Business, Liquidity, and Information Cycles

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Abstract

Stock markets play a dual role: improve capital allocation across firms by conveying information about their fundamentals and provide liquidity to traders by quickly turning stocks into cash. We propose a trading model in which these two roles are endogenously related: if stocks are used more intensively for liquidity, then prices reveal less information. We structurally estimate stock price informativeness for several countries and show that it declines when alternative liquidity sources, such as the banking system, are in distress. To study the real effect of this mechanism, we devise a strategy to integrate our trading module into a dynamic general equilibrium model with heterogeneous firms. We calibrate the model to the US and show that in recessions, prices become more informative and allocation improves, but only if alternative sources of liquidity function normally. Otherwise, traders rely more on stock markets for liquidity, prices become less informative, allocation deteriorates, and the output loss is 43% larger.

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1 Introduction

Stock markets play two critical roles in market economies: allocating resources and providing liquidity. The former comes from the remarkable ability of markets to aggregate information about firms' prospects that is dispersed among traders, usually referred to as *stock price informativeness*. How much agents rely on stock prices to allocate their funds across investment opportunities depends on how informative the prices are. The latter, commonly referred to as *market liquidity*, comes from the easiness of agents buying or selling stocks to satisfy their liquidity needs. How much agents rely on stock prices to obtain liquidity depends on how well financial intermediaries provide liquidity by facilitating credit, commonly referred to as *funding liquidity*.

Despite a rich literature discussing the allocation and liquidity roles of stock markets separately, their interaction is less understood. Does the use of stocks for liquidity enhance or weaken their role in providing information? How does distress in other sources of liquidity, such as the banking sector, affect price informativeness? How does the information content of stock prices vary with business cycles? How do banking crises and other credit market distress contribute to the depth of a recession?

In this paper, we make progress on three fronts. The first is *theoretical*, by building a model of asset trading and information acquisition where both the information and the noise about firms' fundamentals contained in asset prices are partly determined by how much agents use stocks to access liquidity. We then incorporate this module into a dynamic general equilibrium setting with heterogeneous firms where stock price informativeness impacts input allocation and investment. The second front is *empirical*, by structurally estimating stock price informativeness from firm-level panel data for several countries and establishing its cyclical properties. The last front of progress is *quantitative*, by calibrating the general equilibrium model to quantify the role of price informativeness on capital allocation in recessions, with and without funding liquidity distress.

To be more precise about these fronts, our theoretical contribution hinges on extending the seminal insight of Grossman and Stiglitz (1980): the participation of noise traders prevents prices from perfectly revealing available information, hence inducing investors to acquire information at a cost, with prices partially revealing such information. To accommodate the two main roles of stock markets, we instead allow for two types of traders

-day and night- interested in different asset properties -liquidity and fundamentals,- respectively. This structure dispenses from noise traders and creates *endogenous noise* in prices: a high price may indicate that the stock can be easily traded or that the firm has strong fundamentals. The trades from one type of trader act as noise for the other, inducing them to decide whether to acquire information and ultimately determining how much information is conveyed in prices. We show that, in equilibrium, (1) a linear pricing function exists where the relative weights of information on the asset's payoff and liquidity determine price informativeness, and (2) these weights are given by how many day and night traders operate in the market, their information choices, and how aggressively they trade on their information.

We then incorporate this trading module into a real business cycle model with heterogeneous firms. We do this by providing a link between financial and real markets where the linear pricing function is preserved in a non-linear production economy, such that the model remains tractable. Stocks are claims to firms' earnings, which depend on firms' idiosyncratic productivities (i.e., firms' fundamentals). If those productivities were known, investors would allocate capital across firms efficiently. In our model, those productivities are ex-ante unknown, and allocations are based on the investors' best guesses based on stock prices, which are noisy given the participation of day traders. When funding markets malfunction, traders rely more on stocks to access liquidity, price informativeness declines, and real activity gets affected through the ensuing misallocation of resources across firms.

We define price informativeness as how well prices reveal the best available information about firms' profitability. We show that this measure captures the risk investors face when using a stock price as an estimator for the firm's productivity; hence, it is inversely proportional to the extent of misallocation in the economy. Our structural setting allows us to disentangle the components of price informativeness, which are related to the dispersion of both firm productivity and stock liquidity and to their price loadings. Thus, we can trace the fluctuations in price informativeness to the fluctuations of these factors.

Our model delivers a linear relationship between stock prices, firms' earnings, and stock liquidity. We exploit this property on our empirical front. We show that price informativeness can be estimated structurally each year and in each country without having to solve for the full general equilibrium. We implement this structural estimation with panel data from 21 countries and show that price informativeness exhibits cyclicality, but more importantly, it substantially declines in periods of insufficient funding liquidity, such as the Great Recession and COVID-19 pandemic episodes.

Finally, in our quantitative contribution, we measure the relevance of stock price informativeness on the allocation of resources, total investment, and other macro aggregates. We calibrate the parameters of the full model to the United States, assuming the economy is subject to two, possibly correlated, aggregate shocks: one on aggregate productivity and one on funding liquidity. The shock structure is meant to capture recessions with and without distress in the banking sector. We use aggregate moments and moments of the estimated pricing functions to jointly discipline the cost of acquiring information and the dynamics of market liquidity needs while replicating the cyclical properties of the estimated US price informativeness.

Using the calibration, we conduct impulse-response exercises to simulate downturns, with and without distress in funding markets. We show that during recessions without funding liquidity distress, increased uncertainty induces traders to acquire more information, and a "cleansing recession" is observed. When the recession is accompanied by funding liquidity distress, however, the increased stock trading for liquidity purposes reduces the information content in stock prices despite traders acquiring more information. This drop induces a worse allocation of resources and discourages investment, leading to a "sullying recession." In quantitative terms, our exercise shows that an average recession with funding liquidity distress would result in an output decline that is 43% larger and an investment decline that is 56% larger than the same recession without funding liquidity distress. This result indicates a sizeable real effect of banking problems through damaging the allocative role of stock markets.

Based on our calibration, we also explore how the economy would fare when facing a recession with funding liquidity distress under alternative information structures. If information were exogenous, funding liquidity distress would reduce stock price informativeness more drastically, leading to larger misallocation and investment drops, with output declining 30% more than our benchmark with endogenous information. If the cost of information *about a firm's fundamentals* were to decline by half, price informativeness would decline less, leading to output declines that are about 15% smaller. If, instead, the cost of information *about a stock's liquidity* were to decline by half, price informativeness would decline more, leading to output declines that are about 15% larger. These results suggest that regulations facilitating information about firms' profitability make the economy more resilient to recessions with financial shocks. Yet, those facilitating information about stock markets' depth and volume do the opposite.

Literature Review: Our paper lies at the intersection of the literature on price informativeness and the literature on input misallocation. Also on this intersection are David et al. (2016) and David and Venkateswaran (2019), perhaps the closest to our study. The former focuses on the role of informational frictions in resource allocation and measures how much each source of information contributes to productivity gaps. The latter has a larger scope and incorporates many potential frictions that can distort resource allocation in addition to informational frictions. Both analyses provide static measures; hence, they are silent about cyclicality. We additionally focus on stock markets as the main source of information for allocation purposes, highlighting its endogeneity.[1]

We contribute more generally to the literature that analyzes input allocation across firms in recessions. Even though the evidence points to a decline of input *reallocation* during recessions, the extent of input *misallocation* is less clear.[2] We contribute by showing that the seemingly conflicting results should be qualified by whether a recession coincides with distress in financial markets. In doing so, we provide a mechanism that can account for Foster et al. (2016)'s finding that labor reallocation declined during the Great Recession while it improved during the previous US recessions since the 80s. Further, by quantitatively assessing the role of liquidity distress on misallocation, we provide a complementary channel to how financial markets affect business cycles: when banks are in distress, the informational and allocative role of stock markets weakens. Hence, our mechanism adds to the seminal literature on how financial problems magnify fluctuations such as Bernanke (1983), Bernanke and Gertler (1989), and Kiyotaki and Moore (1997).

We also contribute to both the theoretical and empirical literature on price informa-

[1] Goldstein (2023) provides a survey of the evidence about the relevance of stock prices in informing and affecting the decisions of managers, banks, credit rating agencies, and regulators. Also, even though their focus is not misallocation, Dow et al. (2017), Benhabib et al. (2019), and Chousakos et al. (2023) provide theoretical studies of the interactions between real and financial sectors in terms of learning and information acquisition.

[2] Alam (2020) documents a counter-cyclical dispersion of marginal product of capital (MPK) using balance sheet data from North America and Europe. See also Oberfield (2013) and Sandleris and Wright (2014), which document an increased dispersion of MPK during the 1982 Chilean crisis and the 2001 Argentine Crisis, respectively. Flynn and Sastry (2024) and Osotimehin (2019) claim the opposite: the former argues the US public firms' input choices become more careless and volatile, and the latter argues within-sector allocative efficiency among French firms declines in good times.

tiveness. On the theoretical front, the literature usually assumes an exogenous source of noise that prevents prices from being perfectly informative, following the impossibility theorem of Grossman and Stiglitz (1980). Instead, we endogenize the information/noise ratio by assuming two dimensions of information condensed at a single price. The closest papers to ours here are Stein (1987) and Vives (2014).[3] Both use heterogeneity in traders' characteristics (the former on market access and the latter on private valuations) to generate imperfectly informative prices without exogenous noise. We contribute to this work by (*i*) identifying changes in the liquidity role of stocks as a quantitatively relevant driver of endogenous 'noise,' (*ii*) characterizing price informativeness under dual-signal, and (*iii*) proposing a tractable integration with an RBC model where a linear pricing function exists. Furthermore, endogenizing noise yields a novel insight: regulations and innovations that make data on stock liquidity -relative to fundamentals- more accessible can hamper the allocation role of stock prices. This result complements Farboodi and Veldkamp (2020), who focus on uniform improvements in information acquisition technology.

On the empirical front, the literature has provided different strategies to measure price informativeness. Dávila and Parlatore (2018) use time-series regressions to measure price informativeness for each stock, which requires them to make assumptions on how model parameters change over time to keep the cross-sectional variation flexible. We, on the other hand, use cross-sectional regressions to measure the price informativeness of the stock market over time, which requires us to make assumptions on the extent of heterogeneity across stocks to allow parameters to change flexibly over time. Bai et al. (2016), similar to us, analyzes the long-run trend in price informativeness using cross-sectional regressions. They are, however, interested in the ability of prices to predict future earnings (what Bond et al. (2012) refers to as 'forecasting price efficiency'), which is determined jointly by prices' ability to communicate information *and* the quality of such information. In contrast to these papers, we are interested in how prices reflect available information at the time of making investments, *not* its quality (what Bond et al. (2012) refers to as 'revelatory price efficiency'). Lastly, we go beyond the US markets and estimate price informativeness across 21 countries, leveraging data from various crises worldwide.

The paper proceeds as follows. Section 2 introduces a stock trading model with endogenous noise and shows how to integrate it into an otherwise standard RBC model with firm heterogeneity. Section 3 describes the data sources and the empirical strategy

[3]See also Rahi (2021) and Banerjee et al. (2021) for theoretical exercises that expand on Vives (2014).

to compute stock price informativeness and its structural components. Then, it studies the cyclical properties of the estimated series and their relation with measures of funding liquidity distress. Section 4 calibrates the model to the United States. Section 5 uses the calibrated model to assess the quantitative relevance of liquidity distress and information costs on real variables through their impact on price informativeness. Section 6 concludes. All proofs are contained in Appendix A.

2 Model

In this section, we build a stock trading model and incorporate it into a real business cycle model with firm heterogeneity. A firm's stock price provides information on two dimensions: the firm's long-term profitability and the stock's short-term liquidity, which are valued differently by different types of traders. The trading activity of one type of trader masks the information from the other type. Since the real sector uses the stock price as a signal about a firm's long-term profitability for investment purposes, the degree of input misallocation in the economy depends on different trading needs and information production in stock markets. To keep the notation simple, we suppress time subscripts unless necessary.

2.1 Environment

Preferences The economy is populated by a measure one of the traders who live one period and a measure one of identical infinitely lived households.

Traders have constant-absolute-risk-aversion (CARA) preferences where the utility from consuming an amount W is given by $\nu(W) = -e^{-aW}$, with risk aversion $a > 0$. At the start of each period, a fraction γ of newborn traders ('day traders') need to consume early and resort to selling stocks to obtain such consumption, while the rest ('night traders') consume at the end of the day. Changes in the fraction γ are meant to capture changes in funding liquidity, i.e., changes in the ability of banks and other financial intermediaries to provide credit when agents need cash during the day. When funding markets are distressed, more traders need to resort to stocks for liquidity, and γ is larger.

The representative household has constant-relative-risk-aversion (CRRA) preferences with inter-temporal elasticity of substitution $1/\eta$ where the utility from consuming an amount W is given by $u(W) = \frac{W^{1-\eta}-1}{1-\eta}$.

Technology There is a measure one of firms (indexed by i) with profit function:

$$\Pi_i = z_{in}(\bar{K}_i + K_i) - r_i K_i - \xi K_i^2 / 2\bar{K}_i \tag{1}$$

where z_{in} is the productivity of firm i's capital, $\bar{K}_i$ is installed capital, and K_i is rented capital at a price r_i. While $\bar{K}_i$ cannot be changed or reallocated, K_i is determined period-by-period. The last term, a quadratic adjustment cost, introduces curvature to the profit function.

Assets, Endowments, and Market Structure There are three types of assets in the economy: capital, stocks, and foreign bonds.

Households invest in *capital* through a mutual fund and receive a return r. We assume households own the mutual fund, so the interest rate r distributes all the surplus to households. On the other hand, the mutual fund is a price-discriminating monopoly when transacting with firms. In particular, to each firm i, the mutual fund can make a take-it-or-leave-it offer $\{K_i, r_i(z_{in})\}$.[4]

Traders can access two types of assets: *foreign bonds* supplied at exogenous return r^F and firms' shares, i.e., *stocks*. Each share gives ownership of 1 unit of installed capital at the associated firm, making the outstanding share amount of firm i equal to $\bar{K}_i$. Each share i is subject to an exogenous liquidity discount, represented by a decline in the return by z_{id} if sold prematurely; that is, a stock with a high z_{id} is one with low liquidity. Changes in the liquidity discount z_{id} are meant to capture changes in market liquidity, i.e., changes in the losses for a day trader from selling an asset in the middle of the period. These can originate, for instance, from the fee a day trader needs to pay a broker to unload the stock before being able to sell it to newborn traders at the end of the period. Under this interpretation, z_{id} captures the broker's risks and other costs (in terms of expertise,

[4]Note that while K_i cannot be conditioned on z_{in}, the interest rate can be. The conditional interest rate implies that (i) while capital needs to be assigned before z_{in} is observed, the exact payment is determined later to extract all the ex-post surplus, and (ii) there is no default. This market structure guarantees that the profitability of holding a firm's stock is independent of the capital allocated to such a firm, as in Subrahmanyam and Titman (1999), which allows linear pricing function in equilibrium.

inventory, and information) of holding the stock i until the end of the period.

Information The aggregate productivity Z and the fraction of day traders γ define a state $s = \{Z, \gamma\}$, which is public information, and follow a Markov process with joint transition probability $q_{s,s'}$.

The profitability of the firm (z_{in}) consists of three parts: the aggregate productivity shock, a random term that can be learned (θ_{in}), and a random term that cannot be learned ($\tilde{\varepsilon}_{in}$). The learnable component θ_{in} is drawn from a prior distribution $\mathcal{N}(\bar{\theta}_{in}, \sigma^2_{\theta_{in}})$, while the unlearnable component $\tilde{\varepsilon}_{in}$ follows an AR(1) process: $\tilde{\varepsilon}_{in} = \rho\tilde{\varepsilon}^-_{in} + \varepsilon_{in}$ with $\varepsilon_{in} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_{in}})$, where $\tilde{\varepsilon}^-_{in}$ is public information and $\sigma^2_{\varepsilon_{in}}$ is a measure of fundamental uncertainty.[5]

The liquidity discount of the stock (z_{id}) consists of two parts: a random term that can be learned (θ_{id}) and a random term that cannot be learned (ε_{id}). The first component is drawn from $\mathcal{N}(\bar{\theta}_{id}, \sigma^2_{\theta_{id}})$ and the second from a prior distribution $\mathcal{N}(0, \sigma^2_{\varepsilon_{id}})$.

To summarize the distributions of the profitability and liquidity of a firm,

$$\begin{aligned} z_{in} =& Z + \theta_{in} + \tilde{\varepsilon}_{in}, \\ & where \quad \theta_{in} \sim \mathcal{N}(\bar{\theta}_{in}, \sigma^2_{\theta_{in}}), \qquad \tilde{\varepsilon}_{in} = \rho\tilde{\varepsilon}^-_{in} + \varepsilon_{in}, \quad \& \quad \varepsilon_{in} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_{in}}), \\ z_{id} =& \theta_{id} + \varepsilon_{id}, \\ & where \quad \theta_{id} \sim \mathcal{N}(\bar{\theta}_{id}, \sigma^2_{\theta_{id}}) \quad \& \quad \varepsilon_{id} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_{id}}), \end{aligned} \tag{2}$$

where $\theta_{id}, \theta_{in}, \varepsilon_{id}$,and ε_{in} are independently distributed across firms and over time. We allow $\sigma^2_{\theta_{in}}$, $\sigma^2_{\theta_{id}}$, $\sigma^2_{\varepsilon_{in}}$, and $\sigma^2_{\varepsilon_{id}}$ to be functions of aggregate productivity Z.[6]

The fraction of $l \in \{d, n\}$ traders who choose to be informed about stock i is denoted with λ_{il}. Being informed implies for night traders paying $c(\lambda_{in})$ and for day traders paying $c(\lambda_{id})$ to learn $\theta = \{\theta_{id}, \theta_{in}\}$, where $c(\cdot)$ is increasing.[7] The random components ε_{id} and

[5] We capture persistent differences in idiosyncratic returns across stocks through $\tilde{\varepsilon}_{in}$, the component that cannot be learned. Farboodi and Veldkamp (2020), on the other hand, uses the learnable component to capture such dynamics. Unlike their setting, the future pricing function in our setting is stochastic, and the pricing function becomes nonlinear if there is any persistence in the learnable component.

[6] These volatilities are assumed to be only a function of productivity Z and not of liquidity needs γ for two reasons. First, liquidity needs are modeled as shocks on needs for market liquidity that should not affect fundamentals. Second, this choice allows isolating the role of stock markets in capital allocation.

[7] We impose the same cost function for day and traders here for simplicity and relax it when calibrating

ε_{in} are learned for free, but only after their realization. Finally, the mutual fund doesn't have access to this information technology, and similar to uninformed traders, it infers z_{in} purely from observing stock market prices.

Timing Each period starts with the realization of γ, Z and $\theta = \{\theta_{in}, \theta_{id}\}_{i=0}^{1}$. The timing proceeds as follows:

1. Day and night traders simultaneously choose whether to acquire information and buy stocks.
2. Households invest in the mutual fund, which allocates capital across firms after observing all stock prices.
3. $\varepsilon = \{\varepsilon_{in}, \varepsilon_{id}\}_{i=0}^{1}$ is realized, with both θ and ε becoming public information.
4. Day traders sell all their stocks to the broker at a discount z_{id}, consume, and die.
5. Production occurs, firms pay the mutual fund, and the fund pays households.
6. Night traders and the broker sell their stocks to newborn traders, consume, and die.

