instruments thus exhibit complementarity, in that the optimum can be made asymmetric in both dimensions even if the players are ex ante symmetric. Our results are qualitatively robust to (i) affiliated signals and (ii) endogenous information structure. We show that information favoritism can play a useful role in addressing affirmative action objectives.

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1 Introduction

Contest is a widely adopted mechanism that mobilizes focused efforts to achieve stated goals. In a contest, the organizer sets a limited number of prizes and invites entries; contenders sink costly and nonrefundable efforts, and only the frontrunners are rewarded. Governments, firms, nonprofit organizations, and even wealthy individuals often sponsor R&D contests to solicit novel technological solutions, procure innovative products, or encourage scientific breakthroughs (Taylor, 1995; Fullerton and McAfee, 1999; Che and Gale, 2003).[1] The U.S. Department of Defense (DoD) famously operates the Small Business Innovation Research program, which promotes private R&D efforts on military technology, and awards procurement contracts to outstanding innovators (Bhattacharya, 2021). The internal labor markets inside firms provide another analogy (Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Rosen, 1986). Firms induce workers' efforts by scarcely supplied bonus packages, promotions, and opportunities for career advancement, as well as the threat of layoffs. In 2013, 30% of Fortune 500 companies were estimated to reward or punish their employees based on ranking systems inspired by Jack Welch's practice of the "vitality curve."

The ubiquity of contest-like competitive activities has sparked extensive and continuous efforts, in both practice and academic research, to identify feasible and efficient means of administering such mechanisms (see, e.g., Fu and Wu, 2019, 2020; Fang, Noe, and Strack, 2020; Lemus and Marshall, 2021; Hofstetter, Dahl, Aryobsei, and Herrmann, 2021). Two phenomena are broadly observed that inspire contest design.

First, discriminatory measures are often adopted to manipulate the competitive balance of a competition, which either handicap or favor certain contenders. For instance, favorite horses can be requested to carry extra weights in horse racing (Chowdhury, Esteve-González, and Mukherjee, 2023). In firms' succession selection processes, incumbent CEOs and board members may deliberately devote their efforts to grooming preferred candidates, with the celebrated examples of GE and IdeaXerox (Fu and Wu, 2022). Governments in OECD countries strive to support small and medium-sized enterprises (SMEs) through preferential access to public procurement opportunities (OECD, 2018).[2]

Second, contestants can be subject to uncertainty surrounding the nature and environ-

[1]Similarly, in Singapore, the Ministry of Defence (MINDEF) and Singapore Armed Forces sponsor the annual Defense Innovation Challenge, which provides winners with grants and the opportunity to co-develop their solutions with MINDEF. The European Innovation Council administers European Social Innovation Competition schemes to incentivize entrepreneurial solutions that enhance societal well-being, and the Arizona State University Innovation Open encourages student-led ventures to undertake hard-tech R&D.

[2]The reader is referred to OECD (2018), SMEs in Public Procurement: Practices and Strategies for Shared Benefits, OECD Public Governance Reviews, OECD Publishing, Paris, https://doi.org/10.1787/9789264307476-en.

ment of the contest—e.g., the exact value of the prize. Imagine the following scenarios.

(i) Private military contractors, when investing in their prototypes to compete for DoD's procurement contract, may not be fully informed of the exact costs to be incurred when executing the delivery contract—e.g., the efforts to fine-tune and manufacture the product in compliance with DoD's regulations and provisions—which causes uncertainty about the eventual profitability of the contract.

(ii) Workers inside a firm, when competing for a vacancy, are often unaware of the nature and challenge of the new post—e.g., the scope of duties and the resources, and support available from senior management—and its implications for future career advancement.

(iii) The Arizona State University Innovation Open encourages student-led ventures to undertake hard-tech R&D. It awards the winner not only cash prizes but also access to the university's incubator and accelerator program and opportunity to collaborate with the sponsoring companies, which are established firms in relevant areas. However, the option values of these nonmonetary rewards cannot be ascertained without sufficient details—e.g., the resources available from the incubator and the extent of involvement and commitment of the sponsoring companies.

These phenomena spark natural questions for contest design. First, how should a contest designer optimally set biases in contests for efficient incentive provision—i.e., which contestants are to be treated preferentially, and to what extent? Second, how should the designer distribute her information—i.e., should she disclose her information and, if so, to whom?

This paper explores optimal contest design to address the above questions. In contrast to the vast majority of previous studies of contest design, we allow the contest designer to deploy two instruments and choose an optimal combination: (i) a potentially selective information disclosure scheme that may disclose information to one player while concealing it from the other and (ii) a scoring bias that can either discount or inflate a contestant's perceived output in the evaluation process relative to others.

A Snapshot of the Model We consider a contest in which two potentially asymmetric players compete for a prize of a common value. The prize value is unknown and can be either high or low, with a publicly known distribution. Players simultaneously commit to their efforts to vie for the prize—e.g., private military contractors' efforts to develop their prototypes—while the effort incurs a constant marginal cost. The winner is selected through an all-pay auction (see, e.g., Hillman and Riley, 1989; Baye, Kovenock, and De Vries, 1993, 1996; Che and Gale, 1998). The designer evaluates players' entries; each player's effort is converted into a score, and the higher scorer wins. The asymmetry of the players is such that player 1 bears a weakly higher marginal effort cost c_1 than player 2—i.e., $c_1 \geq c_2$—so the latter is the favorite in the contest.

The designer conducts an investigation and acquires an informative binary signal about the underlying prize value, which enables a more precise posterior through Bayesian updating. Prior to the competition, the designer commits to the contest rule, which consists of two elements. First, a disclosure scheme specifies whether the signal is to be disclosed and which player is to receive it. The disclosure scheme is asymmetric when the designer conveys the signal to only one player while concealing it from the other, in which case the recipient is awarded *information favoritism*. For instance, the organizer of a business pitching competition may brief preferred entrepreneurs more elaborately on the funding opportunities available to winning projects. Second, a coefficient is imposed on each player's effort to generate his score. We normalize the coefficient for the underdog—i.e., player 1—to one and that for player 2 to $\delta > 0$, which is called a *scoring bias*. The bias can be interpreted as a nominal judging rule, as well as measures that elevate or discount one's (perceived) output. For instance, the leading candidate for corporate succession is often appointed the president or COO, which endows them access to additional corporate resources and improves their visibility to the board.

We consider two objectives for contest design. The first is the usual maximization of expected *total* effort (see, e.g., Moldovanu and Sela, 2001; Moldovanu, Sela, and Shi, 2007). For instance, the government or a nonprofit organization may use an R&D challenge to fuel the public's interest in a certain area of scientific or technological research; e.g., clean energy. The second is the maximization of the expected *winner*'s effort, which has attracted increasing interest in recent literature (see, e.g., Moldovanu and Sela, 2006; Fu and Wu, 2022). Consider, for instance, a contest sponsored by a pharmaceutical company to procure an innovative ingredient. Only the quality of the winning solution accrues to the benefit of the sponsor. As we show below, the optimal contest design crucially depends on whether the designer aims to maximize the expected total effort or the expected winner's. Intuitively, the difference is driven partly by the fact that the expected total effort is the sum of the *means*, while the expected winner's effort is the *modified first-order statistic,* of the (random) effort variables by the two players.[3]

Summary of Results and Implications The contest game can be viewed as an all-pay auction with interdependent valuations and discrete signal spaces. Siegel (2014) provided the technique for the case with a neutral scoring rule $\delta = 1$. We extend the analysis to characterize the mixed-strategy equilibrium, while allowing for a biased scoring rule $\delta \neq 1$,

[3]It is noteworthy that in our context, the expected winner's effort is not the simple highest effort except for the case of $\delta = 1$. Under a biased scoring rule ($\delta \neq 1$), the winner may not be the one who exerts the highest effort. We thus call the expected winner's effort a modified first-order statistic to reflect the nuance.

which paves the way for the optimal contest design.

Results for the maximization of expected total effort affirm the conventional wisdom that a level playing field fuels competition. The optimum is achieved by an ex post symmetric contest, which sets the scoring bias to $\delta = c_2/c_1$, together with a symmetric disclosure scheme (full disclosure or full concealment). Neither player is awarded information favoritism. The bias $\delta = c_2/c_1$, called the *fair bias* of the contest, precisely offsets the initial advantage of player 2 in terms of the marginal cost of effort.

The optimal contest design departs from the conventional wisdom when the designer aims to maximize the expected winner's effort. In a complete-information all-pay auction, the fair bias maximizes the expected total effort and the expected winner's effort simultaneously (see, e.g., Fu, 2006). In our context, the designer may prefer a *tilting-and-releveling* contest, which creates ex post bilateral asymmetry between players. Specifically, the designer may award one player information favoritism—i.e., disclosing her signal exclusively—while releveling the playing field by letting the scoring bias deviate from the fair level to favor the other. Three observations are noteworthy.

(i) The tilting-and-releveling contest can be optimal even if players are ex ante symmetric.

(ii) When players are ex ante asymmetric, the tilting-and-releveling contest, whenever optimal, awards the underdog, player 1, information favoritism.

(iii) The two instruments—i.e., disclosure scheme and scoring bias—are complementary to each other: Asymmetry never emerges in the optimum in a single dimension; the optimal contest is either ex post symmetric (symmetric disclosure together with fair bias) or ex post asymmetric in terms of both information disclosure and scoring rule.

A tilting-and-releveling contest could enable an upward shift of the winner's effort distribution under certain circumstances. We elaborate on the logic of our results in this simple but counterintuitive case with symmetric players. Specifically, we compare a bilaterally asymmetric contest—which discloses the signal to player 1 and sets a favorable scoring bias $\delta > 1$ for player 2—with a symmetric one, in which the signal is concealed from both players and $\delta = 1$.

Assume a unity marginal effort cost. In the symmetric contest, it is straightforward to infer that their efforts are bounded from above the common expected prize value. We now let the designer "tilt" the contest by disclosing the signal to player 1—which allows him to update his belief about the prize value—while maintaining the neutral scoring rule. It is crucial to note that player 1's mixed strategy would be type-dependent: The bidding supports for low and high types—i.e., player 1 when receiving low and high signals, respectively—do not overlap. Player 2 is the uninformed, so he continues to bid according to the prior, although he takes into account the fact that player 1 receives a signal. The expected winner's effort in this contest falls short of the initially symmetric one. The low type is discouraged by his low valuation. Despite the high valuation, the high type, who is now an ex post favorite relative to the uninformed player 2, has no incentive to raise his maximum effort—i.e., the upper support of his bidding strategy—since his opponent's efforts continue to be bounded by the prior.

Now we let the designer "relevel" the contest by lifting δ above 1. The scoring rule favors player 2, which affects the low and high types of player 1 differently. The bias further squeezes the low type, while diminishing the advantage of the high type. The latter is thus forced to step up his effort. As a result, the high type's bidding support is stretched upward. There exists a unique scoring bias—called the *releveling bias*—under which (i) the low type remains inactive with probability one, (ii) both the high type and the uninformed player 2 remain active with probability one, and (iii) the high type's maximum effort rises to his updated prize valuation.

In summary, the tilting-and-releveling contest may outperform the symmetric one when a high signal is realized, in which case the high-type player 1 may contribute a higher winner's effort that is otherwise impossible. The former contest underperforms the latter when a low signal appears, because the low type would exit the competition. However, the loss is partly made up for by player 2: He bids actively regardless of the realized signal. We identify a condition that summarizes the trade-off and determines the optimum. The same trade-off governs the contest design with asymmetric players and explains the allocation of information favoritism and favorable scoring bias between the weak and strong contenders. Section 3.2.2 delves into the logic in depth.

We extend our model to three alternative settings. In the first, the designer decides whether to provide players with relevant information, and players can acquire interdependent signals from the information (Milgrom and Weber, 1982). The second allows the designer to choose the information structure of her investigation, which corresponds to the concept of the Bayesian persuasion approach pioneered by Kamenica and Gentzkow (2011). The third extension requires that the weaker player win with a minimum probability, which mirrors the practice of affirmative action (Coate and Loury, 1993). We show that our main results are qualitatively robust to these modeling variations with additional insights.

Related Literature In this paper, we consider the optimal design of a contest modeled as an all-pay auction. The contest model can be viewed as a variant of the family of all-pay auctions with interdependent valuations, including those of Krishna and Morgan (1997); Lizzeri and Persico (2000); Siegel (2014); Rentschler and Turocy (2016); Lu and Parreiras (2017); and Chi, Murto, and Välimäki (2019). Our study is primarily linked to two strands of the literature on contest design: (i) (identity-dependent) differential treatments and (ii) information disclosure. To the best of our knowledge, we are the first to allow the designer to fine-tune the two instruments simultaneously and choose their optimal combination.

An enormous amount of scholarly effort has been expended on the optimal way to bias a contest by imposing differentiated treatments (Mealem and Nitzan, 2016). The literature has conventionally espoused the merits of a level playing field for fueling competition and suggested handicapping the favorite to offset initial asymmetry between players—e.g., Epstein, Mealem, and Nitzan (2011); Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Franke, Leininger, and Wasser (2018). A handful of recent studies—e.g., Drugov and Ryvkin (2017); Fu and Wu (2020); Barbieri and Serena (2022); Wasser and Zhang (2023); Echenique and Li (2022)—identify the contexts in which the optimal contest might instead require upsetting the initial balance of the playing field. However, this strand of studies mainly focuses on the optimal construction of identity-dependent treatments and does not involve information disclosure.

The literature has increasingly recognized information disclosure as a valuable addition to a contest designer's toolkit, e.g., Halac, Kartik, and Liu (2017) and Ely, Georgiadis, Khorasani, and Rayo (2022).[4] Lu, Ma, and Wang (2018) and Serena (2022) explore optimal disclosure in contests with contestants of independent and private types. These studies require symmetric disclosure rules, such that the signal from the designer must be public. Wärneryd (2012) considers a common-value contest and allows only a portion of the players to know the prize value. He shows that selectively informing players of the prize value is suboptimal when the designer maximizes total effort. The Bayesian persuasion approach has been applied in a handful of studies to explore optimal information disclosure in contests; e.g., Zhang and Zhou (2016); Chen and Chen (2022); Melo Ponce (2021); and Antsygina and Teteryatnikova (2022). However, these studies focus exclusively on information disclosure in contests.

The rest of the paper proceeds as follows. Section 2 sets up the model and Section 3 carries out the analysis. Section 4 presents three extensions and Section 5 concludes. Appendix A collects the analytical results for Section 4.1. Proofs of our main results are collected in Appendix B.

2 The Model

Two risk-neutral players, indexed by $i \in \mathcal{N} \equiv \{1, 2\}$, compete for a prize of a common value $v \in \{v_H, v_L\}$, with $v_H > v_L > 0$. The high value v_H is realized with a probability

[4] Both studies focus on information disclosure in the course of dynamic contests. This strand of literature also includes Yildirim (2005); Aoyagi (2010); Ederer (2010); and Goltsman and Mukherjee (2011).

$\Pr(v = v_H) =: \mu \in (0,1)$, with the low value v_L to be realized with the complementary probability. Players are initially uninformed about the prize value, while its distribution is common knowledge. They simultaneously exert their efforts $x_i \geq 0$ to win the prize. One player's effort incurs a constant marginal effort cost $c_i > 0$. Without loss of generality, we assume that player 2 is the stronger contender; i.e., $c_1 \geq c_2$.

Winner-selection Mechanism and Scoring Bias The contest designer imposes a scoring bias $\delta_i > 0$ on each player i's effort entry x_i, which generates his score $\delta_i x_i$. We normalize δ_1 to 1 and set $\delta_2 = \delta > 0$. Fixing a set of effort entries $\boldsymbol{x} := (x_1, x_2) \in \mathbb{R}^2_+$, player 1's probability of winning the contest—i.e., the contest success function (CSF)—is given by

$$p_1(x_1, x_2) = \begin{cases} 1, & \text{if } x_1 > \delta x_2, \\ \frac{1}{2}, & \text{if } x_1 = \delta x_2, \\ 0, & \text{if } x_1 < \delta x_2, \end{cases}$$

and player 2 wins with a probability $p_2(x_1, x_2) = 1 - p_1(x_1, x_2)$. That is, a player wins the contest with certainty if his score exceeds that of his opponent. The winner is picked randomly in the event of a tie in scores.

Disclosure Schemes The designer conducts an investigation and obtains a verifiable noisy signal $s \in \{H, L\}$ regarding the prize value v. Specifically, we assume that the signal is drawn as follows:

$$\Pr\left(s = H \,\middle|\, v = v_H\right) = \Pr\left(s = L \,\middle|\, v = v_L\right) = q, \tag{1}$$

where $q \in \left(\frac{1}{2}, 1\right]$ indicates the quality or precision of the signal.[5] In the extreme case with $q = 1$, the signal perfectly reveals the prize value; with $q = 1/2$, the signal is completely uninformative.

The designer commits to her disclosure scheme—i.e., whether and with whom the result of her investigation is to be disclosed to or concealed from. The pure-strategy disclosure scheme can formally be described by $\gamma \in \{CC, CD, DC, DD\}$, where C and D indicate "concealment" and "disclosure," respectively. With $\gamma = CC(DD)$, the realized signal s is conveyed to neither (both) of the players. With $\gamma = CD$, the designer conceals the signal from player 1 while disclosing it to player 2, which awards player 2 *information favoritism*; $\gamma = DC$ is similarly defined, such that player 1 receives the signal privately.

[5] Note that q is exogenous in the baseline model. We will generalize the model and endogenize the information structure using a Bayesian persuasion approach (e.g., Kamenica and Gentzkow, 2011) in Section 4.2.

Contest Design Prior to the contest, the designer chooses the scoring bias $\delta > 0$ in conjunction with her disclosure scheme $\gamma \in \{CC, CD, DC, DD\}$. The design choice can serve two possible design objectives: The designer may maximize either (i) the expected total effort of the contest or (ii) the expected winner's effort in the contest. The former has conventionally been adopted in the vast majority of the contest literature, which resembles revenue maximization in the auction literature. The latter, however, is relevant in a broad array of competitive activities—e.g., architecture competitions, procurement tournaments, R&D competitions, and large public firms' succession horse races—and has attracted increasing attention in recent studies.[6]

3 Analysis

We first characterize the equilibrium under an arbitrary contest scheme (γ, δ), with $\gamma \in \{CC, CD, DC, DD\}$ and $\delta > 0$. We then derive the optimal contest scheme for each design objective.

