Lecture Notes

Macroeconomics - ECON 510a, Fall 2010, Yale University

Real Business Cycles

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1 Introduction

Now we know how a neo-classical model without frictions work, a natural question is whether we can explain business cycles fluctuations in such an environment. If so, there are dramatic consequences in terms of recommendations for stabilization policies.

If the benchmark model without frictions can replicate the stylized facts of the business cycle, the cycles may be just an efficient phenomenon generated by markets in response to certain shocks, such as, for example, productivity shocks. Furthermore, if markets are complete and individuals can diversify risk, Lucas (1987) calculated the welfare cost of the aggregate fluctuations to be negligible.

This is important from a policy perspective since it does not only mean economic fluctuations may be efficient and then stabilization policies are at best irrelevant, but also that the welfare costs of these fluctuations are not important in the first place.

2 Stylized Facts

Since our interest here focuses on variations at business cycle frequencies, the following facts (from King and Rebelo, 1999) are obtained after detrending the series (using a Hodrick-Prescott filter).

1. $var(C) < var(Y) < var(I)$.
2. $var(N) \approx var(Y)$, where $N = nl$, n is employment and l is hours per worker.
3. $var(l) < var(Y)$ and $var(n) \approx var(Y)$.
4. $var(w) < var(Y/N) < var(Y)$.
5. The first order autocorrelation is high for all variables.
6. The contemporaneous correlation with output is generally positive (except for interest rates).

3 A Basic RBC model

This is a baseline RBC model to study the variances that characterize the evolution of aggregate variables over the business cycle. Production is given by

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}$$

where A_t denotes the state of technology in period t. Recall this is a different technology than the one considered in our neo-classical benchmark. In this case this is a labor enhancing productivity.

The transition equation for the capital stock is

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + Y_t - C_t$$

where a fraction of capital δ depreciates between periods and the second equality uses the fact that $Y_t = C_t + I_t$ in the goods market equilibrium.

We have a representative household, whose preferences are given by

$$U_t = \sum_{\tau=t}^{\infty} \sum_{s^\tau | s^t} \beta^\tau \pi(s^\tau)[ln(C_\tau) + bln(1 - N_\tau)]$$

Recall that, differently than in our benchmark, where we denoted consumption and work with small letters, here we're using capital letters since we have a representative household, which can be interpreted as the aggregate consumption and employment if there is a continuum of those households in the economy, with mass 1. Naturally, each household takes prices and the aggregate evolution of capital as given when making decisions, but their aggregate actions determine prices and the aggregate evolution of capital, since all of them are identical.

We assume the evolution of technology is given by

$$\ln A_t = \gamma \ln(A_{t-1}) + \epsilon_t$$

where ϵ_t is an i.i.d. random shock with mean zero.

There is perfect competition for labor and capital. We are not setting explicitly the static problems for firms maximizing their profits, however we know from the benchmark the factors of production are paid their marginal products. Introducing the assumed functional forms and getting rid of explicit notation about different states of the world, factor demands are given by

$$w_t = (1 - \alpha) \left(\frac{K_t}{A_t N_t} \right)^\alpha A_t = (1 - \alpha) \frac{Y_t}{N_t}$$

$$r_t = \alpha \left(\frac{A_t N_t}{K_t} \right)^{1-\alpha} = \alpha \frac{Y_t}{K_t}$$

where $1 + r_t = q_t^{t+1}$. From the household's maximization, using our benchmark results and introducing the functional assumptions, wages should be equal to the marginal rate of substitution between consumption and labor.

$$w_t = \frac{bC_t}{1 - N_t}$$

The Euler equation (that equalizes marginal utility of consumption across periods) between period t and $t-1$ can be expressed as

$$\frac{1}{C_t} = \beta E_t \left[\frac{1 + r_{t+1} - \delta}{C_{t+1}} \right]$$

what can be also expressed in terms of labor

$$\frac{1}{1 - N_t} = \beta w_t E_t \left[\frac{1 + r_{t+1} - \delta}{w_{t+1}(1 - N_{t+1})} \right]$$

As it stands, even when assuming simple functional forms for preferences and technologies, the models cannot be solved analytically. In the problem set, you will be asked to solve a version of this model, using different computational methodologies.

3.1 A Special Case

There is one special case in which actually the model can be be solved analytically. This is the case with full depreciation (this is, $\delta = 1$).

