Supplementary Material

Network Reactions to Banking Regulations

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Formal Analysis of the Phase Transition.

Define $\mathcal{B}(\phi) = BL(\phi) - \alpha\kappa_l'$ and $\varepsilon(d|\phi) = (1-\alpha)Pr_G(R(d|\phi), d-1, 1-\alpha)$. By rearranging terms,

$$V(d|\phi) = \mathcal{B}(\phi) \times \left(d + \frac{AL(\phi)}{\mathcal{B}(\phi)}\right) \times \left(G\big(R(d|\phi), d, 1-\alpha\big) - \frac{\kappa_l}{\mathcal{B}(\phi)}\right)$$
$$+\frac{AL(\phi)\kappa_l}{\mathcal{B}(\phi)} + dBL(\phi)\varepsilon(d|\phi)$$

The term with $\varepsilon(d|\phi)$ is negligible compared to the other terms. $\frac{AL(\phi)\kappa_l}{\mathcal{B}(\phi)}$ is constant with respect to d. As ϕ increases, $\mathcal{B}$ transits from positive to negative at the threshold $\tilde{\phi} = \frac{BD}{\alpha\kappa_l'}$. Around this threshold, the optimal d potentially changes dramatically.

The first problem with a formal proof of the characterization of $d^*(\phi)$ is that it is not obvious if (and how) the change happens as the signs of A, $k - 1 - \frac{AL(\phi)}{\mathcal{B}(\phi)}$ and $1 - \frac{\kappa_l}{\mathcal{B}(\phi)}$ all matter for the full characterization of the solution.

The second problem comes from the discrete structure, which gives rise to the term $\varepsilon(d|\phi)$. Under appropriate assumptions d^* can be characterized using the Chernoff Bounds on tails of the binomial CDF, however, the term with $\varepsilon(d|\phi)$ makes a clean proof cumbersome. Without using Chernoff Bounds, or alike, the discreteness in R also makes analyzing G cumbersome. As d changes, G typically evolves monotonically except at the points that R changes with d.

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The third problem is about the constant term that contains A. When $A \neq 0$, the expression for $R(d|\phi)$ does not linearly scale with d, making the use of Chernoff bounds intractable.

We manage to overcome the first two problems when $A = 0$. Simulations verify the intuition concerning phase transition extends to $A \neq 0$ as well. Moreover, notice that $A = 0$ is the boundary between isolated banks continuing or not continuing so the result encompasses both cases.

The resilience of a bank can be written as

$$
\begin{aligned}
R(d|\phi) = d - 1 - S(d|\phi) &= d - \lceil (F^{-1}(T(d|\phi)) - D - W)/W \rceil \\
&= d - \frac{F^{-1}(T(d|\phi)) - D - W}{W} - \epsilon_1(d|\phi) \\
&= d - d\frac{P\alpha\kappa_l'}{W\alpha\beta L(\phi)} - \frac{PT_0 - S - W}{W} - \epsilon_1(d|\phi) \\
&= d\left(1 - \alpha + \mathcal{B}(\phi)\frac{P}{W\alpha\beta L(\phi)}\right) + \frac{AP}{\alpha\beta W} - \epsilon_1(d|\phi) \\
&= d\left(1 - \alpha + \mathcal{B}(\phi)\frac{P}{W\alpha\beta L(\phi)}\right) + \epsilon_1(d|\phi)
\end{aligned}
$$

where $\epsilon_1(d|\phi) \in [0, 1)$ is a ceiling term that guarantees that $R(d|\phi)$ is an integer.

Proposition 1. *Suppose that $A = 0$. $d^*(\phi) = 0$ for all $\phi > \frac{BD}{\kappa^*}$ where*

$$\kappa^* = max\left\{\kappa_l, \alpha^2\kappa_l', min\left\{\frac{\kappa_l}{1-\alpha}, \alpha\kappa_l'\right\}\right\}.$$

Proof. The condition is equivalent to $\kappa^* > BL$ (we can ignore $\bar{L}$ since it is sufficiently large). If $BL < \alpha^2\kappa_l$ then $R < 0$ which implies that the terms with G in V are equal to 1. Then $V = d(\mathcal{B} - \kappa_l) \leq 0$ for all d. Hence V is maximized at 0. If $BL < \kappa_l$, then $V \leq d(\kappa_l + BL) \leq 0$ for all d. So V is maximized at $d = 0$. If $BL < \frac{\kappa_l}{1-\alpha}$ and $BL < \alpha\kappa_l'$, then $V \leq d\big(-\kappa_l + \mathcal{B}G(R(d), d, 1-\alpha) + BL\varepsilon(d|\phi)\big) \leq d(-\kappa_l + BL(1-\alpha)) \leq 0$ for all d. Thus again, V is maximized at $d = 0$. □