3.1 Equilibrium Results

It is well known that an all-pay auction with complete information or a discrete signal structure, in general, does not possess pure-strategy equilibria (see, e.g., Hillman and Riley, 1989; Baye, Kovenock, and De Vries, 1996; Siegel, 2009, 2010, 2014). Siegel (2014) provides the technique for constructing the unique equilibrium of the contest under a neutral scoring rule; i.e., $\delta = 1$. We adapt his result to our context, which allows for an arbitrary scoring bias $\delta > 0$.

It can be formally verified that for a given contest scheme (γ, δ) and fixed signal quality q, each player's equilibrium strategy in the interim stage contains at most one atom and it must be placed at zero. We describe by a function $b_{is}(x; \gamma, \delta, q)$ the equilibrium bidding strategy of a player i of type s; i.e., when receiving a signal $s \in \{H, L\}$: $b_{is}(0; \gamma, \delta, q)$ gives the *probability* that player i chooses zero effort—i.e., $x = 0$—and remains inactive, while $b_{is}(x; \gamma, \delta, q)$ provides the *probability density* of exerting effort $x > 0$. We omit the subscript s and the argument q if a player $i \in \{1, 2\}$ is not granted access to the signal s, so his equilibrium bidding strategy is given by $b_i(x; \gamma, \delta)$.

We introduce several notations before characterizing the equilibrium. We define $\bar{v} := \mu v_H + (1 - \mu) v_L$, which denotes the ex ante expected prize value. Upon receiving a signal

[6]See, e.g., Moldovanu and Sela, 2006; Barbieri and Serena, 2021; Fu and Wu, 2022; and Wasser and Zhang, 2023).

$s = H$, a player's expected prize value is updated to

$$\hat{v}_H(q) := \frac{\mu q v_H + (1-\mu)(1-q)v_L}{\mu q + (1-\mu)(1-q)}.$$

Similarly, the posterior upon receiving $s = L$ is

$$\hat{v}_L(q) := \frac{\mu(1-q)v_H + (1-\mu)qv_L}{\mu(1-q) + (1-\mu)q}.$$

A signal $s = H$ is realized with an ex ante probability $\hat{\mu}(q) := \mu q + (1-\mu)(1-q)$.

3.1.1 Symmetric Disclosure Schemes

We first characterize the equilibrium under a symmetric information disclosure scheme, i.e., $\gamma \in \{CC, DD\}$, in which case neither player possesses information favoritism.

Proposition 1 ***(Equilibrium Characterization under Symmetric Disclosure)*** *Under $\gamma = DD$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta, q) = \begin{cases} \frac{c_2}{\delta \hat{v}_s(q)}, & \text{if } 0 < x \leq \frac{\delta \hat{v}_s(q)}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{2s}(x; DD, \delta, q) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \text{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_s(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

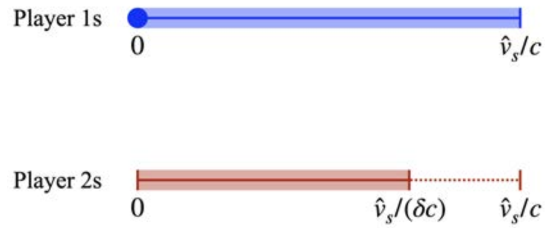
(ii) If $\delta \geq \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta, q) = \begin{cases} 1 - \frac{c_2}{\delta c_1}, & \text{if } x = 0, \\ \frac{c_2}{\delta \hat{v}_s(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{2s}(x; DD, \delta, q) = \begin{cases} \frac{\delta c_1}{\hat{v}_s(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_s(q)}{\delta c_1}, \\ 0, & \text{otherwise.} \end{cases}$$

(iii) The equilibrium bidding strategy under $\gamma = CC$, denoted by $b_i(x; CC, \delta)$, can be obtained by replacing $\hat{v}_s(q)$ with $\bar{v} \equiv \mu v_H + (1-\mu)v_L$ in $b_{is}(x; DD, \delta, q)$.

(a) $\delta = 1$.



(b) $\delta > 1$.

Figure 1: Equilibrium Strategies with Symmetric Players: $\gamma = DD$.

The equilibrium under a symmetric disclosure scheme with discrete signal spaces resembles that in a standard complete-information all-pay auction à la Hillman and Riley (1989) and Baye, Kovenock, and De Vries (1996), and the conventional wisdom in the contest literature remains valid. The scoring bias δ tilts the balance of the playing field and reshapes players' equilibrium bidding strategies. First, one player would remain inactive with a positive probability in the equilibrium unless $\delta = c_2/c_1 \leq 1$. We call c_2/c_1 the *fair bias*, as this perfectly offsets player 2's initial cost advantage. Player 1, as the ex ante weaker contender, would remain inactive with a positive probability whenever δ exceeds the cutoff (Case *ii*), since player 2's advantage remains. In contrast, when δ falls below the cutoff (Case *i*), the favorite, player 2, is excessively handicapped, which discourages him from active bidding.

Second, the scoring bias determines players' maximum efforts. Consider, for instance, the case with $\gamma = DD$ and suppose that $\delta \geq c_2/c_1$. Player 1's effort is bounded by $\hat{v}_s(q)/c_1$; as a result, player 2 has a sure win if he bids $\hat{v}_s(q)/(\delta c_1)$, which sets the upper support of his bidding strategy.

We visualize the equilibrium in Figure 1 in a simple case of symmetric players—with $c_1 = c_2 =: c$—and a disclosure scheme $\gamma = DD$. A neutral scoring rule—i.e., $\delta = 1$—corresponds to the fair bias c_2/c_1 in Proposition 1, in which case both players' efforts are uniformly distributed over the interval $[0, \hat{v}_s/c]$, as Figure 1(a) illustrates. Suppose instead that the designer biases the contest in favor of player 2 by setting $\delta > 1$. Favored player 2 is allowed to slack off, so his maximum effort drops to $\hat{v}_s/(\delta c)$, as Figure 1(b) depicts.

3.1.2 Asymmetric Disclosure Schemes

We now consider the equilibrium under each asymmetric disclosure scheme, i.e., $\gamma = CD$ or DC, in which case one player receives the signal privately.

Proposition 2 ***(Equilibrium Characterization under Asymmetric Disclosure)*** *Under $\gamma = DC$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$*, then*

$$b_{1L}(x; DC, \delta, q) = \begin{cases} \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta, q) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\delta\bar{v}}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \text{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\bar{v}}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If $\frac{c_2}{c_1} \leq \delta \leq \frac{c_2}{\hat{\mu}(q)c_1}$*, then*

$$b_{1L}(x; DC, \delta, q) = \begin{cases} \frac{1}{1-\hat{\mu}(q)}\left(1 - \frac{c_2}{\delta c_1}\right), & \text{if } x = 0, \\ \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta, q) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1} < x \leq \frac{\hat{v}_L(q)}{c_1} + \frac{\delta\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1} < x \leq \frac{\hat{v}_L(q)}{\delta c_1} + \frac{\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(iii) If $\delta > \frac{c_2}{\hat{\mu}(q)c_1}$*, then*

$$b_{1L}(x; DC, \delta, q) = \begin{cases} 1, & \text{if } x = 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta, q) = \begin{cases} 1 - \frac{c_2}{\delta c_1\hat{\mu}(q)}, & \text{if } x = 0, \\ \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_H(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

(a) $\delta = 1$.

(b) $\delta = 1/\hat{\mu}(q)$.

Figure 2: Equilibrium Strategies with Symmetric Players: $\gamma = DC$.

$$b_2(x; DC, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_H(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_H(q)}{\delta c_1}, \\ 0, & \textit{otherwise.} \end{cases}$$

The equilibrium under (CD, δ) *can be obtained similarly.*

Asymmetric disclosure fundamentally changes the nature of the equilibrium. We now elaborate on the nuances in the case with disclosure scheme $\gamma = DC$. Player 1 is informed because of the signal s, and his equilibrium bidding strategy is thus type-dependent. The equilibrium is *monotonic*, in the sense that the support of the high-type player $1H$—i.e., player 1 when receiving $s = H$—does not overlap with that of his low-type counterpart. With $\delta < c_2/c_1$, for instance, the efforts of player $1L$ are uniformly distributed on the support $\big[0, \delta[1 - \hat{\mu}(q)]\hat{v}_L(q)/c_2\big]$, as Proposition 2 shows, while those of player $1H$ are distributed on $\big[\delta[1 - \hat{\mu}(q)]\hat{v}_L(q)/c_2, \delta\bar{v}/c_2\big]$.

Two critical values for δ, the aforementioned fair bias c_2/c_1 and $c_2/[\hat{\mu}(q)c_1]$, are noteworthy under $\gamma = DC$. As in the case with symmetric disclosure, some player remains inactive with a positive probability unless the contest sets a scoring bias at the level of $\delta = c_2/c_1$. For instance, a scoring bias $\delta < c_2/c_1$ excessively handicaps player 2, such that player 2 remains inactive with a positive probability (Case i in Proposition 2).

The balance of the playing field is tilted toward player 2 as δ rises. Note that δ differently affects the strategies of low and high types of player 1. Player $1L$ has a lower updated prize valuation and bears a higher cost, so he is effectively a weaker player compared with player 2 whenever δ exceeds c_2/c_1. A further rise in δ forces him to concede: He is more likely to exert zero effort, and his maximum effort—i.e., the upper bound of his effort support—decreases. In contrast, for δ in the range of $\big[c_2/c_1, c_2/[\hat{\mu}(q)c_1]\big]$, player $1H$ is effectively a stronger player due to his higher updated prize valuation. A larger δ—which implies a less favorable scoring rule—compels him to step up his effort: His maximum bid, $\hat{v}_L(q)/c_1 + \delta\hat{\mu}(q)[\hat{v}_H(q) - \hat{v}_L(q)]/c_2$, would increase accordingly.

The contest effectively "unravels" itself once δ reaches the cutoff $c_2/[\hat{\mu}(q)c_1]$. The low type of player 1 drops out completely—i.e., bidding zero with probability one; the high type's maximum bid reaches $\hat{v}_H(q)/c_1$, which ensures his win but fully extracts his surplus. Player 2 expects only the high-type opponent whenever the competition continues; he maximally bids $[\hat{\mu}(q)\hat{v}_H(q)]/c_2$, which, given the scoring bias $\delta = c_2/[\hat{\mu}(q)c_1]$, matches the maximum bid of player $1H$. A scoring bias $\delta > c_2/[\hat{\mu}(q)c_1]$, however, would discourage even the high-type player 1 and motivate him to remain inactive with a positive probability.

We call $\delta = c_2/[\hat{\mu}(q)c_1]$ the *releveling bias* of the contest under $\gamma = DC$. With a high signal and a disclosure scheme $\gamma = DC$, players 1 and 2 bid actively with probability one and win with equal probability if and only if the scoring bias is set at this level. The releveling bias eliminates the information rent awarded to player 1 by the asymmetric disclosure scheme.

We again illustrate the equilibrium in the simple case of symmetric players—i.e., $c_1 = c_2 =: c$—and a disclosure scheme $\gamma = DC$. Figure 2(a) demonstrates the scenario of a neutral scoring bias $\delta = 1$, which corresponds to the fair bias c_2/c_1 in the general case. As predicted by Proposition 2, the efforts of the informed low type (player $1L$) and those of the high type ($1H$) are uniformly distributed on the support $\big[0, [1 - \hat{\mu}(q)]\hat{v}_L/c\big]$ and $\big[[1 - \hat{\mu}(q)]\hat{v}_L/c, \bar{v}/c\big]$, respectively. In Figure 2(b), the designer sets $\delta = 1/\hat{\mu}(q)$, which corresponds to the releveling bias $c_2/[\hat{\mu}(q)c_1]$ in the general case. The bias removes the low-type player 1 from the competition, which causes the contest to unravel. The upper support of high-type player 1's bidding strategy increases accordingly to $\hat{v}_H/c$.

3.1.3 Expected Total Effort and the Expected Winner's Effort

The equilibrium characterization allows us to derive the optimal contest. Consider a contest with an arbitrary profile of players' marginal effort costs (c_1, c_2), with $c_1, c_2 > 0$. Denote by $TE(\gamma, \delta; c_1, c_2)$ and $WE(\gamma, \delta; c_1, c_2)$, respectively, the expected *total* effort and the expected *winner*'s effort under a contest scheme (γ, δ). Propositions 1 and 2 lead to the following.

Lemma 1 ***(Expected Total Effort under Different Contest Schemes)*** *Fixing a contest scheme (δ, γ) and a profile of marginal effort costs (c_1, c_2), the contest generates an equilibrium expected total effort*

$$TE(CC, \delta; c_1, c_2) = TE(DD, \delta; c_1, c_2) = \begin{cases} \frac{\delta\bar{v}(c_1+c_2)}{2c_2^2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{\bar{v}(c_1+c_2)}{2\delta c_1^2}, & \text{if } \delta \geq \frac{c_2}{c_1} \end{cases} \tag{2}$$

for symmetric disclosure schemes.[7] *Under asymmetric disclosure schemes, the equilibrium*

[7] With a symmetric disclosure scheme, the expected total effort in (2) and the expected winner's effort in (4) are independent of the quality of the signal q and we drop q in $TE(\cdot)$ and $WE(\cdot)$ for $\gamma \in \{CC, DD\}$.

expected total effort of the contest can be obtained as

$$TE(DC, \delta, q; c_1, c_2) = TE(CD, 1/\delta, q; c_2, c_1)$$
$$= \begin{cases} \frac{\delta(c_1+c_2)\left(\hat{v}_L(q)+\hat{\mu}(q)^2\hat{v}_H(q)-\hat{\mu}(q)^2\hat{v}_L(q)\right)}{2{c_2}^2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{c_1+c_2}{2c_1c_2}\left[\frac{c_2}{\delta c_1}\hat{v}_L(q) + \frac{\delta c_1}{c_2}\hat{\mu}(q)^2(\hat{v}_H(q) - \hat{v}_L(q))\right], & \text{if } \frac{c_2}{c_1} \le \delta \le \frac{c_2}{\hat{\mu}(q)c_1}, \\ \frac{(c_1+c_2)\hat{v}_H(q)}{2\delta c_1^2}, & \text{if } \delta > \frac{c_2}{\hat{\mu}(q)c_1}. \end{cases} \tag{3}$$

Further, we derive the equilibrium expected winner's efforts. The following ensues.

Lemma 2 ***(Expected Winner's Effort under Different Contest Schemes)*** *Fixing a contest scheme (δ, γ) and a profile of marginal effort costs (c_1, c_2), the equilibrium expected winner's effort from the contest game is*

$$WE(CC, \delta; c_1, c_2) = WE(DD, \delta; c_1, c_2) = \begin{cases} \frac{\delta\bar{v}(2c_1+3c_2-c_1\delta)}{6{c_2}^2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{\bar{v}(3c_1\delta-c_2+2c_2\delta)}{6{c_1}^2\delta^2}, & \text{if } \delta \ge \frac{c_2}{c_1}, \end{cases} \tag{4}$$

and

$$WE(DC, \delta, q; c_1, c_2) = \begin{cases} \frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_1\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \frac{\delta c_1}{c_2}; c_1, c_2\right), & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_2\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \frac{\delta c_1}{c_2}; c_1, c_2\right), & \text{if } \frac{c_2}{c_1} \le \delta \le \frac{c_2}{\hat{\mu}(q)c_1}, \\ \frac{\hat{v}_H(q)}{6c_1c_2}\mathcal{W}_3\left(\frac{\delta c_1}{c_2}; c_1, c_2\right), & \text{if } \delta > \frac{c_2}{\hat{\mu}(q)c_1}, \end{cases} \tag{5}$$

where $\mathcal{W}_1(\cdot,\cdot,\cdot)$, $\mathcal{W}_2(\cdot,\cdot,\cdot)$, and $\mathcal{W}_3(\cdot)$ are defined as follows:

$$\mathcal{W}_1(u, z, d; c_1, c_2) := -c_2\left(u^3z+1\right)d^2 + \left\{u^2z\left[3(c_1+c_2) - c_1u\right] + 2c_1 + 3c_2\right\}d,$$
$$\mathcal{W}_2(u, z, d; c_1, c_2) := \frac{d^3\left(-u^2\right)z\left[u(c_1+c_2d) - 3(c_1+c_2)\right] + 3c_1d - c_1 + 2c_2d}{d^2},$$
$$\mathcal{W}_3(d; c_1, c_2) := \frac{c_1(3d-1) + 2c_2d}{d^2}.$$

Moreover, we have that $WE(CD, \delta, q; c_1, c_2) = WE(DC, 1/\delta, q; c_2, c_1)$.

Lemmas 1 and 2 pave the way for our analysis of the optimal contest design.

3.2 Optimal Contest

We first derive the optimum for symmetric players—i.e., the case with $c_1 = c_2 > 0$. An in-depth discussion of the symmetric case elucidates the main logic of our analysis, which lays a foundation for the general results of optimal contest design with asymmetric players.

3.2.1 Optimal Contest with Symmetric Players

We are now ready to derive the optimal contest with symmetric players.

Theorem 1 ***(Optimal Contest with Symmetric Players)*** *Fix* $q \in (1/2, 1]$ *and suppose that* $c_1 = c_2 = c > 0$. *The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, 1)$ *and* $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, 1)$ *are an optimal contest scheme.*

(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$, *the optimal contest scheme is* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/\hat{\mu}(q)\big)$ *or* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, \hat{\mu}(q)\big)$; *in the case with* $\hat{\mu}(q)\hat{v}_H(q) \leq 4\hat{v}_L(q)$, *both* $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, 1)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, 1)$ *are an optimal contest scheme.*

Theorem 1 shows that the optima may diverge when varying the design objectives. Theorem 1(i) is intuitive and echoes the conventional wisdom of the contest literature: A level playing field creates competition and fuels effort supply. To maximize expected total effort, the optimal contest maintains the symmetry: A fair bias $\delta = c_2/c_1 = 1$, together with a symmetric disclosure—i.e., $\gamma \in \{CC, DD\}$.

