Bank Runs in the Digital Era

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Abstract

The Digital Era has introduced advanced communication technologies, such as social media and the internet, that have vastly expanded both the accessibility and spread of information about banks' conditions—though often at the cost of accuracy. Online banking has further enabled swift responses to this information. Our findings indicate that while these technological advancements increase the speed and intensity of bank runs, they may also reduce their overall frequency as these platforms are widely adopted. Although individuals are more inclined to withdraw prematurely because they understand many other agents are paying attention to information simultaneously, they tend to be less concerned about the risk of withdrawing too late if they choose to wait.

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JEL Classifications: D83, G14, G21

1 Introduction

Bank runs have been a persistent phenomenon throughout the history of banking systems. They are typically triggered when depositors lose confidence in their bank's financial stability and, perhaps more critically, by the fear that other depositors are losing confidence, too. How information and rumors spread among depositors is crucial in this dynamic.

In the past, bank runs were slow-moving events, taking several days or weeks. Panic spread through word of mouth, long lines outside bank branches, and intermittent media coverage. In recent years, a significant and rapid change in the operation and anatomy of financial systems has taken place with the rise of social media and the ease of withdrawing deposits via mobile apps. These and other developments of the Digital Era are changing the speed at which information flows among investors and the speed at which they can react to such information.[1]

The failures of Silicon Valley Bank, Signature Bank, and First Republic Bank in the United States and Credit Suisse in Switzerland during the spring of 2023 provide stark examples of rapid withdrawals driven by digital technologies. This speed level is unprecedented, not only from a historical perspective but even in comparison to the global financial crisis in 2008, during which retail depositors did not widely utilize social media and mobile banking apps. In response, regulators have expressed concerns about the impact of social media and banking apps on financial stability. But what role do new technologies of the Digital Era play in shaping financial stability?

Academic research is still scarce, but it has been suggested that these developments could make the financial system more fragile. In general, the logic is that digital communication technologies such as social media allow information (and misinformation) to spread rapidly among depositors, leading to heightened fears and accelerated withdrawals, especially considering modern withdrawal technologies. Rumors or negative news can go viral quickly, increasing the likelihood and frequency of a bank run.

This paper argues that digital technologies may play a more nuanced role. We argue that technological progress in communication and withdrawal technologies initially increases the likelihood of bank runs. However, once enough progress has been made, it can result in reducing said likelihood. Nonetheless, runs that do occur tend to be more intense, as banks lose deposits more rapidly during these events.

We propose a model of dynamic bank runs where the value of a bank's assets changes continuously, influencing depositors' incentives to withdraw. If all depositors were always fully aware of this evolution and could react to it, bank runs would display standard coordination characteristics. When agents receive bad news about the bank's assets, they are less inclined to leave their funds because they are worried other agents may have the same concerns. We assume, however, that depositors' attention to the evolution of a bank's assets is intermittent: at any moment, inattentive agents may start paying attention, while conversely, attentive agents may become inattentive. We model the role of digital technologies in enhancing information spreading by increasing the rate at which depositors become attentive to the bank's asset value.[2]

[1] There is evidence that digital devices are the primary source of information among U.S. adults with 86% of them relying on them and 52% selecting them as the preferred option (Pew Research Center, 2021a). In addition, this penetration increases dramatically for people below 50 with social media being the primary source for adults under 30s. For more evidence of social media use as a source to get news check Pew Research Center (2021b,c,d).

[2] A complementary interpretation of the model is that the 'inattention window' represents a period during which the withdrawal technology limits the depositor's ability to act on information. Digital technologies that

Enhanced communication technologies that increase the rate of attentiveness have two effects. On the one hand, an increase in attentiveness raises the number of informed depositors at any given moment, thereby increasing the likelihood of a run (by heightening coordination pressures) and the scale of the run when it occurs. On the other hand, the incentive to withdraw funds out of fear of being inattentive when others start a run diminishes when depositors understand that they are unlikely to remain inattentive for long.

While the first effect is more relevant as coordination failures become essential, the second effect dominates once the mass of informed investors is already significant. The natural implication is that advancements in communication technologies that enhance information flows may eventually reduce financial fragility. In other words, there is an extent of digital progress or, in particular, social media penetration that maximizes financial fragility. Despite the non-linear impact of improved communication and withdrawal technologies on the likelihood of a crisis, their advancement accelerates the pace of a run if it occurs as people become aware of the bank's health and can react to this information faster during a run. In other words, these technological developments affect two dimensions of fragility deferentially; while improved communication technologies such as social media increase the *magnitude of runs*, they may decrease the *frequency of runs.*

This aligns with the pattern observed by Rose (2023), who documented that the most severe runs in recent history before 2022 occurred at Continental in 1984 (with 30% of depositors withdrawing in 10 days) and at Wachovia and Washington Mutual in 2008 (with 4.4% and 10% of deposits withdrawn, respectively, over more than 15 days). In contrast, the most severe run in 2023, with a much larger presence of social media and banking apps, occurred at Silicon Valley Bank, where 25% of deposits were withdrawn in a single day (an additional 62% were scheduled to flow out the next day before the bank was closed).

Further, it is consistent with the evidence presented by Cookson et al. (2023), who study the role of social media in the recent SVB run. The paper suggests that social media had exacerbated the impact on banks during runs, reporting that the stocks of banks with higher Twitter coverage suffered more than those with less social media exposure, even when risk levels were similar. These effects are channeled through the stock market, where information is aggregated. Among riskier banks, the spotlight of social media leads to more considerable stock market losses, creating a new source of contagion.