Proposition 2. *Suppose that $A = 0$. $d^*(\phi) = k - 1$ for all $\phi < \frac{BD}{\kappa^{**}}$ where*

$$\kappa^{**} = max\left\{\frac{\alpha\kappa_l'}{1 - \frac{\alpha}{1-\alpha}\left(\frac{\varepsilon' + \sqrt{\varepsilon'^2 + 8\varepsilon'}}{2}\right)},\right.$$

$$\left.\frac{\alpha\kappa_l' + \kappa_l}{1 - (k-1)Exp\big[-(1-\alpha)(k-2)\varepsilon'\big]}, (1.15)\alpha\kappa_l' + \kappa_l\right\}$$

for some arbitrarily chosen $\varepsilon' > 0$.

Proof. $V = d(\kappa_l - \alpha\kappa_l' G(R, d, 1-\alpha) + BLG(R, d-1, 1-\alpha)) \leq d(\mathcal{B} - \kappa_l + \alpha\kappa_l'\zeta)$ where $\zeta = 1 - G(R, d, 1-\alpha)$. The Chernoff bounds imply that $\zeta \leq \chi = Exp\big(-\frac{\delta_L^2}{2+\delta_L}\mu_d\big)$ where $\mu_d = d(1-\alpha)$ and δ_L is given by $(1+\delta_L)\mu_d = R$ (i.e. $\delta_L = \frac{\alpha}{1-\alpha}\big(1 - \frac{\alpha\kappa_l'}{BL}\big)$).[1] [2]

We first show that if $\frac{\mathcal{B}-\kappa_l}{\alpha\kappa_l'} > 0.15$, then the upper bound for V, $d\big(\mathcal{B} - \kappa_l + \alpha\kappa_l'\chi\big)$, is increasing in d. The derivative of the continuum version of the upper bound in d is $\mathcal{B} - \kappa_l + \alpha\kappa_l'\chi(1-m)$, where $m = d\frac{\delta_L^2(1-\alpha)}{2+\delta_L}$. Notice that $\chi = Exp(-m)$. Also note that $\frac{m'-1}{Exp(m')} < 0.15$ for any value of $m' > 0$. Hence $\frac{\mathcal{B}-\kappa_l}{\alpha\kappa_l'} > 0.15 > (m-1)\chi$ meaning that the derivative is positive.

Then for any $d \leq k-2$. $V(d) \leq d\big(\mathcal{B} - \kappa_l + \alpha\kappa_l'\chi_d\big) \leq (k-2)\big(\mathcal{B} - \kappa_l + \alpha\kappa_l'\chi_{k-2}\big)$ Now we find an appropriate lower bound for $V(k-1)$ and show that $V(d) < V(k-1)$ for any $d \leq k-2$. Since $\varepsilon(d|\phi \geq 0)$, $V(k-1) \geq (k-1)(\kappa_l + \mathcal{B} - \mathcal{B}\chi_{k-1})$. That means we need to prove $\alpha\kappa_l'(k-2)\chi_{k-2} + \mathcal{B}(k-1)\chi_{k-1} \leq \mathcal{B} - \kappa_l$. χ_d is decreasing since $R_d > \mu_d$. Thus, a sufficient condition is $(k-1)\chi_{k-2}(\mathcal{B} + \alpha\kappa_l') \leq \mathcal{B} - \kappa_l$. Now we prove that if $\phi < \frac{BD}{\kappa^{**}}$, the condition holds.

The sufficient condition is equivalent to

$$\frac{\alpha\kappa_l' + \kappa_l}{BL} + (k-1)Exp\bigg(-(1-\alpha)(k-2)\frac{\delta_L^2}{2+\delta_L}\bigg) \leq 1.$$

Both of the summands are in increasing in L. Then if $\frac{\delta_L^2}{2+\delta_L}$ is less than ε' and also

$$\frac{\alpha\kappa_l' + \kappa_l}{BL} + (k-1)Exp\bigg(-(1-\alpha)(k-2)\varepsilon'\bigg) \leq 1,$$

the sufficient condition holds. The first bound on ϕ imples the first (by solving the quadratic), and the second bound on ϕ implies the second. (The third bound on ϕ corresponds to the condition guaranteeing that the upper bound on $V(d)$ is increasing.) □

[1] Here $A = 0$ plays a key role. Otherwise, δ would depend on d and it would make the analysis more complicated.

[2] Here we have ignored the term $\epsilon_1 \in [0, 1)$ which makes no difference to the analysis but it is very costly in notation to explore.