The Collateral Link between Volatility and Risk Sharing*

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Abstract

We show that the effect of aggregate volatility on idiosyncratic risk sharing depends on the nature of collateral that sustains insurance. While aggregate volatility *decreases* the value of private assets—exposed to more variation—it *increases* the value of public assets—more relevant for consumption smoothing. Hence, a more volatile economy weakens risk sharing when the composition of collateral is biased toward private assets. As stable economies encourage private issuance, they may plant the seeds of their own fragility. We empirically show that indeed a more intense use of private assets increases the sensitivity of risk sharing to aggregate volatility.

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1 Introduction

Financial intermediaries rely heavily on interbank markets to insure against idiosyncratic shocks to their assets and/or liabilities. This is evident by the ubiquitous use of derivative contracts to hedge unwanted exposures to certain assets and/or aggregate conditions, and of repo contracts to manage liquidity risk. The smooth functioning of interbank markets, which is then critical for the efficiency and stability of the financial system, depends on the extent of idiosyncratic risk and the possibilities to hedge against them. While *aggregate risk* (time series volatility of aggregate consumption) may affect the exposure of intermediaries to certain shocks (*the demand for insurance*), *counterparty risk* (the danger that one of the parties might default on a promise) may constrain the ability to write insurance contracts against those risks (*the supply of insurance*). Here we show that these two risks are strongly related through valuation effects: aggregate risk affects the value of collateral that is used to relax counterparty risk. But, does aggregate risk improve or impair risk-sharing function of interbank markets? Does it strength or weak financial stability?

The relevance of these questions became apparent during the 2008 crisis, when insurance across banks broke down.[1] Their potential magnitude also becomes clear by the sheer size of interbank markets. The trading of derivative contracts, for instance, amounts to almost nine times the world GDP in 2020, in notional terms.[2] Similarly, the Federal Reserve estimates total repo liabilities at around four trillion dollars in 2020.[3] While most repo contracts are traded over-the-counter and used to hedge directly against idiosyncratic shocks, the lions share of derivative contracts are written conditional on aggregate events (such as interest rate or exchange rate swaps) and are traded among intermediaries to hedge against idiosyncratic exposure to those aggregate conditions.

Behind the massive underwriting of these types of contracts lies the heavy use of collateral to relax pervading counterparty risks.[4] According to the 2014 report of the International Swap and Derivative Association (ISDA), *"The use of collateral agreements is substantial. Among all firms responding to the survey, 91% of all OTC derivatives trades (cleared and*

[1]Heider et al. (2009) and Acharya and Merrouche (2012) show liquidity hoarding by banks active in the interbank market during the 2007/2008 crisis, while several other banks were suffering liquidity shortages.

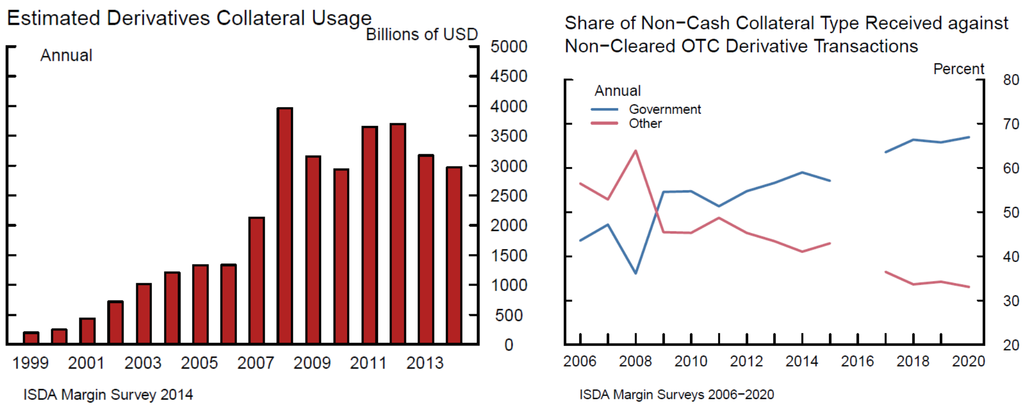
[2]Bank of International Settlements.

[3]Repo liabilities reached a peak of five trillion dollars in March 2008. SEC "Primer on Money Market Funds and the Repo Market," by Baklanova, Kuznits and Tatum, February 18, 2021. Figure 4 of Copeland et al. (2014) shows a dramatic drop in repo volumes after the default of Lehman Brothers, which was concentrated in riskier collateral classes.

[4]Derivatives trade under International Swap and Derivative Association (ISDA) master agreements, which often involve a Credit Support Annex (CSA) that specifies the conditions under which parties must post collateral. Other credit enhancements include ratings triggers that terminate the transaction at market, and third party guarantees.

non-cleared) were subject to a collateral agreements at the end of 2013." The left panel of Figure 1 shows the dramatic increase in collateral posted for derivative contracts since 2000, which duplicated during the 2008 crisis and remains at around four trillion dollars since then. The right panel of Figure 1 shows the contemporaneous change in the composition of non-cash collateral for derivatives, which tilted towards more intensive use of government related assets, from 40% in 2008 to almost 70% in 2020.[5]

Figure 1: Use and Composition of Collateral



Most of the assets, both public (such as government bonds) and private (such as asset backed securities), that are used as *collateral* to smooth *idiosyncratic shocks intratemporally* are also used as *stores of value* to smooth *aggregate shocks intertemporally*. This dual role of public and private assets links aggregate volatility and idiosyncratic risk sharing. The use of assets as collateral affects the extent of idiosyncratic risk sharing in the economy and, thus the intertemporal value of assets. In parallel, the intertemporal value of assets determines their use as collateral and thus the extent of idiosyncratic risk sharing.

We show that the way in which aggregate volatility affects the extent of risk sharing depends on the ratio of private to public assets used as collateral. The reason is that their valuations react to volatility in opposite directions. On the one hand, an increase in aggregate volatility increases the intertemporal price of public assets, the only assets that, because of taxation, can credibly provide future noncontingent payment promises. If intermediaries face high aggregate volatility (that is, when the variance of future aggregate realizations is high), a dollar that pays off in bad periods delivers a high marginal utility, making the promise more valuable. On the other hand, an increase in aggregate volatility reduces the

[5]While cash remains the most prevalent type of collateral, its share declined from 80% in 2008 to 70% in 2020 (ISDA Margin Surveys). In addition, the November 2021 Financial Stability Report, published by the Federal Reserve, shows that over the last few years central clearing counterparties (CCPs) in derivatives markets have heavily relied on non-cash collateral—mainly U.S. Treasuries securities—to manage their credit and liquidity risk (see box Liquidity Vulnerabilities from noncash collateral at central counterparties).

intertemporal price of private assets. The reason is that their payoffs are tied to the evolution of the aggregate economy more closely than government bonds (as discussed and documented by Jiang et al. 2021), and riskier assets become less valuable when the variance of future aggregate realizations is high.

When the ratio of private to public assets used as collateral is relatively low, as in the decade after the 2008 crisis, the value of public assets is more relevant to determine the value of available collateral. In this case, higher volatility implies more valuable collateral on average, and better idiosyncratic insurance—a sort of "positive externality" of volatility on risk sharing. The opposite is true when the ratio of private to public assets is relatively high, as was the case before the crisis in 2008. The defining feature that differentiates assets in our model is not the issuer, but their exposure to aggregate risk. For expositional reasons we will refer to assets with these two different exposures as public and private assets, but in practice there exist public assets that cannot provide noncontingent future promises (e.g., emerging market government bonds) and private assets that can (e.g., supranational debt).

As risk sharing is one of the fundamental roles of interbank markets, and the extensive use of private assets makes the system more fragile to volatility shocks, we study what determines the creation of private collateral, such as securitization. We show that in general creating private collateral is more likely in a stable economic environment. Then, a decrease in aggregate volatility can increase the use of private collateral, turning the financial system increasingly more fragile—in the sense of less risk sharing opportunities—to a sudden increase in aggregate volatility. In short, under quite plausible conditions, *economic stability plants the seeds of its own instability.* While interbank risk sharing may have improved with financial innovation before the crisis in 2008, it may have happened at the cost of making such risk sharing more fragile to sudden increases in aggregate volatility.

Linking aggregate volatility and risk sharing has several additional implications. First, volatility will affect governments' financing costs depending on the use of sovereign debt *and* of private assets as collateral in financial markets.[6] Second, there is an intricate relationship between the creation and valuation of private and public assets. The celebrated crowding out effect of government debt, for instance, may overturn when valuation forces from risk sharing are strong enough to overcome the standard substitution effect on quantities.

Our model also delivers testable implications on the sensitivity of risk sharing to aggregate volatility as a function of the private/public composition of collateral, which we take to the data. Our model maps the *convenience yield*—the additional value assigned to assets net

[6]This effect has become particularly relevant during turbulent times, such as the wake of the COVID-19 pandemic, which represented a sudden, unexpected shock to the economy, affecting all countries and vastly increasing the aggregate volatility financial intermediaries face in the short- and medium-run.

of their payoff risks—to the extent of risk sharing in the economy (less risk sharing leads to a higher convenience yield). We perform an empirical analysis in which we use prices to compute convenience yields to infer the extent of risk sharing, and test whether indeed the convenience yield responds more positively to aggregate volatility in periods in which collateral is dominated by private assets.

More specifically, we perform two tests that differ on the frequency and length of data. First, using low frequency data, we show that the aforementioned sensitivity has increased over time. While the convenience yield was barely reactive to aggregate volatility before the nineties, over the past few decades the relationship has turned positive and significant, consistent with the more recent heightened role of private collateral, as documented by Gorton et al. (2012). Second, using high frequency data, we zoom in on the active period surrounding the crisis in 2008, motivated by the evidence of a reduction in the use of private collateral in derivative contracts after the crisis (see Figure 1). In this case, we show that sensitivity of the convenience yield to aggregate volatility increased dramatically leading up to the crisis, but then declined and remained low afterwards, consistent with a more intensive use of public collateral spurred by regulatory efforts after the crisis.

Related Literature: The literature on collateral is extensive, and highlights several uses, such as backing loans to borrowers with investment projects (such as Kiyotaki and Moore 1997) or liquidity needs (such as Holmstrom and Tirole 1998). Our work belongs to the corner of the literature that highlights the use of collateral to back insurance and other hedging contracts. Krishnamurthy (2003), in the spirit of Kiyotaki and Moore (1997), studies the collateralization of insurance, but with a focus on collateral constrained insurance *against aggregate shocks*, while we focus instead on insurance *against idiosyncratic shocks* and how aggregate volatility affects such risk-sharing through valuation. On a similar setting to his, Di Tella (2017) highlights that downturns coincide with higher idiosyncratic risk. An uncertainty shock concentrates loses in ways that depress asset prices and leads to downturns, creating a feedback effect. Our work focuses instead on the role of ex-ante aggregate risk on the valuation of different types of asset, not upon uncertainty shocks.

We highlights the valuation linkage between aggregate volatility and risk sharing, and is consistent with a rich literature that studies asset prices as a function of aggregate risk, such as Chien and Lustig (2010) and Rampini and Viswanathan (2019), among others. In contrast to the literature that show how collateral constraints arise endogenously, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000), we take those constraints as given and explore the interplay of different types of risk—aggregate or idiosyncratic—and different types of collateral —private or public.

Our result is complementary to Gorton and Ordonez (2021), who also study the dual use of public and private assets as collateral but, in that case, to back productive loans. Their focus is on the role of informational fragility that mounts in the economy as private assets (heterogeneous and plagued by asymmetric information issues) become larger vis-a-vis public assets (more homogeneous and less subject to informational frictions). While that work highlights the *informational fragility of collateral composition for productive reasons*, here we study the *valuation fragility of collateral composition for insurance reasons*. While Gorton and Ordonez (2021) is purely theoretical and silent about asset pricing implications, in this paper we focus on the interaction between private and public asset valuations, we link those valuations to the convenience yield and take it to the data.

There is an equally extensive, but more recent, literature on convenience yields and their implications. The strategies to capture an asset's convenience yield vary. Some papers include the asset directly into the utility function, so the convenience is captured in reduced form by preferences, such as in Nagel (2016). Some others, such as Krishnamurthy and Vissing-Jorgensen (2012), consider settings where agents directly consume the asset's liquidity benefits, decomposing the convenience yield into *liquidity and safety components*. In our paper, an asset's convenience yield has a direct theoretical mapping to the value of the asset to provide insurance through its role as collateral – an *insurance component*.

Our extension to private asset creation is related to Greenwood et al. (2015), Krishnamurthy and Vissing-Jorgensen (2015), and Sunderam (2014), who show that the private sector creates more private liabilities when convenience yields—a proxy for the demand for safe assets in their case—are high. Further, Infante (2020) points out that the creation of private collateral depends on its underlying assets. Here, we go a step further by studying how changes in aggregate volatility directly affect the convenience yield, and thus, incentives for private asset creation not only through quantity but also valuation forces.

Several elements in our model have been validated by the literature. First, the relevance of private assets' valuation for insurance has been documented for housing by Hurst and Stafford (2004), Lustig and Van Nieuwerburgh (2010) and Hryshko et al. (2010). Second, the relation between aggregate volatility and the valuation of public and private assets has been documented by Connolly et al. (2005) and Baele et al. (2010) over recent U.S. history, showing that an increase in volatility appreciates Treasuries and depreciates stocks. Finally, there has been a recent and active literature, such as Jiang et al. (2019), Reis (2021) and Bhandari et al. (2021), identifying the underlying determinants of government bond valuations. Our work shows that the presence, valuation, and use of private assets in interbank markets are important in qualifying these results as they directly impact the valuation of public assets. In particular, our paper shows that government bonds' increased ability to hedge idiosyncratic risks can attenuate or exacerbate the so called "negative beta effect", depending on whether the ratio of private to public assets used as collateral in financial markets is low or high.

Finally, Brumm et al. (2018) quantitatively study how re-using private collateral increases leverage and volatility, while Rampini and Viswanathan (2010) argue that higher collateralizability increases borrowing capacity, leverage, and aggregate volatility. We instead explore the opposite direction, in which aggregate volatility affects the value of both private and public collateral to provide insurance against idiosyncratic shocks, then highlighting that the relationship between leverage and volatility works on both directions and is not trivial.

The paper proceeds as follows. The next section presents a model with aggregate volatility in which public and private assets can be used as collateral to share idiosyncratic risks and as stores of value to smooth aggregate volatility. Section 3 presents a tractable CARA-Normal case that allows for clean comparative statics on the valuation of public and private assets and their use for risk sharing. Section 4 gives agents the ability to create private assets at a cost. In Section 5, we provide empirical evidence on the sensitivity of risk sharing to aggregate volatility and how it has changed over time. Section 6 concludes.

2 Model

In this section, we present a simple model that relates aggregate volatility and the extent of idiosyncratic risk sharing, assuming exogenous supply of private and public assets. We intend to capture financial intermediaries that face idiosyncratic shocks to assets and/or liabilities and that write collateralized insurance contracts with other intermediaries. We will model, however, financial intermediaries in a reduced form that allows to extrapolate results beyond financial markets. We will also abstract from dynamic and production intricacies that would obscure the results by assuming three periods and an endowment economy.

2.1 Environment

Consider a three period ($t \in \{0, 1, 2\}$) endowment economy with two agents, called Raymond (R) and Shirley (S). Both agents have additive separable utility, with each period's consumption utility $u(\cdot)$ and discount factor β. Agents split equally an aggregate endowment, and additionally each agent receives an idiosyncratic endowment shock which is completely offset by the other agent's shock.[7] Specifically, Raymond (Shirley) receives a positive (negative) shock if it "rains" and a negative (positive) shock if it "shines." For simplicity, we

[7] We consider an endowment process instead of a Lucas tree to avoid the possibility of using the tree as collateral.

assume that the probability of rain and shine are both $\frac{1}{2}$. Formally, agent i has the following endowment process:

$$e_{0i} = \frac{Y_0}{2}; \qquad \tilde{e}_{1i} = \frac{\tilde{Y}_1}{2} + \tilde{y}_i; \qquad \tilde{e}_{2i} = \frac{\tilde{Y}_2}{2},$$

where Y_t represents aggregate endowments (the tilde signifies that endowment shocks are t-measurable random variables). For Raymond, $\tilde{y}_i$ is either $\overline{y}$ if it rains or $-\overline{y}$ if it shines. Shirley has the opposing idiosyncratic endowment shock. This particular setting is the simplest to capture financial intermediaries (or investors more generally) that are otherwise identical but face uncorrelated idiosyncratic shocks, so there is room for insurance.[8]

Supply and Demand of Assets: There are three assets in the economy: short-term government bonds, long-term government bonds, and a private asset. While in this section the total *supply* of public and private assets is fixed, the *demand* is determined by agents' optimal portfolio choice. In the interpretation of agents as banks, they start with given asset and liability positions that determine their idiosyncratic exposure to the weather, but can react by changing their portfolio holdings.