Let's start by guessing the propensity to save is constant and then let's verify this is in fact the case. If the propensity to save is constant (this is, $I_t = sY_t$) we have

$$C_t = (1 - s)Y_t$$

Further, since the capital market is in equilibrium (savings equal investment) and capital depreciates immediately, $K_{t+1} = I_t$, then

$$K_{t+1} = sY_t$$

From labor markets clearing,

$$w_t = (1 - \alpha)\frac{Y_t}{N_t} = \frac{bC_t}{1 - N_t}$$

Then,

$$\frac{(1 - \alpha)\frac{Y_t}{N_t}}{(1 - s)Y_t} = \frac{b}{1 - N_t}$$

Thus, if s is constant, then so is labor supply. We get,

$$\frac{(1-\alpha)}{(1-s)} = b\frac{N_t}{1-N_t}$$

Now, we can make the capital market to clear, assuming $\delta = 1$,

$$\frac{1}{(1-s)Y_t} = \beta E_t\left[\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{(1-s)Y_{t+1}}\right] = \beta E_t\left[\frac{\alpha}{s(1-s)Y_t}\right] = \frac{\beta\alpha}{s(1-s)Y_t}$$

which is only fulfilled when

$$s = \beta\alpha$$

Naturally, this solution also satisfies the Euler equation expressed in terms of labor. Check this out!

Now we know s, we can solve for N_t from the market clearing condition

$$N = \frac{1-\alpha}{(1-\beta\alpha)b + (1-\alpha)} < 1 \quad \forall t$$

The, when $\delta = 1$, there is a constant savings rate that solves the model. In this simple case we can still recover some implications.

1. Since $I_t = sY_t$, $var(I_t) = s^2 var(Y_t)$. This implies $var(I_t) < var(Y_t)$, contrary to the stylized fact.

2. $var(N_t) = 0$ is another implausible feature of this simple model. Notice we get this result despite the fact that households are willing to substitute labor intertemporally. The results is due to the offsetting movement of technology and capital. Intertemporal substitution is determined by $E_t\left[\frac{w_t}{w_{t+1}}(1 + r_{t+1} - \delta)\right]$. Suppose there is a positive technology shock today. Then w_t increases relative to $E_t(w_{t+1})$; given r_{t+1} this would tend to increase N_t relative to N_{t+1}. However, a positive technology shock causes firms to acquire more capital so r_{t+1} decreases. These changes exactly offset one another such that labor supply is constant. The implication of constant labor supply is that wages covary too much with output.

3. Log-output follows a second order autoregressive process (AR2). It is sometimes argued that an AR2 representation is a reasonable characterization for

log output. To see that the model generates an AR2 process, remember that $K_t = sY_{t-1}$. Use this to eliminate K in the production function and take logs of $Y_t = K_t^{\alpha}(A_t N_t)^{(1-\alpha)}$

$$\ln Y_t = \alpha \ln s + (1-\alpha)\ln N + \alpha \ln Y_{t-1} + (1-\alpha)\ln A_t \tag{1}$$

Define $\kappa = \alpha \ln s + (1-\alpha)\ln N$, lag one period and solve for $(1-\alpha)\ln A_{t-1}$,

$$(1-\alpha)\ln A_{t-1} = \ln Y_{t-1} - \kappa - \alpha \ln Y_{t-2} \tag{2}$$

Plugging the evolution of technology ($\ln A_t = \gamma \ln(A_{t-1}) + \epsilon_t$) into (1)

$$\ln Y_t = \alpha \ln s + (1-\alpha)\ln N + \alpha \ln Y_{t-1} + (1-\alpha)[\gamma \ln A_{t-1} + \epsilon_t]$$

and using equation (2)

$$\ln Y_t = \kappa(1-\gamma) + (\alpha+\gamma)\ln Y_{t-1} - \gamma\alpha \ln Y_{t-2} + (1-\alpha)\epsilon_t$$

Calling $\varepsilon_t = (1-\alpha)\epsilon_t$ and expressing the above equation in terms of deviation from steady state, we have

$$y_t = (\alpha+\gamma)y_{t-1} - \gamma\alpha y_{t-2} + \varepsilon_t$$

where $y = \ln Y - \ln \bar{Y}$ and $\ln \bar{Y}$ denotes the steady state level of log output. The model has the potential of generating "hump-shaped" responses to shocks. However, the dynamics of the model is crucially dependent on the size of γ. The model has no independent dynamics on its own. Response dynamics is basically equivalent to impulse dynamics. This is a feature of all RBC models based on competitive markets.