2.2 Agents' Problems and Market Clearing

Mutual Fund's Capital Allocation Across Firms Capital is allocated across firms to maximize total returns, which is equivalent to maximizing total welfare given the perfect diversification obtained by the households through the mutual fund. To be more precise, the mutual fund calculates the constrained efficient K_i for firm i according to

$$K_i = \max\left\{\bar{K}_i\left(\frac{E[z_{in}|p] - r}{\xi}\right), 0\right\} \quad \forall i, \tag{3}$$

the model. The cost function increasing in the fraction of informed traders rules out possible equilibrium multiplicity that may arise from complementarities in information acquisition. It can be motivated, for instance, by an upward-sloping supply curve for information (the marginal cost of information is increasing) or by heterogeneous information acquisition costs across traders, with the traders with the lowest cost acquiring the information first. An interior equilibrium exists if the marginal trader is indifferent between acquiring information at a cost or relying on prices for free.

then makes a take-it-or-leave-it offer with a contract $\{K_i, r_i(z_{in})\}$ that satisfies

$$z_{in}K_i - r_i(z_{in})K_i - \xi K_i^2/2\bar{K}_i = 0 \ \ \forall i, z_{in} > 0. \tag{4}$$

Hence, the mutual fund obtains all the ex-post surplus from the rented capital, and the profit of firm i becomes $z_{in}\bar{K}_i$. Night traders receive $z_{in} + p_i'$ for each share of firm i they hold. The first term represents the dividend, and the second represents the resale price. Due to the premature selling of the stock before production occurs, day traders receive $z_{in} - z_{id} + p_i'$.

In an environment where θ_{in} is observable, the fund would choose K_i based on $E[z_{in}|\theta_{in}]$. Hence, 'misallocation' would be due to ε_{in}, which is inevitable. In our setting, however, the mutual fund can only rely on the stock price p_i to infer θ_{in}, hence using $E\left[z_{in}|E[\theta_{in}|p_i]\right]$ to make decisions. For the choice of K_i, θ_{id} is irrelevant, yet a high stock price could stem from a high θ_{in} or a low θ_{id}. In summary, the existence of day traders prevents prices from perfectly revealing θ_{in}.

Traders' Portfolio Choice Problem Traders choose whether to acquire information and how much to demand of each asset, conditional on information or lack thereof. Information acquisition determines how traders form their expectations when making these choices. The portfolio problem of a night trader who is endowed with $\tilde{b}$ foreign bonds is

$$\begin{aligned} &\max_{B_n, \{X_{in}\}_{i\in[0,1]}} E\Big[- exp\Big[- a[(1+r^F)B_n + \int_i X_{in}\big(z_{in} + p_i' - p_i\big)di]\Big]\Big] \\ &s.t. \ \ B_n + \int_i p_i X_{in} di = \tilde{b}, \end{aligned} \tag{5}$$

and the portfolio problem for a day trader is

$$\begin{aligned} &\max_{B_d, \{X_{id}\}_{i\in[0,1]}} E\Big[- exp\Big[- a[(1+r^F)B_d + \int_i X_{id}\big(z_{in} - z_{id} + p_i' - p_i\big)di]\Big]\Big] \\ &s.t. \ \ B_d + \int_i p_i X_{id} di = \tilde{b} \end{aligned} \tag{6}$$

where B_l and $\{X_{il}\}_{i\in[0,1]}$ denote the demands for foreign bonds and stocks of traders $l \in \{d, n\}$. This is a standard portfolio rebalancing problem between safe assets (the foreign bond that pays a risk-free rate of r^F) and risky assets (stocks).

Given the distributional assumptions and the information structure, the stock demand functions by informed and uninformed, day and night, traders are,

$$X_{in}^{I*} = \frac{E[z_{in} + p_i'|\theta] - (1+r^F)p_i}{aVar[z_{in} + p_i'|\theta]} \qquad X_{id}^{I*} = \frac{E[z_{in} - z_{id} + p_i'|\theta] - (1+r^F)p_i}{aVar[z_{in} - z_{id} + p_i'|\theta]}$$
$$X_{in}^{U*} = \frac{E[z_{in} + p_i'|p_i] - (1+r^F)p_i}{aVar[z_{in} + p_i'|p_i]}, \qquad X_{id}^{U*} = \frac{E[z_{in} - z_{id} + p_i'|p_i] - (1+r^F)p_i}{aVar[z_{in} - z_{id} + p_i'|p_i]}, \tag{7}$$

where $X_{in}^{I*}, X_{id}^{I*}, X_{in}^{U*}, and X_{id}^{U*}$ represent the asset demand by night and day traders that are informed (I) and uninformed (U), respectively. Notice that the main difference is informed conditioning expectations on θ and uninformed on p.

Traders' Information Acquisition Problem Let's denote the information acquisition decision of trader j for asset i with $I_{ji} \in \{0,1\}$, and the end-of-period wealth for a night trader j with $W_{nj}(I_j)$. Then,

$$\begin{aligned} W_{nj}(I_j) =& (1+r^F)\left(W_{0j} - \int_i I_{ji} c(\lambda_{in}) di\right) + \\ & \int_i \left(z_{in} + p' - (1+r^F)p_i\right)\left(I_{ji} X_{in}^{I*} + (1-I_{ji}) X_{in}^{U*}\right) di \end{aligned} \tag{8}$$

Let I_j^1 be an arbitrary information acquisition vector where $I_{ji}^1 = 0$ for a specific stock i and I_j^2 be an identical vector, except that $I_{ji}^2 = 1$. The trader would choose I_j^2 over I_j^1, i.e., acquire information about stock i, if and only if $E\left[V\left(W_{nj}(I_j^2)\right) \mid p\right] \geq E\left[V\left(W_{nj}(I_j^1)\right) \mid p\right]$ where p is the vector of stock prices. The same comparison holds for day traders.

Lemma 1. *Let I_j^1 be an arbitrary information acquisition vector where $I_{ji}^1 = 0$ for a specific stock i and I_j^2 be an identical vector, except that $I_{ji}^2 = 1$. For night traders,*

$$\frac{E\left[V\left(W_{nj}(I_j^2)\right) \mid p\right]}{E\left[V\left(W_{nj}(I_j^1)\right) \mid p\right]} = e^{ac(\lambda_{in})}\sqrt{\frac{Var[z_{in} + p_i'|\theta_i]}{Var[z_{in} + p_i'|p_i]}}, \tag{9}$$

and for day traders,

$$\frac{E\left[V\left(W_{dj}(I_j^2)\right) \mid p\right]}{E\left[V\left(W_{dj}(I_j^1)\right) \mid p\right]} = e^{ac(\lambda_{id})}\sqrt{\frac{Var[z_{in} - z_{id} + p_i'|\theta_i]}{Var[z_{in} - z_{id} + p_i'|p_i]}}. \tag{10}$$

Lemma 1 implies that traders decide whether to acquire information about each stock i by comparing the cost of acquiring information with the decrease in the variance of their end-of-period wealth. Define $\psi^{il}(\lambda)$ the benefit of information relative to its cost as,

$$\psi^{in}(\lambda) \equiv e^{ac(\lambda_{in})}\sqrt{\frac{Var[z_{in}+p_i'|\theta_i]}{Var[z_{in}+p_i'|p_i]}}, \quad \text{and} \quad \psi^{id}(\lambda) \equiv e^{ac(\lambda_{id})}\sqrt{\frac{Var[z_{in}-z_{id}+p_i'|\theta_i]}{Var[z_{in}-z_{id}+p_i'|p_i]}}.$$

Corollary 1 provides the equilibrium conditions that allow pinning down λ_{il}.

Corollary 1. *$\psi^{il}(\lambda)$ is monotone in λ_{il} for $l \in \{d, n\}$. Therefore,*
(i) If $\psi^{il}(\lambda) > 1 \ \forall \lambda_{il} \in [0,1]$, all l traders become informed, i.e. $\lambda_{il}^* = 1$.
(ii) If $\psi^{il}(\lambda) < 1 \ \forall \lambda_{il} \in [0,1]$, no l traders become informed, i.e. $\lambda_{il}^* = 0$.
(iii) Otherwise, λ_{il}^* is given by $\psi^{il}(\lambda_{il}^*) = 1$.

Households' Problem The recursive formulation of the representative household's problem is

$$\begin{aligned} H(s,K,k) = \max_{k'} \quad & u\left(k(1+r(s,K)) - k'\right) + \beta \sum_{s'} q_{ss'} H(s',K',k') \\ s.t. \quad & K' = G(K) \end{aligned} \tag{11}$$

where $s = \{\gamma, Z\}$ denotes the aggregate state, $H(.)$ is the value function, β is the discount factor, k is the individual capital holdings and K is the aggregate capital holdings. $G(.)$ represents the household expectations over the future path of the aggregate capital.

Market Clearing Market clearing for the shares of firm i is given by

$$\gamma\left[\lambda_{id}X_{id}^I + (1-\lambda_{id})X_{id}^U\right] + (1-\gamma)\left[\lambda_{in}X_{in}^I + (1-\lambda_{in})X_{in}^U\right] = \bar{K}_i \tag{12}$$

The capital market clearing condition for non-installed capital is

$$\int_i K_i = K, \tag{13}$$

where K is the total capital supplied by households. The mutual fund pays to households r to break even,

$$\sum_i r_i K_i = rK. \tag{14}$$

2.3 Equilibrium

Definition $H, r, k', G, \{K_i, r_i, X^I_{id}, X^U_{id}, X^I_{in}, X^U_{in}, \lambda_{id}, \lambda_{in}, \phi_{i0}, \phi_{i\varepsilon}, \phi_{id}, \phi_{in}, p_i\}_{i \in (0,1)}$ constitute a Linear Rational Expectations Equilibrium such that

1. $X^I_{id}, X^U_{id}, X^I_{in}$, and X^U_{in} solve the traders' problems, as characterized in (7).
2. $\lambda_{id}, \lambda_{in}$ are given by Corollary 1.
3. The stock price of firm i is a linear function of θ_{id} and θ_{in}, i.e., $p_i = \phi_{i0} + \phi_{i\varepsilon}\varepsilon^-_{in} + \phi_{id}\theta_{id} + \phi_{in}\theta_{in}$, where $\phi_{i0}, \phi_{i\varepsilon}, \phi_{id}$, and ϕ_{in} solve the market clearing condition in (12).
4. r, K_i, r_i solve the mutual fund's problem in (3) and (4) and satisfy the capital market clearing condition in (13).
5. k' and H solve the representative consumer's problem in (11).
6. G is consistent with k'.

2.4 Equilibrium Characterization

2.4.1 The Pricing Function

Proposition 1 shows the existence of a linear rational expectations equilibrium.

Proposition 1. *There exists a market price for stock i with the form $p_i = \phi_{i0} + \phi_{i\varepsilon}\varepsilon^-_{in} + \phi_{id}\theta_{id} + \phi_{in}\theta_{in}$ where*

$$\frac{|\phi_{in}|}{|\phi_{id}|} = 1 + \frac{(1-\gamma)\lambda_{in}(\sigma^2_{\varepsilon_{id}} + Var(z_{in} + p'_{in}))}{\gamma\lambda_{id}Var(z_{in} + p'_{in})}. \tag{15}$$

The ratio $\frac{|\phi_{in}|}{|\phi_{id}|}$ captures the relative impact of θ_{in} on the price relative to θ_{id}. Therefore, Proposition 1 provides a simple equation that describes how much can be learned about firms' fundamentals θ_{in}. Corollary 2 shows that this magnitude is determined by the extent of informed trading done by night versus day traders.

Corollary 2. *Ceteris paribus, price becomes less informative about θ_{in} when*
(i) a larger fraction of traders are day traders,

(ii) a larger fraction of day traders are informed compared to night traders,
(iii) day payoff has a smaller residual variance after information acquisition than night payoff.

Corollary 3 provides sufficient conditions for having an interior REE.

Corollary 3. *Let* $\lim_{\lambda_{il}\to 0} c(\lambda_{il}) = 0$ *and* $\lim_{\lambda_{il}\to 1} c(\lambda_{il}) = \bar{C}$ *where* $\bar{C}$ *is large enough. Then, any linear REE is interior, i.e.,* $\lambda_{il} \in (0,1)$.

2.4.2 The Extent of Capital Misallocation

Here, we define and characterize a measure of capital misallocation. The misallocation follows from the mutual fund having to use $E[\theta_{in}|p_i]$ instead of θ_{in} in allocating capital. The expression for the output loss due to misallocation is complicated. Still, it is monotonic in the Bayesian risk associated with using $E[\theta_{in}|p_i]$ as an estimator for θ_{in}. We first introduce a simplification that we maintain in what follows.

Assumption 1. *The parameters* $\bar{K}_i, \bar{\theta}_{in}, \bar{\theta}_{id}, \sigma^2_{\varepsilon_{in}}, \sigma^2_{\varepsilon_{id}}, \sigma^2_{\theta in}$, *and* $\sigma^2_{\theta id}$ *are firm invariant.*

Assumption 1 guarantees that equilibrium fractions of informed investors λ_{in}, λ_{id} and pricing function parameters ϕ_{i0}, $\phi_{i\varepsilon}$, ϕ_{in}, ϕ_{id} are also firm invariant. This assumption allows us to use the cross-sectional variation of prices in estimating price informativeness. This assumption also implies that the only sources of heterogeneity that remain about firms are θ_{in}, θ_{id}, ε_{in}, and ε_{id}.

We treat the mutual fund's problem as one of estimating θ_{in} with $E[\theta_{in}|p_i]$. Under a quadratic loss function, the frequentist risk would equal the summation of a squared bias ($E[E[\theta_{in}|p_i] - \theta_{in}]^2$) and a variance ($\mathrm{Var}[E[\theta_{in}|p_i] - \theta_{in}]$) term. In Proposition 2, we derive (i) the ex-ante and interim (conditional on p_i) risk measures for the mutual fund's estimator and (ii) the ex-post estimation error.

Proposition 2. *Under Assumption 1 and the squared loss function, the mutual fund's estimator is unbiased, and the ex-ante and interim (conditional on* p_i*) risk involved with the inference equals*

$$R(\theta_{in}, E[\theta_{in}|p_i]) = \frac{1}{\frac{1}{\sigma^2_{\theta_n}} + \frac{1}{\sigma^2_{\theta_d}}\left(\frac{\phi_n}{\phi_d}\right)^2}.$$

The estimation error for firm i is given by

$$E[\theta_{in}|p_i] - \theta_{in} = \frac{\frac{(\bar{\theta}_n - \theta_{in})}{\sigma^2_{\theta_n}} + \frac{\phi_n}{\phi_d}\frac{(\theta_{id} - \bar{\theta}_d)}{\sigma^2_{\theta_d}}}{\frac{1}{\sigma^2_{\theta_n}} + \frac{1}{\sigma^2_{\theta_d}}\left(\frac{\phi_n}{\phi_d}\right)^2}$$

As shown in Proposition 2, the frequentist risk is independent of θ_{in}, hence equal to the Bayesian risk.[8] Corollary 4 summarizes what Proposition 2 implies about the characteristics of firms that get under and over-invested.

Corollary 4. *Under Assumption 1 and the squared loss function,*

1. *capital is under-allocated to productive (high θ_{in}) firms and over-allocated to unproductive firms,*
2. *the likelihood of allocating capital to an inefficient firm increases as the signal about its stock's liquidity is more encouraging than expected, and*
3. *the likelihood of not allocating capital to an efficient firm increases as the signal about its stock's liquidity is more discouraging than expected.*

In other words, stocks may be priced higher due to higher long-term value or lower fluctuations in short-term resale value. Thus, compared to the benchmark, firms with lower (higher) than expected θ_{id} shocks are allocated more (less) capital. This prediction is consistent with the evidence provided by Amihud and Levi (2023), that an exogenous decline in a firm's stock liquidity lowers the firm's investment.

Following Goldstein et al. (2014), we define Price Informativeness (PI) as the reduction in the conditional variance of firm profitability after observing the price. We further normalize it with the reduction in the conditional variance of payoffs after acquiring a costly signal. We introduce this normalization to isolate the stock market's ability to communicate traders' information from fundamental shifts in productivity distributions and uncertainty.

[8] The Bayesian risk is defined as $\int R\left(\theta_{in}, E[\theta_{in}|p_i]\right) dF(\theta_{in})$.

Definition The Price Informativeness (PI) measure for firm i is defined as

$$PI_i = \frac{Var[z_{in}] - Var[z_{in}|p_i]}{Var[z_{in}] - Var[z_{in}|\theta_{in}]},$$

where $Var[z_{in}]$ denotes the unconditional variance of z_{in}.

A PI measure close to 0 would indicate that the variance reduction from observing the price is negligible compared to the reduction from acquiring the costly information. A PI measure close to 1 would suggest that observing the price is almost as useful as acquiring the costly signal. The PI measure in our setting boils down to a simple firm-invariant expression that resembles the risk function in Proposition 2.

Corollary 5. *Under Assumption 1, the PI measure equals*

$$PI = 1 - \frac{R(\theta_{in}, E[\theta_{in}|p_i])}{\sigma^2_{\theta_n}} = \frac{1}{1 + \frac{\sigma^2_{\theta_d}}{\sigma^2_{\theta_n}} \left(\frac{\phi_d}{\phi_n}\right)^2}. \tag{16}$$

The PI measure is inversely related to the mutual fund's risk. Hence, the extent of misallocation decreases with the PI measure. Under Assumption 1, the PI measure can be summarized by two parameters ($\sigma^2_{\theta_n}$ and $\sigma^2_{\theta_d}$) and two equilibrium objects (ϕ_n and ϕ_d). Our empirical strategy consists of estimating these four objects using the cross-section of firms in a given country to calculate PI yearly.

3 Measuring Price Informativeness

This section structurally measures Price Informativeness (PI). We first introduce data sources and describe our empirical strategy. Then, we estimate PI by estimating its components as given in (16) annually for each country. Finally, we study the cyclical properties of PI across countries.

3.1 Empirical Strategy and Data Details

3.1.1 Empirical Strategy

PI can be obtained from equation (16) without having to solve for the full equilibrium because traders are short-lived, and the pricing equation is independent of the long-lived households' behavior. The PI series is granular and unrestricted in its cyclical behavior, as we identify them from cross-firm variation in a country-year pair (which we refer to as a market), with pricing parameters freely varying over time in a country.

As is clear from equation (16), PI only depends on four parameters, the variances of the learnable part of stock profitability and stock liquidity ($\sigma^2_{\theta n}$ and $\sigma^2_{\theta d}$), and their price loadings (ϕ_n and ϕ_d). To move forward, we need to construct the signals θ_n and θ_d to obtain the variances and to estimate ϕ_n and ϕ_d from the following stock pricing equation:

$$p_i = \phi_0 + \phi_\varepsilon \varepsilon^-_{in} + \phi_d \theta_{id} + \phi_n \theta_{in}. \tag{17}$$

3.1.2 Data

We use data on stock prices of publicly traded firms from many countries, along with analyst forecasts (to measure expectations) and country-level economic conditions (to measure cycles).

Stock Prices and Fundamentals: We use Worldscope from Thomson/Refinitiv for data on monthly (open, close, high, low) stock prices and yearly fundamentals.[9] Consistent coverage starts around 1985 for the US and by 2000 for most economies.

Analyst Forecasts: We use the Institutional Brokers Estimate System (I/B/E/S) from Refinitiv for daily data on analyst forecasts of earnings-per-share.[10] We access the World-

[9] Worldscope is a leading source of cross-country financials of publicly traded companies. It provides data from 98,000 companies in more than 120 countries, which amounts to 99% of the global market capitalization in 2021. Worldscope uses a standardized template for financial information that corrects for measurable differences in accounting practices across companies and markets. The entries are subject to automated tests that check accounting identities, outliers, and correlations for accuracy. See Reuters (2010) for details on standardization practices, accuracy tests, coverage, and sample selection criteria.