In terms of supply, the government pays short- and long-term government bonds (the two public assets) raising lump-sum taxes on agents in the period the bonds mature. Because of the government's ability to tax agents, these assets will be considered safe—that is, they will always pay at par when they mature. We denote the face value of the total amount of short-term bonds by Θ_0^{Sh} and of long-term bonds by Θ_0. In contrast, we assume the private asset's payoff is proportional to the aggregate endowment process, paying a dividend $\tilde{a}_t = \rho\tilde{Y}_t$, with $\rho \in (0, 1)$ in each period. Initially, each agent is endowed with half of a total private asset supply $\hat{\Theta}_0$.[9]

In terms of demand, in each period $t \in \{0, 1\}$, each agent will demand θ_{ti}^{Sh} of short-term government bonds, θ_{ti} of long-term government bonds, and $\hat{\theta}_{ti}$ of private assets at the market clearing price p_t^{Sh}, p_t, and p_t^a, respectively.

Notice that the difference between public and private assets is given by their payoff

[8]Notice that Raymond and Shirley can be interpreted as risk averse banks in negatively correlated regions. Either their idiosyncratic loans pay or their idiosyncratic depositors withdraw in different states of the "weather." This would induce these banks to write a derivative contract conditional on the weather to insure against those shocks.

[9]There is no loss in generality in assuming that the private asset's payoff is correlated with the aggregate endowment process. In equilibrium, this assumption merely implies that agents exposure to aggregate risk is scaled by $(1 + \rho\hat{\Theta}_0)$: a part attributed to the endowment process 1 and a part attributed to agents' optimal holdings of the private asset $\rho\hat{\Theta}_0$. If the private asset payoff were independent of the endowment process, because of market clearing, agents' optimal portfolio holdings would still create a correlation between their consumption path and asset payoffs. Therefore, absent wealth effects, the introduction of additional risks orthogonal to the risk already embedded in the model does not have different qualitative effects.

relation with the aggregate state, and not by the issuer. We maintain this notation to fix ideas, but we can extrapolate results for "private assets" to assets with payoffs that are positively correlated to the aggregate state and results for "public assets" to assets with payoffs that are not (or are negatively) related to the aggregate state, regardless of who issues those assets.

Risk Sharing and Collateral: At $t = 0$, agents are able to write state-contingent contracts among themselves conditional on the weather. We model these as "Arrow-Debreu" securities that pay one unit of the consumption good depending on whether it rains or shines. Importantly, we assume that agents selling an Arrow-Debreu security (effectively selling insurance for that state of the world), must *fully* collateralize their promise with public or private assets.[10] If we denote by w_i^r and w_i^s—the amount of promises agent i makes in case it rains and shines, respectively (super scripts r and s denote the state "rain" and "shine", respectively)—the need of fully collateralizing the promise implies that

$$w_i^r \leq \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{p}_1^a \hat{\theta}_{0i}; \qquad w_i^s \leq \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{p}_1^a \hat{\theta}_{0i}. \tag{1}$$

A single contract against rain or shine can be thought of as a collateralized insurance, or derivative contract.[11] To capture absolute safety, we assume the ability of an asset to collateralize a claim depends on the worst value it can take (the portion of the payoff that can be pledged in all states of the world).[12] In terms of constraints (1), $\underline{p}_1$ and $\underline{p}_1^a$ are the lowest price the long-term government bond and the private asset can have in $t = 1$, respectively. The parameter α captures the pledgeability of the private relative to public assets.[13]

In what follows, we will denote the market trading price of contingent contracts for when it rains and shines by q^r and q^s, respectively.

[10]Formally, because of collateralizability, these contracts are not real Arrow-Debreu securities. This market incompleteness implies that the first welfare theorem does not hold.

[11]The derivative contract could be also written in terms of an easily observable aggregate variable, such as the price of a commodity. If raining, for instance, is positively correlated with the price of a commodity, Raymond would sell such derivative and Shirley would buy it. Our setting also captures repo contracts. An agent not subject to any idiosyncratic risk could purchase both rain and shine derivative contracts (guaranteeing a given level of consumption) and collect the collateral to back repo liabilities.

[12]This extreme assumption eliminates all credit risk, isolating the role of collateral. This choice can be microfounded, as in Caballero and Farhi (2018) by assuming infinite risk aversion over short intervals.

[13]Even though it is natural to assume that private assets are worse as collateral than public assets ($\alpha < 1$) because of limited pledgeability, informational frictions, etc., this is not necessary for our results. It is not our goal to endogenize α as a function of volatility, but rather focus on valuation effects. For a discussion about endogenizing α see Gorton and Ordonez (2014).

Consumption: We can write agent Raymond's consumption in each period as,

$$c_{0R} = e_{0R} + a_0 \frac{\hat{\Theta}_0}{2} - p_0 \left(\theta_{0R} - \frac{\Theta_0}{2} \right) - p_0^{Sh} \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - p_0^a \left(\hat{\theta}_{0R} - \frac{\hat{\Theta}_0}{2} \right) + q^r w_R^r + q^s w_R^s \quad (2)$$

$$\tilde{c}_{1R} = \tilde{e}_{1R} + \tilde{a}_1 \hat{\theta}_{0R} - \tilde{p}_1 (\theta_{1R} - \theta_{0R}) + \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - \tilde{p}_1^a (\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s \quad (3)$$

$$\tilde{c}_{2R} = \tilde{e}_{2R} + \tilde{a}_2 \hat{\theta}_{1R} + \left(\theta_{1R} - \frac{\Theta_0}{2} \right). \quad (4)$$

Consumption for Shirley takes a symmetric form.

Timing: In $t = 0$, agents choose the amount of government bonds to purchase and contingent contracts to sign, taking into account the need to collateralize these contracts with the assets they hold—that is, satisfy the inequalities of (1). In $t = 1$, agents rebalance their portfolio upon the realization of both the aggregate and idiosyncratic shocks. In $t = 2$, agents consume endowments and proceeds from their portfolio.

Given the symmetry of agents in period 0, each will end up with half the government supply of short- and long-term bonds, which will determine prices p_0^{Sh} and p_0. Each agent, however, has the possibility to rebalance his/her portfolio, demanding, for instance, more long-term bonds or private assets in period 1, once agents' endowments become asymmetric. Taxes are then collected at period 2 to redeem those bonds. Since there are no choices in period $t = 2$, the next subsections characterize backwards the optimal choices in periods $t = 1$ and $t = 0$.

2.2 Equilibrium in $t = 1$

In $t = 1$, after it rains or shines, $\tilde{p}_1 = p_1, \tilde{p}_1^a = p_1^a$, and $\tilde{c}_{1i} = c_{1i}$. Upon this realization, each agent rebalances their portfolio by choosing the optimal amount of long-term bonds (which are now one-period bonds) and private asset holdings. An agent $i \in \{R, S\}$'s first order conditions are

$$\begin{aligned} -p_1 u'(c_{1i}) + \beta \mathbb{E}_1 (u'(\tilde{c}_{2i})) &\leq 0 \\ -p_1^a u'(c_{1i}) + \beta \mathbb{E}_1 (u'(\tilde{c}_{2i}) \tilde{a}_2) &\leq 0. \end{aligned}$$

Raymond, for instance, holds both long-term bonds and private assets in equilibrium if

$$p_1 = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2R}))}{u'(c_{1R})} \tag{5}$$

$$p_1^a = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2R})\tilde{a}_2)}{u'(c_{1R})}, \tag{6}$$

which are the standard intertemporal pricing equations. The condition is identical for Shirley. These prices will depend on the aggregate shock in $t = 1$ and possibly the idiosyncratic shock if agents cannot fully insure. Given the symmetry of the problem, however, the price will be the same for both realizations of the idiosyncratic shock, as one of the two agents will always have the "good shock" and the other the "bad shock".

Denote by θ_{1i}^r and θ_{1i}^s agent i's portfolio in $t = 1$ when it rains or shines, respectively. Using agents' consumption from equations (3) and (4), equilibrium conditions so that both agents hold the long-term government bond and private asset when it rains are,

$$\begin{aligned} &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1R}^r + \theta_{1R}^r - \frac{\Theta_0}{2}))}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1R}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1R}^r - \frac{\hat{\Theta}_0}{2}\right) + (\overline{y} - w_R^r))} = \\ &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1S}^r + \theta_{1S}^r - \frac{\Theta_0}{2}))}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1S}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1S}^r - \frac{\hat{\Theta}_0}{2}\right) - (\overline{y} - w_S^r))} \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1R}^r + \theta_{1R}^r - \frac{\Theta_0}{2})\tilde{a}_2)}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1R}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1R}^r - \frac{\hat{\Theta}_0}{2}\right) + (\overline{y} - w_R^r))} = \\ &\frac{\mathbb{E}_1(u'(\frac{\tilde{Y}_2}{2} + \tilde{a}_2\hat{\theta}_{1S}^r + \theta_{1S}^r - \frac{\Theta_0}{2})\tilde{a}_2)}{u'(\frac{Y_1}{2} + a_1\frac{\hat{\Theta}_0}{2} - p_1\left(\theta_{1S}^r - \frac{\Theta_0}{2}\right) - p_1^a\left(\hat{\theta}_{1S}^r - \frac{\hat{\Theta}_0}{2}\right) - (\overline{y} - w_S^r))}. \end{aligned} \tag{8}$$

Note that, in order to have interesting equilibria, the idiosyncratic shock has to be large enough relative to the exogenous supply of public and private assets so that the maximum amount of insurance is not enough to hedge all idiosyncratic risks. Specifically, whenever (1) binds, we have $\overline{y} - w_i^j > 0$ for state j. Market clearing is given by

$$\theta_{1R}^r + \theta_{1S}^r = \Theta_0; \qquad \hat{\theta}_{1R}^r + \hat{\theta}_{1S}^r = \hat{\Theta}_0.$$

In symmetric equilibria these conditions are identical to when it shines.

Given this possible reoptimization strategy in period 1, denote the optimal continuation value of Raymond's $t = 1$ utility by

$$U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1) = Max_{\{\theta_{1R}, \hat{\theta}_{1R}\}} u(c_{1R}) + \beta \mathbb{E}_1(u(\tilde{c}_{2R})).$$

2.3 Equilibrium in $t = 0$

In $t = 0$, Raymond solves the following maximization problem:

$$Max_{\{\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s\}} u(c_{0R}) + \beta \mathbb{E}_0(U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1)),$$

subject to the constraint (1). Using the envelope condition, this problem leads to the following first order conditions,

$$\begin{aligned}
\theta_{0R}^{Sh} &: -p_0^{Sh} u'(c_{0R}) + \beta \mathbb{E}_0(u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \leq 0 \\
\theta_{0R} &: -p_0 u'(c_{0R}) + \beta \mathbb{E}_0(\tilde{p_1} u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \underline{p}_1 \leq 0
\end{aligned}$$

$$\begin{aligned}
\hat{\theta}_{0R} &: -p_0^a u'(c_{0R}) + \beta \mathbb{E}_0((\tilde{a}_1 + \tilde{p_1^a}) u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \alpha \underline{p}_1^a \leq 0 \\
w_R^r &: q^r u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^r)) - \xi_R^r = 0 \\
w_R^s &: q^s u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^s)) - \xi_R^s = 0,
\end{aligned}$$

where ξ_R^r and ξ_R^s are the Lagrange multipliers associated with the collateral constraint in (1) for w_R^r and w_R^s, respectively; and $\tilde{c}_{tR}^r$ and $\tilde{c}_{tR}^s$ are Raymond's consumption when it rains and shines, respectively. It is natural to assume that in equilibrium, Raymond will buy insurance for when it shines and sell insurance for when it rains. That is, Raymond's collateral constraint will possibly bind only when selling rain insurance. Similarly, Shirley's collateral constraint will possibly bind only when selling shine insurance, thus $\xi_R^s = \xi_S^r = 0$. If those constraints bind, state-specific constraints lead to the following pricing of insurance:

$$\begin{aligned}
q^s &= \frac{\beta}{2} \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R}^s)}{u'(c_{0R})} \right) \\
q^r &= \frac{\beta}{2} \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1S}^r)}{u'(c_{0S})} \right)
\end{aligned}$$

and Lagrange multipliers

$$\begin{aligned}\xi_R^r &= q^r u'(c_{0R}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1R}^r)) \\ \xi_S^s &= q^s u'(c_{0S}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1S}^s))\end{aligned}$$

These conditions show that insurance for a "bad state of the world" (in terms of idiosyncratic shocks) is priced by the agent who needs it most. That is q^j is priced by the agent who suffers a negative idiosyncratic shock in state j. In the symmetric equilibrium, $q^r = q^s$ and $\xi_R^r = \xi_S^s$. In addition, Raymond sells insurance to Shirley and Shirley sells insurance to Raymond—that is, $w_R^r = -w_S^r$ and $w_S^s = -w_R^s$.

To close the model, market clearing is given by

$$\theta_{0R}^{Sh} + \theta_{0S}^{Sh} = \Theta_0^{Sh}; \qquad \theta_{0R} + \theta_{0S} = \Theta_0; \qquad \hat{\theta}_{0R} + \hat{\theta}_{0S} = \hat{\Theta}_0.$$

Pricing and Convenience Yield: Period 0 prices in the symmetric equilibrium are

$$p_0^{Sh} = \beta\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] \tag{9}$$

$$p_0 = \beta\mathbb{E}_0\left(\tilde{p}_1\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \underline{p}_1\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] \tag{10}$$

$$p_0^a = \beta\mathbb{E}_0\left((\tilde{a}_1 + \tilde{p}_1^a)\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \alpha\underline{p}_1^a\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right] \tag{11}$$

Equations (9)–(11) have two components. The first component is the standard asset pricing equalization of intertemporal marginal utilities, and captures the value of assets as *stores of value*. The second component captures the value of assets as *collateral* to improve insurance.[14]

The first component is then related to the $t = 0$ price of a theoretical *risk-free security* that pays par in $t = 1$, in absence of idiosyncratic risks,

$$p_0^{rf} := \beta\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) = \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) + u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right) \tag{12}$$

The pricing effect of the asset as store of value depends on the capital gains expected at period $t = 1$ for long-term bonds ($\tilde{p}_1$) and the dividend and capital gains expected at period $t = 1$ for private assets ($\tilde{a}_1 + \tilde{p}_1^a$).

[14]These pricing equations are consistent with standard results under heterogeneous agents: the agents with the highest marginal rate of substitution prices the asset, as in Alvarez and Jermann (2000). Given that $\mathbb{E}_0\left[u'(\tilde{c}_{1R})\right] = \mathbb{E}_0\left[\frac{u'(\tilde{c}_{1R}^s)+u'(\tilde{c}_{1R}^r)}{2}\right]$, we can rewrite equation (9) as $p_0^{Sh} = \beta\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s)}{u'(c_0)}\right)$, that is, the price of the short-term government bond is proportional to Raymond's marginal consumption when it shines.

The second component is related to the *convenience yield* of having the asset for insurance purposes. The convenience yield in our setting is not related to safety or liquidity properties, but instead by the importance of the asset to improve insurance, and it is simply given by,

$$CY := \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}^s_{1R}) - u'(\tilde{c}^r_{1R})}{u'(c_0)}\right) > 0 \tag{13}$$

This simple expression has an easy interpretation. The convenience yield is equal to the difference in marginal utility when agents suffer a bad idiosyncratic shock relative to a good one. That is, the value of insurance is equal to gap in marginal utilities generated by imperfect insurance against idiosyncratic shocks. In equilibrium, the convenience yield will depend on the amount of safe assets in the economy, their degree of pledgeability, and the size of the idiosyncratic shock. From the expression in equation (13), it is clear that the convenience yield is non-negative (being zero only in the case of perfect insurance, thus no difference between the marginal utility of consuming in shiny or rainy days).

The pricing effect of the convenience yield on the asset depends on how useful the asset is to collateralize the sale of insurance: the worst possible price in $t = 1$ for the long-term bond ($\underline{p}_1$), and the degree of pledgeability and the worst possible price in $t = 1$ for the private asset $\alpha\underline{p}^a_1$.

3 Special Case with Closed-Form Solutions

In this section, we present comparative statics for a special case with simplifying assumptions that allow us to obtain closed-form solutions:

Assumption A1. *Consider a case with the following simplifying assumptions:*

1. *Preferences are characterized by CARA, with risk aversion γ.*
2. $\tilde{Y}_1 = Y_1 = 0$.
3. $\tilde{Y}_2 \sim N(\mu, \sigma^2)$.

Beyond tractability, these assumptions help in focusing our message. In this simpler setting σ^2, which is the variance of aggregate endowment realizations in period 2 and fully captures aggregate volatility. Our goal is to characterize how changes in aggregate risk at $t = 2$ affects insurance contracts at $t = 0$ against idiosyncratic shocks happening at $t = 1$.

3.1 Characterization

This simplified case is useful for the following reasons. First, CARA utility eliminates wealth effects, so agents' optimal risky asset holdings in $t = 1$ do not depend on the idiosyncratic shock, nor do $t = 1$ prices. Second, having a deterministic endowment shock in $t = 1$ means that the $t = 1$ price of the long-term government bond and the private asset are known in $t = 0$. Therefore, the worst-case outcome is merely the price in $t = 1$. Third, normality in the final aggregate shock allows to recover a simple formulation of prices in $t = 1$, facilitating comparative statics.

