

Lecture Notes

Macroeconomics - ECON 510a, Fall 2010, Yale University

Fiscal Policy. Ramsey Taxation.

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1 Fiscal Policy

To study questions of taxation in the neo-classical growth model, we need to extend it to allow for government spending and taxation. Here we study the simplest possible model, where government spending is an exogenous stochastic process $G(s^t)$. We assume that this spending is wasted (thrown into the ocean). It would be possible to amend the model to allow for example for the financing of public goods that benefit all households, but those extensions are not key to make our main points.

The aggregate resource constraint then incorporates government spending so that

$$\sum_{j=1}^J \mu_j c^j(s^t) + K(s^t) - (1 - \delta)K(s^{t-1}) + G(s^t) \leq A(s^t)f(K(s^{t-1}), N(s^t)), \quad \forall s^t$$

Let $T(s^t)$ denote the governments tax revenue in state s^t . The government can issue complete contingent securities $B(s^t)$, so the government budget constraint given state s^t is

$$G(s^t) + B(s^t) \leq T(s^t) + \sum_{s^{t+1} \succ s^t} \frac{q(s^{t+1})}{q(s^t)} B(s^{t+1})$$

Solving this condition forward, the date 0 government budget constraint in terms of initial debt level $B(s^0)$,

$$B(s^0) \leq \sum_{s^t} \frac{q(s^t)}{q(s^0)} [T(s^t) - G(s^t)]$$

The social planning problem is the same as before, with the modified aggregate resource constraint. The decentralization of the optimal allocation in a market economy must in addition specify how the tax revenue is collected. That is the task is to find an allocation, prices and a tax system that maximizes the planner's objective (given as above), subject to the aggregate resource and the government budget constraints, and the relevant household optimality and market-clearing conditions. In order to discuss more specifically the resulting tax implications, we need to make further assumptions about what tax instruments are available.

1.1 Lump Sum Taxes

Suppose first that the government can raise funds by lump sum taxation, asking type j households to contribute $T^j(s^t)$ at event s^t , so that the total tax revenue satisfies

$$T(s^t) = \sum_{j=1}^J \mu_j T^j(s^t)$$

Then, the following striking result emerges,

Ricardian Equivalence: In the Planning problem with government spending and lump-sum taxes, the household allocations only depend on the net present value, not on the time path of their tax obligations.

Under lump-sum taxes the optimal solution to the planning problem is determined just as in the previous section. The decentralization also proceeds along the same lines, but the household budget constraints have to be modified by the tax payments:

$$a^j(s^0) = \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) [c^j(s^t) + T^j(s^t) - \chi_j D(s^t) - w^j(s^t) n^j(s^t)]$$

Therefore, changes in the tax structure that leave the net present value of each house-

holds' tax burden $\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) T^j(s^t)$ the same do not affect households' decisions.

This result is the government counter-part to the Modigliani-Miller Theorem: for a given process of government spending, the time path of lump sum taxes does not affect allocations. In other words, it is irrelevant whether the government uses current lump sum taxes or debt to finance current expenditures, since an increase in current debt is merely carried to the future as an increase in future lump sum taxes, for which households prepare by increasing their savings today.

Again, this result will serve as a benchmark for further discussions of taxation, when lump sum taxes are either not available, or undesirable due for example to distributional concerns. The result in fact crucially relies on the infinite horizon structure - in an OLG model, lowering the current taxes and increasing future taxes imposes an additional tax burden on future, yet unborn generations, which has redistributive effects. More generally, the analysis is abstracting from redistributive considerations by assuming (in the argument above) that there is no shift in tax burdens across types.

1.2 No Lump Sum Taxes: Ramsey Model

More interesting questions arise if lump sum taxes are not feasible, and the planner must raise revenue by taxing labor or capital incomes, which will inevitably distort allocations. This raises the question of how to allocate distortions across time and factors of production.