[10] I/B/E/S collects forecasts about 22,000 active companies in 90 countries from over 18,000 analysts. Each observation is an analyst's forecast announcement regarding a company's balance sheet item for a

scope and the I/B/E/S through Wharton Research Data Services (WRDS), which provides a unique ticker for each company to link the two datasets.[11]

Economic Conditions: To measure funding liquidity distress, we primarily rely on the relative performance of banking stocks over non-financial firms' stocks. We supplement it with banking crisis indicators by Baron et al. (2021) and continuous proxies of funding liquidity from the World Bank. For measuring cyclicality, we rely on the publicly listed firms' average earnings and the growth rate of GDP measures from the World Bank.[12]

3.2 Measuring Signals about Profitability and Liquidity

How do we measure the *signals* of profitability and liquidity observed by informed investors, θ_{id} and θ_{in}? A common practice in the literature is to use realized values, i.e., z_{id} and z_{in}, as proxies for θ_{id} and θ_{in}. However, according to our model, regressing the price p_i on the realized values would lead to biased estimates of the price loadings ϕ because the measurement error would be correlated with the realized earnings. We formalize this drawback in Proposition 3 for $\rho = 0$, such that the unlearnable component is not persistent. The argument generalizes to $\rho > 0$, with more tedious algebra.[13]

Proposition 3. *Let $\rho = 0$. An OLS regression of the price (p_i) on realized values (z_{id} and z_{in}) would give biased estimates of ϕ:*

1. $E[\hat{\phi}_0^B] = \phi_0 + \frac{\bar{\theta}_n \sigma^2_{\varepsilon_n} \phi_n}{\sigma^2_{\varepsilon_n} + \sigma^2_{\theta_n}} + \frac{\bar{\theta}_d \sigma^2_{\varepsilon_d} \phi_d}{\sigma^2_{\varepsilon_d} + \sigma^2_{\theta_d}}$
2. $E[\hat{\phi}_n^B] = \phi_n \left(1 - \frac{\sigma^2_{\varepsilon_n}}{\sigma^2_{\varepsilon_n} + \sigma^2_{\theta_n}}\right)$
3. $E[\hat{\phi}_d^B] = \phi_d \left(1 - \frac{\sigma^2_{\varepsilon_d}}{\sigma^2_{\varepsilon_d} + \sigma^2_{\theta_d}}\right)$

particular horizon. Forecasts are available for various payoff-relevant items, but Earnings-Per-Share (EPS) has the widest coverage.

[11] We use the daily exchange rates provided by I/B/E/S to standardize the currency within a market across firms, time, and variables. See Appendix B.5 for details. Also, see Appendix B.4 for details on how we assign companies to countries, and Appendix B.6 for how we deal with M&As and stock splits.

[12] Taiwan's GDP growth measure is from their National Statistics Bureau. We apply linear detrending to all continuous variables on economic conditions. See Table 14 in Appendix D for summary statistics.

[13] Dávila and Parlatore (2018) discuss a similar bias in a different setting.

The bias becomes larger as the residual uncertainty after acquiring information ($\sigma^2_{\varepsilon_n}$, $\sigma^2_{\varepsilon_d}$) increases.

Given this drawback, we need to rely on signals. This is at the heart of how our price informativeness measure departs from the standard in the literature: we are interested in the extent to which prices reflect the traders' expectations at the time of making decisions, not the eventual realizations.

For θ_{in}, we rely on analyst forecasts for earnings, which, according to equation (2), are,

$$E[z_{in}|\theta_{in}] = Z + \theta_{in} + \rho\tilde{\varepsilon}^{-}_{in}$$

for informed traders. We obtain $E[z_{in}|\theta_{in}]$ using the median one-step-ahead forecast for z_{in} from I/B/E/S for company i (announced within a 15-day window around the date the stock price is documented). The cross-sectional average of $E[z_{in}|\theta_{in}]$ gives Z. The next step is to decompose θ_{in} from $\rho\tilde{\varepsilon}^{-}_{in}$.

First, since we observe the time series $\varepsilon_{in} = z_{in} - E[z_{in}|\theta_{in}]$, we can compute its variance $\sigma^2_{\varepsilon_n}$ and obtain ρ from computing its autocovariance, since

$$\begin{aligned} Cov(F_{it} - E[F_{it}], F_{i,t+1} - E[F_{i,t+1}]) =& Cov(\theta_{int} + \rho\tilde{\varepsilon}_{in,t-1}, \theta_{in,t+1} + \rho(\rho\tilde{\varepsilon}_{in,t-1} + \tilde{\varepsilon}_{int})) \\ =& Cov(\rho\tilde{\varepsilon}_{in,t-1}, \rho^2\tilde{\varepsilon}_{in,t-1}) = \frac{\rho^3\sigma^2_{\varepsilon_n}}{1-\rho^2}. \end{aligned} \tag{18}$$

where F_{it} is the earnings forecast for firm i. Second, once ρ is obtained, $\tilde{\varepsilon}_{in}$ can be approximated via perpetual inventory method using (2) and lagged values of ε_{in}.[14] Third, from the approximated series $\tilde{\varepsilon}_{in}$ we can finally obtain $\theta_{in} = E[z_{in}|\theta_{in}] - Z - \rho\tilde{\varepsilon}^{-}_{in}$.

For θ_{id}, we would like to capture a measure of broker fees' determinants, which are mostly determined by the volatility of stock markets (the risks faced by the broker from holding stocks, potentially buying them high and selling them low). A good measure of this friction would be future volatility, and for its signal, we would ideally use a volatility forecast, which, unfortunately, is not available. Hence, we proxy it with the prior six months of realized price volatility.[15]

[14]We first estimate ρ for markets that have at least 20 companies with past year's data. For the remaining markets, we use the country average if available and, if not, the overall average. For computing $\tilde{\varepsilon}_{in}$, we determine the lag order for each company-date pair based on the availability of past data on ε_{in}, with a maximum of three lags.

[15]To estimate the stock price volatility, we use the measure proposed by Garman and Klass (1980), which

As an illustration of results, Table 1 shows summary statistics for the series of signals and forecast errors we estimated for the United States in 2015.[16]

Table 1: Summary Statistics for the US in 2015, Random Variables

variable	mean	sd	min	median	max
θ_n	0.06	0.06	-0.23	0.06	0.99
θ_d	0.21	0.14	0.05	0.17	1.00
ε_n	-0.01	0.04	-0.82	0.00	0.12
ε_d	0.02	0.10	-0.65	0.03	0.84
$\tilde{\varepsilon}^-$	-0.01	0.05	-0.90	0.00	0.27

Notes: The figures are per unit of asset values constructed by multiplying original figures with the number of outstanding shares and dividing them by the value of their total assets.

3.3 Estimating Pricing Functions

Having measured the time series of signals about profitability and liquidity (θ_{id} and θ_{in}), we proceed to estimate their price loadings for each country-year pair, ϕ_n and ϕ_d, which enter into the calculation of PI as derived in equation (16).

We estimate the pricing function in two steps. We first residualize the normalized stock prices to i) capture the heterogeneity in $\bar{K}_i$ and $\bar{\theta}_{in}$ that is not captured in the model and ii) capture dependencies in the distributions of θ and ε across companies.[17] Then, we estimate the following pricing function using the residualized stock prices for each

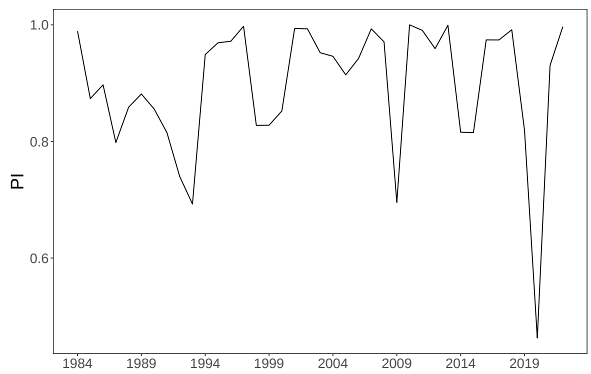
only requires the opening (O), closing (C), highest (H), and lowest (L) prices during the period. In particular, we look at the stock prices over the previous six months to compute

$$\tilde{\sigma}_{it}^2 = 0.511(H_{it} - L_{it})^2 - 0.019[(C_{it} - O_{it})(H_{it} + L_{it} - 2O_{it}) - 2(H_{it} - O_{it})(L_{it} - O_{it})] - 0.383(C_{it} - O_{it})^2$$

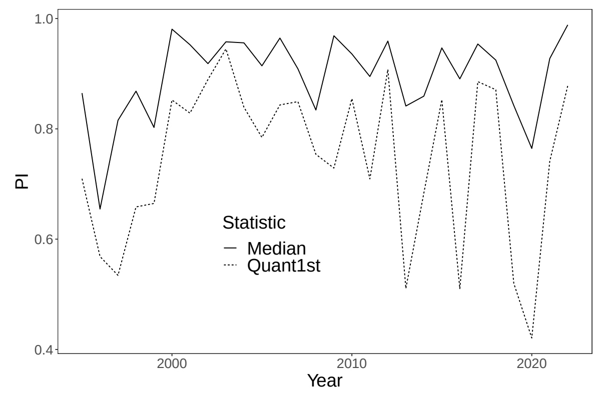
for each ticker i at date t. If the resulting volatility measure exceeds the stock price, we equate it to the stock price, imposing an intuitive limited liability rule on stock ownership. We normalize this measure with the stock price. In Appendix C.2, we provide statistics on the cross-sectional distribution of the range volatility estimates and how they evolve for the median firm.

[16] See Table 13 in Appendix D for the statistics for Japan and the UK.

[17] See Appendix B.1, B.2, and B.3 for a detailed discussion of (1) how we normalize the measures for cross-firm comparability and residualize the prices, (2) the timing assumptions for measuring each object, and (3) sample restrictions we impose, respectively.



(a) PI Estimates for the US



(b) Cross-Country Statistics for PI Estimate

Figure 1: The PI Estimates

Notes: We restrict attention to countries for which PI is estimated for at least 20 years between 1994 and 2022 to get a partially balanced sample. The countries are Australia, Germany, the United Kingdom, France, Japan, Taiwan, Canada, Sweden, and the US.

country-year pair:

$$\hat{p}_i = \beta_0 + \beta_1 \tilde{\varepsilon}_i^- + \beta_2 Range_i + \beta_3 e\hat{p}s_i^f + \upsilon_i, \tag{19}$$

where $\tilde{\varepsilon}_i^-$ for firm i is estimated using forecast errors obtained in Section 3.1.1, $Range_i$ is the range volatility estimate for the stock price in the past six months and is used as the empirical counterpart of θ_{id}. $e\hat{p}s_i^f$ is the forecast for the next announcement of the earnings adjusted for known components and is used as the empirical counterpart of θ_{in}. We treat υ_i as a measurement error that is orthogonal to the regressors. Under Assumption 1, the OLS estimator for β_2 and β_3 provide unbiased estimates of ϕ_d and ϕ_n.

3.4 Price Informativeness Over Time and Across Countries.

We have obtained the four parameters, $\sigma^2_{\theta n}$, $\sigma^2_{\theta d}$, ϕ_n and ϕ_d that we need to compute PI from (16) for 21 countries and 18 years per country in average (a total of 381 country-years pairs).[18] The estimates validate the model's predictions. The model suggests $\phi_n > 0$ and $\phi_d < 0$, although the estimation does not impose this restriction. The former is satisfied in all but one market, while the latter is satisfied in 70% of the markets. Furthermore, the model predicts $\frac{|\phi_n|}{|\phi_d|} > 1$ which is true for 97% of the markets.

[18] See Table 11 in Appendix D for a list of countries and available years in our final sample.

As an illustration of results, Figure 1a presents the PI series for the US. This series is cyclical: it has a time series correlation of 0.51 with the average returns and 0.45 with the GDP growth rate. PI experienced its largest decline in our 40-year sample during the COVID-19 episode, the 2008-09 Great Financial Crisis (GFC), and the late 1980s and early 1990s that surrounded the Savings and Loan (S&L) crisis.[19]

An advantage of our structural estimation of PI is that we can identify the drivers behind these large PI declines, according to equation (16). We show the four components that go into the computation of the PI measure for the U.S. in Figure 2 and show their relevance in the episodes of sizable decline. The COVID-19 pandemic, for instance, was characterized by a jump in the variance of liquidity $\sigma^2_{\theta d}$ and a drop in the price loading of earnings ϕ_n. The Great Financial Crisis, instead, experienced an increase in both the variance of liquidity $\sigma^2_{\theta d}$ and of earnings $\sigma^2_{\theta n}$, but also an increase in the price loading of liquidity ϕ_d. Finally, the long period of distress that surrounded the S&L crisis of 1987-88 was not characterized by a large increase in variances but a sizeable increase in the price loading of liquidity ϕ_d relative to that of earnings ϕ_n.

A more systematic decomposition of these drivers can be obtained by considering the log of a monotonic transformation of PI from equation (16):[20]

$$ln\left(\frac{PI}{1-PI}\right) = ln(\phi_n^2) + ln(\sigma^2_{\theta_n}) - ln(\phi_d^2) - ln(\sigma^2_{\theta_d}). \tag{20}$$

Figure 3a presents this measure (in the solid black line) together with two components: the sum of the price loadings and the variances. The variances (the dotted line) are important for the level of PI but play a relatively small role in its fluctuations relative to the loadings (the dashed line). Figure 3b further decomposes the price loadings. While the fluctuations in the loading of earnings (dashed line) contribute a significant amount, the lion's share of PI fluctuations comes from changes in the loading of liquidity (the dotted line). A natural question that ties this analysis back to our original PI measure is: 'How much would the time series variance of the PI measure decline if the component x is kept

[19] The epicenter of the period of distress on the nation's savings and loan industry was the crisis in 1987-88, but the problems initiated in the early 1980s and extended for a decade as many insolvent thrifts were allowed to remain open, with their financial problems worsening over time. The crisis ended in the early 1990s when the Resolution Trust Corporation (RTC) was terminated.

[20] We thank Liyan Yang for suggesting this intuitive decomposition.

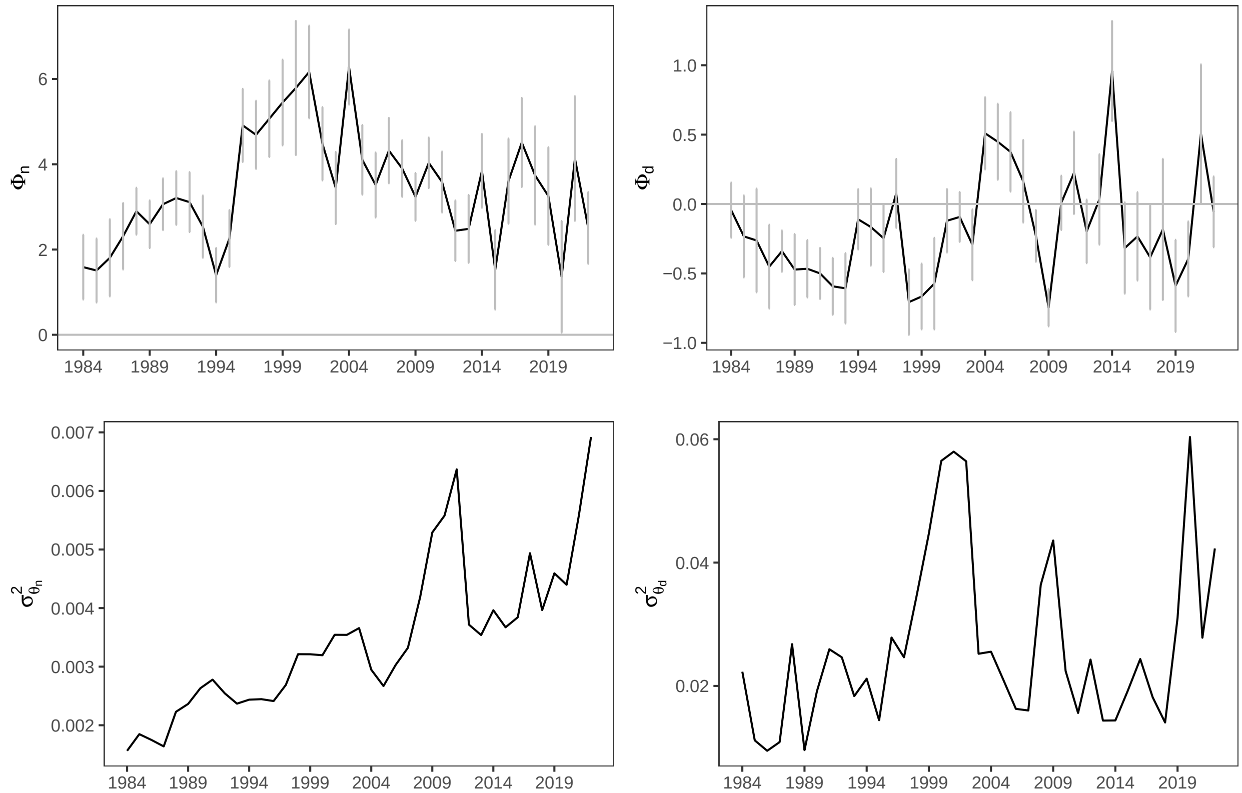


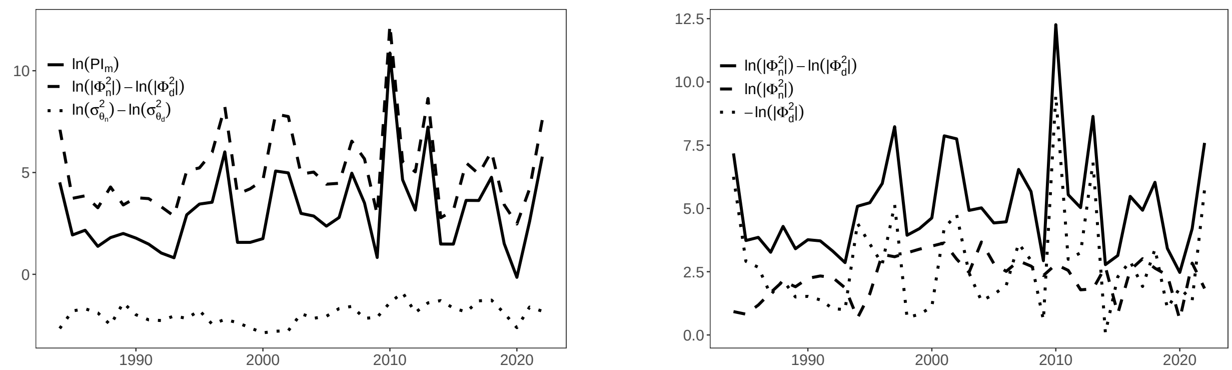
Figure 2: Estimated Price Informativeness Components for the US

fixed at its median value?' The variance would decline by 4%, 21%, and 71% with $\sigma^2_{\theta d}$, ϕ_n, and ϕ_d at their median vales, respectively. If $\sigma^2_{\theta n}$ instead was kept at its median value, the PI variance would increase by 22%.[21]

Figure 1b presents moments from the yearly distribution of PI estimates across the nine countries with observations between 1995 and 2021. The median and the 1st quartile experienced declines around the Great Recession and the COVID-19 pandemic, similar to the US. Other major declines were observed in PI in *i)* Korea around the Asian Financial Crisis in 1997, *ii)* most European economies around the 'Taper Tantrum' and the European sovereign debt crisis in 2013, and *iii)* UK and France around the Brexit decision in 2016. We test the cyclicality of PI more formally using our panel data. In particular, we run the following regression:

$$PI_{it} = \beta_0 + \beta_1 Z_{it} + F_{ci} + \epsilon_{it}, \tag{21}$$

[21] These are, so far, statistical decompositions, not structural ones, as ϕ_n and ϕ_d are themselves functions of $\sigma^2_{\theta n}$ and $\sigma^2_{\theta d}$.



(a) Decomposing PI_m

(b) Decomposing the Price Loadings

Figure 3: Contribution of the Components of PI

where Z_{it} is aggregate productivity of publicly listed firms in country i in year t and is measured by the weighted average of two alternative measures: normalized earnings and growth rate of GDP. PI_{it} is PI estimate and F_{ci} are country fixed-effects. This specification allows us to account for PI's country-specific factors and focus on its cyclicality.

Columns 1 to 4 in Table 2 present the results. There is a positive correlation between both measures of economic activity and the estimated PI series. The correlation becomes larger and more precisely estimated once each country-year observation is weighted by the number of stocks used to estimate PI (columns 3 and 4).

