Fighting Crises with Secrecy
Online Appendix

Gary Gorton* Guillermo Ordoñez†

Abstract

This Appendix contains all proofs of Propositions and Lemmas in the manuscript.

*Yale University and NBER (e-mail: gary.gorton@yale.edu)
†University of Pennsylvania and NBER (e-mail: ordonez@econ.upenn.edu)

1 Proof Proposition 1

We first characterize p^H. The maximum profits that a bank with the highest collateral quality $\overline{p} + \overline{\eta}$ expects when inducing a run, conditional on no other bank facing a run (that is, when $E^r(p) = \overline{p} + \overline{\eta}$), are

$$E_r(\pi|\overline{p} + \overline{\eta}, \overline{p} + \overline{\eta}) = (\overline{p} + \overline{\eta})K^*(qA - 1) - \gamma.$$

while the expected profits when not inducing a run, conditional on no other bank facing a run, are

$$E_{nr}(\pi|\overline{p}) = K_{nr}(\overline{p})(qA - 1)$$

where, from equation (5)

$$K_{nr}(\overline{p}) = \min\left\{K^*, \frac{\gamma}{(1-q)(1-\overline{p})}, \overline{p}C\right\}.$$

There is always a $\overline{p}$ large enough such that $K^* < \frac{\gamma}{(1-q)(1-\overline{p})}$ and $K^* < \overline{p}C$ such that $K_{nr}(\overline{p}) = K^*$. Then no bank would rather face a run and have a positive probability of not being able to invest, given that it can invest at optimal scale without examination. For all $\overline{p} > p^H$, all banks invest without runs, where p^H is defined by $E_r(\pi|p^H + \overline{\eta}, p^H + \overline{\eta}) = E_{nr}(\pi|p^H)$, or

$$(p^H + \overline{\eta})K^*(qA - 1) - \gamma = \frac{\gamma}{(1-q)(1-p^H)}(qA - 1).$$

In this region, $p^* = \overline{p} + \overline{\eta}$, which trivially increases one for one with $\overline{p}$.

We now characterize p^L. The maximum expected profits that a bank with the lowest collateral quality $\overline{p} - \overline{\eta}$ can obtain when avoiding a run when all other banks face runs (that is, $E^{nr}(p) = \overline{p} - \overline{\eta}$) are

$$E_{nr}(\pi|\overline{p} - \overline{\eta}) = K_{nr}(\overline{p} - \overline{\eta})(qA - 1)$$

where, from equation (5)

$$K_{nr}(\overline{p} - \overline{\eta}) = \min\left\{K^*, \frac{\gamma}{(1-q)(1-(\overline{p} - \overline{\eta}))}, (\overline{p} - \overline{\eta})C\right\}.$$

The expected profits when the bank induces a run, conditional on all other banks inducing a run, are

$$E_r(\pi|\overline{p} - \overline{\eta}, \overline{p}) = (\overline{p} - \overline{\eta})\left[K^*(qA - 1) - \frac{\gamma}{\overline{p}}\right].$$

Defining p^L by the point at which $E_r(\pi|p^L - \overline{\eta}, p^L) = E_{nr}(\pi|p^L - \overline{\eta})$, such that

$$(p^L - \overline{\eta})\left[K^*(qA - 1) - \frac{\gamma}{p^L}\right] > \frac{\gamma}{(1-q)(1-(p^L - \overline{\eta}))}(qA - 1),$$

then when $\overline{p} < p_L$ all banks invest such that there is examination of their collaterals. In this region, $p^* = \overline{p} - \overline{\eta}$, which also trivially increases one for one with $\overline{p}$.

In the best equilibrium and by monotonicity, in the intermediate region of $\overline{p}$ the threshold p^* also increases with $\overline{p}$. QED.

2 Proof Proposition 2

Follows from comparing the condition for no information acquisition in the absence of intervention (equation 4) and in the presence of intervention (equation 8), and on the comparative statics in equation (8) with respect to y.

3 Proof Lemma 1

Assume first that the central bank chooses $\widetilde{p} = p_L + \overline{\eta}$, and then no discount even for the bank holding the highest collateral quality, this is $d(p_L + \overline{\eta}, \widetilde{p}) = 0$. Compare equations (10) and (11) for $p = p_L + \overline{\eta}$. It is optimal for such bank to participate, but if this is the only bank participating, then stigma is positive in the sense that participation reveals the highest possible idiosyncratic component. Then, participating is also the optimal strategy for all other banks, which implies that there is no learning from participation and no stigma (i.e., $\chi = 0$), confirming that this is indeed the best sustainable equilibrium.[1] Hence, for $\widetilde{p} \geq p_L + \overline{\eta}$, a fraction $y(\widetilde{p}) = 1$ of banks participate, from equation (4) $\sigma(\widetilde{p}) = 0$ and from equation (8) $K(\widetilde{p}) = K^*$.