Just to a simple practice, consider the evolution of output given that $\alpha = 1/3$ and $\gamma = 0.9$, common numbers used in this literature.

3.2 Solution to the general model

What happens if we allow for depreciation? Consider the other extreme, i.e. when there is no depreciation. Then there will be no investments in the absence of shocks. Suppose there is a positive technology shock. This raises the marginal product of capital so households will undertake some investments (i.e. save) in the next period. Since the savings rate rises expected consumption growth has to be higher than in the above simple case. But from the consumption Euler equation this in turn requires the expected interest rate to be higher; but an increase in interest rates makes the individual substitute labor intertemporally so employment responds to shocks in this case.

More general models than the special case are solved by means of numerical simulation. Parameter values are assigned such that the implications of the model are consistent with some long run stylized facts and micro estimates. Then you simulate a large number of realizations of the model economy, calculate the appropriate variances and covariance, and compare these variances and covariances to those actually observed in the data.

To understand the model, Romer argues in favor of looking at the impulse response functions rather than just presenting the implied variances and covariances. To calculate the impulse-response functions we want to log-linearize the model around a steady state. You will be asked to do this in the Problem Set.

3.2.1 Solution to the general model

A time period is a quarter. The following parameter values are used $\alpha = 1/3$, $\delta = 0.025$, $\gamma = 0.95$, and the steady state labor supply is $\bar{N} = 1/3$. Here go some comments,

1. The shock is, by assumption, very persistent ($\gamma = 0.95$). Capital adjusts gradually and slowly returns to normal. The evolution of output is basically the same as the assumed impulse dynamics. There is consumption smoothing so investment is more volatile than consumption.

2. Since the shock is very persistent, wages adjust smoothly. There is little intertemporal substitution in labor supply induced by wage dynamics. The movements in labor supply are driven by the interest rate.

3. Persistence in this model is generated by the persistence of technology shocks and the sluggish adjustment of capital.

3.2.2 Failures and Proposed Solutions

One of the main failures of the model is that there is too little employment variation. Potential solutions to this problem.

1. Reduce the persistence of technology shocks. This makes the wage fluctuate more and, therefore, there is more intertemporal substitution in labor supply due to wage fluctuations. With a lower value of γ however, fluctuations soon become unrealistically short and sharp.

2. Increase the intertemporal elasticity of substitution in labor supply. This can be achieved by changing preferences, for example in the following way

$$u(C_t, N_t) = \ln C_t + b\frac{(1-N_t)^{1-\theta} - 1}{1-\theta}$$

   If $\theta < 1$, then the intertemporal elasticity is greater than one. This yields more fluctuations in all variables and in employment in particular. However, micro estimates suggest that the elasticity is small and certainly less than unity, this is, for sure $\theta > 1$.

3. Recognize explicitly that the bulk of the fluctuations in total hours come from changes along the extensive margin (the number of employees) rather than the intensive margin (hours per employee). We will explore this possibility next.

3.2.3 Indivisible Labor, or Employment Lotteries

One extension of the baseline RBC model is to assume that changes in total hours come from the fact that individuals move into and out of employment. One extreme version of this assumption is to say that the individual must work a fixed number of hours (N_0 say) given that (s)he is employed; see Hansen (1985).

Let's analyze this model explicitly. The assumption are

1. Each individual either works $N = N_0$ (employed) or $N = 0$ (non-employed). (All individuals would rather work less than N_0, but because of technology they cannot do so.)

2. All individuals can be made better off by the introduction of a lottery where.

   - With probability π, they work N_0 hours and receive wages w_t.
   - With probability $(1-\pi)$ they are non-employed and yet receive w_t. (Thus, there is perfect insurance. This is extreme but can be relaxed with a utility function that is non-separable in consumption and labor supply).

