The formal demography of kinship: A look at the development of a theory

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Why kinship?

Why a formal model?

- birth
- death
- family and kinship

Lots of theory for fertility and mortality

- population growth
- structure
- persistence/extinction
- evolution

For kinship: not so much[1]

[1] "No model, no understanding." Nathan Keyfitz

Kinship: Goodman, Keyfitz, and Pullum 1974

THEORETICAL POPULATION BIOLOGY **5**, 1–27 (1974)

Family Formation and the Frequency of Various Kinship Relationships

Leo A. Goodman

The University of Chicago

Nathan Keyfitz and Thomas W. Pullum

Harvard University

Received January 19, 1970

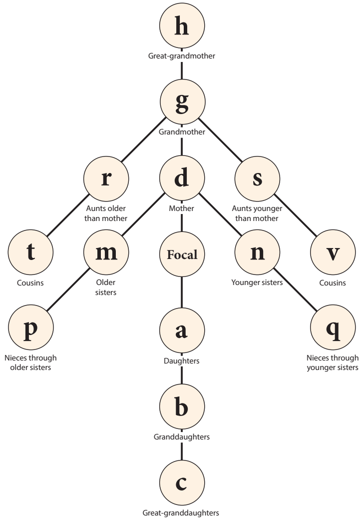
$(l_z/l_y)\, m_z$. In addition, the limits of the corresponding integration have to be altered; instead of α to y, they become y to $a + x + y$. Hence, we have

$$\int_\alpha^\beta \left[\int_\alpha^\beta \left\{\int_y^{a+x+y} \left(\int_\alpha^{a+x+y-z} l_w m_w \, dw\right) \frac{l_z}{l_y} m_z \, dz\right\} W(y)\, dy\right] W(x)\, dx, \qquad (6.2.a)$$

for cousins whose mother is a younger sister of the mother of the girl aged a. The l_y in the denominator could be cancelled with the l_y contained in $W(y)$.

The sum of the two integrals would give the expected number of cousins

The kinship network[2]



[2] Keyfitz and Caswell 2005

Meet Focal

- female (or male) of specified age
- specified mortality and fertility
- distribution π of ages of mothers at birth

The kin of Focal **are a population**

. . . so we might as well model them as one

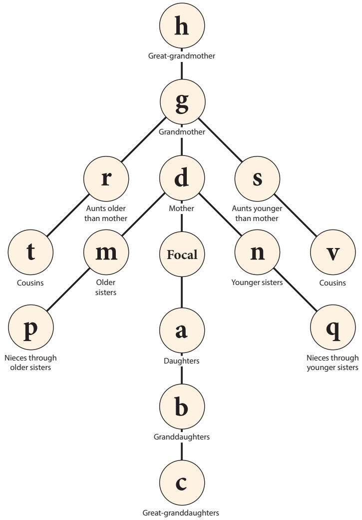
Population projection

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \left[\begin{pmatrix} 0 & 0 & 0 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{pmatrix} + \begin{pmatrix} f_1 & f_2 & f_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t)$$

$$\mathbf{n}(t+1) = (\mathbf{U}+\mathbf{F})\,\mathbf{n}(t)$$

$$\begin{aligned} \mathbf{U} &= \text{survival matrix} \\ \mathbf{F} &= \text{fertility matrix} \end{aligned}$$

The kinship network[3]



[3] Keyfitz and Caswell 2005

Model the kin of Focal as a population

$$
\begin{aligned}
\mathbf{k}(x+1) &= \mathbf{U}\mathbf{k}(x) + \boldsymbol{\beta}(x) \\
\mathbf{k}(0) &= \mathbf{k}_0
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{k}(x) &= \text{age distribution of some type of kin at age } x \text{ of Focal} \\
\mathbf{U} &= \text{survival matrix}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\beta}(x) &= \text{recruitment `subsidy' at age } x \text{ of Focal} \\
&= \begin{cases} \mathbf{0} & \text{no recruitment} \\ \mathbf{F}\,\mathbf{k}^*(x) & \text{recruitment from kin of type } \mathbf{k}^* \end{cases}
\end{aligned}
$$

$$
\mathbf{k}_0 = \text{initial condition (kin at the birth of Focal)}
$$

The pieces

- survival matrix $\mathbf{U}$
- recruitment subsidy β
- initial condition $\mathbf{k}(0)$

Dynamics of kin of Focal: daughters

a(x) = daughters of Focal

- initial condition? Focal has no daughters at birth.