For lower levels of $\widetilde{p}$, this is still an equilibrium as long as the bank with the highest collateral quality finds it optimal to participate, and then all other banks do as well. The critical level $\widetilde{p}_h$ is determined by the point at which the bank with the highest quality (evaluated at $\chi = 0$) is indifferent between participating or not,

$$H(K^*) - d(p_L + \overline{\eta}, \widetilde{p}_h) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \overline{\eta}, \widetilde{p}_h).$$

Using the definitions of $L(p, \widetilde{p})$, $H(K(\widetilde{p}))$ and $d(p, \widetilde{p})$,

$$\widetilde{p}_h = (p_L + \overline{\eta}) - \frac{\varepsilon}{C} \left[(1 - p_L - \overline{\eta})K^*(qA - 1) + \gamma \right]. \qquad \texttt{QED}$$

4 Proof Lemma 2

Assume first the extreme case in which $\widetilde{p} = \widetilde{p}_h$. From the previous proposition, $y(\widetilde{p}_h) = 1$ and $\sigma(\widetilde{p}_h) = 0$. For $\widetilde{p} = \widetilde{p}_h - \epsilon$ (from the definition of $\widetilde{p}_h$),

$$H(K^*) - d(p_L + \overline{\eta}, \widetilde{p}_h - \epsilon) < (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \overline{\eta}, \widetilde{p}_h - \epsilon),$$

[1] Notice that this is only one possible equilibrium. If everybody believes that some banks did not borrow from the discount window, $\chi > 0$, and it may be indeed optimal for those banks not to borrow from the window. This shows how endogenous stigma may induce equilibrium multiplicity and may generate self-confirming collapses in the use of discount windows. Here we focus on the best equilibrium based on intervention, and show its limitations.

and then banks with collateral quality $p_L + \overline{\eta}$ strictly prefer to not participate at the discount window. This implies that $y(\widetilde{p}_h - \epsilon) \equiv Pr(p < p^*_w(\widetilde{p}_h - \epsilon)) < 1$, where $p^*_w(\widetilde{p}_h - \epsilon)$ is given by the indifference condition

$$H(K^*) - d(p^*_w, \widetilde{p}_h - \epsilon) = (1-\varepsilon)H(K^*) + \varepsilon L(p^*_w, \widetilde{p}_h - \epsilon),$$

or

$$d(p^*_w, \widetilde{p}_h - \epsilon) = \varepsilon[H(K^*) - L(p^*_w, \widetilde{p}_h - \epsilon)],$$

where p^*_w declines monotonically as we reduce $\widetilde{p}$. Notice that this construction relies on the conjecture that $\sigma(\widetilde{p}_h - \epsilon) = 0$, but for relatively low ϵ this is the case as long as $y(\widetilde{p}_h - \epsilon)$ and $E^{nw}(p|\widetilde{p}_h - \epsilon)$ are such that

$$K^* < \frac{\gamma}{(1-q)(1-y)(1-E^{nw}(p))}.$$

Define by $\overline{p}^*_w$ the threshold such that $\overline{y}(\overline{p}^*_w)$ is the fraction of banks borrowing from the discount window and $\overline{E}^{nw}(p|\overline{p}^*_w)$ is the expected quality of the non-participating banks' collateral, such that depositors are indifferent between running or not when the bank invests K^*, i.e.,

$$K^* = \frac{\gamma}{(1-q)(1-\overline{y})(1-\overline{E}^{nw}(p))}.$$

The bank with the marginal collateral quality $\overline{p}^*_w(\widetilde{p}_m)$ is determined by

$$H(K^*) - d(\overline{p}^*_w, \widetilde{p}_m) = (1-\varepsilon)H(K^*) + \varepsilon L(\overline{p}^*_w, \widetilde{p}_m),$$

such that

$$\overline{y} = Pr(p < \overline{p}^*_w(\widetilde{p}_m)) \qquad \texttt{and} \qquad \overline{E}^{nw}(p) = E(p|p > \overline{p}^*_w(\widetilde{p}_m)).$$

Finally, the threshold $\overline{p}^*_w$ is well-defined, as both y and $E^{nw}(p)$ monotonically increase in p^*_w, which monotonically decreases in $\widetilde{p}$.