In addition, we extend the main model to understand what would happen if new digital technologies increase the reach of information rather than just the frequency with which people get informed. On the extensive margin, we study what would happen if these new digital technologies would allow a certain measure of "captive" depositors to start monitoring the bank more actively. This heightens coordination pressures increasing both the *magnitude of runs* and the *frequency of runs.* However, on the intensive margin, as the fraction of depositors that has access to an inferior monitoring technology decreases we have a non-monotonic effect once more. Intuitively, as the average depositor is able to monitor the bank more frequently we first get an increased *frequency of runs* due to heightened coordination pressures, to then observe a decrease in it once the mass of informed agents becomes significant. Consistent with our previous result, increasing the fraction of depositors with a superior monitoring technology results in an increased *magnitude of runs.*

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simplify the withdrawal process reduce the duration of this inaction window, yielding similar qualitative outcomes.

At last, we discuss the singularities that social media platforms have as communication devices and how incorporating changes in information precision can affect our main results.

We contribute to models of dynamic runs, (He and Xiong, 2012; Gertler and Kiyotaki, 2015; Amador and Bianchi, 2024), but introduce a dimension of heterogeneity across depositors that we call *asymmetric inattention*. As in He and Xiong (2012), we use a continuous time setting to capture the effects of rapid improvements in communication technologies on the incentives to run by agents that differ not on the available information but on whether they pay attention to such information.

In addition, we contribute to an emerging literature that studies the impact of new technologies on running behavior (Ziebarth, 2013; Koont et al., 2023; Liu et al., 2023; Koont, 2023). Ziebarth (2013) shows that high levels of county-level radio penetration led to larger deposit outflows during the Great Depression. Koont et al. (2023) show that banks with digital platforms that allow for instantaneous withdrawals as well as potentially providing brokerage services have deposits that are less sticky in the sense that they respond more to changes in the Fed Funds Rate. They argue then that digitalization has reduced the franchise value of banks, which explains why SVB was insolvent even before the run. In a related paper, Koont (2023) shows that the adoption of digital banking platforms has increased competition in the deposit and loan markets by allowing banks to branchlessly enter local markets and, as a result, a larger share of surplus is captured by consumers. However, she argues that digitalization may alter financial stability as it leads to a higher share of uninsured deposits in banks' balance sheets and a greater market share of less regulated mid-sized banks. Liu et al. (2023) dissect the Terra-LUNA crash in 2022 and show how the blockchain technology allowed investors to closely monitor others' behavior, increasing the speed of the run. Further, they argue that the complexity of the system put less sophisticated and poorer investors at a greater informational disadvantage, resulting in delayed withdrawals and larger loses. Our paper is closely related to this literature as we aim to rationalize how the introduction of new communication and withdrawal technologies affect running incentives with a particular focus on the Digital Era.

This paper is also related to a mostly empirical literature that studies the real impact that social media activity can have in economics (Bianchi et al., 2021, 2023; Cookson et al., 2023) and in politics (Allcott and Gentzkow, 2017; Fujiwara et al., 2021; Bessone et al., 2022). Although this is a fully theoretical paper, we contribute to understanding the mechanisms through which increased social media activity can affect the likelihood and severity of bank runs and, hence, real outcomes.

In this paper, we focus on the role of new technologies in influencing depositors' decisions. Still, it's important to note that these technologies can also impact the asset side and banks' risk management as pointed out in Koont (2023). For instance, AI tools could be developed to improve liquidity management and monitor withdrawal patterns, potentially lowering the risk of bank runs. However, we do not explore this dimension in our analysis.

The rest of the paper is organized as follows. Section 2 describes and analyses the results of the main model. Section 3 covers an extension where we study the effects of a higher reach of information. Section 4 discusses the features distinguishing social media from other communication technologies and how incorporating changes in information precision can affect our main results. Finally, section 5 concludes.

2 The Digital Era: Social Media and Banking Apps

2.1 Benchmark Model

Improvements in communication technologies affect the frequency and reach of information signals as well as their precision. This section and the following one focus on the former by treating signals as perfectly informative. Later in the paper, we consider how extending the setting to incorporate noisy signals might affect our main results.

The model is in continuous time with an infinite horizon, featuring a unit measure of households or investors and a bank. Households are risk-neutral with identical discount factor ρ and are all endowed with one unit of the consumption good, which they deposit into the bank in period 0. The bank uses these deposits to invest in long-term assets with a diversified portfolio that ensures rents of at least $r > 0$, the interest rate on deposits, in every period. The value of all bank's investments is given by y_t, which follows a geometric Brownian motion with drift μ and volatility σ. That is, the bank's assets follow the stochastic differential equation

$$dy_t = \mu y_t dt + \sigma y_t dW_t$$

where W_t is a standard Wiener process. Note that the bank's assets never mature, capturing the idea that depositors do not get paid when the bank's projects mature but periodically from rents subtracted from all of the bank's investments of different maturities and rates of return. In contrast to He and Xiong (2012), our specification gives rise to the equilibrium multiplicity of the type discussed by Diamond and Dybvig (1983).

On the liability side, risk-neutral depositors decide whether or not to instantly withdraw their deposits at every moment based on information about the bank's assets. This structure corresponds to a typical bank deposit that pays a stable interest rate and is redeemable at anytime. This structure also captures the improvement of new technologies for withdrawing funds at a moment's notice.

Depositors have limited information in the sense that they cannot observe the value of the bank's assets all the time. More precisely, we assume that they observe y_t with exogenous Poisson rate $\lambda > 0$ for $\Delta \sim exp(\delta)$ periods. In other words, depositors become attentive at a rate λ and inattentive at an independent rate δ. These Poisson or exponential processes are independent of the Wiener process driving the bank's assets. We can interpret δ as the entry rate into inattention and λ as the corresponding exit rate.