We analyze this question in the context of the so-called Ramsey Tax Model, in which the planner is restricted to linear taxes on wages and capital income. We further assume (to simplify the analysis) that $J = 1$, i.e. that we have a representative household, and we drop the index j . The tax revenue is then given by:

$$T(s^t) = \tau_w(s^t)w(s^t)n(s^t) + \tau_K(s^t)r(s^t)K(s^{t-1})$$

where $r(s^t) = A(s^t)f_K(K(s^{t-1}), n(s^t))$ denotes the firms' return on capital. For a given tax sequence $\{\tau_w(s^t), \tau_K(s^t)\}$, the equilibrium allocations are characterized by the following equations, which modify the original FOCs to account for the labor and capital

tax distortions. Firms pay capital taxes and maximize dividends.

$$\begin{aligned} \{n(s^t)\} & : \quad w(s^t) = A(s^t)f_n(K(s^{t-1}), n(s^t)) \\ \{K(s^t)\} & : \quad q(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1})\{(1 - \tau_K(s^{t+1}))A(s^{t+1})f_K(K(s^t), N(s^{t+1})) + 1 - \delta\} \end{aligned}$$

which are automatically satisfied by the planning allocation, under our proposed equilibrium prices. Households pay labor taxes and maximize utility subject to a budget constraint.

$$\begin{aligned} \{c(s^t)\} & : \quad q(s^t) = \beta^t \pi(s^t) \frac{u_c(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} \\ \{n(s^t)\} & : \quad (1 - \tau_w(s^t))w(s^t) = - \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^t); n(s^t))} \end{aligned}$$

The Ramsey planning problem consists of choosing a tax sequence $\{\tau_w(\cdot), \tau_K(\cdot)\}$, such that the resulting allocation $\{c(\cdot), n(\cdot), K(\cdot)\}$ and prices $\{w(\cdot), q(\cdot)\}$, such that $\{c(\cdot), n(\cdot), K(\cdot)\}$ and $\{w(\cdot), q(\cdot)\}$ form a competitive equilibrium given the system.

The planner maximizes the objective

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t); n(s^t))$$

and must satisfy the government's inter-temporal budget constraint

$$q(s^0)B(s^0) + \sum_{t=0}^{\infty} \sum_{s^t} q(s^t)G(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q(s^t)[\tau_w(s^t)w(s^t)n(s^t) + \tau_K(s^t)r(s^t)K(s^{t-1})]$$

Instead of considering a two-stage structure, where the planner first announces a tax system and then the households and firms determine the equilibrium, we consider a modified version of the problem, in which the planner announces a tax sequences, an allocation and a set of prices, but is subject to the additional constraint that the allocation and prices form an equilibrium. This modified problem differs from the original one in two respects: First, it de facto gives the government the power to select an equilibrium if the tax system were to generate multiple possible competitive equilibria. In other words, the government is assumed to have a lot of coordination

power. Second, by simply assuming that the governments budget constraint must hold, we are abstracting from the possibility that market participants coordinate on an equilibrium in which the government violates its budget constraint and is forced to default. Formally this is the problem,

$$\max_{\tau_w(\cdot), \tau_K(\cdot), c(\cdot), n(\cdot), K(\cdot), w(\cdot), q(\cdot)} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t); n(s^t))$$

subject to the above FOCs characterizing a competitive equilibrium, the market clearing conditions, and the government's budget constraint. We impose one additional restriction on this problem, namely that the tax on capital holdings in the initial period is zero: $\tau_K(s^0) = 0$. If the planner could choose this tax rate freely, then taxing initial capital holdings would de facto amount to a lump sum tax, since the initial capital stock is already fixed.

We analyze this problem by substituting out the tax and price variables, in order to transform the problem into one of purely determining allocations (just like the original planner's problem). We will be left with the same as the original problem, with an additional implementability constraint that results from the government's budget constraint. For this, notice from the first order condition of labor for households (multiplying by $n(s^t)$)

$$\tau_w(s^t) w(s^t) n(s^t) = w(s^t) n(s^t) + \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^t); n(s^t))} n(s^t)$$

Summing over all dates and states s^t

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) \tau_w(s^t) w(s^t) n(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) w(s^t) n(s^t) + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} n(s^t)$$