Allocations: The benefit of this special set of assumptions is to dramatically simplify equations (7) and (8), as the marginal rates of substitution between $t = 1$ and $t = 2$ must be the same for both agents. Further, given symmetry, we conjecture that the optimal portfolio choice implies each agent holds half of the private asset supply ($\hat{\theta}^r_{1R} = \hat{\theta}^r_{1S} = \hat{\Theta}_0/2$) at the beginning of $t = 1$. Finally, in what follows we assume insurance needs are large enough so that even having all assets collateralized, full insurance is not achieved, that is, $\bar{y} > w := w^r_R = w^s_S = \frac{\Theta^{Sh}}{2} + p_1\frac{\Theta_0}{2} + \alpha p^a_1\frac{\hat{\Theta}_0}{2}$.

Given imperfect insurance, when it rains, Raymond consumes more than when it shines. Still, when it rains Raymond wants to consume the same in period $t = 1$ than in $t = 2$, and then would like to move some of such extra consumption to $t = 2$. Raymond can achieve this goal by buying long-term bonds from Shirley at prevailing prices p_1.[15] Since Shirley faces a bad shock when raining, she would like to sell those bonds to move consumption from $t = 2$ to $t = 1$. Since Raymond wants to equalize consumption in both periods,

$$(\overline{y} - w) - p_1\left(\theta^r_{1R} - \frac{\Theta_0}{2}\right) = \left(\theta^r_{1R} - \frac{\Theta_0}{2}\right),$$

Intuitively, since $\overline{y} > w$, when it rains Raymond will buy long-term bonds, i.e., $\theta^r_{1R} > \frac{\Theta}{2}$. This allows Raymond to move some of the extra consumption $\overline{y} - w$ to $t = 2$. Since the problem for Shirley is symmetrical, when it rains

$$\theta^r_{1R} = \frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2}; \qquad \theta^r_{1S} = -\frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2},$$

[15] Without wealth effects, agents' private asset holdings are their original ones as agents want to maintain their risk profile, and only long-term government bonds are used to smooth the idiosyncratic shock.

Thus, optimal consumption in the symmetric equilibrium is

$$c_{0R} = \frac{Y_0}{2} + a_0 \frac{\hat{\Theta}_0}{2} \tag{14}$$

$$c^r_{1R} = \frac{(\overline{y} - w)}{(1+p_1)}; \qquad c^s_{1R} = -\frac{(\overline{y} - w)}{(1+p_1)} \tag{15}$$

$$\tilde{c}^r_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} + \frac{(\overline{y} - w)}{(1+p_1)}; \qquad \tilde{c}^s_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} - \frac{(\overline{y} - w)}{(1+p_1)}. \tag{16}$$

From inspecting optimal consumption, agents can smooth consumption in $t = 1$ above and beyond what was possible with insurance by rebalancing their portfolios. Raymond, for instance, does not consume the extra amount $\overline{y} - w$ when raining, but something less. In other words, long-term bonds are used both at $t = 0$ as collateral to sustain insurance and at $t = 1$ to trade and obtain further insurance.[16]

In what follows we restrict model parameters for to focus on the more interesting case:

Assumption A2. *There is no full insurance*

$$\overline{y} - \underbrace{\left(\frac{\Theta_0^{Sh}}{2} + p_1 \frac{\Theta_0}{2} + \alpha p_1^a \frac{\hat{\Theta}_0}{2} \right)}_{=w} > 0. \tag{17}$$

There are enough long-term bonds to optimally rebalance portfolios in $t = 1$, i.e., both agents hold the long-term government bond

$$\frac{\Theta_0}{2}(1+p_1) - \left(\overline{y} - \underbrace{\left(\frac{\Theta_0^{Sh}}{2} + p_1 \frac{\Theta_0}{2} + \alpha p_1^a \frac{\hat{\Theta}_0}{2} \right)}_{=w} \right) > 0. \tag{18}$$

Prices: Prices in $t = 1$ can be rewritten purely as a function of parameters given that there are no wealth effects, that $\tilde{a}_t = \rho \tilde{Y}_t$, and that $\tilde{Y}_2$ is normally distributed. From equations

[16] Note that with CARA utility, the difference in consumption between rain and shine cancels out and, thus, does not affect $t = 1$ pricing.

(5) and (6), with CARA utility we have,

$$\begin{aligned} p_1 &= \beta \mathbb{E}_1 \left(\exp\left\{ -\frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \tilde{Y}_2 \right\} \right) \\ &= \beta \exp\left\{ -\frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \mu + \frac{1}{8}\gamma^2 \left(1+\rho\hat{\Theta}_0\right)^2 \sigma^2 \right\} \quad (19) \\ p_1^a &= \beta \mathbb{E}_1 \left(\exp\left\{ -\frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \tilde{Y}_2 \right\} \rho \tilde{Y}_2 \right) \\ &= \rho \left(\mu - \frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \sigma^2 \right) p_1, \quad (20) \end{aligned}$$

All prices at $t = 0$ can be expressed as a function of the convenience yield, (CY in equation (13)), the risk-free rate (p_0^{rf} in equation (12)), and the previous $t = 1$ prices. From equations (9)–(11)

$$\begin{aligned} p_0^{Sh} &= p_0^{rf} + CY \quad (21) \\ p_0 &= p_1 \left(p_0^{rf} + CY \right) \quad (22) \\ p_0^a &= p_1^a \left(p_0^{rf} + \alpha CY \right). \quad (23) \end{aligned}$$

Existence: In what follows, we focus on the case in which the private asset expected return is high enough to have positive pricing but not so high as to make it more valuable than the risk-free rate. Put differently, the private asset's certainty equivalent is less than one, making it less attractive as a store of value. In addition, to put some discipline to the model, we assume that agents' preferences and the private asset's distribution satisfy the Hansen-Jagannathan bounds:

Assumption A3. *Assume the private asset price is positive but lower than the long-term bond price. That is*

$$0 \leq \left(\mu - \frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \sigma^2 \right) \leq 1 \quad (24)$$

and the Hansen-Jagannathan bounds for pricing in $t = 1$ hold.[17] *That is*

$$\left| \left(\mu - \frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \sigma^2 \right) \frac{\gamma}{2} \left(1+\rho\hat{\Theta}_0\right) \right| \leq \exp\left\{ \frac{1}{4}\gamma^2 \left(1+\rho\hat{\Theta}_0\right)^2 \sigma^2 \right\} - 1.$$

Given these closed-form expressions and related assumptions, the next theorem gives the conditions for the existence of a symmetric equilibrium.

Theorem 1 (Existence of Symmetric Equilibrium)**.** *If Assumption A1 and A3 hold,* $\overline{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4}]$, $\beta > \frac{1}{2}$, *and* $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ *is suffi-*

[17]See Lemma 3 in appendix B for the derivation of the Hansen-Jagannathan bound in this context.

ciently small, there exists a symmetric equilibrium characterized by the consumption paths in equations (14)–(16) *and prices in equations* (10)–(11) *and* (19)–(20)*.*

Proof. We only have to ensure that Raymond and Shirley hold both long-term government bonds and private assets in $t = 1$ and that the idiosyncratic shock is large enough so that agents cannot fully hedge their idiosyncratic risk. That is, the inequalities in (17) and (18) hold. From (19), because of condition (24) and $\rho \in (0, 1)$, $p_1, p_1^a \in (0, 1)$. In addition, if $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ is sufficiently small enough $p_1 > \frac{1}{2}$. In effect, using the Hansen-Jagannathan bound of Assumption A3 we know that

$$\begin{aligned}
\ln(2p_1) &= \ln(2\beta) - \frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{4}\gamma\left(1 + \rho\hat{\Theta}_0\right)\sigma^2\right) \\
&= \ln(2\beta) - \frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{2}\gamma\left(1 + \rho\hat{\Theta}_0\right)\sigma^2\right) - \frac{1}{8}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2 \\
&\geq \ln(2\beta) - \left(\exp\left\{\frac{1}{4}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2\right\} + \frac{1}{8}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2 - 1\right) \geq 0,
\end{aligned}$$

where we have use the Hansen-Jagannathan bound, $\log(2\beta) > 0$, and that $g(x) = \exp(x) + x/2 - 1$ is equal to zero when $x = 0$ and strictly increasing. This ensures that if $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ is sufficiently small enough $p_1 > \frac{1}{2}$. Therefore,

$$\overline{y} \geq \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2} > \frac{\Theta_0^{Sh}}{2} + p_1\frac{\Theta_0}{2} + \alpha p_1^a\frac{\hat{\Theta}_0}{2},$$

guaranteeing condition (17), and

$$\overline{y} \leq \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4} < \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2}(1 + 2p_1) + \alpha p_1^a\frac{\hat{\Theta}_0}{2},$$

guaranteeing condition (18).

The parameter space for Theorem 1 is feasible if $\Theta_0 > \alpha\left(1 - \frac{\rho}{2}(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\right)\hat{\Theta}_0$. □

3.2 Comparative Statics

Having characterized the symmetric equilibrium and ensured its existence, we now study how prices and allocations change with changes in the supply of both short- and long-term public assets (Θ_0^{Sh} and Θ_0), the severity of idiosyncratic shocks ($\bar{y}$), the pledgeability of private assets (α), and most importantly, aggregate volatility (σ^2).

Given that asset prices in (21) – (23) are expressed in terms of the traditional risk-free rate and the convenience yield, the following lemma is useful to generalize the model's comparative statics with respect to any of our parameters of interest,

Lemma 1. *(Sensitivity of risk-sharing, risk-free rates and convenience yields to any parameter). Given the equilibrium characterized in Theorem 1, for any model parameter z, we have the following comparative statics,*

$$\begin{aligned}\frac{\partial c^s_{1R}}{\partial z} &= -\frac{\partial c^r_{1R}}{\partial z}\\ \frac{\partial p^{rf}_0}{\partial z} &= -\gamma CY\frac{\partial c^s_{1R}}{\partial z} + \gamma p^{rf}_0\frac{\partial c_{0R}}{\partial z}\\ \frac{\partial CY}{\partial z} &= -\gamma p^{rf}_0\frac{\partial c^s_{1R}}{\partial z} + \gamma CY\frac{\partial c_{0R}}{\partial z}.\end{aligned}$$

Proof. The first equality determines risk-sharing and comes from equations (15). The second and third come from taking the derivative of equations (12) and (13) noting that for CARA utility $u''(z) = -\gamma u'(z)$. □

The interpretation of these sensitivities with respect to any parameter z is informative of the comparative statics with respect to the needs of risk sharing—what we call *risk sharing effects*. Assume, for example, that a change in z increases Raymond's consumption when it shines, and thus reduces the need for risk sharing. This has a direct and an indirect effect. On the one hand, the price of the risk-free bond is lower as agents value less transferring resources from $t = 0$ to $t = 1$, because their exposure to idiosyncratic shocks is lower. This direct effect is proportional to the convenience yield, CY and is captured by the first term in the second equation of the lemma. On the other hand, the convenience yield also declines as agents need less risk sharing across states, because the difference in consumption across states is lower. This indirect effect is proportional to the price of the risk-free bond, p^{rf}_0 and is captured by the first term in the third equation of the Lemma.

3.2.1 Changes in the Supply of Public Assets

Assume the government increases the total amount of both short- and long-term government bonds. Because the government raises lump sum taxes in periods the bonds mature, and because, in $t = 1$, the difference in consumption between rain and shine cancels out, changes in issuance do not change consumption paths, and thus do not affect prices in $t = 1$ (see equations (19)–(20)). The increase in the supply of public assets, however, affects the possibilities of risk sharing. From equations (14)–(16), we have

$$\frac{\partial c_{1R}^{s}}{\partial \Theta_0} = \frac{p_1}{2(1+p_1)} > 0 \qquad \text{and} \qquad \frac{\partial c_{0R}}{\partial \Theta_0} = 0.$$

The case for Θ_0^{Sh} is similar, except that the partial derivatives with respect to consumption are proportional to $\frac{1}{2(1+p_1)}$.

This partial derivative shows that an increase in short- and long-term government bonds increases consumption in the bad state (of Raymond when it shines and of Shirley when it rains) by improving risk sharing. This is a pure collateral effect: the economy has more collateral to sustain insurance promises, which improves hedging against idiosyncratic shocks. This effect shows up in the price of public and private assets in period 0. The next proposition characterizes these changes.

Proposition 1. *(Asset Pricing Effects of the Supply of Public Assets). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to the supply of government bonds, Θ_0^{Sh} and Θ_0,*

$$\begin{array}{lll} \frac{\partial p_0^{Sh}}{\partial \Theta_0^{Sh}} = -\frac{\gamma\left(p_0^{rf}+CY\right)}{2(1+p_1)}, & \frac{\partial p_0}{\partial \Theta_0^{Sh}} = -\frac{\gamma p_1\left(p_0^{rf}+CY\right)}{2(1+p_1)}, & \frac{\partial p_0^{a}}{\partial \Theta_0^{Sh}} = -\frac{\gamma p_1^{a}\left(\alpha p_0^{rf}+CY\right)}{2(1+p_1)} \\ \frac{\partial p_0^{Sh}}{\partial \Theta_0} = -\frac{\gamma p_1\left(p_0^{rf}+CY\right)}{2(1+p_1)}, & \frac{\partial p_0}{\partial \Theta_0} = -\frac{\gamma p_1^2\left(p_0^{rf}+CY\right)}{2(1+p_1)}, & \frac{\partial p_0^{a}}{\partial \Theta_0} = -\frac{\gamma p_1 p_1^{a}\left(\alpha p_0^{rf}+CY\right)}{2(1+p_1)} \end{array}$$

Proof. The result comes from directly applying Lemma 1 to (21)– (23). □

The Proposition shows that having more public assets that improve risk sharing makes all assets, both public and private, less valuable as collateral, reducing their price. Also, because $p_1 < 1$, the effect of short-term government bond supply is larger than the impact of long-term government bond supply. Thus, changes in short-term government bond supply are more effective at increasing risk sharing, making the impact on prices larger.[18]

The effect on the price of the private asset is also proportional to the convenience yield and depends on its collateralizability. Interestingly, even if the private asset were not pledgeable at all (that is, $\alpha = 0$), the increase in idiosyncratic risk would still lower its price, since private assets are also useful to transfer wealth to $t = 1$. Still, a higher α makes the private asset more sensitive to changes in the supply of public assets, because of the extra change in their value as collateral.

[18] In our empirical analysis we corroborate this implication, which is also consistent to Krishnamurthy and Vissing-Jorgensen (2012), Greenwood et al. (2015), and Infante (2020), who show in different ways that convenience yields are more sensitive to changes in T-bill outstanding than changes in longer term U.S. Treasury bonds outstanding.

3.2.2 Changes in Private Asset Pledgeability

Now, assume there is an increase in pledgeability α, perhaps by financial innovation or deregulation. This increase makes risk sharing easier by indirectly increasing the supply of collateral in the economy. As before, prices in $t = 1$ do not change, but allocations, from equations (14)–(16) do. Specifically, we have

$$\frac{\partial c^s_{1R}}{\partial \alpha} = \frac{p^a_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2} > 0 \qquad \texttt{and} \qquad \frac{\partial c_{0R}}{\partial \alpha} = 0.$$

The following proposition summarizes the effect on prices:

Proposition 2. *(Asset Pricing Effects of the Pledgeability of Private Assets). Given the equilibrium characterized in Theorem 1, the initial prices of the short- and long-term government bond and the private asset have the following comparative statics with respect to the pledgeability of private assets α,*

$$\frac{\partial p^{Sh}_0}{\partial \alpha} = -\gamma(p^{rf}_0 + CY)\frac{p^a_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}, \qquad \frac{\partial p_0}{\partial \alpha} = -\gamma p_1(p^{rf}_0 + CY)\frac{p^a_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}, \qquad \frac{\partial p^a_0}{\partial \alpha} = -\gamma p^a_1(\alpha p^{rf}_0 + CY)\frac{p^a_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2} + p^a_1 CY$$

Proof. The result comes from directly applying Lemma 1 to (21)— (23). □

The intuition is parallel to the increase in public assets, but the effect on the private asset price is now ambiguous. On the one hand, private assets become more useful as collateral, becoming more valuable, which operates through $p^a_1 CY$. On the other hand, the implied improvement in risk sharing reduces the value of holding those assets.

3.2.3 Changes in Idiosyncratic Volatility

Assume now that the variance of idiosyncratic shocks increase, which given the binomial structure of idiosyncratic shocks is captured by its size, $\overline{y}$. This increases the need for risk sharing. As with government bonds, because of CARA utility, the realization of the shock does not affect prices in $t = 1$ (see equations (19)–(20)). In terms of allocations, from equations (14)–(16) we have

$$\frac{\partial c^s_{1R}}{\partial \overline{y}} = -\frac{1}{(1+p_1)} > 0 \qquad \texttt{and} \qquad \frac{\partial c_{0R}}{\partial \overline{y}} = 0.$$

The effect of more risk sharing needs on prices is characterized by the next proposition.

Proposition 3. *(Asset Pricing Effects of Idiosyncratic Volatility). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term* *government bond, and private asset have the following comparative statics with respect to the size of idiosyncratic shocks $\overline{y}$,*

$$\frac{\partial p_0^{Sh}}{\partial \overline{y}} = \frac{\gamma \left(p_0^{rf} + CY\right)}{(1+p_1)}, \quad \frac{\partial p_0}{\partial \overline{y}} = \frac{\gamma p_1 \left(p_0^{rf} + CY\right)}{(1+p_1)}, \quad \frac{\partial p_0^a}{\partial \overline{y}} = \frac{\gamma p_1^a \left(\alpha p_0^{rf} + CY\right)}{(1+p_1)}$$

Proof. The result comes from directly applying Lemma 1 to (21)– (23). □

As idiosyncratic shocks become larger, the extent of idiosyncratic insurance decreases, making all assets more valuable as collateral for risk sharing purposes.

