Advanced Macroeconomics I

ECON 525a - Fall 2009

Yale University

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Week 5 - Bubbles

Introduction

- Why a rational representative investor model of asset prices does not generate bubbles?
- Martingale property: LIE (Law of iterated expectations).

Introduction

- Why a rational representative investor model of asset prices does not generate bubbles?
- Martingale property: LIE (Law of iterated expectations).
- This is not the case with heterogeneity, since in general, average expectations fail to satisfy LIE.
- When private information is heterogeneous, agents rely excessively in public signals. Hence
  - Mean price path deviates from consensus liquidation values
  - Prices exhibit inertia.

Fail of LIE with heterogeneous information

- LIE with private information

$$E_{it}\left(E_{i,t+1}(\theta)\right) = E_{it}\left(\theta\right)$$

- LIE with public information

$$E_{t}^{*}\left(E_{t+1}^{*}(\theta)\right) = E_{t}^{*}\left(\theta\right)$$

- LIE fail in averages with asymmetric information

$$\overline{E}_{t}\left(\overline{E}_{t+1}(\theta)\right) \neq \overline{E}_{t}\left(\theta\right)$$

Basics

- Information at all dates:
  - $\theta \sim \mathcal{N}(y, \frac{1}{\alpha})$
  - Signals: $x_i = \theta + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \frac{1}{\beta})$

- Average expectation of average expectations.

$$\overline{E}_t^{T-t}(\theta) \equiv \overline{E}_t\left(\overline{E}_{t+1}\left(...\overline{E}_{T-1}(\theta)\right)\right) = \left(1 - \left(\frac{\beta}{\alpha+\beta}\right)^{T-t}\right) y + \left(\frac{\beta}{\alpha+\beta}\right)^{T-t} \theta$$

- See that

$$\overline{E}_t^{T-t}(\theta) \neq \overline{E}_t(\theta) = \left(1 - \left(\frac{\beta}{\alpha+\beta}\right)\right) y + \left(\frac{\beta}{\alpha+\beta}\right) \theta$$

No learning through prices

- If

$$p_t = \overline{E}_t(p_{t+1})$$

then

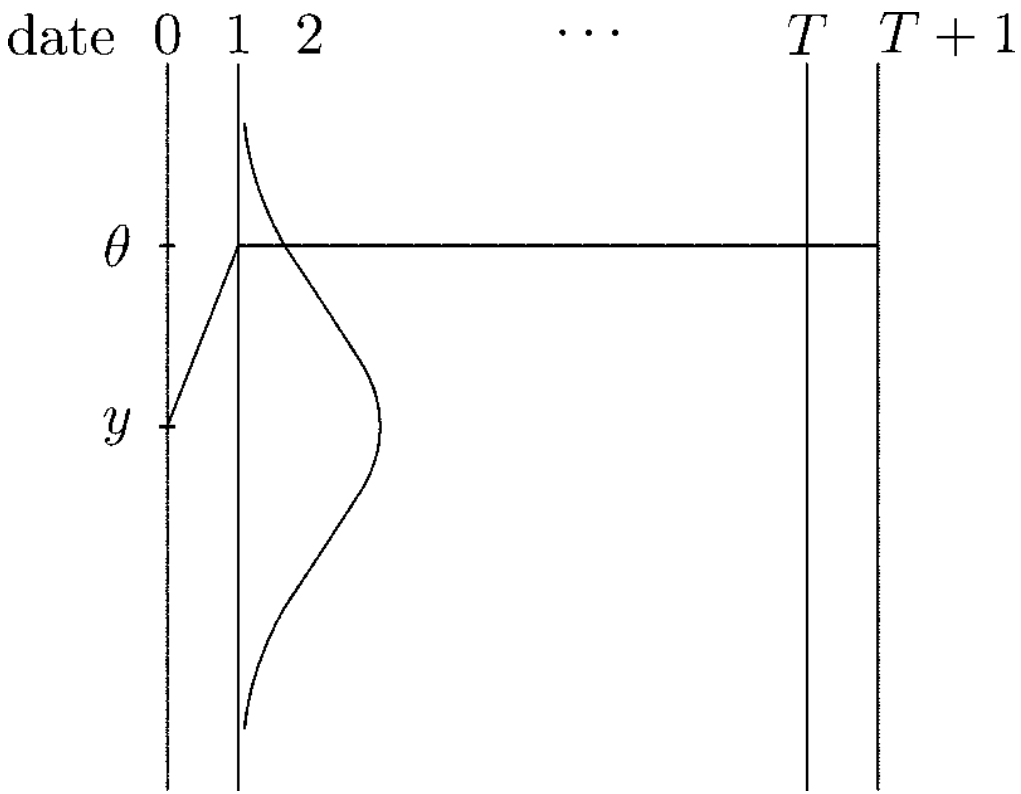
$$p_t = \left(1 - \left(\frac{\beta}{\alpha+\beta}\right)^{T-t}\right) y + \left(\frac{\beta}{\alpha+\beta}\right)^{T-t} \theta$$

