The Supply and Demand for Safe Assets*

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July 2021

Abstract

Safe assets are demanded as stores of value (to smooth consumption intertemporally) and as collateral (to facilitate credit intra-temporally). Some are supplied publicly (government bonds) and some privately (such as asset-backed securities). Private assets are heterogeneous in quality, and information about their quality reduces their safety properties. We show that government bonds discourage both production of (crowding quantity out) and information about (crowding safety in) private assets. Hence, the optimal supply of government bonds need to take into account their dual roles and their impact on the quantity and informational content of private assets.

*We thank Narayana Kocherlakota, Moritz Lenel, Dejanir Silva and an anonymous referee for discussions and very thoughtful feedback. We also thank Fernando Alvarez, Marco Bassetto, Ryan Chahrour, V.V. Chari, Harold Cole, Igor Livshits, Andrew Postlewaite and seminar participants at Wharton, the Philadelphia Fed, the Barcelona GSE 2020 Summer Forum, the 2021 Carnegie-Rochester-NYU Conference on Public Policy, the 2021 SED Meetings and the 2021 NOVA Summer School Conference for comments. The usual waiver of liability applies.

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1 Introduction

Assets that display a stable value are considered "safe" and play an essential role in the economy. Agents demand them for two reasons: as a *store of value* to smooth consumption *inter-temporally* and as *collateral* to smooth consumption *intra-temporally*, for instance by facilitating credit and moving resources from nonproductive to productive agents. The supply of safe assets comes from two sources: public and private. *Public safe assets* are government bonds, promises that the government can make by relying on its taxation power. *Private safe assets* (such as asset-backed securities) take the form of promises that private agents can make by transforming otherwise riskier and potentially non-pledgeable private assets.

Issues surrounding safe assets have become increasingly pressing since the transformation of the financial system from a retail-based banking system to a wholesale banking system, which started in the late 1970s and seems permanent.[1] In this new financial landscape, the use of U.S. Treasuries as safe assets have become more prevalent. Krishnamurthy and Vissing-Jorgensen (2012 and 2015) show that Treasuries have a convenience yield, arising from their safety property. Gorton, Lewellen, and Metrick (2010), Xie (2012), Sunderam (2015) and Lenel (2020) further document that public and private safe assets are *substitutes in terms of quantity*: the private sector produces more private safe assets when the supply of Treasuries declines.

This paper highlights that public and private safe assets are also *complements in terms of safety*. Private assets are not perfect substitutes for public safe assets because they come in heterogeneous and volatile qualities. How closely private assets can substitute for public assets depends on their informational content. If there is no information about the differences and evolution of the assets' qualities, they can be traded as if they were homogeneous and stable, becoming better substitutes for public assets. When such information is generated and revealed, their safety and their role as collateral and store of value declines, a *financial crisis*, in the words of Gorton and Ordonez (2014 and 2020b), if information is suddenly and involves large amounts and categories of private assets.

[1] Gorton, Lewellen, and Metrick (2010) show that as a percentage of the total privately-produced safe debt, demand deposits have fallen from about 80% in the 1950's to 31% now. In contrast, short-term money market instruments rose from 11% to 21% and AAA asset-backed and mortgage-backed securities rose from zero to 18%. More generally, the "shadow banking system," which is the sum of mortgage-backed securities, other asset-backed securities and money-like debt instruments, grew from 11% to 38%, getting larger than demand deposits.

The information content of private assets is not exogenous and depends on the availability of public assets. When the stock of government bonds in the economy is high, agents rely less on the use of private assets to obtain credit and there are fewer incentives to examine their qualities. Safer private assets reduce the consumption variance of agents, in a way *completing markets* by replicating allocations that would be available with insurance contracts, making government bonds less relevant as safe assets. Indeed, as government bonds not only crowd out private assets but also informational financial frictions, their dual effect on the *quantity and safety* of private assets affects the extent to which such bonds are needed in the economy. Here we characterize the optimal supply of bonds that take into account these intricate interactions.

To build these ideas and identify the role of each element on the optimal provision of government bonds, we proceed in steps. We first explore the two roles of safe assets as stores of value and as collateral assuming that only the government supplies safe assets by issuing government bonds. Optimal provision should balance a trade-off: providing bonds to serve as collateral in a given period may distort consumption smoothing across periods. The optimal trade-off is achieved when the intertemporal distortion equalizes the value of collateral in relaxing borrowing constraints.

We then introduce the possibility to create private safe assets. At a cost, agents can transform non-pledgeable and perishable goods into pledgeable and non-perishable assets. Non-perishability allows the private asset to be used as a store of value and the pledgeability allows them to be used as collateral. This possibility reduces the need for the government to provide public safe assets in the optimum. In contrast to government bonds, private assets can potentially be of heterogeneous quality. This is relevant because information about such qualities in credit and asset markets introduces dispersion in investment scale and consumption (i.e., ex-ante risk) increasing the demand for bonds by risk averse agents, both as stores of value and as collateral.

We then endogenize information acquisition about private assets. We show such information is more likely to be produced when government bonds are scarce relative to the needs for collateral and relative to the average quality of projects to be financed and the average quality of private assets that are used as collateral. Furthermore, information acquisition may be sudden and may involve large amounts and categories of private assets, raising rapidly and drastically the need of bonds in the economy.

After characterizing these forces and interactions in a two-period setting, we study their dynamics in an overlapping generations environment, and make two points.

First, we identify conditions under which providing the optimal quantity of bonds is not feasible, a *policy trap*: government bonds may be *too abundant* (their benefits do not compensate the distortions that their provision generate), but reducing their supply may induce information production about private assets, reducing safety in the economy and increasing the need for those bonds, making them *too scarce*. The possibility of facing this trap highlights the difficulty in specifying the optimal amount of government debt without taking into account the stability of private financial markets. The second point is that transitory shocks that reduces the supply of government bonds may increase the incentives to acquire information about private assets persistently, both through the effect of the shock on wealth and on the endogenous creation of private assets.[2]

Our results characterize the optimal provision of government debt, but they also have applications to all policies, fiscal and monetary, that affect the quantity of bonds that circulate in the economy. Central banks, for instance, conduct monetary policy with open market operations that exchange one kind of money, short-term Treasuries, for another kind of money, cash (numeraire in our setting) or quantitative easing programs that provide liquidity by purchasing long-term Treasuries with cash. Our work argues that, in conducting monetary policy, the Central Bank may inadvertently modify the informational content of private safe assets, inducing changes in credit conditions. Hence, macroprudential policy cannot be separated from monetary policy, contrary to the existing literature; see Svensson (2018) and Bernanke (2018)).

This insight has implications for the tools needed to conduct monetary policy. Standard monetary policies that exchange bonds for cash (numeraire) operate through the *size* of the central bank's balance sheet (such as open market operations and quantitative easing), and may not be able to achieve both goals at the same time. If this is not possible, the government needs an additional tool that exchanges government bonds for private assets (not cash), changing the *composition* of the central bank's balance sheet (a recent example was the Term Securities Lending Facility, operated by the Fed during the recent Financial Crisis; see Fleming, Hrung, and Keane (2009)). We call this additional tool the "Bond Exchange Facility" (BEF), with a plausible implementation discussed in Gorton and Ordonez (2020a). While standard monetary policies should focus on implementing optimal intertemporal smoothing, the BEF is needed

[2]The global savings glut is an example of a reduction in the supply of government bonds available in the U.S. before the recent financial crisis. See, e.g., Bernanke (2005).

to guarantee that the safety of private assets is maintained. It may mean that it is optimal for macroprudential reasons for the Central Bank to maintain a balance sheet of enough size to implement BEF when needed.

We are not the first in recognizing that, on top of facilitating savings, government bonds can also relax borrowing constraints.[3] Aiyagari and McGrattan (1998), for instance, study the role of bonds to improve risk-sharing among agents in an incomplete-market setting. They highlight that the cost of providing those bonds comes in the form of i) taxation that distorts incentives and the wealth distribution, and ii) crowding out of private assets. We instead abstract from the sources of distortions (which we introduce in reduced form), and explore more prominently how bonds have the additional role of enhancing the safety of private assets. How well private assets can substitute for bonds to save and to relax borrowing constraints depends on their informational content, and their informational content is determined not only by private assets' fundamentals, but also by the supply of government bonds.