In contrast, Theorem 1(ii) shows that the designer may deliberately create ex post bilateral asymmetry between players with an optimal mixture of asymmetric information disclosure and a bias that deviates from the fair level. That is, the designer tilts the playing field by awarding information favoritism to one player, while releveling the playing field by biasing the contest in favor of the other. As shown in Theorem 1(ii), when the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$ is met, it is optimal for the designer to disclose the signal to player 1 and inflates player 2's score by the releveling bias $\delta = 1/\hat{\mu}(q)$. We call the scheme $(DC, 1/\hat{\mu}(q))$ a *tilting-and-releveling* contest.

A fully symmetric contest, $(CC, 1)$ or $(DD, 1)$, enables maximized participation by players and fully extracts their surplus, which achieves the first-best outcome for the maximization of expected total effort. In the tilting-and-releveling contest, however, the low-type player 1 gives up when a low signal is realized, which causes forgone efforts for the designer and is thus suboptimal. The same may not hold when maximizing the expected winner's effort, in which case only the winner's contribution accrues to the designer's benefit. The tilting-and-releveling contest precludes the low type from the competition, but could more effectively incentivize active players and shift upward the probability mass of their efforts.

To unveil the logic, it is useful to compare the tilting-and-releveling contest $\big(DC, 1/\hat{\mu}(q)\big)$ with the fully symmetric scheme $(CC, 1)$. In the latter, neither player is informed, so they maintain their prior throughout, with an expected prize value $\bar{v}$. Players' efforts are uniformly distributed over the support $[0, \bar{v}/c]$, which generates an expected winner's effort $(2\bar{v})/(3c)$. We then consider the contest $\big(DC, 1/\hat{\mu}(q)\big)$. When a low signal $s = L$ is realized, player 1 gives up. However, the loss is partly compensated for by the contribution of the uninformed player 2, who bids actively regardless. By Proposition 2 and Figure 2(b), player 2's efforts are uniformly distributed over $\big[0, [\hat{\mu}(q)\hat{v}_H(q)]/c\big]$; she wins with probability one, so the expected winner's effort boils down to her average, $[\hat{\mu}(q)\hat{v}_H(q)]/(2c)$. Hence, the contest underperforms $(CC, 1)$ in the event of a low signal. Now imagine that a high signal $s = H$ is realized. Player 2 is privileged by the scoring rule $\delta = 1/\hat{\mu}(q)$, so he bids less aggressively than he does in $(CC, 1)$: An effort of $[\hat{\mu}(q)\hat{v}_H(q)]/c$ ensures a win, which is less than the upper support $\bar{v}/c$ in $(CC, 1)$. However, recall that player $1H$ is an ex post favorite due to his higher updated prize valuation. The unfavorable scoring rule compels him to step up his effort. His maximum effort rises to $\hat{v}_H(q)/c$; this cannot be achieved in the symmetric contest $(CC, 1)$, in which case players' efforts are capped by $\bar{v}/c$. The tilting-and-releveling contest may thus prevail.

A trade-off between $\big(DC, 1/\hat{\mu}(q)\big)$ and $(CC, 1)$ depends on $\hat{\mu}(q)$, $\hat{v}_H(q)$, and $\hat{v}_L(q)$. The former yields a gain when a high signal is realized, which occurs with a probability of $\hat{\mu}(q)$: Hence, $\big(DC, 1/\hat{\mu}(q)\big)$ is more likely to outperform $(CC, 1)$ with a large $\hat{\mu}(q)$. The gain is more significant compared with the loss when the signal prompts substantial upward revision in prize valuation—i.e., from $\bar{v}$ to $\hat{v}_H(q)$—which requires larger $\hat{v}_H(q)$ relative to $\hat{v}_L(q)$. Summing these up gives rise to the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$, and the tilting-and-releveling contest maximizes the expected winner's effort when this requirement is met.

It is worth noting that the two instruments, information favoritism and scoring bias, play *complementary* roles. That is, the optimum is either a fully symmetric contest or a tilting-and-releveling contest that embraces bilateral asymmetry. We highlight the complementarity in the following remark. Suppose that the designer can flexibly deploy only one instrument: She can either manipulate the disclosure scheme while maintaining a neutral scoring rule, or bias the scoring while being limited to symmetric disclosure. The following is obtained.

Remark 1 ***(Unidimensional Contest Design with Symmetric Players)*** *Fix $q \in (1/2, 1]$ and suppose that $c_1 = c_2$. The following statements hold:*

(i) Fix $\delta = 1$. A symmetric disclosure scheme—i.e., $\gamma \in \{CC, DD\}$—maximizes both expected total effort and the expected winner's effort simultaneously.

(ii) Fix $\gamma \in \{CC, DD\}$. The neutral scoring bias—i.e., $\delta = 1$—maximizes both expected total effort and the expected winner's effort simultaneously.

With a neutral scoring rule $\delta = 1$, an asymmetric disclosure scheme cannot force the high-type player 1 to raise his maximum effort above $\bar{v}/c$, as Figure 2(a) illustrates. Similarly, with a symmetric disclosure scheme in place, biasing the scoring rule only reduces competition and allows the favored player to ensure a win with lower maximum effort, as Figure 1(b) shows.

3.2.2 Optimal Contest with Asymmetric Players

We now consider the general case of asymmetric players. In the symmetric case, we show that the designer may devise a tilting-and-releveling contest scheme, such that one player receives information favoritism while the other is favored by the scoring rule. With heterogeneous players, the natural questions would be (i) whether a tilting-and-releveling contest would continue to emerge, and (ii) if it does, which player—the underdog or the favorite—should be the recipient of information favoritism or preferential scoring bias. We obtain the following.

Theorem 2 ***(Optimal Contest with Asymmetric Players)*** *Fix $q \in (1/2, 1]$ and suppose that $c_1 > c_2$. The following statements hold.*

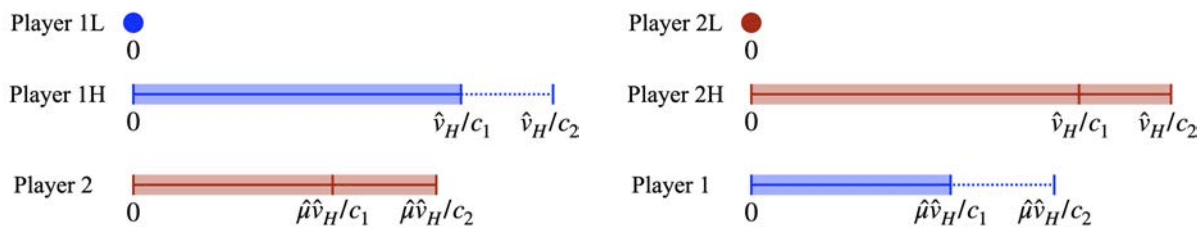
(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ and $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, c_2/c_1)$ are an optimal contest scheme.

(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, the optimal scheme is $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$; in the case with $\hat{\mu}(q)\hat{v}_H(q) \leq \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, c_2/c_1)$ are an optimal contest scheme.

Theorem 2(i), again, affirms the conventional wisdom of leveling the playing field in the contest literature. The fair bias $\delta = c_2/c_1$ perfectly offsets the ex ante asymmetry in the bidders' marginal costs of effort, which, together, with a symmetric disclosure scheme, leads to an *ex post symmetric* contest. By Proposition 1, the fair bias $\delta = c_2/c_1$ uniquely maximizes players' participation, and any deviation will cause one player to remain inactive with a positive probability. This fully extracts players' surplus and maximizes expected total effort.

Again, as Theorem 2(ii) shows, a tilting-and-releveling contest could emerge in the optimum for maximization of the expected winner's effort, in which case the underdog, player 1, is provided information favoritism. Recall by the discussion following Proposition 2 that the releveling bias $\delta = c_2/[\hat{\mu}(q)c_1]$ precludes the low-type player 1 from the competition, while rebalancing the competition between player 2 and the high-type player 1.

The same trade-off for the designer looms large in this context, as in the previously discussed case with symmetric players. Players' efforts evenly accrue to the designer's benefit



(a) $(\gamma, \delta) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$. (b) $(\gamma, \delta) = \left(CD, \frac{\hat{\mu}(q)c_2}{c_1}\right)$.

Figure 3: Equilibrium Strategies under Asymmetric Information Disclosure: $c_1 > c_2$.

when she maximizes expected total effort, while only the winner's effort (i.e., the modified first-order statistic) matters when she is instead concerned about the expected winner's effort. The tilting-and-releveling contest "gives up" the low-type informed player; the loss is compensated for by his better-incentivized high-type counterpart.

Three remarks are in order. First, to understand why information favoritism is given to the weaker player 1, recall that the tilting-and-releveling approach generates a loss compared with $(CC, c_2/c_1)$ when the low signal is realized. Giving up the low-type underdog minimizes the loss, since his higher marginal cost limits the forgone efforts. Despite the exit of the low-type informed player with $s = L$, the uninformed player continues to bid actively and his contribution provides insurance for contest performance, since it mitigates the loss. In a tilting-and-releveling contest, the uninformed player $i \in \{1, 2\}$ randomizes his effort over the interval $\left[0, [\hat{\mu}(q)\hat{v}_H]/c_i\right]$. Picking the favorite—i.e., player 2—obviously maximizes the insurance. Figure 3(a) illustrates the equilibrium in the optimal tilting-and-releveling contest $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$, while Figure 3(b) illustrates its counterpart $\left(CD, [\hat{\mu}(q)c_2]/c_1\right)$ and demonstrates the difference. Note that when the designer's objective is to maximize the winner's effort, the larger the dispersion of the bidders' effort distribution, the better off the designer.

Further, the tilting-and-releveling approach creates a gain compared with $(CC, c_2/c_1)$ when the high signal is realized. The optimal tilting-and-releveling contest $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$ forces the high-type player 1 to bid up to the limit $\hat{v}_H/c_1$—which underperforms the high-type player 2 in the contest $\left(CD, [\hat{\mu}(q)c_2]/c_1\right)$, who can maximally bid $\hat{v}_H/c_2$. However, as stated above, the uninformed player bids up to $\left[\hat{\mu}(q)\hat{v}_H\right]/c_1$ in $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$, which is more than his counterpart in $\left(CD, [\hat{\mu}(q)c_2]/c_1\right)$. Moreover, $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$ creates a closer race between the high-type informed player and the uninformed, which also elevates the expected winner's effort.

Second, the trade-off between $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$ and $(CC, c_2/c_1)$ depends on the comparison between $\hat{\mu}\hat{v}_H$ and $\hat{v}_L$—as in the case with symmetric players—while it can also be moderated by players' asymmetry. Imagine a scenario in which player 1 is substantially weaker than player 2—i.e., when c_1 is substantially larger than c_2. By the rationale laid out above, the loss incurred in the event of low signal is negligible: the forgone efforts of low-type player 1 are limited by his high cost, while the insurance from the uninformed player 2 is amplified because of his low cost. A titling-and-releveling contest thus gains in its appeal. These effects sum up to the condition of $\hat{\mu}(q)\hat{v}_H(q) > [2(c_2/c_1) + 2]\hat{v}_L(q)$ in Theorem 2: Obviously, the more asymmetric the two players—i.e., a smaller c_2/c_1—the more likely the tilting-and-releveling contest to emerge as the optimum.

Third, it is worth noting that the releveling bias $c_2/[\hat{\mu}(q)c_1]$ in Theorem 2(ii) is not necessarily greater than one; the high-type player 1 may remain an underdog in the presence of excessive player asymmetry. However, the releveling bias favors the uninformed relative to the fair bias c_2/c_1; it perfectly relevels the playing field between the high-type player 1 and the uninformed player 2.

4 Extensions

In this section, we discuss three extended settings. In the first, the information disclosed to players generates affiliated signals about the prize value. In the second, the designer can flexibly design the signal structure. Finally, we consider a setting in which the designer is mandated to comply with an affirmative action policy.

4.1 Affiliated Signals

In the baseline model, the designer receives a verifiable signal from her investigation, decides whether/how to disclose it, and the signals the two players observe are thus perfectly correlated with $\gamma = DD$. We now assume that the designer possesses relevant information about the prize value and decides whether to feed her information to the players; players acquire their own signals from the information provided by the designer, and the signals are *affiliated*. For an analogy, the organizer of a business-pitching competition may offer a briefing to competing entrepreneurs about the opportunities for collaborating with sponsoring companies, which provides an informative reference for contenders' judgment of the value of the event. Denote by $\gamma = \widehat{D}\widehat{D}$ the scenario in which the designer provides her information to both players. They each receive a *conditionally independent* private signal s_i as specified in (1) for a given realization of the prize value $v \in \{v_H, v_L\}$. This scenario differs fundamentally from that under $\gamma = DD$: Under the latter, the designer conveys a given signal s to both players; in contrast, players acquire their own signals s_i under $\gamma = \widehat{D}\widehat{D}$, and thus their signals may diverge. Consider the analogy laid out above: Each is given access to the same

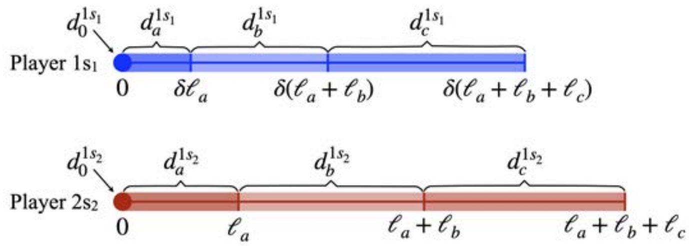


Figure 4: Equilibrium under Full Disclosure with Affiliated Signals: $\delta < 1$.

valuable information, but they may interpret it differently.

With slight abuse of notation, we continue to use $\gamma = CC$, DC, and CD, respectively, to denote the scenarios in which neither of the two players, only player 1, and only player 2 has access to the designer's information. The equilibrium characterization of the cases of $\gamma \in \{CC, DC, CD\}$ is the same as that for their counterparts in the baseline model.

Equilibrium under Full Disclosure with Affiliated Signals We now explore the equilibrium under full disclosure, i.e., $\gamma = \widehat{D}\widehat{D}$. Define $\psi_{s's} := \Pr(s'|s)\mathbb{E}[v|s', s]$, where $\Pr(s'|s)$ is the probability that player $j \neq i$ receives a signal $s' \in \{H, L\}$ conditional on player i receiving a signal $s \in \{H, L\}$, and $\mathbb{E}[v|s', s]$ is the expected prize value conditional on a profile of realized signals (s', s). Specifically, we have that

$$\psi_{HH} := \frac{\mu q^2 v_H + (1-\mu)(1-q)^2 v_L}{\mu q + (1-\mu)(1-q)},$$
$$\psi_{LH} := \frac{q(1-q)\bar{v}}{\mu q + (1-\mu)(1-q)},$$
$$\psi_{HL} := \frac{q(1-q)\bar{v}}{\mu(1-q) + (1-\mu)q},$$
$$\psi_{LL} := \frac{\mu(1-q)^2 v_H + (1-\mu)q^2 v_L}{\mu(1-q) + (1-\mu)q}.$$

Simple algebra would verify that $\psi_{Hs} + \psi_{Ls} = \mathbb{E}[v|s]$ for $s \in \{H, L\}$.

Assuming a neutral scoring bias $\delta = 1$, Siegel (2014) provides a sufficient condition—i.e., the weaker monotonicity condition (WM condition, henceforth)—that ensures the existence and uniqueness of a monotonic equilibrium. The condition boils down to $\psi_{LH} \geq \psi_{LL}$—or equivalently, $q \leq \frac{v_H}{v_H + v_L}$—in our context. When the WM condition fails—i.e., when $q > \frac{v_H}{v_H + v_L}$—Rentschler and Turocy (2016) show in their Example 3(a) that the equilibrium is no longer monotonic; they further characterize the unique equilibrium for the case of $\delta = 1$. We extend their analysis to the general case of arbitrary scoring bias $\delta > 0$ and characterize the equilibrium.

Three remarks are in order, which lay a foundation for our equilibrium analysis. First, it can be verified that each player's equilibrium strategy in the interim stage contains at most one atom and it must be placed at zero. Second, each player bids uniformly over at most three intervals with different densities. Finally, given the linearity of the scoring rule, the bidding intervals for player 1 can be obtained through scaling up (or down) those for player 2 by δ. Figure 4 illustrates the structure of the two players' equilibrium bidding strategies. A player i's equilibrium bidding strategy upon receiving a signal s_i is fully characterized by the combination of (i) the probability of remaining inactive, $d_0^{is_i}$, (ii) the length of each bidding interval, (ℓ_a, ℓ_b, ℓ_c), and (iii) the associated densities on each interval, $(d_a^{is_i}, d_b^{is_i}, d_c^{is_i})$. More formally, the signal-dependent strategy, denoted by $b^{\sharp}_{is_i}\big(x; \widehat{D}\widehat{D}, \delta, q\big)$, can take the form of

$$b^{\sharp}_{is_i}\left(x; \widehat{D}\widehat{D}, \delta, q\right) = \begin{cases} d_0^{is_i}, & \text{if } x = 0, \\ d_a^{is_i}, & \text{if } 0 < x \leq (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})\ell_a, \\ d_b^{is_i}, & \text{if } (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})\ell_a < x \leq (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})(\ell_a + \ell_b), \\ d_c^{is_i}, & \text{if } (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})(\ell_a + \ell_b) \leq (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})(\ell_a + \ell_b + \ell_c), \\ 0, & \text{otherwise.} \end{cases}$$

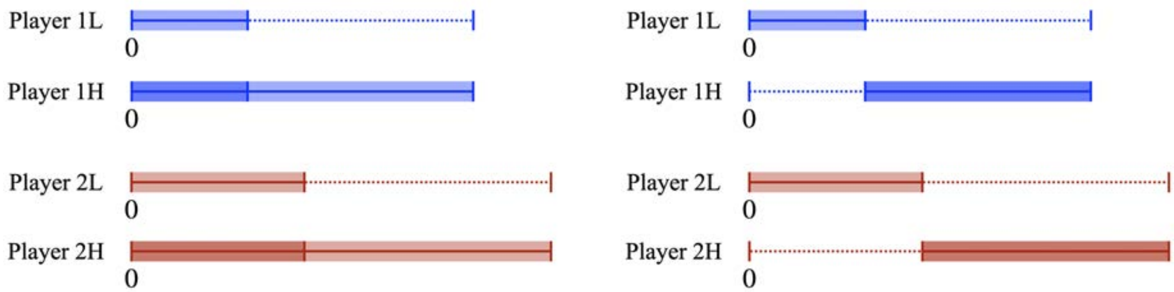
The following result fully characterizes the unique equilibrium under full disclosure with affiliated signals.