$$\mathbf{a}_0 = \mathbf{0}$$

- recruitment? New daughters are the result of reproduction by Focal.

$$\beta(x) = \mathbf{F}\mathbf{e}_x$$

  where

$$\mathbf{e}_x = \begin{pmatrix} 0 & \cdots & 1 & \cdots & 0 \end{pmatrix}^{\mathsf{T}}$$

so

$$\mathbf{a}(x+1) = \mathbf{U}\mathbf{a}(x) + \mathbf{F}\mathbf{e}_x$$

Dynamics of kin of Focal: granddaughters

$\mathbf{b}(x)$ = granddaughters of Focal

- initial condition? Focal has no granddaughters at birth.

$$\mathbf{b}_0 = \mathbf{0}$$

- recruitment? New granddaughters are the result of reproduction by daughters.

$$\beta(x) = \mathbf{F}\mathbf{a}_x$$

so

$$\mathbf{b}(x+1) = \mathbf{U}\mathbf{b}(x) + \mathbf{F}\mathbf{a}_x$$

Dynamics of kin of Focal: mothers

$\mathbf{d}(x)$ = mothers of Focal

- initial condition? Focal has one mother at birth; age unknown but distributed as $\boldsymbol{\pi}$

$$\mathbf{d}_0 = \sum_i \pi_i \mathbf{e}_i \;=\; \boldsymbol{\pi}$$

- recruitment? No new mothers obtained after birth of Focal

$$\boldsymbol{\beta}(x) = \mathbf{0}$$

so

$$\mathbf{d}(x+1) = \mathbf{U}\mathbf{d}(x) + \mathbf{0}$$

Dynamics of kin of Focal: younger sisters

$\mathbf{n}(x)$ = younger sisters of Focal

- initial condition? Focal has no younger sisters at the time of her birth.

$$\mathbf{n}_0 = \mathbf{0}$$

- recruitment? New younger sisters are the children of Focal's mother

$$\beta(x) = \mathbf{F}\mathbf{d}(x)$$

so

$$\mathbf{n}(x+1) = \mathbf{U}\mathbf{n}(x) + \mathbf{F}\mathbf{d}(x)$$

Dynamics of kin of Focal: older sisters

$\mathbf{m}(x)$ = older sisters of Focal

- initial condition? Older sisters of Focal at birth are the children of Focal's mother at the birth of Focal

$$\mathbf{m}_0 = \sum_i \pi_i \mathbf{a}(i)$$

- recruitment subsidy? Focal acquires no new older sisters after she is born

$$\beta(x) = \mathbf{0}$$

so

$$\mathbf{m}(x+1) = \mathbf{U}\mathbf{m}(x) + \mathbf{0}$$

Dynamics of kin of Focal at age x

Symbol	Kin	i.c. $\mathbf{k}_0$	$\beta(x)$
a	daughters	$\mathbf{0}$	$\mathbf{F}\mathbf{e}_x$
b	granddaughters	$\mathbf{0}$	$\mathbf{F}\mathbf{a}(x)$
c	great-granddaughters	$\mathbf{0}$	$\mathbf{F}\mathbf{b}(x)$
d	mothers	$\boldsymbol{\pi}$	$\mathbf{0}$
g	grandmothers	$\sum_i \pi_i \mathbf{d}(i)$	$\mathbf{0}$
h	great-grandmothers	$\sum_i \pi_i \mathbf{g}(i)$	$\mathbf{0}$
m	older sisters	$\sum_i \pi_i \mathbf{a}(i)$	$\mathbf{0}$
n	younger sisters	$\mathbf{0}$	$\mathbf{F}\mathbf{d}(x)$
p	nieces via older sisters	$\sum_i \pi_i \mathbf{b}(i)$	$\mathbf{F}\mathbf{m}(x)$
q	nieces via younger sisters	$\mathbf{0}$	$\mathbf{F}\mathbf{n}(x)$
r	aunts older than mother	$\sum_i \pi_i \mathbf{m}(i)$	$\mathbf{0}$
s	aunts younger than mother	$\sum_i \pi_i \mathbf{n}(i)$	$\mathbf{F}\mathbf{g}(x)$
t	cousins from aunts older than mother	$\sum_i \pi_i \mathbf{p}(i)$	$\mathbf{F}\mathbf{r}(x)$
v	cousins from aunts younger than mother	$\sum_i \pi_i \mathbf{q}(i)$	$\mathbf{F}\mathbf{s}(x)$