Similarly, from the Investment Euler equation (multiplying by $K(s^t)$)

$$\sum_{s^{t+1} \succ s^t} q(s^{t+1}) \tau_K(s^{t+1}) r(s^{t+1}) K(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1}) \{r(s^{t+1}) + 1 - \delta\} K(s^t) - q(s^t) K(s^t)$$

Summing over all dates and states

$$\begin{aligned}
\sum_{t=0}^{\infty} \sum_{s^t} \sum_{s^{t+1} \succ s^t} q(s^{t+1}) \tau_K(s^{t+1}) r(s^{t+1}) K(s^t) &= \sum_{t=0}^{\infty} \sum_{s^t} \sum_{s^{t+1} \succ s^t} q(s^{t+1}) \{r(s^{t+1}) + 1 - \delta\} K(s^t) \\
&\quad - \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) K(s^t) \\
\sum_{t=1}^{\infty} \sum_{s^t} q(s^t) \tau_K(s^t) r(s^t) K(s^{t-1}) &= \sum_{t=1}^{\infty} \sum_{s^t} [q(s^t) \{r(s^t) + 1 - \delta\} - q(s^{t-1})] K(s^{t-1})
\end{aligned}$$

After using the fact that $\tau_K(s^0) = 0$, we find the total tax revenue just in terms of allocations is

$$\begin{aligned}
&\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) [\tau_w(s^t) w(s^t) n(s^t) + \tau_K(s^t) r(s^t) K(s^{t-1})] = \\
&\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) w(s^t) n(s^t) + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} n(s^t) \\
&+ \sum_{t=1}^{\infty} \sum_{s^t} [q(s^t) \{r(s^t) + 1 - \delta\} - q(s^{t-1})] K(s^{t-1})
\end{aligned}$$

Notice that on the right-hand side, we have the sum of the labor and capital distortions, weighted according to the corresponding state prices. Next we make the following assumption:

Assumption 1 *The technology has constant returns to scale (f is homogeneous of degree 1).*

This assumption implies

$$w(s^t) n(s^t) + r(s^t) K(s^{t-1}) = A(s^t) f(K(s^{t-1}), n(s^t))$$

Given this, the government's budget constraint can be rewritten as

$$\begin{aligned}
q(s^0)B(s^0) + \sum_{t=0}^{\infty} \sum_{s^t} q(s^t)G(s^t) &\leq \sum_{t=1}^{\infty} \sum_{s^t} q(s^t)[w(s^t)n(s^t) + (r(s^t) + 1 - \delta)K(s^{t-1}) - K(s^t)] \\
&\quad + q(s^0)[w(s^0)w(s^0) - K(s^0)] \\
&\quad + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} n(s^t) \\
&= \sum_{t=0}^{\infty} \sum_{s^t} q(s^t)[A(s^t)f(K(s^{t-1}), N(s^t)) + (1 - \delta)K(s^{t-1}) - K(s^t)] \\
&\quad + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} n(s^t) - (1 + r(s^0) - \delta)K_{-1}
\end{aligned}$$

Using the aggregate resource constraint

$$q(s^0)[B(s^0) + (1 + r(s^0) - \delta)K_{-1}] \leq \sum_{t=0}^{\infty} \sum_{s^t} q(s^t)c(s^t) + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u_n(c(s^t); n(s^t))}{u_c(c(s^0); n(s^0))} n(s^t)$$

1.3 Transformed Planning Problem

The transformed planning problem after we expressed everything in terms of allocation is

$$\max_{c(\cdot), n(\cdot), K(\cdot)} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t); n(s^t))$$

subject to

$$\begin{aligned}
c(s^t) + G(s^t) &\leq A(s^t)f(K(s^{t-1}), n(s^t)) + (1 - \delta)K(s^{t-1}) - K(s^t) \\
B(s^0) + (1 + f(K_{-1}, n(s^0)) - \delta)K_{-1} &\leq \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u_c(c(s^t), n(s^t))c(s^t) + u_n(c(s^t), n(s^t))n(s^t)]
\end{aligned}$$

The first set of constraints is the resource condition (with Lagrange multiplier $\lambda(s^t)$), the second the implementability constraint (with Lagrange multiplier χ) that we had derived above (after we substitute out $q(s^t)$ from this condition).