3.3 Changes in Aggregate Volatility

In this subsection, we present our main result. We assume an increase in aggregate volatility, which is captured by an increase in the variance of period 2 aggregate realizations, σ^2. In contrast to the previous cases, this generates a direct impact on prices at $t = 1$. From equations (19)–(20),

$$\begin{aligned}
\frac{\partial p_1}{\partial \sigma^2} &= \frac{\gamma^2}{8}\left(1+\rho\hat{\Theta}_0\right)^2 p_1 \\
\frac{\partial p_1^a}{\partial \sigma^2} &= \rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1 \\
&= \frac{p_1^a}{p_1}\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1
\end{aligned}$$

The $t = 1$ price of the long-term bond always increases when volatility increases. This can be interpreted as a standard "negative beta" effect of government bonds: as aggregate volatility increases, the need to smooth consumption intertemporally from $t = 1$ to $t = 2$ increases, making long-term government bonds more valuable.

The $t = 1$ price effect on the private asset is, however, more intricate. On the one hand, similar to the long-term government bond, there is a "negative beta" effect proportional to the private asset's certainty equivalent (the first term). On the other hand, more volatility implies that the private asset is less desirable per se, as it encompass part of the aggregate risk, putting downward pressure on its price (the second term). We focus on the economically interesting case in which the second effect dominates and the private asset's price declines with aggregate volatility, $\frac{\partial p_1^a}{\partial \sigma^2} < 0$. The next assumption characterizes the parametric condition for this to happen

Assumption A4. *Assume aggregate volatility depresses the price of private assets,* $\frac{\partial p_1^a}{\partial \sigma^2} < 0$*,* *which is guaranteed if*

$$\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)<1. \tag{25}$$

This assumption is ensured for $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small.[19] We highlight later the role of this natural assumption for the effect of aggregate volatility on risk sharing.

Remark of the plausibility of Assumption A4: *Even though this assumption seems natural, usually it is difficult to test given the lack of a purely exogenous shock on aggregate volatility. The recent crisis caused by the outbreak of the COVID-19 virus constitutes, however, a unique shock to aggregate volatility and higher future uncertainty—exogenous, unexpected, significant, without an end in sight, and truly aggregate as it affects all countries at once. We exploit this unique event to test these pricing assumptions. In Figure A.1 of Appendix A, we use VIX as a measure of aggregate volatility, which was relatively stable during 2018 and 2019 and indeed exhibited a large and sudden increase starting in February 2020 with the COVID-19 outbreak. As the VIX was stable, the spread between public and private yields was roughly constant. In February of 2020 the behavior of public and private yields suddenly moved in opposite directions, consistent with Assumption A4.*

How do these changes in $t=1$ prices affect allocations, in particular the extent of risk sharing in the economy? From equations (14)–(16), we have

$$\begin{aligned}\frac{\partial c^s_{1R}}{\partial\sigma^2} &= \underbrace{\frac{1}{(1+p_1)}\left[\frac{\partial p_1}{\partial\sigma^2}\left(\frac{\Theta_0}{2}+\frac{(\overline{y}-w)}{(1+p_1)}\right)+\alpha\frac{\partial p_1^a}{\partial\sigma^2}\frac{\hat{\Theta}_0}{2}\right]}_{\text{Valuation Effect}} \\ &= \frac{1}{(1+p_1)^2}\left[\overline{y}-\frac{\Theta_0^{Sh}}{2}+\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}\end{aligned} \tag{26}$$

and $\frac{\partial c_{0R}}{\partial\sigma^2}=0$. The effect of higher $t=2$ aggregate volatility on Raymond's consumption when it shines comes through changes in the price of assets in $t=1$, since those assets are used as collateral in $t=0$ to back promises that mitigate the effects of idiosyncratic shocks. That is, *risk sharing is affected by aggregate volatility purely by a valuation effect.*

[19] Note that the parameter space assumed in Proposition 4 is non-empty. In effect, the equilibrium in Theorem 1 exists if $\Theta_0>\alpha\left(1-\frac{1}{2}\frac{p_1^a}{p_1}\right)\hat{\Theta}_0$. This condition can simultaneously hold with $\alpha\frac{p_1^a}{p_1}\hat{\Theta}_0>\Theta_0$ if $\frac{p_1^a}{p_1}>\frac{2}{3}$, which depends on the total amount of collateralizability of the private asset's certainty equivalent relative to the amount of long-term government bonds. Using the Hansen-Jagannathan bounds, condition (25) holds if $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma<\sqrt{\ln(3)}$.

The overall impact is mixed. On the one hand, the price of long-term bonds in $t = 1$ increases, improving risk sharing (both by using them as collateral and by selling them in case of a bad shock). On the other hand, the price of private assets (under Assumption A4) in $t = 1$ decreases, weakening risk sharing (by using them as collateral). The net impact depends on the relative amount of public and private assets used as collateral, which itself depends on the supply of assets and the private assets' certainty equivalence and pledgablility. If the economy relies heavily on private assets, overall consumption in the bad state decreases, reducing risk sharing. Finally, because aggregate volatility in $t = 2$ does not affect the price of short-term government bonds in $t = 1$, it does not affect risk sharing through the use of short term bonds as collateral. These observations are summarized as follows

Proposition 4. *(Risk Sharing Effects of Aggregate Volatility). Given the equilibrium characterized in Theorem 1 and Assumption A4, if more private assets are used as collateral than long-term government bonds—that is,* $\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$ *(in terms of parameters this condition is* $\alpha\rho(\mu - \frac{\gamma}{2}(1 + \rho\hat{\Theta}_0)\sigma^2)\hat{\Theta}_0 > \Theta_0$*)— then an increase in aggregate volatility* σ^2 *reduces risk sharing—that is,* $\frac{\partial c_{1R}^s}{\partial \sigma^2} < 0$*. Moreover, the decrease in risk sharing is larger if the private asset is more pledgeable—that is,* $\frac{\partial^2 c_{1R}^s}{\partial \alpha \partial \sigma^2} < 0$*.*

Proof. See Appendix B. □

When agents rely more on private assets than on public long-term assets for risk sharing, the increase in aggregate volatility decreases insurance through a decrease in aggregate collateralizability, as the most relevant asset to hedge idiosyncratic risk becomes less valuable. In addition, the proposition also shows that, when the private asset becomes more useful as collateral, captured by α, then the negative sensitivity of risk sharing to aggregate volatility becomes even stronger. Intuitively, when private assets become more important as collateral, a reduction in their price triggered by an increase in aggregate volatility becomes more pervasive for risk sharing. Finally, notice that relaxing Assumption A4 implies that the price of both public and private assets would, perhaps counterfactually, increase with aggregate volatility, unconditionally improving risk sharing in the economy.

While Proposition 4 shows the impact of aggregate volatility on allocations, Proposition 5 shows its impact on $t = 0$ asset prices.

Proposition 5. *(Asset Pricing Effects of Aggregate Volatility). Given the equilibrium characterized in Theorem 1, the initial prices of the short-term government bond, long-term* *government bond, and private asset have the following comparative statics with respect to* σ^2*,*

$$\begin{aligned}
\frac{\partial p_0^{Sh}}{\partial \sigma^2} &= -\gamma(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} \\
\frac{\partial p_0}{\partial \sigma^2} &= -\gamma p_1(p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} + (p_0^{rf} + CY)\frac{\partial p_1}{\partial \sigma^2} \\
\frac{\partial p_0^a}{\partial \sigma^2} &= -\gamma p_1^a(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \sigma^2} + (p_0^{rf} + \alpha CY)\frac{\partial p_1^a}{\partial \sigma^2}.
\end{aligned}$$

Proof. The result comes from directly applying Lemma 1 to (21)– (23). □

Proposition 5 shows that the effect of aggregate volatility on $t = 0$ prices depends on two forces: a direct effect on the asset itself and an indirect effect on facilitating risk sharing.

The direct, asset-specific effect of aggregate volatility on long-term government bonds and private assets comes from their value changing in $t = 1$. While the value of long-term bonds increases with volatility, the value of private assets decreases (under Assumption A4). The indirect effect of aggregate volatility on risk sharing depends on the composition of collateral. If there are more private assets used as collateral, then there is less idiosyncratic insurance and the $t = 0$ value of all securities are higher as the convenience yield increases. These results underscore that the composition of collateral is important to understand the overall impact of aggregate volatility on risk sharing and asset prices. A relevant question, then, is what determines such composition? In the next section, we endogeneize the private creation of assets and collateral.

Remark on the generalization to other sources of valuation: Even though we have focused on how aggregate risk affects risk-sharing through the valuation of collateral, there are other aggregate changes that may have similar differential effects on public and private assets. One example is the expected endowment level μ in period 2. While the value of private assets will tend to decrease with worse prospects, the value of public assets will tend to increase given the higher relevance assigned to bringing consumption to the second period. Then, if interbank markets rely more on private assets, bad news about economic activity (a recession looming, for instance) will be detrimental for risk-sharing and may cause financial instability. These insights can be extended to other sources of shocks (changes in taxation, shocks to sovereign debt positions, etc) that move the prices of public and private assets in opposite directions.

4 Private Asset Creation

In this section, we entertain the idea that Raymond and Shirley have the ability to create private assets at a cost. The goal is providing conditions under which supplying public assets can either crowd out or crowd in private assets (through quantities and their valuations). We also provide conditions under which a stable economy induces the creation and use of private assets to share risk, and then makes the risk sharing more fragile to aggregate risk.

4.1 Model with private asset creation

Assume the cost of producing x units of private assets is $C(x)$ in terms of consumption goods, with $C', C'' > 0$. This cost is meant to capture both technological (such as the costs of financial innovation, securitization, managing information, etc.) and regulatory (such as constraints on the use of private assets by regulated financial institutions) costs to create and use private assets in financial contracts. Agents incur this cost before choosing their portfolio in $t = 0$ and sell these assets (perhaps to themselves) at the equilibrium price p_0^a. Focusing on Raymond, his consumption path changes as follows,

$$\begin{aligned}
c_{0R} &= e_{0R} + a_0 \frac{\hat{\Theta}_0}{2} - p_0 \left(\theta_{0R} - \frac{\Theta_0}{2} \right) - p_0^{Sh} \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - p_0^a \left(\hat{\theta}_{0R} - \left(\frac{\hat{\Theta}_0}{2} + x_R \right) \right) \\
&\quad + q^r w_R^r + q^s w_R^s - C(x_R) \\
\tilde{c}_{1R} &= \tilde{e}_{1R} + \tilde{a}_1 \hat{\theta}_{0R} - \tilde{p}_1 (\theta_{1R} - \theta_{0R}) + \left(\theta_{0R}^{Sh} - \frac{\Theta_0^{Sh}}{2} \right) - \tilde{p}_1^a (\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s \\
\tilde{c}_{2R} &= \tilde{e}_{2R} + \tilde{a}_2 \hat{\theta}_{1R} + \left(\theta_{1R} - \frac{\Theta_0}{2} \right)
\end{aligned}$$

where x_R is the amount of private assets Raymond creates. If no agent internalizes creation effects on prices, through the envelope condition, Raymond's optimal production of assets is determined by the following condition

$$C'(x_R^*) = p_0^a. \tag{27}$$

Thus, given the problem's symmetry (Shirley faces the same problem at $t = 0$), the total stock of private assets is given by $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$, and all the previous pricing equations hold simply replacing $\hat{\Theta}_0$ with $\hat{\Theta}$.

Exploiting again the specific case under Assumption A1, optimal consumption paths are

$$c_{0R} = \frac{Y_0}{2} + a_0\frac{\hat{\Theta}}{2} - C(x_R^*) \tag{28}$$

$$c_{1R}^r = \frac{(\overline{y} - w)}{(1+p_1)}; \qquad c_{1R}^s = -\frac{(\overline{y} - w)}{(1+p_1)} \tag{29}$$

$$\tilde{c}_{2R}^r = \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}}{2} + \frac{(\overline{y} - w)}{(1+p_1)}; \qquad \tilde{c}_{2R}^s = \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}}{2} - \frac{(\overline{y} - w)}{(1+p_1)}. \tag{30}$$

If we also adopt the parameter restrictions on agents' preferences and the private asset's distribution, described in Assumption A3, we have the following equilibrium characterization:

Theorem 2 (Existence of Symmetric Equilibrium with Private Asset Creation)**.** *If Assumption A1 and A3 hold,* $\overline{y} \in (\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4})$*,* $\beta > \frac{1}{2}$*,* $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ *is sufficiently small enough, and* $C'(\cdot)$ *is sufficiently large enough, there exists a symmetric equilibrium characterized by the consumption paths in equations* (28)–(30)*, prices in equations* (10)–(11) *and* (19)–(20)*, and the total amount of safe asset creation is given by* $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$*, where* x_R^* *solves* (27)*.*

Proof. The proof is exactly as before, except that we have to ensure that the total amount of private assets $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$ is such that

$$\overline{y} \in \left[\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}}{4}\right]$$

The equilibrium is characterized by the following system of equations:

$$T_1 := C'(x_R^*) - p_0^a = 0 \tag{31}$$

$$\begin{aligned} T_2 &:= p_0^a - \left[\beta\mathbb{E}_0\left(p_1^a\frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \alpha p_1^a\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right]\right] \\ &= p_0^a - p_1^a\left(p_0^{rf} + \alpha CY\right) = 0 \end{aligned} \tag{32}$$

which is guaranteed by the relevant bounds when marginal cost C' is sufficiently high. □

Having established the existence of symmetric equilibria with private asset creation, the following lemma provides the comparative statics of any model parameter to private asset creation and prices.

Lemma 2. *(Private Asset Creation and Prices). Given the equilibrium characterized in Theorem 2, for any model parameter z, we have the following comparative statics on private* *assets creation and prices*

$$\begin{pmatrix} \frac{\partial x_R^*}{\partial z} \\ \frac{\partial p_0^a}{\partial z} \end{pmatrix} = \frac{1}{|D|} \begin{pmatrix} 1 \\ C''(x_R) \end{pmatrix} \frac{\partial \left(p_1^a (p_0^{rf} + \alpha CY) \right)}{\partial z}$$

where $\frac{\partial \left(p_1^a (p_0^{rf} + \alpha CY) \right)}{\partial z}$ *is the partial equilibrium sensitivity of prices without safe asset creation (as obtained from combining Lemma 1 and equation* (23)*) and* $|D| = C''(x_R) - 2\frac{\partial p_0^a}{\partial \hat{\Theta}_0}$ *with*

$$\frac{\partial p_0^a}{\partial \hat{\Theta}_0} \;=\; -\gamma p_1^a (\alpha p_0^{rf} + CY) \frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + (p_0^{rf} + \alpha CY) \frac{\partial p_1^a}{\partial \hat{\Theta}_0} + \gamma p_1^a (p_0^{rf} + \alpha CY) \frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \tag{33}$$

the partial derivative of the private asset price p_0^a *to private asset supply.*

Proof. See Appendix B. □

The effect of a model's parameter on private asset creation is determined by $|D| = C''(x_R) - 2\frac{\partial p_0^a}{\partial \hat{\Theta}}$. The first component captures how fast the marginal cost of producing private assets changes with production. The second component captures how fast the marginal benefit of producing private assets changes with production. The expression $|D|$ is positive when the left-hand side of equation (27) increases faster than the right-hand side as there is more production, and negative otherwise. Hence, $|D|$ captures the *change in the net marginal cost of private asset creation*. While the most intuitive case is that the overall cost increases with production (this is $|D| > 0$, as usual with convex production costs), the role of private assets as collateral and their interaction with risk-sharing may flip this net marginal cost.

While the first component of $|D|$ is technological and assumed positive, the second component is characterized in equation (33), which encodes equilibrium effects that has three elements: The first is a *direct consumption element*, $\frac{\partial c_{0R}}{\partial \hat{\Theta}} = \frac{\rho Y_0}{2} > 0$. The second is a *supply element*, $\frac{\partial p_1^a}{\partial \hat{\Theta}} < 0$, which captures the reduction of prices from having more private assets. The third is a *risk-sharing element*, $\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0}$, which is more closely related to our mechanism. This last element involves both valuation and quantity effects, which operate in opposing directions. To be more precise,

$$\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} \;=\; \underbrace{\frac{1}{(1+p_1)} \left[\frac{\partial p_1}{\partial \hat{\Theta}_0} \frac{\Theta_0}{2} + \alpha \frac{\partial p_1^a}{\partial \hat{\Theta}_0} \frac{\hat{\Theta}}{2} + \frac{(\overline{y} - w)}{(1+p_1)} \frac{\partial p_1}{\partial \hat{\Theta}_0} \right]}_{\text{Valuation Effect}} + \underbrace{\frac{\alpha p_1^a}{2(1+p_1)}}_{\text{Quantity Effect}}. \tag{34}$$

On the one hand, more private assets provide more collateral and sustain more risk sharing, *a positive quantity effect.* This quantity effect depends on the private asset's collateralizability. On the other hand, more private assets reduce their value and their usefulness as collateral, *a negative valuation effect.*[20] The net effect on risk sharing depends on how much the economy relies on long-term assets and their pledgeability. For example, if there were many pledgeable long-term assets in the economy (high $\Theta_0, \hat{\Theta}_0$), and private assets were not very pledgable (low α), then the valuation effect would dominate, counterintuitively resulting in less risk sharing.[21]

4.2 Government Bond Supply and Private Asset Creation

It is commonly understood that more provision of government bonds crowds out private assets, as they tend to be substitutes on their uses as store of value and collateral, disincentivizing their production. From Lemma 2, crowding out is formally captured when $\frac{\partial x_R^*}{\partial \hat{\Theta}_0} < 0$. Since $\frac{\partial p_1^a}{\partial \hat{\Theta}_0} < 0$ (see footnote 20), this is the case when $|D| > 0$. As we discussed above, this condition is fulfilled when the net marginal cost of producing private assets is increasing.