In columns 5 and 6, we ask whether the PI measure is related to funding liquidity conditions in the economy. We construct a measure, 'Banking Stock Performance,' which is the ratio of the median normalized stock price of publicly listed banks to that of publicly listed non-financial firms. This measure captures how the banking sector is doing relative to the rest of the economy and proxies the capacity of the banking system to provide liquidity through credit: the lower this variable, the higher the reliance of agents on stock markets to access liquidity. Table 2 shows that PI tends to be higher when the banking sector is doing relatively well compared to the rest of the economy. Consistent with our model, stock prices become more informative about firms' fundamentals when stock markets have a lesser role in liquidity provision. In other words, *stock markets reveal more information about firms when banks are healthier.*

Table 2: The Cyclicality of the Price Informativeness Measures

	PI					
	(1)	(2)	(3)	(4)	(5)	(6)
GDP Growth Rate	0.54		2.56***			
	(0.48)		(0.37)			
Avg Earnings		3.51***		8.31***		
		(1.35)		(0.99)		
Banking Stock Perf.					0.57	0.87***
					(0.47)	(0.30)
Range	'84-'22	'84-'22	'84-'22	'84-'22	'84-'22	'84-'22
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Weights	No	No	Yes	Yes	No	Yes
Observations	344	344	344	344	319	319

Notes: In (3), (4), and (6), each country-year observation is weighted with the number of stocks used in the estimation of the PI measure. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

4 Model Calibration

In this section, we calibrate the model to the US economy, where each model period is one year. We use the fluctuations in Price Informativeness (PI) estimated in the previous section to infer the structural parameters of its sources. Finally, we use the calibrated model to obtain the evolution of the fraction of day traders, γ, which we do not observe.

4.1 Calibration Strategy

In the previous section, we have estimated the time series of the means ($\bar{\theta}_n, \bar{\theta}_d$) and the variances ($\sigma^2_{\theta_n}$, $\sigma^2_{\theta_d}$, $\sigma^2_{\varepsilon_n}$, and $\sigma^2_{\varepsilon_d}$) associated with the stock returns. In this section, we finalize the model's parameterization in two steps. First, we externally calibrate standard parameters. Second, we internally calibrate the parameters of the information cost functions, which involves estimating the latent series for γ and discretizing it as part of the model's aggregate state.

Externally Calibrated Parameters For households, we set the intertemporal elasticity of substitution $1/\eta$ equal to 0.5. For traders, following Farboodi and Veldkamp (2020), we set the absolute risk aversion parameter a equal to 0.05. Additionally, we set the depreciation rate δ equal to 0.1, the risk-free interest rate available to the traders r^F equal to 0.02, and the autocorrelation of the unlearnable part of profitability, ρ, to 0.[22]

Series of Liquidity Needs Equation (15) provides a formula for liquidity needs captured by the fraction of daily traders, γ. Estimating γ requires the knowledge of the ratio of the pricing coefficients (estimated in Section 3), $Var(z_{in} + p'_{in})$ and λ_n/λ_d.

Estimating $Var(z_{in} + p'_{in})$ is nontrivial because part of the uncertainty is about future prices and hence future pricing coefficients. To estimate the conditional variance of the future payoff, we estimate a first-order auto-regressive process for the estimated pricing coefficient vector $\hat{\Phi}$:

$$\hat{\Phi}_t = B\hat{\Phi}_{t-1} + U + W_t, \tag{22}$$

where $\hat{\Phi}_t = [\hat{\phi}_0\ \hat{\phi}_\varepsilon\ \hat{\phi}_n\ \hat{\phi}_d]$, B is a 4×4 diagonal matrix that controls the persistence, U is a 4×1 vector that stores the constant terms, and W_t is a 4×1 error term where $W_t \sim MVN(0, Q)$ and Q is the associated variance-covariance matrix. Once estimated, (22) provides a simple formula for $Var(z_{in} + p'_{in})$.

After recovering $Var(z_{in} + p'_{in})$, we use Corollary 2 to back out λ_n/λ_d. In particular, the following equations hold in an interior solution to the information acquisition problem:

$$e^{ac_n(\lambda_n)} \operatorname{Var}\left(\epsilon_{in} + p'_i\right) = \operatorname{Var}\left(\epsilon_{in} + p'_i\right) + \frac{\sigma^2_{\theta_n}}{1 + \frac{\sigma^2_{\theta_n}}{\sigma^2_{\theta_d}}\left(\frac{\phi_n}{\phi_d}\right)^2}, \tag{23}$$

$$e^{ac_d(\lambda_d)} \left(\sigma^2_{\varepsilon_d} + \operatorname{Var}\left(\epsilon_{in} + p'_i\right)\right) = \left(\sigma^2_{\varepsilon_d} + \operatorname{Var}\left(\epsilon_{in} + p'_i\right)\right) + \frac{\sigma^2_{\theta_n}}{1 + \frac{\sigma^2_{\theta_n}}{\sigma^2_{\theta_d}}\left(\frac{\phi_n}{\phi_d}\right)^2} + \frac{\sigma^2_{\theta_d}}{1 + \frac{\sigma^2_{\theta_d}}{\sigma^2_{\theta_n}}\left(\frac{\phi_d}{\phi_n}\right)^2}. \tag{24}$$

[22]If $\rho \neq 0$ the $\tilde{\epsilon}_i$ never reaches an ergodic distribution under aggregate shocks, and its distribution becomes an additional state variable. Even though accounting for a potential correlation is necessary for interpreting the data, it wouldn't play a significant role in our model due to the static capital allocation across companies.

An advantage of our setting is that equations (23) and (24) allow recovering λ_n and λ_d *for given* cost functions $c_n(.)$ and $c_d(.)$. Using the estimated values for λ_n/λ_d and $Var(z_{in} + p'_{in})$, and (15), we can recover the time series for γ. The challenge, however, is to discipline the information cost functions. We next discuss our strategy.

Information Cost Functions We parametrize the information cost function as follows:

$$c_j(\lambda_j) = \nu_j \left(\frac{1}{1-\lambda_j}\right)^{\psi_j} - \nu_j \tag{25}$$

for $j \in \{d, n\}$, which satisfies $c_j(0) = 0$ and $\lim_{x\to 1} c_j(x) = \infty$. This functional form simplifies the equilibrium computation by mapping each nonnegative cost value to a unique $\lambda_j \in [0, 1)$. For each set of parameters, $\{\nu_d, \nu_n, \psi_d, \psi_n\}$, the stock trading module provides a mapping between the pricing parameters (ϕ) and the fraction of informed agents (λ) through equations (12), (23) and (24). Our estimation strategy relies on choosing these four parameters to match the moments of the pricing function in booms and busts. In particular, we focus on seven moments implied by the estimated pricing function: (1) the level of PI in low productivity periods, (2) the change in PI after aggregate shocks, (3) the average fractions of day and night traders that are informed, and (4) the growth in the information acquisition activities during busts.[23]

The level of the cost of acquiring information depends on ν_d and ν_n, while its curvature is determined by ψ_d and ψ_n. The average levels of λ_n and λ_d are hence directly informative about ν_d and ν_n, together with the average level of PI, which depends on λ_n and λ_d. The changes in the level of PI and information acquisition activities, on the other hand, tell us the extent to which day and night traders respond to economic changes: a low ψ_d (ψ_n) makes it easier for a day (night) trader to scale their information acquisition, tilting the prices to reflect more of θ_d (θ_n).

Discretizing the Aggregate States We have to estimate the process of the aggregate state defined as $s = \{Z, \gamma\}$. We first remove a linear trend from the original series of the logarithm of earnings per share Z and liquidity needs γ. Then, we estimate a vector-auto-

[23] To be precise λ_d and λ_n are not moments observed directly from the data. However, they can be inferred from the estimated pricing function for a given cost function. Given the estimated parameters, matching the PI boils down to matching ϕ_n/ϕ_d. The reason why the ratio doesn't exactly pin down λ_d and λ_n is the $Var(z_{in} + p'_i)$ term, which depends on the entire pricing function and how it changes from booms to busts. In other words, the estimated parameters of the pricing function in booms and busts provide the two missing moments in this estimation.

regression (VAR) and discretize it with a 4-state Markov chain (2 levels for each state) following Gospodinov and Lkhagvasuren (2014). We assign each year as a high or low Z given the estimated Z grid and compute the average levels of $\sigma^2_{\theta_n}$, $\sigma^2_{\theta_d}$, $\sigma^2_{\varepsilon_n}$, and $\sigma^2_{\varepsilon_d}$ in those years. These averages constitute their levels in high and low Z states in the model.

Estimation Algorithm We start with a guess $\{\nu^0_d, \nu^0_n, \psi^0_d, \psi^0_n\}$ and take the next steps:

1. Start iteration k with $\{\nu^k_d, \nu^k_n, \psi^k_d, \psi^k_n\}$. Use (23) and (24) with estimated series for pricing coefficients and parameters to infer $\lambda^k_{d,data}$ and $\lambda^k_{n,data}$ series.

2. Invert (15) to infer γ series.

3. Follow the discretization procedure discussed above to estimate a VAR for $\{Z, \gamma\}$ and discretize it into a Markov Chain. Compute the values for $\sigma^2_{\theta_n}$, $\sigma^2_{\theta_d}$, $\sigma^2_{\varepsilon_n}$, and $\sigma^2_{\varepsilon_d}$ associated with each Z level.

4. Compute the stock market equilibrium and simulate the economy for the model moments M^k_{sim} (See Table 4 for a list of moments).

5. Compare M^k_{sim} with M^k_{data}. If the discrepancy is below the threshold, stop. If not, go back to step 1 with new $\{\nu^{k+1}_d, \nu^{k+1}_n, \psi^{k+1}_d, \psi^{k+1}_n\}$.

After having estimated the cost function parameters, we estimate the discount factor to generate a 2% risk-free interest rate.[24] Because we directly estimated the level of productivity from earnings data, we have a degree of freedom to set the scale of the economy. So, we normalize the adjustment cost $\bar{k}/\xi$ to achieve an average capital level of 1.

4.2 Calibration Results

The calibrated liquidity series γ for the United States and the underlying cost functions are depicted in Figures 4 and 5, respectively. The first panel of Figure 4 shows the estimated liquidity series for the U.S., suggesting elevated transitory reliance on stock markets for liquidity purposes (high γ) around the Great Recession and the COVID-19 pandemics and a persistent period of high reliance surrounding the period of S&L distress,

[24]This interest rate is not the return households receive from capital (r) since the realized return accrues all the surplus to the households. Instead, we evaluate the counterfactual interest rate in the model that would clear the markets in a competitive setting.

from 1985 to 1995. While the average fraction of traders with liquidity needs after 2000 ranges around 40%, these three events reach levels above 80%. These results are suggestively consistent with the periods of a sizeable decline in PI. The second panel of Figure 4 shows the estimated series of liquidity needs for the median of developed economies using the estimated information costs for the US. Naturally, these series are less volatile than for a single country but display similar patterns of increase during periods of global stress, like the COVID-19 pandemic or the European Sovereign Debt crisis.

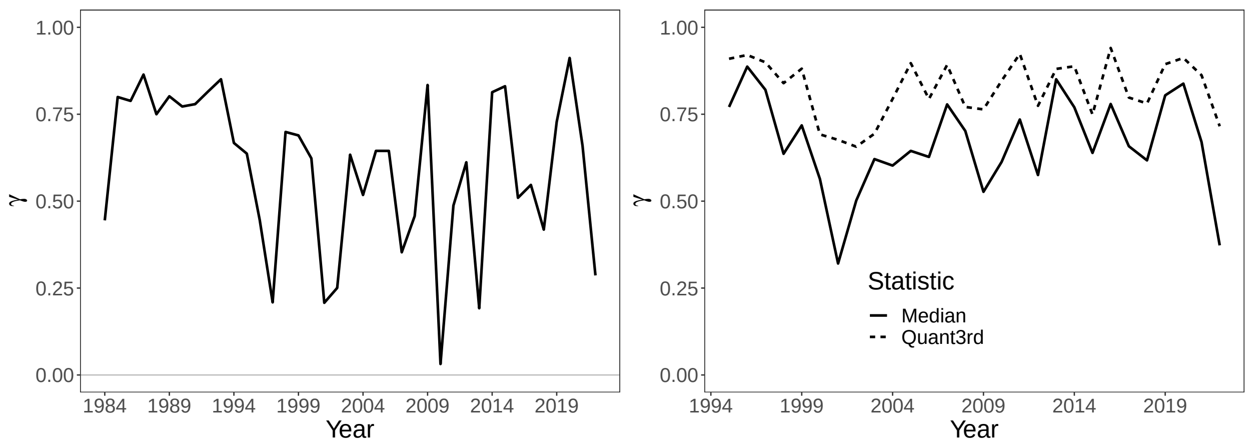


Figure 4: γ Series Implied by the Estimated Cost Function Notes: The left panel presents the estimated γ series for the United States, while the right panel presents the cross-sectional moments from a panel of countries using the cost function estimate from the US. To get a partially balanced sample, we restrict attention to countries where a PI is estimated for at least 20 years between 1994 and 2022. The countries are Australia, Germany, United Kingdom, France, Japan, Taiwan, Canada, Sweden, and the US.

Figure 5 demonstrates the shape of the estimated cost functions for day and night traders. The estimated cost function indicates that the cost of learning about the liquidity of stocks is much higher and steeper than that of learning about their fundamental payoffs. This result is disciplined by two observations: (*i*) the post-information acquisition uncertainty in stock liquidity ($\sigma^2_{\epsilon_d}$) is much higher than the uncertainty in fundamental payoffs ($\sigma^2_{\epsilon_n}$), and (*ii*) the price uncertainty ($Var(\epsilon_{in} + p'_i)$) is much larger than the dispersion of signals ($\sigma^2_{\theta_n}$ and $\sigma^2_{\theta_d}$). As a result, (23) and (24) indicate that the night traders must be paying a higher cost. Under symmetric cost functions, paying a higher cost would imply more information acquisition by night traders and a high PI. However, this scenario would be inconsistent with the information acquisition motives of the day traders in equilibrium: day traders should be willing to pay more than night traders, given the additional uncertainty they face. Furthermore, the increase in information acquisition in recessions would be as large as 200% as new day traders would acquire much more information than the old night traders. Hence, the cost function should be more restrictive for day traders to match the level of PI and be steep enough to prevent large swings in information acquisitions between booms and busts.

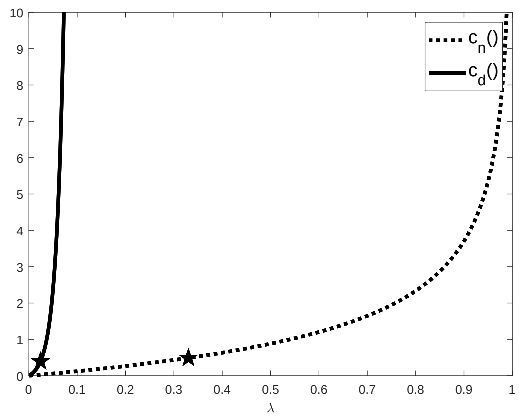


Figure 5: Estimated Information Acquisition Cost Functions
Notes: The solid and the dashed lines represent $c_n()$ and $c_d()$, respectively. The stars refer to the median λ and associated costs in the stochastic steady state.

Table 3 summarizes the four aggregate states resulting from discretizing the (Z, γ) series and the years our method assigns to each aggregate state. We also show the standard deviations of signals and forecast errors in each state. The Z value fluctuates between 0.051 and 0.063 while γ fluctuates between 0.38 and 0.80. The values for Z are consistent with median earnings per share fluctuations over time. We interpret the values of γ as indicating that roughly two-thirds of the traders focus exclusively on the long-run earnings of stocks when funding liquidity markets operate well, while about 80% consider the short-run price fluctuations when funding liquidity dries up.

Table 3: Estimated Aggregate State Levels

s	Z	γ	σ_{θ_n}	σ_{θ_d}	σ_{ϵ_n}	σ_{ϵ_d}	Years
1	0.051	0.38	0.058	0.18	0.036	0.14	’96,’97, ’01, ’02, ’13, ’16
2	0.051	0.80	0.058	0.18	0.036	0.14	’89, ’90, ’91-’94, ’98-’00, ’03, ’09, ’12, ’15, ’17, ’19-’21
3	0.063	0.38	0.059	0.14	0.038	0.12	’84,’95,’04,’07,’08,’10,’11,’18,’22
4	0.063	0.80	0.059	0.14	0.038	0.12	’85-’88,’05,’06,’14

Notes: The last column shows the years with Z and γ estimates closest to the values associated with each aggregate state. The values of $\sigma^2_{\theta_n}$, $\sigma^2_{\theta_d}$, $\sigma^2_{\varepsilon_n}$, and $\sigma^2_{\varepsilon_d}$ are estimated by taking the averages over the years associated with the Z values.

Table 4 summarizes all calibrated parameters, both externally and internally. We set PI corresponding to the aggregate state $\{\overline{Z}, \underline{\gamma}\}$, i.e., $\underline{PI}$, as a benchmark. *Ceteris paribus*, PI declines when the economy transitions to a state with higher market liquidity needs and increases when the economy transitions to a state with lower aggregate productivity. If both productivity declines and banking gets in distress, the second effect prevails, and PI declines. Our calibrated model matches this pattern.

Table 4: Externally and Internally Calibrated Parameters

Parameter	Value	Moment	Parameter	Value	Moment	Model	Target
$\bar{k}/\xi$	600	Normalized	ν_n	4.22	λ_d	0.06	0.02
η	2	External	ν_d	0.12	λ_n	0.29	0.33
a	0.05	External	ψ_n	0.27	$\underline{PI}$	0.87	0.87
δ	0.1	External	ψ_d	58.6	$\Delta PI_{\bar{z}\underline{\gamma}\to\underline{z}\underline{\gamma}}$	0.02	0.01
					$\Delta PI_{\bar{z}\underline{\gamma}\to\bar{z}\bar{\gamma}}$	-0.16	-0.11
					$\Delta PI_{\bar{z}\underline{\gamma}\to\underline{z}\bar{\gamma}}$	-0.10	-0.16
					$\Delta\lambda$	0.17	0.22
r^F	0.02	External	β	0.955	Real Interest Rate	0.02	0.02

Notes: λ_d and λ_n are computed as the average values in the ergodic distribution of aggregate states. The data counterpart is computed as the median value in the inferred series. $\underline{PI}$ is the PI level averaged over low Z states in the ergodic distribution. Its data counterpart is computed as the average PI level in designated low Z years. The ΔPI terms are computed as percentage changes in PI measures in an IRF exercise where the economy moves from a long sequence of high Z low γ states. Their data counterpart is computed as the percentage differences in average PI levels across years with different aggregate state designations. The $\Delta\lambda$ is computed as the percentage change in aggregate (day and night) fraction of informed traders in an IRF exercise where the economy moves from a long sequence of high Z low γ states to a state of low Z high γ. Its data counterpart is the percentage difference in employment in the NAICS 52394 (Portfolio management and investment advice) industry during recessions and booms (CES Survey, BLS).

5 Quantitative Relevance of Stock Price Informativeness

This section uses the calibrated model to assess the quantitative relevance of stock price informativeness on resource misallocation. The first exercise estimates an impulse response function for a recessionary shock with and without a liquidity shock. This comparison means to capture recessions with and without distress in financial markets and informs us how an increase in market liquidity needs amplifies or dampens the impact of a recessionary shock through changes in stock price informativeness. The second exercise compares the aggregate effects of recessions with financial distress in alternative economies, one with lower information costs and another in which traders receive information exogenously. While the first economy is useful for evaluating disclosure policies, the second shows the quantitative importance of modeling information as a choice.

5.1 Allocation Effects of Productivity and Liquidity Shocks

We simulate our economy for a long time with a high Z and low γ (state 3 in Table 3).[25] Then, we introduce a one-period recession *with* financial distress: a decline in Z *and* an increase in γ (a transition to state 2 in Table 3). This *dual shock* captures the major downturns of the US economy during our estimation period: the Great Recession in 2009 and the COVID-19 recession in 2020. We then compare these results to the aggregate effects of a recession *without* liquidity distress: a drop in Z that is *not* accompanied by an increase in γ (a transition to state 1 in Table 3). This *single shock* captures recessions without clear liquidity problems, such as in 2001. Figure 6 presents the results of these two different aggregate shocks: the solid line in the first case and the dotted line in the second case.