These assumptions are meant to describe an average depositor who is not constantly monitoring the bank but can do so occasionally when their jobs and other obligations allow her. Hence, the depositor is getting informed through social media outlets or the internet for an average of $1/\delta$ periods.[3] This also implies a strictly positive measure of agents monitoring the bank's assets at any time. In particular, there is a stationary fraction of depositors who are tracking the bank equal to $N(\lambda, \delta) = \lambda(\lambda + \delta)^{-1}$, which results from equating the entry rate of $(1 - N)\lambda$ with the exit rate of $N\delta$.

For simplicity, we assume that depositors do not make withdrawals when they do not receive information about the bank's health (besides the fact that it still hasn't failed). This means

[3]The assumption that the length of the attentive window Δ is random and exponentially distributed makes the model stationary since, at any time, the expected time until inattention is $1/\delta$. Further, it reflects the idea that people do not know how long they will be able to monitor the bank's assets since external random factors or shocks restrict their attention. The same applies for the inattentive window.

that the attention window coincides with the decision window, where at any time the agent decides whether to keep waiting or not expecting to still be attentive for some time. We will discuss later the reasoning and importance of this assumption.

At any time, a fraction $1 - N(\lambda, \delta)$ of the depositors will not have information about the bank's assets and, hence, will not make a withdrawal decision. In addition, we assume that depositors do not see others run against the bank and are not allowed to signal their running behavior or intentions. These two assumptions are motivated by the advent of online banking, which allows people to instantly withdraw their deposits from their cell phones or computers without anyone else knowing.[4] We abstract here from agents using new technologies to communicate their withdrawal decisions.[5]

In case of a run, we assume the bank can look for a credit line from the interbank market, which is not guaranteed. When a measure X of depositors withdraw, the bank may fail to obtain a credit line at a Poisson rate (independent of any other stochastic process mentioned) θX, where θ measures the availability of the credit lines. We also assume that, when the conditions for a run disappear, there is no constraint from attracting depositors to the initial level.

That banks borrow to face runs captures two main ideas. The first one is technological: at any short time interval $[t, t + dt]$ a bank fails with probability $\theta X dt$ when facing a run of X depositors, which implies that the probability that the bank fails at any point in time is equal to zero. In other words, depositors can withdraw their money instantly from their cellphones or computers, knowing that the probability that the bank fails when they decide to transfer their money out of the bank is zero.[6] The second idea is institutional: banks can access short-term credit lines from other private entities or the Central Bank, but this liquidity may dry up.

Note that the failure rate varies over time since the total outflow of deposits, X, changes during a run. The first ones to withdraw are the fraction of attentive agents $N(\lambda, \delta)$, but while assets haven't recovered, a measure $(1 - N)\lambda dt$ decides whether to withdraw their deposits knowing that the bank is in distress. If all aware depositors decide to run during these times, at least a total of

$$N + (1 - N)\left(1 - e^{-\lambda T}\right) \tag{1}$$

deposits will have left the bank T periods within the run. This makes the model non-stationary. We assume then that the failure rate only depends on the first N depositors that instantly secure their deposits when the bank is distressed.[7] As will be clear later, this assumption does not affect the key mechanisms of the model that we focus on. Moreover, our primary concern is equilibrium behavior when λ is large and, hence, the second term is negligible.

[4]Although this feature is more recent for the average depositor (certainly since the early 2000s), large corporations and corporate customers have already had this ability since the late 70s. Further, even for households, the technology available in the 80s (Rose, 2023) did not delay their withdrawals more than just a couple of hours or a day.

As it will be clear later, the effects of changes in the possibility of making withdrawals parallel those of a higher frequency of information arrival.

[5]We discuss the possible implications of a greater easiness to coordinate among depositors provided by new technologies in section 4.1.

[6]This contemplates another key difference with He and Xiong (2012), where the fact that the bank rarely fails (it does so with a Poisson intensity of $\theta\delta$) is due to a small fraction δdt of credits maturing at any time interval of size dt.

[7]This assumption is analogous to assuming the bank has trouble refinancing sudden withdrawals, but once in distress, it can get secured funding for the remaining.

When credit lines fail, the bank falls into bankruptcy and must liquidate its investments to repay its depositors. All depositors, those who were running and those who did not, receive a payment of $\min\{1, \alpha y\}$, where $\alpha \in (0, 1)$ is the recovery rate of the bank's assets.

We do not make any additional parameter restriction besides $\alpha \in (0, 1)$ and $r > \rho$. In what follows, we discuss the possibility of equilibrium multiplicity and explain the main forces that drive running incentives in the model. In the subsequent section, we solve the model and use it to understand how improvements in information technologies can affect equilibrium running behavior.

2.2 Modeling technological improvements in the Digital Era

Since $r > \rho$, the model consistently exhibits a no-run equilibrium where all depositors always keep their deposits in the bank. We show conditions under which they would withdraw if other depositors withdraw, in the spirit of standard bank runs as in Diamond and Dybvig (1983).

We are interested in how new technological developments such as online banking and the introduction of social media affect equilibrium outcomes. We think of new communication technologies (i.e. social media, cellphones, the internet, etc.) as allowing people to receive information about the bank's well-being at a higher frequency $\uparrow \lambda$, potentially increasing the noise of the available information. The latter is motivated by the well-documented appearance and prevalence of what is called "Fake News" articles,[8] together with a conjectured increase in the number of journalism errors due to the rise in the speed with which news is produced and a change in composition towards a less qualified average information producer.[9] This is a way to capture the idea that there is a trade-off between more frequent information and higher noise with the introduction of social media and the internet. In addition, we expect improvements in communication technologies to increase the reach of information in the sense that people who were not getting informed at all in the past now do so. This is because getting informed becomes less costly in terms of time and money.