Finally, even when there is no examination and then stigma on the equilibrium path, there would be stigma in case participation is leaked or voluntarily disclosed. Stigma, $\chi(p, \widetilde{p})$, is the cost in terms of a higher probability of a run in the second period coming from information that the bank participated at the discount window in the first period. Intuitively, since banks participate when their collateral quality is lower than average (specifically $p < p^*_w$), revelation of participation would make those banks more prone to be examined in the second period. Formally, *in the second period*, the bank will suffer a run in the second period and obtain $H(K^*)$ only with probability p and lose the cost of information γ if equation (4) is not fulfilled at optimal scale K^*, this is if

$$K^* > \frac{\gamma}{(1-q)(1-E^w(p|\widetilde{p}, \bar{p}_{t=2}))} \qquad \texttt{or} \qquad E^w(p|\widetilde{p}, \bar{p}_{t=2}) < 1 - \frac{(1-q)K^*}{\gamma}$$

where $E^w(p|\widetilde{p}, \bar{p}_{t=2})$ is the expectation of p in the second period, conditional on having participated in the discount window in the first period and conditional on the second period displaying an aggregate component $\bar{p}_{t=2}$. Since we have assumed that $\bar{p}_{t=2} = p_H > p^H$ we can rewrite this expectation as

$$E^w(p|\widetilde{p}_m, \bar{p}_{t=2}) = p_H + E^w(\eta|\widetilde{p}).$$

in which case the bank would lose in net $(1-p)K^*(qA-1)+\gamma$.

This implies that stigma is given by

$$\chi(p,\widetilde{p}) = \begin{cases} 0 & \texttt{if} \quad E^w(\eta|\widetilde{p}) \geq 1 - p_H - \frac{(1-q)K^*}{\gamma} \\ (1-p)K^*(qA-1)+\gamma & \texttt{if} \quad E^w(\eta|\widetilde{p}) < 1 - p_H - \frac{(1-q)K^*}{\gamma}. \end{cases}$$

QED.

5 Proof Lemma 3

Assume first the extreme case where $\widetilde{p} = \widetilde{p}_m$. From the previous lemma, $y(\widetilde{p}_m) = \overline{y}$ and $\sigma(\widetilde{p}_h) = 0$. For $\widetilde{p} = \widetilde{p}_m - \epsilon$, the bank that is indifferent about borrowing from the discount window has slightly lower quality, $p^*_w(\widetilde{p}_m - \epsilon) < p^*_w(\widetilde{p}_m)$. Then $y(\widetilde{p}_m - \epsilon) < \overline{y}$ and $E^{nw}(p|\widetilde{p}_m - \epsilon) < \overline{E}^{nw}(p)$, and there are incentives to run if investment is K^*, as there are relatively few participants at the discount window (low y) and the collateral of those not participating at the discount windows are worse in expectation (low $E^{nw}(p)$). Formally,

$$K^* > \frac{\gamma}{(1-q)(1-y(\widetilde{p}_m-\epsilon))(1-E^{nw}(p|\widetilde{p}_m-\epsilon))}.$$

One possibility for banks to prevent runs, $\sigma(\widetilde{p}) = 0$, is to scale back the investment in the project to $K(p^*_w) < K^*$, to avoid information acquisition. The size of the investment K, however, also determines y, as $p^*_w(\widetilde{p}_m - \epsilon)$ is pinned down by the condition

$$d(p^*_w, \widetilde{p}_m - \epsilon) = \varepsilon[H(K(p^*_w)) - L(p^*_w, \widetilde{p}_m - \epsilon)].$$

A lower K relaxes the constraint and reduces the incentive to run, but at the same time reduces p^*_w for a given $\widetilde{p}$, increasing the incentive to run. Intuitively, for a given discount, a reduction in the gains of borrowing from the discount window (from lower $H(K)$) reduces the quality of the marginal bank which is indifferent between borrowing or not, i.e., reducing p^*_w further.