In the case of the dual shock to productivity and liquidity needs (solid lines), the participation of more day traders absconds the fundamental value of firms. As a response to more uncertain fundamentals, the benefits of acquiring information also increase for night traders. Despite more information acquisition overall, Price Informativeness (PI) ultimately declines. The decline in aggregate productivity and the increased misallocation induced by lower PI discourages investments and magnifies the decline in output and investment caused simply by a drop in Z.

How about a recession without financial distress? This case of a single shock to productivity (dotted lines) also generates a decline in output and investment, but only half as severe as if it were accompanied by financial distress. As in the previous case, the increase in uncertainty σ_{θ_d} that accompanies a recession induces day traders to produce more information, but the higher σ_{ϵ_d} makes them trade less aggressively on their information. This combination generates less 'noise' in markets and makes stock prices more informative about fundamentals. The increase in PI improves the allocation of capital, partially compensating for the negative productivity shock, which otherwise would lead to an output loss twice as large.[26]

[25] We use 1000 grid points for capital. For deterministic IRFs, we simulate the economy for 220 periods and discard the first 195. For stochastic simulations, we simulate 300 economies for 200 periods and discard the first 100 periods. We present the averages of time series statistics across these economies.

[26] PI would be unaffected, for instance, if the economy only experiences a drop in Z from 0.063 to 0.051 (a pure recession as in Table 3) but without any other change. Because capital allocation is independent of Z, the size of the initial drop in output would be the same as the drop in Z, i.e., 19%.

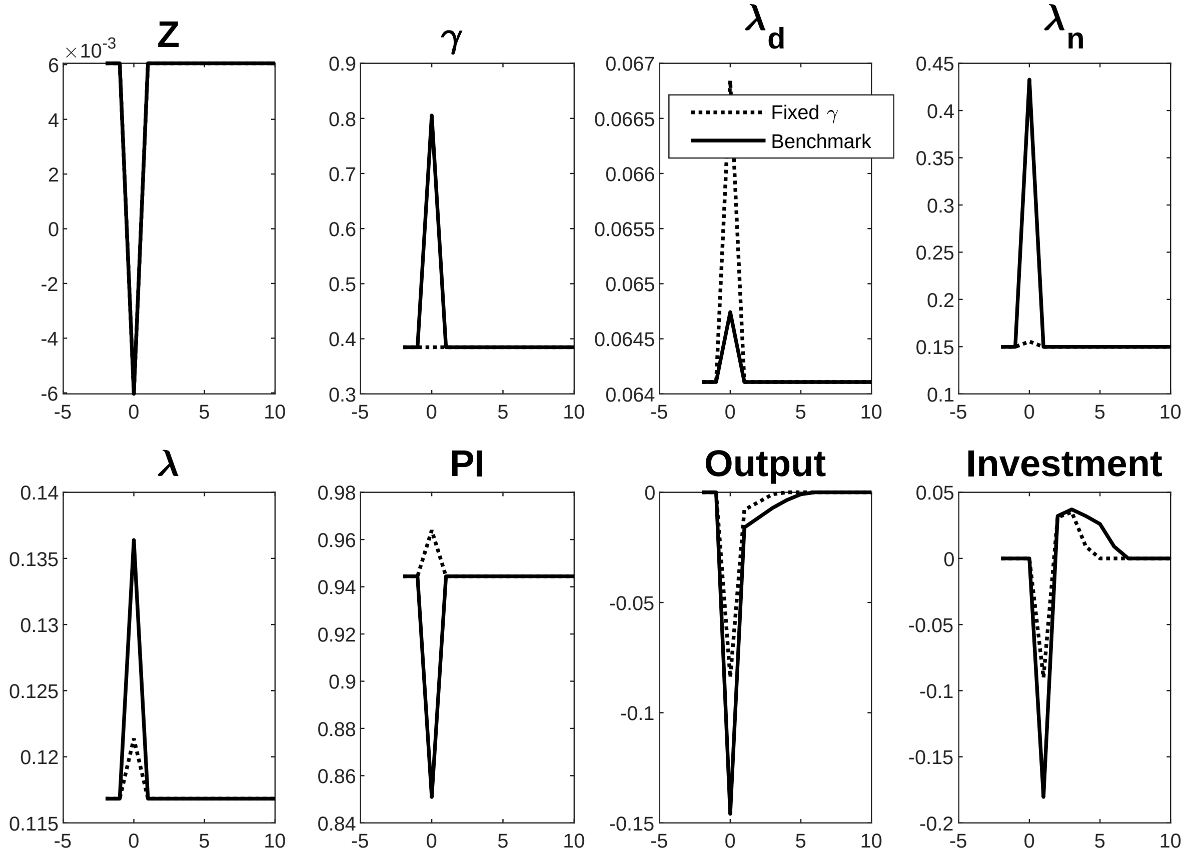


Figure 6: Impulse Response Functions
Notes: The first two panels provide the shocks that hit the economy under the benchmark and the counterfactual recession scenarios. λ denotes the aggregate fraction of traders that acquire information. Output and investment values represent the percentage changes from the pre-shock values.

5.1.1 Testable Implications

Our calibrated model generates a relation between shocks, PI, and allocations that we can empirically test across countries.

First, our calibration shows that PI goes up with productivity shocks, goes down with liquidity shocks, and reacts more to a standard liquidity shock than a standard productivity shock. To study the sensitivity of PI to productivity and liquidity shocks, we estimate:

$$PI_{ct} = \beta_0 + \beta_1 Z_{ct} + \beta_2 y_{ct} + F_c + F_t + \epsilon_{ct}, \tag{26}$$

where Z_{ct} represents the aggregate productivity process of publicly listed firms and is measured by the weighted average of normalized earnings. PI_{ct} is PI estimate from (19). y_{ct} represents several proxies of the extent to which stock markets are used to face liquidity needs in country c and year t. We use the 'Banking Stock Performance' variable, which we presented in Table 2, and several other proxies for funding liquidity, such as binary measures of banking panics and banking equity crises (constructed by Baron et al. (2021)), and continuous measures, such as capital-asset ratios of the banking sector, loan spreads of the banks (lending rate minus treasury bill rate), and the ratio of non-performing loans to total gross loans. In low-liquidity environments, such as the Great Recession, the capital asset ratio is expected to be low, while the loan spreads and non-performing ratios are expected to be high. Finally, F_c and F_t are country and year fixed effects.[27]

Table 5 presents the results. Consistent with our calibration, less liquidity provided by banking and financial sectors is associated with less PI across all specifications. A one standard deviation increase in the capital-to-asset ratio of the banking system, for example, is associated with a 0.2 increase (0.83 standard deviations) in PI. That is, when banks are better armed to provide credit, stock prices become more informative. However, once conditioned on the level of economy-wide liquidity, lower aggregate productivity is associated with high PI, which is not statistically significant.[28]

Second, misallocation from a drop in PI takes a particular shape in our model: less PI induces firms' fundamentals to look more alike, reducing the dispersion of investment. We test this implication by comparing the dispersion of capital expenditures in a particular country-year pair against the level of PI in Table 6. Regardless of how different country-year pairs are weighted, higher PI is correlated to a higher dispersion in capital expenditures, even after controlling for the impact of the cycle.

Our result of a recession without financial distress is reminiscent of 'cleansing recessions,' but for different reasons. In that literature, recessions reduce the cost of reallocating resources to more productive activities. In our case, recessions increase stock market informativeness and allow for better resource allocation. Our work, however, highlights that 'cleansing recessions' can turn into 'sullying recessions' when accompanied by heightened liquidity concerns. Recessions accompanied by a weakness in the banking sector become periods of lower PI and worse allocation of resources. In prior literature,

[27] See Table 14 in Appendix D for summary statistics of the variables.

[28] Most estimates are robust to using the average earnings as the measure of economic activity (see Table 15) or to weighting observations with the number of stocks used in the estimation (see Table 16).

Table 5: Price Informativeness and Economic Conditions

	PI					
	Banking Stock Perf.	Bank Capital to Assets	Bank Loan Spreads	Non-performing Loans	Banking Panic	Banking Equity Crisis
	(1)	(2)	(3)	(4)	(5)	(6)
GDP Growth	−0.94	−0.60	−0.40	−1.24	−1.41	−1.44
	(0.83)	(1.14)	(1.36)	(1.42)	(1.01)	(1.01)
Liq. Measure	0.81	0.10^{**}	-0.02^{***}	-0.03^{***}	-0.14^{*}	−0.02
	(0.50)	(0.04)	(0.01)	(0.01)	(0.08)	(0.08)
Range	'84-'22	'05-'22	'84-'22	'05-'22	'84-'16	'84-'16
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	319	180	185	185	244	244

Notes: In each regression, the dependent variable is the PI. Column labels refer to the liquidity measure used in each regression. Both country and year fixed effects are included. The standard errors are clustered at the country level. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

tighter borrowing constraints, time-varying risk premia, counter-cyclical adverse selection in the market for used capital, and managers' incentives to hide reallocation needs during recessions have been proposed as potential mechanisms for counter-cyclical misallocation.[29] Ours is a novel mechanism that can generate higher or lower misallocation depending on whether a recession coincides with distress in the financial sector.

Third, according to our model, information acquisition increases during downturns with or without funding liquidity distress. This is consistent with the findings of Loh and Stulz (2018) and Jiang et al. (2015). The former documents that financial analysts produce longer and more frequent reports, and their reports have a larger price impact in bad times. The latter documents that management earnings forecasts become more frequent during recessions. Our model rationalizes these patterns as endogenous responses to less informative stock prices.

[29] See Ordonez (2013), Khan and Thomas (2013), Fajgelbaum et al. (2017) and Straub and Ulbricht (2023) for tighter financial constraints, David et al. (2022) for time-varying risk premia, Fuchs et al. (2016) for adverse selection, and Eisfeldt and Rampini (2008) for managerial incentives.

Table 6: Price Informativeness and Capital Allocation

	Dispersion of Investment			
	(1)	(2)	(3)	(4)
PI	0.004**	0.004**	0.004*	0.005**
	(0.002)	(0.002)	(0.002)	(0.002)
GDP Growth		0.036**		0.058*
		(0.018)		(0.031)
Range	1984-2022	1984-2022	1984-2022	1984-2022
Fixed Effects	Yes	Yes	Yes	Yes
Weights	No	No	Yes	Yes
Observations	344	344	344	344

Notes: In each regression, the dependent variable is the standard deviation of normalized capital expenditures. Both country and year fixed effects are included. In columns (3) and (4), each country-year observation is weighted with the number of stocks used to estimate the PI measure. The standard errors are clustered at the country level for the unweighted regressions. *p<0.1; **p<0.05; ***p<0.01

5.2 Alternative Information Structures

In the previous exercise, we analyzed the quantitative response of PI to productivity and liquidity shocks and its aggregate consequences. Here, we compare these reactions with those of alternative economies. The first column of Table 7a shows the changes in total information acquisition ($\lambda = \gamma\lambda_d + (1-\gamma)\lambda_n$), PI, output, and investment when the economy suffers the dual shock on productivity and liquidity needs. These changes are expressed as percentages relative to the benchmark aggregate state given by high Z and low γ. In this baseline economy, for instance, a dual shock that reduces Z and increases γ reduces PI by 10% and output by almost 15%. These numbers replicate the magnitudes of the changes depicted by the solid lines of Figure 6.

One alternative economy assumes that a fraction of traders receive signals exogenously, captured in the second column of Table 7a, where λ does not change when the economy suffers the dual shock.[30] This economy would suffer a 63% reduction in PI (six times larger than the benchmark with endogenous information), leading to a severe misallocation that reduces investment and output by more than 30%. This result highlights

[30]This fixed level is the average λ value that arises in the boom periods of the simulated benchmark economy.

the quantitative relevance of endogenous information acquisition. In the economy, agents react to lower information content in stock markets by acquiring more information and partially offsetting such reduction. Information acquisition provides a stabilizing force to the funding liquidity distress.

Another alternative economy we consider is one with lower information costs. Since two types of traders acquire different information in our setting, we consider two situations, shown in the last two columns of Table 7a. If night traders can acquire information at half the cost (third column), they would acquire more information; PI would decline only 6.5% instead of 10%, leading output to decline 12.5% instead of 14.6%, and investment to decline 13% instead of 18%. In contrast, if day traders can acquire information at half the cost (last column), they would also acquire more information, PI would decline by 11.7% instead of 10%, leading output to decline 15.5% instead of 14.6%, and investment to decline 19% instead of 18%.

An economy with lower information costs for night traders, ν_n, corresponds, for instance, to improvements in the Securities and Exchange Commission disclosure regulations, with more information requirements to be filed with each prospectus, more frequent filings, or firm-specific statistics publications. In contrast, lower information cost for day traders, ν_d, corresponds to regulations that disclose information about market transactions, such as recent changes that disclose stress tests, the use of Central Counterparties (CCPs), disclosures about trading positions or the use of discount windows. Our analysis shows that regulations that facilitate information about firms' profitability make the economy more resilient to recessions with financial shocks. In contrast, those that facilitate information about markets' operations do the opposite.

While Table 7a shows how alternative economies fare when facing both productivity and liquidity shocks *on impact*, Table 7b shows how the aggregates from these alternative economies would differ in their stochastic steady state. For instance, economies with lower information costs would display more information acquisition by all traders overall. However, if information about fundamentals is cheaper, the economy displays higher levels of investment, consumption, and output in a stochastic steady state, with the opposite happening if information about markets' operations and assets' liquidity is cheaper.

Table 7: Counterfactual Estimates

Moments	Baseline	Fixed λ	low ν_n	low ν_d
$\Delta\lambda_{\overline{z}\underline{\gamma}\to\underline{z}\overline{\gamma}}$	0.167	0	0.127	0.158
$\Delta PI_{\overline{z}\underline{\gamma}\to\underline{z}\overline{\gamma}}$	-0.099	-0.628	-0.065	-0.117
$\Delta Y_{\overline{z}\underline{\gamma}\to\underline{z}\overline{\gamma}}$	-0.146	-0.343	-0.125	-0.155
$\Delta Inv_{\overline{z}\underline{\gamma}\to\underline{z}\overline{\gamma}}$	-0.180	-0.381	-0.130	-0.190

(a) Impulse Response Functions

Moments	Baseline	Fixed λ	low ν_n	low ν_d
Y	0.168	0.129	0.186	0.159
C	0.066	0.041	0.074	0.062
Inv	0.102	0.088	0.112	0.096
R	0.122	0.103	0.124	0.122
PI	0.868	0.600	0.913	0.849
λ_n	0.288	0.150	0.372	0.309
λ_d	0.065	0.086	0.065	0.075

(b) Stochastic Steady State Averages

Notes: In the left-hand-side panel, each number denotes the percentage change at the moment when the economy moves from a long sequence of $\overline{z}\underline{\gamma}$ states to a $\underline{z}\overline{\gamma}$ state. For the fixed γ scenario, this is equivalent to moving to $\underline{z}\overline{\gamma}$. R denotes the gross interest rate that would clear the market in a competitive economy.

6 Conclusions

Stock markets contribute to the economy in two distinct ways. One is conveying information about the best use of resources through prices. The other is allowing agents to access liquidity quickly by trading stocks. Here, we have explored how changes in the relevance of the latter affect the performance of the former. When banks fall short, agents rely more on stocks to access liquidity, deteriorating their information revelation role. To connect the two roles of stock markets, we have provided a model of price formation with endogenous information acquisition about a firm's fundamentals and its stock's liquidity.

We provide a novel measure of stock price informativeness that isolates how well stocks reveal traders' private information from how good that information is. This measure captures the ability of prices to reveal available information, which is useful for making investment decisions, and not their forecasting ability, which mechanically goes down before unexpected events. Since our measure is structural, we can measure changes in price informativeness for many countries and periods and decompose the sources behind those fluctuations. We show that the informative role of stock markets suffers when funding liquidity is distressed, and market liquidity becomes more relevant. To assess the quantitative effect of a reduction in price informativeness in periods of banking and funding liquidity distress, we embed our stock trading setting into an otherwise standard real business cycle model with heterogeneous firms.

Once calibrated, we show that stocks provide less information during recessions accompanied by financial distress, and the corresponding capital misallocation considerably magnifies the recession's impact. Our counterfactuals further suggest that facilitating access to information about firms' fundamentals would lead to higher levels of output and consumption with reduced fluctuations while facilitating access to liquidity-relevant information, such as trading volumes or brokers' details, does the opposite.

Our study highlights a novel link between the functioning of credit markets and stock markets. Even though these markets provide similar services, like liquidity and information, some argue banks are more efficient in supplying liquidity and stocks in revealing information (see discussion in Gorton and Ordonez (2023)). We have argued here that when banks are in distress, stocks supply liquidity at the expense of their role in revealing information and guiding resource allocations.

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A Proofs

Proof of Lemma 1. The proof extends the corresponding proof in Grossman and Stiglitz (1980) to this environment. Since end-of-period wealth is additive in information acquisition costs and payoffs across stocks, proving the result for a single-asset case is sufficient. First, notice that the end-of-period wealth for informed and uninformed agents can be written as

$$\begin{aligned} W^{n,i}_{I,j} =& r\left(W_{oj} - c\left(\lambda^i_n\right)\right) + [(z_{in} + p'_i) - (1+r^I)p_i]X^n_{iI} \\ W^{n,i}_{U,j} =& rW_{oj} + [(z_{in} + p'_i) - (1+r^I)p_i]X^n_{iU} \end{aligned} \tag{27}$$

The expected value of being informed for a night trader j can be written as

$$E[V(W^{n,i}_{I,j}) \mid p] = E[e^{-aW^{n,i}_{I,j}} \mid p_i] = -exp\left(-aE\left[E[W^{n,i}_{I,j} \mid \theta] - \frac{a}{2}\operatorname{Var}\left[W^{n,i}_{I,j} \mid \theta\right]\Big| p_i\right]\right) \tag{28}$$

Combining Equations (7) and (27), we can write

$$E[W^{n,i}_{I,j} \mid \theta] = r\left(W_{oj} - c\left(\lambda^i_n\right)\right) + \frac{[Z + \tilde{\theta}_{in} + E[p'_i] - (1+r^I)p_i]^2}{a\operatorname{Var}(z_{in} + p'_i \mid \theta)} \tag{29}$$

$$\operatorname{Var}[W^{n,i}_{I,j} \mid \theta] = \frac{[Z + \tilde{\theta}_{in} + E[p'_i] - (1+r^I)p_i]^2}{a^2\operatorname{Var}(z_{in} + p'_i \mid \theta)} \tag{30}$$

since W_{0j} and p_i are not random given θ. Thus, we can rewrite Equation 28 as

$$\begin{aligned} E[V(W^{n,i}_{I,j}) \mid p] =& - exp\left[-ar(W_{oj} - c(\lambda^i_n)) - \frac{[Z + \tilde{\theta}_{in} + E[p'_i] - (1+r^I)p_i]^2}{2\operatorname{Var}(z_{in} + p'_i \mid \theta)}\right] \\ =& - exp\left[-ar(W_{oj} - c(\lambda^i_n))\right] \times \\ & E\left[exp\left(\frac{-1}{2\operatorname{Var}(z_{in} + p'_i \mid \theta)}\left[Z + \tilde{\theta}_{in} + E[p'_i] - (1+r^I)p_i\right]^2\right) \mid p_i\right] \end{aligned} \tag{31}$$

Now define

$$h_{in} := \text{Var}[\tilde{\theta}_{in} \mid p]$$
$$g_{in} := \frac{Z + \tilde{\theta}_{in} + E[p'_i] - (1 + r^I)p_i}{\sqrt{h_{in}}}$$

so, $E[V(W^{n,i}_{I,j}) \mid p]$ can be rewritten as

$$E[V(W^{n,i}_{I,j}) \mid p] = e^{ac^n(X_{in})} V(rW_{oj}) E_s \left[exp \left(\frac{-h_{in}}{\text{Var}(z_{in} + p'_i \mid \theta)} g^2_{in} \right) \mid p_i \right] \tag{32}$$

Since p_i is a linear function of θ, conditional on p_i, $\tilde{\theta}_{in}$ is normally distributed. Therefore, g^2_{in} is distributed with Chi-squared. Hence, moment generating function of g^2_{in} has the form:[31]

$$E[e^{-t(g_{in})^2} | p] = \frac{1}{\sqrt{1+2t}} exp \left(\frac{-t(E[g_{in} \mid p])^2}{1+2t} \right). \tag{33}$$

Now we can rewrite,

$$\begin{aligned} E[V(W^{n,i}_{I,j}) \mid p] =& \frac{1}{\sqrt{1 + \frac{h_{in}}{\text{Var}(z_{in} + p'_i | \theta)}}} exp \left(\frac{-h_{in} E[g_{in} \mid p]^2}{2\left(\text{Var}(z_{in} + p'_i \mid \theta) + h_{in}\right)} \right) \\ =& \frac{1}{\sqrt{1 + \frac{h_{in}}{\text{Var}(z_{in} + p'_i | \theta)}}} exp \left(\frac{-\left(Z + E[\tilde{\theta}_{in} \mid p_i] + E[p'_i] - (1 + r^I)p_i \right)^2}{2\left(\text{Var}(z_{in} + p'_i \mid \theta) + h_{in}\right)} \right) \end{aligned} \tag{34}$$

Furthermore, notice that

$$\frac{\text{Var}\left(z_{in} + p'_i \mid \theta\right)}{\text{Var}\left(z_{in} + p'_i \mid p\right)} = \frac{\text{Var}\left(z_{in} + p'_i \mid \theta\right)}{\text{Var}\left(z_{in} + p'_i \mid \theta\right) + h_{in}} = \frac{1}{1 + \frac{h_{in}}{\text{Var}\left(z_{in} + p'_i | \theta\right)}} \tag{35}$$

[31] For this to work, we need $\text{Var}(z_{in} + p'_i \mid \theta)$ to be deterministic given p_i, i.e., $\text{Var}[\text{Var}(z_{in} + p'_i \mid \theta) \mid p_i] = 0$. $\text{Var}(z_{in} + p'_i \mid \theta) = \text{Var}(\varepsilon_{in} + p'_i)$ is not a function of θ or a function of p_i.