We expect that improvements in communication technologies will have several effects. The first one is that every time the bank's assets have low values, more people will run as more of them will be monitoring it (higher λ and reach). This, in turn, reinforces running incentives further. The second effect is that as uninformed people can monitor the bank's health more frequently (higher λ) they face lower running risk every time they decide not to run, decreasing running incentives. The intuition is simple: if you can only watch the bank's assets somewhat infrequently (say, once per day), waiting until the next opportunity can be extremely risky if

[8]There is evidence that the most popular "Fake News" are more widely shared than any mainstream news stories and that people report to believe them (Silverman, 2016; Silverman and Singer-Vine, 2016). Further, people who use social media as their main source of news are more likely to hear conspiracy theories and wrong facts about politics (Mitchell et al., 2020). For "Fake News" shared in the recent SVB run check Cookson et al. (2023), for understanding the motivations to produce "Fake News" check Allcott and Gentzkow (2017), for recent trends check Allcott et al. (2019).

[9]We call it "information producer" to include cases of people who inform others but are not necessarily considered journalists. The idea of the composition change comes from the fact that the entry cost into producing and sharing news has become zero, with social media providing a free access sharing platform (Allcott and Gentzkow, 2017). In addition, the increase in speed for news production comes from the desire to be the one breaking the news, facing higher competition, and knowing that anyone can post new information anytime. This leads to less fact-checking and mistakes due to fast decision-making. For more evidence on information mistrust in social media check Pew Research Center (2021c).

the bank's assets are already low. Should the assets decline further, others monitoring the bank might run, potentially causing a failure before you can react. In contrast, if you know you'll likely monitor the bank again soon, holding your deposits is less risky since there's less time for the asset value to drop enough to trigger a run while you're inattentive. For example, if you only observe the news in the morning, you have to make a decision factoring all that can happen throughout the day. However, if you can check the news at lunch or every two hours, you're only concerned with what might happen in that shorter window. Since a bank is more likely to fail over the course of a day than in the next two hours, the *inattention risk* is much lower in the latter scenario. Which of these two latter effects dominates will determine whether depositors will be more willing to run or not in equilibrium.

In addition, if this more frequent information comes with extra noise, other effects should be considered. Increasing the noise of the signal received by depositors has two effects. On the one hand, it makes it harder for agents to infer the actual value of the bank's assets, giving an incentive to wait and gather more information before withdrawing. The intuition is again simple: noisy information in social media and the internet makes you wait and check if you see it repeatedly to reconfirm that it is accurate. On the other hand, it increases the probability that the bank's assets are not as valuable as signaled, implying that the bank may not be able to repay as much in case of failure.

Alternatively, λ can be interpreted as determining the frequency with which agents are able to withdraw in the full information case, rather than just opening the information window. This is strongly affected by the ease of withdrawal, which has increased significantly with the advent of smartphones and online banking services. The effects on equilibrium outcomes remain essentially the same as those discussed earlier.

To understand all these effects, we start by solving the main model focusing on changing the frequency of information arrival λ, to then extend it and study the remaining effects related to reach. Later on, we discuss how introducing changes in the precision of information might affect our main results.

2.3 Solving the model

We restrict our attention to monotone strategies and use a guess-and-verify approach to find a stationary equilibrium. Specifically, we guess that there will be an equilibrium threshold $\tilde{y}$ such that attentive depositors will run if and only if the value of the bank's assets is below or equal to this threshold.

Now, we proceed to find a stationary monotone equilibrium. Let Δ_k be the random monitoring time after the Poisson jump at T_k that makes a depositor attentive. The relevant information available to a depositor at time t is[10]

$$\mathcal{F}_t = \begin{cases} \sigma\left(T_k, \Delta_k, y_s : T_k \le s \le T_k + \Delta_k, k \le L_t\right) & \text{if } T_{L_t} + \Delta_{L_t} \le t \\ \sigma\left(T_k, \Delta_{k-1}, y_s : (T_k \le s \le T_k + \Delta_k \text{ for } k < L_t) \cup (T_{L_t} \le s \le t), k \le L_t\right) & \text{otherwise} \end{cases}$$

with $L_t = \max\{k \in \mathbb{N} : T_k \le t\}$, $\Delta_0 = \emptyset$ and $\mathbb{F} = (\mathcal{F}_t)_{t \ge 0}$. The first component accounts for the information set when the depositor is not monitoring the bank, while the second corresponds

[10] We omit from this information that the agent also has information about the bank not having failed in the past. This information will never be useful when the agent can potentially make a withdrawal decision (either when monitoring or at the instant where it becomes inattentive) as it is always up to date with the bank's attributes y.

to that when it is. Note that each depositor will have a unique set of information as the timing Poisson jumps are idiosyncratic and iid. We omit to make explicit this distinction for simplicity. Let $\mathcal{T}$ be the set of $\mathbb{F}-$stopping times. Given the process for the random time at which the bank fails τ_θ, governed by the others' optimal stopping decisions, the problem of each agent that is attentive at t is given by

$$V^A = \max_{\tau \in \mathcal{T}} \mathbb{E} \left\{ \int_t^{\min\{\tau,\tau_\theta,\tau_\delta\}} e^{-\rho(s-t)} r ds + e^{-\rho(\min\{\tau,\tau_\theta,\tau_\delta\}-t)} \left[\min\left(1, \alpha y_{\tau_\theta}\right) I_{\{\min\{\tau,\tau_\delta\}>\tau_\theta\}} \right.\right.$$
$$\left.\left. + I_{\{\tau<\min\{\tau_\theta,\tau_\delta\}\}} + V^I\left(y_{\tau_\delta}\right) I_{\{\tau_\delta<\min\{\tau_\theta,\tau\}\}} \right] \mid \mathcal{F}_t \right\}$$

