

# Advanced Macroeconomics I

## ECON 525a - Fall 2009

### Yale University

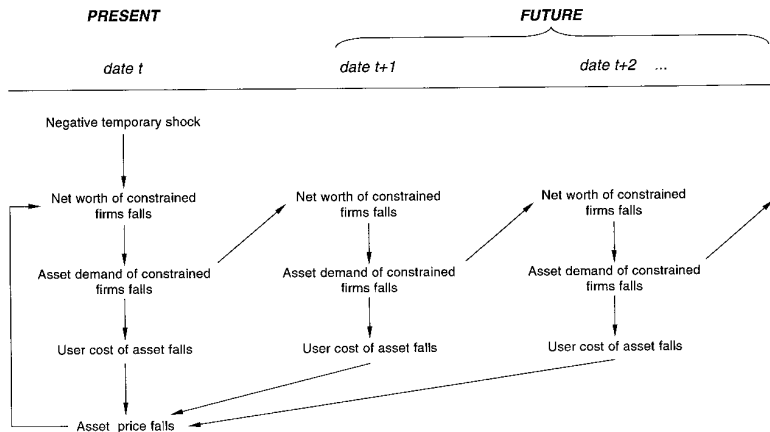
**Guillermo L. Ordoñez**

Week 4

# Introduction

- Credit frictions  $\rightarrow$  amplification & persistence of shocks
- Two roles for capital
  - Factor of production
  - Collateral for loans
- Negative productivity shock
  - Reduces output; reduces value of collateral
  - Reduces borrowing, which reduces output further
  - "Multiplier" effects amplifies losses

# Mechanism Summary



# Agents

- Farmers. measure 1

$$E_t \sum_{s=0}^{\infty} \beta^s x_{t+s}$$

- Gathers, measure  $m$

$$E_t \sum_{s=0}^{\infty} \beta'^s x'_{t+s}$$

- Farmers *more* impatient ( $\beta < \beta'$ )  
(will imply that Farmers are the borrowers in equilibrium)
- Both use land  $k_t$  to produce fruit
- Value of land  $k_t q_t$  used as collateral

# Farmers

- Farmers' production function for fruit

$$y_{t+1} = (a + c)k_t$$

$ak_t$  = sellable fruit

$ck_t$  = "bruised fruit" which must be consumed

- Assume  $c > a(\frac{1}{\beta} - 1)$

(in equilibrium farmer wants to consume  $ck_t$  and sell  $ak_t$ )

## Farmers (constrained)

- Can borrow  $b_t$  at rate  $R$
- Borrowing Constraint (from inalienability of farmers' human capital)

$$Rb_t \leq q_{t+1}k_t$$

- Farmers' "flow of funds" constraint

$$(a + c)k_{t-1} + b_t + q_t k_{t-1} = x_t + Rb_{t-1} + q_t k_t$$

$x_t$  is consumption of fruit

## Gatherers (unconstrained)

- They do not have specific skills to threat not paying.
- Gatherers' production function for fruit

$$y'_{t+1} = G(k'_t)$$

$G(\cdot)$  has decreasing returns to scale

- Gatherers' budget constraint

$$G(k'_{t-1}) + b'_t + q_t k'_{t-1} = x'_t + Rb'_{t-1} + q_t k'_t$$

$x'_t$  is consumption of fruit

# Equilibrium

- Sequences of land prices, allocations of land, debt, consumption for farmers and gatherers

$$\{q_t, k_t, k'_t, b_t, b'_t, x_t, x'_t\}$$

such that everyone's optimizing and markets clearing.

- No uncertainty: perfect foresight



## Equilibrium Results: Farmers

- Farmers always borrow the maximum and invest in land

$$b_t = q_{t+1}k_t/R \quad \text{and} \quad x_t = ck_{t-1}$$

- Implied optimal land holdings

$$k_t = \frac{1}{q_t - q_{t+1}/R} \underbrace{[(a + q_t)k_{t-1} - Rb_{t-1}]}_{\text{net worth}}$$

$$u_t \equiv q_t - q_{t+1}/R = \text{"down payment"}$$

- Farmers spend entire net worth on difference between price of new land  $q_t$  and amount against which they can borrow against each unit of land  $q_{t+1}/R$*

## Equilibrium Results: Gatherers

- Gatherer's demand for land

$$G'(k'_t)/R = u_t = \underbrace{q_t - (q_{t+1}/R)}_{\text{user cost}}$$

## Farmers in the Aggregate

- Farmer aggregate landholding & borrowing

$$K_t = \frac{1}{u_t} [(a + q_t)K_{t-1} - RB_{t-1}]$$

$$B_t = \frac{1}{R} q_{t+1} K_t$$

# Market Clearing

- Land market resource constraint

$$mk'_t + K_t = \bar{K}$$

- Land market clearing

$$u_t = q_t - q_{t+1}/R = G' \left( \underbrace{\frac{1}{m}(\bar{K} - K_t)}_{k'} \right) / R$$