The first order conditions for all $s^t \neq s^0$

$$\begin{aligned} \{K(s^t)\} &: \quad \lambda(s^t) = \sum_{s^{t+1} \succ s^t} \lambda(s^{t+1}) \{A(s^{t+1})f_K(K(s^t), n(s^{t+1})) + 1 - \delta\} \\ \{n^j(s^t)\} &: \quad \lambda(s^t)A(s^t)f_n(K(s^{t-1}), n(s^t)) = -\beta^t \pi(s^t)v_n(c^j(s^t); n^j(s^t)) \\ \{c^j(s^t)\} &: \quad \lambda(s^t) = \beta^t \pi(s^t)v_c(c^j(s^t); n^j(s^t)) \end{aligned}$$

where $v(c; n) = u(c; n) + \chi(u_c(c; n)c + u_n(c; n)n)$. The function $v(c(s^t); n(s^t))$ modifies the utility function u by the shadow cost of the tax distortions. The first-order conditions are exactly the same as in the original undistorted planning problem, except that $v(c(s^t); n(s^t))$ replaces $u(c(s^t); n(s^t))$. We now impose the following assumption on preferences:

Assumption 2 *The utility function is $u(c; n) = \frac{1}{1-\sigma}c^{1-\sigma} - \phi(n)$*

This formulation imposes additive separability between consumption and leisure, and a constant inter-temporal elasticity of substitution for consumption.

Result by Chamley (1986) and Judd (1985). Under Assumption 2, the optimal capital distortion is zero, for all $s^t \neq s^0$.

Comparing the FOC for capital with the equilibrium FOC, we already observe that capital accumulation must be undistorted if the multipliers on the resource constraints, $\lambda(s^t)$, are proportional to the state prices $q(s^t)$. For this, in turn, it suffices that for some γ , $v(c(s^t); n(s^t)) = \gamma u(c(s^t); n(s^t))$, for all s^t . But under assumption 2, this condition is satisfied since $v_c = u_c(1 + \chi + \chi\sigma)$.

To gain some intuition for this result, start once again from a small open economy, in which the planner takes state prices $q(s^t)$ as exogenously given, and faces the inter-temporal budget constraint

$$\sum_{s^t} q(s^t)[A(s^t)f(K(s^{t-1}), n(s^t)) + (1 - \delta)K(s^{t-1}) - K(s^t) - c(s^t) - G(s^t)] \geq 0$$

instead of the period-by-period resource constraints. In that case, we find the same FOCs with $\lambda(s^t) = \lambda q(s^t)$, where λ is the multiplier on the inter-temporal budget constraint, i.e. the shadow value of resources is proportional to the state prices. It then

follows directly that the planner has no incentives to distort capital accumulation in order to finance government expenditures. In the closed economy, any distortion to capital accumulation therefore results from a desire to manipulate state prices (relative to shadow values), in order to affect the real value of debt. Under Assumption 2, the manipulation incentive vanishes, since v_c is proportional to u_c , by assumption. Away from assumption 2, any manipulation incentives are tied to fluctuations in v_c/u_c , but such fluctuations have to be small, if the economy is fluctuating around a steady-state equilibrium.

The key is therefore in understanding why, for given exogenous state prices, the planner finds it optimal not to distort capital accumulation. For this, notice that the net present value of total tax revenue of the government can be rewritten as (again normalizing the marginal utility of consumption in the first period):

$$\begin{aligned} \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) T(s^t) &= \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) [A(s^t) f(K(s^{t-1}), n(s^t)) + (1 - \delta) K(s^{t-1}) - K(s^t)] \\ &\quad + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u_n(c(s^t); n(s^t)) n(s^t) - (1 - r(s^0) - \delta) K_{-1} \end{aligned}$$

The first term on the RHS corresponds to the NPV of aggregate output, net of investment costs. The second term on the RHS can be rewritten as

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u_n(c(s^t); n(s^t)) n(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) (1 - \tau_w(s^t)) w(s^t) n(s^t)$$

and therefore corresponds to after tax labor income. The NPV of government revenues is therefore equivalent to the NPV of aggregate output, minus investment costs and after tax payments to the representative household. Therefore, the Ramsey planning problem is equivalent to a problem in which the government owns the capital stock and the firms, determines investments in the capital stock and after-tax wages, acting as a de facto monopsonist in the labor market. But if the government directly owns the capital stock, it has no incentive to distort its accumulation.