Other papers explore the intricate relation between the dual role of government bonds and the functioning of markets and the presence of other assets. Kocherlakota (2003) shows that government bonds, when illiquid (positive nominal return), allow agents with different intertemporal marginal rates of substitution not only to store value but also to realize additional intertemporal gains to trade. Infante and Ordonez (2020) study the roles of government bonds for saving and for risk sharing, and show that the effect of aggregate volatility on the extent of insurance in the economy critically depends on the composition of public and private safe assets. In contrast, we study the creation and informational linkages between government bonds and private assets when both can fulfill the two roles as stores of value and as collateral.

The paper proceeds as follows. In Section 2 we introduce the basic model with no information frictions, with two sources of demand for safe assets – store of value and collateral – and two sources of supply – public bonds and privately-produced assets. In Section 3 we introduce private assets of heterogeneous quality and information frictions, showing that they are only "safe" as long as no information about their quality is produced and revealed. In Section 4 we extend the model to an overlapping generations setting and study dynamics, showing the potential for a policy trap and for the need to combine monetary and macroprudential policies. Section 5 concludes.

[3]Other examples include Woodford (1990), Holmström and Tirole (1998), and more recently Venkateswaran and Wright (2013), Angeletos, Collard, and Dellas (2016), Azzimonti and Yared (2019).

2 Two Period Model Without Information Frictions

In this section we introduce the two sources of demand for safe assets (store of value and collateral) and the two sources of supply (public and private). First, as a benchmark, we consider a simple setting with only public supply (government bonds) and a single role (store of value). Then, we add the second role (collateral) and show that the two roles interact – the use of bonds as collateral may increase or decrease their needs as store of value. Finally, we introduce assets of *homogeneous quality* that can be produced privately, that can also serve both roles and that are in competition with government bonds. Here the optimal provision of bonds is a function of how costly it is to produce private assets and how well they fulfill the two roles of safe assets.

2.1 Optimal Bond Policy when Bonds are not Collateral

Assume households live for two periods in an endowment economy. Endowment in the first period is Y_1 and endowment in the second period is Y_2. The consumption of numeraire good gives households utility $U(C)$, which is separable, strictly concave and satisfies the Inada conditions. The discount between periods is β.

Unconstrained Optimum: Assume a planner with a storage technology that allows the planner to move resources freely across periods. The planner's problem is then:

$$\max_{C_1,C_2} U(C_1) + \beta U(C_2)$$

subject to

$$C_1 + C_2 \leq Y_1 + Y_2$$

which is simply characterized by the Euler equation

$$U'(C_1) = \beta U'(C_2) \tag{1}$$

Equilibrium and Optimum Implementation: Imagine now that households cannot transfer resources across periods but the government has access to a storage technology that can do it freely (as assumed before for the planner). This setting displays extreme incomplete markets, such as in the absence of a government, agents can only consume in autarky (their endowments in each period).

The government's storage technology is just a shortcut for the government's taxation power in a more involved setting in which Ricardian Equivalence does not hold.[4] Here we rely on a reduced-form distortionary taxation when operating the storage technology (discussed more in detail by Auerbach and Kotlikoff (1987) and McGrattan (1994)). More specifically, the government sells (or buys) Treasury bonds B in the first period at an equilibrium price P, which can be positive (buy) or negative (sell). If the government's promise in the second period exceeds what the government collects in the first period (this is $B > PB$), the difference has to be raised by taxing second-period households' endowments, Y_2. We assume that taxing endowments is distortionary in that it destroys χ units of endowment per unit of tax. If the government's promise in the second period is less than collected in the first period (this is $B < PB$), we assume the extra resources are "thrown into the ocean" at no cost.

The household problem in this economy becomes

$$\max_{C_1, C_2, B} U(C_1) + \beta U(C_2)$$

subject to

$$\begin{aligned} C_1 + PB &\leq Y_1 \\ C_2 &\leq Y_2 + B - T \\ T &= (1 + \chi) \max\{0, (1 - P)B\}. \end{aligned}$$

Plugging taxes into the second-period households' budget constraint, the maximization problem becomes:

$$\begin{aligned} C_1 + PB &\leq Y_1 \\ C_2 &\leq Y_2 + Bm. \end{aligned}$$

[4]One example would be, as in Barro (1974), an overlapping generations environment without altruism (as documented by Altonji, Hayashi, and Kotlikoff (1992, 1996 and 1997)) in which every generation has $Y_1 > 0$ and $Y_2 = 0$ and the government taxes the young to give to the old. We explore this possibility later in the paper. Another example would be a setting with two coexisting types of households, one with complete access to credit markets, and the other (the one we model here) without access. The government could tax the group with access and give to the group without access. The effect of incomplete markets on the failure of the Ricardian Equivalence has been explored by Aiyagari (1994) and Heathcote (2005) among others.

with the distortion formally defined as

$$m = \min\{1, P - \chi(1-P)\}. \tag{2}$$

The Euler equation is then,

$$PU'(C_1) = \beta m U'(C_2) \tag{3}$$

Optimal Bond Price and Quantity: Comparing the planner's condition (1) with the households' condition (3), the unconstrained optimum is implemented when $P = m$.

This implies that the *optimal debt quantity* is B_S^* such that equation (3) is satisfied when $C_1 = Y_1 - B_S^*$ and $C_2 = Y_2 + B_S^*$, then purely determined by preferences, endowments and the discount. In contrast, the *optimal debt pricing* is $P = m$, pinned down by the distortions in the way a government can move resources across periods.

To focus on the optimal debt quantity and its relation with private assets and their safety, we have assumed tax distortions such that $P = 1$ always, regardless of B_S^*. To see this notice that, if bonds were abundant and $P < 1$, then $\frac{P}{m} = \frac{P}{P-\chi(1-P)} > 1$,[5] which implies that the government has to distort consumption in the second period with taxes, forcing it to also inefficiently reduce consumption in the first period. In contrast, if bonds were scarce, and $P > 1$, then $\frac{P}{m} = \frac{P}{1} > 1$, which implies that the government extracts more in the first period than what it delivers in the second period, wasting resources and reducing consumption excessively. Notice also that, in the case where taxation is not distortionary (this is $\chi = 0$), $m = 1$ and any price $P \leq 1$ is consistent with optimal implementation, a version of Ricardian Equivalence in our setting.

This result in optimal pricing is not general, just expositionally convenient. If distortions were a function of the amount of debt (this is, $m(B_S)$), the optimal quantity of debt would still be B_S^* as above, but its optimal pricing would be $P = m(B_S^*)$. If, for instance, $m(B_S)$ is an increasing function of the amount of debt, the higher is the optimal quantity of debt, the larger the price that the government should target to implement it.[6] We will use this result later, when we discuss the optimal debt quantity when the information content of private assets is endogenous.

[5] When $\chi > \frac{P}{1-P}$, then $\frac{P}{m} < 0$, which also distorts consumption.

[6] This is perhaps the most natural assumption, as an increase in the amount of outstanding government debt tends to increase the probability of default and the cost of holding debt.

Log Example (bonds only as store of value): Take an example with $U(C) = \log(C)$. The Euler equation, hence the households' demand for bonds, is

$$P\frac{1}{Y_1 - PB} = \beta\frac{m}{Y_2 + Bm} \qquad \Longrightarrow \qquad B = \frac{\beta Y_1 - \frac{P}{m}Y_2}{(1+\beta)P},$$

decreasing in the price P. The unconstrained optimum is characterized by a supply of bonds B_S that implements $\frac{P}{m} = 1$, this is

$$B_S^* = \frac{\beta Y_1 - Y_2}{(1+\beta)}. \tag{4}$$

As can be seen, the optimal supply of bonds increases with Y_1 and decreases with Y_2. If $\beta Y_1 > Y_2$ the government would like to sell bonds in the first period (in exchange for numeraire) to implement the unconstrained optimum by reducing the consumption of households in the first period and increasing it in the second period. If $\beta Y_1 < Y_2$ the government would like to buy bonds from households in the first period to implement the unconstrained optimum, this time by increasing consumption in the first period with resources coming from the second period (or from another unmodelled group in the same period).

Relation with Monetary Policy: The planner's optimal $P \equiv \frac{1}{1+r} = 1$ can be implemented by a monetary authority that exchanges Treasury bonds (through open market operations or quantitative easing) and sets $1 + r = 1$ (a Friedman rule, given our assumption that creating bonds is costless for the government, as storage of numeraire follows a costless technology). If the central bank expects Y_2 to be very low with respect to Y_1 it would like to increase interest rates (to move consumption from the first to the second period) by selling Treasuries in exchange for money (numeraire in our case) and reducing P. The opposite happens when Y_2 is expected to be large relative to Y_1. Then the central bank would like to reduce rates by buying Treasuries. The amount of bonds given in equation (4) is what implements the optimal consumption path in the economy.