3. All individuals have the same utility ex ante (i.e. prior to the lottery), but individuals who are required to work (by the draw of the lottery) are worse off ex-post. Expected utility is given by,

$$\begin{aligned} u^e &= E_t[\ln C_t + b\ln(1-N_t)] \\ &= \ln C_t + b[\pi_t \ln(1-N_0) + (1-\pi_t)\ln(1)] \\ &= \ln C_t + b\pi_t \ln(1-N_0) \end{aligned}$$

   The control variables for the agents are C_t and π_t rather than C_t and N_t as it was earlier.

4. In equilibrium, we have $N_t = \pi_t N_0$, so $\pi_t = N_t/N_0 < 1$. Therefore, preferences can be written as,

$$u^e = \ln C_t + \frac{b\ln(1-N_0)}{N_0} N_t$$

   A critical assumption is $\frac{b\ln(1-N_0)}{N_0} < 0$. Thus, instantaneous utility is linear in N_t and the intertemporal elasticity of substitution is infinite. So with this alternative formulation of the model average labor supply varies a lot more intertemporally (but the intertemporal elasticity of labor supply at the individual level is still in line with the estimates from micro data).

3.3 The Success and Failures of RBC

The major success of the RBC program is perhaps that their methodologies has become standard practice in dynamic macroeconomics. The RBC program has changed the state of play. Researchers now work with dynamic stochastic general equilibrium models with optimizing agents. Into these models they may introduce frictions, among which we will see later commitment and information problems.

Since Prescott (1986), there have been four types of objections raised against the prototype RBC model

1. Technological shocks are represented by Solow residuals. The Solow residuals (a measure of our ignorance.) are the residuals (u_t) from a regression of the following kind
$$\Delta \ln Y_t = \alpha \Delta \ln K_t + (1 - \alpha) \Delta \ln N_t + u_t$$
This is probably a poor measure of technology shocks as such. For instance, the Solow residual can be predicted using such observables as the political party of the president, military spending, and oil price movements. Moreover, the volatility of the Solow residual implies the economy will be in technological regress around 40 % of the time, which seems implausible.

2. One of the central propagation mechanisms intertemporal substitution in labor supply is empirically irrelevant. Micro estimates suggest that the intertemporal elasticity of substitution is small.

3. The evidence suggests that monetary shocks have important real effects. This suggests that nominal rigidities and imperfections are important.

4. There is no independent propagation of shocks in the model. The dynamics of assumed impulses and the response of log output are essentially the same. There is some new literature that tries to introduce financial markets as a source of magnification of real shocks and as a way to make them more persistent (Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997)).

In a kind of defence of RBC, King and Rebelo (1999) argue that some straightforward alterations of the model make it more palatable. They argue that

1. Introducing variable capital utilization significantly reduces the problems associated technological regress. With variable capital utilization even small shocks may produce fairly large responses in output. Technological regress occurs only 1 percent of the time.

2. Introducing indivisible labor increases the intertemporal elasticity of labor supply (see above).

4 And now...what?

There are still two open and important questions that have been challenging RBC models. The first is explaining the behavior of asset prices. Mehra and Prescott (85) have shown the RBC model that explains relatively well macro variables is at odds with explaining asset prices. We will see this in detail in the next lecture.

The second important and haunting question is what generates business cycles? What are those technological shocks? How is it possible that real shocks, for example have created a Great Depression or the recent financial crisis?

The main source of shocks, Prescott argues, is exogenous technological changes that affect the TFP. However, TFP shocks may not be purely technological and exogenous, but can well be endogenous (such as monetary and fiscal shocks). Furthermore there may be forces that endogenously magnify small technological shocks, such as variable capital utilization, variability in labor effort, changes in markup, etc, that introduce wedges between TFP and true technological shocks.

The reason most people is skeptic about the interpretation of TFP shocks as generated by technological shocks is that it means recessions should be times of important technological regress. There has been a growing literature proposing different sources of shocks that manifest themselves as changes in TFP. These are

1. Oil Shocks.

2. Investment Specific Technical Change.

3. Fiscal Shocks.

4. Monetary Shocks.
5. Multiple Equilibrium Models (based on externalities, increasing economies of scale and monopolistic competition). Beliefs can generate business cycles and create strong internal persistence.
6. News shocks.
7. Innovation and TFP.

Finally, let me mention an important under-exploited question is what generates the comovement of variables across different sectors that we observe in the data. Is the TFP shock common across industries? Is there an exogenous input-output link? Is there an endogenous link?