The Collateral Link Between Volatility and Risk Sharing *

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Abstract

We show that the effect of aggregate volatility on idiosyncratic risk sharing depends on the nature of collateral sustaining insurance. While volatility *increases* the value of public assets—more useful for consumption smoothing—it *decreases* the value of private assets—more exposed to aggregate variation. Hence, a more volatile economy weakens risk sharing when collateral composition is biased toward private assets. When applied to financial intermediaries that rely heavily on private collateral to share risks, aggregate instability is more likely to induce financial instability. We empirically show that the sensitivity of risk sharing to aggregate volatility indeed depends on the collateral composition as predicted by the theory.

Keywords: collateral, aggregate volatility, financial stability, risk sharing, convenience yield.

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1 Introduction

Financial intermediaries rely heavily on interbank markets to insure against idiosyncratic shocks to their assets and/or liabilities. This is evident by the ubiquitous use of derivative contracts to hedge unwanted exposures to certain assets and/or aggregate conditions, and repo contracts to manage liquidity risk. The smooth functioning of interbank markets, critical for the efficiency and stability of the financial system, depends on the extent of idiosyncratic risk and the possibilities to hedge against them. While *aggregate volatility* (time series volatility of aggregate consumption) may affect the exposure of intermediaries to certain shocks (*the demand for insurance*), *counterparty risk* (the danger that one of the parties might default on a promise) may constrain the ability to write insurance contracts against those risks (*the supply of insurance*). Here, we show that these two risks are strongly related through valuation effects: aggregate volatility affects the collateral value used to relax counterparty risk. But do aggregate volatility changes improve or impair interbank markets' risk-sharing function? Does it strengthen or weaken financial stability?

The relevance of these questions became apparent during the 2008 crisis when insurance across banks broke down.[1] Their potential magnitude also becomes clear by the sheer size of interbank markets. The trading of derivative contracts amounts to almost nine times the world GDP in 2020, in notional terms.[2] Similarly, total repo liabilities reached $4.7 trillion dollars during March 2020.[3] While many repo contracts are traded to hedge directly against idiosyncratic shocks, the lion's share of derivative contracts are written conditional on aggregate events (such as interest rate or exchange rate swaps) and are traded among intermediaries to hedge against idiosyncratic exposure to those aggregate conditions.

Behind the massive underwriting of these contracts lies the heavy use of collateral to relax pervading counterparty risks.[4] According to the 2014 report of the International Swap and Derivative Association (ISDA), *"The use of collateral agreements is substantial. Among all firms responding to the survey, 91% of all OTC derivatives trades (cleared and non-cleared) were subject to a collateral agreements at the end of 2013."* The left panel of Figure 1 shows

[1]Heider et al. (2009) and Acharya and Merrouche (2012) show liquidity hoarding in the interbank market during the 2007/2008 crisis, in the midst of several liquidity shortages.

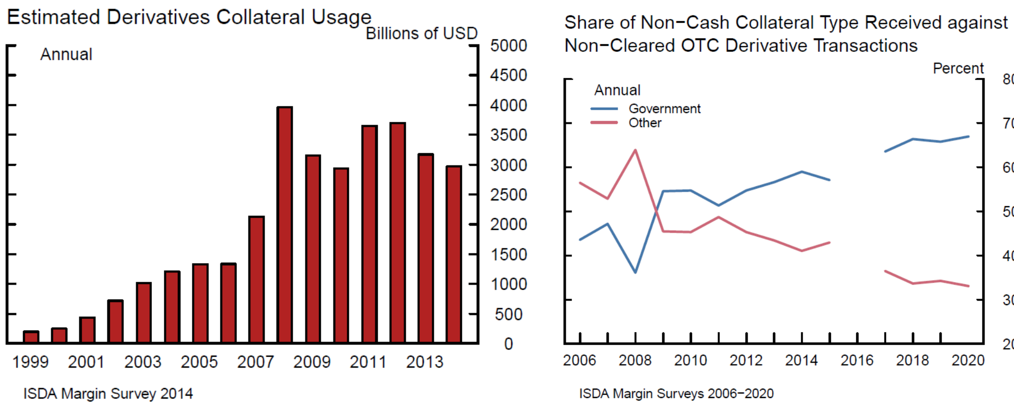
[2]Bank of International Settlements.

[3]Financial Accounts of the United States. Repo liabilities reached a peak of five trillion dollars in March 2008. SEC "Primer on Money Market Funds and the Repo Market," by Baklanova, Kuznits and Tatum, February 18, 2021. Figure 4 of Copeland et al. (2014) shows a dramatic drop in repo volumes after the default of Lehman Brothers, which was concentrated in riskier collateral classes.

[4]Derivatives trade under International Swap and Derivative Association (ISDA) master agreements, which often involve a Credit Support Annex (CSA) that specifies the conditions under which parties must post collateral. Other credit enhancements include rating triggers that terminate the transaction and third-party guarantees.

the dramatic increase in collateral posted for derivative contracts since 2000, which doubled during the 2008 crisis and remains at around four trillion dollars. The right panel of Figure 1 shows the contemporaneous change in the composition of non-cash collateral for derivatives, which tilted towards more intensive use of government-related assets, from 40% in 2008 to almost 70% in 2020.[5]

Figure 1: Use and Composition of Collateral



Most of the assets, both public (such as government bonds) and private (such as asset-backed securities), that are used as *collateral* to smooth *idiosyncratic shocks intratemporally* are also used as *stores of value* to smooth *aggregate shocks intertemporally.* This dual role of public and private assets links aggregate volatility and idiosyncratic risk sharing. Using assets as collateral affects the extent of idiosyncratic risk sharing in the economy and, thus, the intertemporal value of assets. In parallel, the intertemporal value of assets determines their use as collateral and, hence, the extent of idiosyncratic risk sharing.

We construct a simple theoretical framework to show how aggregate volatility shocks affect the extent of risk sharing depending on the ratio of private to public assets used as collateral. The reason is that their valuations react to volatility in opposite directions. On the one hand, an increase in aggregate volatility increases the intertemporal price of public assets, the only assets that, because of taxation, can credibly provide future noncontingent payment promises. If intermediaries face high aggregate volatility (when the variance of future aggregate realizations is high), a dollar that pays off in bad periods delivers a high marginal utility, making the promise more valuable. On the other hand, an increase in aggregate volatility reduces the intertemporal price of private assets. The reason is that

[5]While cash remains the most prevalent type of collateral, its share declined from 80% in 2008 to 70% in 2020 (ISDA Margin Surveys). In addition, the November 2021 Financial Stability Report, published by the Federal Reserve, shows that over the last few years, central clearing counterparties (CCPs) in derivatives markets have heavily relied on non-cash collateral—mainly U.S. Treasuries securities—to manage their credit and liquidity risk (see box Liquidity Vulnerabilities from noncash collateral at central counterparties).

their payoffs are tied to the evolution of the aggregate economy more closely than government bonds (as discussed and documented by Jiang et al. (2021)), and riskier assets become less valuable when the variance of future aggregate realizations is high.

The defining feature differentiating assets in our model is not the issuer (public or private) but their exposure to aggregate volatility. Still, just for expositional reasons, we will refer to assets with these two different exposures as public and private assets. In practice, some public assets cannot provide noncontingent future promises (e.g., emerging market government bonds) while some private assets can (e.g., supranational debt). From that vantage point, when the ratio of private to public assets used as collateral is relatively low, as in the decade after the 2008 crisis, the value of public assets is more relevant to determining the value of collateral. In this case, higher volatility implies more valuable collateral on average and better idiosyncratic insurance—a sort of "positive externality" of volatility on risk sharing. The opposite is true when the ratio of private to public assets is relatively high, as was the case before the crisis in 2008.

While these insights are particularly relevant for the role of collateral for insurance motives in interbank markets, the intuition extends to more general contexts. For example, home equity loans—a contract intended to hedge future idiosyncratic wealth shocks— depend on the value of the underlying home used as collateral. To the extent that home valuations decline with aggregate volatility, a more volatile environment also reduces the ability of home equity loans to hedge idiosyncratic wealth shocks. More generally, to the extent that collateral is used to back any contract between counterparties, the value of those contracts will depend on the sensitivity of the underlying collateral to aggregate uncertainty. Our mechanism also generalizes toward other aggregate shocks besides aggregate volatility. When any aggregate shock affects the valuation of different assets used as collateral for insurance in opposite directions, then the collateral composition matters for the impact of those shocks on risk sharing.

Linking aggregate volatility and risk sharing has several additional implications. First, volatility affects governments' financing costs depending on the use of sovereign debt *and* of private assets as collateral in financial markets.[6] Second, creating private collateral is more likely in a stable economic environment, which encourages their wider use as collateral and results in the system becoming increasingly more fragile—in the sense of fewer risk-sharing opportunities—to a sudden increase in aggregate volatility. In short, *economic stability may plant the seeds of its instability.*

Our model guides us on how to measure the convenience yield in the data. When evalu-

[6]This effect became particularly relevant during turbulent times, such as the wake of the COVID-19 pandemic, which represented a sudden, unexpected shock to the economy.

ated in a standard CARA-Normal setup, our model also delivers testable implications on the sensitivity of risk sharing to aggregate volatility as a function of the private/public composition of collateral, which we take to the data. We are able to map the *convenience yield*—the additional value assigned to assets net of their payoff risks—to the extent of risk sharing in the economy (less risk sharing leads to a higher convenience yield). We perform an empirical analysis using rates to compute the convenience yield and test whether it responds more positively to aggregate volatility in periods when collateral is tilted towards private assets.

Using high-frequency data, we study the period surrounding the crisis in 2008, motivated by the evidence of a reduction in the use of private collateral in derivative contracts after the crisis (see Figure 1). We show that the sensitivity of the convenience yield to aggregate volatility increased dramatically leading up to the crisis, but then declined and remained low afterwards, consistent with a more intensive use of public collateral spurred by regulatory efforts after the crisis.[7]

Related Literature: The literature on collateral is extensive and highlights several uses, such as backing loans to borrowers with investment projects (such as Kiyotaki and Moore 1997) or liquidity needs (such as Holmstrom and Tirole 1998). Our work belongs to the corner of the literature, highlighting the use of collateral to back insurance and other hedging contracts. Krishnamurthy (2003), in the spirit of Kiyotaki and Moore (1997), studies the collateralization of insurance, but with a focus on collateral-constrained insurance *against aggregate shocks*. In contrast, we focus instead on insurance *against idiosyncratic shocks* and how aggregate volatility affects such risk-sharing through valuation. In a similar setting, Di Tella (2017) highlights that downturns coincide with higher idiosyncratic risk. An uncertainty shock concentrates losses in ways that depress asset prices and lead to downturns, creating a feedback effect. Our work focuses instead on the role of ex-ante aggregate risk on the valuation of different types of assets, not upon uncertainty shocks.

We highlight the valuation linkage between aggregate volatility and risk sharing, consistent with a rich literature that studies asset prices as a function of aggregate risk, such as Chien and Lustig (2010) and Rampini and Viswanathan (2019), among others. In contrast to the literature that shows how collateral constraints arise endogenously, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000), we take those constraints as given and explore the interplay of different types of risk—aggregate or idiosyncratic—and different kinds of collateral—private or public.

Our result is complementary to Gorton and Ordonez (2022), who also study the dual

[7] As a robustness check, we also perform a low-frequency analysis over a longer time horizon. We show this sensitivity has increased in the last decades, consistent with the more intensive use of private assets as collateral in the U.S., as documented by Gorton et al. (2012).

use of public and private assets as collateral but, in that case, to back productive loans. They focus on the role of informational fragility that mounts in the economy as private assets (heterogeneous and plagued by asymmetric information issues) become larger vis-a-vis public assets (more homogeneous and less subject to informational frictions). While that work highlights the *informational fragility of collateral composition for productive reasons*, here we study the *valuation fragility of collateral composition for insurance reasons*. While Gorton and Ordonez (2022) is purely theoretical and silent about asset pricing implications, in this paper, we focus on the interaction between private and public asset valuations, link those valuations to the convenience yield, and take it to the data.

There is an equally extensive but more recent literature on convenience yields and their implications. The strategies to capture an asset's convenience yield vary. Some papers include the asset directly into the utility function, so preferences capture the convenience in reduced form, such as in Nagel (2016). Some others, such as Krishnamurthy and Vissing-Jorgensen (2012), consider settings where agents directly consume the asset's liquidity benefits, decomposing the convenience yield into *liquidity and safety components*. Others still, such as Lagos (2010), explicitly model how assets are useful to overcome trading frictions in decentralized markets, which endogenously gives rise to a *trading component* of the convenience yield. Our paper contributes to this literature by providing a novel micro founded *insurance component* of the convenience yield and obtaining testable implications on how it co-moves with aggregate volatility. Specifically, an asset's convenience yield has a direct theoretical mapping to the asset's value to provide idiosyncratic insurance through its role as collateral.

We take the supply of safe assets as exogenous but provide an extension with private asset creation in the Appendix. Consistent with Greenwood et al. (2015), Krishnamurthy and Vissing-Jorgensen (2015), and Sunderam (2014), we show that the private sector creates more private liabilities when convenience yields—a proxy for the demand for safe assets in their case—are high. Further, Infante (2020) points out that creating safe assets depends on its underlying collateral. None of these papers explicitly model the financial stability implications of private asset creation, which we capture by the reduction in the cross-insurance role of interbank markets.

The literature has validated several elements of our model. First, the relevance of private assets' valuation for insurance has been documented for housing by Hurst and Stafford (2004), Lustig and Van Nieuwerburgh (2010) and Hryshko et al. (2010). Second, the relation between aggregate volatility and the valuation of public and private assets has been documented by Connolly et al. (2005) and Baele et al. (2010) over recent U.S. history, showing that an increase in volatility appreciates Treasuries and depreciates stocks. Finally, there has been a recent and active literature, such as Jiang et al. (2019), Reis (2021) and Bhandari et al. (2021), identifying the underlying determinants of government bond valuations. Our work shows that the presence, valuation, and use of private assets in interbank markets are important in qualifying these results as they directly impact the valuation of public assets. In particular, our paper shows that government bonds' increased ability to hedge idiosyncratic risks can attenuate or exacerbate the so-called "negative beta effect", depending on whether the ratio of private to public assets used as collateral in financial markets is low or high.

Finally, Brumm et al. (2018) quantitatively study how re-using private collateral increases leverage and volatility, while Rampini and Viswanathan (2010) argue that higher collateralizability increases borrowing capacity, leverage, and aggregate volatility. Instead, we explore the opposite direction, in which aggregate volatility affects the value of private and public collateral to provide insurance against idiosyncratic shocks, and then highlight that the relationship between leverage and volatility works on both directions.

The paper proceeds as follows. The next section presents a model with aggregate volatility in which public and private assets can be used as collateral to share idiosyncratic risks and as stores of value to smooth aggregate volatility. Section 3 presents a tractable CARA-Normal case that allows for clean comparative statics on the valuation of public and private assets and their use for risk sharing. In Section 4, we provide empirical evidence on the sensitivity of risk sharing to aggregate volatility and how it has changed over time. Section 5 concludes.

2 Model

In this section, we present a simple model that relates aggregate volatility and the extent of idiosyncratic risk sharing, assuming an exogenous supply of private and public assets. We intend to capture financial intermediaries that face idiosyncratic shocks to assets and/or liabilities and that write collateralized insurance contracts with other intermediaries. In this context, financial risk-sharing becomes a critical tool for financial stability. We will model, however, financial intermediaries in reduced form, allowing us to extrapolate results beyond financial markets. Our model is a three-period endowment economy, which will enable us to abstract from dynamic and production intricacies that would obscure the results without adding to the determinants of the relation between aggregate risk and sharing idiosyncratic risk and then between aggregate stability and financial stability.