where $V^I(\cdot)$ reflects the value function of a depositor when it becomes inattentive. Note that the value function reflects all relevant components for a depositor. First, as long as the bank remains alive an attentive depositor receives rents $r > \rho$ before withdrawing. Second, in case the depositor cashes out its deposits before it becomes inattentive or the bank fails, it recovers them fully.[11] Third, if the bank fails before the depositor becomes inattentive or withdraws its money, it receives $\min\{1, \alpha y\}$, which could be significantly below 1. At last, the depositor faces the risk of becoming inattentive before withdrawing its deposits or the bank failing and receiving

$$V^I\left(y_t\right) = \mathbb{E}\left[\int_t^{\min\{\tau_\lambda,\tau_\theta\}} e^{-\rho(s-t)} r ds + e^{-\rho(\min\{\tau_\lambda,\tau_\theta\}-t)} \left[\min\left(1, \alpha y_{\tau_\theta}\right) I_{\{\tau_\lambda>\tau_\theta\}} + I_{\{\tau_\lambda<\tau_\theta\}} V^A(y_{\tau_\lambda}) \right] \mid \mathcal{F}_t \right]$$

which reflects that while an inattentive agent receives rents $r > \rho$ as long as the bank is alive, it does not have the option to withdraw, being considerably exposed to the risk of failure. In contrast with V^A, the entry rate into attention, λ, directly affects V^I.

Given that others choose this common strategy, any particular agent will choose a threshold $\bar{y}$ to run the bank as long as $y \leq \bar{y}$ and will not run otherwise. Then the value function of an uninformed agent in the continuation region (i.e., $y_t > \bar{y}$) is the unique C^2 solution to the following differential equation

$$\rho V^A(y_t;\tilde{y}) = r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \delta\left[V^I(y_t;\tilde{y}) - V^A(y_t;\tilde{y})\right] + \theta N(\lambda,\delta) I_{\{y_t\leq\tilde{y}\}} \left[\min\{1,\alpha y_t\} - V^A(y_t;\tilde{y})\right]$$

with $\lim_{y\to\bar{y}^+} V^A(y;\tilde{y}) = 1$. This follows from applying Feynman-Kac's Theorem.[12] This differential equation perfectly captures the main benefits and risks of keeping your deposits in the bank. First, you earn a rate $r > \rho$ as long as you stay and the bank survives. Second, at a rate δ you can become inattentive, receiving a negative payoff of $\left[V^I(y_t;\tilde{y}) - V^A(y_t;\tilde{y})\right]$. Third, you face the risk of the bank failing at a rate θN only in case its assets have a low enough value that makes others run, suffering a change in the value of $\left[\min\{1,\alpha y_t\} - V^A(y_t;\tilde{y})\right]$, which can be substantially negative if y is low.

We provide the details of the solution in Appendix A. If it exists, a symmetric equilibrium threshold is given by

$$\tilde{y} = \frac{(\alpha_1 - \iota_3)\left(\frac{r}{\rho} - 1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2-\alpha_1} - \iota_3\left(1 - \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}\right)}{(1-\iota_3)\frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}}$$

[11] The assumption that the outside option has a value of one is without loss of generality in this setting. Introducing an outside option valued $1 + \bar{r} > 1$ per deposit would only change the cutoff values at which depositors choose to put their money in the bank at $t = 0$ or withdraw it at any other time. We assume $\bar{r} = 0$ for simplicity, as it does not affect any of the qualitative results.

[12] To be more precise, in Feynman-Kac's terms, we define the optimal stopping time to be $\tau = \inf\{t > 0 : y_t \notin G\}$ where $G = (\bar{y}, \infty)$. Further, the only assumption needed for this is that $V^I(\cdot)$ be continuous, which will be the case in an equilibrium of this sort.

in case it is below α^{-1} (i.e. the recovery value) and

$$\tilde{y} = \left[\frac{(\alpha_1 - \iota_3)\left(\frac{r}{\rho} - 1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2-\alpha_1} - \iota_3\left(1 - \frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}\right)}{\iota_3\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)} + (1-\iota_3)\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}} \right]^{\frac{1}{\iota_4}} \alpha^{-1}$$

otherwise. The values of the constants α_1, ι_3, ι_4 and β_2 are defined in the appendix.

In figure 2.3 we can see the three possible path's for the bank's assets and the corresponding equilibrium outcomes. This figure uses the fact that all agents decide to run if the bank's assets are below a certain threshold $\tilde{y}$, while the bank replaces its short-term credit obtained in the run for new depositors as soon as its assets recover. The shaded area marks the values of y for which agents decide to run and, hence, the bank faces the risk of failure. Once the bank's assets get valued below $\tilde{y}$, the depositors that are monitoring the bank decide to run. This is because they are concerned about the risk entailed by other depositors running in this region. This includes a measure $N(\lambda, \delta)$ of depositors who instantly withdraw their deposits as soon as y touches the threshold and a fraction λdt per unit of time of the remaining uninformed agents who continuously become aware of this situation and decide to run. The latter is reflected in the second plot that shows how the bank's deposits evolve, decreasing gradually during the run after the initial sudden withdrawal and suddenly recovering as soon as the bank is in good standing again.

These paths show the three possible outcomes for the bank's assets. The green line is one where the bank's assets are almost always above the thresholds, not being in distress for long and, hence, the bank survives. The golden path is where the bank's assets grow initially but then decrease, exposing the bank to a run. Nevertheless, the bank can obtain short-run financing to sustain itself during the run until the assets eventually recover, and then it manages to survive. The final path, shown in blue, represents a bank experiencing prolonged distress. Although there is a moment when it appears the bank might recover, its assets decline in value once more, ultimately leading to failure at τ_θ.

This illustrates the model's possible outcomes and hints at the main incentives that interplay in it. We proceed to prove equilibrium existence and uniqueness and perform comparative statics.

