Advanced Macroeconomics I

ECON 525a - Fall 2009

Yale University

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Week 2

Question

- Why is debt the primary source of external finance?
- Gale and Hellwig show this is the case with ex-post hidden information with costly state verification.

Environment

- E and L are risk neutral. E has all bargaining power.
- E has zero wealth. L has deep pockets. Risk free rate 0.
- E has monopoly access to a project that costs I.
- The project generates a random profit $y \in [y_L, y_H]$, with cdf F and pdf f, smooth.
- The project has a positive NPV.

$$\int_{y_L}^{y_H} yf(y)dy > I$$

Information Structure

- Hidden Information: Cash flow privately observed by E.
- Costly State Verification: L observes the realized y only by paying an auditing cost C.

Contracting Problem

- E maximizes expected payoffs subject to L breaking even.
- Assumption: **Deterministic audits** $a(y) = \{0, 1\}$ (not WLOG)
- WLOG, confine attention to direct mechanism design.
- Contract: $\{P(y), a(y)\}$ enforced by a court **without renegotiation**.

  - $P : Y \rightarrow \mathbb{R}$ (payment for each report $\hat{y}$)
  - $a : Y \rightarrow \{0, 1\}$ (audit decision at each report $\hat{y}$)

Contracting Problem

- E maximizes

$$\int_{y_L}^{y_H} [y - P(y)] f(y) dy$$

subject to

LL: (limited liability)

$$P(y) \leq y, \quad \textit{for all } y$$

PC: (participation constraint)

$$\int_{y_L}^{y_H} [P(y) - a(y)C] f(y) dy = I$$

IC: When $a(y) = 0$, $P(y) = R$ (constant since no info about y).

  - $P(y) = (1 - a(y))R + a(y)S(y) = R - a(y)[R - S(y)]$
  - $a(y)S(y) \leq R$

Contracting Problem

- Hence E decides $a(y)$ and $S(y)$, (pinning down R uniquely from PC).
- We can rewrite the problem as an optimal control problem, deciding the $\hat{y}$ to stop auditing and the payments when auditing $S(y)$.

Contracting Problem as Optimal Control

- Define

$$H(y) \equiv \int_{y_L}^{y} [P(\tilde{y}) - a(\tilde{y})C] f(\tilde{y}) d\tilde{y}$$

- Hence, PC can be written as $H(y_H) = I$

Contracting Problem as Optimal Control

$$\max_{a(y),S(y)} \int_{y_L}^{y_H} [y - R + a(y)(R - S(y))]f(y)dy$$

subject to PC

$$H'(y) = f(y)[R - a(y)(R - S(y) + C)]$$

$$H(y_L) = 0 \qquad H(y_H) = I$$

LL: $y - R + a(y)[R - S(y)] \geq 0$

IC: $R - a(y)S(y) \geq 0$

Contracting Problem as Optimal Control

$$
\begin{aligned}
\mathcal{L} \;&=\; f(y)[y - R + a(y)(R - S(y))] + \pi f(y)[R - a(y)(R - S(y) + C)] \\
&\quad + \lambda[y - R + a(y)(R - S(y))] + \mu[R - a(y)S(y)]
\end{aligned}
$$

Take derivatives

$$
\frac{\partial \mathcal{L}}{\partial a(y)} = (f(y) + \lambda)[R - S(y)] - \pi f(y)[R - S(y) + C] - \mu S(y)
$$

$$
\frac{\partial \mathcal{L}}{\partial S(y)} = a(y)[f(y)\pi - f(y) - \lambda - \mu]
$$

Solution $a(y) = 1$

- If $a(y) = 1$, $\frac{\partial \mathcal{L}}{\partial a(y)} > 0$,

$$[R - S(y)][\lambda + f(y)(1 - \pi)] > \pi f(y) C + \mu S(y) \geq 0$$

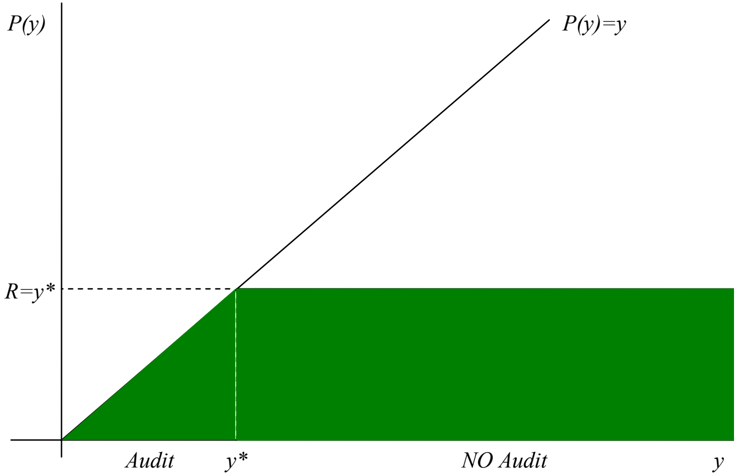
- Hence $R - S(y) > 0$ and $\mu = 0$.
- We know, $\pi' = -\frac{\partial \mathcal{L}}{\partial H} = 0, \forall y$. This implies $\pi(y) = \pi^*, \forall y$.
- Furthermore, from $\frac{\partial \mathcal{L}}{\partial S(y)} \geq 0$, $\pi^* \geq 1$ (in fact, strict).
- Since $\lambda^* > f(y)(\pi^* - 1) \geq 0$, then $S(y) = y$