Our general equilibrium focus may flip these intuitive results when private asset valuations are strong enough. Specifically, if private asset supply reduces equilibrium prices, by compressing the convenience yield, relative to the increase in marginal costs of production, then $|D| < 0$. Intuitively, an increase in government bonds reduces their their value as collateral and their effectiveness for risk sharing. If this induced collateral scarcity (in terms of government bond value) reduces risk sharing enough, the implied increase in convenience yields may make private assets more valuable, inducing more production.

This result shows the importance of studying valuation effects of assets that are substitutes in providing several functions—in this case, as a store of value or as collateral—in general equilibrium. Even though public and private assets are substitutes as collateral, the endogenous valuation may turn them to complements with supply.

4.3 Economic Stability and Private Asset Creation

Private asset creation also responds to changes in aggregate volatility. We focus here on the most intuitive case in which public assets crowd out private assets (as we discussed above, $|D| > 0$). From Lemma 2, the sensitivity of private asset creation to volatility depends on the partial equilibrium sensitivity of p_0^a to σ^2, which as shown in Proposition 5, depends on the convenience yield component of valuation. Under Assumption A4, we know that a more volatile environment leads to a lower price of private assets in $t = 1$. In addition,

[20]Formally, $\frac{\partial p_1}{\partial \hat{\Theta}_0} = -\frac{\gamma}{2} p_1^a < 0$ and $\frac{\partial p_1^a}{\partial \hat{\Theta}_0} = -\frac{\gamma}{2} \frac{(p_1^a)^2}{p_1} - \frac{\gamma}{2} \rho^2 \sigma^2 < 0$.

[21]More formally, this is the case when $\gamma \left(\frac{\Theta_0}{2} + \alpha \frac{p_1^a}{p_1} \frac{\hat{\Theta}_0}{2} \right) > \alpha$.

if government bonds are abundant, aggregate volatility also improves risk sharing, which compresses convenience yields, and further reduces private asset valuations. This result is summarized in the following Proposition.

Proposition 6. *(Aggregate Volatility and Private Asset Production). Given the equilibrium characterization in Theorem 2 and Assumption A4, there is a sufficiently convex production cost ($C''(\dot{)}$ is sufficiently large) and enough public assets used as collateral relative to existing private assets, that private asset creation decreases with aggregate volatility (i.e., $\frac{\partial x_R}{\partial \sigma^2} < 0$).*

Proof. First, a C'' sufficiently large guarantees that $|D| > 0$. Because of Assumption A4, $\frac{\partial p_a^1}{\partial \sigma^2} < 0$. From Proposition 4, enough public collateral relative to existing private collateral induces risk sharing to improve with aggregate volatility (this is $\frac{\partial CY}{\partial \sigma^2} > 0$, the negative beta effect of public assets). □

This Proposition shows the conditions under which, as an economy becomes more stable (this is, with less aggregate volatility), there is more production of private assets, which adds to the available stock of private collateral. We provide conditions purely based on the technological production of private assets and the relative use of public assets as collateral. While the high convexity of production costs guarantees interior solutions (and well-behaved comparative statics), the extensive use of public assets as collateral trumps the relevance of convenience yields on the valuation of private assets.

This result is relevant for several reasons. First, it highlights that private assets can be heavily created as the economy becomes more stable, increasing the importance of private assets as collateral. While beneficial in stable times, the economy's higher reliance to private assets make risk sharing more fragile (this is, more likely to suffer) in case of an increase in aggregate risk. Second, this result is more prevalent when private assets do not have a large convenience yield component in their valuation and when their production is indeed more complicated (in the sense of the cost convexity). This implies that regulations that disincentivize the creation and use of private assets as collateral, would create this link between stability and the endogenous creation use of private assets as collateral.

5 Empirical Analysis

Our main theoretical result is that the effect of increased aggregate volatility on risk sharing depends on the extent to which agents rely on private or public assets to collateralize their insurance contracts. While measuring the relative share of private to public assets, and their usefulness as collateral, is challenging (see for instance Gorton et al. 2012), our model predicts that this share determines the sensitivity of risk sharing to aggregate volatility.

Unfortunately, measuring risk sharing is also challenging, but we can use the convenience yield as a proxy that captures the relevance of risk sharing in the economy: when risk sharing is either not needed or easily provided, the value of assets as collateral is low, and thus better risk sharing implies a lower convenience yield.

Proposition 5 states that if the amount private collateral is larger than the amount of public collateral ($\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$) then an increase in aggregate volatility decreases risk sharing ($\frac{\partial c_{1R}^s}{\partial \sigma^2} < 0$). Moreover, this sensitivity decreases as the private asset becomes more useful as collateral ($\frac{\partial^2 c_{1R}^s}{\partial \alpha \partial \sigma^2} < 0$). We can test these sensitivities by studying the correlation between the convenience yield and measures of aggregate volatility. More formally, as the convenience yield is inversely related to risk sharing, our testable implication is:

Proposition 7. *(Testable Implications Based on the Convenience Yield). Given the equilibrium characterized in Theorem 1, if more private assets are used as collateral than long-term government bonds—that is, $\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$—then an increase in aggregate volatility σ^2 increases the convenience yield—that is,*

$$\frac{\partial CY}{\partial \sigma^2} = -\gamma p_0^{Sh} \frac{\partial c_{1R}^s}{\partial \sigma^2} > 0.$$

Moreover, if $\alpha p_1^a \hat{\Theta}_0 < 2$, then the increase in convenience yield is larger if the private asset is more collateralizable, that is,

$$\frac{\partial^2 CY}{\partial \alpha \partial \sigma^2} = \gamma p_0^{Sh} \frac{p_1^a}{(1+p_1)} \frac{\hat{\Theta}_0}{2} \frac{\partial c_{1R}^s}{\partial \sigma^2} - \gamma p_0^{Sh} \frac{\partial^2 c_{1R}^s}{\partial \alpha \partial \sigma^2} > 0.$$

Proof. See Appendix B. □

Proposition 7 states that an increase in aggregate volatility increases the convenience yield when private assets are heavily used as collateral, and even more so when they are more pledgable.[22] Taken literally, an unexpected increase in aggregate volatility tomorrow affects the convenience yield today.[23] These observations motivate us to estimate the following

[22] The condition $\alpha p_1^a \hat{\Theta}_0 > p_1 \Theta_0$ in Proposition 7 is necessary but not sufficient, implying a positive sensitivity between the convenience yield and aggregate volatility whenever more private assets are used as collateral. This sensitivity can be interpreted as a supply shock to the total amount of safe assets. Specifically, if the share of private assets is larger, a change in valuations following an increase in aggregate volatility effectively reduces the total amount of collateral in the economy, and thus, reduces the total amount of idiosyncratic insurance, increasing the convenience yield. The model does not have a clear prediction when the share of public assets is larger.

[23] Our stylized three-period model is not designed to capture price changes in response to a fully dynamic, infinite horizon, volatility process. The model is intended to capture how changes in the volatility of future payoffs affect agents' exposure to idiosyncratic risk in the near term—that is, how future aggregate volatility affects risk sharing today.

empirical model:

$$\Delta CY_t = \beta_0 + \beta_V \Delta VIX_t + \sum \gamma_j \Delta CY_{t-j} + \beta_F \Delta FedFunds_t + \eta FedFunds_{t-1} + \theta \Delta Gov_t + \epsilon_t, \quad (35)$$

where ΔCY_t is first differences of well-known empirical measures of the safe asset convenience yield and ΔVIX_t is first difference of the Chicago Board Options Exchange VIX index, a measure of implied volatilities of S&P500 index options, to capture changes in aggregate volatility. Lagged changes of the convenience yield are used to control for serial autocorrelation.[24] This empirical specification is inspired by Nagel (2016), who shows that the level of the convenience yield depends on the level of rates, as it depends on the opportunity cost of holding money. In particular, Nagel shows that, once you control for the level of rates, government asset supply loses its statistical significance in explaining the safe asset convenience yield. Therefore, we also control for changes in the level of rates and changes in government bond supply. Importantly, our specification differs as we focus on changes in the convenience yield, rather than its level; however, we also control for the lagged level of rates which may capture different sensitivities due to the changes in opportunity cost of holding money.

The mechanism in our model operates through changes in the valuations of public and private asset, which effectively alters the *supply of collateral*, and thus, idiosyncratic insurance. From this interpretation, we expect β_V in equation (35) to be positive in times when the share of private asset collateral is larger than that of public asset collateral.

We then estimate the empirical model in equation (35) over different time periods that differ on the production and use of private assets as collateral. One of those changes evolved in the long term, spanning several decades, and was given by a process of slow financial innovation and deregulation.[25] The other happened more drastically over a short period and goes in the opposite direction, driven by the Global Financial Crisis (GFC) that put the use of private assets as collateral under distress and was promptly followed by tight regulations.[26] We conjecture that the sensitivity of the convenience yield to changes in aggregate volatility is higher in the 90s and 2000s when compared with the decades immediately after World War II, when less private assets were used as collateral. Moreover, this sensitivity increased rapidly in the 2000s leading toward the financial crisis, after which it declined as new regulations were implemented.

[24] We have also considered specifications with levels of VIX and CY as additional independent variables. The main insights remain and results are available upon request.

[25] For example, in the 1980s repos were excluded from automatic stay, contributing to the prevalence of these types of contracts.

[26] For example, the Liquidity Coverage Ratio places a larger regulatory burden on private assets that are used to back financial firms' liabilities.

5.1 Controlling for an Alternative Explanation

Our mechanism operated through supply of collateral, but an alternative explanation is that, for some reason, changes in aggregate volatility modifies the *demand of collateral*. That is, in times when aggregate volatility is high, agents' need more to hedge idiosyncratic shocks, and thus the convenience yield increases, all else constant. From this perspective, different sensitivities across time periods could merely capture changes in the demand for safety that are time varying, and β_V in equation (35) could differ across periods regardless of the share of private and public collateral. Appendix D extends the model to show this alternative.

In order to control for this alternative explanation we exploit an instrument that isolates changes in supply, which allows to estimate the demand sensitivity in each period. Following Infante (2020), we consider changes in the total amount of T-bills with less than one month to mature.[27] Since four-week T-bill rates are typically below overnight general collateral repo rates, it is very unlikely that a firm would raise outside funding to finance their short-term T-bills positions, since that trade would involve a negative carry, then providing an exogenous source of supply change. With the use of this instrument we can measure whether the demand for safe assets induced by aggregate volatility differs across periods. That is, if changes in future volatility alter the demand for safe assets, we would expect the sensitivity of the convenience yield to changes in short-term T-bills outstanding to differ across periods as aggregate volatility changes. This leads us to estimate the following empirical model

$$\begin{aligned}\Delta CY_t \quad = \quad & \beta_0 + \sum \gamma_j \Delta CY_{t-j} + \beta_V \Delta VIX_t + \beta_F \Delta FedFunds_t + \eta FedFunds_{t-1} + \theta \Delta Gov_t \\ & + \varphi \Delta VIX_t \times \Delta log(ShortTBillsOut_t) + \epsilon_t. \qquad (36)\end{aligned}$$

where $\Delta log(ShortTBillsOut_t)$ is first differences of the total amount of T-bill with less than one month to mature.

In what follows, we first estimate model (35) using monthly data from 1950 to 2011 to capture the long-run evolution of private assets as collateral since World War II, documented by Gorton et al. (2012). We then estimate model (36) using overlapping daily data from 2001 to 2020 to capture the rapid increase in the use of private assets as collateral leading to the Great Recession and the large collapse of such use afterwards.

[27]Infante (2020) shows that changes in short-term T-bills affect the level of the convenience yield. In this paper we test if changes in short-term T-bills also affect changes in the convenience yield. The exogeneity of T-bill issuance is reinforced by the fact that it is known in advance, and the U.S. Treasury does not respond to changes in market rates.

5.2 Longer-Term Analysis

For the long-term analysis, we use the same data as Nagel (2016).[28] The convenience yield is measured as the spread between the banker's acceptance and the three-month T-bills spread (BA/T-bill spread). The VIX index is only available from 1990 onward, but earlier time periods are estimated using the projection of the VIX on realized S&P Index volatility. We winsorize the changes in convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers. The interest rate is the federal funds rate, and the government's supply of bonds is captured by the total amount of T-bill outstanding and total U.S. debt relative to GDP. This post-war data *frequency is monthly, from January 1950 to December 2011.*[29] Details of the data can be found in Nagel (2016).

In this estimation, we study how the slow and persistent process of financial innovation, financial engineering (such as securitization), and financial deregulation, which according to Gorton et al. (2012) generated an increase in the relative share of private to public assets used as collateral, has changed the sensitivity of the convenience yield to aggregate volatility. Inspired by Proposition (7), we expect β_V to be larger in the more recent decades. To capture this long-term change in sensitivity, we estimate model (35) splitting the sample in 1990.

Table 1 shows the estimates using data before 1990 and using data after 1990. The results show that the statistical significance of ΔVIX_t is indeed much larger in the latter part of the sample. That is, in the more recent decades, when the economy faces an increase in aggregate volatility the convenience yield increases significantly. From the eyes of our model, this happens because of the economy's higher reliance on private assets, which reduces their value with an increase in aggregate volatility.

5.3 Shorter-term Analysis

For the shorter-term analysis that covers the most recent period, we use the same data as Infante (2020). The convenience yield is measured by the spread between the one-month overnight index swap (OIS) rate downloaded from Bloomberg, and the four-week T-bill rate, downloaded from the Federal Reserve H.15 Statistical Release. We again winsorize the changes in the convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers.[30] Government supply is captured by the total amount of T-bill outstanding and

[28]This dataset is available on Nagel's website.

[29]Nagel (2016) provides convenience yield data from January 1920, however Gorton et al. (2012) show that the increase in private safe assets began at the start of the 1950s.

[30]We also drop observations on quarter-end dates, and two days surrounding quarter-end, to exclude any changes in short-term rates driven by financial firms' window dressing behavior. See Infante (2020) for more details.

Table 1: Convenience Yield and Volatility: Pre- and Post- 1990

	Pre-1990	Post-1990	Pre-1990	Post-1990
$\Delta FedFunds_t$	0.197***	0.107***	0.196***	0.082**
	(0.030)	(0.036)	(0.030)	(0.034)
$FedFunds_{t-1}$	0.003	0.001	0.004	0.000
	(0.002)	(0.002)	(0.002)	(0.002)
ΔVIX_t	0.005	0.007***	0.005	0.008***
	(0.003)	(0.002)	(0.003)	(0.003)
$\Delta log(TBillsOut_t/GDP_t)$			-0.267*	-0.409**
			(0.150)	(0.166)
$\Delta log(USTNotesOut_t/GDP_t)$			-1.200	-0.324
			(0.801)	(0.278)
P-value	0.815	0.100	0.775	0.138
Adj RSq	0.199	0.110	0.207	0.132
N obs	476	264	476	264

Note: This table shows the empirical results of equation (35) using monthly average data. The convenience yield measure is the spread between the monthly average 3-month bankers acceptance and the monthly average 3-month T-bills. ΔVIX_t is the first difference of the monthly average VIX Index, $\Delta FedFunds_t$ is the first difference of the monthly average federal funds rate, and $FedFunds_{t-1}$ is the lagged monthly average federal funds rate. $\Delta log(TbillOut_t/GDP_t)$ is the log difference of total outstanding of T-bills to GDP, and $\Delta log(Debt_t/GDP_t)$ is the log difference of total U.S. debt to GDP. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from January 1950 to December 2011. The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 12 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

total amount of Treasury notes and bonds outstanding, published by TreasuryDirect. As discussed in section 5.1, exogenous changes in the supply of safe assets is captured by changes in T-bills outstanding with less than one month to mature, which allows us to isolate the sensitivity of demand. These series are constructed using Treasury auction results published by TreasuryDirect.[31] The data *frequency is daily and runs from August 2004 and April 2020.* Here we estimate the sensitivity of 5-day changes using overlapping data to reduce the impact of high frequency variations.[32]

In this estimation, we study how the more stringent regulatory landscape implemented after the Great Recession, which in principle reduced private asset creation and made the use of private assets as collateral more difficult, affected how the convenience yield reacts to changes in aggregate volatility. Again, inspired by Proposition 7 we expect the coefficient

[31] We would like to thank staff in the Division of Monetary Affairs at the Federal Reserve Board for sourcing and organizing the data from TreasuryDirect.