If $H(K(p^*_w))$ declines faster than $L(p^*_w)$, then no participant will go to the discount window if, at the lowest possible p, which is $p_L - \overline{\eta}$,

$$H(K(p_L - \overline{\eta})) - L(p_L - \overline{\eta}, p_L) < 0$$

which we have assumed in the definition of a crisis (p_L is low enough such that all banks would rather face examination than restricting their investments to avoid runs). In words, it is not in the best interests of banks to discourage runs by reducing the size of their investments in the project, which is in contrast to what happens in the absence of intervention. Our result here comes from the endogenous participation of banks at the discount window. By reducing K, the effect of a lower y in inducing information acquisition is stronger than the effect of a lower K in discouraging information acquisition, thus increasing on net the incentives for depositors to examine a bank's collateral as K declines.

Under these conditions, the equilibrium should involve either the discount window sustaining an investment of K^* (when a fraction $\overline{y}$ of banks borrows from the discount window) or no participation in the discount window at all, which replicates the allocation without intervention. To maintain the fraction $\overline{y}$ constant in this region as $\widetilde{p}$ declines, the marginal bank with collateral quality $p^*_w(\widetilde{p}_m)$ should be indifferent between borrowing from the discount window or not. This is achievable only if depositors choose to run with some probability and examine the portfolio of banks as $\widetilde{p}$ declines, as this increases the incentives to have bonds in the portfolio.

As the fraction of banks participating at the discount windows is constant at $\bar{y}$, depositors are indeed indifferent between running or not, and $\sigma(\widetilde{p}) > 0$ is an equilibrium. To determine $\sigma(\widetilde{p})$ in equilibrium we next discuss the determination of endogenous stigma.

With positive information acquisition ($\sigma(\widetilde{p}) > 0$) there is stigma when the depositor discovers a bank's participation at the discount window. The reason there is stigma is that those banks borrowing from the discount window are the ones with relatively low collateral quality (relatively low η_i). Once a bank is stigmatized, it may face withdrawals during normal times in the second period. To be more precise about the endogeneity of stigma, once back in normal times, the bank will face a run when investing at the optimal scale of production if

$$K^* > \frac{\gamma}{(1-q)(1-E^w(p))},$$

and the bank will not suffer a run in the second period based on an indifference condition that pins down p^*_2 in the second period where

$$E_r(\pi|p^*_2, E^r(p|p < p^*_w)) = E_r(\pi|E^{nr}(p|p < p^*_w)).$$

We denote by $K^w(p, \widetilde{p})$ the investment size that a bank with a collateral of quality p can obtain in the second period conditional on it having been revealed that the bank borrowed from the discount window in the first period.

Then, stigma is given by

$$\chi(p, \widetilde{p}) = [K^* - K^w(p, \widetilde{p})](qA - 1),$$

where χ is an increasing function of the discount (a decreasing function of $\widetilde{p}$). As the discount increases, p_w^* decreases, $y(p_w^*)$ decreases and $E^w(p)$ decreases. This leads to a decline in K^w and then an increase in stigma from going to the discount window.

Given $\overline{y}$, to maintain the investment size K^* without triggering information, the indifference of the marginal bank $\overline{p}_w^*$ pins down the probability the depositor runs. This is $E^{nw}(\pi|p_w^*) = E^w(\pi|p_w^*)$, which implies

$$\sigma L(\overline{p}_w^*, \widetilde{p}) + (1-\sigma)[(1-\varepsilon)H(K^*) + \varepsilon L(\overline{p}_w^*, \widetilde{p})] = [H(K^*) - d(\overline{p}_w^*, \widetilde{p})] - \sigma\chi(\overline{p}_w^*, \widetilde{p})$$

and then

$$\sigma(\widetilde{p}) = \frac{d(\overline{p}_w^*, \widetilde{p}) - \varepsilon[H(K^*) - L(\overline{p}_w^*, \widetilde{p})]}{(1-\varepsilon)[H(K^*) - L(\overline{p}_w^*, \widetilde{p})] - \chi(\overline{p}_w^*, \widetilde{p})}. \tag{1}$$

Finally, depositors randomize between running and not running given that the bank is investing K^* in the project, and a bank with collateral quality $\overline{p}_w^*$ is indifferent between borrowing from the discount window or not. QED.