2.3.1 Equilibrium Existence and Uniqueness

We focus on equilibrium existence and uniqueness before we study how changing certain parameters affects equilibrium outcomes. The following two propositions determine the main results.

Proposition. *There is no stationary asymmetric equilibrium under monotone strategies.*

Proposition. *An equilibrium threshold $\tilde{y} > 0$ exists, and it is the unique stationary equilibrium under monotone strategies if and only if*

$$r < \frac{\rho + \theta N(\lambda,\delta) + \frac{\delta+\lambda}{\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))}{\left(1 + \frac{\delta+\lambda}{\rho\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))\right)} \in (0, \rho + \theta N(\lambda,\delta))$$

The intuition is that if r were to be too large, then even for extremely low values of y, knowing that everyone else is running and the bank may fail eventually, the agent still prefers

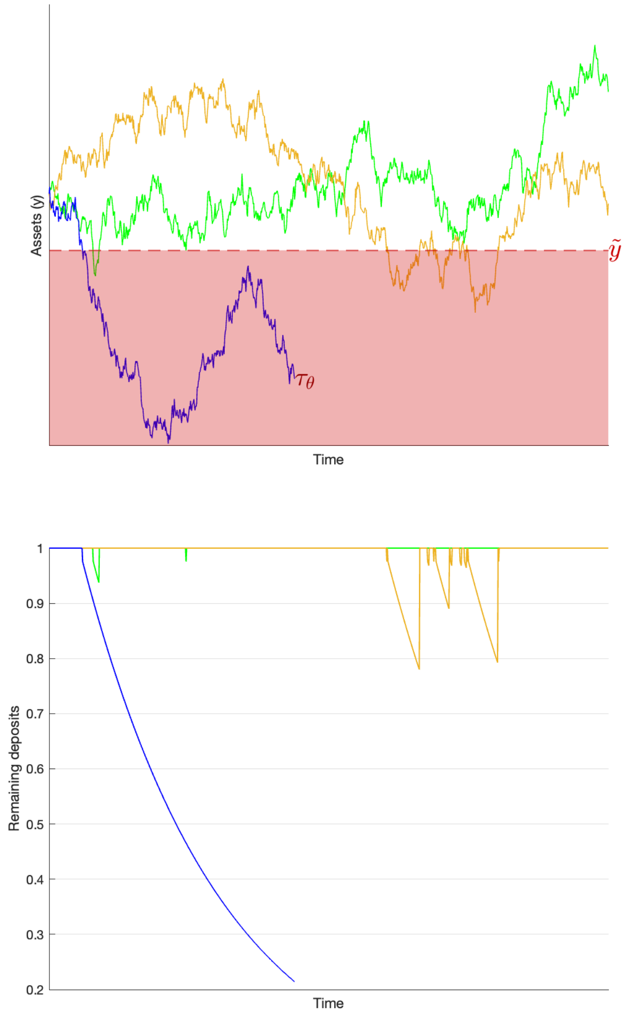

Figure 1: Simulated paths for the bank's assets and the corresponding equilibrium outcomes in an economy where depositors run when y is below $\tilde{y}$. Shaded areas represent the regions where the bank might fail at a random time that we denote τ_θ as it depends on the running intensity and the availability of credit lines θ.

to keep his deposits for a little longer in the bank to benefit from the expected rents. Note that this condition depends on λ since it affects the risk of failure under a run event and the frequency with which the agent gets to make a withdrawal decision. When the agent evaluates the decision to wait a little longer, it has to consider the trade-off between gaining rents $r - \rho$ and the risk of bank failure $\theta N(\lambda, \delta)$ in case others run. Note that $\theta N(\lambda, \delta)$ is the expected loss if the bank fails, and y is close to zero. Although $r < \rho + \theta N(\lambda, \delta)$ is a necessary condition, so that waiting for a little longer is never optimal when y is close to zero, guaranteeing the existence of an equilibrium, it is not sufficient. This is because an agent in this model also has to consider that in the close future, he might become inattentive and, hence, unable to make a withdrawal decision for a while. This results in a tighter bound for the interest rate r, which will also depend on δ and the parameters that determine the evolution of y.

2.4 Non-monotonic Bank Run Frequency

We are interested in understanding how the equilibrium running threshold changes with λ, the frequency with which information arrives to the general public. We focus on steady state outcomes. The following proposition illustrates the key result.

Proposition. *The equilibrium running threshold is non-monotonic in λ. For low enough values of λ, $\tilde{y}$ increases with λ. For high enough values of λ, $\tilde{y}$ decreases with λ.*

This proposition shows that under any calibration which provides a strictly positive equilibrium threshold $\tilde{y}$, the effect of changes in λ on it is non-monotonic. Decreasing λ eventually leads to the threshold getting closer and closer to 0, while increasing it will ultimately result in the same endpoint as λ becomes large. This is illustrated in figure 2, where we plot the equilibrium threshold for different values of λ for what we call the "standard calibration".[13] The intuition for this result uncovers the two main mechanisms we want to illustrate in this paper.

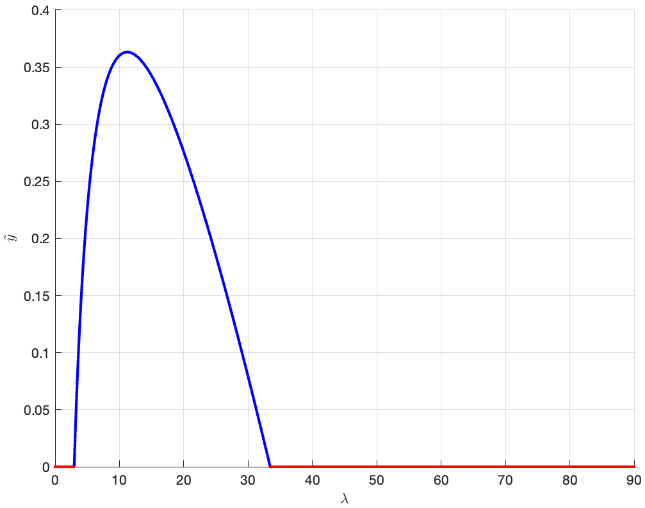

Figure 2: Equilibrium threshold $\tilde{y}$ as a function of λ. Other parameters are according to the standard calibration. The red line corresponds to the threshold being at zero, which means that the only stationary equilibrium possible under said parametrization is that of no runs.