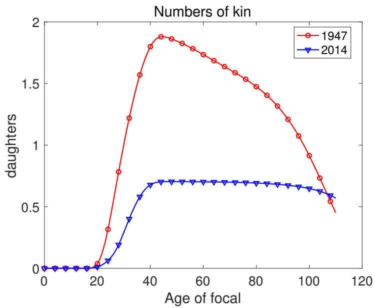
Example: Japan 1947 and 2014[4]

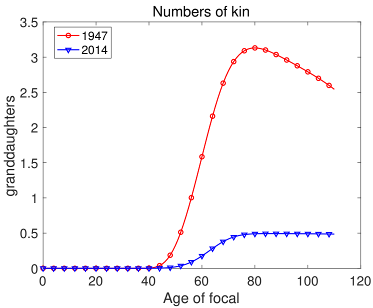
	1947	2014	%
life exp	54	87	$+61\%$
TFR	4.6	1.4	-70%
R_0	1.7	0.7	-59%

[4] Caswell 2019 Demographic Research

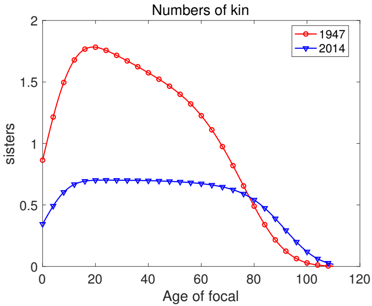
Numbers of kin

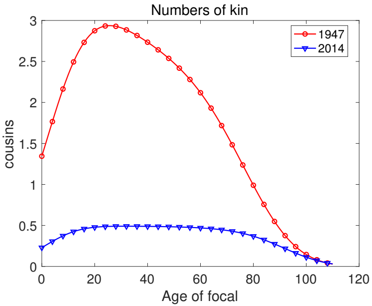
daughters and granddaughters



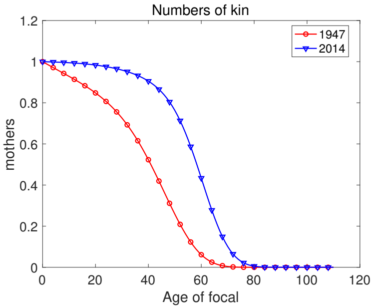


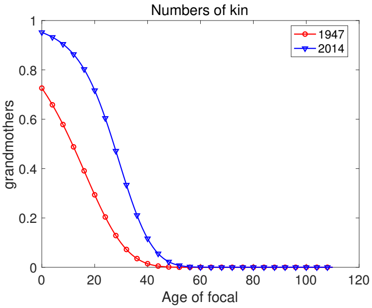
Numbers of kin: sisters and cousins





Numbers of kin: mothers and grandmothers





Beyond age distributions: death and bereavement

$$\tilde{\mathbf{k}}(x) = \left(\frac{\mathbf{k}_{\text{living}}}{\mathbf{k}_{\text{dead}}} \right)(x)$$

Then

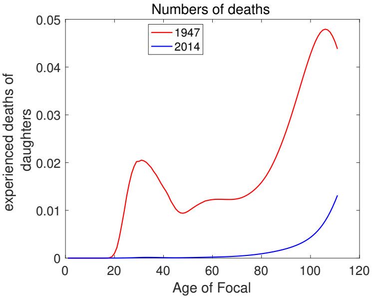
$$\tilde{\mathbf{k}}(x+1) = \begin{cases} \left(\begin{array}{c|c} \mathbf{U} & \mathbf{0} \\ \hline \mathbf{M} & \mathbf{0} \end{array} \right) \tilde{\mathbf{k}}(x) + \tilde{\beta}(x) & \textbf{experienced} \text{ at age } x \\ \\ \left(\begin{array}{c|c} \mathbf{U} & \mathbf{0} \\ \hline \mathbf{M} & \mathbf{I} \end{array} \right) \tilde{\mathbf{k}}(x) + \tilde{\beta}(x) & \textbf{cumulative} \text{ to age } x \end{cases}$$

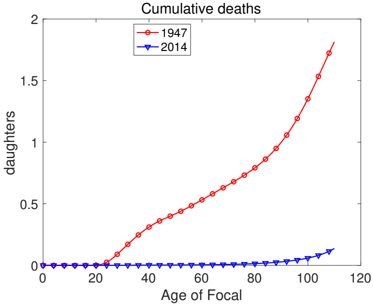
where

$$\mathbf{M} = \text{diag}(\mathbf{q})$$

- can be extended to causes of death or ages at death

Deaths of kin: daughters





Multistate kinship models: age and something else[5]

$$
\begin{aligned}
s &= \text{number of `stages'} \\
\omega &= \text{number of age classes}
\end{aligned}
$$