Proposition 3 ***(Equilibrium under Full Disclosure with Affiliated Signals)*** *Fix* (c_1, c_2) *and* $\delta > 0$. *Consider the scenario of* $\gamma = \widehat{D}\widehat{D}$ *under which the designer provides both players with her information and each receives a conditionally independent private signal of the prize value. If* $\psi_{LH} < \psi_{LL}$*—or equivalently, if* $q > v_H/(v_H + v_L)$*—the equilibrium is nonmonotonic and is given by Table A1 in Appendix A; otherwise, the equilibrium is monotonic and is given by Table A2 in Appendix A.*

Optimal Contest We first consider the optimal scoring bias with $\gamma = \widehat{D}\widehat{D}$.

Lemma 3 ***(Optimal Scoring Bias under Full Disclosure with Affiliated Signals)*** *Fix* $q \in (1/2, 1]$ *and* (c_1, c_2) *and* $\gamma = \widehat{D}\widehat{D}$. *The fair bias* $\delta = c_2/c_1$ *maximizes both the expected total effort and the expected winner's effort.*

Lemma 3 is largely consistent with the observations of Remark 1. With evenly distributed information under $\widehat{D}\widehat{D}$, the designer has no incentive to upset the competitive balance regardless of her objective, so she adopts the fair bias—i.e., setting $\delta = c_2/c_1$—to achieve ex post symmetry. Figures 5(a) and 5(b) depict the respective equilibria under the scoring bias $\delta = c_2/c_1$ when the WM condition is satisfied and is violated.



(a) Nonmonotonic Equilibrium: $\psi_{LH} < \psi_{LL}$.

(b) Monotonic Equilibrium: $\psi_{LH} \geq \psi_{LL}$.

Figure 5: Equilibrium Strategies under Full Disclosure with Affiliated Signals: $c_1 > c_2$ and $\delta = c_2/c_1$.

We now explore the optimal contest when the designer decides on both (i) information favoritism and (ii) scoring bias. Combining Lemmas 1 to 3 yields the following:

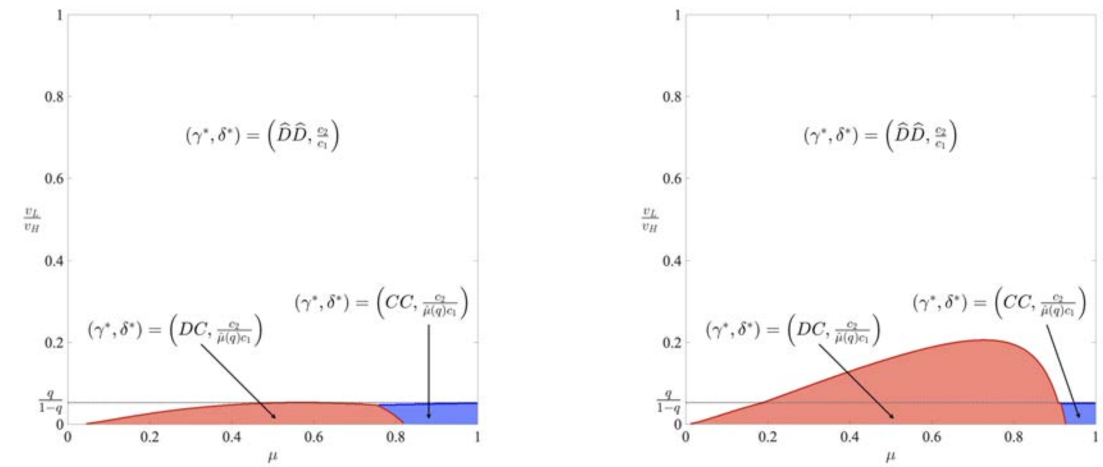
Theorem 3 ***(Optimal Contest with Affiliated Signals)*** *Fix $q \in (1/2, 1]$. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then in the case in which $q < v_H/v_L$, the optimal scheme is $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$; in the case in which $q \geq v_H/(v_H+v_L)$, both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ and $(\gamma^*_{TE}, \delta^*_{TE}) = (\widehat{D}\widehat{D}, c_2/c_1)$ are optimal.

(ii) If the designer aims to maximize the expected winner's effort, then one of the following schemes is optimal: $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, c_2/[\hat{\mu}(q)c_1]\big)$, $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ or $(\gamma^*_{WE}, \delta^*_{WE}) = (\widehat{D}\widehat{D}, c_2/c_1)$.

Theorem 3(i) demonstrates the robustness of Theorem 2(i): The designer always prefers an ex post symmetric contest—i.e., symmetrically distributed information and a fair bias ($\delta = c_2/c_1$)—when she aims to maximize the expected total effort.

In the baseline model, full concealment (CC) and full disclosure (DD) generate the same amount of expected total effort. Such (revenue) equivalence may dissolve in the current case with independent investigation. It is important to note that each player receives a conditionally independent private signal under $\widehat{D}\widehat{D}$, which gives rise to a common-value all-pay auction. The usual *winner's curse* ensues in a monotonic equilibrium and disincentivizes the players. As a result, CC may outperform $\widehat{D}\widehat{D}$ because players are immune to the winner's curse under CC. The dominance of CC requires $q \geq v_H/(v_H + v_L)$, in which case a monotonic equilibrium emerges. Otherwise, however, the equivalence still holds. In this case, the equilibrium is nonmonotonic, in the sense that players' equilibrium strategies have overlapping bidding supports when they receive diverging signals. As a result, a player with



(a) Symmetric Players: $(c_1, c_2, q) = (1, 1, 0.95)$ (b) Asymmetric Players: $(c_1, c_2, q) = (1, 0.1, 0.95)$

Figure 6: Winner's Effort-maximizing Contest Scheme.

a low signal may still win the contest in the case in which his opponent receives a high signal, (see Figure 5(a)), which attenuates the winner's curse.

Theorem 3(ii) demonstrates the robustness of Theorem 2(ii). To maximize the expected winner's effort, the designer may adopt the tilting-and-rebalancing contest scheme; as in Section 3.2, the underdog—i.e., player 1—is awarded information favoritism, while the releveling bias $c_2/[\hat{\mu}(q)c_1]$ is imposed on player 2.

Figures 6(a) and 6(b) illustrate the optimum in the $(\mu, v_L/v_H)$ space for the maximization of the expected winner's effort. Again, it is noteworthy that the two ex post symmetric contests—i.e., contests $\big(CC, c_2/[\hat{\mu}(q)c_1]\big)$ and $\big(\widehat{D}\widehat{D}, c_2/[\hat{\mu}(q)c_1]\big)$—are not equivalent, in contrast to that in the baseline model. As shown in Figures 6(a) and 6(b), $\big(CC, c_2/[\hat{\mu}(q)c_1]\big)$ outperforms $\big(\widehat{D}\widehat{D}, c_2/[\hat{\mu}(q)c_1]\big)$ and emerges in the optimum when $q < v_H/(v_H + v_L)$—or equivalently, $v_L/v_H < q/(1-q)$—and μ is large. The former condition leads to monotonic equilibrium under $\widehat{D}\widehat{D}$, such that the winner's curse looms large. The latter condition ensures that the disincentive caused by the winner's curse is sufficiently strong: A player's ex post belief about the prize value drops more significantly when his opponent receives a low signal if his initial prior is more optimistic—i.e., with a larger μ.

4.2 Endogenous Information Structure

We now let the designer flexibly design and precommit to the information structure of her investigation. She is endowed with full control over the amount of information to be revealed through the investigation and the form of the signal to be disclosed to players. This corresponds to the concept of the Bayesian persuasion approach in the literature pioneered by Kamenica and Gentzkow (2011).

An information structure consists of a signal space $\mathcal{S}$ and a pair of likelihood distributions $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ over $\mathcal{S}$. For instance, the information structure depicted in Section 2 involves a binary signal space $\mathcal{S} = \{H, L\}$ and a conditional likelihood distribution for each underlying state—i.e., v_H or v_L—parameterized by a variable q [see Equation (1)].

More formally, the designer sets the scoring bias $\delta > 0$, the information disclosure policy $\gamma \in \{CC, CD, DC, DD\}$, and the information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ of the signal to be communicated to players, to maximize expected total effort or the expected winner's effort. We can show that it is without loss of generality to consider a binary signal space in our setting, i.e., $\mathcal{S} = \{H, L\}$.[8] Fixing an information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$, denote the expected prize value conditional on s—i.e., $\mathbb{E}(v|s)$—by v_s^π. Without loss of generality, we assume that realization of high signal $s = H$ gives rise to a higher expected prize value, i.e., $v_H^\pi \geq v_L^\pi$. In addition, define $\mu^\pi := \Pr(s = H)$.

The equilibrium characterization in this setting is similar to that in the baseline case. Specifically, the equilibrium characterization, as well as the expressions of the expected total effort and the expected winner's effort in the equilibrium, can be obtained from Propositions 1 and 2 and Lemmas 1 and 2 by replacing $\hat{v}_H(q)$, $\hat{v}_L(q)$, and $\hat{\mu}(q)$ with v_H^π, v_L^π, and μ^π, respectively.

To identify the optimum, we first find the optimal scoring bias δ under given arbitrary information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ and disclosure policy γ. Next, we solve for the optimal information structure holding fixed the disclosure scheme γ and then compare different disclosure schemes to obtain the optimum. It is straightforward to verify that designing the information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ is equivalent to choosing the tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies the following constraint:[9]

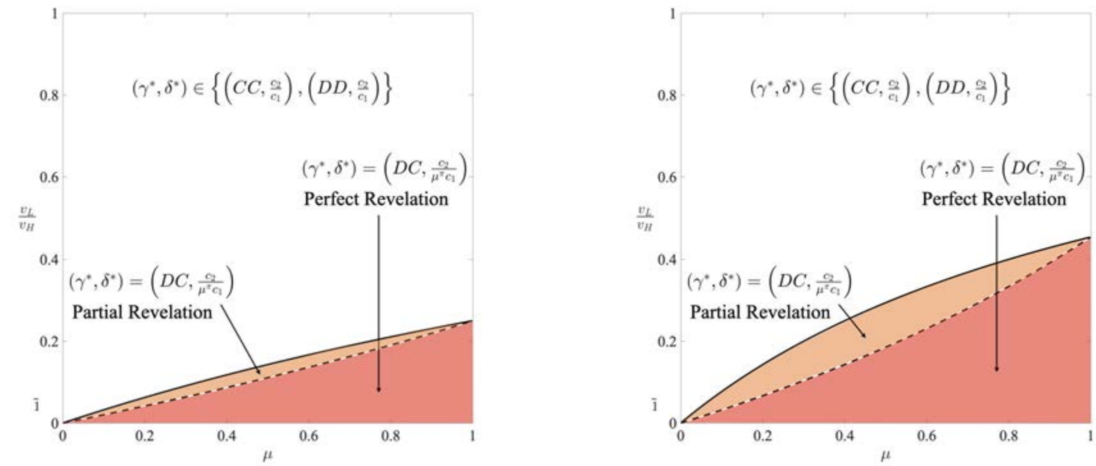
$$v_H \geq v_H^\pi > \bar{v} > v_L^\pi \geq v_L, \text{ and } \mu^\pi v_H^\pi + (1 - \mu^\pi) v_L^\pi = \bar{v}. \tag{6}$$

The above condition is identical to the one in Kamenica and Gentzkow (2011) and is commonly referred to as the Bayes-plausibility constraint. Therefore, to search for the optimal information structure, we simply express the designer's objective—either maximization of the expected total effort or that of the expected winner's effort—as a function of $(v_H^\pi, v_L^\pi, \mu^\pi)$ and optimize over $(v_H^\pi, v_L^\pi, \mu^\pi)$ subject to constraint (6). The following result ensues.

Theorem 4 ***(Optimal Contest with Endogenous Information Structure)*** *Suppose that $c_1 \geq c_2$. Consider the joint design of scoring bias $\delta > 0$, disclosure scheme γ, and*

[8] Details are available from the authors upon request.

[9] We require that $\pi(s|v)$ not be completely uninformative—i.e., $v_H^\pi > v_L^\pi$. If a completely uninformative information structure is desirable—i.e., $v_H^\pi = v_L^\pi$—the designer can simply choose $\gamma = CC$ to conceal the signal from both players.



(a) Symmetric Players: $(c_1, c_2) = (1, 1)$ (b) Asymmetric Players: $(c_1, c_2) = (1, 0.1)$

Figure 7: Winner's Effort-maximizing Contest Scheme with Endogenous Information Structure.

information structure $\left\{\pi(\cdot|v_H), \pi(\cdot|v_L)\right\}$. *The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, c_2/c_1)$ *with any information structure* $\left\{\pi(\cdot|v_H), \pi(\cdot|v_L)\right\}$ *are an optimal contest scheme.*

(ii) If the designer aims to maximize the expected winner's effort, then

(a) in the case in which $\bar{v}/v_L > 2c_2/c_1 + 2$*, the optimal contest scheme consists of* $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DC, c_2/(\mu^\pi c_1)\right)$ *and*

$$\pi\left(H|v_H\right) = 1, \pi\left(H|v_L\right) = \begin{cases} 0, & \text{if } \frac{\bar{v}}{v_L} \geq 4 - 2\mu + 2\frac{c_2}{c_1}, \\ \frac{4-2\mu+2\frac{c_2}{c_1} - \frac{\bar{v}}{v_L}}{2(1-\mu)}, & \text{if } 2\frac{c_2}{c_1} + 2 < \frac{\bar{v}}{v_L} < 4 - 2\mu + 2\frac{c_2}{c_1}; \end{cases}$$

(b) in the case in which $\bar{v}/v_L \leq 2c_2/c_1 + 2$*, both* $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, c_2/c_1)$ *with an arbitrary information structure* $\left\{\pi(\cdot|v_H), \pi(\cdot|v_L)\right\}$ *are an optimal contest scheme.*

Our main predictions in the baseline model are qualitatively robust to endogenizing the information structure. Theorem 4(i) states that when maximizing expected total effort, the optimal contest requires symmetric distribution of information and can accommodate any information structure. The designer does not benefit from the freedom to set the information structure.

By Theorem 4(ii), the designer may, again, resort to a tilting-and-releveling contest when maximizing the expected winner's effort. In contrast to maximization of the expected total effort, the prevailing information structure does affect the resultant performance of the contest when the condition $\bar{v}/v_L > 2c_2/c_1 + 2$ is met, in which case a particular information structure emerges in the optimum. When the ratio $\bar{v}/v_L$ is sufficiently large—i.e., $\bar{v}/v_L \geq 4 - 2\mu + 2c_2/c_1$—the optimum requires that the signal be perfectly informative—i.e., $\pi(H|v_H) = \pi(L|v_L) = 1$ (perfect revelation); as in the baseline model, the weaker contender, player 1, exclusively receives the signal. When the ratio falls in the intermediate range—i.e., $\bar{v}/v_L \in (2c_2/c_1 + 2, 4 - 2\mu + 2c_2/c_1)$—a noisy investigation emerges as the optimum (partial revelation).

Theorem 4(ii) can similarly be interpreted in light of the rationale outlined in the baseline setting. Asymmetric disclosure is optimal when the probability of realizing a high signal is high and a signal triggers substantial revision of expected prize value, which requires large μ^π and v_H^π/v_L^π. Therefore, a closer look at (6) indicates that the designer should perfectly reveal the state v_L—i.e., set $v_L^\pi = v_L$—as predicted in Theorem 4(ii). Further, rearranging the Bayes-plausibility constraint (6) yields

$$\frac{v_H^\pi}{v_L^\pi} = \frac{v_H^\pi}{v_L} = 1 + \frac{\bar{v} - v_L}{v_L} \times \frac{1}{\mu^\pi},$$

which unveils the designer's trade-off of increasing μ^π versus increasing v_H^π/v_L^π. When the term $(\bar{v} - v_L)/v_L$—or the term $\bar{v}/v_L$—is large, an increase in μ^π would lead to a large decrease in v_H^π/v_L^π. This compels the designer to increase v_H^π/v_L^π, which implies a perfectly informative high signal for the optimum—i.e., $v_H^\pi = v_H$—and perfect revelation in the optimum. The trade-off leads to partial revelation for a moderate value of $\bar{v}/v_L$, as in Theorem 4(ii). Figure 7 illustrates the optimum in the $(\mu, v_L/v_H)$ space for maximization of the expected winner's effort.

4.3 Affirmative Action Policy

Next, we extend the model to incorporate an affirmative action policy.[10] Suppose that the two players are heterogeneous, i.e., $c_1 > c_2$. The affirmative action policy is interpreted as a mandate that requires the underdog—i.e., player 1—to win with a probability no less than a certain threshold $\eta \in [0, 1/2]$. The requirement can be interpreted, for instance, as (employment) quotas that aim to maintain diverse and inclusive workforce (Coate and Loury, 1993; Chan and Eyster, 2003; Fryer and Loury, 2013; Bodoh-Creed and Hickman,

[10] For studies of affirmative action and discrimination within tournament/contest settings, see Schotter and Weigelt (1992); Cornell and Welch (1996); Fryer and Loury (2005); Fu (2006); Franke (2012); Bodoh-Creed and Hickman (2018); Estevan, Gall, and Morin (2019); Echenique and Li (2022).

2018).[11] As a result, the contest design is rendered a constrained optimization problem. Our baseline analysis in Section 3 corresponds to the case of $\eta = 0$ such that the constraint does not bind. A closer look at the equilibria characterized in Propositions 1 and 2 yields the following:

Lemma 4 ***(Expected Equilibrium Winning Probabilities)*** *Absent the affirmative action policy, player 1's expected equilibrium winning probability is*

$$WP_1(\delta) = \begin{cases} 1 - \frac{\delta c_1}{2c_2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{c_2}{2\delta c_1}, & \text{if } \delta \geq \frac{c_2}{c_1}, \end{cases}$$

irrespective of the prevailing disclosure policy $\gamma \in \{CC, CD, DC, DD\}$*; player 2's expected equilibrium winning probability is* $WP_2(\delta) = 1 - WP_1(\delta)$.