Hence, we can rewrite it as

$$E\left[\exp\left(\frac{-h_{in}}{\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)}g_{in}^2\right)\middle|\, p\right]=\sqrt{\frac{\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)}{\mathrm{Var}\left(z_{in}+p_i'\mid p_i\right)}}\times \exp\left(\frac{-\left(Z+E\left[\tilde{\theta}_{in}\mid p_i\right]+E\left[p_i'\right]-(1+r^I)p_i\right)^2}{2\left(\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)+\mathrm{Var}\left[\theta_{in}\mid p_i\right]\right)}\right) \tag{36}$$

Then,

$$E\left[V(W_I^{n,i}j)\mid p\right]=e^{ac(\lambda_{in})}V(rW_{oj})\sqrt{\frac{\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)}{\mathrm{Var}\left(z_{in}+p_i'\mid p_i\right)}}\times \exp\left(\frac{-\left(Z+E\left[\tilde{\theta}_{in}\mid p_i\right]+E\left[p_i'\right]-(1+r^I)p_i\right)^2}{2\left(\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)+\mathrm{Var}\left[\theta_{in}\mid p_i\right]\right)}\right) \tag{37}$$

Following similar steps for the value of being uninformed yields

$$E\left[V\left(W_{u,j}^{n,i}\right)\mid p\right]=V\left(rW_{o_j}\right)\exp\left(\frac{-\left(Z+E\left[\tilde{\theta}_{in}\mid p_i\right]+E\left[p_i'\right]-(1+r^I)p_i\right)^2}{2\left(\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)+\mathrm{Var}\left[\theta_{in}\mid p_i\right]\right)}\right) \tag{38}$$

Therefore,

$$\frac{E\left[V\left(W_{I,j}^{n,i}\right)\mid p\right]}{E\left[V\left(W_{u,j}^{n,i}\right)\mid p\right]}=e^{ac(\lambda_{in})}\sqrt{\frac{\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)}{\mathrm{Var}\left(z_{in}+p_i'\mid\theta\right)+\mathrm{Var}\left(\theta_{in}\mid p_i\right)}}$$

□

Proof of Proposition 1. Conjecture a linear price function for each aggregate state s:

$$p_i^s = \phi_{i0}^s + \phi_{in}^s \theta_{in} + \phi_{id}^s \theta_{id} + \phi_{i\varepsilon}^s \tilde{\varepsilon}_{in}^- \tag{39}$$

Then, the signals uninformed traders will use from observing the price can be drawn from $(p_i^s - \phi_{i0}^s - \phi_{i\varepsilon}^s \tilde{\varepsilon}_{in}^- - \phi_{id}^s \theta_{id})/\phi_{in}$ and $(p_i^s - \phi_{i0}^s - \phi_{i\varepsilon}^s \tilde{\varepsilon}_{in}^- - \phi_{in}^s \theta_{in})/\phi_{id}$ for θ_{in} and θ_{id}, respectively. Since the prior distributions are Gaussian and the signal is a linear function of a Gaussian random variable, the posterior distribution for z_{in} is also Gaussian with mean and variance:

$$E_s\left[z_{in} \mid p_i\right] = Z + \frac{\frac{\bar{\theta}_{in}}{\sigma_{\theta_{in}}^2} + \frac{1}{\sigma_{\theta_{id}}^2}\left(\frac{\phi_{in}^s}{\phi_{id}^s}\right)^2\left(\frac{p_i - \phi_{i0}^s - \phi_{id}^s \bar{\theta}_{id} - \phi_{i\varepsilon}^s \tilde{\varepsilon}_{in}^-}{\phi_{in}^s}\right)}{\frac{1}{\sigma_{\theta_{in}}^2} + \frac{1}{\sigma_{\theta_{id}}^2}\left(\frac{\phi_{in}^s}{\phi_{id}^s}\right)^2} + \rho \tilde{\varepsilon}_{in}^- \tag{40}$$

$$\mathrm{Var}_s\left[z_{in} \mid p_i\right] = \left(\frac{1}{\sigma_{\theta_{in}}^2} + \frac{1}{\sigma_{\theta_{id}}^2}\left(\frac{\phi_{in}^s}{\phi_{id}^s}\right)^2\right)^{-1} + \sigma_{\varepsilon_{in}}^2 \tag{41}$$

Using these, we can write down the expectation and the variance for the total payoff from holding one share of the firm i:

$$E_s\left[z_{in} + p_i' \mid p_i\right] = E_s\left[z_{in} \mid p_i\right] + \sum_{s'} q_{ss'}[\phi_{io}^{s'} + \phi_{in}^{s'} \bar{\theta}_{in} + \phi_{id}^{s'} \bar{\theta}_{id} + \phi_{i\varepsilon}^{s'} \rho \tilde{\varepsilon}_{in}^-] \tag{42}$$

$$\begin{aligned} \mathrm{Var}_s\left[z_{in} + p_i' \mid p_i\right] &= \mathrm{Var}_s\left(\theta_{in} \mid p_i\right) + \mathrm{Var}_s\left(\varepsilon_{in} + p_i'\right) \\ &= \left(\frac{1}{\sigma_{\theta_{in}}^2} + \frac{1}{\sigma_{\theta_{id}}^2}\left(\frac{\phi_{in}^s}{\phi_{id}^s}\right)^2\right)^{-1} \\ &\quad + \mathrm{Var}_s\left(\phi_{0i}^{s'} + \phi_{di}^{s'} \theta_{di} + \phi_{ni}^{s\prime} \theta_{ni} + \left(1 + \phi_{\varepsilon i}^{s'} \rho^2\right) \varepsilon_{in} + \phi_{\varepsilon i}^{s'} \rho \tilde{\varepsilon}_{in}^-\right) \end{aligned} \tag{43}$$

Using these, we can rewrite the market clearing condition as

$$(1-\gamma)\lambda_{in}^{s}\left[\frac{Z+\tilde{\theta}_{in}+E_s\left[p_i'\right]-(1+r^I)p_i}{a\,\mathrm{Var}_s\left(\varepsilon_{in}+p_i'\right)}\right]+\gamma\lambda_{id}^{s}\left[\frac{Z+\tilde{\theta}_{in}-\theta_{id}+E_s[p_i']-(1+r^I)p_i}{a\left(\sigma_{\varepsilon_{id}}^2+\mathrm{Var}_s\left(\varepsilon_{in}+p_i'\right)\right)}\right]+$$
$$(1-\gamma)\left(1-\lambda_{in}^{s}\right)\left[\frac{E_s\left[z_{in}\mid p_i\right]+E_s\left[p_i'\right]-(1+r^I)p_i}{a\left(\mathrm{Var}_s\left(\varepsilon_{in}+p_i'\right)+\mathrm{Var}_s\left(\theta_{in}\mid p_i\right)\right)}\right]+$$
$$\gamma\left(1-\lambda_{id}^{s}\right)\left[\frac{E_s\left[z_{in}-z_{id}\mid p_i\right]+E_s\left[p_i'\right]-(1+r^I)p_i}{a\left(\mathrm{Var}_s\left(\varepsilon_{in}+p_i'\right)+\mathrm{Var}_s\left(\theta_{in}-\theta_{id}\mid p_i\right)\right)}\right]=\overline{K_i} \tag{44}$$

We suppress the aggregate state s in the rest of the proof to declutter the notation. First, denote

$$\begin{aligned} \chi_1&=\frac{\gamma\lambda_{id}}{a\left(\sigma_{\varepsilon_{id}}^2+\mathrm{Var}\left(\varepsilon_{in}+p_i'\right)\right)} & \chi_3&=\frac{\gamma\left(1-\lambda_{id}\right)}{a\left(\sigma_{\varepsilon_{id}}^2+\mathrm{Var}\left(\varepsilon_{in}+p_i'\right)+\mathrm{Var}\left(\theta_{in}-\theta_{id}\mid p_i\right)\right)} \\ \chi_2&=\frac{(1-\gamma)\lambda_{in}}{a\,\mathrm{Var}\left(\varepsilon_{in}+p_i'\right)}, & \chi_4&=\frac{(1-\gamma)\left(1-\lambda_{in}\right)}{a\left(\mathrm{Var}\left(\varepsilon_{in}+p_i'\right)+\mathrm{Var}\left(\theta_{in}\mid p_i\right)\right)} \end{aligned} \tag{45}$$

and $\chi=(\chi_1+\chi_2+\chi_3+\chi_4)$. One can rearrange the terms to get

$$\begin{aligned} &(\chi_1+\chi_2)\left(\theta_{in}+\rho\tilde{\varepsilon}_{in}^{-}\right)-\chi_1\theta_{id}+\chi\left(Z+\sum_{s'}\left[\phi_{i0}^{s'}+\phi_{in}^{s'}\bar{\theta}_n+\phi_{id}^{s'}\bar{\theta}_d+\phi_{i\varepsilon}^{s'}\rho\tilde{\varepsilon}_{in}^{-}\right]q_{ss'}\right) \\ &+(\chi_3+\chi_4)\left(\rho\tilde{\varepsilon}_{in}^{-}+\frac{\frac{\bar{\theta}_{in}}{\sigma_{\theta_{in}}^2}+\frac{1}{\sigma_{\theta_{id}}^2}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\left(\frac{p_i-\phi_{i0}-\phi_{id}\bar{\theta}_{id}-\phi_{i\varepsilon}\tilde{\varepsilon}_{in}^{-}}{\phi_{in}}\right)}{\frac{1}{\sigma_{\theta_{in}}^2}+\frac{1}{\sigma_{\theta id}^2}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}\right) \\ &-\chi_3\frac{\frac{\bar{\theta}_{id}}{\sigma_{\theta_{id}}^2}+\frac{1}{\sigma_{\theta_{in}}}\left(\frac{\phi_{id}}{\phi_{in}}\right)^2\left(\frac{p_i-\phi_{i0}-\phi_{in}\bar{\theta}_{in}-\phi_{i\varepsilon}\tilde{\varepsilon}_{in}^{-}}{\phi_{id}}\right)}{\frac{1}{\sigma_{\theta_{id}}^2}+\frac{1}{\sigma_{\theta_{in}}^2}\left(\frac{\phi_{id}}{\phi_{in}}\right)^2}-\chi(1+r^I)p_i=\bar{K}_i \end{aligned} \tag{46}$$

Next, we rearrange terms to leave p_i alone:

$$\underbrace{\left[(1+r^I)\chi+\chi_3\frac{\left(\frac{\phi_{id}}{\phi_{in}}\right)^2\frac{1}{\sigma^2_{\theta_{in}}}\operatorname{Var}\left[\theta_{id}\mid p_i\right]}{\phi_{id}}-(\chi_3+\chi_4)\frac{\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\frac{1}{\sigma_{\theta_{id}}}\operatorname{Var}\left[\theta_{in}\mid p_i\right]}{\phi_{in}}\right]}_{\tilde{\phi}}p_i=$$

$$\underbrace{(\chi_1+\chi_2)}_{\tilde{\phi}^s_{in}}\theta_{in}-\underbrace{\chi_1}_{\tilde{\phi}^s_{id}}\theta_{id}+\left[\rho\chi\left(1+\sum_s q_{ss'}\phi^{s'}_{i\varepsilon}\right)-(\chi_3+\chi_4)\left(\frac{\operatorname{Var}\left[\theta_{in}\mid p_i\right]}{\sigma^2_{\theta id}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\frac{\phi_{i\varepsilon}}{\phi_{in}}\right)\right.$$
$$\left.+\chi_3\frac{\operatorname{Var}\left[\theta_{id}\mid p_i\right]}{\sigma^2_{\theta in}}\left(\frac{\phi_{id}}{\phi_{in}}\right)^2\frac{\phi_{i\varepsilon}}{\phi_{id}}\right]\tilde{\varepsilon}^-_{in}+\chi\left(Z+\sum_{s'}\left[\phi^{s'}_{i0}+\phi^{s'}_{in}\bar{\theta}_{in}+\phi^{s'}_{id}\bar{\theta}_{id}\right]q_{ss'}\right)+$$
$$+\chi_3\operatorname{Var}\left[\theta_{id}\mid p_i\right]\left[\frac{\phi_{0i}+\phi_{ni}\bar{\theta}_{in}}{\phi_{di}}\left(\frac{\phi_{id}}{\phi_{in}}\right)^2\frac{1}{\sigma^2_{\theta_{in}}}-\frac{\bar{\theta}_{id}}{\sigma^2_{\theta_{id}}}\right]-\bar{K}_i$$
$$-(\chi_3+\chi_4)\operatorname{Var}\left[\theta_{in}\mid p_i\right]\left[\frac{\phi^s_{0i}+\phi^s_{id}\bar{\theta}_{id}}{\phi_{in}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\frac{1}{\sigma^2_{\theta_{id}}}-\frac{\bar{\theta}_{in}}{\sigma^2_{\theta_{in}}}\right] \tag{47}$$

In (47), $\phi^s_{in}=\frac{\tilde{\phi}^s_{in}}{\tilde{\phi}}$ and $\phi^s_{id}=\frac{\tilde{\phi}^s_{id}}{\tilde{\phi}}$. Hence, the expression in the Proposition follows. □

Proof of Proposition 2. The ex-ante bias associated with the hedge fund's estimator can be written as:

$$\begin{aligned}\left|E\left[E[\theta_{in}\mid p_i]-\theta_{in}\right]\right| &= \left|\frac{\frac{\bar{\theta}_{in}}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\left(\frac{p_i-\phi_{i0}-\phi_{id}\bar{\theta}_{id}-\phi_{i\varepsilon}\tilde{\varepsilon}^-_{in}}{\phi_{in}}\right)}{\frac{1}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}-\bar{\theta}_{in}\right| \\ &= \left|\frac{\frac{\bar{\theta}_{in}}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\left(\frac{\phi_{i0}+\phi_{id}\bar{\theta}_{id}+\phi_{in}\bar{\theta}_{in}+\phi_{i\varepsilon}\tilde{\varepsilon}^-_{in}-\phi_{i0}-\phi_{id}\bar{\theta}_{id}-\phi_{i\varepsilon}\tilde{\varepsilon}^-_{in}}{\phi_{in}}\right)}{\frac{1}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}-\bar{\theta}_{in}\right| \\ &= \left|\frac{\frac{\bar{\theta}_{in}}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\bar{\theta}_{in}}{\frac{1}{\sigma^2_{\theta_{in}}}+\frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}-\bar{\theta}_{in}\right|=0.\end{aligned}$$

The variance associated with the estimator can be written as:

$$\text{Var}\left[E[\theta_{in} \mid p_i] - \theta_{in}\right] = \text{Var}\left[\frac{\frac{\overline{\theta}_{in}}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\left(\frac{\phi_{id}(\theta_{id}-\overline{\theta}_{id})+\phi_{in}\theta_{in}}{\phi_{in}}\right)}{\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2} - \theta_{in}\right]$$

$$= \frac{\text{Var}\left[\frac{\overline{\theta}_{in}}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\left(\frac{\phi_{id}\theta_{id}}{\phi_{in}}\right) - \frac{\theta_{in}}{\sigma^2_{\theta_{in}}}\right]}{\left(\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\right)^2}$$

$$= \frac{\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}{\left(\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2\right)^2} = \frac{1}{\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}.$$

Lastly, under the mean-squared error loss function,

$$R\left(\theta_{in}, E[\theta_{in}|p_i]\right) = \left|E\left[E[\theta_{in} \mid p_i] - \theta_{in}\right]\right|^2 + \text{Var}\left[E[\theta_{in} \mid p_i] - \theta_{in}\right].$$

Hence

$$R\left(\theta_{in}, E[\theta_{in}|p_i]\right) = 0 + \frac{1}{\frac{1}{\sigma^2_{\theta_{in}}} + \frac{1}{\sigma^2_{\theta_{id}}}\left(\frac{\phi_{in}}{\phi_{id}}\right)^2}. \tag{48}$$

□

Proof of Proposition 3. The pricing function under $\rho = 0$ becomes

$$\begin{aligned} P_i =& \Phi_o + \Phi_n\theta_{in} + \Phi_d\theta_{id} \\ =& \Phi_o + \Phi_n z_{in} + \Phi_d z_{id} - \Phi_n\varepsilon_{in} - \Phi_d\varepsilon_{id}. \end{aligned}$$

Hence, when the price is regressed on θ_{in} and θ_{id}, the error term becomes $\nu_i = -\Phi_n\varepsilon_{in} - \Phi_d\varepsilon_{id}$, which is correlated with z_{in}. Let $\tilde{Z}_i$=[1 $\quad z_{in} \quad z_{id}$] ,$\tilde{\Phi} = [\Phi_o \quad \Phi_n \quad \Phi_d]$. Then

$$\hat{\Phi}_{OLS} = \tilde{\Phi} - \Phi_n\left(\frac{1}{n}\sum \tilde{Z}_i\tilde{Z}_i'\right)^{-1}\left(\frac{1}{n}\sum \tilde{Z}_i\varepsilon_{in}\right) - \Phi_d\left(\frac{1}{n}\sum \tilde{Z}_i\tilde{Z}_i{}'\right)^{-1}\left(\frac{1}{n}\sum \tilde{Z}_i\varepsilon_{id}\right).$$

First, because θ_{in}, θ_{id}, and ε_{in} are independent, the second term on the right-hand side can be decomposed as:

$$\left(\frac{1}{n}\sum \tilde{Z}_i\tilde{Z}_i'\right)^{-1}\left(\frac{1}{n}\sum \tilde{Z}_i\varepsilon_{in}\right) = \left(\frac{1}{n}\sum \tilde{Z}_iZ_i'\right)^{-1}\left(\frac{1}{n}\sum[\varepsilon_{in} \quad \theta_{in}\varepsilon_{in}+\varepsilon_{in}^2 \quad \theta_{id}\varepsilon_{in}+\varepsilon_{id}\varepsilon_{in}]\right)$$

$$\xrightarrow{\text{p}} \underbrace{\begin{bmatrix} 1 & \overline{Z_n} & \overline{Z_d} \\ \overline{Z_n} & \overline{Z_nZ_n} & \overline{Z_nZ_d} \\ \overline{Z_d} & \overline{Z_nZ_d} & \overline{Z_dZ_d} \end{bmatrix}^{-1}}_{\mathbf{Z}^{-1}} \begin{bmatrix} 0 \\ \sigma^2_{\varepsilon_n} \\ 0 \end{bmatrix},$$

where $\xrightarrow{\text{p}}$ denotes convergence in probability and $\overline{X_n}$ denotes $E[X_i]$. We can further write

$$Z^{-1} = \frac{1}{\det(z)} \begin{bmatrix} \sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}+\overline{\theta}_n^2(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}+\overline{\theta}_d^2)-\overline{\theta}_n^2\overline{\theta}_d^2 & -(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})\overline{\theta}_n^2 & -\theta_d(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}) \\ -(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})\overline{\theta}_n & \sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d} & 0 \\ -\theta_d(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}) & 0 & \sigma^2_{\theta_n}+\sigma^2_{\varepsilon_n} \end{bmatrix}.$$