Result (Tax Smoothing): *The optimal sequence of labor taxes smoothes labor distortions over time.*

From the planner's FOC for labor, we have

$$A(s^t)f_n(K(s^{t-1}), n(s^t)) = -\frac{v_n(c(s^t); n(s^t))}{v_c(c(s^t); n(s^t))} = -\frac{u_n(c(s^t); n(s^t))}{u_c(c(s^t); n(s^t))} \frac{1 + \chi + \chi \frac{u_{nn}(c(s^t); n(s^t))n(s^t)}{u_n(c(s^t); n(s^t))}}{1 + \chi + \chi \frac{u_{cc}(c(s^t); n(s^t))c(s^t)}{u_c(c(s^t); n(s^t))}}$$

Since $\frac{u_{cc}}{u_c}c < 0$ and $\frac{u_{nn}}{u_n}n > 0$ (u is increasing, concave in c and decreasing, concave in n), it follows that,

$$A(s^t)f_n(K(s^{t-1}), n(s^t)) > -\frac{u_n(c(s^t); n(s^t))}{u_c(c(s^t); n(s^t))}$$

The 'labor wedge' term smoothes these distortions over time; if u is additively separable, and $\frac{u_{cc}}{u_c}c$ and $\frac{u_{nn}}{u_n}n$ are constants (assumption 2, along with the functional form $\phi(n) = \gamma n^\psi$), then the labor wedge is positive and constant over time. Thus, the resulting labor wedge is constant over time and across states.

The intuition for this tax-smoothing result can be explained as follows: suppose there was some state at which labor income was undistorted, and some other state at which it was distorted. Then, there is room for a welfare gain: by introducing an infinitesimal labor tax of size $\epsilon > 0$ in the undistorted state, the government raises some revenue, but the welfare loss from distorting labor at that state are of second order (i.e. proportional to ϵ^2), due to the lack of distortions. The government can use this revenue to reduce labor taxes in a distorted state, causing a first-order welfare gain, which must dominate the loss of a marginal distortion.

Remarks

- A slightly different way to think about the intuition for the zero capital tax result is to consider the tradeoff between taxing capital and taxing labor. An increase in the tax on capital reduces the capital labor ratio, which reduces wages, and therefore labor income and labor tax revenues. The planner will always find it optimal to reduce capital taxes and replace them by an increase in labor taxes, and can do so without reducing overall revenues, because of the resulting increase in wages and labor tax revenues.
- All the above derivations relied on the constant returns to scale assumption. This assumption implies that all output is allocated as factor payments to either labor or capital. With decreasing returns to scale, a similar analysis as above

applies, provided that the government can either use dividend taxation as an additional instrument, or acquire all shares of firm in the market. This leads to the same scenario as above, namely that the government earns the NPV of output net of investments and after tax wage payments. Here the tradeoff is even more explicit: suppose the government owns the capital stock, or taxes dividend payments at a constant rate that is arbitrarily close to 1 - in both cases, the government can then de facto appropriate the rents to capital without distorting investment incentives. But then appropriating these rents (or owning the capital directly) and leaving investment incentives undistorted is a more effective way to raise revenue than taxing capital income, which will allow for at best a partial transfer of the rents, and comes at the cost of distorting accumulation.

- In the entire discussion above, we have focused on capital taxes for all $s^t \neq s^0$. At the initial node, the previous capital stock is pre-determined, so a tax on capital amounts to a lump sum tax on the initial capital income. This points to a general issue with commitment to capital taxes. Ideally, the government would like to commit ex ante to not taxing capital at any future period. However, once the period is reached, the government has a strong incentive to deviate from its original plan, since the previous investment in capital is now sunk, and the government can reduce distortions by fully expropriating the capital holders ex post.