2.2 Optimal Bond Policy with Bonds as Collateral

Now we extend the endowment setting to include production with credit needs. More specifically, a fraction x of households have an investment opportunity at the beginning of the second period, after the endowment Y_2 is obtained but before consumption takes place. This investment opportunity transforms l unit of endowment good per household into Al^α units of endowment good at the end of the second period. The rest of the households $1-x$ do not have investment opportunities. Whether a household is productive or not is realized at the beginning of the second period.

The unconstrained optimal scale of production for each productive household is given by $\max_l[Al^\alpha - l]$, or $l^* = [\alpha A]^{\frac{1}{1-\alpha}}$. To simplify the cases in terms of feasibility, in what follows, we assume that $Y_2 < l^* < \frac{Y_2}{x}$. In other words, the endowment of each household is not enough to finance production at the optimal scale, but the endowments of all households are sufficient.

Unconstrained Optimum: If the planner has a technology to transfer resources *both across periods and across agents*, given our previous feasibility assumption, it is clear that the unconstrained optimum is given by allowing productive agents to operate at optimal scale l^* (using own endowment and endowment transferred from nonproductive agents at the beginning of the second period), equalizing marginal utilities of consumption between productive and non-productive agents in the second period (transferring numeraire at the end of the second period), and smoothing intertemporal consumption based on the Euler equation (1).

Constrained Optimum: Here we study the more interesting case in which the planner can move resources across periods, but is *constrained* to move resources across agents. This implies that *i*) projects can only be operated by productive agents with their own endowments (either current or saved from the first period) at the *beginning of the second period* and *ii*) there is no cross-insurance of consumption between productive and non-productive agents at the *end of the second period*.

Formally, this constrained problem is given by,

$$\max_{C_1,C_{p,2},C_{np,2},l} U(C_1) + \beta[xU(C_{p,2}) + (1-x)U(C_{np,2})]$$

subject to

$$
\begin{aligned}
C_1 + C_{p,2} &\leq Y_1 + Y_2 + \widehat{Y}_2 \\
C_1 + C_{np,2} &\leq Y_1 + Y_2 \\
\widehat{Y}_2 &= [Al^{\alpha} - l] \\
l &\leq Y_2 + (Y_1 - C_1).
\end{aligned}
$$

Denoting by λ the Lagrange multiplier for budget constraints and μ for the credit constraint, first order conditions are:

$$
\begin{aligned}
\{C_1\} &: \quad U'(C_1) = \lambda_{p,2} + \lambda_{np,2} + \mu \\
\{C_{p,2}\} &: \quad \beta x U'(C_{p,2}) = \lambda_{p,2} \\
\{C_{np,2}\} &: \quad \beta(1-x)U'(C_{np,2}) = \lambda_{np,2} \\
\{l\} &: \quad \lambda_{p,2}(\alpha A l^{\alpha-1} - 1) = \mu
\end{aligned}
$$

Defining the marginal return of the project as

$$
R(l) = \alpha A l^{\alpha-1} - 1,
$$

with $R(l) > 0$ and $R'(l) < 0$ in equilibrium, and $R(l^*) = 0$. Combining these conditions gives

$$
U'(C_1) - \beta \mathbb{E}(U'(C_2)) = \beta x U'(C_{p,2}) R(l). \tag{5}
$$

This equation highlights the planner's main trade-off when constrained from moving resources across agents with different investment opportunities. The planner equalizes the intertemporal smoothing distortions (the difference between $U'(C_1)$ and $\beta\mathbb{E}(U'(C_2))$) with the increase in extra second period's production obtained by moving resources to the second period (x productive agents will be able to produce an extra numeraire $R(l)$, which they value at $\beta U'(C_{p,2})$ at the margin).[7]

Intuitively, in the absence of investment opportunities, the planner just equalizes marginal utilities in both periods (as in the previous setting). When investment opportunities exist, however, there is an extra gain from moving resources to the second period to sustain investment that is restricted by second-period endowments. This

[7] Notice that, if there were enough resources for each productive agent to implement l^* in the second period, with own endowments and savings, then $R(l^*) = 0$ and the planner would not introduce additional intertemporal distortions.

leads the planner to distort consumption smoothing by consuming less in the first period in order to be able to produce and consume more in the second period. At the same time, however, this extra consumption reduces the marginal utility of consumption in the second period, which reduces the gains from moving consumption to the second period. This highlights the intricate relation between intertemporal and intratemporal smoothing in the constrained optimum.

Equilibrium and Constrained Optimum Implementation: The unconstrained optimum could be implemented if credit and insurance markets were complete. If output from investment opportunities were pledgeable, for instance, then productive agents would be able to borrow l^* to produce at optimal scale. If ex-ante insurance contracts were present, agents could insure against being nonproductive in the second period.

Here we assume market incompleteness that prevents frictionless transfer of resources across agents in the second period, and study how the government could implement the constrained planner's allocation by providing government bonds in the first period. In particular, the main assumption is that numeraire is not pledgeable. This implies that *i)* there are credit frictions in which nonproductive agents would not be willing to transfer resources to productive agents at the beginning of the second period and *ii)* insurance contracts that promised to transfer resources from productive to nonproductive agents at the end of the second period are not implementable.

As we want to capture the dual role of government bonds as store a of value and as collateral, in what follows we assume that, in contrast to numeraire, which can be absconded with by agents, government bonds are pledgeable. Hence, productive agents can borrow numeraire from non-productive agents using bonds as collateral in order to overcome the credit friction. At this point, since production is deterministic and bonds are of known quality, productive agents can either sell bonds or borrow using bonds as collateral, and obtain the same allocation. We show later that, when information acquisition about the asset is endogenous, using bonds as collateral is superior to selling them to obtain investment funds.

The household problem in the first period, knowing that in the second period it may become a productive agent with probability x, is:

$$\max_{C_1, C_{p,2}, C_{np,2}, l, B} U(C_1) + \beta[xU(C_{p,2}) + (1-x)U(C_{np,2})]$$

subject to

$$\begin{aligned} C_1 + PB &\leq Y_1 \\ C_{p,2} &\leq Y_2 + Bm + \widehat{Y}_2 \\ C_{np,2} &\leq Y_2 + Bm \\ \widehat{Y}_2 &= [Al^{\alpha} - l] \\ l &\leq Y_2 + Bm. \end{aligned}$$

where m is the bond consumption after taxation distortions from equation (2). Obtaining and combining first-order conditions we obtain the condition that determines the demand for bonds in equilibrium,

$$PU'(C_1) - \beta m \mathbb{E}(U'(C_2)) = \beta m x U'(C_{p,2}) R(l). \tag{6}$$

This equation clarifies how the two roles of government bonds interact. First, they are useful as collateral (higher right hand side), which increases the demand for bonds. Second, their use as collateral increases consumption and reduces marginal utilities in the second period (higher left hand side), which reduces the demand for bonds.

The constrained optimum can again be implemented if and only if $P = m = 1$. To obtain the supply of government bonds that implements such a price, we need to equalize the supply with the demand of bonds from equation (6) at $P = 1$.

Log Example (bonds used also as collateral): To compare with the initial benchmark, take the case of log utilities, such that equation (6) becomes

$$P \frac{1}{Y_1 - PB} - \beta m \left[\frac{x}{A(Y_2 + Bm)^{\alpha}} + \frac{1-x}{Y_2 + Bm} \right] = \beta m \frac{x}{A(Y_2 + Bm)^{\alpha}} (\alpha A (Y_2 + Bm)^{\alpha - 1} - 1)$$

$$P \frac{1}{Y_1 - PB} = \beta m \left[\frac{x\alpha}{Y_2 + Bm} + \frac{1-x}{Y_2 + Bm} \right]$$

or, defining $\widehat{\beta} \equiv \beta(1 - x(1-\alpha)) < \beta$

$$P \frac{1}{Y_1 - PB} = \widehat{\beta} \frac{m}{Y_2 + Bm} \qquad \Longrightarrow \qquad B = \frac{\widehat{\beta} Y_1 - \frac{P}{m} Y_2}{(1 + \widehat{\beta}) P}. \tag{7}$$

The extra use of bonds as collateral allows i) for more production in the second period, and then ii) a reduction of the second period's marginal utility of consumption. In the particular case of log utilities the second effect dominates and the use of bonds as collateral effectively lowers households' patience. Compared to the setting in which bonds are not used as collateral there is a marginally lower demand for bonds, and the optimum is implemented by choosing the supply of bonds B_S such that $P = 1$. This is

$$B_S^* = \frac{\widehat{\beta} Y_1 - Y_2}{(1+\widehat{\beta})} \qquad \texttt{with} \;\; \widehat{\beta} = \beta(1 - x(1-\alpha)). \tag{8}$$

2.3 Optimal Bond Policy with Production of Private Assets

So far we have assumed that only government bonds provide a vehicle to move consumption intertemporally and to use as pledgeable promises to sustain credit in the economy. Here, we allow agents to produce private assets that can also serve as a store of value and collateral, possibly only partially (in case a unit of private asset sustains $\phi < 1$ units of collateral).[8] Formally, households can produce Z private assets available in the second period at a cost of Z^γ in terms of numeraire in the first period, where $\gamma > 1$. This technology effectively transforms numeraire, making it *storable* (useful as a store of value) and *pledgeable* (useful as collateral).