By Lemma 4, players' ex ante winning probabilities are independent of the disclosure scheme γ in the equilibrium and depend only on the prevailing scoring bias δ. The quota constraint can then be formally expressed as $WP_1(\delta) \geq \eta$. The following result ensues.

Theorem 5 ***(Optimal Contest under an Affirmative Action Policy)*** *Suppose that the affirmative action policy requires that the winning probability of the underdog—i.e., player 1—be no less than* $\eta \in [0, 1/2]$*. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then the optimal contest with affirmative action coincides with the unconstrained optimum. That is, both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, c_2/c_1)$ *are an optimal contest scheme under the affirmative action constraint.*

(ii) If the designer aims to maximize the expected winner's effort, then

(a) in the case in which $\hat{\mu}(q)\hat{v}_H(q) > \big(2\frac{c_1}{c_2} + 2\big)\hat{v}_L(q)$*, the optimal contest scheme is*

$$(\gamma^*_{WE}, \delta^*_{WE}) = \begin{cases} (DC, \frac{c_2}{\hat{\mu}(q)c_1}), & \text{if } 0 \leq \eta \leq \frac{\hat{\mu}(q)}{2}, \\ (DC, \delta^\dagger), & \text{if } \frac{\hat{\mu}(q)}{2} < \eta \leq \frac{c_2}{2\delta^\dagger c_1}, \\ \Big(CD, \frac{\hat{\mu}(q)c_2}{c_1}\Big), & \text{if } \frac{c_2}{2\delta^\dagger c_1} < \eta \leq \frac{1}{2}, \end{cases}$$

where $\delta^\dagger \in \big[\frac{c_2}{c_1}, \frac{c_2}{\hat{\mu}(q)c_1}\big]$ *uniquely solves* $WE\big(DC, \delta, q; c_1, c_2\big) = WE\big(CD, \frac{\hat{\mu}(q)c_2}{c_1}, q; c_1, c_2\big)$*;*

[11] See Fang and Moro (2011) and Onuchic (2022) for comprehensive surveys of theoretical studies of statistical discrimination and affirmative action.

(b) in the case in which $\left(2\frac{c_2}{c_1}+2\right)\hat{v}_L(q) < \hat{\mu}(q)\hat{v}_H(q) \leq \left(2\frac{c_1}{c_2}+2\right)\hat{v}_L(q)$*, the optimal contest scheme is*

$$(\gamma^*_{WE}, \delta^*_{WE}) = \begin{cases} (DC, \frac{c_2}{\hat{\mu}(q)c_1}), & \text{if } 0 \leq \eta \leq \frac{\hat{\mu}(q)}{2}, \\ (DC, \delta^{\ddagger}), & \text{if } \frac{\hat{\mu}(q)}{2} < \eta \leq \frac{c_2}{2\delta^{\ddagger}c_1}, \\ \left(CC, \frac{c_2}{c_1}\right) \text{ or } \left(DD, \frac{c_2}{c_1}\right), & \text{if } \frac{c_2}{2\delta^{\ddagger}c_1} < \eta \leq \frac{1}{2}, \end{cases}$$

where $\delta^{\ddagger} \in \left[\frac{c_2}{c_1}, \frac{c_2}{\hat{\mu}(q)c_1}\right]$ *uniquely solves* $WE\left(DC, \delta, q; c_1, c_2\right) = WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right)$*;*

(c) in the case in which $\hat{\mu}(q)\hat{v}_H(q) \leq \left(2\frac{c_2}{c_1}+2\right)\hat{v}_L(q)$*, both* $(\gamma^*_{WE}, \delta^*_{WE}) = \left(CC, \frac{c_2}{c_1}\right)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DD, \frac{c_2}{c_1}\right)$ *are an optimal contest scheme.*

Theorem 5(i) is intuitive. Absent the affirmative action policy, the ex post symmetric contest—which consists of a fair bias $\delta = c_2/c_1$ and a symmetric disclosure scheme $\gamma \in \{CC, DD\}$—maximizes expected total effort. The contest perfectly levels the playing field in the sense that players have equal chance to win (see Lemma 4). Imposing an affirmative action policy does not affect the resultant optimum because the constraint $WP_1(\delta) \geq \eta$ does not bind.

Theorem 5(ii) shows that a tilting-and-releveling contest may still maximize the expected winner's effort. In case (a) of Theorem 5(ii), the optimum involves a disclosure scheme $\gamma = DC$ and the favorite, player 2, receives the signal privately. The rationale laid out in Section 3.2 continues to shed light on the trade-off in this alternative setting between a tilting-and-releveling contest and an ex post symmetric contest. Theorem 5 delineates conditions for the optimality of tilting-and-releveling contests similar to Theorem 2.

However, it is noteworthy that the opposite may emerge in the optimum when an affirmative action policy is imposed. When the condition $\hat{\mu}(q)\hat{v}_H(q) \geq \left(2c_1/c_2+2\right)\hat{v}_L(q)$ is met, the constrained optimum can be achieved by a tilting-and-releveling contest $\left(CD, [\hat{\mu}(q)c_2]/c_1\right)$.

To see the logic, note that the contest $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$ may violate the affirmative action quota constraint, whereas $\left(CD, [\hat{\mu}(q)c_2]/c_1\right)$ does not. By Lemma 4, player 1 wins with a probability of $\hat{\mu}(q)/2 < 1/2$ under the former and with a probability of $1 - \hat{\mu}(q)/2 > 1/2$ under the latter. When η is small—i.e., $\eta \leq \hat{\mu}(q)/2$—the quota constraint does not bind in the contest $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$, so it continues to be optimal, as predicted in Theorem 2. When η is in an intermediate range, the designer is required to lower the scoring bias from $c_2/[\hat{\mu}(q)c_1]$ to $\delta^{\dagger}$, which squeezes the favorite to satisfy the constraint. A large η, however, would cripple the reoptimization under $\gamma = DC$. The designer will turn to the quota-proof disclosure scheme CD and award information favoritism to the favorite instead.

5 Concluding Remarks

This paper studies the optimal design of a contest in which two potentially asymmetric players compete for a common-valued prize. We allow the contest designer to choose a combination of two instruments—an information disclosure scheme and a scoring bias—to advance her interests. We show that the optimum depends on the design objective. When she aims to maximize expected total effort, the optimum embraces the conventional wisdom of leveling the playing field: Information is symmetrically distributed, and a fair scoring bias offsets the initial asymmetry between players. However, the contest that maximizes the expected winner's effort can create bilateral asymmetry: The designer discloses an informative signal to one player privately, while compensating the other by a relatively favorable scoring rule, which we call a *tilting-and-releveling* contest. She may do so even if players are ex ante symmetric. We further show that the results are qualitatively robust to extensions to (i) affiliated signals and (ii) endogenous information structure. Finally, we show that information favoritism can play a useful role in addressing affirmative action policy objectives.

Our paper is one of the first in the contest literature to examine the optimal combination of multiple design instruments. It is noteworthy that the two instruments demonstrate complementarity, in that the optimal contest requires either ex post full symmetry or bilateral asymmetry. Our results generate novel implications for contest design. Optimal bilateral asymmetry would not arise in the conventional settings of unidimensional contest design and sheds fresh light on the debate regarding the relationship between (a) symmetry and the performance of a contest.

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Appendix A Equilibrium Characterization Under Full Disclosure with Affiliated Signals

is	d_0^i	d_a^i	d_b^i	d_c^i	ℓ_a	ℓ_b	ℓ_c
Case 1(a): $\psi_{LH} < \psi_{LL}$ and $\frac{\delta c_1}{c_2} \leq 1$							
$1H$	0	0	$\frac{c_2(\psi_{LL}-\psi_{LH})}{\delta(\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH})}$	$\frac{c_2}{\delta\psi_{HH}}$	0	$\frac{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}{c_2(\psi_{HH}-\psi_{HL})}$	$\frac{\psi_{HH}}{c_2} \times \frac{\psi_{HH}+\psi_{LH}-\psi_{HL}-\psi_{LL}}{\psi_{HH}-\psi_{HL}}$
$1L$	0	0	$\frac{c_2(\psi_{HH}-\psi_{HL})}{\delta(\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH})}$	0			
$2H$	$1-\frac{\delta c_1}{c_2}$	0	$\frac{\delta c_1(\psi_{LL}-\psi_{LH})}{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}$	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	$1-\frac{\delta c_1}{c_2}$	0	$\frac{\delta c_1(\psi_{HH}-\psi_{HL})}{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}$	0			
Case 1(b): $\psi_{LH} < \psi_{LL}$ and $\frac{\delta c_1}{c_2} > 1$							
$1H$	$1-\frac{c_2}{\delta c_1}$	0	$\frac{c_2(\psi_{LL}-\psi_{LH})}{\delta(\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH})}$	$\frac{c_2}{\delta\psi_{HH}}$	0	$\frac{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}{\delta c_1(\psi_{HH}-\psi_{HL})}$	$\frac{\psi_{HH}}{\delta c_1} \times \frac{\psi_{HH}+\psi_{LH}-\psi_{HL}-\psi_{LL}}{\psi_{HH}-\psi_{HL}}$
$1L$	$1-\frac{c_2}{\delta c_1}$	0	$\frac{c_2(\psi_{HH}-\psi_{HL})}{\delta(\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH})}$	0			
$2H$	0	0	$\frac{\delta c_1(\psi_{LL}-\psi_{LH})}{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}$	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	0	0	$\frac{\delta c_1(\psi_{HH}-\psi_{HL})}{\psi_{HH}\psi_{LL}-\psi_{HL}\psi_{LH}}$	0			

Table A1: Nonmonotonic Equilibrium: $\psi_{LH} < \psi_{LL}$.

is	d_0^i	d_a^i	d_b^i	d_c^i	ℓ_a	ℓ_b	ℓ_c
Case 2(a): $\psi_{LH} \geq \psi_{LL}$ and $\frac{\delta c_1}{c_2} \leq \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}$							
$1H$	0	0	0	$\frac{c_2}{\delta\psi_{HH}}$	0	$\frac{\psi_{LH}}{c_2}$	$\frac{\psi_{HH}}{c_2}$
$1L$	0	0	$\frac{c_2}{\delta\psi_{LH}}$	0			
$2H$	$1-\frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}\frac{\delta c_1}{c_2}$	0	$\frac{\delta c_1}{\psi_{HL}}$	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	1	0	0	0			
Case 2(b): $\psi_{LH} \geq \psi_{LL}$ and $1 \geq \frac{\delta c_1}{c_2} > \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}$							
$1H$	0	0	0	$\frac{c_2}{\delta\psi_{HH}}$	$\frac{\psi_{LL}}{c_2}\left(1-\frac{c_2-\delta c_1}{\delta c_1}\frac{\psi_{HL}}{\psi_{LH}}\right)$	$\frac{\psi_{HL}}{\delta c_1}\left(1-\frac{\delta c_1}{c_2}\right)$	$\frac{\psi_{HH}}{c_2}$
$1L$	0	$\frac{c_2}{\delta\psi_{LL}}$	$\frac{c_2}{\delta\psi_{LH}}$	0			
$2H$	0	0	$\frac{\delta c_1}{\psi_{HL}}$	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	$\left(1-\frac{\delta c_1}{c_2}\right)\frac{\psi_{HL}+\psi_{LH}}{\psi_{LH}}$	$\frac{\delta c_1}{\psi_{LL}}$	0	0			
Case 2(c): $\psi_{LH} \geq \psi_{LL}$ and $\frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}} \geq \frac{\delta c_1}{c_2} > 1$							
$1H$	0	0	$\frac{c_2}{\delta\psi_{HL}}$	$\frac{c_2}{\delta\psi_{HH}}$	$\frac{\psi_{LL}}{\delta c_1}\left(1-\frac{\delta c_1-c_2}{c_2}\frac{\psi_{HL}}{\psi_{LH}}\right)$	$\frac{\psi_{HL}}{c_2}\left(1-\frac{c_2}{\delta c_1}\right)$	$\frac{\psi_{HH}}{\delta c_1}$
$1L$	$\frac{\delta c_1-c_2}{\delta c_1}\frac{\psi_{HL}+\psi_{LH}}{\psi_{LH}}$	$\frac{c_2}{\delta\psi_{LL}}$	0	0			
$2H$	0	0	0	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	0	$\frac{\delta c_1}{\psi_{LL}}$	$\frac{\delta c_1}{\psi_{LH}}$	0			
Case 2(d): $\psi_{LH} \geq \psi_{LL}$ and $\frac{\delta c_1}{c_2} > \frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}$							
$1H$	$1-\frac{c_2}{\delta c_1}\frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}$	0	$\frac{c_2}{\delta\psi_{HL}}$	$\frac{c_2}{\delta\psi_{HH}}$	0	$\frac{\psi_{LH}}{\delta c_1}$	$\frac{\psi_{HH}}{\delta c_1}$
$1L$	1	0	0	0			
$2H$	0	0	0	$\frac{\delta c_1}{\psi_{HH}}$			
$2L$	0	0	$\frac{\delta c_1}{\psi_{LH}}$	0			

Table A2: Monotonic Equilibrium: $\psi_{LH} \geq \psi_{LL}$.

Appendix B Proofs

Proofs of Propositions 1 and 2 and Lemmas 1 and 2

Proof. It can be verified that the strategy profiles provided in Propositions 1 and 2 constitute an equilibrium under $\gamma \in \{CC, DD\}$ and $\gamma \in \{DC, CD\}$, respectively. The equilibrium uniqueness in Proposition 1 follows from Hillman and Riley (1989) and Baye, Kovenock, and De Vries (1996), and that in Proposition 2 follows from Siegel (2014). Lemmas 1 and 2 follow immediately the equilibrium characterizations in Propositions 1 and 2. ■

Proof of Theorem 1

Proof. See the proof of Theorem 2. ■

Proof of Theorem 2

Proof. We first prove part (i) of the theorem. From (2), it is straightforward to verify that $\delta = \frac{c_2}{c_1}$ maximizes $TE(CC, \delta; c_1, c_2)$ and $TE(DD, \delta; c_1, c_2)$, and the maximum expected total effort is $\frac{(c_1+c_2)\bar{v}}{2c_1c_2}$. Similarly, from (3), it can be verified that either $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$ maximizes $TE(DC, \delta, q; c_1, c_2)$. Moreover, we have that

$$\begin{aligned} TE\left(DC, \frac{c_2}{c_1}, q; c_1, c_2\right) &= \frac{(c_1+c_2)\left\{\hat{\mu}^2(q)\hat{v}_H(q) + \left[1-\hat{\mu}^2(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right), \end{aligned}$$

and

$$\begin{aligned} TE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}, q; c_1, c_2\right) &= \frac{(c_1+c_2)\hat{\mu}(q)\hat{v}_H(q)}{2c_1c_2} \\ &< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right). \end{aligned}$$

Therefore, choosing $\gamma \in \{CC, DD\}$ with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort to the designer than choosing $\gamma = DC$ with any $\delta > 0$. Recall $TE(DC, \delta, q; c_1, c_2) = TE(CD, 1/\delta, q; c_2, c_1)$ from Lemma 1. This immediately implies that choosing $\gamma \in \{CC, DD\}$

with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort for the designer than choosing $\gamma = CD$ with any $\delta > 0$.

Next, we prove part (ii). It is useful to prove an intermediate result.

Lemma 5 *Fix $q \in (1/2, 1]$. $WE(DC, \delta, q; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$.*

Proof. Fix $u \in (0,1)$ and $z \in \mathbb{R}_{++}$. First, for $d \in (0,1)$, we have that

$$\frac{\partial \mathcal{W}_1(u,z,d;c_1,c_2)}{\partial d} = u^2 z \left[(3-u)c_1 + (3-2u)c_2\right] + (2c_1 + c_2) + 2\left(c_2 u^3 z + c_2\right)(1-d) > 0.$$

Therefore, $\mathcal{W}_1(u,z,d;c_1,c_2)$ is increasing in d for $d \in (0,1)$.

Next, we show that $\mathcal{W}_2(u,z,d;c_1,c_2)$, with $d \in [1, 1/\mu]$, is maximized at $d = 1$ or $d = 1/u$. Simple algebra would verify that

$$\frac{\partial \mathcal{W}_2(u,z,d;c_1,c_2)}{\partial d} = \frac{\left[zu^2\mathcal{W}_4(u,d;c_1,c_2) - 1\right](3c_1 d + 2c_2 d - 2c_1)}{d^3},$$

where $\mathcal{W}_4(u,d;c_1,c_2) := \frac{3(c_1+c_2) - u(c_1+2c_2 d)}{3c_1 d + 2c_2 d - 2c_1} d^3$. Note that

$$\frac{\partial \mathcal{W}_4(u,d;c_1,c_2)}{\partial d} = \frac{6d^2\mathcal{W}_5(u,d;c_1,c_2)}{\left[c_1(3d-2) + 2c_2 d\right]^2},$$

where $\mathcal{W}_5(u,d;c_1,c_2) := -c_2 u(3c_1 + 2c_2)d^2 + \left[c_1^2(3-u) + c_1 c_2(2u+5) + 2c_2^2\right] d + c_1\left[c_1 u - 3(c_1+c_2)\right]$. Note that $\mathcal{W}_5(u,d;c_1,c_2)$ is concave in d, which implies that

$$\mathcal{W}_5(u,d;c_1,c_2) \geq \min\left\{\mathcal{W}_5(u,1;c_1,c_2), \mathcal{W}_5(u,1/u;c_1,c_2)\right\}, \text{ for } d \in [1, 1/\mu];$$

together with $\mathcal{W}_5(u,1;c_1,c_2) = 2c_2(c_1+c_2) - c_2 u(c_1 + 2c_2) > 0$ and $\mathcal{W}_5(u,1/u;c_1,c_2) = \frac{c_1(c_1(3-u)(1-u) + c_2(2-u))}{u} > 0$, we can conclude that $\mathcal{W}_5(u,d;c_1,c_2) > 0$. As a result, $\frac{\partial \mathcal{W}_4(u,d;c_1,c_2)}{\partial d} > 0$ and thus $\mathcal{W}_4(u,d;c_1,c_2)$ is increasing in d for $d \in [1, 1/u]$, which in turn implies that

$$\frac{\partial \mathcal{W}_2(u,z,d;c_1,c_2)}{\partial d} \gtreqless 0 \Leftrightarrow zu^2\mathcal{W}_4(u,d;c_1,c_2) \gtreqless 1.$$

Therefore, $\mathcal{W}_2(u,z,d;c_1,c_2)$ is either monotonic or U-shaped in $d \in [1, 1/u]$. This implies that $\mathcal{W}_2(u,z,d;c_1,c_2)$ is maximized at $d = 1$ or $d = 1/u$.