2.1 Environment

Consider a three period ($t \in \{0, 1, 2\}$) endowment economy with two agents, called Raymond (R) and Shirley (S). Both agents have additive separable preferences, with each period's consumption utility $u(\cdot)$ and discount factor β. Agents split the aggregate endowment equally. Each agent also receives an idiosyncratic endowment shock at $t = 1$, completely offset by the other agent's shock. To fix ideas, Raymond (Shirley) receives a positive (negative) shock if it "rains" and a negative (positive) shock if it "shines." The probabilities of rain and shine are simply $\frac{1}{2}$ (independent of the aggregate endowment). Hence agent i endowment is:

$$e_{0i} = \frac{Y_0}{2}; \qquad \tilde{e}_{1i} = \frac{\tilde{Y}_1}{2} + \tilde{y}_i; \qquad \tilde{e}_{2i} = \frac{\tilde{Y}_2}{2},$$

where Y_t represents aggregate endowments and the tilde signifies that endowment shocks are t-measurable random variables. For Raymond, $\tilde{y}_i$ is either $\overline{y}$ if it rains or $-\overline{y}$ if it shines. Shirley has the opposing idiosyncratic endowment shock.

This is the most straightforward setting to capture financial intermediaries (or investors more generally) that are otherwise identical but face negatively correlated idiosyncratic shocks, so there is room for insurance. For instance, Raymond and Shirley can be interpreted as risk-averse banks in negatively correlated regions, such as when their idiosyncratic loans pay or their idiosyncratic depositors withdraw in different states of the "weather." This would induce these banks to write derivative contracts conditional on the weather to insure themselves. Their health and stability depend on the possibility of trading assets and writing insurance contracts.

Assets: The economy has three assets: short-term government bonds, long-term government bonds, and a private asset. While the total *supply* of public and private assets is fixed, agents' optimal portfolio choice determines their *demand*.[8] In the interpretation of agents as banks, they start with given asset and liability positions that determine their idiosyncratic exposure to the weather but can write insurance contracts and rebalance their portfolio once the weather outcome is realized.

In terms of supply, we denote the exogenous face value of the total amount of short-term government bonds by Θ_0^{Sh}, of long-term government bonds by Θ_0, and of private assets by $\hat{\Theta}_0$. The government pays short- and long-term government bonds (the two public assets), raising lump-sum taxes on agents when the bonds mature. Because of the government's ability to tax agents, these assets will be considered safe— i.e., they always pay at par

[8]In Appendix F we study the case in which agents can create private assets and their supply, which internalizes the value of assets as collateral, is endogenous.

when they mature. We assume households are endowed symmetrically with the private asset, which has a risky payoff proportional to the aggregate endowment process, paying a dividend $\tilde{a}_t = \rho \tilde{Y}_t$, with $\rho \in (0,1)$ in each period.

In terms of demand, in each period $t \in \{0,1\}$, each agent will choose to purchase θ_{ti}^{Sh} of short-term government bonds, θ_{ti} of long-term government bonds, and $\hat{\theta}_{ti}$ of private assets at the market clearing price p_t^{Sh}, p_t, and $\hat{p}_t$, respectively. In period $t = 1$, these choices will be conditional on the realization of the aggregate and idiosyncratic shocks.

Notice that the difference between public and private assets is given by how their payoffs relate to the aggregate state and not by the issuer. We maintain this denomination to fix ideas, but "private assets" are more generally assets with payoffs that are positively correlated to the aggregate state, and "public assets" are assets with a fixed payoff, regardless of the aggregate state of the world or the issuer.

Risk Sharing and Collateral: At $t = 0$, agents can write state-contingent contracts among themselves, conditional on the weather. We model these as "Arrow-Debreu" securities that pay one unit of the consumption good depending on whether it rains or shines. Importantly, we assume that agents selling an Arrow-Debreu security (effectively selling insurance for that state of the world) must *fully* collateralize their promise with public and/or private assets. If we denote by w_i^j the amount of promises agent i makes in case the weather outcome is $j \in \{r, s\}$ —where r and s denote the state "rain" and "shine", respectively—the need of fully collateralize these promises implies

$$w_i^r \le \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{\hat{p}}_1 \hat{\theta}_{0i}; \qquad w_i^s \le \theta_{0i}^{Sh} + \underline{p}_1 \theta_{0i} + \alpha \underline{\hat{p}}_1 \hat{\theta}_{0i}. \tag{1}$$

A single contract against rain or shine can be thought of as a collateralized insurance or derivative contract.[9] To capture absolute safety, we assume the ability of an asset to collateralize a claim depends on the worst value it can take (the portion of the payoff that can be pledged in all states of the world).[10] In the constraints in (1), $\underline{p}_1$ and $\underline{\hat{p}}_1$ are the lowest price the long-term government bond and the private asset can have in $t = 1$, respectively. The parameter α captures the pledgeability of the private relative to public assets. It is natural to assume that private assets have a lower collateral value than public ones, hence $\alpha < 1$.

[9] The derivative contract could also be written in terms of an easily observable aggregate variable, such as the price of a commodity. If rain, for instance, is positively correlated with the price of a commodity, Raymond would sell such a derivative, and Shirley would buy it. Our setting also captures repo contracts. An agent not subject to any idiosyncratic risk could purchase both rain and shine derivative contracts (guaranteeing a given level of consumption) and collect the collateral to back both contracts. This is equivalent to a secured loan backed by a financial security, i.e., a repo.

[10] This extreme assumption eliminates all credit risk, isolating the role of collateral valuation. This choice can be microfounded, as in Caballero and Farhi (2018), by assuming infinite risk aversion over short intervals.

This is, for instance, because of their more limited pledgeability, informational frictions, etc. Our results do not depend, however, on this relative inferiority of private assets as collateral, and all results go through with $\alpha = 1$.[11] We denote the equilibrium price of contingent contracts for when it rains and shines by q^r and q^s, respectively.

Consumption: Raymond's consumption in each period is,

$$c_{0R} = e_{0R} + a_0 \frac{\hat{\Theta}_0}{2} - p_0 \theta_{0R} - p_0^{Sh} \theta_{0R}^{Sh} - \hat{p}_0 \left(\hat{\theta}_{0R} - \frac{\hat{\Theta}_0}{2} \right) + q^r w_R^r + q^s w_R^s + \frac{T_0}{2} \tag{2}$$

$$\tilde{c}_{1R} = \tilde{e}_{1R} + \tilde{a}_1 \hat{\theta}_{0R} - \tilde{p}_1(\theta_{1R} - \theta_{0R}) + \theta_{0R}^{Sh} - \tilde{\hat{p}}_1(\hat{\theta}_{1R} - \hat{\theta}_{0R}) - w_R^r 1^r - w_R^s 1^s + \frac{T_1}{2} \tag{3}$$

$$\tilde{c}_{2R} = \tilde{e}_{2R} + \tilde{a}_2 \hat{\theta}_{1R} + \theta_{1R} + \frac{T_2}{2}, \tag{4}$$

where T_t are aggregate lump-sum transfers (negative values are taxes) from the government. In the baseline model, we assume no distortionary taxation, so the government returns lump-sum what it raises in each period. That is, $T_0 = p_0^{Sh} \Theta_0^{Sh} + p_0 \Theta_0$, $T_1 = -\Theta_0^{Sh}$ and $T_2 = -\Theta_0$.[12] Consumption for Shirley takes a symmetric form.

Timing: In $t = 0$, agents choose the amount of government bonds to purchase and contingent contracts to sign, taking into account the need to collateralize these contracts with the assets they hold and purchase—that is, satisfy the inequalities of (1). In $t = 1$, agents rebalance their portfolio upon the realization of both the aggregate and idiosyncratic shocks. In $t = 2$, agents consume endowments and proceeds from their portfolios.

Given the symmetry of agents in period $t = 0$, each will end up with half of the supply of total assets. Each agent, however, can rebalance their portfolio, buying or selling, for instance, more long-term bonds or private assets in period $t = 1$ once agents' endowments become asymmetric if full insurance is not achieved. Taxes are then collected at period $t = 2$ to redeem government bonds. Since there are no choices in period $t = 2$, the following subsections characterize the optimal choices in periods $t = 1$ and $t = 0$, respectively.

2.2 Equilibrium in $t = 1$

In $t = 1$, after it rains or shines, each agent rebalances their portfolio by choosing the optimal amount of long-term bonds (now one-period bonds) and private asset holdings to smooth consumption.

[11] It is beyond the scope of this paper to microfound α, but rather to focus on α valuation effects. For a discussion about endogenizing α, see Gorton and Ordonez (2014).

[12] Alternative tax schemes do not modify the central insight of the paper, and are available upon request from the authors.

In what follows, we will focus on an "interior equilibrium" in which both agents hold both the long-term bond and private asset at $t = 1$ regardless of the idiosyncratic shock. By avoiding the characterization of corner solutions that complicate the analysis, the demand for long-term government bonds and private assets is determined by

$$p_1 = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2R}))}{u'(c_{1R})} = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2S}))}{u'(c_{1S})} \tag{5}$$

$$\hat{p}_1 = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2R})\tilde{a}_2)}{u'(c_{1R})} = \frac{\beta \mathbb{E}_1(u'(\tilde{c}_{2S})\tilde{a}_2)}{u'(c_{1S})},, \tag{6}$$

where the $\mathbb{E}_1(\cdot)$ operator is the expectations over aggregate risk in period 2. These are standard intertemporal pricing equations when both agents hold both assets, such that allocations equalize the stochastic discount factor of agents with different idiosyncratic shocks.[13] Given the symmetry of the problem, however, the price will be the same for both realizations of the idiosyncratic shock, as one of the two agents will always have the "good shock" and the other the "bad shock."

Combining these prices with equation 3, it is clear that $u'(c_{1i})$, for both agents $i \in \{R, S\}$, depend on prices for two reasons: First because prices in $t = 1$ determine the extent of insurance written in $t = 0$ through equation 1. Second, agents can sell or buy assets at these prices in $t = 1$ after the realization of their respective idiosyncratic shocks.

Market clearing at $t = 1$ in both markets are

$$\theta^j_{1R} + \theta^j_{1S} = \Theta_0 \qquad \texttt{and} \qquad \hat{\theta}^j_{1R} + \hat{\theta}^j_{1S} = \hat{\Theta}_0; \qquad \forall j \in \{r, s\}.$$

The main insight from the characterization in $t = 1$ is that any asset price increase helps risk sharing in two ways: 1) by allowing agents to better insurance ex-ante and 2) by allowing agents to sell assets in better conditions when facing a negative shock ex-post. The question then hinges on how prices depend on aggregate volatility and the composition of those assets in agents' portfolios. Further, as we will see, these effects on risk sharing feedback to prices, breaking the standard link between aggregate volatility and prices. In the next part, we characterize prices at $t = 0$, which conveys information about the value of assets as both storage and collateral and point towards a way to measure the link between aggregate volatility and risk sharing.

[13] We defer the full derivation of $t = 1$ and $t = 0$ pricing equations to Appendix C.

2.3 Equilibrium in $t = 0$

In $t = 0$, agents optimize their consumption paths given the expected equilibrium in $t = 1$, subject to the constraints in equations (1).

It is natural to assume that in equilibrium, Raymond will buy insurance for when it shines and sell insurance for when it rains. That is, Raymond's collateral constraint will possibly bind only when he sells rain insurance in $t = 0$. Similarly, Shirley's collateral constraint may bind only when she sells shine insurance. As usual, insurance is priced by the agent who buys it (and needs it the most). In the symmetric equilibrium, Raymond sells insurance against rain to Shirley, and Shirley sells insurance against shine to Raymond—that is, $w_R^r = -w_S^r$ and $w_S^s = -w_R^s$.

From Raymond's perspective, denoting $\tilde{c}_{1R}^r$ and $\tilde{c}_{1R}^s$ Raymond's consumption conditional on rain and shine in $t = 1$, respectively; period $t = 0$ prices in the symmetric interior equilibrium come from first-order conditions,

$$p_0^{Sh} = \beta \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R})}{u'(c_{0R})} \right) + \left[\frac{\beta}{2} \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_{0R})} \right) \right] \tag{7}$$

$$p_0 = \beta \mathbb{E}_0 \left(\tilde{p}_1 \frac{u'(\tilde{c}_{1R})}{u'(c_{0R})} \right) + \underline{p}_1 \left[\frac{\beta}{2} \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_{0R})} \right) \right] \tag{8}$$

$$\hat{p}_0 = \beta \mathbb{E}_0 \left((\tilde{a}_1 + \tilde{\hat{p}}_1) \frac{u'(\tilde{c}_{1R})}{u'(c_{0R})} \right) + \alpha \underline{\hat{p}}_1 \left[\frac{\beta}{2} \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_{0R})} \right) \right], \tag{9}$$

where the expectations operator $\mathbb{E}_0(\cdot)$ is only over aggregate uncertainty (since idiosyncratic uncertainty is independent of aggregate uncertainty and is explicit in the expression by separating consumption when rain and shine with probabilities 1/2). These pricing equations are consistent with standard results under heterogeneous agents: the agents with the highest marginal rate of substitution price the asset, as in Alvarez and Jermann (2000). For Raymond, given that $\mathbb{E}_0 [u'(\tilde{c}_{1R})] = \mathbb{E}_0 \left[\frac{u'(\tilde{c}_{1R}^s) + u'(\tilde{c}_{1R}^r)}{2} \right]$, we can rewrite equation (7) as $p_0^{Sh} = \beta \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R}^s)}{u'(c_{0R})} \right)$. That is, the price of the short-term government bond is proportional to Raymond's marginal consumption when it shines. Empirically, the expression in (7) can be interpreted as the price of a short-term risk-free bond, such as a U.S. Treasury T-bill, used as collateral on derivative contracts.

Equations (7)–(9) have two components. The first component is the standard asset pricing equalization of intertemporal marginal utilities and captures the value of assets as *stores of value*. This is the price of a theoretical *risk-free security* that pays par in $t = 1$, in absence of idiosyncratic risks,

$$p_0^{rf} := \beta \mathbb{E}_0 \left(\frac{u'(\tilde{c}_{1R})}{u'(c_{0R})} \right) \tag{10}$$

Alternatively, the expression in (10) captures the price of a risk-free instrument that cannot be used as collateral (for instance because of lack of pledgeablity), and thus does not exhibit a collateral premium. Empirically, it can be interpreted as the price of a risk-free derivative contract, such as the overnight index swap (OIS).

The second component captures the value of assets as *collateral* to improve risk sharing, and it is then related to the *convenience yield* of "safe assets". However, in this context, the convenience yield is not related to safety or liquidity properties, as in most literature, but instead to the role of the asset in facilitating insurance, and it is formally given by,

$$CY := \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}^s_{1R}) - u'(\tilde{c}^r_{1R})}{u'(c_{0R})}\right) > 0. \tag{11}$$

This simple expression has a straightforward interpretation. The convenience yield is equal to the difference in marginal utility when agents suffer a bad idiosyncratic shock relative to a good one. It captures the value of missing insurance, which is equal to the gap of marginal utilities generated by consumption that remains contingent on idiosyncratic shocks. In the interior equilibrium, the convenience yield will depend on the amount of safe assets in the economy, their degree of pledgeability, and the size of idiosyncratic shocks. From the expression in equation (11), it is clear that the convenience yield is non-negative and is zero only if perfect insurance eliminates any difference between the marginal utility of consuming when it shines or rains.