Constrained Optimum: We solve for a planner's optimum when the planner cannot impose a transfer of resources between productive and non-productive agents, but has the previously described technology to create assets. The constrained problem is:

$$\max_{C_1, C_{p,2}, C_{np,2}, l, Z} U(C_1) + \beta[xU(C_{p,2}) + (1-x)U(C_{np,2})]$$

subject to

$$\begin{aligned} C_1 + C_{p,2} + Z^\gamma &\leq Y_1 + Y_2 + Z + \widehat{Y}_2 \\ C_1 + C_{np,2} + Z^\gamma &\leq Y_1 + Y_2 + Z \\ \widehat{Y}_2 &= [Al^\alpha - l] \\ l &\leq Y_2 + (Y_1 - C_1 - Z^\gamma) + \phi Z. \end{aligned}$$

[8]Again, private assets can be directly sold at the beginning of the second period to increase the scale of production. In that interpretation $\phi < 1$ would capture imperfect liquidity of the asset (the probability of finding a buyer).

First order conditions are

$$
\begin{aligned}
\{C_1\} &: U'(C_1) = \lambda_{p,2} + \lambda_{np,2} + \mu \\
\{C_{p,2}\} &: \beta x U'(C_{p,2}) = \lambda_{p,2} \\
\{C_{np,2}\} &: \beta(1-x)U'(C_{np,2}) = \lambda_{np,2} \\
\{l\} &: \lambda_{p,2}(\alpha A l^{\alpha-1} - 1) = \mu \\
\{Z\} &: (\lambda_{p,2} + \lambda_{np,2})(\gamma Z^{\gamma-1} - 1) + \mu.\gamma Z^{\gamma-1} = \phi\mu
\end{aligned}
$$

Combining these conditions, the planner's optimal choice on how to allocate consumption across periods is given by

$$U'(C_1) - \beta\mathbb{E}(U'(C_2)) = \beta x U'(C_{p,2})R(l), \tag{9}$$

and the planner's optimal choice of asset production is given by:

$$\gamma Z^{\gamma-1}U'(C_1) - \beta\mathbb{E}(U'(C_2)) = \beta x\phi U'(C_{p,2})R(l) \tag{10}$$

Equations (9) and (10) characterize the optimal choices for the planner between delaying consumption and generating assets. Both alternatives smooth consumption intertemporally and increase production in the second period. Equation (9) is the already discussed trade-off of delaying consumption: the benefit of more resources to invest in the second period at the cost of distorting consumption intertemporally. Equation (10) displays a similar trade-off of producing assets: the benefit of more resources to invest in the second period (but scaled down by $\phi < 1$) at the cost of distorting consumption intertemporally (but with a marginal cost in terms of numeraire in the first period of $\gamma Z^{\gamma-1}$).[9]

Combining these two equations gives us the combination between delayed consumption and production of assets that the planner would like to achieve to smooth consumption and increase production. Subtracting (10) from (9), the constrained optimal production of private assets Z^* satisfies

$$\gamma(Z^*)^{\gamma-1} = 1 - (1-\phi)\frac{x\beta U'(C_{p,2})R(l)}{U'(C_1)}. \tag{11}$$

[9] Notice that the marginal benefit of creating private assets is smaller than delaying consumption as $\phi < 1$, and then $\gamma Z^{\gamma-1} < 1$. A decline in the use of private assets to facilitate credit, ϕ, translates into a reduction in the production of private assets, Z.

Equilibrium and Constrained Optimum Implementation: Now we solve for the equilibrium and for the optimal amount of government bonds that implement the constrained optimum. The households' problem is:

$$\max_{C_1, C_{p,2}, C_{np,2}, l, B, Z} U(C_1) + \beta[xU(C_{p,2}) + (1-x)U(C_{np,2})]$$

subject to

$$\begin{aligned} C_1 + PB + Z^\gamma &\leq Y_1 \\ C_{p,2} &\leq Y_2 + Bm + Z + \widehat{Y}_2 \\ C_{np,2} &\leq Y_2 + Bm + Z \\ \widehat{Y}_2 &= [Al^\alpha - l] \\ l &\leq Y_2 + Bm + \phi Z \end{aligned}$$

From the first order condition for bonds,

$$PU'(C_1) - \beta m \mathbb{E}(U'(C_2)) = \beta m x U'(C_{p,2}) R(l) \tag{12}$$

and from the first order condition for private assets,

$$\gamma Z^{\gamma-1} U'(C_1) - \beta \mathbb{E}(U'(C_2)) = \beta x \phi U'(C_{p,2}) R(l). \tag{13}$$

From conditions (12) and (13), the equilibrium production of private assets is:

$$\gamma (Z^{eq})^{\gamma-1} = \frac{P}{m} - (1-\phi) \frac{x \beta U'(C_{p,2}) R(l)}{U'(C_1)}. \tag{14}$$

Notice that even when the agents have the possibility of buying bonds to smooth consumption and to use as collateral, they would still produce some private assets since their production function is convex and the marginal cost of production is zero at $Z = 0$. Agents' production of private assets is increasing in P and, comparing with equation (11), it is clear that the optimal implementation requires $P = m = 1$.

Log Example (with private assets): Just for comparison purposes with previous subsections assume log preferences and $\phi = 1$ (with $\phi < 1$ there is no closed-form solution of the optimal bond supply, but the logic remains). The bond demand is:

$$P\frac{1}{Y_1 - PB - (Z^{eq})^\gamma} = \beta m \left[\frac{x(1+\alpha Al^{\alpha-1}-1)}{Y_2 + Bm + Z^{eq} + Al^\alpha - l} + \frac{1-x}{Y_2 + Bm + Z^{eq}}\right]$$

where $l = Y_2 + Bm + Z^{eq}$, and from equation (14), $Z^{eq} = \left[\frac{P}{\gamma m}\right]^{\frac{1}{\gamma-1}}$. Then,

$$B = \frac{\widehat{\beta}Y_1 - \frac{P}{m}Y_2 - (\widehat{\beta}+\gamma)\left[\frac{P}{\gamma m}\right]^{\frac{\gamma}{\gamma-1}}}{(1+\widehat{\beta})P},$$

where, as in the previous section, $\widehat{\beta} = \beta(1-x(1-\alpha))$. The demand for bonds decreases with the bonds' price and it is higher without a technology that creates private assets.

The optimum is implemented with $P = m = 1$, which can be done by providing bonds B_S^* such that,

$$B_S^* = \frac{\widehat{\beta}Y_1 - Y_2 - (\widehat{\beta}+\gamma)\left[\frac{1}{\gamma}\right]^{\frac{\gamma}{\gamma-1}}}{(1+\widehat{\beta})}, \tag{15}$$

which is lower than the supply of bonds needed in the absence of private assets.

Comparing the optimal bond supply (15) with the optimal supply (8) in the absence of private assets, it is clear their creation reduces the demand for bonds (as private assets provide an alternative channel to store value and to facilitate credit), proportionally to the cost of generating private assets.

3 Two Period Model With Information Frictions

Now we assume that private assets come in two qualities. A fraction $\bar{p}$ are good assets, with value $Z_G = \kappa_G Z > Z$, and the rest are bad, with value $Z_B = 0$, with $\bar{p}\kappa_G = 1$, so the expected value of private assets is still Z.