Finally, for $d > 1$, we have that

$$\frac{\partial \mathcal{W}_3(d;c_1,c_2)}{\partial d} = -\frac{3c_1 d - 2c_1 + 2c_2 d}{d^3} < 0,$$

which implies that $\mathcal{W}_3(d;c_1,c_2)$ is decreasing in d for $d > 1$.

In summary, (i) $\mathcal{W}_1(u,z,d;c_1,c_2)$ is increasing in d for $d \in (0,1)$; (ii) $\mathcal{W}_2(u,z,d;c_1,c_2)$ is maximized at $d=1$ or $d=1/u$; and (iii) $\mathcal{W}_3(d;c_1,c_2)$ is decreasing in d for $d>1$. All together, these facts imply that $WE(DC,\delta,q;c_1,c_2)$ is maximized at $\delta=\frac{c_2}{c_1}$ or $\delta=\frac{c_2}{\hat{\mu}(q)c_1}$, which concludes the proof. ■

For $\gamma \in \{CC, DD\}$, we have that

$$\frac{\partial WE(CC,\delta;c_1,c_2)}{\partial\delta} = \frac{\partial WE(DD,\delta;c_1,c_2)}{\partial\delta} = \begin{cases} \frac{(3c_2-2c_1\delta+2c_1)\bar{v}}{6c_2^2} > 0, & \text{if } \delta < \frac{c_2}{c_1}; \\ -\frac{(3c_1\delta-2c_2+2c_2\delta)\bar{v}}{6c_1^2\delta^3} < 0, & \text{if } \delta \geq \frac{c_2}{c_1}. \end{cases}$$

Therefore, $WE(CC,\delta;c_1,c_2)$ and $WE(DD,\delta;c_1,c_2)$ are both maximized at $\delta=\frac{c_2}{c_1}$. The maximum expected winner's effort is $\frac{(c_1+c_2)\bar{v}}{3c_1c_2}$.

Further, fixing $q \in (1/2,1]$, it follows from Lemma 5 that $WE(DC,\delta,q;c_1,c_2)$ is maximized at $\delta=\frac{c_2}{c_1}$ or $\delta=\frac{c_2}{\hat{\mu}(q)c_1}$. Carrying out the algebra, we can obtain that

$$\begin{aligned} WE\left(DC,\frac{c_2}{c_1},q;c_1,c_2\right) &= \frac{(c_1+c_2)\left\{2\bar{v}-\left[2-\hat{\mu}(q)\right]\left[1-\hat{\mu}(q)\right]\hat{\mu}(q)\left[\hat{v}_H(q)-\hat{v}_L(q)\right]\right\}}{6c_1c_2} \\ &< \frac{(c_1+c_2)\bar{v}}{3c_1c_2} = WE\left(CC,\frac{c_2}{c_1};c_1,c_2\right), \end{aligned}$$

and

$$WE\left(DC,\frac{c_2}{\hat{\mu}(q)c_1},q;c_1,c_2\right) = \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2+c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2}.$$

Further, by Lemma 2, we have that $WE(CD,\delta,q;c_1,c_2) = WE(DC,1/\delta,q;c_2,c_1)$; together with the above analysis, we can conclude that $WE(CD,\delta,q;c_1,c_2)$ is maximized at $\delta=\frac{c_2}{c_1}$ or $\delta=\frac{\hat{\mu}(q)c_2}{c_1}$. Moreover, we have that

$$WE\left(CD,\frac{c_2}{c_1},q;c_1,c_2\right) = WE\left(DC,\frac{c_1}{c_2},q;c_2,c_1\right) = WE\left(DC,\frac{c_2}{c_1},q;c_1,c_2\right) < WE\left(CC,\frac{c_2}{c_1};c_1,c_2\right),$$

and

$$\begin{aligned} WE\left(CD,\frac{\hat{\mu}(q)c_2}{c_1},q;c_1,c_2\right) &= \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_1+c_2\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} \\ &< \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2+c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} = WE\left(DC,\frac{c_2}{\hat{\mu}(q)c_1},q;c_1,c_2\right), \end{aligned}$$

where the strict inequality follows from $c_1 \geq c_2$ and $3 - \hat{\mu}(q) > 2$. As a result, $\gamma = CD$ would not arise in the optimum.

In summary, fixing $q \in (1/2, 1]$, the expected winner's effort from the contest is maximized by either $(\gamma, \delta) = \left(CC \text{ or } DD, \frac{c_2}{c_1}\right)$ or $(\gamma, \delta) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$. Carrying out the algebra, we have that

$$WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) - WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}, q; c_1, c_2\right) = \frac{\left[1 - \hat{\mu}(q)\right] \times \left[2(c_1 + c_2)\hat{v}_L(q) - c_1\hat{\mu}(q)\hat{v}_H(q)\right]}{6c_1c_2}.$$

It can be verified that $WE\big(CC, \frac{c_2}{c_1}; c_1, c_2\big) > WE\big(DC, \frac{c_2}{\hat{\mu}(q)c_1}, q; c_1, c_2\big)$ is equivalent to $\hat{\mu}(q)\hat{v}_H(q) < \big(2\frac{c_2}{c_1} + 2\big)\hat{v}_L(q)$, which concludes the proof. ■

Proofs of Proposition 3

Proof. It can be verified that the strategy profile provided in Proposition 3 constitutes an equilibrium under $\gamma = \widehat{D}\widehat{D}$. The equilibrium uniqueness for the case of $\psi_{LH} < \psi_{LL}$ follows from Rentschler and Turocy (2016) and that for the case of $\psi_{LH} \geq \psi_{LL}$ follows from Siegel (2014). ■

Proof of Lemma 3

Proof. From the equilibrium characterization in Proposition 3, we can derive the expected total effort and the expected winner's effort with $\gamma = \widehat{D}\widehat{D}$—which we denote by $TE^{\sharp}(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$ and $WE^{\sharp}(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$, respectively—as the following:

$$TE^{\sharp}(\widehat{D}\widehat{D}, \delta, q; c_1, c_2) = \begin{cases} TE_{1(a)}, & \text{if } q > \frac{v_H}{v_H + v_L} \text{ and } \delta \leq \frac{c_2}{c_1}, \\ TE_{1(b)}, & \text{if } q > \frac{v_H}{v_H + v_L} \text{ and } \delta > \frac{c_2}{c_1}, \\ TE_{2(a)}, & \text{if } q \leq \frac{v_H}{v_H + v_L} \text{ and } \delta \leq \frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL} + \psi_{LH}}, \\ TE_{2(b)}, & \text{if } q \leq \frac{v_H}{v_H + v_L} \text{ and } \frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL} + \psi_{LH}} < \delta \leq \frac{c_2}{c_1}, \\ TE_{2(c)}, & \text{if } q \leq \frac{v_H}{v_H + v_L} \text{ and } \frac{c_2}{c_1} < \delta \leq \frac{c_2}{c_1} \times \frac{\psi_{HL} + \psi_{LH}}{\psi_{HL}}, \\ TE_{2(d)}, & \text{if } q \leq \frac{v_H}{v_H + v_L} \text{ and } \delta > \frac{c_2}{c_1} \times \frac{\psi_{HL} + \psi_{LH}}{\psi_{HL}}, \end{cases} \tag{7}$$

and

$$WE^{\sharp}(\widehat{D}\widehat{D},\delta,q;c_1,c_2)=\begin{cases}WE_{1(a)}, & \text{if } q>\frac{v_H}{v_H+v_L} \text{ and } \delta\le\frac{c_2}{c_1},\\ WE_{1(b)}, & \text{if } q>\frac{v_H}{v_H+v_L} \text{ and } \delta>\frac{c_2}{c_1},\\ WE_{2(a)}, & \text{if } q\le\frac{v_H}{v_H+v_L} \text{ and } \delta\le\frac{c_2}{c_1}\times\frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}},\\ WE_{2(b)}, & \text{if } q\le\frac{v_H}{v_H+v_L} \text{ and } \frac{c_2}{c_1}\times\frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}<\delta\le\frac{c_2}{c_1},\\ WE_{2(c)}, & \text{if } q\le\frac{v_H}{v_H+v_L} \text{ and } \frac{c_2}{c_1}<\delta\le\frac{c_2}{c_1}\times\frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}},\\ WE_{2(d)}, & \text{if } q\le\frac{v_H}{v_H+v_L} \text{ and } \delta>\frac{c_2}{c_1}\times\frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}},\end{cases} \tag{8}$$

where

$$TE_{1(a)}:=\frac{\delta\bar{v}(c_1+c_2)}{2c_2^2},$$

$$TE_{1(b)}:=\frac{\bar{v}(c_1+c_2)}{2\delta c_1^2},$$

$$TE_{2(a)}:=\frac{\delta(c_1+c_2)\left\{\mu q v_H\left[\mu q+(2-\mu)(1-q)\right]+(1-\mu)(1-q)v_L(1-\mu+2\mu q)\right\}}{2c_2^2\left[\mu q+(1-\mu)(1-q)\right]},$$

$$TE_{2(b)}:=\frac{(c_1+c_2)\left(c_1^2\delta^2\mathcal{D}+c_2^2\mathcal{E}\right)}{2c_1^2c_2^2\delta(-2\mu q+q+\mu)^2}, \text{ with}$$

$$\mathcal{D}:=\mu v_H\left[1-q(2-2q+2\mu q-\mu)(1+q+\mu-2\mu q)\right] + (1-\mu)v_L\left[q^2-(1-q)(2q-1)\mu(q+\mu-2\mu q)\right],$$

$$\mathcal{E}:=(2q-1)(1-\mu)\mu(2\mu q-q-\mu+1)\left[(1-q)v_H-qv_L\right],$$

$$TE_{2(c)}:=\frac{(c_1+c_2)\left(c_1^2\delta^2\mathcal{E}+c_2^2\mathcal{D}\right)}{2c_1^2c_2^2\delta\left(-2\mu q+q+\mu\right)^2},$$

$$TE_{2(d)}:=\frac{(c_1+c_2)\left\{\mu q v_H\left[\mu q+(2-\mu)(1-q)\right]+(1-\mu)(1-q)v_L(1-\mu+2\mu q)\right\}}{2\delta c_1^2\left[\mu q+(1-\mu)(1-q)\right]};$$

$WE_{1(a)}$, $WE_{1(b)}$, $WE_{2(a)}$, $WE_{2(b)}$, $WE_{2(c)}$, and $WE_{2(d)}$ will be provided later in the proof.

Expected Total Effort Maximization. The analysis is trivial for the case of $q>\frac{v_H}{v_H+v_L}$ from the expressions of $TE_{1(a)}$ and $TE_{1(b)}$, and it remains to consider the case of $q\le\frac{v_H}{v_H+v_L}$.

It is straightforward to verify that $TE_{2(a)}$ increases with δ. Next, recall $TE_{2(b)}=\frac{(c_1+c_2)(c_1^2\delta^2\mathcal{D}+c_2^2\mathcal{E})}{2c_1^2c_2^2\delta(-2\mu q+q+\mu)^2}$. It can be verified that $\mathcal{D}>0$ for all $(q,\mu)\in(1/2,1]\times(0,1)$, which implies that $TE_{2(b)}$ is either increasing in δ ($\mathcal{E}\ge0$) or U-shaped in δ ($\mathcal{E}<0$). Further, we can show that $TE_{2(b)}\big(\delta=\frac{c_2}{c_1}\big)>TE_{2(b)}\big(\delta=\frac{c_2}{c_1}\times\frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}\big)$. These facts together imply that $\delta=\frac{c_2}{c_1}$ maximizes $TE^{\sharp}(\widehat{D}\widehat{D},\delta,q;c_1,c_2)$ for $\delta\in\big(0,\frac{c_2}{c_1}\big]$.

Next, note that fixing $\tilde{\delta} \in (0,\infty)$, $TE^{\sharp}\big(\widehat{D}\widehat{D}, \frac{c_2}{c_1}\tilde{\delta}, q; c_1, c_2\big) = TE^{\sharp}\big(\widehat{D}\widehat{D}, \frac{c_2}{c_1}\frac{1}{\tilde{\delta}}, q; c_1, c_2\big)$. Therefore, $\delta = \frac{c_2}{c_1}$ maximizes $TE^{\sharp}(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$ in (7) for $\delta \in \big(0,\infty\big)$.

Expected Winner's Effort Maximization. We consider the following two cases depending on q relative to $\frac{v_H}{v_H+v_L}$.

Case I: $q > \frac{v_H}{v_H+v_L}$. $WE_{1(a)}$ in expression (8) is given by

$$WE_{1(a)} = \frac{\delta\left(c_1\delta\mathcal{J} + \mathcal{K}\right)}{6c_2^2\left(1-q-\mu+2\mu q\right)\big[qv_H-(1-q)v_L\big]^2},$$

where

$$\begin{aligned}
\mathcal{J} :=& -\bigg\{(1-q-\mu+2\mu q)\Big[(2q-1)^2v_L^3 + (2q-1)\big[2q+\mu(2q-1)\big]v_L^2(v_H-v_L) \\
&+ q\big[q+2\mu(2q-1)\big]v_L(v_H-v_L)^2\Big] + \mu q\left[(1-q)^2+(2q-1)\mu\right](v_H-v_L)^3\bigg\}, \\
\mathcal{K} :=& c_1\mathcal{K}_1 + c_2\mathcal{K}_2, \text{ with} \\
\mathcal{K}_1 :=& -qv_H^2v_L\Big[\left(2q^2+q-1\right)\mu^2 + (5-8q)\mu q + 2(q-1)q + \mu\Big] \\
&+ (q-1)v_Hv_L^2\Big\{2q^2\left[\mu(\mu+2)-2\right] - q\left(5\mu^2+\mu-4\right) + 2(\mu-1)\mu\Big\} \\
&+ \mu q v_H^3\Big\{q\left[q(6\mu-4)-5\mu+5\right]+\mu-1\Big\} \\
&- (q-1)(\mu-1)v_L^3\Big\{\left[q(6q-7)+2\right]\mu - 2(q-1)^2\Big\}, \\
\mathcal{K}_2 :=& 3\left[q(2\mu-1)-\mu+1\right]\big[v_L-q(v_H+v_L)\big]^2\left[\mu(v_H-v_L)+v_L\right].
\end{aligned}$$

We first show that $WE_{1(a)}$ is increasing in δ for $\delta \le \frac{c_2}{c_1}$. Note that $WE_{1(a)}$ is quadratic in δ and it can be verified that $\mathcal{J} < 0$. Therefore, it suffices to show that $\frac{\mathcal{K}}{-2c_1\mathcal{J}} > \frac{c_2}{c_1}$, which is equivalent to $2c_2\mathcal{J} + \mathcal{K} > 0$. Carrying out the algebra, we have that

$$\begin{aligned}
2c_2\mathcal{J}+\mathcal{K} =& (2c_1+c_2)(1-q-\mu+2\mu q)v_L \times \left\{\begin{array}{c} (2q-1)^2v_L^2 \\ +(2q-1)\big[2q+\mu(2q-1)\big]v_L(v_H-v_L) \\ +q\big[q+2\mu(2q-1)\big](v_H-v_L)^2 \end{array}\right\} \\
&+ \mu q(v_H-v_L)^3\left(c_1\mathcal{R}_1 + c_2\mathcal{R}_2\right),
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{R}_1 :=& q\big[q(6\mu-4)-5\mu+5\big]+\mu-1, \\
\mathcal{R}_2 :=& q\big[q(6\mu-5)-7\mu+7\big]+2(\mu-1).
\end{aligned}$$

It can be verified that $\mathcal{R}_1 > 0$ and $\mathcal{R}_2 > 0$, from which we can conclude that $2c_2\mathcal{J}+\mathcal{K} > 0$ and thus $WE_{1(a)}$ is increasing in δ for $0 < \delta < \frac{c_2}{c_1}$.

$WE_{1(b)}$ in expression (8) is given by

$$WE_{1(b)} = \frac{c_2\mathcal{J} + \delta\left(c_1\mathcal{K}_2 + c_2\mathcal{K}_1\right)}{6c_1^2\delta^2\left(1-q-\mu+2\mu q\right)\left[qv_H - (1-q)v_L\right]^2}.$$

Next, we show that $WE_{1(b)}$ is decreasing in δ for $\frac{\delta c_1}{c_2} > 1$. For notational convenience, let $\hat{\delta} := \frac{\delta c_1}{c_2}$. We can rewrite $WE_{1(a)}$ and $WE_{1(b)}$ as the following:

$$WE_{1(a)} = \mathcal{S}\left(c_1, c_2, \hat{\delta}\right) := \frac{\hat{\delta}\left(c_2\hat{\delta}\mathcal{J} + \mathcal{K}_1 c_1 + \mathcal{K}_2 c_2\right)}{6c_1c_2\left(1-q-\mu+2\mu q\right)\left[qv_H - (1-q)v_L\right]^2}, \tag{9}$$

$$WE_{1(b)} = \frac{c_1\mathcal{J} + \hat{\delta}\left(c_1\mathcal{K}_2 + c_2\mathcal{K}_1\right)}{6c_1c_2\hat{\delta}^2\left(1-q-\mu+2\mu q\right)\left[qv_H - (1-q)v_L\right]^2} = \mathcal{S}\left(c_2, c_1, 1/\hat{\delta}\right). \tag{10}$$

We have shown that $\mathcal{S}(c_1, c_2, \hat{\delta})$ increases with $\hat{\delta} \in (0,1)$; together with (9) and (10), we can conclude that $\mathcal{S}(c_2, c_1, 1/\hat{\delta})$ increases in $1/\hat{\delta}$ for $1/\hat{\delta} \in (0,1)$, which in turn implies that $WE_{1(b)}$ decreases in $\hat{\delta} \in (1,\infty)$.