- How to obtain the equation for p_t?
- How to deal with learning from past prices?

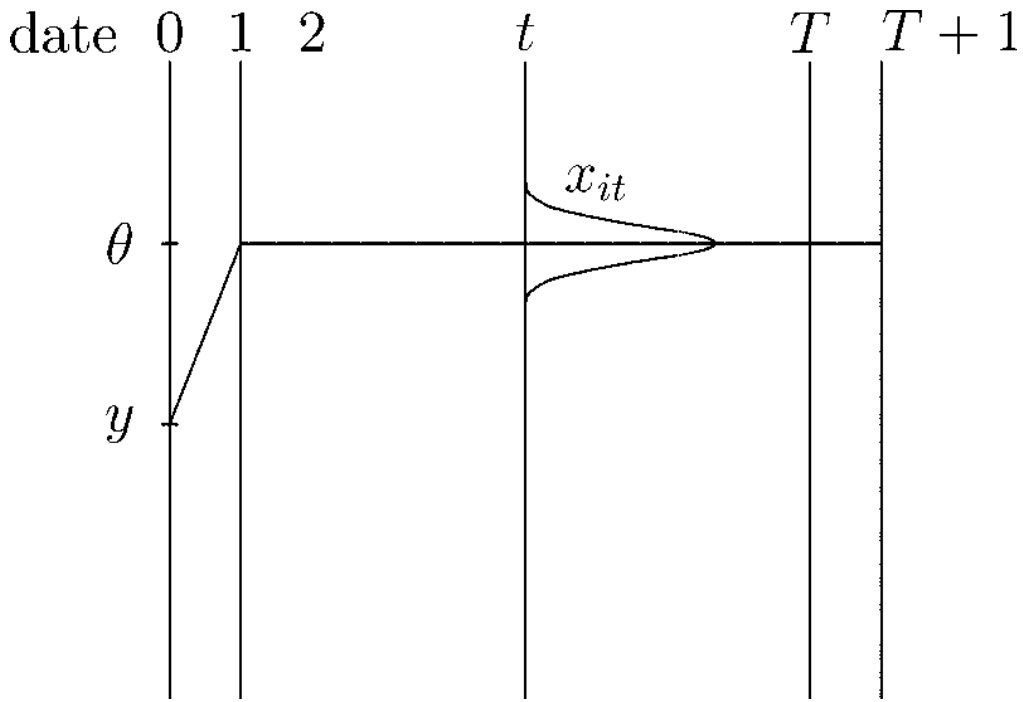
Model

- Single risky asset, liquidated at $T+1$ but traded from 1 to T.
- Liquidation value θ is determined before date 1. $\theta \sim \mathcal{N}(y, \frac{1}{\alpha})$
- Overlapping generation of no wealth constrained traders, each living for two periods and consuming in the second period. $u(c) = -e^{-\frac{c}{\tau}}$
- Information set: $\{y, p_1, p_2, .., p_t, x_{it}\}$ where $x_{it} = \theta + \epsilon_{it}$ and $\epsilon_{it} \sim \mathcal{N}(0, \frac{1}{\beta})$
- Each period exogenous net supply of assets $s_t \sim \mathcal{N}(0, \frac{1}{\gamma})$

Path of fundamental value



Private Information



Price at date T

- Trader i's demand at date T

$$D_{iT} = \frac{\tau}{V_{iT}(\theta)} \left(E_{iT}(\theta) - p_T \right)$$

- Market clearing is given by

$$D_T = \frac{\tau}{V_T(\theta)} \left(\overline{E}_T(\theta) - p_T \right) = s_T$$