6 Proof Lemma 4

From equation (1), $\sigma(p_w^*) = 1$ for $d(\overline{p}_w^*, \widetilde{p}) \geq H(K^*) - L(\overline{p}_w^*, \widetilde{p}) - \chi(\overline{p}_w^*, \widetilde{p})$. Define $\widetilde{p}_l$ to be the discount that solves this condition with equality. Evaluating $E_{nw}(\pi)$ and $E_(\pi)$ at $\sigma(\widetilde{p}) = 1$ given a project of size K^* and at the threshold $\overline{p}_w^*$ a bank with collateral p_w^* goes to the discount window whenever

$$H(K^*) - d(\overline{p}_w^*, \widetilde{p}) - \chi(\overline{p}_w^*, \widetilde{p}) > L(\overline{p}_w^*, \widetilde{p})$$

which is never the case in this region by the previous condition. Then $y(\widetilde{p}) = 0$ and then it is indeed optimal for depositors to run, that is $\sigma(\widetilde{p}) = 1$. QED.

7 Proof Lemma 5

Define $\widetilde{p}_T$ by

$$H(K^*) - d(p_L + \overline{\eta}, \widetilde{p}_T) = L(p_L + \overline{\eta}, \widetilde{p}_T).$$

For all $\widetilde{p} > \widetilde{p}_T$

$$H(K^*) - d(p_L + \overline{\eta}, \widetilde{p}) > L(p_L + \overline{\eta}, \widetilde{p})$$

and the bank with collateral of quality $p_L + \overline{\eta}$ strictly prefers to borrow from the discount window. Notice that as all banks participate, $\chi(p, \widetilde{p}_T) = 0$ for all p.

For all $\widetilde{p} < \widetilde{p}_T$, the fraction of participating banks y_T in a transparent window is given by the threshold p_{wT}^* such that $H(K^*) - d(p_{wT}^*, \widetilde{p}) - \chi(p_{wT}^*, \widetilde{p}) = L(p_{wT}^*, \widetilde{p})$ and $y_T = Pr(p < p_{wT}^*)$.

Recall the condition that pins down $\widetilde{p}_h$ in Lemma 1 is

$$H(K^*) - d(p_L + \overline{\eta}, \widetilde{p}_h) = (1-\varepsilon)H(K^*) + \varepsilon L(p_L + \overline{\eta}, \widetilde{p}_h).$$

Then, as the right-hand side is larger than in the condition above that pins down $\widetilde{p}_T$, the discount has to be smaller and $\widetilde{p}_T < \widetilde{p}_h$. QED.

8 Derivation of Intervention Distortions

As banks keep the net gains from production, welfare is an aggregation of the payoffs of all banks with different quality p, given by

$$\begin{aligned} W(\widetilde{p}) &= \int_p \left[\mathbb{I}_w[H + B - pC] + (1-\mathbb{I}_w)[\sigma L(p) + (1-\sigma)((1-\varepsilon)H + \varepsilon L(p))]\right] dF(p) \\ &+ \int_p \mathbb{I}_w[q(pC - B) + (1-q)(\phi pC - B)]dF(p) \\ &+ \int_p \left[\mathbb{I}_w[q(H - \sigma\chi(p)) - (1-q)\delta(1-\phi)pC] + (1-\mathbb{I}_w)H\right] dF(p), \end{aligned}$$

where $\mathbb{I}_w$ is an indicator function that takes the value 1 if the bank participates at the discount window and 0 otherwise.

Taking integrals and rewriting the expression,

$$\begin{aligned} W(\widetilde{p}) &= y(H + B - E^w(p)C) + (1-y)[\sigma\widehat{L} + (1-\sigma)((1-\varepsilon)H + \varepsilon\widehat{L})] \\ &+ y[q(E^w(p)C - B) + (1-q)(\phi E^w(p)C - B)] \\ &+ y[q(H - \sigma\widehat{\chi}) - (1-q)\delta(1-\phi)E^w(p)C] + (1-y)H, \end{aligned}$$

where $H \equiv H(K^*)$, $\widehat{L} \equiv \int_{p|nw} L(p, \widetilde{p})dp$ and $\widehat{\chi} \equiv \int_p \chi(p, \widetilde{p})dp$.

The first two terms (the first line) represent the welfare of banks in the crisis period. A fraction y of banks borrow from the discount window leading to a production of H and exchanging private collateral for bonds at an average discount of $B - E^w(p)C$. A fraction $1-y$ of banks do not participate and their investments lead to a production level that depends on whether they suffered a run or if there was an informational leak. This first line of the welfare function can be rewritten as

$$H - y(E^w(p)C - B) - (1-y)(\varepsilon + \sigma(1-\varepsilon))(H - \widehat{L}).$$

