Advanced Macroeconomics I

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Week 6 - Global Games

Main ideas

- Many models based on indeterminacy and sunspots, assuming
  - Economic fundamentals are common knowledge.
  - Agents are certain about each other's behavior in equilibrium.
- Payoffs depend on actions, motivated by beliefs.
- Global Games: Uncertainty about others' beliefs lead to uniqueness.
- One's beliefs are pinned down by the knowledge of fundamentals and that other agents are rational.

Application - Diamond and Dybvig

- Standard Diamond-Dybvig model with the following simplifying assumptions.
  - Discount rate is 1. Only the bank can invest in the illiquid project.
  - Illiquid project generates $R > 1$ at period 2.
  - If a proportion x are withdrawn in period 1, the rate of return is reduced to $Re^{-\ell}$
  - If $0 < r = log(R) < 1$, the rate of return can be written as $e^{r-\ell}$
- From the social optimum, $c_1^* = 1$ and $c_2^* = r$

Payoffs and Multiplicity

- Multiple equilibria

	Withdraw	NOT withdraw
$\ell = 0$	0	$r > 0$
$\ell = 1$	0	$r - 1 < 0$

Uncertainty and Uniqueness

- Suppose $r \sim \mathcal{N}(\bar{r}, \frac{1}{\alpha})$, where $0 < \bar{r} < 1$
- Imprecise signals about r: $x_i = r + \epsilon_i$ where $\epsilon_i \sim \mathcal{N}(0, \frac{1}{\beta})$
- From Bayesian rule, the updated belief upon observing x_i is

$$\rho_i = E(r|x_i) = \frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta}$$

- Furthermore, the ex-post distribution of r is,

$$r|\rho_i \sim \mathcal{N}\left(\rho_i, \frac{1}{\alpha + \beta}\right)$$

Uncertainty and Uniqueness

- However, more than updating the fundamental it is also important to infer beliefs (and hence actions) of others.
- Others' signals ($x_j = r + \epsilon_j$), conditional on updated beliefs about fundamentals are

$$x_j | \rho_i \sim \mathcal{N}\left(\rho_i, \frac{1}{\alpha+\beta} + \frac{1}{\beta}\right)$$

Main question

- When a depositor i has posterior belief ρ_i, what is the probability that i attaches to some other depositor j have a posterior belief lower than himself?

$$
\begin{aligned}
Pr(\rho_j < \rho_i | \rho_i) &= Pr\left(x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) | \rho_i\right) \\
&= \Phi(\sqrt{\gamma}(\rho_i - \bar{r}))
\end{aligned}
$$

where

$$
\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}
$$

Uniqueness

Proposition

Provided that $\gamma \leq 2\pi$, there is a unique equilibrium where every patient agent withdraws if and only if $\rho < \rho^*$, where

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r}))$$

In the limit, as $\gamma \to 0$, $\rho^* \to \frac{1}{2}$

Uniqueness

- ρ^* is the switching point at which the agent is indifferent between withdraw or not. This is, $E_r\left[r - \ell|\rho^*\right] = 0$

$$E_r(r|\rho^*) = Pr(\rho_j < \rho^*|\rho^*)$$

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r}))$$

Uniqueness

- The equilibrium will be unique as long as the slope of $\Phi(\sqrt{\gamma}(\rho^* - \overline{r}))$ is less than 1.
- This slope is just the density and achieves a maximum of $\sqrt{\frac{\gamma}{2\pi}}$ at $\overline{r}$.
- The sufficient condition for uniqueness is then $\gamma \leq 2\pi$, which happens when β is big enough with respect to α

Observable implications

- A depositor withdraws whenever $\rho_i < \rho^*$. This means.

$$x^*(\rho^*, \overline{r}) = \frac{\alpha + \beta}{\beta}\rho^* - \frac{\alpha}{\beta}\overline{r}$$

- Hence, we can obtain the equilibrium fraction of withdraws for each realization r

$$\ell(r) = Pr(x_i < x^*(\rho^*, \overline{r})|r) = \Phi\left(\sqrt{\beta}(x^*(\rho^*, \overline{r}) - r)\right)$$

Observable implications

- Gorton (1988), for example, shows fundamentals play a key role in explaining bank runs.
- Withdrawal is high when the return is low.
- Payoff relevant fundamentals generate self-fulfilling equilibrium.

Applications

- Bank runs
- Currency crises
- Riots
- Risk Taking in Credit Markets
- Debt Pricing

Limitations

- Aggregation of information through prices (Atkeson, 00)