Standard debt is optimal

- For all y s.t. $a^*(y) = 1$, $P^*(y) = y$
- For all y s.t. $a^*(y) = 0$, $P^*(y) = R$
- Hence, audit whenever $y < R$ and get $P(y) = y$. For $y \geq R$, no auditing and payment is $P(y) = R$.

- **Standard Debt Contract is Optimal.**

Standard Debt Contract



Issues

- Why restrict to $a \in \{0, 1\}$? It is more efficient to introduce public randomization. (Mookherjee and Png, QJE, 89).
- Not renegotiation proof.
- Krasa and Villamil (Ecta, 00) show the standard debt contract is optimal when renegotiation is possible.

Delegated Monitoring

- Without intermediaries there is a duplication of monitoring efforts.
- Monitors can also lie. Who monitor the monitor?
- Important to recognize delegation costs.
- Diversification is key despite risk neutrality for all the agents.

Model

- N risk neutral E's with 0 initial wealth. A project costs $1 and produces $y \in [0, \infty)$ with expectation greater than 1.
- Many risk neutral L. Each has available $\$\frac{1}{m} < 1$. Risk free rate $=0$.
- Only E can freely observe the realization of y. L should pay C to observe y.
- Monitoring represents a non-monetary cost to E. **Non critical**, as shown by Williamson (JME, 86)

Optimal Contract is Standard Debt Contract

- Each L get $\rho(y) = min\{y/m, R\}$
- E gets

$$Pr(y \geq R)E_y(y - R|y \geq R) - Pr(y < R)mC$$

- For mC big enough, there is no loan (underinvestment).

Delegation to a bank for a unique loan

- Standard debt contract is optimal between the bank and E.
- C is paid only once in case of monitoring.
- Standard debt contract is also optimal between depositors L and the bank (since depositors cannot observe the payment from E to the bank).
- This means, if the bank only intervenes for one loan, We have the same costs as without intermediary, plus one C.

Delegation to a bank for many loans

- Return to the bank when $\rho(y, R) = min\{y, R\}$

$$\sum_{n=1}^{N} \rho(y_n) - I_{(y_n < R)} C$$

- Return to depositors $\rho(\widetilde{y}) = min\{\widetilde{y}, R_N\}$, where

$$\widetilde{y} = \frac{1}{mN} \sum_{n=1}^{N} \left[R I_{(y_n \geq R)} + (y_n - C) I_{(y_n < R)} \right]$$

Then

$$\int \min\{\widetilde{y}, R_N\} g_N(\widetilde{y}) d\widetilde{y} - C G_N(R_N)$$

where $G_N(R_N)$ is the probability of bank default.

Delegation to a bank for many loans

- Assumption: $E(\tilde{y}) > \frac{1}{m}$.
- As $N \to \infty$, there is no monitoring to the bank, since $G_N(R_N) \to 0$.
- Hence, depositors do not need to monitor an infinitely large intermediary, who can achieve 1 per project with probability 1.

Delegation to a bank for many loans

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- Hence, depositors do not need to monitor an infinitely large intermediary, who can achieve 1 per project with probability 1.
- **Diversification relaxes the "monitoring the monitor" problem.**