In summary, $WE^{\sharp}(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ for $q > \frac{v_H}{v_H+v_L}$.

Case II: $\boldsymbol{q \leq \frac{v_H}{v_H+v_L}}$. It suffices to show that (i) $WE_{2(a)}$ is strictly increasing in δ for $\delta \in \left(0, \frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}\right]$; (ii) $WE_{2(b)}$ is maximized at $\delta = \frac{c_2}{c_1}$ for $\delta \in \left(\frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}, \frac{c_2}{c_1}\right]$; (iii) $WE_{2(c)}$ is maximized at $\delta = \frac{c_2}{c_1}$ for $\delta \in \left[\frac{c_2}{c_1}, \frac{c_2}{c_1} \times \frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}\right]$; and (iv) $WE_{2(d)}$ strictly decreases with δ for $\delta \in \left(\frac{c_2}{c_1} \times \frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}, \infty\right)$.

$WE_{2(a)}$ in expression (8) is given by

$$WE_{2(a)} = \frac{\delta\left[\mathcal{F}v_L + \mathcal{G}v_H + c_1\delta(\mathcal{M}_1 v_L + \mathcal{M}_2 v_H)\right]}{6(c_2)^2\left(1-\mu-q+2\mu q\right)^2},$$

where

$$\mathcal{F} := \mu q \times \left\{\begin{array}{c} (2q-1)(q-1)\mu\left[c_1(q-6) - 9c_2\right] \\ -(\mu-2\mu q)^2\left[c_1 q - 3(c_1+c_2)\right] + (q-1)^2\left[c_1(2q+3) + 6c_2\right] \end{array}\right\},$$

$$\mathcal{G} := (1-\mu)(1-q) \times \left\{\begin{array}{c} -(2q-1)\mu\left[c_1(3q^2+q-4) + 3c_2(q-2)\right] \\ +(\mu-2\mu q)^2\left[c_1(q+2) + 3c_2\right] - (q-1)(2c_1+3c_2) \end{array}\right\},$$

$$\mathcal{M}_1 := \mu q\left\{(1-2q)^2(2q-3)\mu^2 + q\left[(27-10q)q - 23\right]\mu + q\left[q(2q-7)+8\right] + 6\mu - 3\right\},$$

$$\mathcal{M}_2 := -(1-q)(1-\mu)\left\{(1-2q)^2(2q+1)\mu^2 + q\left[(5-6q)q+3\right]\mu - q - 2\mu + 1\right\}.$$

We first show that $WE_{2(a)}$ is increasing in δ for $\delta \leq \frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}$. Note that $WE_{2(a)}$ is quadratic in δ and it can be verified that $\mathcal{M}_1 < 0$ and $\mathcal{M}_2 < 0$. Therefore, it suffices to show that

$$-\frac{\mathcal{F}v_L + \mathcal{G}v_H}{2c_1(\mathcal{M}_1 v_L + \mathcal{M}_2 v_H)} > \frac{c_2}{c_1},$$

which is equivalent to

$$\begin{aligned}\mathcal{H}(v_L, v_H) :&= 2c_2\left(\mathcal{M}_1 v_L + \mathcal{M}_2 v_H\right) + \mathcal{F}v_L + \mathcal{G}v_H \\ &= \mu q v_H\left(\mathcal{M}_3 c_1 + \mathcal{M}_4 c_2\right) + (1-\mu)(1-q)v_L\left(\mathcal{M}_5 c_1 + \mathcal{M}_6 c_2\right) > 0,\end{aligned}$$

where

$$\begin{aligned}\mathcal{M}_3 &:= (q-6)(2q-1)(q-1)\mu + (3-q)(\mu-2\mu q)^2 + (2q+3)(q-1)^2, \\ \mathcal{M}_4 &:= (3-10q)(2q-1)(q-1)\mu + (4q-3)(\mu-2\mu q)^2 + 4q(q-1)^2, \\ \mathcal{M}_5 &:= (1-2q)\left(3q^2+q-4\right)\mu + (q+2)(\mu-2\mu q)^2 - 2q + 2, \\ \mathcal{M}_6 &:= 4q^3(3-4\mu)\mu + 4q^2\mu(5\mu-4) + q\left[(9-8\mu)\mu - 1\right] + (\mu-1)^2.\end{aligned}$$

It can be verified that $\mathcal{M}_3 > 0$, $\mathcal{M}_4 > 0$, $\mu q\mathcal{M}_3 + (1-\mu)(1-q)\mathcal{M}_5 > 0$, and $\mu q\mathcal{M}_4 + (1-\mu)(1-q)\mathcal{M}_6 > 0$. Therefore,

$$\begin{aligned}\mathcal{H}(v_L, v_H) &= \mu q v_H\left(\mathcal{M}_3 c_1 + \mathcal{M}_4 c_2\right) + (1-\mu)(1-q)v_L\left(\mathcal{M}_5 c_1 + \mathcal{M}_6 c_2\right) \\ &> \mu q v_L\left(\mathcal{M}_3 c_1 + \mathcal{M}_4 c_2\right) + (1-\mu)(1-q)v_L\left(\mathcal{M}_5 c_1 + \mathcal{M}_6 c_2\right) \\ &> c_1 v_L\left[\mu q\mathcal{M}_3 + (1-\mu)(1-q)\mathcal{M}_5\right] + c_2 v_L\left[\mu q\mathcal{M}_4 + (1-\mu)(1-q)\mathcal{M}_6\right] > 0,\end{aligned}$$

which in turn implies that $WE_{2(a)}$ is increasing in δ for $\delta \in \left(0, \frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}\right]$.

Next, recall $\hat{\delta} \equiv \frac{\delta c_1}{c_2}$. $WE_{2(b)}$ in expression (8) is given by

$$WE_{2(b)} = \mathcal{Q} - \frac{(1-\hat{\delta})\left\{\begin{array}{c}\mu(v_H - v_L)\left[c_1(\mathcal{M}_7\hat{\delta}^2 + \mathcal{M}_8\hat{\delta} + \mathcal{M}_9) + \hat{\delta}c_2(\mathcal{M}_{10}\hat{\delta}^2 + \mathcal{M}_{11}\hat{\delta} + \mathcal{M}_{12})\right] \\ +v_L\left[c_1(\mathcal{M}_{13}\hat{\delta}^2 + \mathcal{M}_{14}\hat{\delta} + \mathcal{M}_{15}) + \hat{\delta}c_2(\mathcal{M}_{16}\hat{\delta}^2 + \mathcal{M}_{17}\hat{\delta} + \mathcal{M}_{18})\right]\end{array}\right\}}{6c_1c_2\hat{\delta}^2(q+\mu-2\mu q)^4(1-q-\mu+2\mu q)}, \quad (11)$$

where

$$\mathcal{Q} := \frac{(c_1+c_2)\left\{\begin{array}{c} \mu v_H \left[\begin{array}{c} \big[q(q+2)-1\big](\mu-2\mu q)^2 \\ -q\big[q(2q-3)(4q+7)+17\big]\mu \\ +q\Big\{2q\big[q(2q-1)-4\big]+9\Big\}+(4\mu-3) \end{array}\right] \\ -(\mu-1)v_L\left[\begin{array}{c} \big[(q-4)q+2\big](\mu-2\mu q)^2 \\ +(q-2)(2q-1)(3q-1)\mu+2(q-1)q \end{array}\right] \end{array}\right\}}{6c_1c_2\big[q(2\mu-1)-\mu\big]\big[q(2\mu-1)-\mu+1\big]},$$

$$\begin{aligned}
\mathcal{M}_7 :=& -(2q-1)^3\Big\{q\big[q(4q^2-26q-5)+21\big]-6\Big\}\mu^3 \\
&+(2q-1)\Big\{q\big[q(5q^4+7q^3-22q+11)+3\big]-2\Big\}\mu \\
&+\big[(4q-15)q^2+3\big](\mu-2\mu q)^4-(q-1)q^2\big(2q^4+3q^3-4q+2\big) \\
&-(q-3)q(2q-1)^5\mu^5-(2q^5+20q^4+8q^3-38q^2+17q-1)(\mu-2\mu q)^2, \\
\mathcal{M}_8 :=& (1-q)^2(2q-1)(1-\mu)\big[q^2+2(1-2q)^2\mu^2+(7q-6q^2-2)\mu\big] \\
&\times\big[q(2\mu-1)-\mu-2\big]\big[q(2\mu-1)-\mu+1\big], \\
\mathcal{M}_9 :=& (1-q)^2(2q-1)(1-\mu)\Big\{q^2+2(1-2q)^2\mu^2+\big[(7-6q)q-2\big]\mu\Big\}\big[q(2\mu-1)-\mu+1\big]^2, \\
\mathcal{M}_{10} :=& (2q-1)\big[(11q^4-23q^3+23q-13)q^2+1\big]\mu-\big[(11q-18)q^2+3\big](\mu-2\mu q)^4 \\
&-(q-1)^2q^2\big[(q-1)q(2q+1)+1\big]+q(2q-3)(2q-1)^5\mu^5 \\
&+(2q-1)^3\big[(q-1)q(23q^2-17q-21)-6\big]\mu^3 \\
&-\big[q(q+1)(23q^3-66q^2+59q-19)+2\big](\mu-2\mu q)^2 \\
\mathcal{M}_{11} :=& -(q-1)q^2\big(2q^4+3q^3-4q+2\big)+(2q-1)^5(2q^2-3q+3)\mu^5 \\
&+(2q-1)^3(23q^4-37q^3+47q^2-24q+3)\mu^3-(11q^3-18q^2+21q-6)(\mu-2\mu q)^4 \\
&+(q-1)q(22q^5-5q^4+15q^3-11q^2-3q+10)\mu \\
&-(23q^5-31q^4+38q^3-26q^2+2q+2)(\mu-2\mu q)^2+2\mu \\
\mathcal{M}_{12} :=& (q-1)(2q-1)(\mu-1)\big[q(2\mu-1)-\mu+1\big]^2 \\
&\times\big[-(q+2)q^2+(1-2q)^2(q-4)\mu^2+(14q-9)\mu q+\mu\big], \\
\mathcal{M}_{13} :=& -2q^2(2q-1)\big[(2q^2+q-6)q^2+2\big]\mu+(10q^3+18q^2-3q-4)(\mu-2\mu q)^4 \\
&+(14q^5+8q^4-18q^3-6q^2+q+2)(\mu-2\mu q)^2-2(q-1)q^4 \\
&+2(1-2q)^6\mu^6-(2q-1)^5(2q^2+10q-1)\mu^5 \\
&-(2q-1)^3(18q^4+16q^3-10q^2-10q+1)\mu^3, \\
\mathcal{M}_{14} :=& (1-2q)^2(\mu-1)\mu\big[(q-1)q^2+2(1-2q)^2\mu^2-(2q-1)(q^2+3q-2)\mu\big] \\
&\times\big[q(2\mu-1)-\mu-2\big]\big[q(2\mu-1)-\mu+1\big], \\
\mathcal{M}_{15} :=& (2q-1)^2(1-\mu)\mu(-2\mu q+q+\mu-1)^2
\end{aligned}$$

$$\times \left[-(q-1)q^2 - 2(1-2q)^2\mu^2 + (q-1)(2q+7)\mu q + 2\mu\right],$$

$$\begin{aligned}\mathcal{M}_{16} :=& q^2(2q-1)\left[(2q^2+q-6)q^2+2\right]\mu + (q-1)q^4 - (1-2q)^6\mu^6 \\ &+ (2q-1)^5(q^2+5q+1)\mu^5 + (2q-1)^3(9q^4+8q^3+7q^2-14q+2)\mu^3 \\ &- (5q^3+9q^2+6q-5)(\mu-2\mu q)^4 - \left[q^3(q+1)(7q-3) - 9q^2+2q+1\right](\mu-2\mu q)^2,\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{17} :=& (2q-1)^3(9q^4-4q^3-2q^2+q+2)\mu^3 + q^2\left[(4q^2-12q+17)q^3 - 11q + 4\right]\mu \\ &- 2(q-1)q^4 - (1-2q)^6\mu^6 + (2q-1)^5(q^2+5q-2)\mu^5 \\ &- (5q^3+6q^2-6q+1)(\mu-2\mu q)^4 - (7q^5-11q^4+6q^3+3q^2+2q-2)(\mu-2\mu q)^2,\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{18} :=& (1-2q)^2(\mu-1)\mu\left[-(2q+1)q^2 - 4(1-2q)^2\mu^2 + 2(2q^2+5q-4)\mu q + \mu\right] \\ &\times \left[q(2\mu-1) - \mu + 1\right]^2.\end{aligned}$$

Note that $\mathcal{Q}$ is independent of δ and thus $\hat{\delta}$, and it can be verified that $WE_{2(b)}\big(\delta = \frac{c_2}{c_1}\big) = \mathcal{Q}$. It remains to show that $\mathcal{Q} > WE_{2(b)}$ for all $\delta \in \big(\frac{c_2}{c_1} \times \frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}, \frac{c_2}{c_1}\big)$, or equivalently, for $\hat{\delta} \in \big(\frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}}, 1\big)$. Note that $\frac{\psi_{HL}}{\psi_{HL}+\psi_{LH}} = \mu q + (1-\mu)(1-q) \equiv \hat{\mu}(q)$. Fix $\hat{\delta} \in \big(\hat{\mu}(q), 1\big)$. The following can be verified:

(a) $\mathcal{M}_7 > 0$ and $(\mathcal{M}_8)^2 - 4\mathcal{M}_7\mathcal{M}_9 < 0$, which implies that $\mathcal{M}_7\hat{\delta}^2 + \mathcal{M}_8\hat{\delta} + \mathcal{M}_9 > 0$.

(b) $\mathcal{M}_{10} < 0$, $\mathcal{M}_{10}\big[\hat{\mu}(q)\big]^2 + \mathcal{M}_{11}\big[\hat{\mu}(q)\big] + \mathcal{M}_{12} > 0$, and $\mathcal{M}_{10} + \mathcal{M}_{11} + \mathcal{M}_{12} > 0$, which implies that $\mathcal{M}_{10}\hat{\delta}^2 + \mathcal{M}_{11}\hat{\delta} + \mathcal{M}_{12} > 0$.

(c) $\mathcal{M}_{13} > 0$ and $\mathcal{M}_{14}^2 - 4\mathcal{M}_{13}\mathcal{M}_{15} < 0$, which implies that $\mathcal{M}_{13}\hat{\delta}^2 + \mathcal{M}_{14}\hat{\delta} + \mathcal{M}_{15} > 0$.

(d) $\mathcal{M}_{16} < 0$, $\mathcal{M}_{16}\big[\hat{\mu}(q)\big]^2 + \mathcal{M}_{17}\big[\hat{\mu}(q)\big] + \mathcal{M}_{18} > 0$, and $\mathcal{M}_{16} + \mathcal{M}_{17} + \mathcal{M}_{18} > 0$, which implies that $\mathcal{M}_{16}\hat{\delta}^2 + \mathcal{M}_{17}\hat{\delta} + \mathcal{M}_{18} > 0$.

The above conditions, together with (11), imply that $\mathcal{Q} > WE_{2(b)}$ for all $\hat{\delta} \in \big(\hat{\mu}(q), 1\big)$.