While CY is a common "insurance component" in the price of all assets, its effect on the price of a given asset will be determined by the asset's intermediate price and its value as collateral, α. Indeed, if aggregate volatility also increases the extent of asymmetric information and adverse selection (as in Kurlat 2013 or Bigio 2015) or induces information acquisition (as in Gorton and Ordonez, 2022), then α would decline with aggregate volatility (private assets will be discounted more when used as collateral). In this case, we may observe an increase in the convenience yield of public assets (coming from an increase in CY) and a decrease in the convenience yield of private assets if the decline in α is stronger than the increase in CY. This implies that a given asset's convenience yield can decline even if CY increases if α declines enough.

To close the model, market clearing in the three markets at $t = 0$ are

$$\theta^{Sh}_{0R} + \theta^{Sh}_{0S} = \Theta^{Sh}_0; \qquad \theta_{0R} + \theta_{0S} = \Theta_0; \qquad \hat{\theta}_{0R} + \hat{\theta}_{0S} = \hat{\Theta}_0.$$

3 The Storage - Collateral Link: CARA-Normal Setting

The previous general setting highlights the two components in the value of an asset as a store of value (captured by the risk-free price) and as collateral (captured by the convenience yield). It also shows the intricate relationship between these two roles through expectations and marginal utilities. We have, however, characterized the equilibrium for arbitrary preferences and an arbitrary stochastic process for aggregate endowments. To make progress on how the two roles interact, in this section, we assume preferences and endowment processes that are standard and tractable enough to isolate the relationship between storage and collateral analytically:

Assumption A1. *Consider a case with the following simplifying assumptions:*

1. *Preferences are characterized by CARA, with risk aversion* γ.
2. $\tilde{Y}_1 = Y_1 = 0$.
3. $\tilde{Y}_2 \sim N(\mu, \sigma^2)$.

Beyond tractability, these assumptions help to focus on the link between aggregate volatility and risk sharing. In this more straightforward setting, σ^2 is the variance of aggregate endowment realizations in period 2 and fully captures aggregate volatility. Our goal is to characterize how changes in aggregate risk in $t = 2$ affect the risk-sharing of idiosyncratic risk that can be sustained in $t = 1$ with collateralized insurance contracts written in $t = 0$.

This simplified case is useful for the following reasons. First, CARA preferences eliminate wealth effects, so agents' optimal risky asset holdings in $t = 1$ do not depend on the idiosyncratic shock, nor do $t = 1$ prices. Second, a deterministic endowment in $t = 1$ implies that the prices of the long-term government bond and of the private asset at $t = 1$ are known in $t = 0$. Therefore, the worst possible value of the collateral is merely its price in $t = 1$.[14] Third, Gaussian shocks on $t = 2$ endowments generate closed-form prices in $t = 1$, facilitating comparative statics.

3.1 Characterization

As in the previous section, we restrict attention to an interior equilibrium with partial insurance to make our analysis interesting and comparative statics tractable. First, we

[14] These two assumptions further allow us to abstract from changes in the wealth distribution that aggregate volatility shocks may generate and that may also influence the marginal utility of idiosyncratic shocks, hence risk-sharing needs.

assume assets are not enough to collateralize insurance entirely; otherwise, there is no sense in which insurance varies with aggregate volatility. Second, agents always hold both public and private assets in equilibrium, regardless of the realization of the idiosyncratic shock. We next characterize allocations and prices in this equilibrium. **Allocations:** In an interior

equilibrium with partial insurance, the marginal rates of substitution between $t = 1$ and $t = 2$ must be the same for both agents (from equations (5) and (6)). Given the model's symmetry, we can conjecture that the optimal portfolio choice implies that each agent holds half of all assets in $t = 0$. Our first restriction formally implies that insurance needs are so large that even if agents use all of the assets as collateral, full insurance is not achieved,

$$\overline{y} - \underbrace{\left(\frac{\Theta_0^{Sh}}{2} + p_1 \frac{\Theta_0}{2} + \alpha \hat{p}_1 \frac{\hat{\Theta}_0}{2} \right)}_{:=w} > 0, \tag{12}$$

where w is defined at the value of both agents' initial portfolio in $t = 0$. Given imperfect insurance, Raymond consumes more when it rains than when it shines. Still, when it rains, Raymond wants to move some of such extra consumption to $t = 2$ and does it by buying long-term bonds from Shirley at prevailing prices p_1.[15] Since Shirley faces a bad shock when raining, she is willing to sell those bonds to bring consumption from $t = 2$ to $t = 1$. Since Raymond wants to equalize consumption in both periods,

$$(\overline{y} - w) - p_1 \left(\theta_{1R}^r - \frac{\Theta_0}{2} \right) = \left(\theta_{1R}^r - \frac{\Theta_0}{2} \right),$$

Intuitively, since $\overline{y} > w$, when it rains, Raymond buys extra long-term bonds, i.e., $\theta_{1R}^r > \frac{\Theta_0}{2}$. This rebalancing allows Raymond to move some extra consumption $\overline{y} - w$ to $t = 2$. Since the problem for Shirley is symmetric, when it rains, we can express the portfolio at $t = 1$ in the interior equilibrium as

$$\theta_{1R}^r = \frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2} < \Theta_0; \qquad \theta_{1S}^r = -\frac{(\overline{y} - w)}{1 + p_1} + \frac{\Theta_0}{2} > 0, \tag{13}$$

An interior equilibrium requires that Shirley not sell all her long-term bonds to Raymond when rebalancing her portfolio in $t = 1$ after it rains.

From equation (3), this portfolio implies that when Raymond is unable to fully insure

[15] Without wealth effects, agents' private asset holdings are their original ones as agents want to maintain their risk profile and only long-term government bonds are used to smooth the idiosyncratic shock.

idiosyncratic shocks, his optimal consumption in $t = 1$ is,

$$c^r_{1R} = \frac{(\overline{y} - w)}{(1 + p_1)}; \qquad\qquad c^s_{1R} = -\frac{(\overline{y} - w)}{(1 + p_1)}. \tag{14}$$

By rebalancing portfolios, agents can smooth consumption of $t = 1$ above and beyond what is possible with insurance.[16] Raymond, for instance, does not consume the extra amount $\bar{y} - w$ when raining, but something less. In other words, in $t = 0$, long-term bonds are used as collateral to sustain insurance, and in $t = 1$, long-term bonds are used to trade and smooth consumption. Hence, contingent consumption in equation (14) captures the extent of risk-sharing. Full insurance, for instance, is a benchmark with $c^r_{1R} = c^s_{1R} = 0$.

This result confirms our general insight that an increase in p_1 improves risk sharing in two ways, improving insurance ex-ante (increase in w) and increasing revenue when selling upon bad news (increase in $1 + p_1$). This second force is not present for private assets because in the CARA-Normal case, agents' attitude toward aggregate risk does not change with wealth. Thus, the risk-adjusted return for holding the private asset after the realization of the idiosyncratic shock is the same as in $t = 0$, resulting in private assets not being rebalanced in $t = 1$.

Prices: In $t = 1$ prices can be expressed in closed form given that there are no wealth effects, that $\tilde{a}_t = \rho\tilde{Y}_t$, and that $\tilde{Y}_2$ is normally distributed. From equations (5) and (6),

$$p_1 = \beta \exp\left\{-\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\mu + \frac{1}{8}\gamma^2\left(1 + \rho\hat{\Theta}_0\right)^2\sigma^2\right\} \tag{15}$$

$$\hat{p}_1 = \rho\left(\mu - \frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma^2\right)p_1. \tag{16}$$

Similarly, from equations (7)–(9), prices in $t = 0$ are in closed form

$$p^{Sh}_0 = p^{rf}_0 + CY \tag{17}$$

$$p_0 = p_1\left(p^{rf}_0 + CY\right) \tag{18}$$

$$\hat{p}_0 = \hat{p}_1\left(p^{rf}_0 + \alpha CY\right). \tag{19}$$

The full derivation of the interior equilibrium with partial insurance and the sufficient primitive parametric conditions to guarantee its existence are characterized in Appendix A.

[16] In Assumption A1 we have normalized to 0 the endowment in $t = 1$, hence under perfect insurance optimal consumption is 0 for both agents in $t = 1$. With partial insurance, Raymond's (Shirley's) optimal consumption when it shines (rains) is negative in $t = 1$.

3.2 Comparative Statics

Having characterized an interior symmetric equilibrium with partial insurance, we now study how prices and allocations react to changes in the supply of long-term public assets (Θ_0), the severity of idiosyncratic shocks ($\bar{y}$), the pledgeability of private assets (α), and most importantly, aggregate volatility (σ^2). Given that asset prices in (17) – (19) are just expressed in terms of a risk-free rate and a convenience yield, the following lemma expresses the model's comparative statics with respect to an arbitrary parameter of interest, which we denote generically as $z \in \{\Theta_0, \bar{y}, \alpha, \sigma^2\}$.

Lemma 1. *(Sensitivity of risk-sharing, risk-free rates, and convenience yields to an arbitrary parameter z). In an interior equilibrium with partial insurance, the comparative statics of consumption and price components for an arbitrary parameter z are,*

$$\frac{\partial c_{1R}^s}{\partial z} = -\frac{\partial c_{1R}^r}{\partial z} \tag{20}$$

$$\frac{\partial p_0^{rf}}{\partial z} = -\gamma CY \frac{\partial c_{1R}^s}{\partial z} + \gamma p_0^{rf} \frac{\partial c_{0R}}{\partial z} \tag{21}$$

$$\frac{\partial CY}{\partial z} = -\gamma p_0^{rf} \frac{\partial c_{1R}^s}{\partial z} + \gamma CY \frac{\partial c_{0R}}{\partial z}. \tag{22}$$

Proof. The first equality determines risk-sharing and comes from equations (14). The second and third come from taking the derivative of equations (10) and (11), given the observation that with CARA utility $u''(z) = -\gamma u'(z)$. □

These results are informative on the interaction between an asset's values for storage and collateral. Assume, for example, that a change in z increases Raymond's consumption when it shines and thus reduces the need for risk sharing. This has a direct and an indirect effect. On the one hand, the price of the risk-free bond declines as agents value less, transferring resources from $t = 0$ to $t = 1$ because their exposure to idiosyncratic shocks is lower. This direct effect is proportional to the convenience yield, CY, and is captured by the first term in equation (21). On the other hand, the convenience yield also declines as agents do not need to share risks so much. This indirect effect is proportional to the price of the risk-free bond, p_0^{rf}, and is captured by the first term in equation (22).

The comparative statics to changes in the supply of public assets, idiosyncratic volatility, and private assets' pledgability (that is, $z \in \{\Theta_0, \bar{y}, \alpha\}$) are easily derived from Lemma 1 because they do not affect equilibrium prices in $t = 1$, nor do they change consumption in $t = 0$. However, they do change the amount of idiosyncratic insurance agents can hedge, shown in equation (14). If the change in parameters increases (decreases) the amount of risk sharing, that is, makes the expressions in (14) smaller (larger) in absolute value, then prices in $t = 0$ will decrease (increase). Intuitively, because there is more (less) risk sharing in the economy, the value of assets as collateral declines (increases). For instance, more government bonds or more pledgeability of private assets increases the amount of risk sharing by increasing w. In contrast, larger idiosyncratic shocks increase the need for risk sharing. These comparative statics are summarized in the following proposition.

Proposition 1. *(Asset Pricing Effects of the Supply of Public Assets, Private Asset Pledgeability, and Idiosyncratic Volatility). In an interior equilibrium with partial insurance, $t = 0$ prices of long-term government bonds and private assets have the following comparative statics with respect to the supply of government bonds, Θ_0, the pledgeability of private assets α, and the size of the idiosyncratic shock $\overline{y}$*

$$\begin{array}{ll}
\frac{\partial p_0}{\partial \Theta_0} = -\frac{\gamma p_1^2\left(p_0^{rf}+CY\right)}{2(1+p_1)}, & \frac{\partial \hat{p}_0}{\partial \Theta_0} = -\frac{\gamma p_1 \hat{p}_1\left(\alpha p_0^{rf}+CY\right)}{2(1+p_1)} \\
\frac{\partial p_0}{\partial \alpha} = -\frac{\gamma p_1 \hat{p}_1 (p_0^{rf}+CY)}{(1+p_1)}\frac{\hat{\Theta}_0}{2}, & \frac{\partial \hat{p}_0}{\partial \alpha} = -\frac{\gamma (\hat{p}_1)^2 (\alpha p_0^{rf}+CY)}{(1+p_1)}\frac{\hat{\Theta}_0}{2} + \hat{p}_1 CY \\
\frac{\partial p_0}{\partial \overline{y}} = \frac{\gamma p_1\left(p_0^{rf}+CY\right)}{(1+p_1)}, & \frac{\partial \hat{p}_0}{\partial \overline{y}} = \frac{\gamma \hat{p}_1\left(\alpha p_0^{rf}+CY\right)}{(1+p_1)}.
\end{array}$$

Proof. The result comes from applying Lemma 1 to (18)– (19), and noting that the partial derivatives of c_{0R} with respect to Θ_0, α, and $\overline{y}$ are zero. □

The results in Proposition 1 operate through changes in the economy's need and availability of idiosyncratic insurance. It is trivial that, as there are more long-term government bonds to collateralize insurance or as the size of idiosyncratic shocks decline and insurance is less needed, the price of those bonds decline.[17] The comparative statics with respect to changes in the pledgeability α of private assets is more nuanced. On the one hand, private assets become more useful as collateral, which operates through $\hat{p}_1 CY$. On the other hand, as there is more collateral and better risk sharing, the convenience yield declines. Interestingly, even if the private asset were not pledgeable (that is, $\alpha = 0$), an increase in risk sharing still lowers its price since private assets are also useful to transfer wealth to $t = 1$, captured by the term accompanying CY.

[17] Comparative statics with respect to Θ_0^{Sh} are identical to those with respect to Θ_0, but divided by p_1. Similarly, comparative statics of p_0^{Sh} are identical to those of p_0, divided by p_1. That is, comparative statics with respect to, and of, long- or short-term bonds are proportional to the value of those bonds in $t = 1$.

3.2.1 Changes in Aggregate Volatility

In this subsection, we present our main result. In contrast to the previous analysis, an aggregate volatility shock, which is captured by an increase in the variance of aggregate endowments at $t = 2$, directly impacts prices at $t = 1$. From (15) and (16),

$$\begin{aligned}\frac{\partial p_1}{\partial \sigma^2} &= \frac{\gamma^2}{8}\left(1+\rho\hat{\Theta}_0\right)^2 p_1 \\ \frac{\partial \hat{p}_1}{\partial \sigma^2} &= \rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1 \\ &= \frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial \sigma^2} - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\rho p_1.\end{aligned}$$

The price of the long-term bond at $t = 1$ always increases with volatility. This is a standard "negative beta" effect of government bonds: as aggregate volatility increases, the need to smooth consumption intertemporally from $t = 1$ to $t = 2$ increases, making long-term government bonds with a certain payment more valuable. The price of the private asset at $t = 1$ reacts to volatility more intricately. On the one hand, similar to the long-term government bond, there is a "negative beta" effect proportional to the private asset's certainty equivalence (the first term). On the other hand, the private asset is less valuable per se, as its payoffs are now riskier, putting downward pressure on its price (the second term).