[32] Appendix E shows the results for daily changes, which are qualitatively similar to the analysis with weekly changes.

on ΔVIX_t to be positive and significant only before the Great Recession.[33] To capture the change in sensitivity, we estimate model (35), splitting the sample in 2009.

5.3.1 Sensitivity of Private and Public Assets

Before studying the sensitivity of the convenience yield to aggregate volatility, we first document the underlying sensitivity of private and public asset valuations to volatility. Because our main mechanism operates as a supply effect via asset valuations, it is instructive to first verify that in fact public assets have a negative sensitivity to aggregate volatility—they increase in value when the future become more uncertain, the celebrated "negative beta" effect—while private assets have a positive one—they decrease in value in the face of future uncertainty. Put differently, these results verify that Assumption A4 in fact holds for private assets, while public ones have opposite sensitivity.

Inspired by equation (35), Table 2 shows the sensitivity of changes in 10-year U.S. Treasury, Agency MBS, investment grade corporate bond, and high yield bond yields to changes in ΔVIX. Consistent with existing literature, the 10-year U.S. Treasury yield increases as volatility increase, confirming the negative beta effect. From the table we can appreciated that as the risk of the asset class increases—from Agency MBS bonds to high yield corporate bonds—the sensitivity of changes in yields to aggregate volatility monotonically decrease. In particular, consistent with Assumption A4, high yield corporate bond yields significantly decreases (in a statistical sense) as aggregate volatility increases.

The results in Table 2 is consistent with an spectrum of asset riskiness, which captures the supply effects in our model.[34] This may raise the question regarding the prevalence of the use of risky assets as collateral. While the evidence presented in Figure 1 does not provide details on the composition of private collateral, other sources point to the important role of riskier asset classes in collateralized markets. For example, the Federal Reserve's 2021 Financial Stability Report shows that some CCPs, such as the Options Clearing Corporation, have a sizable share equity and mutual fund collateral. Intuitively, these collateral decisions may be appropriate to manage idiosyncratic risks that a particular CCP may face, but they are subject to the valuation effect in response to aggregate risk that we put forth in this paper. Furthermore, evidence from U.S. repo markets shows that the use of riskier collateral is substantial in different repo market segments. For example, data from the Federal Reserve Bank of New York shows a sizable amount of tri-party repo activity with nonfedwire

[33]In this section, $\Delta x_t = x_t - x_{t-5}$, the first difference operator with five lags.

[34]The regression results reflect the average sensitivity to changes in volatility, but these may be differ in times of severe market stress. In effect, a close inspection of Figure A.1 shows that during the market turmoil of March 2020, MBS yields and corporate bond yields tended to increase, while Treasury yields decreased.

Table 2: **Yields and Volatility**

	Δ10-year UST	ΔMBS	ΔIG Corp Bonds	ΔHY Corp Bonds
$\Delta FedFunds_t$	-0.065	-0.021	-0.087*	0.005
	(0.047)	(0.056)	(0.051)	(0.071)
$FedFunds_{t-5}$	-0.002	-0.002	0.001	-0.001
	(0.002)	(0.003)	(0.002)	(0.003)
$\Delta log(ShortTBillsOut_t)$	-0.005	0.024	0.077	0.105
	(0.064)	(0.068)	(0.064)	(0.112)
$\Delta log(USTNotesOut_t)$	-1.777**	-2.242**	-3.078***	-3.353*
	(0.807)	(0.949)	(0.957)	(2.020)
ΔVIX_t	-0.012***	-0.007***	-0.002	0.038***
	(0.002)	(0.002)	(0.002)	(0.003)
P-value	0.340	0.184	0.018	0.000
Adj RSq	0.099	0.037	0.051	0.340
N obs	2406	2404	2604	2604

Note: This table shows the empirical results of equation (36) using overlapping daily data. The dependent variables are the 5-day changes in the 10-year U.S. Treasury yield, the . ΔVIX_t is the 5-day first difference of the VIX Index, $\Delta FedFunds_t$ is the 5-day first difference of the federal funds rate, and $FedFunds_{t-5}$ is the 5-day lag of the federal funds rate. $\Delta log(ShTbillsOut_t)$ is the 5-day log difference of Treasury bills outstanding with maturity less than one month, and $\Delta log(USTNotesOut_t)$ is the 5-day log difference of total U.S. Treasury notes and bonds outstanding. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to April 2020. Estimates exclude quarter-end dates (and ± 2 days surrounding quarter-end). The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

eligible securities, which include high yield corporate bonds and equities. These observations underscore the relevance risk private securities to the modern financial infrastructure.

5.3.2 Sensitivity of Convenience Yield Before and After 2009

Table 3 shows three versions of model (35), using data before 2009 and after 2009. The first version only controls for $\Delta log(ShortTBillsOut_t)$, to test whether the demand for safe assets is more or less sensitive to exogenous changes in supply before and after 2009. The second specification includes the interaction term $\Delta VIX_t \times \Delta log(ShortTBillsOut_t)$ to test whether the sensitivity of the demand for safe assets changes with the VIX. Finally, the third specification directly considers changes aggregate volatility ΔVIX_t which isolates the valuation effect of collateral over idiosyncratic insurance.

The first two estimates in Table 3 show that $\Delta log(ShortTBillsOut_t)$ is statistically significant for both time periods, and larger before 2009. This indicates that before 2009 this measure of the convenience yield was more responsive to changes in the supply of public assets. The second two estimates show that there is no statistically significant relationship between the convenience yield and the interaction term. This indicates that aggregate volatility does not change how convenience yields react to public supply of safe assets, showing that the alternative explanation that volatility affects deferentially the need for insurance is not present in the data. Finally, the last two estimates show that ΔVIX_t is only positive and statistically significant before the Great Recession. From the perspective of our model, this indicates that before 2009 an increase in aggregate volatility reduced the supply of collateral, consistent with with idea of a larger share of private collateral. However, this effect waned after 2009, consistent with post-crisis regulatory efforts to reduced the financial system's reliance on private assets.

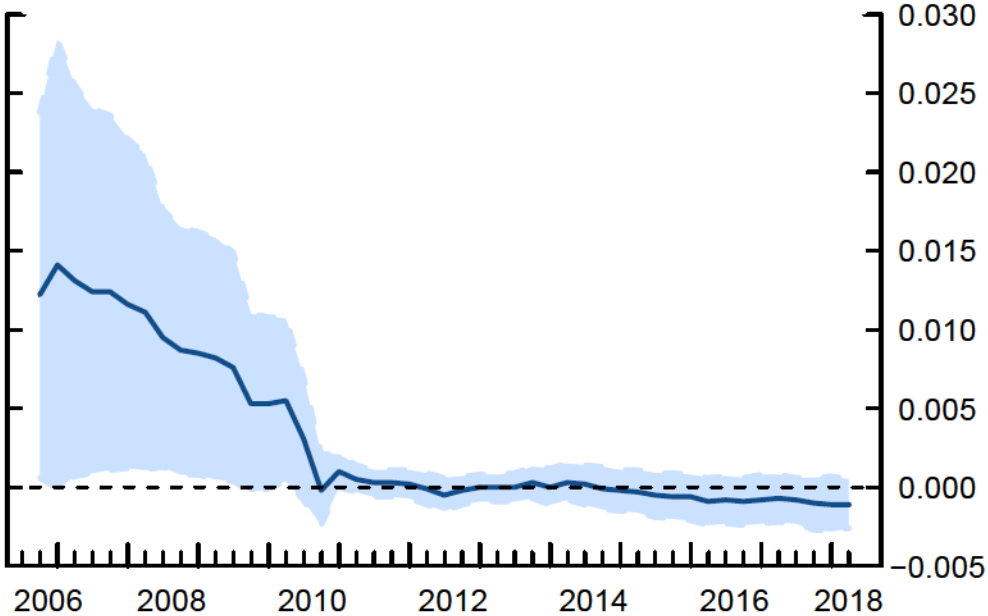
Table 3: Convenience Yield and Volatility: Pre- and Post- 2009

	Pre- 2009	Post- 2009	Pre- 2009	Post- 2009	Pre- 2009	Post- 2009
$\Delta FedFunds_t$	-0.156*	-0.117***	-0.155**	-0.118***	-0.167**	-0.120***
	(0.080)	(0.030)	(0.078)	(0.030)	(0.076)	(0.030)
$FedFunds_{t-5}$	0.013**	-0.002	0.013**	-0.002	0.012*	-0.001
	(0.007)	(0.003)	(0.007)	(0.003)	(0.006)	(0.003)
$\Delta log(ShortTBillsOut_t)$	-0.663***	-0.098***	-0.665***	-0.099***	-0.681***	-0.099***
	(0.168)	(0.024)	(0.166)	(0.025)	(0.165)	(0.025)
$\Delta log(USTNotesOut_t)$	-1.479	0.311	-1.493	0.303	-1.340	0.317
	(3.368)	(0.447)	(3.366)	(0.446)	(3.202)	(0.450)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t$			0.015	0.005	0.012	0.006
			(0.061)	(0.007)	(0.050)	(0.007)
ΔVIX_t					0.008**	-0.001
					(0.003)	(0.001)
P-value	0.349	0.000	0.353	0.000	0.320	0.000
Adj RSq	0.107	0.106	0.106	0.106	0.125	0.107
N obs	682	1724	682	1723	682	1723

Note: This table shows the empirical results of equation (36) using overlapping daily data. ΔVIX_t is the 5-day first difference of the VIX Index, $\Delta FedFunds_t$ is the 5-day first difference of the federal funds rate, and $FedFunds_{t-5}$ is the 5-day lag of the federal funds rate. $\Delta log(ShTbillsOut_t)$ is the 5-day log difference of Treasury bills outstanding with maturity less than one month, and $\Delta log(USTNotesOut_t)$ is the 5-day log difference of total U.S. Treasury notes and bonds outstanding. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to April 2020. Estimates exclude quarter-end dates (and $\pm$ 2 days surrounding quarter-end). The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

We can further exploit the high frequency data to estimate the model in shorter time intervals and see the evolution of ΔCY_t's sensitivity to ΔVIX_t. Specifically, in each quarter,

Figure 2: Five-Day Sensitivity of ΔCY_t to ΔVIX_t



Note: The solid line shows the point estimate of the 5-day estimation of full model in equation (36) using daily data and ± 2 years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.

we estimate the empirical model (35) using plus and minus two years of data. With this strategy, we can keep track of the changes in sensitivities over time.

Figure 2 shows the results. We can observe that the point estimate on ΔVIX_t is positive and statistically significant at the end of 2006. Arguably, this period is the pinnacle of the securitization boom that began in the previous decade. We would expect that this period also coincides with an increase in financial engineering, which allowed agents to use more private assets as collateral. After the crisis, the point estimate begins to decline, turning insignificant at the end of 2011, around the time when new regulatory initiatives took hold and financial firms' ability to use private collateral was less attractive. From the lens of our model, the results in Figure 2 suggest that before the onset of the crisis, the economy relied heavily on private assets as collateral, a trend which persistently reversed thereafter.

Remark on the sensitivity of convenience yields in other countries: While the focus of our empirical analysis is in the United States, one may wonder whether the sensitivity of risk sharing to aggregate volatility has changed in other countries, especially for those traditionally considered as safe issuers. One may conjecture that the use of private collateral may not have been as prevalent in other countries.[35] This suggests that empirical

[35]For example, Mancini et al. (2016) document that acceptable collateral baskets in cleared interbank

models such as (35) may not capture a positive sensitivity between ΔCY_t and ΔVIX_t, as prescribed by our model.

To test this hypothesis, we study the convenience yield in German bond markets. Specifically, we consider the spread between the 3-month Euro OverNight Index Average (EONIA) swap rate and the 3-month German T-bill rate as a measure of convenience yield in Germany. We then regress changes in this convenience yield on the Euro Stoxx 50 volatility index, the European counterpart of the VIX. The lack of data availability only permits us to run the high frequency analysis from January 2007 to April 2020, without high frequency government issuance controls. However, the results in Table E.2 of the appendix show there is no statistical relationship between ΔCY_t and ΔVIX_t across the two available sub-sample periods. From the lens of our model, this suggests that in German financial markets, public assets make up a large share of collateral used for risk sharing.

6 Concluding Remarks

We have characterized the relationship between aggregate volatility, which determines the cyclical properties of the economy, and risk sharing, which determines its distributional properties. As both assets are used for intra- as well as intertemporal reasons, aggregate volatility can either improve or weaken risk sharing depending on the composition of private and public assets that are used as collateral to sustain insurance promises. The main linkage is then given by the valuation of collateral, as aggregate volatility affects the valuation of private and public assets in opposite directions. An economy that relies more on private assets to collateralize risk sharing sees insurance decline when aggregate volatility increases.

Our model then generates testable implications that relate aggregate volatility and risk sharing depending on the intensity of using private assets as collateral. We overcome the difficulty to measure risk sharing by using the convenience yield of safe assets as a proxy and testing its sensitivity to changes in aggregate volatility. We provide empirical evidence that this sensitivity has increased over the second half of the 21st century, and dramatically so during early 2000s. This trend, however, has sharply reversed after the Great Recession. From the prism of our model, this suggests that the U.S. economy's reliance on private collateral, and thus the added fragility that comes with it, has increased during the second half of the 21st century (consistent with financial innovation and financial deregulation) but declined after the Great Recession, a period indeed characterized by stricter regulations.

We also show that, because of the valuation implications of aggregate volatility, if the

European repo markets consist of the high quality government collateral admitted by open market operation of the European Central Bank.

economy is less reliant on private collateral, a more stable economy may prompt a higher production of private assets, endogenously making them more relevant to back insurance contracts. As such, economic stability endogenously induces a higher dependence on private collateral, making risk sharing more fragile to shocks to aggregate volatility. In short, stability creates a more fertile ground for fragility, planting the seeds of its own instability. Since, as discussed initially, financial intermediaries are among the largest players in trading public and private assets to back derivative contracts, this last insight provides a novel element—the public/private composition of collateral—that policymakers should follow when assessing the fragility of the economy and when imposing macroprudential safeguards. This is particularly relevant given the importance of insurance contracts in financial and interbank markets to steer distress scenarios.

References

Acharya, V. and Merrouche, O. (2012), 'Precautionary hoarding of liquidity and interbank markets: Evidence from the subprime crisis', *Review of Finance* **17**(1), 107–160.

Alvarez, F. and Jermann, U. J. (2000), 'Efficiency, equilibrium, and asset pricing with risk of default', *Econometrica* **68**(4), 775–797.

Baele, L., Bekaert, G. and Inghelbrecht, K. (2010), 'The determinants of stock and bond return comovements', *The Review of Financial Studies* **23**(6), 2374–2428.

Bhandari, A., Evans, D., Golosov, M. and Sargent, T. (2021), Managing public portfolios. Working Paper Chicago.

Brumm, J., Grill, M., Kubler, F. and Schmedders, K. (2018), 'Re-use of collateral: leverage, volatility, and welfare'. ECB Working Paper 2218.

Caballero, R. J. and Farhi, E. (2018), 'The safety trap', *Review of Economic Studies* **85**(1), 223–274.

Chien, Y. and Lustig, H. (2010), 'The market price of aggregate risk and the wealth distribution', *Review of Financial Studies* **23**(4), 1596–1650.

Connolly, R., Stivers, C. and Sun, L. (2005), 'Stock market uncertainty and the stock-bond return relation', *Journal of Financial and Quantitative Analysis* **40**(1), 161–194.

Copeland, A., Martin, A. and Walker, M. (2014), 'Repo runs: evidence from the tri-party repo market', *Journal of Finance* **69**(6), 2343–2380.

Di Tella, S. (2017), 'Uncertainty shocks and balance sheet recessions', *Journal of Political Economy* **125**(6), 2038–2081.

Gorton, G., Lewellen, S. and Metrick, A. (2012), 'The safe-asset share', *American Economic Review* **102**(3), 101–106.

Gorton, G. and Ordonez, G. (2014), 'Collateral crises', *American Economic Review* **104**(2), 343–378.

Gorton, G. and Ordonez, G. (2021), The supply and demand for safe assets. Forthcoming, Journal of Monetary Economics.

Greenwood, R., Hanson, S. G. and Stein, J. C. (2015), 'A comparative-advantage approach to government debt maturity', *Journal of Finance* **70**(4), 1683–1722.

Heider, F., Hoerova, M. and Holthausen, C. (2009), Liquidity hoarding and interbank market spreads: The role of counterparty risk. ECB Working Paper 1126.

Holmstrom, B. and Tirole, J. (1998), 'Private and public supply of liquidity', *Journal of Political Economy* **106**(1), 1–40.

Hryshko, D., Luengo-Prado, M. J. and Sorensen, B. (2010), 'House prices and risk sharing', *Journal of Monetary Economics* **57**, 975–987.

Hurst, E. and Stafford, F. (2004), 'Home is where the equity is: mortgage refinancing and household consumption', *Journal of Money, Credit and Banking* **36**(6), 985–1014.