$WE_{2(c)}$ in expression (8) is given by

$$WE_{2(c)} = \mathcal{Q} + \frac{(\hat{\delta}-1)\left\{\begin{array}{c}\mu(v_H - v_L)\left[c_1(\mathcal{M}_{19}\hat{\delta}^2 + \mathcal{M}_{20}\hat{\delta} + \mathcal{M}_{21}) + \hat{\delta}c_2(\mathcal{M}_{22}\hat{\delta}^2 + \mathcal{M}_{23}\hat{\delta} + \mathcal{M}_{24})\right] \\ + v_L\left[c_1(\mathcal{M}_{25}\hat{\delta}^2 + \mathcal{M}_{26}\hat{\delta} + \mathcal{M}_{27}) + \hat{\delta}c_2(\mathcal{M}_{28}\hat{\delta}^2 + \mathcal{M}_{29}\hat{\delta} + \mathcal{M}_{30})\right]\end{array}\right\}}{6c_1c_2\hat{\delta}^2(q+\mu-2\mu q)^4(1-q-\mu+2\mu q)},$$

where

$$\begin{aligned}\mathcal{M}_{19} :=& (1-q)(2q-1)(1-\mu)\left[-(q+2)q^2 + (1-2q)^2(q-4)\mu^2 + (14q-9)\mu q + \mu\right] \\ &\times \left[q(2\mu-1) - \mu + 1\right]^2,\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{20} :=& (1-q)q^2(2q^4+3q^3-4q+2) + (2q-1)^5(2q^2-3q+3)\mu^5 \\ &+ (2q-1)^3(23q^4-37q^3+47q^2-24q+3)\mu^3\end{aligned}$$

$$
\begin{aligned}
&+ (q-1)q(22q^5 - 5q^4 + 15q^3 - 11q^2 - 3q + 10)\mu \\
&- (11q^3 - 18q^2 + 21q - 6)(\mu - 2\mu q)^4 \\
&- (23q^5 - 31q^4 + 38q^3 - 26q^2 + 2q + 2)(\mu - 2\mu q)^2 + 2\mu, \\
\mathcal{M}_{21} :=& (2q-1)\left(11q^6 - 23q^5 + 23q^3 - 13q^2 + 1\right)\mu \\
&- (11q^3 - 18q^2 + 3)(\mu - 2\mu q)^4 - (q-1)^2 q^2 \left[(q-1)q(2q+1) + 1\right] \\
&+ q(2q-3)(2q-1)^5\mu^5 + (2q-1)^3\left[(q-1)q(23q^2 - 17q - 21) - 6\right]\mu^3 \\
&- \left[q(q+1)(23q^3 - 66q^2 + 59q - 19) + 2\right](\mu - 2\mu q)^2, \\
\mathcal{M}_{22} :=& (1-q)^2(2q-1)(1-\mu)\left[q^2 + 2(1-2q)^2\mu^2 + (7-6q)\mu q - 2\mu\right]\left[q(2\mu-1) - \mu + 1\right]^2, \\
\mathcal{M}_{23} :=& (1-q)^2(2q-1)(1-\mu)\left[q^2 + 2(1-2q)^2\mu^2 + (7q - 6q^2 - 2)\mu\right] \\
&\times \left[q(2\mu-1) - \mu - 2\right]\left[q(2\mu-1) - \mu + 1\right], \\
\mathcal{M}_{24} :=& -(2q-1)^3\left\{q\left[q(4q^2 - 26q - 5) + 21\right] - 6\right\}\mu^3 \\
&+ (2q-1)\left\{q\left[q(5q^4 + 7q^3 - 22q + 11) + 3\right] - 2\right\}\mu \\
&+ \left[(4q-15)q^2 + 3\right](\mu - 2\mu q)^4 - (q-1)q^2\left(2q^4 + 3q^3 - 4q + 2\right) \\
&- (q-3)q(2q-1)^5\mu^5 - (2q^5 + 20q^4 + 8q^3 - 38q^2 + 17q - 1)(\mu - 2\mu q)^2, \\
\mathcal{M}_{25} :=& (1-2q)^2(\mu-1)\mu\left[-(2q+1)q^2 - 4(1-2q)^2\mu^2 + 2(2q^2 + 5q - 4)\mu q + \mu\right] \\
&\times \left[q(2\mu-1) - \mu + 1\right]^2, \\
\mathcal{M}_{26} :=& (2q-1)^3(9q^4 - 4q^3 - 2q^2 + q + 2)\mu^3 + q^2(2q-1)(2q^4 - 5q^3 + 6q^2 + 3q - 4)\mu \\
&- 2(q-1)q^4 - (1-2q)^6\mu^6 + (2q-1)^5(q^2 + 5q - 2)\mu^5 - q(5q^2 + 6q - 6)(\mu - 2\mu q)^4 \\
&- (\mu - 2\mu q)^4 + (2 - 7q^5 - 11q^4 + 6q^3 + 3q^2 + 2q)(\mu - 2\mu q)^2, \\
\mathcal{M}_{27} :=& q^2(2q-1)(2q^4 + q^3 - 6q^2 + 2)\mu + (q-1)q^4 - (1-2q)^6\mu^6 \\
&+ (2q-1)^5(q^2 + 5q + 1)\mu^5 + (2q-1)^3(9q^4 + 8q^3 + 7q^2 - 14q + 2)\mu^3 \\
&- (5q^3 + 9q^2 + 6q - 5)(\mu - 2\mu q)^4 - \left[q^3(q+1)(7q-3) - 9q^2 + 2q + 1\right](\mu - 2\mu q)^2, \\
\mathcal{M}_{28} :=& (2q-1)^2(1-\mu)\mu\left[-(q-1)q^2 - 2(1-2q)^2\mu^2 + (q-1)(2q+7)\mu q + 2\mu\right] \\
&\times \left[q(2\mu-1) - \mu + 1\right]^2, \\
\mathcal{M}_{29} :=& (1-2q)^2(\mu-1)\mu\left[(q-1)q^2 + 2(1-2q)^2\mu^2 - (2q-1)(q^2 + 3q - 2)\mu\right] \\
&\times \left[q(2\mu-1) - \mu - 2\right]\left[q(2\mu-1) - \mu + 1\right], \\
\mathcal{M}_{30} :=& -2q^2(2q-1)(2q^4 + q^3 - 6q^2 + 2)\mu + (10q^3 + 18q^2 - 3q - 4)(\mu - 2\mu q)^4 \\
&+ (14q^5 + 8q^4 - 18q^3 - 6q^2 + q + 2)(\mu - 2\mu q)^2 - 2(q-1)q^4 + 2(1-2q)^6\mu^6 \\
&- (2q-1)^5(2q^2 + 10q - 1)\mu^5 - (2q-1)^3(18q^4 + 16q^3 - 10q^2 - 10q + 1)\mu^3.
\end{aligned}
$$

$WE_{2(d)}$ in expression (8) is given by

$$WE_{2(d)} = \frac{c_1 \left(v_H \mathcal{M}_{31} + v_L \mathcal{M}_{32}\right) + \mathcal{M}_{33}\hat{\delta}}{6c_1c_2\hat{\delta}^2 \left(1-\mu-q+2\mu q\right)^2},$$

where

$$\begin{aligned}
\mathcal{M}_{31} :=& \mu q \left\{(1-2q)^2(2q-3)\mu^2 + q\left[(27-10q)q-23\right]\mu + q\left[q(2q-7)+8\right] + 6\mu - 3\right\}, \\
\mathcal{M}_{32} :=& (1-\mu)(1-q)\left[-(1-2q)^2(2q+1)\mu^2 + (q-1)(2q-1)(3q+2)\mu + q - 1\right], \\
\mathcal{M}_{33} :=& 3c_1\left[q(2\mu-1)-\mu+1\right]\left\{\mu q v_H\left[2q(\mu-1)-\mu+2\right] + (q-1)(\mu-1)v_L((2q-1)\mu+1)\right\} \\
&+ c_2(q-1)(\mu-1)v_L\left[(-6q^3+q^2+9q-4)\mu + (1-2q)^2(q+2)\mu^2 - 2q + 2\right] \\
&- c_2\mu q v_H\left[(1-2q)^2(q-3)\mu^2 - (q-6)(2q-1)(q-1)\mu - (2q+3)(q-1)^2\right].
\end{aligned}$$

Similarly, we can show that $\mathcal{Q} - WE_{2(c)} > 0$ for all $\delta \in \left(\frac{c_2}{c_1}, \frac{c_2}{c_1} \times \frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}\right]$ and $WE_{2(d)}$ strictly decreases with δ for $\delta \in \left(\frac{c_2}{c_1} \times \frac{\psi_{HL}+\psi_{LH}}{\psi_{HL}}, \infty\right)$. This concludes the proof.

■

Proof of Theorem 3

Proof. We first prove part (i) of the theorem. From the proof of Theorem 2, $(\gamma, \delta) = (CC, \frac{c_2}{c_1})$ maximizes expected total effort over $(\gamma, \delta) \in \{DC, CD, CC\} \times \mathbb{R}_{++}$ and the corresponding maximum value is $\frac{(c_1+c_2)\bar{v}}{2c_1c_2}$.

Suppose $q \geq \frac{v_H}{v_H+v_L}$. By Lemma 3, $TE^\sharp(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$ is maximized by $\delta = c_2/c_1$ and the corresponding maximum value is $\frac{(c_1+c_2)\bar{v}}{2c_1c_2}$. Therefore, both $(CC, \frac{c_2}{c_1})$ and $(\widehat{D}\widehat{D}, \frac{c_2}{c_1})$ maximize expected total effort from the contest.

Suppose $q < \frac{v_H}{v_H+v_L}$. Again, by Lemma 3, $TE^\sharp(\widehat{D}\widehat{D}, \delta, q; c_1, c_2)$ is maximized by $\delta = c_2/c_1$. Carrying out the algebra, we can obtain that

$$\frac{(c_1+c_2)\bar{v}}{2c_1c_2} - TE^\sharp\left(DD, \frac{c_2}{c_1}, q; c_1, c_2\right) = \frac{(c_1+c_2)}{c_1c_2} \times \frac{(2q-1)(1-\mu)\mu\left[v_H - q(v_H+v_L)\right]}{q+\mu-2\mu q} > 0.$$

Therefore, $(CC, \frac{c_2}{c_1})$ maximizes expected total effort from the contest.

Next, we prove part (ii). By Theorem 2, $(CC, \frac{c_2}{c_1})$ or $(DC, \frac{c_2}{[c_1\hat{\mu}(q)]})$ maximizes the expected winner's effort over $(\gamma, \delta) \in \{DC, CD, CC\} \times \mathbb{R}_{++}$. Further, by Lemma 3, $WE^\sharp(\widehat{D}\widehat{D}, \delta, q)$ is maximized at $\delta = \frac{c_2}{c_1}$. These facts together conclude the proof. ■

Proof of Theorem 4

Proof. The proof of part (i) of the theorem closely follows that of Theorem 2(i) and it remains to prove part (ii). It is useful to prove an intermediate result.

Lemma 6 *Suppose that $\gamma = DC$. Fix an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (6) and let the designer set the scoring bias $\delta > 0$. Then the expected winner's effort from the contest is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\mu^\pi c_1}$.*

Proof. The proof closely follows that of Lemma 5 and is omitted for brevity. ■

Following the same steps in the proof of Theorem 2, we can show that for an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (6), the expected winner's effort from the contest is maximized by $(\delta, \gamma) = (c_2/c_1, CC)$, $(\delta, \gamma) = (c_2/c_1, DD)$, or $(\delta, \gamma) = \left(\frac{c_2}{\mu^\pi c_1}, DC\right)$. The first two contest schemes generate an expected winner's effort of $\frac{c_1+c_2}{3c_1c_2}\bar{v}$, while the third one generates an expected winner's effort of $\frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}$. The optimization problem thus boils down to

$$\max_{\{v_L^\pi, v_H^\pi, \mu^\pi\}} WE^\pi := \max\left\{\frac{c_1+c_2}{3c_1c_2}\bar{v}, \frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}\right\} \text{ s.t. } (6).$$

It is straightforward to verify that $\frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}$ is increasing in $\mu^\pi \in (0,1)$. By (6), we can obtain that

$$\mu^\pi = \frac{\bar{v} - v_L^\pi}{v_H^\pi - v_L^\pi} = 1 - \frac{v_H^\pi - \bar{v}}{v_H^\pi - v_L^\pi},$$

which implies that μ^π is decreasing in v_L^π. Therefore, $v_L^\pi = v_L$ in the optimum. Plugging $v_L^\pi = v_L$ into (6) yields that $v_H^\pi = v_L + \frac{\bar{v}-v_L}{\mu^\pi}$. Replacing (v_L^π, v_H^π) with $(v_L, v_L + \frac{\bar{v}-v_L}{\mu^\pi})$ in WE^π, the above maximization problem can be further simplified as

$$\max_{\mu^\pi \in [\mu, 1)} \max\left\{\frac{c_1+c_2}{3c_1c_2}\bar{v}, \mathcal{V}(\mu^\pi)\right\},$$

where

$$\mathcal{V}(\mu^\pi) := \frac{-c_1 v_L(\mu^\pi)^2 + \left[(4c_1+2c_2)v_L - c_1\bar{v}\right]\mu^\pi + (3c_1+2c_2)(\bar{v}-v_L)}{6c_1c_2},$$

and the constraint $\mu^\pi \geq \mu$ is due to the constraint $v_H^\pi = v_L + \frac{\bar{v}-v_L}{\mu^\pi} \leq v_H$ imposed in (6).

Note that $\mathcal{V}(1) = \frac{c_1+c_2}{3c_1c_2}\bar{v}$ and $\mathcal{V}(\mu^\pi)$ is quadratic and inverted U-shaped in μ^π. Therefore, $\max_{\mu^\pi \in [\mu,1)} \mathcal{V}(\mu^\pi) > \frac{c_1+c_2}{3c_1c_2}\bar{v}$ if and only if

$$\frac{(4c_1+2c_2)v_L - c_1\bar{v}}{2c_1v_L} < 1 \iff \frac{\bar{v}}{v_L} > 2 + 2\frac{c_2}{c_1}.$$

In this case, $\mu^\pi = \max\left\{\mu, \frac{(4c_1+2c_2)v_L - c_1\bar{v}}{2c_1v_L}\right\}$ in the optimum.

In summary, if $\frac{\bar{v}}{v_L} > 2 + 2\frac{c_2}{c_1}$, the expected winner's effort is maximized by a contest scheme with $(\delta, \gamma) = \left(\frac{c_2}{\mu^\pi c_1}, DC\right)$ and

$$\pi\left(H|v_H\right) = 1, \pi\left(H|v_L\right) = \begin{cases} 0, & \text{if } \frac{\bar{v}}{v_L} \geq 4 - 2\mu + 2\frac{c_2}{c_1}, \\ \frac{4-2\mu+2\frac{c_2}{c_1} - \frac{\bar{v}}{v_L}}{2(1-\mu)}, & \text{if } 2 + 2\frac{c_2}{c_1} < \frac{\bar{v}}{v_L} < 4 - 2\mu + 2\frac{c_2}{c_1}; \end{cases}$$

otherwise, it is maximized by $(\delta, \gamma) = (c_2/c_1, CC)$ or $(\delta, \gamma) = (c_2/c_1, DD)$ with an arbitrary information structure $\left\{\pi(\cdot|v_H), \pi(\cdot|v_L)\right\}$. This concludes the proof. ■

Proof of Lemma 4

Proof. The lemma follows immediately from the equilibrium characterizations established in Propositions 1 and 2, and is omitted for brevity. ■

Proof of Theorem 5

Proof. We first prove part (i) of the theorem. By Lemma 4, the quota constraint $WP_1(\delta) \geq \eta$, with $\eta \in (0, 1/2]$, can be simplified as $\delta \leq \bar{\delta} := c_2/(2\eta c_1) \in [c_2/c_1, \infty)$. Recall that absent the affirmative action policy, the optimal contest scheme $(\gamma, \delta) = (CC, c_2/c_1)$ or $(\gamma, \delta) = (DD, c_2/c_1)$ maximizes expected total effort. It is straightforward to verify that $\bar{\delta} \geq c_2/c_1$, $(\gamma, \delta) = (CC, c_2/c_1)$ and $(\gamma, \delta) = (DD, c_2/c_1)$, and thus the quota constraint is satisfied.

Next, we prove part (ii). Consider the following three cases:

Case I: $\boldsymbol{\hat{\mu}(q)\hat{v}_H(q) \geq \left(2\frac{c_1}{c_2} + 2\right)\hat{v}_L(q).}$ From the proof of Theorem 2, we have that

$$WE\left(DC, \frac{c_2}{c_1}, q; c_1, c_2\right) < WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) \leq WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}, q; c_1, c_2\right). \quad (12)$$

Moreover, from the proof of Lemma 5, we know that (i) $WE\left(DC, \delta, q; c_1, c_2\right)$ is increasing in $\delta \in (0, c_2/c_1)$; (ii) $WE\left(DC, \delta, q; c_1, c_2\right)$ is either monotonic or U-shaped in δ for $\delta \in \left(c_2/c_1, c_2/[\hat{\mu}(q)c_1]\right)$; and (iii) $WE\left(DC, \delta, q; c_1, c_2\right)$ decreases with δ for $\delta \in \left(c_2/[\hat{\mu}(q)c_1], \infty\right)$. Together with (12), we can further conclude that $WE\left(DC, \delta, q; c_1, c_2\right)$ is either increasing or U-shaped in $\delta \in \left(c_2/c_1, c_2/[\hat{\mu}(q)c_1]\right)$.

Similarly, from the proofs of Theorem 2 and Lemma 5, we can establish that

$$WE\left(CD, \frac{c_2}{c_1}, q; c_1, c_2\right) < WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) \leq WE\left(CD, \frac{\hat{\mu}(q)c_2}{c_1}, q; c_1, c_2\right). \quad (13)$$

Moreover, we know that (i) $WE(CD, \delta, q; c_1, c_2)$ is increasing in δ for $\delta \in (0, \hat{\mu}(q)c_2/c_1)$; (ii) $WE(CD, \delta, q; c_1, c_2)$ is decreasing or U-shaped in δ for $\delta \in \left(\hat{\mu}(q)c_2/c_1, c_2/c_1\right)$; and (iii) $WE(CD, \delta, q; c_1, c_2)$ is decreasing in δ for $\in (c_2/c_1, \infty)$.

By (12), (13), and the monotonicity of $WE\left(DC, \delta, q; c_1, c_2\right)$ and $WE\left(CD, \delta, q; c_1, c_2\right)$, we can pin down the optimal contest scheme as stated in case (a) of Theorem 5(ii).

Case II: $\boldsymbol{\left(2\frac{c_2}{c_1}+2\right)\hat{v}_L(q) \leq \hat{\mu}(q)\hat{v}_H(q) < \left(2\frac{c_1}{c_2}+2\right)\hat{v}_L(q)}$. The analysis of $WE(DC, \delta, q; c_1, c_2)$ for this case is the same as that for Case I. For $WE(CD, \delta, q; c_1, c_2)$, it is straightforward to verify that

$$\max\left\{WE\left(CD, \frac{c_2}{c_1}, q; c_1, c_2\right), WE\left(CD, \frac{\hat{\mu}(q)c_2}{c_1}, q; c_1, c_2\right)\right\} < WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right).$$

Further, it follows from the proof of Theorem 2 that $WE(CD, \delta, q; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{\hat{\mu}(q)c_2}{c_1}$. Therefore, $(\gamma, \delta) = (CC, c_2/c_1)$ dominates $\gamma = CD$ with any $\delta > 0$ in terms of the expected winner's effort.

From the above analysis, we only need to compare the expected winner's effort under $(\gamma, \delta) = (CC, c_2/c_1)$ and the maximum expected winner's effort that can be achieved under $\gamma = DC$ in the presence of the quota constraint $\delta \leq \bar{\delta}$. By (12) and the monotonicity of $WE(DC, \delta, q; c_1, c_2)$, we can pin down the optimal contest scheme as stated in case (b) of Theorem 5(ii).

Case III: $\boldsymbol{\hat{\mu}(q)\hat{v}_H(q) < \left(2\frac{c_2}{c_1}+2\right)\hat{v}_L(q)}$. From the proof of Theorem 2, $(\gamma, \delta) = (CC, c_2/c_1)$ dominates $\gamma = DC$ and $\gamma = CD$ with any δ in WE maximization. Further, the quota constraint under $(\gamma, \delta) = (CC, c_2/c_1)$ is satisfied. Therefore, the optimal contest scheme with an affirmative action policy is $(\gamma, \delta) = (CC, c_2/c_1)$ or $(\gamma, \delta) = (DD, c_2/c_1)$.

■