How do these changes in asset valuations at $t = 1$ affect allocations, particularly the extent of risk sharing? While it is immediate that $\frac{\partial c_{0R}}{\partial \sigma^2} = 0$, from (14), we have

$$\begin{aligned}\frac{\partial c^s_{1R}}{\partial \sigma^2} &= \frac{1}{(1+p_1)}\left[\underbrace{\frac{\partial p_1}{\partial \sigma^2}\left(\frac{\Theta_0}{2} + \frac{(\overline{y}-w)}{(1+p_1)}\right)}_{Valuation\ public\ assets} + \underbrace{\alpha\frac{\partial \hat{p}_1}{\partial \sigma^2}\frac{\hat{\Theta}_0}{2}}_{Valuation\ private\ assets}\right] \\ &= \frac{1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial \sigma^2} - \frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}\end{aligned} \tag{23}$$

Risk sharing improves if this derivative is positive and weakens if it is negative. The effect of aggregate volatility on Raymond's consumption when it shines comes through changes in the price of assets in $t = 1$, as those assets are insurance collateral in $t = 0$. In few words, *risk sharing is affected by aggregate volatility purely through the valuation of collateral.*

If the price of private assets increases with aggregate volatility, then risk-sharing and the financial stability that depends on it improves with aggregate volatility. If, instead, the price of private assets decreases, the overall impact is nontrivial. On the one hand, there is a positive *valuation of public assets effect*. The price of long-term bonds in $t = 1$ always increases with aggregate volatility, improving risk sharing (both by using them as collateral and selling them in case of a bad shock). On the other hand, there is a negative *valuation of private assets effect*, weakening risk sharing. This result highlights the importance of collateral composition and its components' sensitivity to aggregate volatility on assessing how aggregate risk affects the extent of insurance against idiosyncratic risk.

Data, however, suggests that this nontrivial effect of collateral composition on risk sharing is quite likely. Private riskier assets seem to suffer from price declines when aggregate volatility increases. The following Proposition formalizes the discussion of this case in terms of endogenous prices. In the Appendix B, we provide empirical evidence that private asset prices tend to decline with aggregate volatility, a sensitivity that is more pronounced the riskier they are, and discuss the primitive parametric conditions under which this case arises in our setting.

Proposition 2. *(Risk Sharing Effects of Aggregate Volatility). In an interior equilibrium with partial insurance, if* $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ *is sufficiently small so that private asset prices decline with aggregate volatility—that is,* $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$*—and more private assets are used as collateral than long-term government bonds—that is,* $\alpha \hat{p}_1 \hat{\Theta}_0 > p_1 \Theta_0$*— then an increase in aggregate volatility* σ^2 *reduces risk sharing—that is,* $\frac{\partial c^s_{1R}}{\partial \sigma^2} < 0$*. Moreover, the decrease in risk sharing is larger if the private asset is more pledgeable—that is,* $\frac{\partial^2 c^s_{1R}}{\partial \alpha \partial \sigma^2} < 0$*.*

Proof. See Appendix C. □

Intuitively, unless all assets used as collateral become more valuable when future aggregate conditions become more uncertain, the net impact on risk-sharing depends on the relative amount of public and private assets used as collateral, which itself depends on the supply of assets and the private assets' pledgability. If the economy relies heavily on private assets, risk sharing weakens when aggregate volatility increases. Finally, because aggregate volatility does not affect the price of short-term government bonds in $t = 1$, using short-term bonds as collateral is immune to changes in future aggregate volatility. These results point towards the need to consider the composition of assets used as collateral in assessing how financial stability reacts to aggregate uncertainty.

Remark on the generalization to other sources of valuation: *Even though we have focused on how aggregate volatility affects risk-sharing through the valuation of collateral, other aggregate changes may have similar differential effects on public and private assets, and in general, the different components of the collateral portfolio. One example is the expected endowment level* μ *in period 2. While the value of private assets will tend to decrease with* *worse prospects, the value of public assets will tend to increase given the higher relevance assigned to bringing consumption to the second period. Then, if interbank markets rely more on private assets, bad news about economic activity (a recession looming, for instance) will be detrimental to risk-sharing and may cause financial instability. More generally, these insights can be extended to other sources of shocks (changes in taxation, shocks to sovereign debt positions, etc) in more complex asset-pricing models. The key ingredients is that changes in the valuation of assets determine the amount of risk sharing through a collateral channel and that shocks can change the prices of public and private assets in opposite directions. In these types of settings, regardless of the underlying model, the composition of collateral will have implications over the amount of risk sharing available to agents in the economy and, ultimately, financial stability.*

4 Empirical Analysis

Our model shows that the sensitivity of risk sharing to aggregate volatility is not independent of the collateral composition. In the specific case of CARA-Normal, for instance, Proposition 2 states that if the price of private assets declines with aggregate volatility and the amount of private collateral is larger than the amount of public collateral ($\alpha\hat{p}_1\hat{\Theta}_0 > p_1\Theta_0$), then an increase in aggregate volatility decreases risk sharing ($\frac{\partial c_{1R}^s}{\partial\sigma^2} < 0$). Moreover, this sensitivity decreases as private assets become more useful as collateral ($\frac{\partial^2 c_{1R}^s}{\partial\alpha\partial\sigma^2} < 0$). This result has implications on how convenience yields react to changes in volatility, as the next Proposition characterizes,

Proposition 3. *(Convenience Yield and Aggregate Volatility with CARA-Normal). In an interior equilibrium with partial insurance, if* $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ *is sufficiently small and more private assets are used as collateral than long-term government bonds—that is,* $\alpha\hat{p}_1\hat{\Theta}_0 > p_1\Theta_0$*—then an increase in aggregate volatility* σ^2 *increases the convenience yield—that is,*

$$\frac{\partial CY}{\partial\sigma^2} = -\gamma p_0^{rf}\frac{\partial c_{1R}^s}{\partial\sigma^2} > 0.$$

Moreover, the increase in convenience yield is larger if the private asset is more pledgable, that is, $\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} > 0$.

Proof. See Appendix C. □

Since our model provides a precise mapping between the extent of risk sharing and assets' convenience yield and a clear guideline on how to measure such convenience yield, we can test its sensitivity to aggregate risk by studying its correlation with measures of aggregate volatility in periods that differ on the share of private to public assets used as collateral.

4.1 Empirical Strategy

As discussed, the first condition in the Proposition 3, which ensures that private asset valuations decline with an increase in future volatility, holds empirically. Hence, with standard preferences, an increase in aggregate volatility would increase the convenience yield when private assets are heavily used as collateral. Specifically, an unexpected increase in aggregate volatility tomorrow affects the convenience yield today. These observations motivate the estimation of the following empirical model:

$$\begin{aligned}\Delta CY_t \;=\;& \beta_0 + \beta_V \Delta VIX_t + \sum \gamma_j \Delta CY_{t-j} + \beta_F \Delta FedFunds_t + \eta FedFunds_{t-1} \\ & + \theta \Delta Gov_t + \varphi \Delta VIX_t \times \Delta log(ShortTBillsOut_t) + \epsilon_t, \qquad (24)\end{aligned}$$

where ΔCY_t is the first difference of an empirical measure of assets' convenience yields, and ΔVIX_t is the first difference of an empirical measure of expected aggregate volatility. In the model, the convenience yield in equation (11) is obtained by deducting the risk-free rate in equation (10) from the price of the short-term bond in equation (7). In the data, we measure this theoretical object as the spread between the one-month OIS rate downloaded from Bloomberg (a proxy for risk-free rate) and the four-week T-bill rate, downloaded from the Federal Reserve H.15 Statistical Release (a proxy for the short-term bond).[18] We measure aggregate volatility with the Chicago Board Options Exchange VIX index, a measure of the implied volatility of S&P500 index options.[19]

The mechanism in our model operates through changes in the valuations of public and private asset, which effectively alters the *supply of collateral*, and thus, idiosyncratic insurance. From this interpretation, we expect β_V in equation (24) to be positive when private assets are used widely and easily as collateral.

We estimate the model using *daily data from August 2004 to March 2020*.[20] We exploit that in the period preceding the Global Financial Crisis (GFC) in 2009 there was a heavy

[18] Even though this convenience yield measure has been used previously in the literature (see, for instance Sunderam (2014)), in our case, it is informed by our setting. The model also implies that the cleanest CY measure is obtained using the rate on T-bills. Computing instead the convenience yield of private assets would also provide information about how α changes with aggregate volatility.

[19] We winsorize the changes in the convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers. We also drop observations on quarter-end dates and two days surrounding quarter-end to exclude any changes in short-term rates driven by financial firms' window-dressing behavior. See Infante (2020) for more details.

[20] We use 5-day changes with overlapping data to reduce the impact of high-frequency variations.

use of private assets as collateral, which were seriously under distress during the crisis. Furthermore, tighter regulations promptly followed the crisis and limited their use.[21] We interpret these changes as a persistent decline in α after 2009. Hence, based on our model, we conjecture that the sensitivity of the convenience yield to changes in aggregate volatility was high before the GFC and declined drastically afterward.

We introduce several controls that may also affect the evolution of the convenience yield differentially in these two periods. First, we control for serial autocorrelation using two lagged changes of the convenience yield. Second, we control for changes in fed fund rates $\Delta FedFunds_t$ and of government bond supply ΔGov_t (measured by log changes in short-term T-bills outstanding with less than one month to mature and log changes in Treasury notes and bonds outstanding), inspired by the specification of Nagel (2016), who shows these are related with convenience yields in levels, by affecting the opportunity cost of holding money and the availability of government bonds. Importantly, our specification differs as we focus on changes rather than levels; however, we also control for the lagged level of rates $FedFunds_{t-1}$ which may capture differences in the opportunity cost of holding money.[22] Finally, we introduce an interaction between changes in VIX and changes in the supply of short-term T-bills as an additional control that allows us to identify whether volatility may have affected the price impact of collateral on convenience yields differentially in the two periods.[23]

4.2 Sensitivity of Convenience Yield Before and After 2009

Here, we present the empirical analysis of our main hypothesis. We study how the more stringent regulatory landscape implemented after the GFC, which in principle reduced the use of private assets as collateral, affected how the convenience yield reacted to changes in aggregate volatility. Again, inspired by Proposition 3 we expect the coefficient on ΔVIX_t to be positive and significant only before the GFC.[24]

To capture the change in sensitivity after the GFC, we include an indicator variable equal to one after January 2009. Table 1 shows the results from three empirical model

[21] For example, the Liquidity Coverage Ratio places a larger regulatory burden on private assets that are used to back financial firms' liabilities.

[22] We have also considered specifications with levels of VIX and CY as additional independent variables. The main insights remain, and results are available upon request.

[23] For this particular specification, we exploit short-term T-bills as an instrument that isolates changes in supply, as in Infante (2020). These series are constructed using Treasury auction results published by TreasuryDirect. The exogeneity of T-bill issuance is reinforced by the fact that it is known in advance, and the U.S. Treasury does not respond to changes in market rates. Also, since four-week T-bill rates are typically below overnight general collateral repo rates, it is very unlikely that a firm would raise outside funding to finance their short-term T-bill positions since that trade would involve a negative carry.

[24] In this section, $\Delta x_t = x_t - x_{t-5}$, the first difference operator with five lags.

specifications. The first specification measures the sensitivity of the convenience yield to future aggregate volatility throughout the sample. There, we observe that unconditionally, there is a positive statistically significant sensitivity, which—through the lens of our model—suggests an increase in VIX reduces the value of collateral and depresses supply. We also find that an increase in short-term T-bill increases supply, hence reduces the convenience yield.

The second specification includes the January 2009 dummy, which shows that the sensitivity is concentrated in the first part of the sample. Specifically, before 2009, there was a positive and statistically significant sensitivity of the convenience yield to increases in future aggregate volatility, but this sensitivity was reversed after 2009, when it disappeared altogether. Finally, the third specification is identical to the second, but including the interaction between future aggregate volatility and changes in short-term T-bill supply: $\Delta log(ShortTBillsOut_t) \times \Delta VIX_t$. This regression shows there is no statistically significant relationship between the convenience yield and the interaction term, either before or after 2009, indicating that the VIX does not change the price impact of exogenous changes in collateral supply, and this is true both before and after the GFC.

Our model rationalizes the reduction in the sensitivity of convenience yields to aggregate volatility by a decline in the sensitivity of collateral valuation to changes in aggregate volatility, a *collateral supply effect*. This result would also be consistent with an exogenous decline in the sensitivity of risk-sharing needs to changes in aggregate volatility, a *collateral demand effect*.[25] Even though we cannot directly rule out that possibility without controlling directly by idiosyncratic risk, existing literature indeed suggests that, if anything, after the GFC there was an increase in the sensitivity of risk-sharing needs to changes in aggregate volatility.[26] This evidence indeed reinforces the plausibility of our proposed mechanism.

We can further exploit the high-frequency data to estimate the model in shorter intervals and see the evolution of ΔCY_t's sensitivity to ΔVIX_t. Specifically, in each quarter, we estimate the empirical model in (24) using plus and minus two years of data. With this strategy, we can keep track of the changes in sensitivities over time.

Figure 2 shows the results. We can observe that the point estimate on ΔVIX_t is positive

[25] Appendix D extends the model to accommodate this alternative, insurance demand-driven, mechanism.

[26] Based on risk-sharing needs in commodity markets, for instance, Acharya et al. (2013) show that post-2008 crisis, the risk tolerance of speculators became more limited, hence intensifying the interaction between volatility and hedging demand. Specifically, the paper finds that in a high-volatility environment, the sensitivity of the futures risk premium to producers' hedging demand increased by 45-60%. Similarly, Cheng et al. (2015) show that financial traders (such as hedge funds) and commercial hedgers became more sensitive to changes in the VIX after the crisis. Financial traders who previously held large long positions in commodity futures were more likely to reduce these positions during periods of high volatility. When financial traders reduce their positions due to a spike in the VIX, hedgers end up holding more risk than they otherwise would, increasing their need for insurance.