First, when λ is small enough, raising it results in a considerable increase in the number of agents monitoring the bank at any time (i.e., $N(\lambda, \delta)$ rises). This means that we have an increase in *running intensity*, which we define as the amount of money the bank needs to obtain in the short run to cover the withdrawals. This results in a higher probability of a bank failure in the event of a run, making agents more cautious and, hence, less willing to keep their deposits at the bank. For this reason, the equilibrium threshold rises. This is perhaps the most

[13] We take one unit of time to be a day and choose most of the parameters to roughly match those in He and Xiong (2012). Unless otherwise specified we use for the remaining figures $r = (1 + 7\%)^{\frac{1}{365}} - 1$, $\rho = \mu = (1 + 15\%)^{\frac{1}{365}} - 1$, $\alpha = 0.55$, $\sigma = 0.0105$, $\lambda = 8$, $\delta = 48$, $\nu = 95\%$, and $\theta = 7^{-1}(5\% + 95\% \cdot N)^{-1}$. The parameters freely chosen here are the entry and exit rates into attention, while we calibrate θ using the specification of the extended model in 3 that has a 5% share of expert depositors. The values of λ and δ are determine an expected length of the inattentive and attentive windows of 3 and 0.5 hours respectively.

straightforward effect usually present in the general discussion about the impact of new communication technologies such as social media on running events. As information disseminates faster to the general public, more and more people withdraw their deposits when bad news emerge, making keeping one's deposits in the bank more risky. Likewise, decreasing λ enough eliminates the possibility of a run as the *running intensity* becomes so negligible that the bank is not vulnerable during a run, removing individual incentives to run even if all other agents decide to do so in equilibrium. Hence, at first, increases in the frequency of information arrival generate the possibility of a run equilibrium in this model by raising rollover concerns.

However, an increase in λ also affects the frequency with which any particular agent receives information about the bank and, hence, gets to make a withdrawal decision. As a result, an increase in λ decreases running incentives for uninformed agents as they expect to be able to make a withdrawal decision early in the future in case they become inattentive, exposing them to lower risk if they decide to keep their deposits in the bank. To see this, note that if a depositor decides not to withdraw his money, he is exposed to the risk of becoming inattentive and unable to make a withdrawal decision until the Poisson process hits back at a frequency of λ. During that time, the bank's assets might become so much less valuable that other depositors may run against it without the agent noticing this on time. Then, an increase in λ decreases the exposure to such risk as the agent expects to be inattentive for a shorter period and, hence, it is less likely both that the bank's assets will become bad enough in between and that the bank would fail in the event of a run before he can withdraw his money. This effect, which we coin as a decrease in exposure to *inattention risk*, is analogous to a decrease in exposure to *rollover risk* as defined by He and Xiong (2012).

The interaction of these two effects determines the non-monotonic relationship between the equilibrium threshold and λ. Our results show that the first effect is stronger for low values of λ, and the second one is stronger for high values of λ. The mechanism by which the introduction of new communication technologies decreases the exposure to *inattention risk* and, thus lower running incentives, is missing in the general discussion and turns out to be crucial once enough progress has been made according to this model. As illustrated in figure 2, if λ increases enough, the exposure to *inattention risk* decreases until the running equilibrium is eliminated. Even though a significant fraction of agents are monitoring the bank at any time, making it considerably vulnerable in the event of a run, depositors are not so concerned about this as they expect to be attentive once the bank is in distress and, hence, able to make a withdrawal before suffering significant losses. This incentivizes keeping deposits at the bank for longer to earn rents, which further decreases *innatention risk* and the importance of the higher *running risk* for others up to the point that no agent runs in equilibrium.

One may think this result is due to the particular way we modeled the failure rate under a running event for banks in distress, which is proportional to $N(\lambda, \delta)$ a strictly increasing and bounded function of λ. Yet, this result is robust to introducing the running intensity of inattentive agents separately from the share of attentive agents into the failure rate in a variety of ways such as defining $N(\lambda, \delta) \equiv \lambda(\lambda + \delta)^{-1} + \lambda$ or using any increasing and concave function $f(\lambda)$ that satisfies $f(0) = 0$ to map running intensity to the failure probability.[14] We can use the following two propositions to explain this.

[14] Concavity is not a required condition as it holds for some strictly convex functions such as $f(\lambda) = \lambda^{3/2}$. However, we have not proved it works under more general conditions yet.