3.1 The Role of Information

In what follows we compare two information alternatives, with and without information about private assets at the beginning of the second period, and show that the economy's welfare is higher when information is not revealed until consumption takes place at the end of the second period.

When information gets revealed at the beginning of the second period, there are two sources of risk. One is the "productivity shock" (also present above and which we assumed was non-insurable). This shock determines whether the agent has access to an investment opportunity or not, which we denote by $i \in \{p, np\}$, with $q_p = x$ and $\widehat{Y}_{np,2} = 0$. The other, which is a new source of risk that is introduced by information revelation, is a "private collateral shock" that determines whether the private asset is good or bad, which we denote by $j \in \{G, B\}$, with $q_G = \bar{p}$ and $Z_B = 0$.

Using this, more general, notation the households' problem can be written as:

$$\max_{C_1, C_{ij,2}, l_j, B, Z} U(C_1) + \beta \sum_{i,j} q_i q_j U(C_{ij,2})$$

subject to

$$\begin{aligned} C_1 + PB + Z^\gamma &\leq Y_1 \\ C_{ij,2} &\leq Y_2 + Bm + \kappa_j Z + \widehat{Y}_{ij,2} && \forall i, j \\ \widehat{Y}_{pj,2} &= \left[A l_j^\alpha - l_j\right] \\ l_j &\leq Y_2 + Bm + \phi \kappa_j Z && \forall j. \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \{C_1\} &: U'(C_1) = \lambda \\ \{C_{ij,2}\} &: \beta q_i q_j U'(C_{ij,2}) = \lambda_{ij,2} \\ \{l_j\} &: \lambda_{pj,2}(\alpha A l_j^{\alpha-1} - 1) = \mu_j \\ \{B\} &: P\lambda - m \sum_{i,j} \lambda_{ij,2} = m \sum_j \mu_j \\ \{Z\} &: \gamma Z^{\gamma-1} \lambda - \sum_{i,j} \kappa_j \lambda_{ij,2} = \sum_j \phi \kappa_j \mu_j. \end{aligned}$$

Notice that $\sum_{i,j} \lambda_{ij,2} = \beta \mathbb{E}(U'(C_{ij,2}))$. There is more risk than in the case with a single collateral type as the return on the project, $R_j \equiv (\alpha A l_j^{\alpha-1} - 1)$ is stochastic.

The demand for bonds equalizes their cost and benefits,

$$PU'(C_1) - \beta m \mathbb{E}(U'(C_{ij,2})) = \beta m x \sum_j q_j U'(C_{pj,2}) R_j, \tag{16}$$

which is also the case for creation of private assets,

$$\gamma Z^{\gamma-1} U'(C_1) - \beta \mathbb{E}(\kappa_j U'(C_{ij,2})) = \beta x \phi \sum_j q_j \kappa_j U'(C_{pj,2}) R_j. \tag{17}$$

Notice that the benefit of producing an additional unit of private asset is smaller than the benefit of buying an additional unit of government bonds, for two reasons. First, the heterogeneity in the quality of private assets introduces consumption risk in the second period compared to bonds, then generating a lower expected utility when agents are risk averse (i.e., $\beta \mathbb{E}(\kappa_j U'(C_{ij,2})) < \beta \mathbb{E}(U'(C_{ij,2}))$. Second, private assets are worse collateral, not only because of $\phi < 1$, but also because of the extra risk generated by their heterogeneity, which is also inferior when agents are risk averse (i.e., $\sum_j q_j \kappa_j U'(C_{pj,2}) R_j < \sum_j q_j U'(C_{pj,2}) R_j$). In other words, when agents are risk averse, assets that generate future consumption risk are less valuable as store of value.

If information about the quality of private assets is only revealed upon consumption at the end of the second period, and not at the credit stage, there is a reduction in consumption variance in the second period. There are three messages that arise from comparing these different information environments. First, fixing the amounts of bonds and private assets in the economy, welfare is lower with information simply because agents face higher utility uncertainty. Second, the benefits of buying bonds and creating private assets increase when information is revealed as the second period is more uncertain. This comes from applying Jensen's inequality, given risk aversion (concavity of the utility function) and decreasing marginal returns (concavity of the production function). Formally, $\mathbb{E}(U'(C_{ij,2})R(Z_j)) > U'(\mathbb{E}C_{ij,2})R(\mathbb{E}Z_j)$. Finally, information increases the benefits of bonds more than it increases the benefits of private assets. This comes from comparing information-equations (16) with (17). The reason is that private assets are scarcer in states of the world in which they are more valuable, both because they provide low consumption and make poor collateral and then low production. More formally, $\mathbb{E}(U'(C_{ij,2})R(Z_j)) > \mathbb{E}(\kappa_j U'(C_{ij,2})R(Z_j))$.

We summarize this discussion in the following Proposition

Proposition 1 *Information about private assets of heterogeneous quality makes government bonds more valuable as collateral (relative to private assets) and more valuable as store of value (as future consumption is riskier).*

This comparison highlights the pervasive role of information about heterogeneous collateral in credit markets. If there is no information, all productive agents obtain a loan for a collateral of expected value $\widehat{\kappa}_G Z = Z$. If there is information, some productive agents would obtain a larger loan based on collateral $\kappa_G Z$ (potentially having excessive collateral once the loan implements the optimal scale l^*) while some others would not get to produce. With risk aversion and decreasing marginal returns this risk reduces welfare, intuitively because information destroys the cross-insurance that ignorance can provide. In other words, ignorance transforms assets into "safe collateral," which is beneficial in terms of total consumption in the economy.

3.2 Information Acquisition

Now we add two extensions to the previous setting, so we can study the conditions under which the economy will be in the first situation (with information revealed at the beginning of the second period, before lending takes place) or the second situation (with information revealed at the end of the second period upon consumption and after production has already happened).

In contrast to the previous setting, in which the project outcome was assumed deterministic, now we want to study the possibility that a lender (a non-productive agent in our setting) ends up with the asset in case of default, and as such may want to privately acquire information about its quality. First, we assume the project fails with probability d. Second, we assume that a lender can acquire information about the quality of a private asset at the beginning of the second period, at a *utility cost* of ψ.

We have to be more explicit now about the credit protocol. We assume there is random matching between a lender and a borrower and that $x > 1/2$, such that the borrower has all the bargaining power. The possibility of default not only adds another source of risk to the borrower (who is the residual claimant on the project) but also exposes the lender to the risk of receiving private collateral of low quality in the case of default.

In a match between agents, negotiating a loan of size l, the part that is not covered by bonds and needs to be covered by private assets is $l - Y_2 - Bm$. Assuming the borrower has a private asset of perceived quality p_b, then the expected value of such a private asset is $p_b \kappa_G Z$. This implies that the borrower has to finance a fraction $f = \frac{l - Y_2 - Bm}{p_b \kappa_G Z} \leq 1$ of the asset to get a loan l. Naturally, $f = 1$ as long as $R(l) > 0$ in equilibrium, which occurs when there is not enough collateral to reach l^*.

Under what conditions can the information-insensitivity of collateral (superior in terms of welfare) be sustained in equilibrium? This happens when the lender's expected utility from acquiring information (net of the cost) is less than the lender's expected utility from not acquiring information. This is, when,

$$\mathbb{E}U(C^I_{np,2}) - \mathbb{E}U(C^U_{np,2}) \leq \psi.$$

When not acquiring information, the lender faces the risk of consuming a bad private asset upon default. Indeed, avoiding the state in which there is a default and the borrower's private asset is bad is the only one in which information is valuable, as it allows the lender to walk away from the contract. In the case of repayment, information about the collateral quality is irrelevant. Then, in case of receiving good quality collateral, or in the case of repayment, there is the same level of utility regardless of whether information was produced or not. Then, *no information* is a sustainable equilibrium if and only if

$$d(1 - p_b)[U(C^I_{np,2}) - U(C^U_{np,2})] \leq \psi. \tag{18}$$

where

$$\begin{aligned} C^I_{np,2} &= Y_2 + Bm + p_l \kappa_G Z \\ C^U_{np,2} &= Y_2 + Bm + p_l \kappa_G Z - (l - Y_2 - Bm). \end{aligned}$$

In other words, the gain from information production comes from the lender's possibility to walk away from the contract upon finding out that the collateral is bad quality and avoiding a loss of consumption by the size of the loan backed by the private asset in case of default. Note that if agents were risk neutral this condition would boil down to subtracting $C^U_{np,2}$ from $C^I_{np,2}$ and the condition would become

$$l \leq \frac{\psi}{d(1-p_b)} + Y_2 + Bm,$$

which is similar to the conditions in Gorton and Ordonez (2014 and 2020b), but considering the use of government bonds and own funds in obtaining the loan.