Infante, S. (2020), 'Private money creation with safe assets and term premia', *Journal of Financial Economics* **136**(3), 828–856.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S. and Xiaolan, M. (2019), The u.s. public debt valu- ation puzzle. NBER Working Paper 26583.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S. and Xiaolan, M. (2021), Manufacturing risk-free government debt. Unpublished working paper. Stanford University.

Kehoe, T. J. and Levine, D. K. (1993), 'Debt-constrained asset markets', *The Review of Economic Studies* **60**(4), 865–888.

Kiyotaki, N. and Moore, J. (1997), 'Credit cycles', *Journal of political economy* **105**(2), 211–248.

Krishnamurthy, A. (2003), 'Collateral constraints and the amplification mechanism', *Journal of Economic Theory* **111**(2), 277–292.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2012), 'The aggregate demand for Treasury debt', *Journal of Political Economy* **120**(2), 233–267.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2015), 'The impact of Treasury supply on financial sector lending and stability', *Journal of Financial Economics* **118**(3), 571–600.

Lustig, H. and Van Nieuwerburgh, S. (2010), 'How much does household collateral constrain regional risk sharing?', *Review of Economic Dynamics* **13**(2), 265–294.

Mancini, L., Ranaldo, A. and Wrampelmeyer, J. (2016), 'The euro interbank repo market', *The Review of Financial Studies* **29**(7), 1747–1779.

Nagel, S. (2016), 'The liquidity premium of near-money assets', *Quarterly Journal of Economics* **131**(4), 1927–1971.

Rampini, A. A. and Viswanathan, S. (2010), 'Collateral, risk management, and the distribution of debt capacity', *The Journal of Finance* **65**(6), 2293–2322.

Rampini, A. and Viswanathan, S. (2019), 'Financial intermediary capita', *Review of Economic Studies* **86**, 413–455.

Reis, R. (2021), The constraint on public debt when r¡g but g¡m. Working Paper LSE.

Sunderam, A. (2014), 'Money creation and the shadow banking system', *Review of Financial Studies* **28**(4), 939–977.

Appendix

A Figures to back Assumption A4

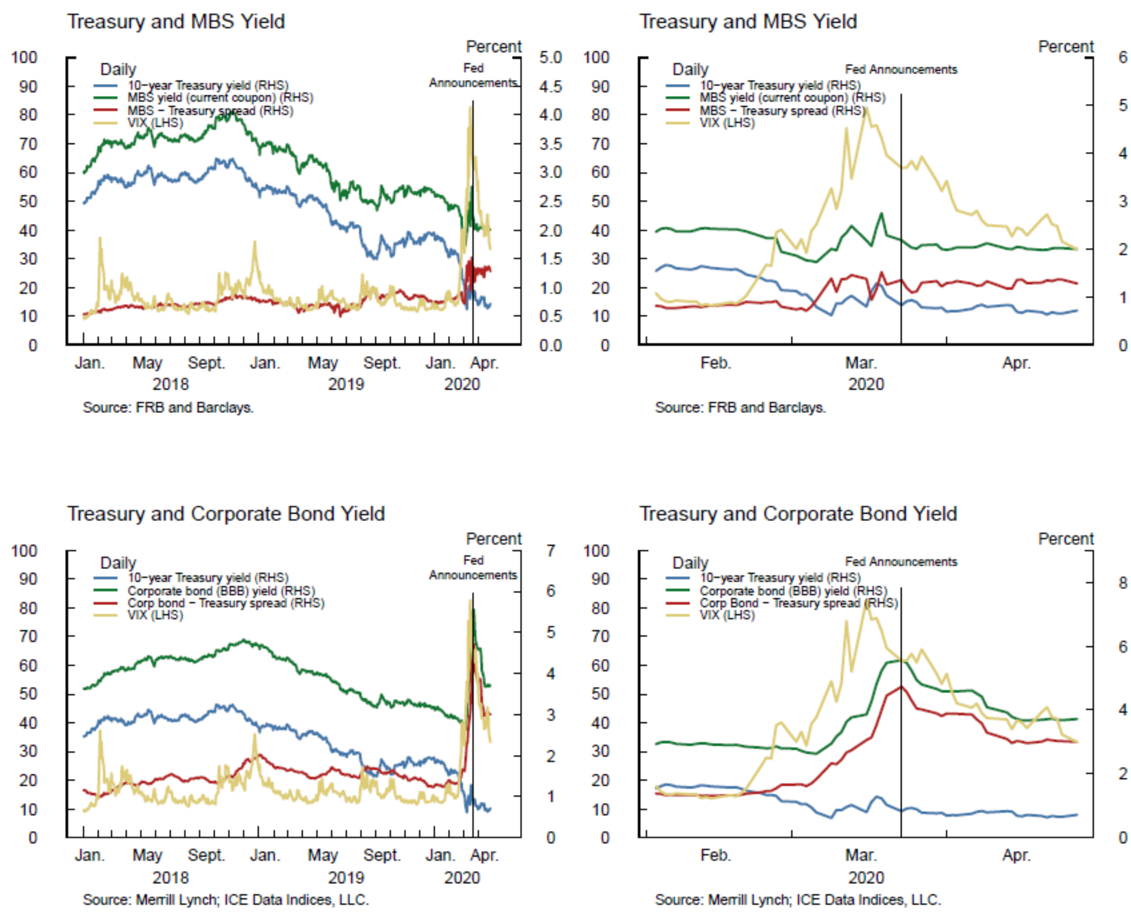


Figure A.1: Ten-Year Treasury, Agency MBS, and Investment-Grade Corporate Bond Yields; Spreads Relative to the Ten-year Treasury Yield and VIX Index
The top two panels show the Treasury and Agency MBS yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The bottom two panels show the Treasury and investment-grade corporate bond yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The tripwire indicates the date the Federal Reserve announced expanded asset purchases and new funding facilities on March 23, 2020.

B Proofs

Proof of Proposition 4

From Theorem 1's hypothesis, we know that equation (18) holds, thus $\overline{y} - \frac{\Theta_0^{Sh}}{2} \leq (1+p_1)\frac{\Theta_0}{2} + p_1\frac{\Theta_0}{2} + \alpha p_1^a\frac{\hat{\Theta}_0}{2}$, and form the Proposition's hypothesis, $\Theta_0 \leq \alpha\frac{p_1^a}{p_1}\hat{\Theta}_0$. Thus, from equation (26), we have

$$\begin{aligned}
\frac{\partial c_{1R}^s}{\partial\sigma^2} &\leq \frac{1}{(1+p_1)}\alpha\frac{p_1^a}{p_1}\frac{3}{2}\hat{\Theta}_0\frac{\partial p_1}{\partial\sigma^2} - \frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2} \\
&= \frac{\alpha}{(1+p_1)}\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)p_1\hat{\Theta}_0\left[\frac{3}{2}\frac{p_1^a}{p_1}\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho,\right]
\end{aligned}$$

where the second equality replaces the expression for $\frac{\partial p_1}{\partial\sigma^2}$. Replacing the expression for p_1^a/p_1, the term in the square bracket is bounded by

$$\frac{3}{2}\rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho \quad\leq\quad \frac{3}{2}\rho\left[\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - \frac{5}{3},\right]$$

where we use the Hansen-Jagannathan bound. Thus, for $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small, we have, $\frac{\partial c_{1R}^s}{\partial\sigma^2} \leq 0$.

Finally, we have to show that $\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} \leq 0$. From the expression in equation (26), we have

$$\begin{aligned}
\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} &= \frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2} - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right] \\
&= \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)\frac{\gamma}{4}(1+\rho\hat{\Theta}_0) - (1+p_1)\right] \\
&\leq \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\frac{1}{2}\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1\right) - (1+p_1),\right]
\end{aligned}$$

also using the Hansen-Jagannathan bound. Thus, for $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small we have, $\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} \leq 0$.

■

Proof of Lemma 2

Invoking the implicit function theorem, we have

$$\begin{pmatrix} \frac{\partial x_R}{\partial\Theta_0} \\ \frac{\partial p_0^a}{\partial\Theta_0} \end{pmatrix} = -\underbrace{\begin{bmatrix} \frac{\partial T_1}{\partial x_R} & \frac{\partial T_1}{\partial p_0^a} \\ \frac{\partial T_2}{\partial x_R} & \frac{\partial T_2}{\partial p_0^a} \end{bmatrix}^{-1}}_{:=D^{-1}} \begin{pmatrix} \frac{\partial T_1}{\partial\Theta_0} \\ \frac{\partial T_2}{\partial\Theta_0} \end{pmatrix}$$

We first have to characterize the partial derivatives with respect to the endogenous variables. These are

$$\frac{\partial T_1}{\partial x_R} = C''(x_R); \qquad \frac{\partial T_1}{\partial p_0^a} = -1; \qquad \frac{\partial T_2}{\partial x_R} = -\frac{\partial p_0^a}{\partial\hat{\Theta}}\frac{\partial\hat{\Theta}}{\partial x_R}; \qquad \frac{\partial T_2}{\partial p_0^a} = 1$$

where with a slight abuse of notation, $\frac{\partial p_0^a}{\partial\hat{\Theta}_0}$ is the partial derivative of the $t=0$ price of the private asset in the original model. Specifically,

$$
\begin{aligned}
\frac{\partial p_1^a}{\partial \hat{\Theta}_0} &= -\gamma p_1^a(\alpha p_0^{rf}+CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0}+(p_0^{rf}+\alpha CY)\frac{\partial p_1^a}{\partial \hat{\Theta}_0}+\gamma p_1^a(p_0^{rf}+\alpha CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \\
&= -\gamma p_1^a(\alpha p_0^{rf}+CY)\left[-\frac{\gamma p_1^a}{2(1+p_1)^2}\left[\overline{y}-\frac{\Theta_0^{Sh}}{2}+\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]-\frac{\gamma\alpha p_1}{2(1+p_1)}\rho^2\sigma^2\frac{\hat{\Theta}}{2}+\frac{\alpha p_1^a}{2(1+p_1)}\right] \\
&\quad +(p_0^{rf}+\alpha CY)\left[-\frac{\gamma}{2}\rho\left(\mu-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)p_1^a-\frac{\gamma}{2}\rho^2\sigma^2+\gamma p_1^a\left(\frac{\rho Y_0}{2}-C'(x_R^*)\right)\right]
\end{aligned}
$$

Therefore, we have that

$$|D| = C''(x_R) - 2\frac{\partial p_0^a}{\partial \hat{\Theta}}$$

Inspecting the derivatives of exogenous variables, note that $\frac{\partial T_1}{\partial z}=0$ and $\frac{\partial T_2}{\partial z}=-\frac{\partial p_1^a\left(p_0^{rf}+\alpha CY\right)}{\partial z}$ is merely the partial equilibrium sensitivities characterized by the model without endogenous safe asset creation giving the Lemma's result.

Proof of Proposition 7

The proof of $\frac{\partial CY}{\partial \sigma^2}>0$ is a direct consequence of Lemma 1 and Proposition 4.

The expression for $\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2}$ comes from taking the derivative of $\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2}$ then using Proposition 2. Thus, using the expression in equation (26) and in the proof of Proposition 4, we have

$$
\begin{aligned}
\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} &= \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{p_1^a}{(1+p_1)^2}\left[\overline{y}-\frac{\Theta_0^{Sh}}{2}+\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1 p_1^a\frac{\hat{\Theta}_0}{2}\right. \\
&\quad \left. -\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\} \\
&\geq \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{p_1^a}{(1+p_1)}\left[\frac{\Theta_0}{2}+\alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1 p_1^a\frac{\hat{\Theta}_0}{2}\right. \\
&\quad \left. -\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\} \\
&\geq \gamma p_0^{Sh}\frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{\alpha p_1^a}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[2\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]-\left[\frac{1}{(1+p_1)}\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\}\geq 0
\end{aligned}
$$

where we used the condition in (8) and the fact that $\alpha p_1^a\hat{\Theta}_0 > p_1\Theta$. Because of the Hansen-Jagannathan bound, the term accompanying $\frac{p_1^a}{p_1}\frac{\partial p_1}{\partial\sigma^2}$ can be made arbitrarily small. Because $\alpha p_1^a\hat{\Theta}_0 < 2$, we have the result.

■

In order to put some discipline on the model, it is important to choose parameters that satisfy then Hansen-Jagannathan bound. The following Lemma characterizes the Hansen-Jagannathan bounds in period $t=1$ of the model.

Lemma 3. *The Hansen-Jagannathan bounds for the pricing in $t=1$ is given by*

$$\left|\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\right| \leq \exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}-1.$$

Proof. Given the optimal consumption paths in (15) and (16), the stochastic discount factor is

$$\tilde{S} = \beta\exp\left\{-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\tilde{Y}_2\right\}$$

and the $t=1$ prices for the for the risk free and risky asset ($\tilde{a}_2 = \rho\tilde{Y}_2$) is,

$$\begin{aligned} p_1 &= \mathbb{E}(\tilde{S}) \\ &= \beta \exp\left\{-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\mu + \frac{1}{8}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} \\ p_1^a &= \mathbb{E}(\tilde{S}\rho\tilde{Y}_2) \\ &= \rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)p_1. \end{aligned}$$

which written in terms of excess returns implies that $\mathbb{E}\left(\tilde{S}\left(\frac{\rho\tilde{Y}_2}{p_1^a} - \frac{1}{p_1}\right)\right) = 0$. Therefore, the Hansen-Jagannathan bound requires that

$$\left|\mathbb{E}(\tilde{S})\mathbb{E}\left(\frac{\rho\tilde{Y}_2}{p_1^a} - \frac{1}{p_1}\right)\right| \leq \mathbb{V}(\tilde{S})\mathbb{V}\left(\frac{\rho\tilde{Y}_2}{p_1^a}\right)$$

where

$$\begin{aligned} \mathbb{V}(\tilde{S}) &= \mathbb{E}(\tilde{S}^2) - \mathbb{E}(\tilde{S})^2 \\ &= \beta^2\mathbb{E}\left(\exp\{-\gamma(1+\rho\hat{\Theta}_0)\}\tilde{Y}_2\right) - p_1^2 \\ &= \beta^2\mathbb{E}\left(\exp\left\{-\gamma\left(1+\rho\hat{\Theta}_0\right)\mu + \frac{1}{2}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}\right) - p_1^2 \\ &= p_1^2\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1\right) \end{aligned}$$

Thus, the bound can be rewritten as

$$\left|\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\right| \leq \exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1. \tag{B.1}$$

□

C Alternative Government Tax Schemes

In this appendix, we explore the impact of implementing different government policies to tax agents. The motivation is to capture the interaction between altering agents' intertemporal smoothing through taxation and their risk sharing. We show that, if taxes can change the path of consumption, it may have valuation effects on collateral, which affects risk sharing.

Specifically, because the government is the only agent in the economy that can store wealth by raising and holding funds through bond issuance and repayment, it can directly alter agents' consumption paths. In this sense, the role of the government is to store agents' wealth for future periods, as their bonds are the only (safe) way agents can carry wealth from one period to the next, and choose how much agents can transfer. This assumption can be interpreted as a shortcut to the assumption that the government has access to markets that the agents cannot, such as foreign investors.

Therefore, in this case, we can write Raymond's consumption processes—which is identical to Shirley's—in each period as

$$\check{c}_{0R} = e_{0R} + a_0\frac{\hat{\Theta}_0}{2} - \check{p}_0\theta_{0R} - \check{p}_0^{Sh}\theta_{0R}^{Sh} - \check{p}_0^a\left(\hat{\theta}_{0R} - \frac{\hat{\Theta}_0}{2}\right) + q^r w_R^r + q^s w_R^s + \frac{T_0}{2} \tag{C.2}$$

$$\tilde{\check{c}}_{1R} = \tilde{e}_{1R} + \tilde{a}_1\hat{\theta}_{0R} - \tilde{\check{p}}_1(\theta_{1R} - \theta_{0R}) + \theta_{0R}^{Sh} - \tilde{\check{p}}_1^a(\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s + \frac{T_1}{2} \tag{C.3}$$

$$\tilde{\check{c}}_{2R} = \tilde{e}_{2R} + \tilde{a}_2\hat{\theta}_{1R} + \theta_{1R} + \frac{T_2}{2}, \tag{C.4}$$

where we have used $\check{p}_t^{Sh}, \check{p}_t$ and $\check{p}_t^a$ for the equilibrium prices for the short-, long-term bond, and private asset, respectively.[36] In this case, T_0, T_1, and T_2 are aggregate lump sum transfers to agents (negative values are taxes). Note that these consumption equations are identical to the original model (equations (2)–(4)), except that the government returns what it raises (plus interest) when short- and long-term government bonds mature and manages its transfers to agents to balance its budget.

We assume that the government must have enough funds to make payments intertemporally. That is, in each period, the government must have enough funds to make final bond payments and transfers. Specifically,

$$\begin{array}{lrl} t=0: & T_0 & \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \\ t=1: & \Theta_0^{Sh} + T_1 + T_0 & \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \\ t=2: & \Theta_0^{Sh} + \Theta_0 + T_0 + T_1 + T_2 & = \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 \end{array}$$

where the last equality ensures that the government has to balance its aggregate budget in $t = 2$. These inequalities imply that the government uses its storage technology to transfer aggregate consumption from one period to the next, but it must be able to fulfill its promises in each period. To simplify the analysis, we will restrict the governments choice set by assuming that the government fully balances its budget in $t = 1$. This implies that $T_2 = -\Theta_0$, $T_0 = \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0 - T_1 - \Theta_0^{Sh}$, and thus the initial financing constraint implies that $T_0 \leq \check{p}_0^{Sh}\Theta_0^{Sh} + \check{p}_0\Theta_0$.[37]

Thus, in absence of idiosyncratic shocks (i.e., $\overline{y} = 0$), the direct impact of the government's issuance and tax policy is on how it affects the cost to transfer wealth from one period to the next.