Then, we can characterize the term as

$$\left(\frac{1}{n}\sum \tilde{Z}_i\tilde{Z}_i'\right)^{-1}\left(\frac{1}{n}\sum \tilde{Z}_i\varepsilon_{in}\right) = Z^{-1}\begin{bmatrix} 0 \\ \sigma^2_{\varepsilon_n} \\ 0 \end{bmatrix} = \frac{1}{det(Z)}\begin{bmatrix} \sigma^2_{\varepsilon_n}(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})\overline{\theta}_n \\ \sigma^2_{\varepsilon_n}(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}) \\ 0 \end{bmatrix},$$

where

$$
\begin{aligned}
det(z) =& 1\left((\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}+\overline{\theta}^2_n)(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}+\overline{\theta}^2_d)-\overline{\theta}^2_n\overline{\theta}^2_d\right)-\overline{\theta}_n\left(\overline{\theta_n}(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}+\overline{\theta}^2_d)-\overline{\theta_d\theta_n\theta_d}\right)\\
&+\overline{\theta_d}\left(\overline{\theta_n\theta_n\theta_d}-\overline{\theta}_d(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}+\overline{\theta}^2_n)\right)\\
=&\sigma^2_{\varepsilon_n}\sigma^2_{\varepsilon_d}+\sigma^2_{\varepsilon_n}\sigma^2_{\theta_d}+\sigma^2_{\varepsilon_n}\overline{\theta}^2_d+\sigma^2_{\varepsilon_d}\sigma^2_{\theta_n}+\sigma^2_{\varepsilon_d}\overline{\theta}^2_n+\sigma^2_{\theta_n}\sigma^2_{\theta_d}+\sigma^2\theta_d\overline{\theta}^2_d+\sigma^2{}_{\theta_d}\overline{\theta}^2_n\\
&-\sigma^2_{\varepsilon_d}\sigma^2_{\varepsilon_n}-\sigma^2_{\theta_d}\overline{\theta}^2_n-\overline{\theta}^2_d\overline{\theta}^2_n+\overline{\theta}^2_n\overline{\theta}^2_d+\overline{\theta}^2_d\overline{\theta}^2_n-\sigma^2_{\varepsilon n}\overline{\theta}^2_d-\sigma^2_{\theta_n}\overline{\theta}^2_d-\overline{\theta}^2_n\overline{\theta}^2_d\\
=&\sigma^2_{\varepsilon_n}\sigma^2_{\varepsilon_d}+\sigma^2_{\varepsilon_n}\sigma^2_{\theta_d}+\sigma^2_{\varepsilon_d}\sigma^2_{\theta_n}+\sigma^2_{\theta_n}\sigma^2_{\theta_d}.
\end{aligned}
$$

Following similar steps would yield:

$$
\left(\frac{1}{n}\sum\tilde{Z}_i\tilde{Z}'_i\right)^{-1}\left(\frac{1}{n}\sum\tilde{Z}_i\varepsilon_{id}\right)=\frac{1}{det(Z)}\begin{bmatrix}\sigma^2_{\varepsilon_d}(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n})\overline{\theta}_d\\ \\ 0\\ \\ \sigma^2_{\varepsilon_d}(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n})\end{bmatrix}.
$$

Therefore,

$$
\hat{\Phi}_{OLS}=\widetilde{\Phi}\;-\;\frac{1}{(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n})(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})}\begin{bmatrix}-\sigma^2_{\varepsilon_n}(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})\overline{\theta}_n\Phi_n-\sigma^2_{\varepsilon_d}(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n})\overline{\theta}_d\Phi_d\\ \\ \sigma^2_{\varepsilon_n}(\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d})\Phi_n\\ \\ \sigma^2_{\varepsilon_d}(\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n})\Phi_d\end{bmatrix}
$$

$$
=\begin{bmatrix}\Phi_o+\frac{\overline{\theta}_n\sigma^2_{\varepsilon_n}\Phi_n}{\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}}+\frac{\overline{\theta}_d\sigma^2_{\varepsilon_d}\Phi_d}{\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}}\\ \\ \Phi_n(1-\frac{\sigma^2_{\varepsilon_n}}{\sigma^2_{\varepsilon_n}+\sigma^2_{\theta_n}})\\ \\ \Phi_d(1-\frac{\sigma^2_{\varepsilon_d}}{\sigma^2_{\varepsilon_d}+\sigma^2_{\theta_d}})\end{bmatrix}.
$$

□

B Data Cleaning and Ajustments

This section describes our methodology for defining an economy (or a market) and the steps we take to standardize data within an economy.

B.1 Data Adjustments

We make two adjustments before estimating the pricing functions. First, in the model, the stock-level shocks ($\theta_{in}, \theta_{id}, \varepsilon_{in}, \varepsilon_{id}$) are assumed to be independently distributed across firms. While providing tractability, this assumption rules out any correlation across stocks beyond the one driven by the aggregate shock. Additionally, to ensure Φ does not vary across stocks, we assume the expected earnings ($\bar{\theta}_{in}$) and installed capital $\bar{K}_i$ do not vary across firms within a market. To accommodate departures from these assumptions in the data, we perform a factor analysis to residualize stock prices from common factors, past earnings, and total assets.

The factor analysis involves running the following regression for each stock i in market m (a country-year pair) at date t,

$$R_{it} = \alpha_i + \beta_{1i} MR_{mt} + \beta_{2i} SMB_{mt} + \beta_{3i} HML_{mt} + \epsilon_{it} \tag{49}$$

using monthly observations from $t-23$ to t where $R_{it} = (p_{it} - p_{it-1})/p_{it-1}$. We construct the three Fama-French factors for each market-date pair based on a balanced panel of stock prices from the past 24 months. The market return (MR) is constructed by looking at the month-to-month change in aggregate market cap. The small-minus-big (SMB) is the difference in the aggregate returns of the top and bottom 30% stocks in terms of market cap. The high-minus-low (HML) is the difference in the aggregate returns of the top and bottom 30% stocks in terms of book-to-market ratio.[32] We then use the estimates for β_{1i}, β_{2i}, and β_{3i}, which represent the factor loadings (the 'betas') for firm i, to residualize the prices by regressing them on second-order polynomials of the estimated betas, latest eps announcement (representing $\bar{\theta}_{in}$), and total assets (representing $\bar{K}_i$).

Second, in the model, we assume each stock provides ownership of one unit of in-

[32]See Figure 14 in Appendix D for the time series of the estimated factors for the US

stalled capital in the firm. In the data, however, the meaning of a single share, hence the stock price and earnings-per-share (eps), differ across firms. To make these variables comparable across firms and consistent with our model, we transform the factor-adjusted stock prices, eps, and eps forecasts to per-unit-of-asset values. We do this by multiplying the original value by the number of outstanding shares of the firm during a year and dividing it by the value of the Total Assets reported at the end of that year.

Table 8 shows summary statistics for the series we used in this Section to estimate pricing functions for the US in 2015. See Table 12 in Appendix D for the statistics for Japan and the UK.

Table 8: Summary Statistics for the US in 2015, Pricing Function

variable	mean	sd	min	median	max
ALPHA	0.00	0.03	-0.09	0.00	0.77
BMarketReturn	1.08	1.00	-18.53	1.00	6.79
BSMB	0.10	0.37	-1.75	0.08	5.69
BHML	-0.04	1.52	-45.22	-0.16	7.13
p	1.52	1.25	0.30	1.12	6.14
$\bar{K}_i$	11,709.18	34,237.83	19.85	2,122.20	552,257.00
$\bar{\theta}_n$	0.05	0.06	-0.57	0.05	0.15

Notes: The p and $\bar{\theta}_n$ figures are per unit of asset values constructed by multiplying original figures with the number of outstanding shares and dividing them by the value of their total assets. Total assets ($\bar{K}_i$) are given in thousands of US dollars.

B.2 Data Timing

Our data sources are of varying frequencies, and the accounting years (AY) differ across firms, which introduces several timing challenges. First, while the data on stock prices is monthly, data on company fundamentals is yearly. Second, flow variables, such as earnings, refer to flows during the AY, while stock variables, such as total assets, refer to values at the end of the AY.[33] Third, the eps forecast announcements are available daily even though the relevant target dates, by construction, are yearly.

To tackle these challenges, we consider the stock price for each stock i six months

[33]Furthermore, the accounting year generally differs across firms, and fundamentals are publicly announced a couple of months after the last day of the accounting year.

before the respective firm's AY ends. Call this date D_{it} where t refers to the associated year. For each stock-year pair it, we use the stock price at D_{it} to represent the model object p_i. Next, we map the median of the analysts' forecasts that are announced within a 15-day band around D_{it} for the current year eps to $Z + \theta_{in} + \rho\tilde{\varepsilon}_{in}^{-}$. The realized value for the same eps is then mapped to $Z + \theta_{in} + \rho\tilde{\varepsilon}_{in}^{-} + \varepsilon_{it}$.[34]

Figure 7 provides an example for a firm i whose previous AY ended in December 1995. Then, p_i is measured in June 1996. For θ_{in}, we use the forecasts announced around June 1996 for the earnings during the AY that ends in December 1996 (eps_i^f). The latest announcement for earnings (eps_i^a) on December 1995 represents what's publicly known when p_i is determined. For θ_{id}, we look at the realized range volatility between December 1995 and June 1996 ($Range_i$). The realized range volatility between June 1996 and December 1996 provides z_{id}.

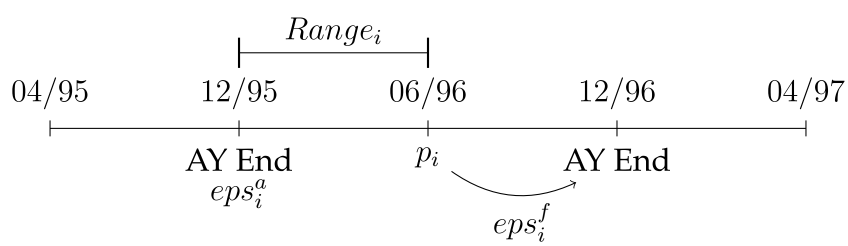

Figure 7: A Timing Example

This choice has implications for the interpretation of coefficients. Since the end of the accounting year is December 31st for most companies, the yearly estimates will generally refer to the stock prices and forecasts around June of the corresponding year. Hence, the effects of a major event before June, e.g., Bernanke's 'Taper Tantrum' in May 2013, are expected to be seen in the estimates for 2013. On the other hand, the effects of a major event after June, e.g., the collapse of Lehman Brothers in September 2008, are expected to be seen in the estimates for 2009.[35]

[34] If the stock prices were sampled at the same date for all firms, the traders' information set would differ firm by firm. Instead, we sample prices at different points in time to make sure that *i)* prices are equally spaced within the respective firm's AY and *ii)* the previous year's fundamentals are already announced, i.e., the stock prices reflect traders' knowledge of $\tilde{\varepsilon}_{in}^{-}$. If no earnings forecasts are available six months prior, we use five and seven months prior, in that order. See Appendix C.1 for the associated robustness checks for the timing assumptions.

[35] We drop Australia from our sample because the fiscal year for the majority of companies ends in July, leading to the price being evaluated around January 1st and indeterminacy regarding which year's real activity the price would be associated with.

B.3 Sample Restrictions

First, to guarantee an unambiguous match between the relevant monthly stock prices and the yearly fundamentals for each firm, we remove observations for which *i)* firms are in the finance/insurance sectors or ever cross-listed in multiple stock exchanges, *ii)* the listed accounting year-end dates are inconsistent (more than 12 months ahead) with the date of the stock price, or *iii)* the financial statements are announced earlier than the end of the reported accounting year or after more than six months. Second, to exclude firms that promise short-run losses with a possibility of abnormally high earnings in the long run, we remove observations where the company's earnings forecast indicates losses larger than 10% of the total value of its assets.[36] Third, to run the Fama-French analysis, we need monthly stock prices available six months before and after, and we require the associated market to have more than 30 stocks that constitute a balanced panel with at least 12 months of stock price data available in the past 24 months. Finally, we winsorize the adjusted earnings forecasts, earnings, and stock prices at a 5% level to deal with stock anomalies and potential inaccuracies in data.

B.4 Market Assignment

There are multiple ways of defining the relevant stock market for a given economy. While both I/B/E/S and Worldscope assign each company to a country, this assignment is inconsistent. Worldscope, before 2013, assigned companies based on "... country of major operations revenue of the company and if not determined by operations then country of headquarters", while after 2013, the assignment was based on the primary listing of the company. On the other hand, I/B/E/S assigns companies based on "country of domicile". The assigned country does not always match between the two datasets. To overcome these inconsistencies, we reassign each company to a country based on the location of the stock exchange in which the company's shares are traded.

First, we remove all stocks that have multiple nation or industry assignments in Worldscope. Second, we remove stocks that are cross-listed in multiple exchanges.[37] Third, we

[36]These observations are predominantly pharmaceutical companies that run consistent losses for several years. The correlation between earnings and stock price forecasts is negative for these firms, while it is close to 1 for the rest of the sample.

[37]I/B/E/S and Worldscope sometimes use different identifiers for the stocks of the same company in

Table 9: Discrepancy Between the Original and the Exchange-Based Country Assignments

#	IBES Assign.	Stock Exch. Assign.	#	WS Assign.	Stock Exch. Assign.
0.13	Brazil	United States	0.07	China	Hong Kong
0.12	Netherlands	United States	0.03	Netherlands	United Kingdom
0.09	China	Hong Kong	0.03	Finland	United States
0.08	Switzerland	United States	0.03	Netherlands	United States
0.06	Finland	United States	0.02	Netherlands	Poland
0.05	United Kingdom	United States	0.02	Switzerland	Germany
0.05	Hong Kong	United States	0.02	Netherlands	Germany
0.04	France	United States	0.01	China	United States
0.04	Germany	United States	0.01	Finland	United Kingdom
0.03	Netherlands	United Kingdom	0.01	Finland	Sweden

Notes: The table shows ten pairs of countries with the highest fraction of discrepancy between the original and the stock exchange-led assignments in our final sample. Each number represents the fraction of the companies with the original assignment (I/B/E/S or WS) reassigned to another country based on where they trade.

link stock exchanges to countries using the bridge provided by Worldscope and aggregate the exchanges within a single country. If the stock exchange information is missing, listed as 'others,' or the stock is traded over the counter, we assign the stock to a market based on the Worldscope nation assignment.

We prefer grouping companies based on the stock exchange because the stock prices are determined primarily by that country's traders and their liquidity needs. Hence, measuring PI necessitates grouping companies based on who owns and trades their shares. Regardless, in most cases, the exchange-country disconnect is minimal. Table 9 provides the fraction of companies that are reassigned based on their stock exchange and the country assigned to them by I/B/E/S or Worldscope.

B.5 Exchange Rate Adjustments

In this section, we describe how we standardize the exchange rates for each market to allow cross-sectional and time-series comparability. I/B/E/S and Worldscope provide the currency used in each entry, while I/B/E/S further provides daily exchange rates

different exchanges. For example, we've noticed an instance where "SUEZ" and "SUEZ LYONNAISE DES EAUX" refer to the same company but trade on different markets and have different I/B/E/S tickers ("SZE" and "@LYE"). While Worldscope retains data for both, I/B/E/S only selects and collects the forecast for "@LYE." Unfortunately, it is not possible to systematically deal with these instances.

throughout its sample coverage.

First, we determine the dominant currency for each market using the most commonly used currency across its stocks in Worldscope. Second, we convert all values in a market (prices, actuals, forecasts, etc.) to the dominant currency using the date of the closest available exchange rate. Third, for countries that have adopted the Euro as their exchange rate, we convert all numbers to the country's original currency using the exchange rate at the time of adoption.

We validate our steps by comparing a random sample of our adjusted series with other sources that already present the data in the destination exchange rate.

B.6 Mergers and Acquisition

Mergers and acquisitions (M&A) and stock splits create challenges for time-series compatibility of data by changing what the company consists of from one year to the other. Both I/B/E/S and Worldscope describe how they handle M&As and stock splits in their respective guidebooks. We went through the raw data for ten well-known cases,[38] and all ten were consistent with the explanations:

1. The acquiring company retains its I/B/E/S ticker and entity name, continuing to report stock data under these identifiers. The data recording for the acquired company is discontinued.

2. In the case of a merger, the newly formed entity continues using the I/B/E/S ticker of one of the original companies and adopts a new entity name. The I/B/E/S ticker of the other company ceases to record data.

3. For stock splits, both I/B/E/S and Worldscope databases adjust their records to reflect the new stock size, ensuring consistency across reported values.

[38] These are Pfizer's acquisition of Warner-Lamber in 2000, Vodafone's acquisition of Mannesmann in 2000, Exxon and Mobil merger in 1999, Glaxo Wellcome and SmithKline Beecham merger in 2000, Gaz de France and Suez merger in 2008, Dow Chemical and DuPont merger in 2009 and the following division into spinoffs in 2019, Heinz and Kraft merger in 2015, United Technologies and Raytheon merger in 2019, Apple's stock split in 2020, and Tesla's stock split in 2022. See Footnote 37 for a distinct issue we noticed during this exercise.

C Robustness Checks

C.1 Monthly Variation in Forecasts

In the baseline analysis, we focus on earnings forecasts that are made six months prior to the fiscal year-end date. This ensures that six months have passed since the end of the prior fiscal year; hence, the associated earnings announcements are already made for most firms. Therefore, the prices at that point already carry the information from the previous year's earnings.

If the forecasts change substantially month-to-month, then our results would be sensitive to the timing assumptions made. Here, we show that the month-to-month variation in earnings forecasts is relatively small. We focus on the forecasts of companies in the US and Japan and set the monthly forecast for a stock as the median forecast made within 15 days of the beginning of the month. We restrict attention to firms whose fiscal years end in the usual months -January in the US and March in Japan- and to forecasts made 4 to 8 months before the fiscal year-end. We only include firms whose forecasts are announced in all months.

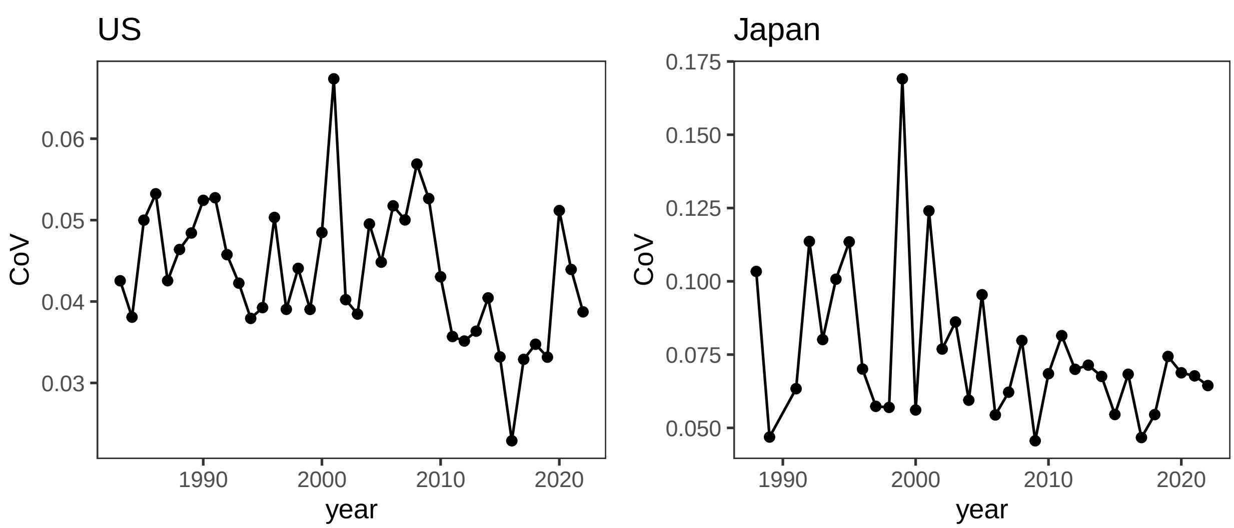

Figure 8: Month-to-Month Coefficient of Variation of Earnings-per-Share Forecasts for the Median Firm

Figure 8 shows the monthly variation in forecasts for the median stock. In the US, the monthly standard deviation for the median stock is almost always below 6% of the mean, while it's mostly below 10% for Japan.