The next proposition shows comparative statics for the determinants of information acquisition in our setting with risk aversion.

Proposition 2 *The incentives to acquire information about private assets used as collateral increase with the size of the loan sustained by private collateral (l given B) and decrease with the amount of bonds used as collateral (B given l). The incentives also increase with the probability of default (d) and decrease with the probability the asset is of good quality (p_b) and with the cost of information production (ψ).*

Proof Denote the net incentives to acquire information as

$$\Pi = d(1-p_b)[U(C^I_{np,2}) - U(C^U_{np,2})] - \psi$$

Then

$$\frac{\partial \Pi}{\partial l} = d(1-p_b)U'(C^U_{np,2}) > 0$$

and

$$\frac{\partial \Pi}{\partial B} = dm(1-p_b)[U'(C^I_{np,2}) - U'(C^U_{np,2})] - dm(1-p_b)U'(C^U_{np,2}) < 0.$$

There are two reasons why the incentives to acquire information decrease with B. The first argument shows that, fixing the use of collateral (that is, fixing $l - Y_2 - Bm$), the lender is "relatively richer" and has fewer incentives to explore such collateral. The second argument shows that, fixing the size of the loan (i.e., fixing l), using more bonds reduces the use of private collateral, hence the incentives to explore it.

$$\frac{\partial \Pi}{\partial d} = (1-p_b)[U(C^I_{np,2}) - U(C^U_{np,2})] > 0$$

$$\frac{\partial \Pi}{\partial \psi} = -1 < 0$$

Finally,

$$\frac{\partial \Pi}{\partial p_b} = -d[U(C^I_{np,2}) - U(C^U_{np,2})] < 0.$$

Q.E.D.

This proposition shows that a heavy use of bonds to sustain a given loan size discourages lenders from acquiring information about private collateral. To see this, notice that the incentives to acquire information increase in $d(1-p_b)$ and in $l - Y_2 - Bm$.

Remark on the Superior Use of Private Assets as Collateral: With endogenous information acquisition, productive agents would strictly prefer to borrow using private assets as collateral instead of selling those assets to raise investment funds. If instead a productive agent sells the private asset, the buyer's incentives to acquire information about its quality is given by equation (18), but without d, since the buyer would always take possession of the asset. Then selling private assets of uncertain quality would encourage information acquisition more than using it as collateral, which ex-ante reduces the value of those assets to obtain investment funds. This result highlights an informational justification for using assets as collateral instead of selling them to raise funds.

4 Insights from Overlapping Generations

Above we used a two-period model to highlight how the optimal provision of government bonds should accommodate their dual role as store of value and as collateral and in affecting the creation and (informational) safety of private assets. Here we propose an overlapping-generation (OLG) extension to study the dynamics of private asset creation and safety in response to government bond changes. We convey two messages. First, the optimal policy in steady state may face a policy trap that requires combining conventional policies (such as open market operations and quantitative easing, or OMO and QE) and unconventional policies (what we call bond exchange facility, or BEF). Second, a transitory decline in government bonds (driven by external demand shocks or other fiscal considerations) may have long-term consequences in terms of financial fragility.

4.1 Environment

Each generation lives for two periods. In each calendar period $t \in \{0, 1, ...\}$ there coexists a young generation and an old one. An agent is born young at date t with endowment Y_1 units of numeraire. At the beginning of the period she invests in private assets, which we denote as a flow z_t that adds to the stock Z_t. Then the agent buys a one-period maturity bond, B_t. At the end of its youth the agent randomly matches with an old individual, who sells their private asset at its perceived fundamental value $p\kappa_G(1-\delta)Z_t$, where δ is the depreciation rate of private assets, independent of their quality, and p is the perceived probability that such an asset is good.[10] The buyer's previous investment, z_t, inherits the quality of the asset purchased.[11]

To clarify how we envision investment quality, consider the following example. At the beginning of their youth, agents invest in infrastructure z_t and at the end of their youth they buy a depreciated building of size $(1-\delta)Z_t$ that is located on a specific acre of land. The total size of the building will then be $Z_{t+1} = (1-\delta)Z_t + z_t$. The acre where the building is located can be of good quality (in the sense that it boosts the value of the building by κ_G) or bad quality (the land is in such a bad location that the value of the whole building on that acre is 0). It is in this sense that the investment of an agent inherits the quality of the asset she buys.

During the transition from period t to $t+1$, a fraction $1-\lambda$ of land (the location of the building in the previous example) experiences an idiosyncratic shock that resets the quality, which can become good with probability $\bar{p}$. This implies that there is depreciation of information about the quality of an asset at a rate $1-\lambda$ (in case information is not replenished in the economy). Note that the actual amounts of good and bad quality assets do not change. This process generates three possible beliefs about the quality of private assets, $p = 0$ (the asset is known to be bad), $p = 1$ (the asset is known to be good) and $p = \bar{p}$ (the asset is of unknown quality). More generally, the possible beliefs about purchased assets are $k \in \{0, \bar{p}, 1\}$, which can transition to $j \in \{0, \bar{p}, 1\}$ when traded, according to the process of idiosyncratic shocks and information acquisition. We call a *crisis* the situation in which a large fraction of assets of unknown quality is investigated, and their quality discovered, before granting credit takes place.

[10]Buyers acquire assets at the fundamental value because we assume they have all bargaining power. Assuming otherwise would imply that the price of private assets would also include their use as store of value and collateral. This alternative would add an additional layer of interactions between the supply of government bonds and the value of private assets. For an analysis of this alternative valuation layer see Infante and Ordonez (2020).

[11]The particular timing in which young households first invest and buy bonds and then buy assets implies that investments and portfolios are not conditional on the quality of traded assets. Reversing the timing (first buying the assets and then investing and buying bonds) would just introduce additional sources of heterogeneity across agents but would not affect aggregate results and dynamics.

When the agent gets old, he receives an endowment of Y_2 and draws a productivity (i.e., whether he has an investment opportunity or not). Each productive old agent borrows funds from an unproductive old agent using his bonds and private assets as collateral. When used to obtain credit the value of the private asset is $\widehat{Z}_{t+1} = p\kappa_G Z_{t+1}$, where $Z_{t+1} = (1-\delta)Z_t + z_t$ and p is either $\bar{p}$ if quality is unknown, 0 if known to be bad or 1 if known to be good.

At the end of the period, project outcomes are realized, loan contracts are fulfilled, and the old agents redeem their bonds and sell their private asset at the price determined by information (or lack thereof) about the private asset during the credit stage. Furthermore, we allow all agents to obtain an additional endowment for consumption at the end of the period, X. As will become clear, without this extra consumption, and given that the project can fail, no productive agent would borrow up to the constraint for fear of defaulting and consuming 0. In other words, Inada conditions would prevent collateral constraints from binding. We will go on to assume that X is large enough such that the borrowers would like to use all his available assets at the beginning of the period as collateral.

In this extended setting, the government can access resources from future generations. Since we assume one-period bonds, if at the calendar period $t-1$ the government promises a young generation B_{t-1} when old and sells those bonds at a price $P_{t-1} < 1$, the extra resources $T_t = (1 - P_{t-1})B_{t-1}$ can be obtained *i)* as before from old agents, so their net endowment when old becomes $Y_2 - T_t$ or *ii)* from the next generation, which then will have a lower endowment when young of $Y_1 - T_t$. To explore the possible dynamic distortion, we dispense from distortionary taxation m that we explored before and assume the second possibility. We will discuss the implications later when discussing the constrained optimum.