Table 1: Convenience Yield and Volatility: Pre- and Post- 2009

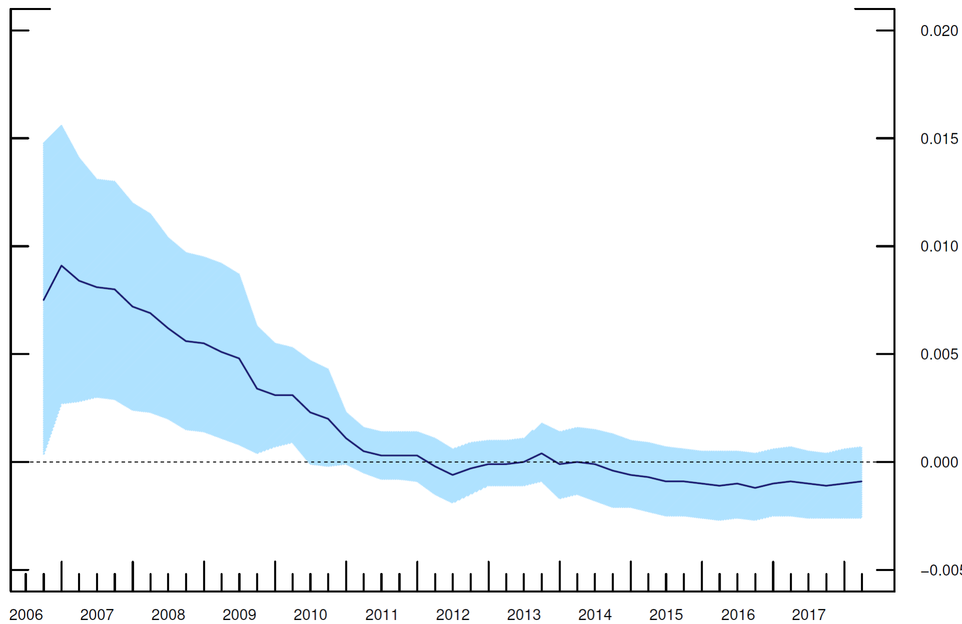
ΔVIX_t	0.002*	0.009***	0.008**
	(0.001)	(0.003)	(0.003)
$\Delta log(ShortTBillsOut_t)$	-0.284***	-0.662***	-0.665***
	(0.073)	(0.154)	(0.152)
$\Delta VIX_t \times 1_{2009}$		-0.009***	-0.009**
		(0.003)	(0.003)
$\Delta log(ShortTBillsOut_t) \times 1_{2009}$		0.595***	0.597***
		(0.151)	(0.150)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t$			0.021
			(0.053)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t \times 1_{2009}$			-0.015
			(0.053)
$\Delta VIX_t + \Delta VIX_t \times 1_{2009}$		-0.000	-0.000
		(0.001)	(0.001)
$\Delta log(ShortTBillsOut_t) + \Delta log(ShortTBillsOut_t) \times 1_{2009}$		-0.067**	-0.069**
		(0.029)	(0.029)
P-value	0.769	0.424	0.431
Adj RSq	0.065	0.112	0.112
N obs	2405	2405	2405
N obs	2405	2405	2405

Note: This table shows the empirical results of equation (24) using overlapping daily data. ΔVIX_t is the 5-day first difference of the VIX Index, and $\Delta log(ShTbillsOut_t)$ is the 5-day log difference of Treasury bills outstanding with maturity less than one month. 1_{2009} is an indicator variable equal to one after January 1, 2009. We also use $\Delta FedFunds_t$ the 5-day first difference of the federal funds rate, $FedFunds_{t-5}$ the 5-day lag of the federal funds rate, $\Delta log(USTNotesOut_t)$ is the 5-day log difference of total U.S. Treasury notes and bonds outstanding, and two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to March 2020. Estimates exclude quarter-end dates (and $\pm$ two days surrounding quarter-end). The dependent variable and ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

and statistically significant at the end of 2006. This period is the pinnacle of the securitization boom that began in the previous decade. We would expect that this period also coincides with an increase in financial engineering, which allowed agents to use more private assets as collateral. Interestingly, the heightened sensitivity in the first year of our sample comes right after the passage of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005, which significantly changed the bankruptcy treatment of repos backed by mortgage-related securities. This change encouraged dealers to increase credit supply financed by repos, as shown by Lewis (2023).[27] From the lens of our model, the increased reliance on repos backed

[27] This Act expanded exemption from automatic stay repos backed by private-label mortgage collateral, granting cash lenders to immediately access the underlying collateral in case of a borrower default. Repos

Figure 2: Five-Day Sensitivity of ΔCY_t to ΔVIX_t



Note: The solid line shows the point estimate of the 5-day estimation of the full model in equation (24) using daily data and ± two years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.

by private-label MBS increased the financial system's exposure to aggregate volatility shocks.

After the crisis, the point estimate began to decline, turning insignificant at the end of 2011, when new regulatory initiatives took hold, and financial firms' ability to use private collateral was less attractive. From the lens of our model, the results in Figure 2 suggest that before the onset of the crisis, the economy relied heavily on private assets as collateral, a trend which persistently reversed after that.

4.3 Robustness

We have performed a series of robustness tests. Here, we describe the main results, but the detailed analysis can be found in Appendix E.

First, we perform our analysis using low-frequency monthly data since 1950. We show that the sensitivity of the convenience yield to changes in aggregate volatility is higher in the 90s and 2000s when compared with the decades immediately after World War II, consistent with a secular increase in the use of private collateral driven by a process of financial innovation and deregulation, as documented by Gorton et al. (2012) (see Appendix

backed by U.S. Treasuries and Agency securities were exempted from the automatic stay in the Bankruptcy Amendments Act of 1984.

E.1).[28]

Second, we extend the sample period to March 2023, capturing the large uncertainty change introduced by the breakout of the COVID-19 pandemic, and we use one-day differences rather than five-day differences (see Appendix E.2). We still obtain our main changes in the sensitivity of convenience yields to aggregate volatility as predicted by the model.

5 Concluding Remarks

We have characterized the relationship between aggregate volatility, which determines the cyclical properties of the economy, and risk sharing, which determines its distributional properties. As assets are both used for intratemporal and intertemporal reasons, aggregate volatility can either improve or weaken risk sharing depending on the composition of private and public assets used as collateral to sustain insurance promises. The valuation of collateral then gives the main linkage, as aggregate volatility affects the valuation of private and public assets in opposite directions. An economy that relies more on private assets to collateralize risk sharing sees insurance decline when aggregate volatility increases. More generally, however, this intuition carries over to other shocks affecting the valuation of different types of collateral in opposite directions.

Our model then relates aggregate volatility and risk sharing, depending on the intensity of using private assets as collateral. We overcome the difficulty of measuring risk sharing by using the convenience yield of safe assets as a proxy and testing its sensitivity to changes in aggregate volatility. We provide empirical evidence that this sensitivity has sharply declined after the Great Recession, alongside a sharp increase in the use of public assets as collateral. From the prism of our model, this suggests that the U.S. economy's reliance on government bonds as collateral after the Great Recession has strengthened financial stability by reinforcing interbank insurance.

Our model highlights the dangers of excessive reliance on private assets as collateral and the importance of regulation to incorporate this dimension into macroprudential policies. When financial intermediaries rely more and more heavily on private assets to sustain interbank relations, a shock on aggregate volatility would curtail insurance across banks, hinder the provision of funding liquidity, and introduce financial instabilities.

[28]For example, in the 1980s repos were excluded from automatic stay, contributing to the prevalence of these types of contracts.

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Appendix

A Sufficient Primitive Parametric Conditions for the Existence of an Interior Equilibrium with Partial Insurance.

In our simple setting, we can provide a complete characterization of the primitive parameters that generate an interior equilibrium with partial insurance, as we discuss in the main text. In the interior equilibrium without full insurance, Raymond's optimal consumption in $t = 0$ is

$$c_{0R} = \frac{Y_0}{2} + a_0 \frac{\hat{\Theta}_0}{2}, \tag{A.1}$$

his optimal consumption in $t = 1$ is given by equation (3), and his optimal consumption in $t = 2$ is

$$\tilde{c}^r_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} + \frac{(\overline{y} - w)}{(1 + p_1)}; \qquad \tilde{c}^s_{2R} = \frac{\tilde{Y}_2}{2} + \tilde{a}_2 \frac{\hat{\Theta}_0}{2} - \frac{(\overline{y} - w)}{(1 + p_1)}. \tag{A.2}$$

Having characterized agents' optimal consumption paths, we introduce parametric constraints such that preferences and returns are consistent with prices in equilibrium. First, we restrict attention to parameters within the Hansen-Jagannathan bound (correlation between SDF and returns less than 1). This constraint limits the set of returns that can be priced, given a discount factor. The following Lemma characterizes this bound in period $t = 1$ in our specific setting.

Lemma 2. *The Hansen-Jagannathan bound for pricing in $t = 1$ is given by*

$$\left| \left(\mu - \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0 \right) \sigma^2 \right) \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0 \right) \right| \leq \exp \left\{ \frac{1}{4} \gamma^2 \left(1 + \rho \hat{\Theta}_0 \right)^2 \sigma^2 \right\} - 1. \tag{A.3}$$

Proof. The stochastic discount factor in $t = 1$ is

$$\tilde{S} = \beta \exp \left\{ -\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0 \right) \tilde{Y}_2 \right\}$$

and $t = 1$ prices for risk free and risky assets ($\tilde{a}_2 = \rho \tilde{Y}_2$) are,

$$\begin{aligned} p_1 &= \mathbb{E}(\tilde{S}) = \beta \exp \left\{ -\frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0 \right) \mu + \frac{1}{8} \gamma^2 \left(1 + \rho \hat{\Theta}_0 \right)^2 \sigma^2 \right\} \\ \hat{p}_1 &= \mathbb{E}(\tilde{S} \rho \tilde{Y}_2) = \rho \left(\mu - \frac{\gamma}{2} \left(1 + \rho \hat{\Theta}_0 \right) \sigma^2 \right) p_1, \end{aligned}$$

which, written in terms of excess returns, implies that $\mathbb{E}\left(\tilde{S}\left(\frac{\rho\tilde{Y}_2}{\hat{p}_1}-\frac{1}{p_1}\right)\right)=0$. Therefore, the Hansen-Jagannathan bound requires that

$$\left|\mathbb{E}(\tilde{S})\mathbb{E}\left(\frac{\rho\tilde{Y}_2}{\hat{p}_1}-\frac{1}{p_1}\right)\right|\leq\mathbb{V}(\tilde{S})\mathbb{V}\left(\frac{\rho\tilde{Y}_2}{\hat{p}_1}\right)$$

where

$$\begin{aligned}\mathbb{V}(\tilde{S}) &= \mathbb{E}(\tilde{S}^2)-\mathbb{E}(\tilde{S})^2\\ &= \beta^2\mathbb{E}\left(\exp\{-\gamma(1+\rho\hat{\Theta}_0)\}\tilde{Y}_2\right)-p_1^2\\ &= \beta^2\mathbb{E}\left(\exp\left\{-\gamma\left(1+\rho\hat{\Theta}_0\right)\mu+\frac{1}{2}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}\right)-p_1^2\\ &= p_1^2\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}-1\right)\end{aligned}$$

Thus, the bound can be rewritten as

$$\left|\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\right|\leq\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}-1.$$

□

The first restriction for an interior equilibrium with partial insurance is that agents hold both public and private assets, which can only happen if the private asset price is positive and lower than the long-term bond price. This condition restricts the return and variance of the private asset relative to agents' risk tolerance. Formally,

$$0\leq\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\leq 1 \tag{A.4}$$

Within this set of parameters, we can set bounds for asset prices at $t=1$. From the above characterization of p_1 and $\hat{p}_1$, and the parameter restriction in A.4, we have that $p_1,\hat{p}_1\in(0,1)$.

Given these parameters, we have to additionally guarantee partial insurance from condition (12) and an interior equilibrium in $t=1$ regardless of the idiosyncratic shock realization.

To achieve this result note that imposing conditions (A.3) and (A.4) gives,

$$
\begin{aligned}
\ln(2p_1) &= \ln(2\beta) - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{4}\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right) \\
&= \ln(2\beta) - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\left(\mu - \frac{1}{2}\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right) - \frac{1}{8}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2 \\
&\geq \ln(2\beta) - \left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} + \frac{1}{8}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2 - 1\right) \geq 0.
\end{aligned}
$$

The last inequality holds if $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ is sufficiently small because $\log(2\beta) > 0$ and $g(x) = \exp(x) + x/2 - 1$ is strictly increasing and equal to zero when $x = 0$, hence $p_1 > \frac{1}{2}$.

To have partial insurance, we restrict the size of the idiosyncratic shock $\bar{y}$ to be large enough such that

$$
\bar{y} \geq \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2} > \frac{\Theta_0^{Sh}}{2} + p_1\frac{\Theta_0}{2} + \alpha\hat{p}_1\frac{\hat{\Theta}_0}{2} \equiv w, \tag{A.5}
$$

guaranteeing partial insurance from condition (12).

We can also restrict the size of the idiosyncratic shock $\bar{y}$ to be small enough such that

$$
\begin{aligned}
\bar{y} &\leq \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4} \\
&< \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2}(1+2p_1) + \alpha\hat{p}_1\frac{\hat{\Theta}_0}{2} \equiv \frac{\Theta_0}{2}(1+p_1) + w,
\end{aligned} \tag{A.6}
$$

guaranteeing an interior equilibrium from condition (13).

Thus, conditions (A.5) and (A.6) need to hold jointly, determining the primitive sufficient conditions for the existence of an interior equilibrium with partial insurance that makes returns and preferences consistent with pricing. When $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ is sufficiently small, there is always a size of the idiosyncratic shock in which such equilibrium exists as long as

$$
\Theta_0 > \alpha\left(1 - \frac{\rho}{2}(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\right)\hat{\Theta}_0.
$$

B Empirical Evidence of the Sensitivity of Private and Public Assets to Aggregate Volatility

We document here how the prices of different assets react to aggregate volatility. We will show that, as the theory predicts, public assets have a negative sensitivity to aggregate volatility—they increase in value when the future becomes more uncertain, the celebrated "negative beta" effect—while private assets have a positive one—they decrease in value in the face of future uncertainty— being stronger the riskier they are.

Table B.1 shows the sensitivity of changes in 10-year U.S. Treasury, Agency MBS, investment grade corporate bond, and high yield bond yields to changes in ΔVIX, a typical proxy for aggregate volatility. Consistent with existing literature, the 10-year U.S. Treasury yield increases as volatility increases, confirming the negative beta effect. As the risk of the asset class increases—from Agency MBS bonds to high yield corporate bonds—the sensitivity of changes in yields to aggregate volatility decreases. In particular, high-yield corporate bond yields significantly decrease (in a statistical sense) as aggregate volatility increases.

Table B.1: Yields and Volatility

	Δ10-year UST	ΔMBS	ΔIG Corp Bonds	ΔHY Corp Bonds
$\Delta FedFunds_t$	-0.065	-0.021	-0.087*	0.005
	(0.047)	(0.056)	(0.051)	(0.071)
$FedFunds_{t-5}$	-0.002	-0.002	0.001	-0.001
	(0.002)	(0.003)	(0.002)	(0.003)
$\Delta log(ShortTBillsOut_t)$	-0.005	0.024	0.077	0.105
	(0.064)	(0.068)	(0.064)	(0.112)
$\Delta log(USTNotesOut_t)$	-1.777**	-2.242**	-3.078***	-3.353*
	(0.807)	(0.949)	(0.957)	(2.020)
ΔVIX_t	-0.012***	-0.007***	-0.002	0.038***
	(0.002)	(0.002)	(0.002)	(0.003)
P-value	0.340	0.184	0.018	0.000
Adj RSq	0.099	0.037	0.051	0.340
N obs	2406	2404	2604	2604

Note: This table shows the empirical results of equation (24) using overlapping daily data. The dependent variables are the 5-day changes in the 10-year U.S. Treasury yield, Agency MBS, investment grade corporate bond, and high-yield bond yields. ΔVIX_t is the 5-day first difference of the VIX Index, $\Delta FedFunds_t$ is the 5-day first difference of the federal funds rate, and $FedFunds_{t-5}$ is the 5-day lag of the federal funds rate. $\Delta log(ShTbillsOut_t)$ is the 5-day log difference of Treasury bills outstanding with maturity less than one month, and $\Delta log(USTNotesOut_t)$ is the 5-day log difference of total U.S. Treasury notes and bonds outstanding. Two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to March 2020. Estimates exclude quarter-end dates (and ± two days surrounding quarter-end). The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

This spectrum of asset riskiness is behind the composition of collateral used in the economy. While using government bonds as collateral is evident, the use of riskier asset classes in collateralized markets is also prevalent. For example, the Federal Reserve's 2021 Financial Stability Report shows that some CCPs, such as the Options Clearing Corporation, have a sizable share of equity and mutual fund collateral. Intuitively, these collateral decisions may be appropriate to manage idiosyncratic risks that a particular CCP may face. Still, they are subject to the valuation effect in response to the aggregate risk that we put forth in this paper. Furthermore, evidence from U.S. repo markets shows that riskier collateral is substantial in different repo market segments. For example, data from the Federal Reserve Bank of New York shows a sizable amount of tri-party repo activity with nonfedwire eligible securities, which include high-yield corporate bonds and equities.