The third term (the second line) represents the welfare for the government. From the fraction y of banks borrowing from the discount window, a fraction q has their private collateral seized, which delivers $E^w(p)C$ in expectation while a fraction $1-q$ defaults and the private collateral has to be liquidated, recovering just $\phi E^w(p)C$. Still in both cases the government has to repay B for the bonds. This second line of the welfare function can be rewritten as

$$y(E^w(p)C - B) - y(1-q)(1-\phi)E^w(p)C.$$

The last term (the third line) captures investments in the second period (no discount). Those banks that did not borrow from the discount window and those that did without their participation being revealed can borrow without triggering a run in the second period (then leading to production level H). Those banks that participated and their participation was revealed (because they were examined) can potentially suffer a run in the second period because of stigma (captured by L). Finally, the government has to repay (facing inefficiency costs $1-\phi$ and distortionary taxation costs δ) the bonds that could not be covered by liquidating private assets in the previous period. The third line can then be rewritten as

$$H - y[q\sigma\widehat{\chi} + (1-q)\delta(1-\phi)E^w(p)C].$$

Adding (and canceling) terms, total welfare is

$$W(\widetilde{p}) = 2H - (1-y)(\varepsilon + \sigma(1-\varepsilon))(H - \widehat{L}) - y[(1-q)(1+\delta)(1-\phi)E^w(p)C - q\sigma\widehat{\chi}].$$

Since the unconstrained welfare is $2H$ in both periods, we can denote the distortion from the crisis as equation (14).

The first component shows the distortion that comes from lower output from banks that did not participate, either due to leaks or runs. The second component shows the costs of the distortionary taxation that is needed to cover deposits from defaulting banks, and that cannot be covered by liquidating the private collateral. Finally, the third component shows the lower production in the second period that arises from stigma – banks who were discovered borrowing from the discount window and then were revealed to have collateral of relatively lower quality – being more likely to suffer a run and produce less in the second period.

9 Proof Proposition 5

Here we compare the distortions for different levels of discount $\widetilde{p}$ (in the different regions that we characterized in the previous section) for different disclosure policies.

We consider first the parametric case for which $\widetilde{p}_T < \widetilde{p}_l$. Under transparency, all banks participate when $\widetilde{p} \geq \widetilde{p}_T$. This implies that $y = 1$, $\chi = 0$ and $E^w(p) = p_L$. Distortions in this range are then

$$Dist(\widetilde{p}|Tr) = (1+\delta)(1-q)(1-\phi)p_L C.$$

In contrast, for all $\widetilde{p} < \widetilde{p}_T$, the discount window is not sustainable and distortions are the same as without intervention, $H - L \equiv (1-p_L)K^*(qA-1) + \gamma$.

Under opacity, distortions also depend on the equilibrium regions. In the "very low" discount region all banks participate, then $y = 1$ and $\chi = 0$, and distortions are the same as under transparency (in the region $\widetilde{p} \geq \widetilde{p}_T$ above).

In the "low" discount region, $y < 1$ but $\sigma = 0$, then from equation (14)

$$Dist(\widetilde{p}|Op) = (1-y)(H-\widehat{L})\varepsilon + y(1+\delta)(1-q)(1-\phi)E^w(p|Op)C.$$

Since $yE^w(p|Op) < p_L$ and $H - L > H - \widehat{L}$, the sufficient condition for the distortion under opacity is lower than the distortion under transparency is

$$(1+\delta)(1-q)(1-\phi)p_L C > \varepsilon(H-L).$$

In the "intermediate" discount region, $y = \overline{y}$, but $\sigma > 0$ and increases with the discount. From equation (14) it is clear that in this region the distortion increases with σ and then with the discount. While the fraction of banks participating is fixed, there are more runs and then more banks not participating end up producing less, both in the first period (less deposits) and the second period (more stigma).

Finally, in the "high" discount region, the discount window is not sustainable under opacity, so distortions are the same as without intervention, $H - L$.

Summarizing, as the discount increases, distortions under opacity are fixed in the very low discount region, decrease in the low discount region, increase in the intermediate discount region and reach the maximum in the high discount region. In contrast, distortions under transparency are fixed whenever the discount window is sustainable, as either all banks participate or neither does. This implies the optimal discount is $\widetilde{p}_m$ under opacity.

Consider finally the parametric case for which $\widetilde{p}_T \geq \widetilde{p}_l$. In this case the discount window under opacity collapses at lower discount levels than under transparency. Still the optimal policy is a discount of $\widetilde{p}_m$ under opacity because $\widetilde{p}_T < \widetilde{p}_h$, as shown in Lemma 5. QED.