4.2 Characterization

The problem of an individual born in calendar period t depends on the stock of capital accumulated in the economy up to t. As productive agents suffer the additional risk of project failure, denote $A_s \in \{A, 0\}$ with $s \in \{success, failure\}$ and $Pr(failure) = d$.

$$\max_{C_{k,1t}, C_{ijs,2t+1}, l_{j,t+1}, B_t, z_t} \sum_k q_k U(C_{k,1t}) + \beta \sum_{i,j,s} q_i q_j q_s U(C_{ijs,2t+1})$$

subject to

$$
\begin{aligned}
C_{k,1t} + z_t^{\gamma} + P_t B_t + p_k \kappa_G (1-\delta) Z_t &\leq Y_1 - (1 - P_{t-1}) B_{t-1} \\
C_{ijs,2t+1} &\leq Y_2 + B_t + (1-\delta)\widehat{Z}_{j,t+1} + \widehat{Y}_{ijs,2t+1} + X \\
\widehat{Z}_{j,t+1} &= p_j \kappa_G [(1-\delta) Z_t + z_t] \\
\widehat{Y}_{pjs,2t+1} &= \left[A_s l_{j,t+1}^{\alpha} - l_{j,t+1}\right] \\
l_{j,t+1} &\leq Y_2 + B_t + \phi \widehat{Z}_{j,t+1}.
\end{aligned}
$$

Besides the explicit reference to the calendar period, first-order conditions are the same as those that characterize the solution in the two-period model with heterogeneous private collateral and probability of default. Assuming throughout that collateral constraints bind, or $\mu_{j,t+1} > 0$,

$$(1-d)U'(C_{pj,2t+1}^{success})R_{j,t+1} > dU'(C_{pj,2t+1}^{failure})$$

which is just a technical condition on the size of X (should be large enough).

The demand for bonds is characterized by,

$$P_t \mathbb{E}_{k,t}(U'(C_{k,1t})) = \beta \mathbb{E}_{ijs,t+1}(U'(C_{ijs,2t+1})) + \beta q_p \mathbb{E}_{js,t+1}[U'(C_{pjs,2t+1})R_{js,t+1}], \tag{19}$$

where $R_{js,t+1} = \alpha A_s l_{j,t+1}^{\alpha-1} - 1$ is the realized return of the project.

The creation of private assets is characterized by,

$$\gamma z_t^{\gamma-1} \mathbb{E}_{k,t}(U'(C_{k,1t})) = \beta \mathbb{E}_{ijs,t+1}(p_j \kappa_G U'(C_{ijs,2t+1})) + \beta q_p \phi \mathbb{E}_{js,t+1}[p_j \kappa_G U'(C_{pjs,2t+1})R_{js,t+1}]. \tag{20}$$

The main difference between the two-period setting and this overlapping generation structure is given by the links across periods. There are three links. The first is given by the law of motion of the *quantity of private assets* in the economy. This link, of course, would be eliminated by assuming $\delta = 1$ (all private assets have to be created every period). The second link is given by the evolution of the *belief distribution about private assets quality*, which is driven both by information acquisition in credit markets and by the process of idiosyncratic shocks. This link would be eliminated by assuming $\lambda = 0$ (this is all private assets are good with probability $\bar{p}$ every period).[12] The third link is the one imposed by the government budget constraint on their one-period bonds. This link would be eliminated if $P_t = 1$ in all t.

In what follows we characterize the steady state of this economy, and discuss optimal implementation, focusing on the determinants of private assets' information content.

4.3 Information Regimes in Steady State

As we assume there is no population or economic growth, assume a constant provision of bonds, B_{SS}, in steady state, so we can eliminate the calendar period notation. From the condition that determines the demand of bonds (19),

$$P\mathbb{E}_k(U'(C_{k,1})) = \beta\mathbb{E}_{ijs}(U'(C_{ijs,2})) + \beta q_p \mathbb{E}_{js}[U'(C_{pjs,2})R_{js}],$$

and from the investment condition (20),

$$\gamma z^{\gamma-1}\mathbb{E}_k(U'(Ck,1)) = \beta\mathbb{E}_{ijs}(p_j\kappa_G U'(C_{ijs,2})) + \beta q_p \phi \mathbb{E}_{js}[p_j\kappa_G U'(C_{pjs,2})R_{js}].$$

In steady state the quantity of private assets is constant,

$$(1-\delta)Z + z = Z \qquad \Longrightarrow \qquad z = \delta Z.$$

The private assets' quality distribution is also constant in steady state, and there are two possibilities. The first possibility is that, in steady state, there is no information acquisition about collateral of uncertain quality $\bar{p}$. Then

$$k = j = \bar{p} \qquad with\ prob.\ 1.$$

This *information-insensitive steady state* is the same as the two-period model without heterogeneity (as $\bar{p}\kappa_G = 1$), as characterized by equations (12) and (13).

The second possibility is that, in steady state, there is information acquisition about

[12] Another, minor, difference with the two-period setting is that agents buy some private assets. Since the matching seller is of random quality, consumption in the first period is also stochastic.

collateral of uncertain quality $\bar{p}$. Then

$$k = j = \begin{cases} 1 & \textit{with prob. } \bar{p} \\ 0 & \textit{with prob. } 1 - \bar{p}. \end{cases}$$

This *information-sensitive steady state* displays idiosyncratic risk, as in the two-period model with known heterogeneity, as characterized by equations (16) and (17).

Comparing these two regimes, as discussed in section 3.1, the demand for government bonds is higher when there is information about private assets, as it increases the uncertainty when old and makes bonds more valuable to move consumption from youth to old age.

Remark on Dynamic Efficiency: Notice that the steady state can be dynamically inefficient, as usual in OLG settings, depending on the weight the planner assigns to different generations.[13] In such a case, when the decentralized equilibrium displays over-production of private assets, a government could implement the planner's solution by crowding out private assets with enough supply of government bonds such that $P < 1$ (this is making the target "safe" interest rate positive to encourage savings on bonds instead of private assets). As we're interested in isolating the role of government bond provision on the informational content of private assets, and to facilitate the comparison with our two-period benchmark, we assume in what follows that the planner's weights across generations are such that both informational steady states are dynamically efficient, and then the provision of government bonds that implement the optimal allocation is $P = 1$.

Remark on the Interaction of the Informational Regime and Optimal Bond Pricing: Notice that the optimal supply of government bonds depends on whether the steady state is information sensitive or insensitive. As the demand for bonds is larger in the first case, the optimal provision of bonds is also larger. If providing more bonds is costlier for the government (for instance because the storage technology cost is an increasing function of the quantity of bonds), then the optimal pricing of debt would be higher (the optimal interest rate lower) when private assets are information intensive.

Endogenous Information Acquisition in Steady State: Whether the steady state is

[13] There is a rich literature on the different conditions for OLG models to display dynamic inefficiency. See Diamond (1965) and Abel et al. (1989).

information-sensitive or information-insensitive depends on whether lenders want to acquire information about collateral of uncertain quality $\bar{p}$. In contrast to the two-period setting, here the intertemporal trading of private assets across generations changes the state in which information is valuable. While lenders are not concerned about ending up with bad collateral in case of default (as they can sell the collateral at its expected value to the next generation, regardless of its quality), since information acquisition is private and certifiable, lenders can gain if they obtain a good quality asset upon default (as they can sell the good private asset at a higher price to the next generation). Formally, no information is acquired if

$$d\bar{p}[U(C^I_{np,2}) - U(C^U_{np,2})] \leq \psi \tag{21}$$

with

$$C^U_{np,2} = Y_2 + B + Z \qquad \texttt{and} \qquad C^I_{np,2} = Y_2 + B + Z + \frac{(1-\bar{p})}{\bar{p}}(l - Y_2 - B)$$

when evaluated at the optimum, $P = 1$. This condition is a modified version of equation (18), with the same comparative statics as in Proposition 2. The next Corollary shows that there is an amount of bonds $\underline{B}$ below which lenders have incentives to acquire information about private collateral.