- No bubbles in land price:  $\lim_{s \rightarrow \infty} E_t(R^{-s} q_{t+s}) = 0$

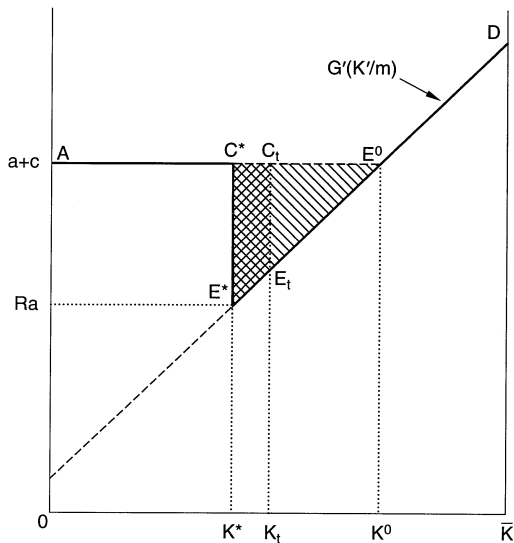
# Steady State

$$u^* = (1 - 1/R)q^* = a$$

$$u^* = G' \left( \frac{1}{m}(\bar{K} - K^*) \right) / R$$

$$(R - 1)B^* = aK^*$$

## Steady State



## One-time Productivity Shock *with Credit Constraints*

- Say  $y_{t+1} = (1 + \Delta)(a + c)k_t$
- Period of shock (period  $t$ )

$$u(K_t)K_t = (a + \Delta a + q_t - q^*)K^*$$

- Subsequent periods (periods  $t + s$ ,  $s = 1, 2, \dots$ )

$$u(K_t)K_t = (a + \Delta a + q_t - q^*)K^*$$

## One-time Productivity Shock *with Credit Constraints*

- Log-linearize around steady state
- Define for variable  $X_t$  the proportional change from steady state

$$\hat{X}_t = \frac{X_t - X^*}{X^*}$$

- Period of shock (period  $t$ )

$$(1 + 1/\eta)\hat{K}_t = \Delta + \frac{R}{R-1}\hat{q}_t$$

- Subsequent periods (periods  $t + s$ ,  $s = 1, 2, \dots$ )

$$(1 + 1/\eta)\hat{K}_{t+s} = \hat{K}_{t+s-1}$$

where  $\eta$  denotes elasticity of land supply of gatherers to user cost



## Response of Land Price & Land Holdings

- Land price response

$$\hat{q}_t = \frac{1}{\eta} \Delta$$

- Overall land holding response

$$\hat{K}_t = \underbrace{\frac{1}{1 + \frac{1}{\eta}} \left( 1 + \frac{R}{R-1} \frac{1}{\eta} \right)}_{>1} \Delta$$

## Response of Land Price & Land Holdings

- Land price response

$$\hat{q}_t = \frac{1}{\eta} \Delta$$

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$$\hat{K}_t = \underbrace{\frac{1}{1 + \frac{1}{\eta}} \left(1 + \frac{R}{R-1} \frac{1}{\eta}\right)}_{>1} \Delta$$

- Say  $\eta = 1$ ,  $R = 1.05$

$$\hat{K}_t \approx 11\Delta$$

## Static Response of Land Price & Land Holdings

- Land price response

$$\hat{q}_t|_{q_{t+1}=q^*} = \frac{1}{\eta} \underbrace{\frac{R-1}{R}}_{<1} \Delta$$

- Overall land holding response

$$\hat{K}_t|_{q_{t+1}=q^*} = \Delta$$

## Response of Output & Productivity

$$\hat{Y}_{t+s} = \underbrace{\frac{a+c-Ra}{a+c}}_{\text{Productivity diff.}} \underbrace{\frac{(a+c)K^*}{Y^*}}_{\text{Farmers' share}} \hat{K}_{t+s-1}$$

# Net Worth Shock

- One time reduction in debt obligations
- Increases net worth
- Farmer increases leverage, production
- Another view of Bernanke-Paulson policies?

## One-time Productivity Shock *at First-Best Steady State*

- Say  $y_{t+1} = (1 + \Delta)(a + c)k_t$
- Output rises by  $\Delta$
- Net worth rises
- But prices  $q^0$  unaffected; land  $k^0$  unaffected
- No change to future variables

# Conclusions

- Firms' productive capital also used as collateral
- Amplification of real shocks through lower collateral value of capital
- Real effects of lower asset values

## Critiques/Comments

- Kocherlakota (QR, 2000): Quantitative importance likely to be small if land & capital share less than 0.4
- Andres Arias (WP, 2005): Calibrated RBC model with KM credit constraints deliver small amplification effects
- Does this work through "investment wedge?" or TFP, or both?
- *Real effects of housing/stock bubbles*