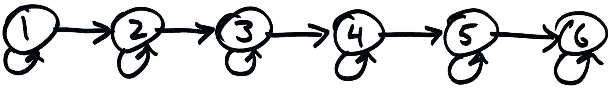
Joint age×stage structure of kin of type **k**:

$$
\widetilde{\mathbf{k}}(x) = \left(\begin{array}{c} k_{11} \\ \vdots \\ k_{s1} \\ \hline \vdots \\ \hline k_{1\omega} \\ \vdots \\ k_{s\omega} \end{array}\right)(x) \quad = \quad \left(\begin{array}{c} \mathbf{k}_1 \\ \hline \mathbf{k}_2 \\ \hline \vdots \\ \hline \mathbf{k}_\omega \end{array}\right)(x)
$$

[5] Caswell 2020, Demographic Research

An example: age and parity in Slovakia

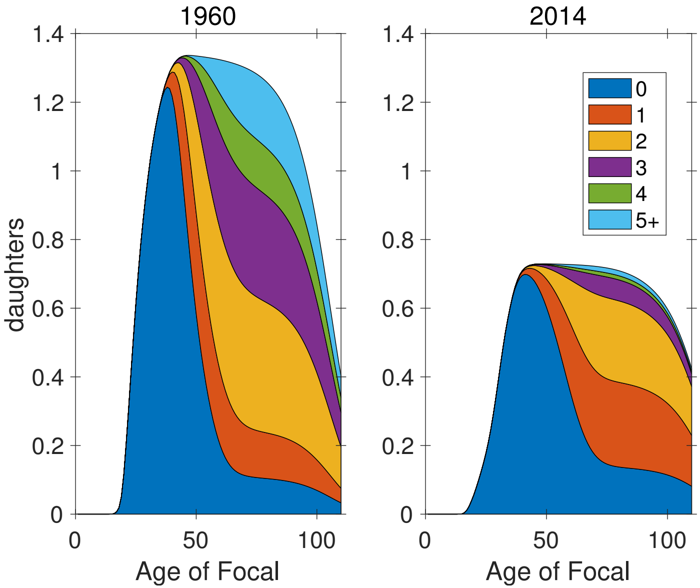
Stage (parity) transition structure:



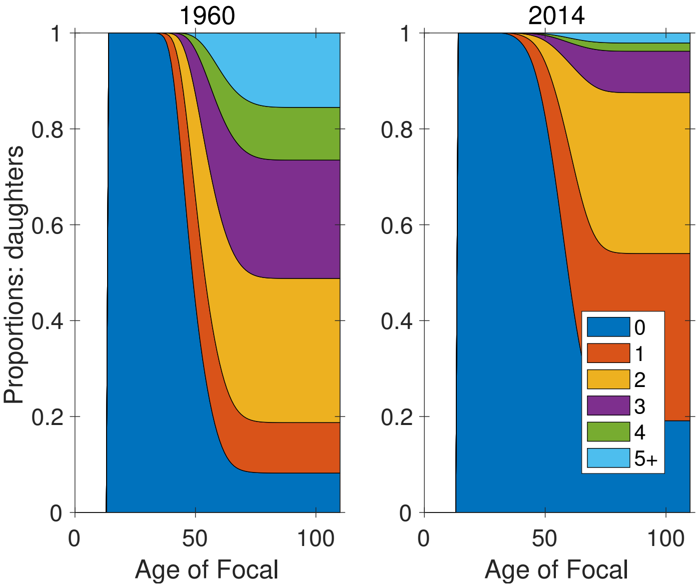
Slovakia 1960 – 2014

	1960	2014	
TFR	3.6	1.5	−63%
Life expectancy	62	80	+29%

Slovakia: marginal partity structure of daughters



Slovakia: proportional parity structure of daughters



Extending the formal theory

- basic theory (Caswell 2019)
- age$\times$stage multistate models (Caswell 2020)
- time-varying models (Caswell and Song 2021)
- two-sex models (Caswell 2022)[6]

and, of course, more in development

- stochastic models
- dynamics and persistence of lineages
- kinship for animals
- bereavement and causes of death
- and so on

[6] These citations get redundant; the series is published in Demographic Research if you want to find it

Thank you.