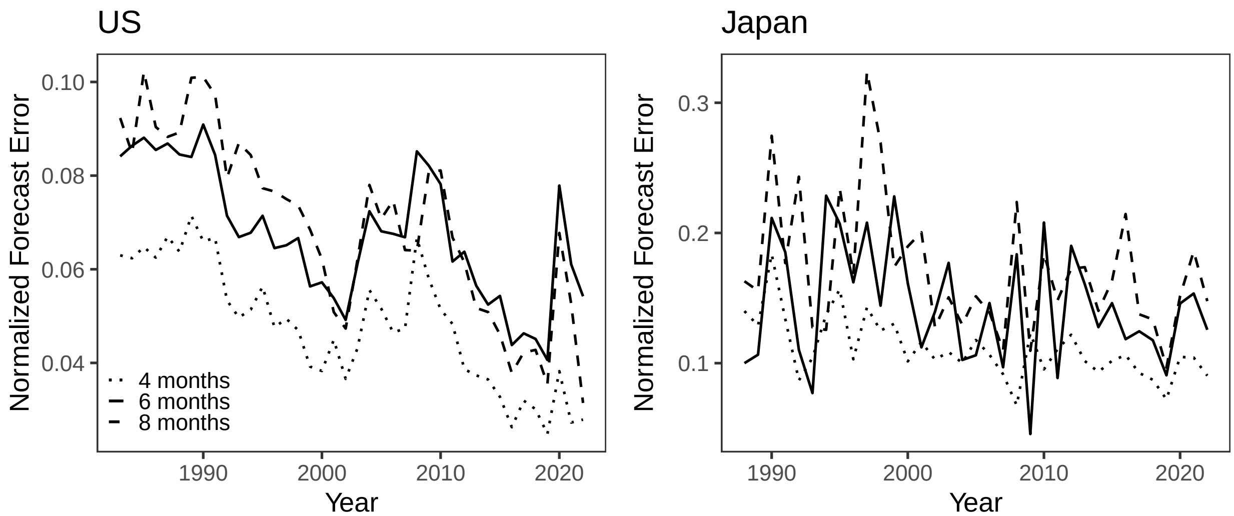

Figure 9: Forecast Error for Earnings-per-Share Forecasts for the Median Firm
Notes: The dotted, solid, and dashed lines represent the normalized errors for forecasts made 8, 6, and 4 months before the fiscal year-end date for the median stock, respectively.

Figure 9 plots the normalized forecast error, i.e., the absolute forecast error divided by the realized value, for the median stock. For both the US and Japan, the forecast error declines as the forecast date becomes closer to the fiscal year's end, with few exceptions. However, the improvement for the median stock mostly stays small.

C.2 Range Volatility Measures

In this section, we provide summary statistics on the range volatility measure we use. Figure 10 depicts the median firm's normalized range volatility measures for Japan and the US. We restrict attention to years where there are at least 40 stocks with monthly price data that allows the estimation of range volatility. Both measures are relatively stable, with high volatility episodes in 2001, 2008, and 2020.

The experience of the median firm is representative of the majority of the firms in both stock markets. Table 10 provides the cross-sectional summary statistics in 2018 for Japan and the US. The interquartile range is similar to the time series variation for the median firm.

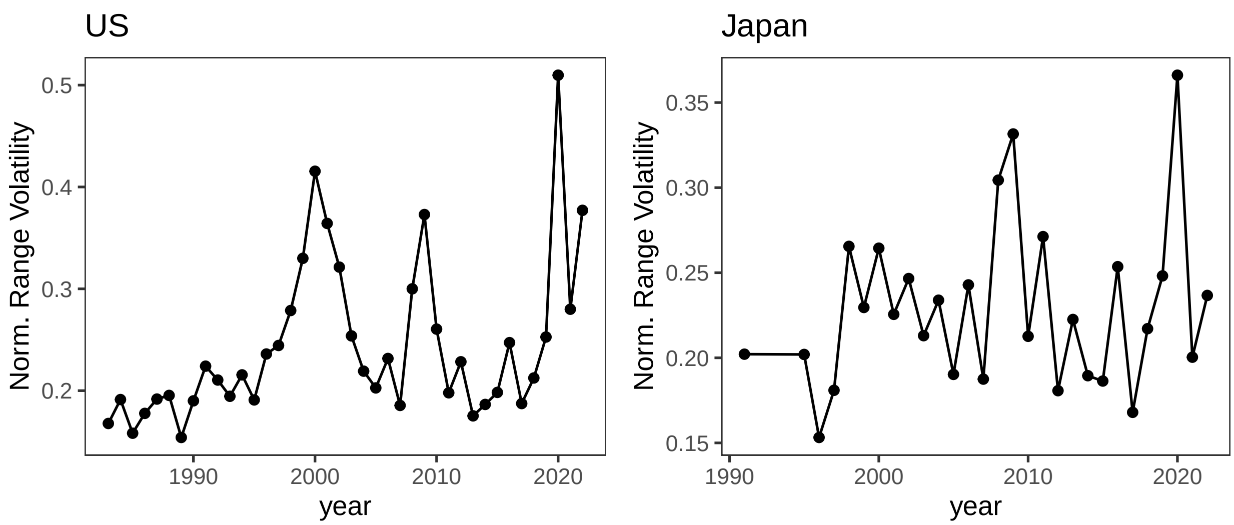

Figure 10: Normalized Range Volatilities for the Median Firm

Table 10: The Summary Statistics for the Normalized Range Volatilities in 2018

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
US	0.01	0.07	0.11	0.13	0.16	2.37
Japan	0.03	0.06	0.11	0.13	0.15	0.74

C.3 Distribution of signals and forecast errors: United States

Here, we report the parameters determining the distribution of signals and forecast errors for the United States as an illustration of the results. Figure 11 shows the time series of the forecast error variance ($\sigma^2_{\varepsilon_n}$ and $\sigma^2_{\varepsilon_d}$). Figure 12 shows the time series of the signal averages ($\bar{\theta_n}$ and $\bar{\theta_d}$), and Figure 13 shows ($\sigma^2_{\theta_n}$ and $\sigma^2_{\theta_d}$) that are needed to construct the measure of PI.

D Additional Figures and Tables

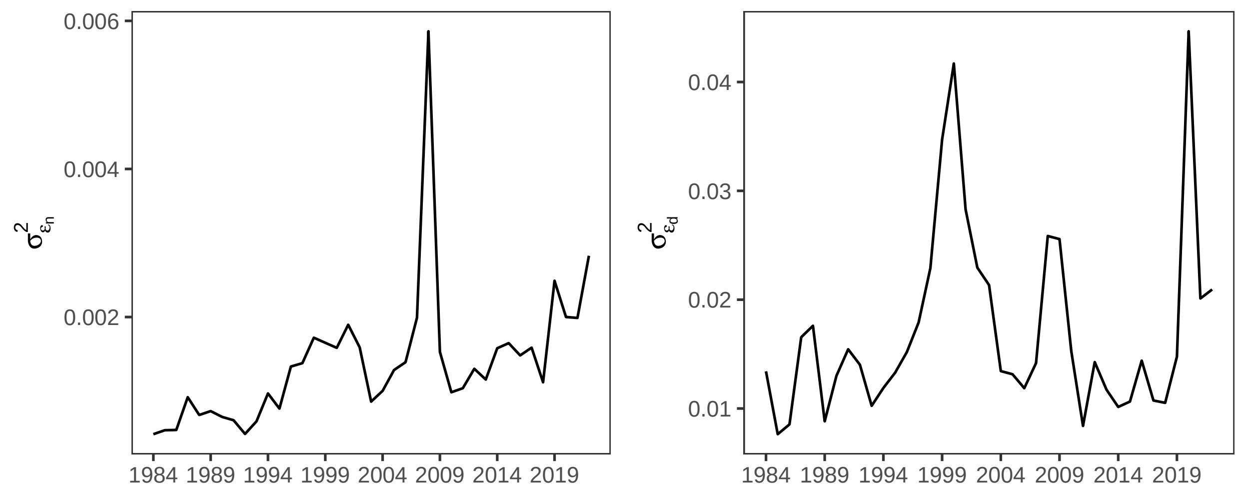

Figure 11: Forecast Error Variance Estimates in the US for a Balanced Panel

Figure 12: Median Earnings and Volatility Signals in the US for a Balanced Panel

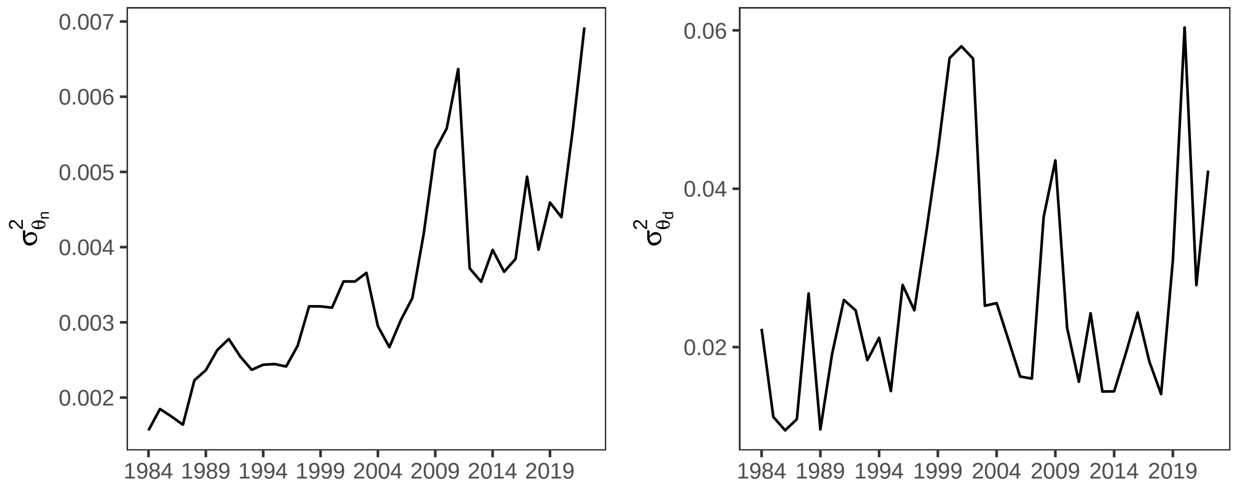

Figure 13: Variances of the Earnings and Volatility Signals in the US for a Balanced Panel

Table 11: List of Countries and the Data Coverage

CountryName	IBESCode	YearRange	CountryName	IBESCode	YearRange
Germany	ED	2000 - 2022	Japan	FJ	1998 - 2022
France	EF	2000 - 2022	South Korea	FK	1994 - 2022
Poland	EG	2018 - 2018	Malaysia	FM	1997 - 2022
Italy	EI	2004 - 2021	Thailand	FT	2004 - 2022
Netherlands	EN	2000 - 2002	Brazil	LB	2009 - 2022
Switzerland	ES	2001 - 2022	Canada	NC	1991 - 2022
Turkey	ET	2000 - 2000	Finland	SF	2008 - 2022
United Kingdom	EX	1992 - 2022	Norway	SN	2010 - 2010
Taiwan	FA	1999 - 2022	Sweden	SS	1999 - 2022
China	FC	2006 - 2022	United States	US	1984 - 2022
Hong Kong	FH	2004 - 2022			

Table 12: Summary Statistics for Japan and the UK in 2015, Pricing Function

variable	mean	sd	min	median	max
ALPHA	0.01	0.03	-0.13	0.01	0.15
BMarketReturn	1.12	1.15	-3.63	0.96	6.58
BSMB	0.51	1.02	-1.59	0.34	4.59
BHML	-0.04	1.31	-8.04	0.03	5.58
p	1.19	1.19	0.22	0.72	5.49
$\bar{K}_i$	242,029.32	583,495.49	2,080.62	56,076.00	4,427,773.00
$\bar{\theta}_n$	0.04	0.03	-0.01	0.04	0.10

(a) Japan Estimates

variable	mean	sd	min	median	max
ALPHA	0.01	0.02	-0.05	0.01	0.09
BMarketReturn	0.78	0.71	-1.48	0.77	4.26
BSMB	0.10	0.46	-0.98	0.02	3.75
BHML	0.02	0.51	-1.22	-0.07	1.93
p	1.44	1.13	0.30	1.06	4.91
$\bar{K}_i$	4,597.32	15,712.71	9.94	676.25	176,474.59
$\bar{\theta}_n$	0.07	0.06	-0.24	0.07	0.21

(b) The UK Estimates

Notes: The p and $\bar{\theta}_n$ figures are per unit of asset values constructed by multiplying original figures with the number of outstanding shares and dividing them by the value of their total assets. Total assets ($\bar{K}_i$) are given in thousands of Japanese yen and Pound sterling.

Table 13: Summary Statistics for Japan and the UK in 2015, Random Variables

variable	mean	sd	min	median	max
θ_n	0.05	0.04	-0.03	0.04	0.17
θ_d	0.22	0.16	0.03	0.18	1.00
ε_n	0.00	0.02	-0.16	0.00	0.06
ε_d	0.04	0.12	-0.59	0.03	0.51
$\tilde{\varepsilon}^-$	0.00	0.02	-0.13	0.00	0.05

(a) Japan Estimates

variable	mean	sd	min	median	max
θ_n	0.08	0.06	-0.22	0.07	0.31
θ_d	0.20	0.11	0.04	0.17	0.91
ε_n	0.00	0.04	-0.33	0.00	0.28
ε_d	-0.01	0.08	-0.51	0.00	0.34
$\tilde{\varepsilon}^-$	-0.01	0.04	-0.33	0.00	0.25

(b) The UK Estimates

Notes: The figures are per unit of asset values constructed by multiplying original figures with the number of outstanding shares and dividing them by the value of their total assets.

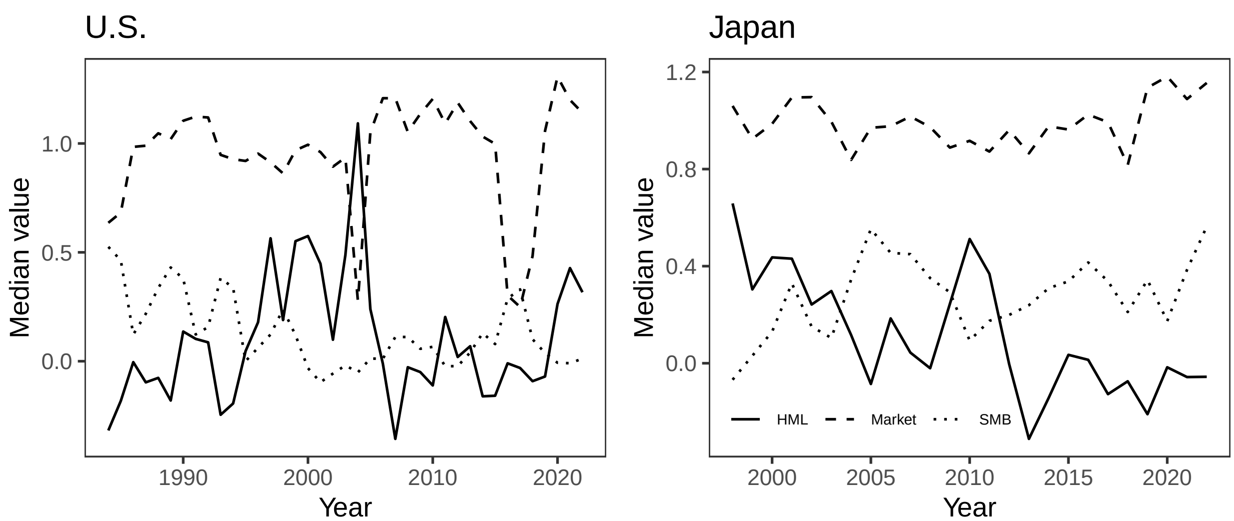

Figure 14: Factor Loadings for the Median Stock from the Fama-French Estimation
Notes: The dashed, solid, and dotted lines refer to the beta estimates for the Market return, High-Minus-Low, and Small-Minus-Big factors. We restrict attention to years where there are at least 40 stocks with monthly price data that allows the estimation of the factors.

Table 14: Summary Statistics on Economic Conditions in A Panel of Countries

Statistic	N	Mean	St. Dev.	Min	Max
Bank Capital to Assets	180	6.865	1.954	4.109	10.565
Bank Loan Spreads	185	4.039	6.645	−0.032	39.216
Non-performing Loans	185	2.471	2.509	−0.090	16.911
Banking Panic	244	0.041	0.199	0	1
Banking Equity Crisis	244	0.037	0.189	0	1
PI	344	0.820	0.232	0.014	1.000
Avg Earnings	344	0.053	0.016	0.017	0.097
GDP Growth Rate	344	0.026	0.031	−0.102	0.120
Banking Stock Performance	319	0.102	0.049	0.011	0.293

Notes: The average earnings denote the weighted average of the normalized earnings. The liquidity crisis indicators, Banking Panic, and Banking Equity Crisis are from Baron et al. (2021). The continuous liquidity measures, Bank Capital to Asset, Bank Loan Spreads, and Non-performing Loans are from the World Bank. The authors estimate the PI measure.

Table 15: Price Informativeness and Economic Conditions, Alternative Measure

	PI					
	Banking Stock Perf.	Bank Capital to Assets	Bank Loan Spreads	Non-performing Loans	Banking Panic	Banking Equity Crisis
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Earnings	0.46	2.16	−0.75	2.27*	−1.41	−1.24
	(1.44)	(1.41)	(2.62)	(1.38)	(1.74)	(1.71)
Liq. Measure	0.71	0.09**	−0.02***	−0.03***	−0.14*	−0.02
	(0.46)	(0.04)	(0.01)	(0.00)	(0.08)	(0.07)
Range	1984-2022	2005-2022	1984-2022	2005-2022	1984-2016	1984-2016
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	319	180	185	185	244	244

Notes: In each regression, the dependent variable is PI. Column labels refer to the liquidity measure used in each regression. The standard errors are clustered at the country level. *p<0.1; **p<0.05; ***p<0.01

Table 16: Price Informativeness and Economic Conditions, Weighted Least Squares

	PI					
	Banking Stock Perf.	Bank Capital to Assets	Bank Loan Spreads	Non-performing Loans	Banking Panic	Banking Equity Crisis
	(1)	(2)	(3)	(4)	(5)	(6)
GDP Growth	-2.28^{***}	-3.22^{***}	-2.20^{*}	-3.97^{***}	-2.25^{***}	-2.22^{***}
	(0.70)	(1.04)	(1.23)	(1.09)	(0.78)	(0.78)
Liq. Measure	1.32^{***}	0.21^{***}	0.04^{*}	-0.08^{***}	-0.06	-0.06
	(0.43)	(0.04)	(0.02)	(0.02)	(0.04)	(0.04)
Range	1984-2022	2005-2022	1984-2022	2005-2022	1984-2016	1984-2016
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	319	180	185	185	244	244

Notes: In each regression, the dependent variable is PI. Column labels refer to the liquidity measure used in each regression. Each country-year observation is weighted with the number of stocks used to estimate the PI measure. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$