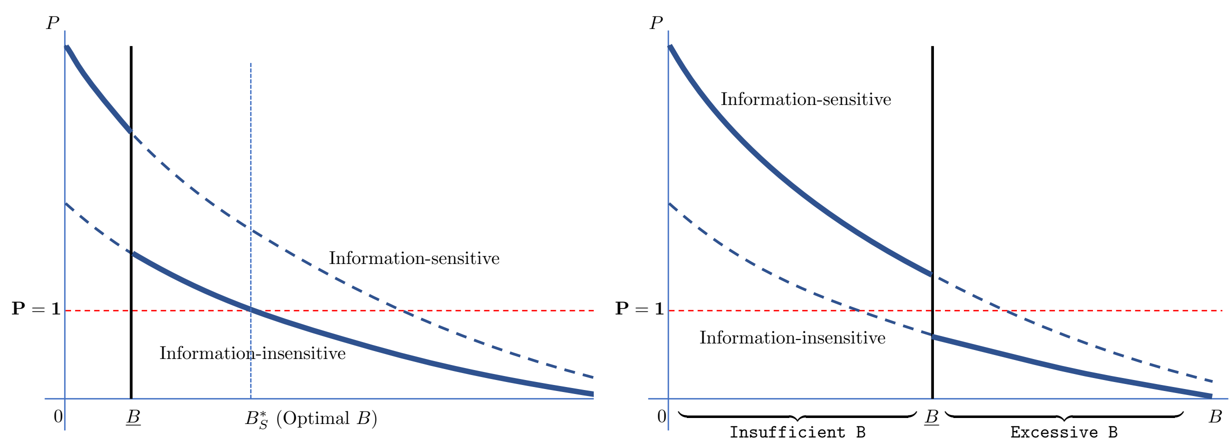
Corollary 1 *There is a* $\underline{B}(\underset{+}{d}, \underset{+}{l}, \underset{-}{Y_2}, \underset{-}{\psi}, \underset{-}{\bar{p}})$*, such that there is information acquisition about private collateral if* $B < \underline{B}$*, and there is no information if* $B \geq \underline{B}$*.*

4.4 The Bond Policy Trap

From Proposition 2, the information-insensitive steady state is more likely when there are more bonds in the economy, both because lenders are wealthier in the second period (there is less utility value from information) and because borrowers rely less on private collateral (there is less at stake from acquiring information). From Corollary 1, given a supply of bonds, such a steady state is more likely the better is collateral on average (high $\bar{p}$), the more likely the project succeeds (low d) and the higher is the examination cost of private assets (high ψ).

Both panels in Figure 1 show that the demand for bonds is larger in the information-sensitive regime, as discussed above. The sudden change in informational regime around a critical supply of bonds $\underline{B}$ may be irrelevant for the implementation of optimal monetary policy (as in the first panel of Figure 1, in which B_S^* is to the right of $\underline{B}$) but may also induce a policy trap, as in the second panel of Figure 1. In such case, when the economic fundamentals are such that the discontinuity represented by $\underline{B}$ lies on the point at which $P = 1$, then there are either too few bonds (with scarcity of collateral in the economy and too little production) or too many bonds (with distortionary taxation in a two-period model or dynamic distortions in this OLG setting).

Figure 1: Optimal Policy and Policy Trap



Monetary Policy Trap: In case the optimal amount of bonds circulating in the economy is determined through monetary policy, and the economy is in an informational trap, an unconventional policy conducted through a Bond Exchange Facility that exchanges bonds for private assets intra-period (as, for example, a repo contract between the government and agents) can always get rid of the discontinuity by relaxing the information sensitivity of private assets, and then allowing conventional monetary policy to implement the optimum.

The main reason for this result is that conventional policies exchange bonds for numeraire (open market operations and quantitative easing), so providing more bonds as collateral relaxes pressures for information production at the cost of reducing numeraire to consume when supplying the bonds. Offering agents the opportunity to borrow a government bond in exchange for private assets can both provide bonds to sustain no information acquisition in the second period, without reducing consumption when selling the bonds (this is, without affecting its price in terms of numeraire).

Proposition 3 *When the quality of private collateral and/or projects is good enough (high $\bar{p}$ and low d), conventional policy is enough to implement the constrained optimum. When the quality of private collateral or investments decline, the government may need to resort to using BEF (unconventional policy) to fulfill both financial stability and consumption smoothing.*

Details on the optimal design of a BEF are discussed in Gorton and Ordonez (2020a), who show how BEF is useful to face crises. Here we highlight its relevance in preventing crises and in recovering the implementation of optimal monetary policy.

4.5 Transition upon shocks to the supply of bonds

Assume the economy is in a steady state such that $P = 1$ and there is no information acquisition in credit markets. In this section we analyze how the economy fares when, at periods $\hat{t} \in \{t....T\}$, B_{SS} temporarily declines to $\widehat{B}$ (for instance because of foreign shocks that increase the foreign demand for bonds, making them scarcer domestically).

On impact, this transitory negative shock to the supply of bonds implies that $P_t > 1$, which reduces on impact the demand of bonds when young, compared to steady state.[14] As the production of private assets took place in period t by the time the shock to the supply of bonds happens, there is a reduction in the wealth available to the old in $t + 1$ (fewer available bonds to consume in $t + 1$). This implies that, not only are there fewer assets to sustain credit in the next period (a *credit crunch* because there are fewer bonds available), but there is also an increase in the incentives to acquire information about private collateral, potentially inducing a *crisis*. Recall that information is not acquired (equation 21) as long as

$$d\bar{p}[U(C^I_{np,2}) - U(C^U_{np,2})] \leq \psi$$

where

$$C^U_{np,2} = Y_2 + \widehat{B} + Z_{SS}$$

$$C^I_{np,2} = Y_2 + \widehat{B} + Z_{SS} + \frac{(1-\bar{p})}{\bar{p}} Z_{SS}$$

[14]For simplicity we maintain the assumption that the extra resources the government obtains from $P > 1$ are thrown to the ocean and then do not affect future generations' consumption.

This inequality is less likely to hold in steady state, as $t+1$ lenders are relatively poorer and the potential gains of investigating private collateral are larger in terms of marginal utility. This is a pure wealth effect. Lenders are more afraid of lending.

If the scarcity of bonds persists in $t+1$, young $t+1$ agents react by producing more private assets than in steady state (quantity crowd in). Then $Z_{t+1} > Z_{SS}$, and then $l_{t+2} > l_{t+1}$. The higher supply of private assets has the double effect of increasing credit and the wealth of lenders in period $t+2$, compared to period $t+1$. Even though at $t+2$ lenders are wealthier than in $t+1$, and then less interested about acquiring information about private collateral, there is also more use of private assets as collateral. With log-preferences, for instance, the second effect dominates. Hence, when bonds at t and $t+1$ are scarce, fragility is larger on impact (at $t+1$), but even larger subsequently (at $t+2$). If the new, lower, level of government bonds is permanent, the economy moves to a new steady state with fewer bonds, more private assets and more fragility. Whether the new steady state is information-sensitive or insensitive depends on parameters and the extent of bond supply scarcity.

When, at a later period $T+1$, bonds return to the original steady state level (the source of the shock disappears, for instance), the economy takes time to return to the original level of fragility. The reason is that the economy has accumulated a relatively large volume of private assets, which takes time to depreciate. This implies that a transitory shock to the supply of bond can have long lasting consequences in terms of fragility through the accumulation of private collateral. In this instance, the use of BEF, by taking private assets "out of circulation" and replacing them with bonds as collateral, also has the important role of speeding up the transition of the economy to a less fragile environment once the transitory shock has concluded or is under way.

5 Conclusion

Economic agents demand safe assets to use as stores of value and as collateral. The main difference between public and private safe assets is that the latter may come in heterogeneous and volatile qualities. Here we show that information about individual assets' qualities may reduce their safety and make them less useful for both purposes. When information acquisition is a choice, there are conditions (notably the availability of public safe assets) under which such information is not produced and private assets are indeed safe assets. Those conditions, however, may change and a large volume of private assets' qualities may get examined – *a crisis* ensues.

We have explored the optimal supply of government bonds, which needs to accommodate both sources of demand and to consider its effects on the creation of and on information production about these assets, this is on their quantity and safety. The government should strive to induce optimal intertemporal consumption smoothing in the economy while providing enough collateral to avoid too much reliance on private asset creation so to maintain their safety properties. Government bonds not only complete markets themselves, but also enhance private assets in doing so.

When applying this insight to the conduct of monetary policy, we show it cannot be isolated from macroprudential policy and financial instability concerns. Monetary policy may require taking government bonds out of the economy. But if this policy generates a scarcity of government bonds as collateral, the private sector may react by creating more private safe assets, increasing the fragility of the financial system to changes in information. The goals of facilitating consumption smoothing while minimizing the likelihood of a financial distress are two goals that may be mutually exclusive when the average quality of private assets is low or the probability of default in the economy is high. In this circumstance the Central Bank should rely on the use of alternative policies that contemporaneously exchanges private assets for government bonds (in a repo-type operation).

In our setting acquisition and revelation of information about private assets is unequivocally damaging. Agents would obtain higher welfare if information is only revealed upon consumption, or not revealed at all. The reason is that information only adds uncertainty and cannot be exploited for allocation purposes or to improve economic decisions (such as improvement consumption and investment choices, or private assets quality). Information benefits would add interesting trade-offs (as highlighted by Gaballo and Ordonez (2021) in a more general setting), which are outside the scope of this paper but are critical on characterizing further the optimal amount of government debt given the role of government bonds in shaping the informational environment in financial markets.

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