It is easy to show that in this context, under assumptions A1 and A3, Raymond and Shirley's optimal portfolios are just as in the original model. Specifically, Raymond (Shirley) sells rain (shine) insurance to Shirley (Raymond); and agents hold half of the private asset supply in all periods, half of the government's issuance in $t = 0$, and rebalance their long-term government bond holdings in $t = 1$ to smooth their idiosyncratic risk exposure. Incorporating the government's tax plan, as a function of T_1, agents optimal consumption is

$$\begin{aligned} \check{c}_{0R} &= \frac{Y_0}{2} + a_0\frac{\hat{\Theta}_0}{2} - \frac{\Theta_0^{Sh}}{2} - \frac{T_1}{2} \\ \check{c}_{1R}^r &= \frac{(\overline{y}-w)}{(1+\check{p}_1)} + \frac{\Theta_0^{Sh}}{2} + \frac{T_1}{2}; \qquad \check{c}_{1R}^s = -\frac{(\overline{y}-w)}{(1+\check{p}_1)} + \frac{\Theta_0^{Sh}}{2} + \frac{T_1}{2} \\ \tilde{\check{c}}_{2R}^r &= \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}; \qquad \tilde{\check{c}}_{2R}^s = \frac{\tilde{Y}_2}{2} + \tilde{a}_2\frac{\hat{\Theta}_0}{2} - \frac{(\overline{y}-w)}{(1+\check{p}_1)}. \end{aligned}$$

In this version of the model, optimal consumption in $t = 1$ has a component attributed to idiosyncratic risk and a component attributed to the government's tax scheme. Thus, in contrast to the original model, changes in c_{1R}^s do not merely reflect the degree of risk sharing.

The equilibrium $\check{p}_t^{Sh}, \check{p}_t$ and $\check{p}_t^a$ take the same functional form as (5)–(6) for $t = 1$ and (9)–(11) for $t = 0$, however the final expressions will differ because of agents' new optimal consumption paths.

$$\begin{aligned} \check{p}_1 &= \beta\mathbb{E}_1\left(\exp\left\{-\frac{\gamma}{2}\left[\left(1+\rho\hat{\Theta}_0\right)\tilde{Y}_2 - T_1 - \Theta_0^{Sh}\right]\right\}\right) \\ &= \beta\exp\left\{-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\mu + \frac{1}{8}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2 + \frac{\gamma}{2}\left(T_1+\Theta_0^{Sh}\right)\right\} \\ &= p_1\exp\left\{\frac{\gamma}{2}\left(T_1+\Theta_0^{Sh}\right)\right\} \qquad (C.5) \\ \check{p}_1^a &= \beta\mathbb{E}_1\left(\exp\left\{-\frac{\gamma}{2}\left[\left(1+\rho\hat{\Theta}_0\right)\tilde{Y}_2 - T_1 - \Theta_0^{Sh}\right]\right\}\rho\tilde{Y}_2\right) \\ &= \rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\check{p}_1. \qquad (C.6) \end{aligned}$$

[36] To alleviate excessive notation, all other variables in this model extension take the same form.
[37] This set up nests the original model, where $T_0 = p_0^{Sh}\Theta_0^{Sh} + p_0\Theta_0$ and $T_1 = -\Theta_0^{Sh}$.

That is, prices in this model are proportional to the prices in the original ones but scaled by the relative distortion from the government's tax policy. A larger lump sum transfer in $t = 1$ increases consumption in $t = 1$, and thus increase the need for intertemporal smoothing between $t = 1$ to $t = 2$, putting upward pressure on $t = 1$ prices.

It is easy to check that the same arguments in the proof of Theorem 1 still hold, thus, if $\overline{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4}]$, $\beta > \frac{1}{2}$, and $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small, then there exist a symmetric equilibrium.

Moreover, the convenience yield takes the same functional form as before. Thus, the effect of the governments alternate tax policy on the $t = 0$ prices can be expressed as

$$\begin{aligned}
\check{p}_0^{Sh} &= (\check{p}_0^{rf} + \check{CY}) = p_0^{Sh}\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\} \\
\check{p}_0 &= \check{p}_1(\check{p}_0^{rf} + \check{CY}) = p_0\exp\left\{-\frac{\gamma}{2}\left(T_1 + \Theta_0^{Sh}\right)\right\} \\
\check{p}_0^a &= \check{p}_1^a(\check{p}_0^{rf} + \alpha\check{CY}) = p_0^a\exp\left\{-\frac{\gamma}{2}\left(T_1 + \Theta_0^{Sh}\right)\right\},
\end{aligned}$$

where $\check{CY} = CY\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and $\check{p}_0^{rf} = p_0^{rf}\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ are the convenience yield and the price of the risk-free security in absence of idiosyncratic risk.

The effect of different tax policies in $t = 0$ is the opposite to what happens in $t = 1$. As lump sum transfers in $t = 1$ increases, there is more consumption in $t = 1$ and less in $t = 0$. In this case, the government is effectively forcing agents to save more, making it less attractive to do so, putting downward pressure on prices.

Thus, the equilibrium in the case of alternative tax plans are the same as in the original model, scaled by the direct effect of said tax plan. This implies that the comparative statics of all non-governmental variables are as before, scaled by the tax distortion. The only important difference are the comparative statics with respect to the government's $t = 1$ lump sum tax decision. These decisions not only have an effect on agents' consumption smoothing across time, but also on the amount of risk sharing. Specifically, we have

$$\frac{\partial\check{c}_{1R}^s}{\partial T_1} = \underbrace{\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right]}_{\text{Valuation Effect}} + \frac{1}{2}$$

and $\frac{\partial\check{c}_{0R}}{\partial T_1} = -\frac{1}{2}$. The direct effect due to changes in agents' consumption smoothing is capture by the last term: $1/2$. The effect on agents' risk sharing comes through a pure valuation effect: an increase in T_1 increases the price of the long-term bond and the private asset, augmenting agents' ability to hedge idiosyncratic risks. This leads to the following result,

Proposition 8. *Given the equilibrium characterized in Theorem 1 with an alternative tax plan, the initial prices of the short-term government bond, long-term government bond, and private asset have the following comparative statics with respect to* T_1*,*

$$\begin{aligned}
\frac{\partial\check{p}_0^{Sh}}{\partial T_1} &= -\gamma(\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right] + 1\right) \\
\frac{\partial\check{p}_0}{\partial T_1} &= -\gamma\check{p}_1(\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right] + \frac{1}{2}\right) \\
\frac{\partial\check{p}_0^a}{\partial T_1} &= -\gamma\check{p}_1^a(\alpha\check{p}_0^{rf} + \check{CY})\left(\frac{\gamma}{2(1+\check{p}_1)}\left[\check{p}_1\frac{\Theta_0}{2} + \alpha\check{p}_1\frac{\hat{\Theta}_0}{2} + \frac{(\overline{y}-w)}{(1+\check{p}_1)}\check{p}_1\right]\right) - \frac{\gamma}{2}\check{p}_1^a(\check{p}_0^{rf} + \alpha\check{CY})
\end{aligned}$$

Proof of Proposition

The result comes from noting that $\check{CY} = CY\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and $\check{p}_0^{rf} = p_0^{rf}\exp\left\{-\gamma\left(T_1 + \Theta_0^{Sh}\right)\right\}$ and observing that $\frac{\partial\check{c}_{1R}^s}{\partial T_1} = \frac{\partial c_{1R}^s}{\partial T_1} + \frac{1}{2}$ (where CY, p_0^{rf}, and c_{1R}^s are as in the original model), and applying Lemma 1.

■

The increase in lump sum transfers in T_1 unequivocally makes all assets less valuable in $t = 0$. The direct effect is an increase (decrease) in aggregate consumption in $t = 1$ ($t = 0$), which reduces the need to transfer wealth from $t = 0$ to $t = 1$ and thus reduces $t = 0$ prices. This effect is somewhat muted by the increase in $t = 1$ prices, which affects both long-term bonds and private assets. These effects are somewhat mechanical and well understood. The novel change is the valuation effect on risk sharing. By making prices higher in $t = 1$, assets are more pledgeable, allowing for more risk sharing, making the assets less valuable in $t = 0$.

This indicates that the government can improve the amount of risk sharing by either altering the amount of government bonds or how they pay for them. An important element in these results is that the agents cannot transfer resources intertemporally to undo the effects of government taxes. The only way agents can react is through their demand for government bonds, thus affecting their price and the valuation effect. While the power of the government to change the path of consumption may seem extreme, this assumption should be taken as capturing incomplete markets, or other frictions in which taxation affects paths of consumption in equilibrium.

D Direct Effect of Aggregate Volatility on the Extent of Idiosyncratic Risk

In this appendix, we explore the case in which an increase in volatility also increases the level of agents' idiosyncratic risk. That is, the level of future volatility in $t = 2$ affects the magnitude of the idiosyncratic shock in $t = 1$,

$$\hat{y} = \overline{y} + \eta\sigma^2.$$

This specification captures the notion that higher future volatility may increase the need for idiosyncratic risk sharing, captured by the parameter η. For σ (or η) sufficiently small enough, $\hat{y}$ satisfies the conditions of Theorem 1, ensuring there exists an equilibrium. [38] In this version of the model prices in $t = 1$ are as in the original one (equations (19) and (20)), because in the case without wealth effects (CARA utility) the realization of the idiosyncratic shock does not change prices.

Denoting the equilibrium consumption processes by $\hat{c}_{it}$, from equation (15) we have that

$$\hat{c}^s_{1R} = c^s_{1R} - \frac{\eta\sigma^2}{1+p_1}.$$

Therefore, from equation (26), we have that

$$\frac{\partial \hat{c}^s_{1R}}{\partial \sigma^2} = \frac{\partial c^s_{1R}}{\partial \sigma^2} - \frac{\eta}{(1+p_1)} \underbrace{\left[1 - \frac{\sigma^2}{(1+p_1)}\frac{\partial p_1}{\partial \sigma^2}\right]}_{>0, \text{ for } \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma \text{ small.}} \tag{D.7}$$

$$= \frac{1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{p_1^a}{p_1}\frac{\hat{\Theta}_0}{2} + \eta\sigma^2\right]\frac{\partial p_1}{\partial \sigma^2}$$
$$-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2} - \frac{\eta}{(1+p_1)} \tag{D.8}$$

The expression in equation (D.7) confirms that if the size of the idiosyncratic shock is proportional to aggregate volatility, then an increase in aggregate volatility increases decreases the amount of risk sharing. Thus, under the conditions of Proposition 4, if there are more private assets used as collateral, an increase in volatility would lead to an even large reduction in risk sharing.

[38] Specifically, that $\hat{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4}]$.

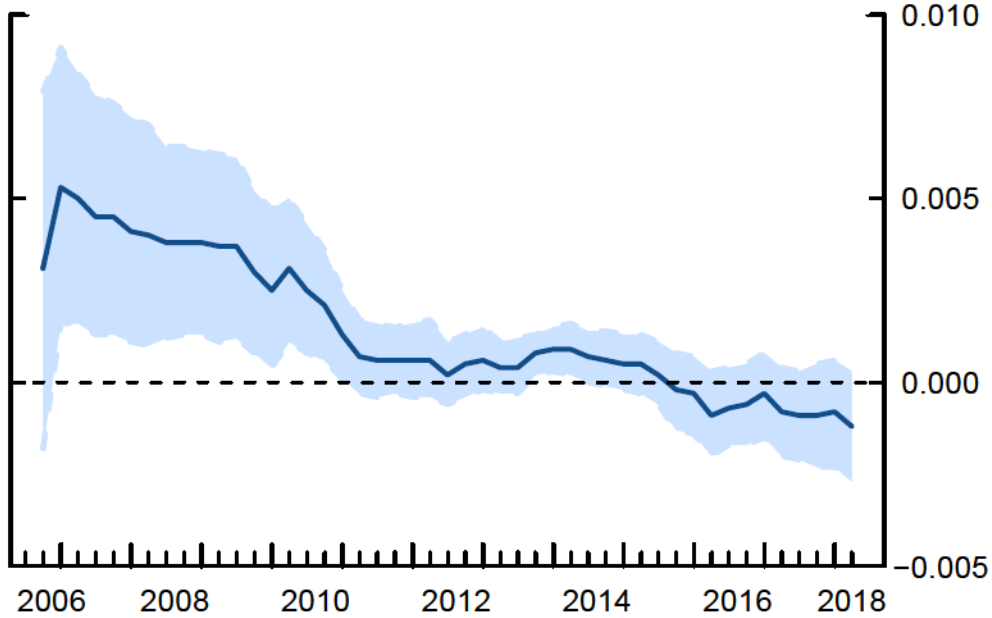


Figure E.2: One-Day Sensitivity of ΔCY_t to ΔVIX_t

The solid line shows the point estimate of the 1-day estimation of fully model in equation (36) using daily data and ± 2 years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.

This model extension we directly characterizes the intuition behind the alternative explanation of the empirical results in Section 5. Specifically, following the same steps as in Proposition 7, in this model the sensitivity of of the convenience yield is given by

$$\frac{\partial CY}{\partial \sigma^2} = -\gamma p_0^{Sh} \frac{\partial \hat{c}_{1R}^s}{\partial \sigma^2}.$$

Thus, for a high enough sensitivity of idiosyncratic risk to aggregate volatility, η, the convenience yield is increasing in future volatility.

E Additional Empirical Analysis — Daily Frequency and European Repo Markets

In this appendix, we show the results for the same empirical strategy described in section 5.3, but use one-day differences rather than five-day differences. This appendix also shows the results from estimating the main empirical model in German bond markets. The results in Table E.1 are qualitatively similar to those in Table 3. There is a positive and statistically significant relationship between changes in the convenience yield and changes in the VIX in the early part of the sample, before the GFC. After the GFC, the relationship loses its statistical power. The results in Figure E.2 are qualitatively similar to those in Figure 2.[39] Sensitivity of ΔVIX_t on CY_t is positive and statistically significant toward the end of 2006 and loses significance thereafter.

[39]The scales on both figures are the same to simplify the comparison.

Table E.1: Volatility versus Convenience Yield Pre- and Post- 2009

	Pre- 2009	Post- 2009	Pre- 2009	Post- 2009	Pre- 2009	Post- 2009
$\Delta FedFunds_t$	0.025	-0.040*	0.027	-0.038*	0.016	-0.038*
	(0.039)	(0.022)	(0.041)	(0.023)	(0.038)	(0.022)
$FedFunds_{t-1}$	0.001	-0.001	0.001	-0.001	0.001	-0.001
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
$\Delta log(ShortTBillsOut_t)$	-0.171***	-0.052***	-0.169***	-0.051***	-0.171***	-0.051***
	(0.032)	(0.009)	(0.031)	(0.009)	(0.030)	(0.009)
$\Delta log(USTNotesOut_t)$	-1.176	-0.032	-1.135	-0.032	-1.188	-0.008
	(1.591)	(0.246)	(1.589)	(0.247)	(1.548)	(0.246)
$\Delta VIX_t \times \Delta log(ShortTBillsOut_t)$			0.025	0.001	0.026	0.001
			(0.025)	(0.006)	(0.025)	(0.006)
ΔVIX_t					0.005***	-0.001
					(0.002)	(0.001)
P-value	0.001	0.000	0.001	0.000	0.001	0.000
Adj RSq	0.122	0.150	0.124	0.144	0.135	0.145
N obs	765	1948	765	1945	765	1945

Note: This table shows the empirical results of equation (36) using daily data. The convenience yield measure is the spread between the 1-month overnight index swap rate and the 4-week Treasury bills rate. ΔVIX_t is the first difference of the VIX Index, $\Delta FedFunds_t$ is the first difference of federal funds rate, and $FedFunds_{t-1}$ is the 1-day lag of the federal funds rate. $\Delta log(ShTbillsOut_t)$ is the log difference of Treasury bills outstanding with maturity less than one month, and $\Delta log(USTNotesOut_t)$ is the log difference of total U.S. Treasury notes and bonds outstanding. Four lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to April 2020. Estimates exclude quarter-end dates. The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table E.2: EU Volatility and German Convenience Yield Pre- and Post- 2009

	Pre- 2009	Post- 2009
ΔVIX_t	-0.001	0.000
	(0.002)	(0.001)
P-value	0.036	0.000
Adj RSq	0.037	0.066
N obs	334	1943

Note: This table shows the empirical results of equation (36) using overlapping daily data. ΔCY_t is the 5-day first difference of the spread between the 3-month Euro OverNight Index Average (EONIA) swap rate and the 3-month German T-bill rate. ΔVIX_t is the 5-day first difference of the Euro Stoxx 50 volatility index. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. Estimates exclude quarter-end dates (and ± 2 days surrounding quarter-end). The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.