Interpreting these sensitivities is usually challenging, given the lack of a purely exogenous shock to aggregate volatility. The recent crisis caused by the outbreak of the COVID-19 virus constitutes, however, a unique shock to aggregate volatility and higher future uncertainty—exogenous, unexpected, significant, without an end in sight, and truly aggregate as it affects all countries at once. In Figure B.1, we use the VIX as a measure of aggregate volatility, which was relatively stable during 2018 and 2019 and indeed exhibited a large and sudden increase starting in February 2020 with the COVID-19 outbreak. As the VIX was stable, the spread between public and private yields was roughly constant. In February of 2020, the behavior of public and private yields suddenly moved in opposite directions.

These empirical regularities further restrict the set of parameters that makes our setting consistent with data, in particular, such that aggregate volatility depresses the price of private assets, $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$. This is the case when $\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma$ is sufficiently small (already a condition for an interior equilibrium with partial insurance), which together with the Hansen-Jagannathan bound (Lemma 2) results in

$$\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right) < 1, \tag{B.7}$$

which ensures that $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$.

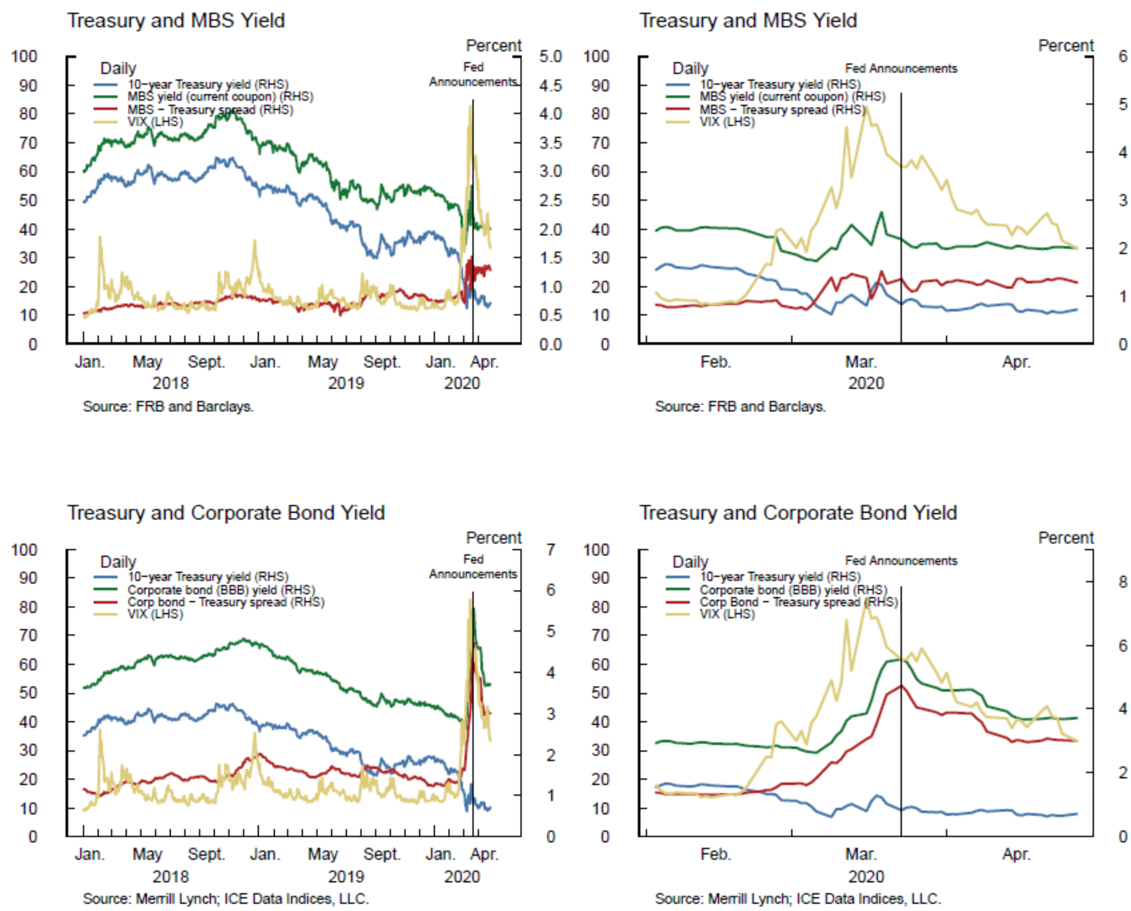

Figure B.1: Ten-Year Treasury, Agency MBS, and Investment-Grade Corporate Bond Yields; Spreads Relative to the Ten-year Treasury Yield and VIX Index

The top two panels show the Treasury and Agency MBS yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The bottom two panels show the Treasury and investment-grade corporate bond yields, their spread, and the VIX index during January 2018–April 2020 and February 2020–April 2020. The tripwire indicates the date the Federal Reserve announced expanded asset purchases and new funding facilities on March 23, 2020.

C Proofs

C.1 Derivation of pricing equations.

In $t = 1$, after it rains or shines, $\tilde{p}_1 = p_1, \tilde{\hat{p}}_1 = \hat{p}_1$, and $\tilde{c}_{1i} = c_{1i}$. Upon this realization, each agent rebalances their portfolio by choosing the optimal amount of long-term bonds (now one-period bonds) and private asset holdings. Specifically, agent $i \in \{R, S\}$'s first order conditions are

$$\begin{aligned} -p_1 u'(c_{1i}) + \beta \mathbb{E}_1(u'(\tilde{c}_{2i})) &\leq 0 \\ -\hat{p}_1 u'(c_{1i}) + \beta \mathbb{E}_1(u'(\tilde{c}_{2i})\tilde{a}_2) &\leq 0. \end{aligned}$$

If both agents hold long-term bonds and private assets, these inequalities hold with equality for both agents, giving rise to equation (5) and (6).

Given the reoptimization strategy in period 1, denote the optimal continuation value of Raymond's $t = 1$ utility by

$$U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1) = Max_{\{\theta_{1R}, \hat{\theta}_{1R}\}} u(c_{1R}) + \beta \mathbb{E}_1(u(\tilde{c}_{2R})).$$

In $t = 0$, each agent optimizes their consumption paths given the expected equilibrium in $t = 1$, subject to the constraints in equations (1). Specifically, Raymond's maximization problem is given by:

$$Max_{\{\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s\}} u(c_{0R}) + \beta \mathbb{E}_0(U_R(\theta_{0R}^{Sh}, \theta_{0R}, \hat{\theta}_{0R}, w_R^r, w_R^s; \tilde{Y}_1)),$$

subject to the constraint (1). Using the envelope condition, this problem leads to the following first-order conditions,

$$\begin{aligned} \theta_{0R}^{Sh} &: \quad -p_0^{Sh} u'(c_{0R}) + \beta \mathbb{E}_0(u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s) \leq 0 \\ \theta_{0R} &: \quad -p_0 u'(c_{0R}) + \beta \mathbb{E}_0(\tilde{p}_1 u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s)\underline{p}_1 \leq 0 \\ \hat{\theta}_{0R} &: \quad -\hat{p}_0 u'(c_{0R}) + \beta \mathbb{E}_0((\tilde{a}_1 + \tilde{\hat{p}}_1) u'(\tilde{c}_{1R})) + (\xi_R^r + \xi_R^s)\alpha \underline{\hat{p}}_1 \leq 0 \\ w_R^r &: \quad q^r u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^r)) - \xi_R^r = 0 \\ w_R^s &: \quad q^s u'(c_{0R}) - \frac{\beta}{2} \mathbb{E}_0(u'(\tilde{c}_{1R}^s)) - \xi_R^s = 0, \end{aligned}$$

where ξ_R^r and ξ_R^s are the Lagrange multipliers associated with the collateral constraint in (1) for w_R^r and w_R^s, respectively; and $\tilde{c}_{tR}^r$ and $\tilde{c}_{tR}^s$ are Raymond's consumption when it rains and shines, respectively. In equilibrium, Raymond will buy insurance for when it shines and sell insurance for when it rains. That is, Raymond's collateral constraint will possibly bind only when it rains in $t = 1$. Similarly, Shirley's collateral constraint will possibly bind only when it shines in $t = 1$. Thus $\xi_R^s = \xi_S^r = 0$. If those constraints bind, state-specific constraints lead to the following pricing of insurance contracts:

$$q^s = \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s)}{u'(c_{0R})}\right), \quad q^r = \frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1S}^r)}{u'(c_{0S})}\right)$$

and Lagrange multipliers

$$\xi_R^r = q^r u'(c_{0R}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1R}^r)), \quad \xi_S^s = q^s u'(c_{0S}) - \frac{\beta}{2}\mathbb{E}_0(u'(\tilde{c}_{1S}^s)).$$

If both agents hold short-, long-term bonds, and private assets, the first three inequalities hold with equality, generating equations (7), (8), and (9).

C.2 Proof of Proposition 2

In the interior symmetric equilibrium with partial insurance, condition (13) holds, thus $\overline{y} - \frac{\Theta_0^{Sh}}{2} \leq (1+p_1)\frac{\Theta_0}{2} + p_1\frac{\Theta_0}{2} + \alpha\hat{p}_1\frac{\hat{\Theta}_0}{2}$. Using the Proposition's condition $\Theta_0 \leq \alpha\frac{\hat{p}_1}{p_1}\hat{\Theta}_0$, from equation (23), we have

$$\begin{aligned}\frac{\partial c_{1R}^s}{\partial \sigma^2} &\leq \frac{1}{(1+p_1)}\alpha\frac{\hat{p}_1}{p_1}\frac{3}{2}\hat{\Theta}_0\frac{\partial p_1}{\partial \sigma^2} - \frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2} \\ &= \frac{\alpha}{(1+p_1)}\frac{\gamma}{4}\left(1+\rho\hat{\Theta}_0\right)p_1\hat{\Theta}_0\left[\frac{3}{2}\frac{\hat{p}_1}{p_1}\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho,\right]\end{aligned}$$

where the second equality replaces the expression for $\frac{\partial p_1}{\partial \sigma^2}$. Replacing the expression for $\hat{p}_1/p_1$, the Hansen-Jagannathan bound A.3 puts a constraint to the term in the square bracket

$$\frac{3}{2}\rho\left(\mu - \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right) - \rho \leq \frac{3}{2}\rho\left[\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - \frac{5}{3},\right]$$

Since $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ is sufficiently small (condition for an interior equilibrium), we have, $\frac{\partial c_{1R}^s}{\partial \sigma^2} \leq 0$.

Finally, we have to show that $\frac{\partial^2 c^s_{1R}}{\partial\alpha\partial\sigma^2} \leq 0$. From the expression in equation (23), we have

$$
\begin{aligned}
\frac{\partial^2 c^s_{1R}}{\partial\alpha\partial\sigma^2} &= \frac{1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[\frac{1}{(1+p_1)}\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2} - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right] \\
&= \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)\frac{\gamma}{4}(1+\rho\hat{\Theta}_0) - (1+p_1)\right] \\
&\leq \rho\frac{p_1}{(1+p_1)^2}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\frac{\hat{\Theta}_0}{2}\left[\frac{1}{2}\left(\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\} - 1\right) - (1+p_1),\right] < 0
\end{aligned}
$$

also using the Hansen-Jagannathan bound and sufficiently small $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$.

■

C.3 Proof of Proposition 3

The proof of $\frac{\partial CY}{\partial\sigma^2} > 0$ is a direct consequence of Lemma 1 and Proposition 2.

The expression for $\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2}$ comes from taking the derivative of $\frac{\partial CY}{\partial\sigma^2}$ and using Lemma 1. Specifically, using the expression in equation (23) and the proof of Proposition 2, we have

$$
\begin{aligned}
\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} &= \gamma CY\frac{\gamma\hat{p}_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\frac{\partial c^s_{1R}}{\partial\sigma^2} - \gamma p_0^{rf}\frac{\partial^2 c^s_{1R}}{\partial\alpha\partial\sigma^2} \\
&= \frac{\gamma CY}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[\frac{\gamma\hat{p}_1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2} - \frac{\alpha\gamma\hat{p}_1}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}\right] \\
&\quad - \left[\frac{\gamma p_0^{rf}}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left[\frac{1}{(1+p_1)}\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2} - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right] \\
&= \frac{\gamma}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{\gamma CY\hat{p}_1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\partial p_1}{\partial\sigma^2} - \frac{\alpha\gamma CY\hat{p}_1}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}\right. \\
&\quad \left. - \left[\frac{p_0^{rf}}{(1+p_1)}\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2} - \frac{\gamma p_0^{rf}}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\}
\end{aligned}
$$

Because of equation equation (12), $\overline{y} - \frac{\Theta_0^{Sh}}{2} > p_1\frac{\Theta_0}{2} + \alpha\hat{p}_1\frac{\hat{\Theta}_0}{2}$, therefore

$$
\begin{aligned}
\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} \quad &\geq \quad \frac{\gamma}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\frac{\gamma CY p_1}{(1+p_1)}\left[\frac{\Theta_0}{2}+\alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right]\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\alpha\gamma CY\hat{p}_1}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2}\right. \\
&\quad \left. -\left[\frac{p_0^{rf}}{(1+p_1)}\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2}-\frac{\gamma p_0^{rf}}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right]\right\} \\
&= \quad \frac{\gamma}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\left\{\left(\frac{\gamma CY p_1}{(1+p_1)}\left[\frac{\Theta_0}{2}+\alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right]-\frac{p_0^{rf}}{(1+p_1)}\right)\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2}\right. \\
&\quad \left. -\left[\frac{\alpha\gamma CY\hat{p}_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2}-p_0^{rf}\right]\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\right\}.
\end{aligned}
$$

As before, using the Hansen-Jagannathan bound, $\frac{\hat{p}_1}{p_1}\frac{\partial p_1}{\partial\sigma^2}$ can be made arbitrarily small for a sufficiently small $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ (condition for an interior equilibrium). From the pricing equations (10) and (11) it is clear that $p_0^{rf} > CY$, and again using the Hansen-Jagannathan bound we have that,

$$
\begin{aligned}
\frac{\alpha\gamma\hat{p}_1}{(1+p_1)}\frac{\hat{\Theta}_0}{2} \quad &= \quad \frac{\alpha\gamma}{(1+p_1)}\frac{\hat{\Theta}_0}{2}\rho\left(\mu-\frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma^2\right)p_1 \\
&\leq \quad \frac{\alpha\rho\hat{\Theta}_0}{\left(1+\rho\hat{\Theta}_0\right)}\left[\exp\left\{\frac{1}{4}\gamma^2\left(1+\rho\hat{\Theta}_0\right)^2\sigma^2\right\}-1\right]\frac{p_1}{(1+p_1)}
\end{aligned}
$$

which is strictly less than 1 for $\gamma\left(1+\rho\hat{\Theta}_0\right)\sigma$ sufficiently small, which ensures that $\frac{\partial^2 CY}{\partial\alpha\partial\sigma^2} > 0$ completing the proof.