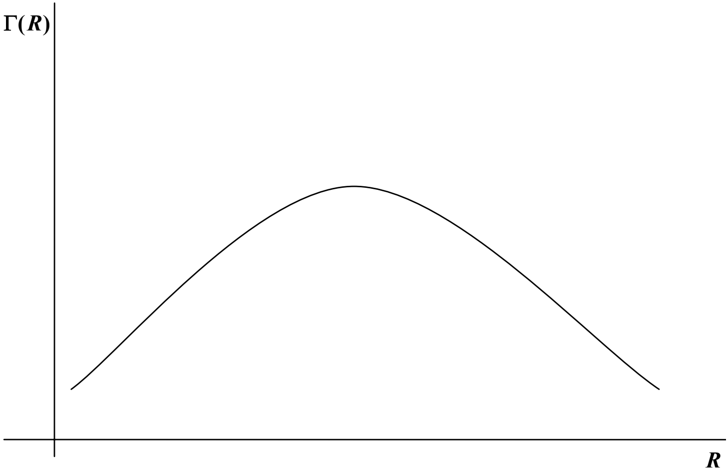
Extensions

- Results are not that strong when projects' results are correlated.
- What if the cost of monitoring a larger bank is higher? Optimal Bank Size. Trade off between monitoring and diversification.
- Yet another theory of optimal bank size. Trade off between bank capital and diversification.
- With risk aversion the result is naturally stronger.

Main ideas

- Credit rationing (and unemployment) may not be a disequilibrium event.
- Higher interest rates
  - Attract borrowers less likely to pay (adverse selection).
  - Induce borrowers to take more risks (moral hazard).
- Hence, the expected return by the bank may increase less rapidly that the interest rate.
- Banks may deny loans to borrowers who are observationally indistinguishable than those who receive loans.

Graphical idea



Model

- Here I will focus on the main idea without moral hazard and without collateral.
- E need \$1 from L to start a project.
- Projects pay $y \sim F(., \theta)$
- Two types of projects $\theta = \{\theta_G, \theta_B\}$, only known by E.
- Standard Debt Contract:
  - Profits to L: $\gamma(y, R) = min\{y, R\}$
  - Profits to E: $\pi(y, R) = max\{0, y - R\}$

Two cases in terms of profits distributions

- Define $\Gamma(R|\theta) = E_y\left[\gamma(y, R)|\theta\right]$
- Define $\Pi(R|\theta) = E_y\left[\pi(y, R)|\theta\right]$
- All projects need to generate a minimum $\overline{\Pi}$.
- Define $\overline{R}(\theta)$ such that $\Pi(\overline{R}|\theta) = \overline{\Pi}$
- We will consider two different cases with opposite results

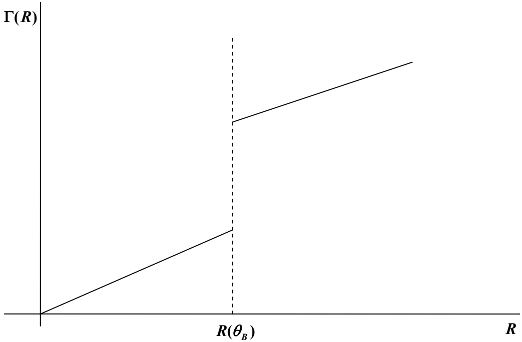
First Order Stochastic Dominance (FOSD)

$$F(y|\theta_G) \leq F(y|\theta_B) \quad \textit{for all } y$$

- For L: $\Gamma(R|\theta_G) \geq \Gamma(R|\theta_B)$
- For E: $\Pi(R|\theta_G) \geq \Pi(R|\theta_B)$
- Hence, $\overline{R}(\theta_G) \geq \overline{R}(\theta_B)$

First Order Stochastic Dominance (FOSD)

$$\Gamma(R) = Pr(\theta_G)\Gamma(R|\theta_G) + Pr(\theta_B)\Gamma(R|\theta_B)$$



Second Order Stochastic Dominance (SOSD)

$$E(y|\theta_G) = E(y|\theta_B)$$
$$\int_l^a F(y|\theta_G)dy \leq \int_l^a F(y|\theta_B)dy \quad \textit{for all } a$$

- For L: $\Gamma(R|\theta_G) \geq \Gamma(R|\theta_B)$
- Property: If X SOSD Y, then $E(h(X)) \leq E(h(Y))$ for all convex function h.
- For E: Since $\pi(y, R)$ is convex, $\Pi(R|\theta_G) \leq \Pi(R|\theta_B)$
- Hence, $\overline{R}(\theta_G) \leq \overline{R}(\theta_B)$

Second Order Stochastic Dominance (SOSD)

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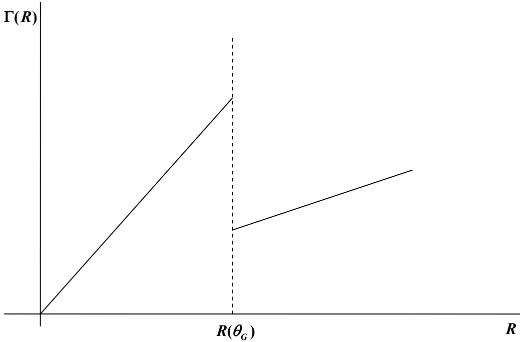

Credit Rationing