- Then, the price at date T is

$$p_T = \overline{E}_T(\theta) - \frac{V_T(\theta)}{\tau} s_T$$

Price at date t

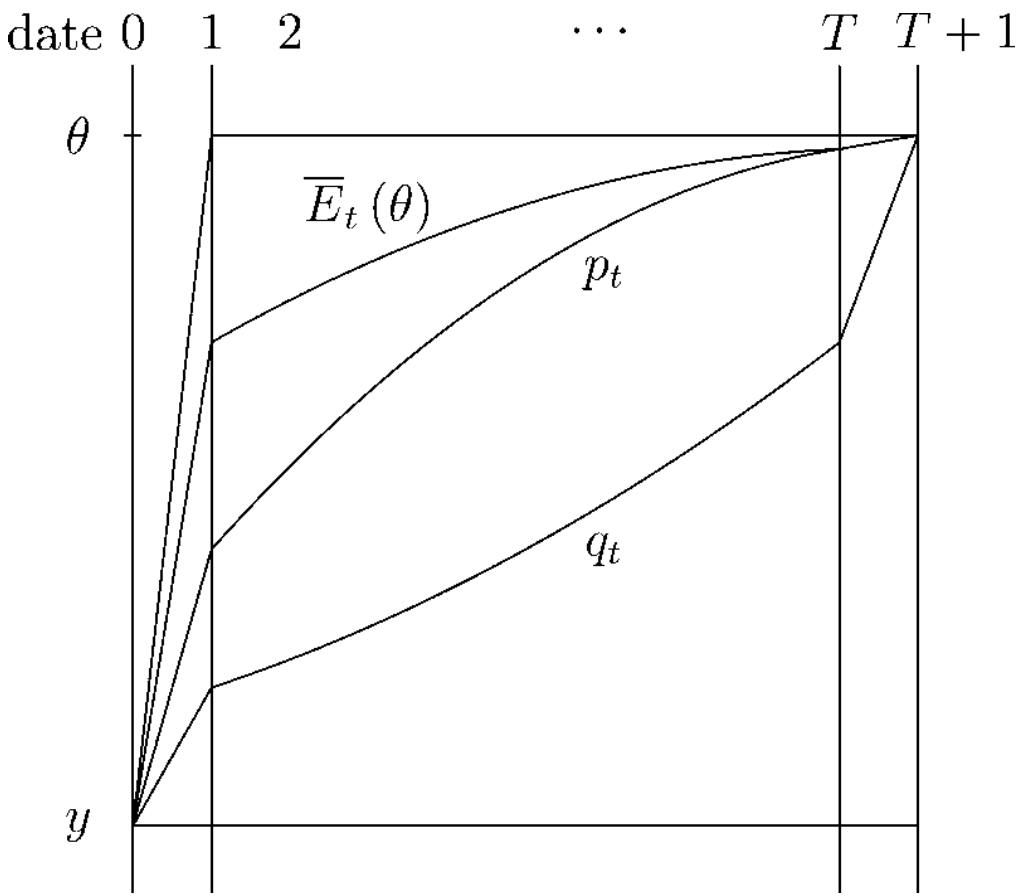
- The asset price at date $T-1$ is

$$p_{T-1} = \overline{E}_{T-1}(p_T) - \frac{V_{T-1}(p_T)}{\tau} s_{T-1} = \overline{E}_{T-1}\overline{E}_T(\theta) - \frac{V_{T-1}(p_T)}{\tau} s_{T-1}$$

- The asset price at a general date t is

$$p_t = \overline{E}_t \overline{E}_{t+1} ... \overline{E}_T(\theta) - \frac{V_t(p_{t+1})}{\tau} s_t$$

Main results



Main results

- Prices deviate systematically from the average expectation of the fundamental value of the asset.
- Inertia of prices.

- Intuition: Excessive weight assigned to the public signal y and previous prices.

Main results

- For risk neutral traders or infinitely precise signals, prices are fully revealing of the fundamental value. This is $p_t \longrightarrow \overline{E}_t(\theta) \neq \theta$.

- As investors become very risk averse ($\tau \longrightarrow 0$), they are less aggressive and prices are less informative. This is $p_t \longrightarrow q_t$

Main ideas

- Rational arbitrageurs may know the price of an asset exceeds the fundamental and still decide not to sell.
- The key is they do not know when the bubble will burst, where it is required a critical mass of speculators to do it.
- Main elements for this to work:
  - Dispersion of opinions among arbitrageurs.
  - Need for coordination.