■

D Direct Effect of Aggregate Volatility on Idiosyncratic Risk

In this appendix, we explore the case in which an increase in volatility also increases agents' idiosyncratic risk. That is, the level of future volatility in $t = 2$ affects the magnitude of the idiosyncratic shock in $t = 1$,

$$\hat{y} = \overline{y} + \eta\sigma^2.$$

This specification captures the notion that higher future volatility may increase the need for idiosyncratic risk sharing, captured by the parameter η. For σ (or η) sufficiently small enough, $\hat{y}$ satisfies same conditions as in the baseline model that ensure the existence of an interior equilibrium with partial insurance.[29] In this version of the model, prices in $t = 1$ are as in the original one (equations (15) and (16)) because in the case without wealth effects (CARA utility) the realization of the idiosyncratic shock does not change prices. Denoting the equilibrium consumption processes by $\hat{c}_{it}$, from equation (14) we have that

$$\hat{c}^s_{1R} = c^s_{1R} - \frac{\eta\sigma^2}{1+p_1}.$$

Therefore, from equation (23), we have that

$$\frac{\partial \hat{c}^s_{1R}}{\partial \sigma^2} = \frac{\partial c^s_{1R}}{\partial \sigma^2} - \frac{\eta}{(1+p_1)} \underbrace{\left[1 - \frac{\sigma^2}{(1+p_1)}\frac{\partial p_1}{\partial \sigma^2}\right]}_{>0, \text{ for } \frac{\gamma}{2}\left(1+\rho\hat{\Theta}_0\right)\sigma \text{ small.}} \tag{D.8}$$

$$= \frac{1}{(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2} + \eta\sigma^2\right]\frac{\partial p_1}{\partial \sigma^2}$$

$$-\frac{\alpha}{(1+p_1)}\frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\rho p_1\frac{\hat{\Theta}_0}{2} - \frac{\eta}{(1+p_1)} \tag{D.9}$$

The expression in equation (D.8) confirms that if the size of the idiosyncratic shock is proportional to aggregate volatility, then an increase in aggregate volatility decreases the amount of risk sharing. Thus, under the conditions of Proposition 2, if there are more private assets used as collateral, an increase in volatility leads to even larger reductions in risk sharing.

This model extension directly characterizes the intuition behind the alternative explanation of the empirical results in Section 4. Specifically, following the same steps as in Proposition 3, in this model, the sensitivity of of the convenience yield is given by

$$\frac{\partial CY}{\partial \sigma^2} = -\gamma p_0^{rf}\frac{\partial \hat{c}^s_{1R}}{\partial \sigma^2}.$$

Thus, for a high enough sensitivity of idiosyncratic risk to aggregate volatility, η, the convenience yield is increasing in future volatility.

[29]Specifically, that $\hat{y} \in [\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}_0}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}_0}{4}]$.

E Robustness

E.1 Long-Term Analysis

We use the data in Nagel (2016) for the long-term analysis.[30] The convenience yield is measured as the spread between the banker's acceptance and the three-month T-bills spread (BA/T-bill spread). The VIX index is only available from 1990 onward, but earlier periods are estimated using the projection of the VIX on realized S&P Index volatility. We winsorize the changes in convenience yield and VIX at the 1^{st} and 99^{th} percentile to control for outliers. The interest rate is the federal funds rate, and the government's supply of bonds is captured by the total amount of T-bill outstanding and total U.S. debt relative to GDP. This post-war data *frequency is monthly, from January 1952 to December 2011.*[31] Further data details can be found in Nagel (2016).

To capture how the share of public to private assets has changed over time, we use updated quarterly series from Gorton et al. (2012) provided by Almadani et al. (2020), which measure the amount of safe assets issued by U.S. federal, state, and local governments and the amount safe assets issued by the financial sector, both as a share of total assets in the U.S. economy.[32] That is the share of public and private safe assets in the United States. Figure E.2 shows that since 1952, the growth in the share of private assets has outpaced the share of public assets, peaking in the second quarter of 2009 and then sharply reversing. These trends are consistent with a change in the prevalence of private vs. public collateral circa 2009.

To leverage these data, we modify the empirical model in (24) to study how changes in the relative share of private to public safe assets affect the sensitivity of the convenience yield to changes in future volatility. From the lens of our model, if the use of private safe assets is larger than public safe assets, i.e, $\hat{p}_0\hat{\Theta}_0/p_0\Theta_0$ is high; then we would expect the convenience yield to increase with volatility. Specifically, we estimate

$$\begin{aligned}\Delta CY_t \;&=\; \beta_0 + \theta\Delta Gov_t + \sum \gamma_j \Delta CY_{t-j} + \beta_F \Delta FedFunds_t + \eta FedFunds_{t-1} \\ &\quad + \beta_V \Delta VIX_t + \delta \frac{Fin\,SafeAssets}{Gov\,SafeAsset}_{q(t)-1} \\ &\quad + \delta' \frac{Fin\,SafeAssets}{Gov\,SafeAsset}_{q(t)-1} \times \Delta VIX_t + \epsilon_t, \qquad \text{(E.10)}\end{aligned}$$

[30] This dataset is available on Nagel's website.

[31] Nagel (2016) provides convenience yield data from January 1920, however we use data from Almadani et al. (2020) which has the share of public and private safe assets in the economy starting in 1952 (see Figure E.2).

[32] We would like to thank Michael Batty and Wayne Passmore for providing us with the updated series at a quarterly frequency.

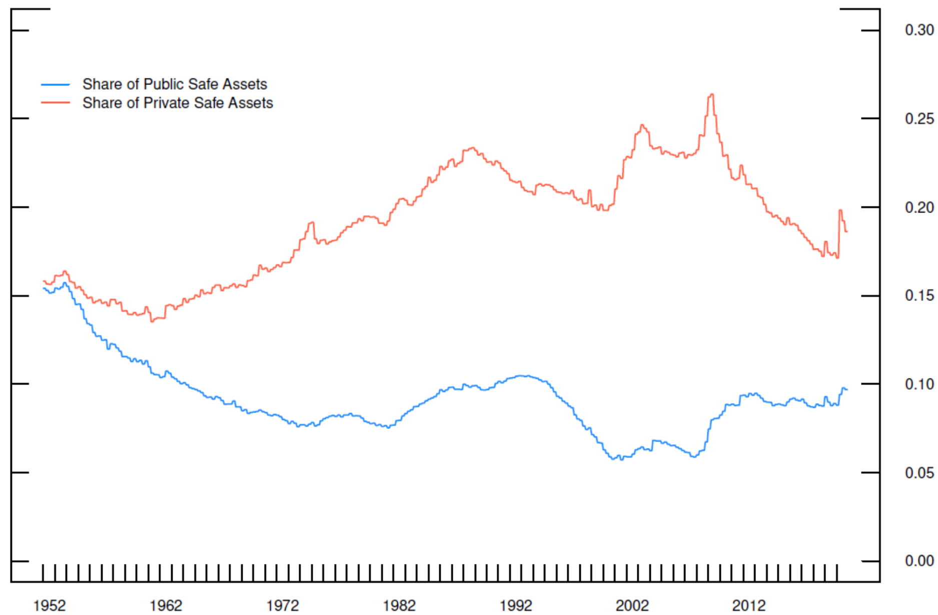

Figure E.2: Safe assets issued by the financial sector and safe assets issued by the government as a share of total assets in the U.S. economy
The figure shows the share of safe assets issued by U.S. federal, state, and local governments (i.e., the share of public safe assets) and the amount of safe assets issued by the financial sector (i.e., the share of private safe assets), both as a share of total assets in the U.S. economy.

where $Fin\,SafeAssets_{q(t)-1}$ is safe assets issued by the financial sector as a share of total assets in the U.S. economy and $Gov\,SafeAsset_{q(t)-1}$ is safe assets issued by U.S. federal, state, and local governments as a share of total assets in the U.S. economy; both measured at the end of the previous quarter.

Table E.2 shows the estimates of equation (E.10), highlighting the interaction term between the share of private to public assets and changes in the VIX. Consistent with the insights from Proposition 3, the regression with government controls shows that if there are more private assets available in the economy (i.e., a higher private to public safe asset share), changes in future aggregate volatility would decrease the amount of risk sharing and put upward pressure on the convenience yield. The total average effect of changes in the VIX and its interaction with private to public safe asset share is positive and statistically significant in both specifications. From our model's eyes, this happens because of the economy's increased reliance on private assets, which reduces the convenience yield in response to an increase in aggregate volatility.

Table E.2: Convenience Yield and Volatility Interacted with Fraction of Share Safe Assets Issued by Financial and Government Sector

ΔVIX_t	-0.001	-0.002
	(0.005)	(0.005)
$\frac{Fin\,SafeAssets}{Gov\,SafeAssets}_{q(1)-1}$	0.002	0.006
	(0.005)	(0.006)
$\Delta VIX_t \times \frac{Fin\,SafeAssets}{Gov\,SafeAssets}_{q(1)-1}$	0.003	0.004*
	(0.002)	(0.002)
$\Delta VIX_t + \Delta VIX_t \times avg\left(\frac{Fin\,SafeAssets}{Gov\,SafeAssets}_{q(1)-1}\right)$	0.006**	0.006**
	(0.003)	(0.003)
Government Controls	N	Y
P-value	0.665	0.653
Adj RSq	0.189	0.199
N obs	720	720

Note: This table shows the empirical results of equation (E.10) using monthly average data. The convenience yield measure is the spread between the monthly average 3-month bankers' acceptance and the monthly average 3-month T-bills. ΔVIX_t is the first difference of the monthly average VIX Index and $\frac{Fin\,SafeAssets}{Gov\,SafeAssets}_{q(1)-1}$ is the ratio between safe assets issued by the financial sector and safe assets issued by U.S. federal, state, and local governments, both as a share of total assets in the U.S. economy in the previous quarter-end. $avg(\cdot)$ is the operator that takes sample average. We also use $\Delta FedFunds_t$ the first difference of the monthly average federal funds rate, $FedFunds_{t-1}$ the lagged monthly average federal funds rate, and two lags of the dependent variable are included as controls (not shown) with reported p-values of lags equal to zero. Government controls are $\Delta log(TbillOut_t/GDP_t)$ the monthly log difference of total outstanding T-bills to GDP, $\Delta log(Debt_t/GDP_t)$ the monthly log difference of total U.S. debt to GDP (not shown). The sample runs from January 1952 to December 2011. The dependent variable and the ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 11 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

E.2 Expanding Sample and the Frequency

Following the same empirical strategy used to create Figure 2, Figure E.3 exploits the high-frequency data to estimate the model at a daily frequency over shorter time intervals to see the evolution of ΔCY_t's sensitivity to ΔVIX_t. The results for 1- and 5-day changes are qualitatively similar.[33] Specifically, the sensitivity of ΔVIX_t on ΔCY_t is positive and statistically significant toward the end of 2006 and loses significance after that.

The results in Table E.3—which expands the sample period to March 2023—and E.4—which uses daily changes—are qualitatively similar to those in Table 1. There is a positive and statistically significant relationship between changes in the convenience yield and changes in the VIX in the early part of the sample, before the GFC. After the GFC, the relationship loses its statistical power.

[33]The scales on both figures are the same to simplify the comparison.

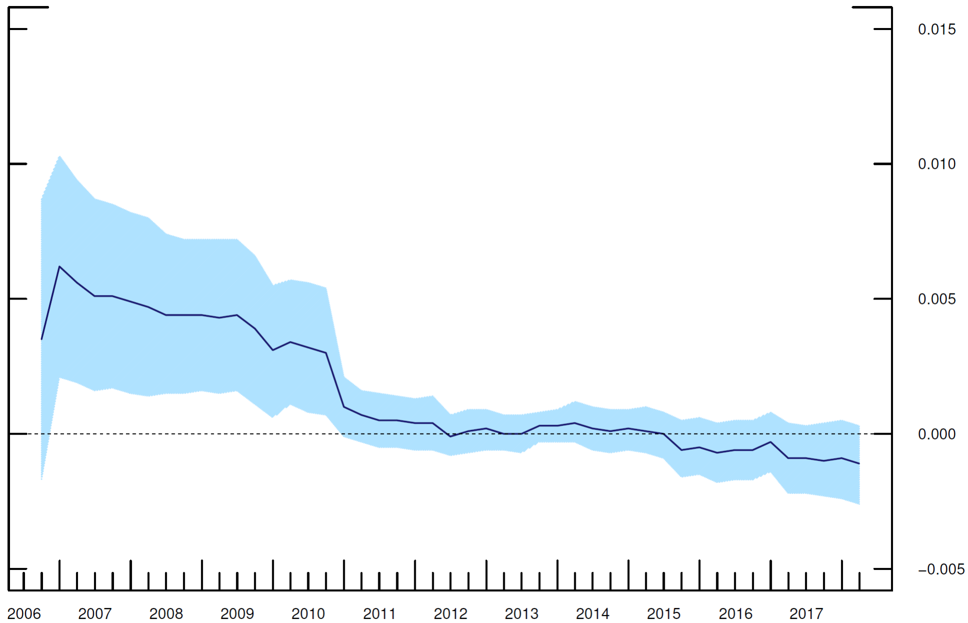

Figure E.3: One-Day Sensitivity of ΔCY_t to ΔVIX_t

The solid line shows the point estimate of the 1-day estimation of the full model in equation (24) using daily data and ± two years of data each quarter. The shaded region shows the 95% confidence interval of each estimate.

F Private Asset Creation

In this appendix, we entertain the idea that Raymond and Shirley can create private assets at a cost. The goal is to provide conditions under which supplying public assets can either crowd out or crowd in private assets (through quantities and their valuations). We also provide conditions under which a stable economy induces the creation and use of private assets to share risk and then makes the risk sharing more fragile to aggregate risk.

F.1 Model with private asset creation

Assume the cost of producing x units of private assets is $C(x)$ in terms of consumption goods, with $C', C'' > 0$. This cost is meant to capture both technological (such as the costs of financial innovation, securitization, managing information, etc.) and regulatory (such as constraints on using private assets by regulated financial institutions) costs to create and use private assets in financial contracts. Agents incur this cost before choosing their portfolio in $t = 0$ and sell these assets (perhaps to themselves) at the equilibrium price $\hat{p}_0$. In this case, Raymond's consumption is as in equations (2) — (4), except that in $t = 0$ Raymond incurs the cost $C(x_R)$ of issuing x_R assets and receives the proceeds, $\hat{p}_0 x_R$, from selling those

Table E.3: Volatility and Convenience Yield: Pre- and Post- 2009
(Sample Period: August 2004 to March 2023)

ΔVIX_t	0.002**	0.008**	0.008**
	(0.001)	(0.003)	(0.003)
$\Delta log(ShortTBillsOut_t)$	-0.252***	-0.624***	-0.626***
	(0.060)	(0.142)	(0.141)
$\Delta VIX_t \times 1_{2009}$		-0.008**	-0.007**
		(0.003)	(0.003)
$\Delta log(ShortTBillsOut_t) \times 1_{2009}$		0.554***	0.557***
		(0.141)	(0.141)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t$			0.020
			(0.047)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t \times 1_{2009}$			-0.027
			(0.048)
$\Delta VIX_t + \Delta VIX_t \times 1_{2009}$		0.000	0.000
		(0.001)	(0.001)
$\Delta log(ShortTBillsOut_t) + \Delta log(ShortTBillsOut_t) \times 1_{2009}$		-0.070	-0.069
		(0.030)	(0.030)
P-value	0.500	0.228	0.231
Adj RSq	0.059	0.104	0.105
N obs	2861	2861	2861

Note: This table shows the empirical results of equation (24) using overlapping daily data. ΔVIX_t is the 5-day first difference of the VIX Index, and $\Delta log(ShTbillsOut_t)$ is the 5-day log difference of Treasury bills outstanding with maturity less than one month. 1_{2009} is an indicator variable equal to one after January 1, 2009. We also use $\Delta FedFunds_t$ the 5-day first difference of the federal funds rate, $FedFunds_{t-5}$ the 5-day lag of the federal funds rate, $\Delta log(USTNotesOut_t)$ is the 5-day log difference of total U.S. Treasury notes and bonds outstanding, and two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to March 2023. Estimates exclude quarter-end dates (and ± two days surrounding quarter-end). The dependent variable and ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

assets. If agents do not internalize the price effect of creating assets, Raymond's optimal production of assets is determined by the following condition

$$C'(x_R^*) = \hat{p}_0. \tag{F.11}$$

Thus, given the problem's symmetry (Shirley faces the same issue at $t = 0$), the total stock of private assets is given by $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$, and all the previous pricing equations hold simply replacing $\hat{\Theta}_0$ with $\hat{\Theta}$.

As with the model with no private asset creation, we focus on an interior equilibrium with partial insurance in which there is imperfect risk sharing, and both agents hold the

Table E.4: Volatility versus Convenience Yield Pre- and Post- 2009
(Daily Frequency)

ΔVIX_t	0.001	0.006***	0.006***
	(0.001)	(0.002)	(0.002)
$\Delta log(ShortTBillsOut_t)$	-0.097***	-0.188***	-0.186***
	(0.011)	(0.028)	(0.028)
$\Delta VIX_t \times 1_{2009}$		-0.006***	-0.006***
		(0.002)	(0.002)
$\Delta log(ShortTBillsOut_t) \times 1_{2009}$		0.130***	0.127***
		(0.029)	(0.029)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t$			0.033
			(0.024)
$\Delta log(ShortTBillsOut_t) \times \Delta VIX_t \times 1_{2009}$			-0.032
			(0.024)
$\Delta VIX_t + \Delta VIX_t \times 1_{2009}$		-0.000	-0.000
		(0.001)	(0.001)
$\Delta log(ShortTBillsOut_t) + \Delta log(ShortTBillsOut_t) \times 1_{2009}$		-0.059***	-0.059***
		(0.009)	(0.009)
P-value	0.004	0.025	0.035
Adj RSq	0.073	0.098	0.102
N obs	3053	3053	3053

Note: This table uses daily data to show the empirical results of equation (24). ΔVIX_t is the 1-day first difference of the VIX Index, and $\Delta log(ShTbillsOut_t)$ is the 1-day log difference of Treasury bills outstanding with maturity less than one month. 1_{2009} is an indicator variable equal to one after January 1, 2009. We also use $\Delta FedFunds_t$ the 1-day first difference of the federal funds rate, $FedFunds_{t-1}$ the 1-day lag of the federal funds rate, $\Delta log(USTNotesOut_t)$ is the 1-day log difference of total U.S. Treasury notes and bonds outstanding, and two lags of the dependent variable are included as controls (not shown), with reported p-values of lags equal to zero. The sample runs from August 2004 to March 2020. Estimates exclude quarter-end dates (and $\pm$ two days surrounding quarter-end). The dependent variable and ΔVIX are winsorized at the 1% and 99%. Newey-West standard errors with 21 lags are reported. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

long-term bond after an idiosyncratic shock, that is, conditions (12) and (13) hold.

Existence is guaranteed with similar arguments as in the case with no private asset creation, except that we have to ensure that the total amount of private assets $\hat{\Theta} = \hat{\Theta}_0 + 2x_R^*$ is such that

$$\overline{y} \in \left[\frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{\Theta}}{2}, \frac{\Theta_0^{Sh}}{2} + \Theta_0 + \alpha\rho(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2)\frac{\hat{\Theta}}{4} \right],$$

which are the same conditions as in (A.5) and(A.5), but replacing $\hat{\Theta}_0$ by $\hat{\Theta}$.

In this case, the equilibrium is characterized by the following system of equations:

$$\begin{aligned} T_1 &:= C'(x_R^*) - \hat{p}_0 = 0 && \text{(F.12)} \\ T_2 &:= \hat{p}_0 - \left[\beta\mathbb{E}_0\left(\hat{p}_1 \frac{u'(\tilde{c}_{1R})}{u'(c_0)}\right) + \alpha\hat{p}_1\left[\frac{\beta}{2}\mathbb{E}_0\left(\frac{u'(\tilde{c}_{1R}^s) - u'(\tilde{c}_{1R}^r)}{u'(c_0)}\right)\right]\right] \\ &= \hat{p}_0 - \hat{p}_1\left(p_0^{rf} + \alpha CY\right) = 0 && \text{(F.13)} \end{aligned}$$

which is guaranteed by the relevant bounds when marginal cost C' is sufficiently high.

Having established the existence of symmetric equilibria with private asset creation, we provide comparative statics on the creation of private assets and their prices relative to an arbitrary parameter z.

Lemma 3. *(Private Asset Creation and Prices). In an interior equilibrium with partial insurance and private asset creation, the comparative statics on private asset creation and prices for an arbitrary parameter z are,*

$$\begin{pmatrix} \frac{\partial x_R^*}{\partial z} \\ \frac{\partial \hat{p}_0}{\partial z} \end{pmatrix} = \frac{1}{|D|}\begin{pmatrix} 1 \\ C''(x_R) \end{pmatrix} \frac{\partial\left(\hat{p}_1(p_0^{rf} + \alpha CY)\right)}{\partial z}$$

where $\frac{\partial\left(\hat{p}_1(p_0^{rf}+\alpha CY)\right)}{\partial z}$ is the partial equilibrium sensitivity of prices without safe asset creation (as obtained from combining Lemma 1 and equation (19)*) and $|D| = C''(x_R) - 2\frac{\partial \hat{p}_0}{\partial \hat{\Theta}_0}$ with*

$$\frac{\partial \hat{p}_0}{\partial \hat{\Theta}_0} = -\gamma\hat{p}_1(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + (p_0^{rf} + \alpha CY)\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0} + \gamma\hat{p}_1(p_0^{rf} + \alpha CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \quad \text{(F.14)}$$

the partial derivative of the private asset price $\hat{p}_0$ to private asset supply.

Proof. Proof of Lemma 3

Invoking the implicit function theorem, we have

$$\begin{pmatrix} \frac{\partial x_R}{\partial \Theta_0} \\ \frac{\partial \hat{p}_0}{\partial \Theta_0} \end{pmatrix} = -\underbrace{\begin{bmatrix} \frac{\partial T_1}{\partial x_R} & \frac{\partial T_1}{\partial \hat{p}_0} \\ \frac{\partial T_2}{\partial x_R} & \frac{\partial T_2}{\partial \hat{p}_0} \end{bmatrix}^{-1}}_{:=D^{-1}} \begin{pmatrix} \frac{\partial T_1}{\partial \Theta_0} \\ \frac{\partial T_2}{\partial \Theta_0} \end{pmatrix}$$

We first have to characterize the partial derivatives with respect to the endogenous variables. These are

$$\frac{\partial T_1}{\partial x_R} = C''(x_R); \qquad \frac{\partial T_1}{\partial \hat{p}_0} = -1; \qquad \frac{\partial T_2}{\partial x_R} = -\frac{\partial \hat{p}_0}{\partial \hat{\Theta}}\frac{\partial \hat{\Theta}}{\partial x_R}; \qquad \frac{\partial T_2}{\partial \hat{p}_0} = 1$$

where with a slight abuse of notation, $\frac{\partial \hat{p}_0}{\partial \hat{\Theta}_0}$ is the partial derivative of the $t = 0$ price of the private asset in the original model. Specifically,

$$\begin{aligned}
\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0} &= -\gamma \hat{p}_1(\alpha p_0^{rf} + CY)\frac{\partial c_{1R}^s}{\partial \hat{\Theta}_0} + (p_0^{rf} + \alpha CY)\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0} + \gamma \hat{p}_1(p_0^{rf} + \alpha CY)\frac{\partial c_{0R}}{\partial \hat{\Theta}_0} \\
&= -\gamma \hat{p}_1(\alpha p_0^{rf} + CY)\left[-\frac{\gamma \hat{p}_1}{2(1+p_1)^2}\left[\overline{y} - \frac{\Theta_0^{Sh}}{2} + \frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right] - \frac{\gamma \alpha p_1}{2(1+p_1)}\rho^2\sigma^2\frac{\hat{\Theta}}{2} + \frac{\alpha \hat{p}_1}{2(1+p_1)}\right] \\
&\quad + (p_0^{rf} + \alpha CY)\left[-\frac{\gamma}{2}\rho\left(\mu - \frac{\gamma}{2}(1+\rho\hat{\Theta}_0)\sigma^2\right)\hat{p}_1 - \frac{\gamma}{2}\rho^2\sigma^2 + \gamma \hat{p}_1\left(\frac{\rho Y_0}{2} - C'(x_R^*)\right)\right]
\end{aligned}$$

Therefore, we have that

$$|D| = C''(x_R) - 2\frac{\partial \hat{p}_0}{\partial \hat{\Theta}}$$

From these derivatives note that $\frac{\partial T_1}{\partial z} = 0$ and $\frac{\partial T_2}{\partial z} = -\frac{\partial \hat{p}_1\left(p_0^{rf} + \alpha CY\right)}{\partial z}$ is merely the partial equilibrium sensitivities characterized by the model without endogenous safe asset creation giving the Lemma's result. □

The effect of an arbitrary parameter on private asset creation is determined by $|D| = C''(x_R) - 2\frac{\partial \hat{p}_0}{\partial \hat{\Theta}}$. The first component captures how fast the marginal cost of producing private assets changes with production. The second component captures how fast the marginal benefit of producing private assets changes with production. The expression $|D|$ is positive when the left-hand side of equation (F.11) increases faster than the right-hand side as there is more production, and negative otherwise. Hence, $|D|$ captures the *change in the net marginal cost of private asset creation*. While the most intuitive case is that the overall cost increases with production (this is $|D| > 0$, as usual with convex production costs), the role of private assets as collateral and their interaction with risk-sharing may flip this net marginal cost.

While the first component of $|D|$ is technological and assumed positive, the second component is characterized in equation (F.14), which encodes equilibrium effects that have three elements: The first is a *direct consumption element*, $\frac{\partial c_{0R}}{\partial \hat{\Theta}} = \frac{\rho Y_0}{2} > 0$. The second is a *supply element*, $\frac{\partial \hat{p}_1}{\partial \hat{\Theta}} < 0$, which captures the reduction of prices from having more private assets. The third is a *risk-sharing element*, $\frac{\partial c^s_{1R}}{\partial \hat{\Theta}_0}$, which is more closely related to our mechanism. This last element involves valuation and quantity effects, which operate in opposing directions. To be more precise,

$$\frac{\partial c^s_{1R}}{\partial \hat{\Theta}_0} = \underbrace{\frac{1}{(1+p_1)}\left[\frac{\partial p_1}{\partial \hat{\Theta}_0}\frac{\Theta_0}{2} + \alpha\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0}\frac{\hat{\Theta}}{2} + \frac{(\overline{y}-w)}{(1+p_1)}\frac{\partial p_1}{\partial \hat{\Theta}_0}\right]}_{\text{Valuation Effect}} + \underbrace{\frac{\alpha \hat{p}_1}{2(1+p_1)}}_{\text{Quantity Effect}}. \tag{F.15}$$

On the one hand, more private assets provide more collateral and sustain more risk sharing, *a positive quantity effect*. This quantity effect depends on the private asset's collateralizability. On the other hand, more private assets reduce their value and usefulness as collateral, *a negative valuation effect*.[34] The net effect of risk sharing depends on how much the economy relies on long-term assets and their pledgeability. For example, if there were many pledgeable long-term assets in the economy (high $\Theta_0, \hat{\Theta}_0$), and private assets were not very pledgable (low α), then the valuation effect would dominate, counterintuitively resulting in less risk sharing.[35]

F.2 Government Bond Supply and Private Asset Creation

It is commonly understood that the provision of government bonds crowds out private assets, as they tend to be substitutes for their use as a store of value and collateral, disincentivizing their production. From Lemma 3, crowding out is formally captured when $\frac{\partial x^*_R}{\partial \hat{\Theta}_0} < 0$. Since $\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0} < 0$, this is the case when $|D| > 0$. As discussed above, this condition is fulfilled when the net marginal cost of producing private assets increases.

These intuitive results, however, may flip in general equilibrium when private asset valuations are strong enough. Specifically, if the supply of private assets depresses equilibrium prices in $t = 1$, relative to the increase in marginal production costs, then $|D| < 0$. Intuitively, more government bonds reduce their value as collateral and their effectiveness for risk sharing. Suppose this induced collateral scarcity (in terms of government bond value) reduces risk sharing enough. In that case, the implied increase in convenience yields may

[34]Formally, $\frac{\partial p_1}{\partial \hat{\Theta}_0} = -\frac{\gamma}{2}\hat{p}_1 < 0$ and $\frac{\partial \hat{p}_1}{\partial \hat{\Theta}_0} = -\frac{\gamma}{2}\frac{(\hat{p}_1)^2}{p_1} - \frac{\gamma}{2}\rho^2\sigma^2 < 0$.

[35]More formally, this is the case when $\gamma\left(\frac{\Theta_0}{2} + \alpha\frac{\hat{p}_1}{p_1}\frac{\hat{\Theta}_0}{2}\right) > \alpha$.

make private assets more valuable, inducing more of their creation.

This result shows the importance of studying the valuation effects of assets that are substituted in providing several functions—in this case, as a store of value or as collateral—in general equilibrium. Even though public and private assets are substitutes as collateral, their endogenous valuation may turn them into complements.

F.3 Economic Stability and Private Asset Creation

Private asset creation also responds to changes in aggregate volatility. We focus here on the most intuitive case in which public assets crowd out private assets (as discussed above, $|D| > 0$). From Lemma 3, the sensitivity of private asset creation to volatility depends on the partial equilibrium sensitivity of $\hat{p}_0$ to σ^2, which as shown in Proposition 2, depends on the convenience yield component of the valuation. If $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$ holds, we know that a more volatile environment leads to a lower price of private assets in $t = 1$. In addition, if government bonds are abundant, aggregate volatility also improves risk sharing, compresses convenience yields, and further reduces private asset valuations. This result is summarized in the following Proposition.

Proposition 4. *(Aggregate Volatility and Private Asset Production). In an interior equilibrium with partial insurance and private asset creation, if* $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ *is sufficiently small so that* $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$*, there is a sufficiently convex production cost (*$C''(\dot{})$ *is sufficiently large) and enough public assets used as collateral relative to existing private assets, that private asset creation decreases with aggregate volatility (i.e.,* $\frac{\partial x_R}{\partial \sigma^2} < 0$*).*

Proof. First, a C'' sufficiently large guarantees that $|D| > 0$. Because $\frac{\gamma}{2}\left(1 + \rho\hat{\Theta}_0\right)\sigma$ is sufficiently small so that $\frac{\partial \hat{p}_1}{\partial \sigma^2} < 0$. From Proposition 2, enough public collateral relative to existing private collateral induces risk sharing to improve with aggregate volatility (this is $\frac{\partial CY}{\partial \sigma^2} > 0$). □

This Proposition shows the conditions under which, as an economy becomes more stable (with less aggregate volatility), there is more production of private assets, which adds to the available stock of private collateral. We provide conditions purely based on the technological production of private assets and the relative use of public assets as collateral. While the high convexity of production costs guarantees interior solutions (and well-behaved comparative statics), the extensive use of public assets as collateral trumps the relevance of convenience yields on the valuation of private assets.

This result is relevant for several reasons. First, it highlights that private assets can be heavily created as the economy becomes more stable, increasing the importance of private assets as collateral. While beneficial in stable times, the economy's higher reliance on private assets makes risk sharing more fragile (this is more likely to suffer) in case of an increase in aggregate risk. Second, this result is more prevalent when private assets' production is indeed more complicated (in the sense of the cost convexity).