Proposition. *Keeping N fixed, the equilibrium threshold decreases with λ. Furthermore, $\lim_{\lambda \to \infty} \tilde{y} = -\infty$.*

Proposition. *The equilibrium threshold increases with N for low values of N. In addition, $\lim_{N \to \infty} \tilde{y} \propto \alpha^{-1}$.*

The first shows that a reduced exposure to *inattention risk*, while keeping *running intensity* constant, decreases the frequency of runs. Similarly, the second proposition demonstrates that increasing *running intensity*, while keeping the exposure to *inattention risk* constant, leads to a higher frequency of runs when N is sufficiently small.[15] However, the effect of increasing the latter on the equilibrium threshold is somewhat bounded since a high enough failure rate takes the equilibrium threshold close to the recovery value as it would be in any static classic bank run model. Intuitively, agents will not run for asset values above the recovery value as it is expected that the bank will fail almost instantly during a run so that no capital losses are incurred in waiting. This further explains why the equilibrium threshold will converge to a value below α^{-1}. Even when others run if $y \leq \alpha^{-1}$ a depositor has increasing incentives to wait bit longer as N grows large since the expected losses from waiting become negligible relative to the expected rents. This implies that there is a limit to how much a higher *running intensity* can affect equilibrium running behavior. In addition, decreasing the exposure to *inattention risk* features no such limit, and the greater the *running intensity*, the more critical it becomes to minimize this risk. This illustrates why the second effect dominates for large values of λ for several specifications of the failure rate in this model. In particular, it suggests that introducing time-varying specifications (stationary or not) on the failure rate will not affect our main results.[16]

Further, an additional concern may be that δ, which measures the expected amount of time that a depositor devotes to getting informed when he can is fixed in this comparative statics. In general, one would think that if an agent cannot obtain information about the bank's assets often (low λ), it will then spend more time on average (low δ) to get informed when he can. This means that there is a positive relation between δ and λ, which will then make the increase in N as λ increases less strong. This would reduce the effect of raising λ on *running intensity*, reaching the decreasing part of $\tilde{y}(\lambda)$ earlier. We can show that as long as the relationship between δ and λ is strictly concave or δ is bounded away from zero and from above, our main result remains valid.[17] The boundedness from above is easy to justify. Then, our results are also robust to endogenous responses of δ to changes in λ.

So far, we have been comparing steady-state outcomes. Yet, an increase in λ will gradually increase the fraction of attentive uninformed agents N until it reaches its final higher value, implying that the impact on *running intensity* will be small or even negligible at the beginning of the transition. This means that the decreased exposure to *inattention risk* will dominate, and

[15] As shown in the model of He and Xiong (2012) it will not be true here that the equilibrium threshold always increases with N or θ if σ is high enough.

[16] Note that introducing a failure rate such as that in (1) will increase the risk of failure in general but more so for low values of λ. Given that $\lim_{\theta \to \infty} \tilde{y} \propto \alpha^{-1}$ we will have that the increased exposure to *inattention risk* will have a smaller effect on $\tilde{y}$ when λ is small. When λ is large, it will not change significantly, strengthening our results.

[17] We have not tried to prove that the results hold under convexity of this relationship. One reason is that having strict convexity N may present strange features.

$\tilde{y}$ will decrease with λ at the beginning of the transition, even if the starting point is small.[18] However, as N increases, the first effect might become more important if the endpoint for λ is not large so that $\tilde{y}$ will start to increase.

2.5 Comparative Statics

We now study how the effects discussed change when the economy is highly volatile compared to the standard calibration in figure 3. An increase in volatility affects the comparative statics in three ways. At first, it makes the initial effect of a higher *running intensity* significantly stronger as, once it becomes not negligible and running concerns emerge, the "rat race" among depositors makes the threshold skyrocket given the higher risk entailed in keeping their deposits. This is because their best response function to $\tilde{y}$ is shifted upwards, as agents want to hedge against the behavior of others more than before for any given failure rate. However, the tipping point at which the decreased exposure to *inattention risk* dominates is reached earlier. The intuition for this result is that increases in σ increase the relative importance of *inattention risk* with respect to a change in *running intensity* once rollover concerns become relevant in equilibrium. This is because an increased volatility makes the agent much more concerned of what can happen in the inattentive window than when it is monitoring the bank. The former is only relevant as long as the chance of failure is non-negligible. At last, the point at which the threshold reaches zero is delayed as rollover concerns remain strong given the higher volatility of assets.

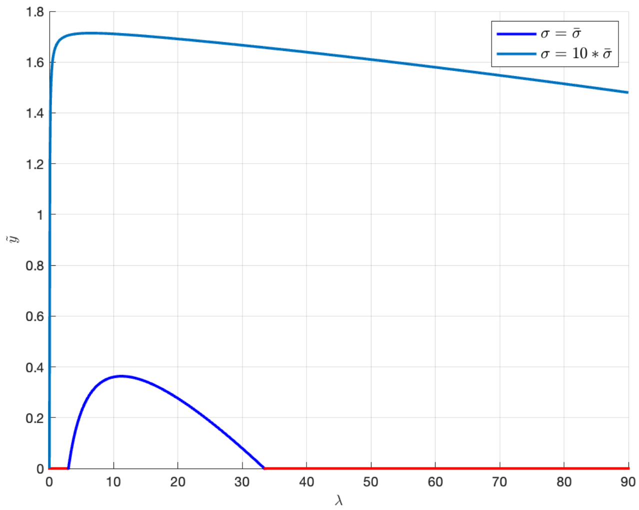

Figure 3: Equilibrium threshold $\tilde{y}$ as a function of λ for different volatility levels. All other parameters remain as in the standard calibration. The red line corresponds to the threshold being at zero, which means that the only equilibrium possible under said parametrization is that of no runs. Both curves reach zero for λ close enough to zero.

This shows how the effect of increases in the frequency of information arrival on banks can be asymmetric. While its impact on raising running incentives at the beginning and decreasing

[18] In case we use the non-stationary discussed specification, it would still be true that the first effect will be less strong, but as there will be an instantaneous impact on the withdrawals λdt per unit of time affecting the failure rate considerably for low values of λ, it might be that it dominates anyway at the beginning of the transition.

them later on is less beneficial for volatile banks, it can happen that raising λ makes "stable" banks more fragile while reducing the frequency of runs on those with volatile portfolios.

We perform the same exercise for banks with different liquidity levels in plot 4. The results are similar to those discussed in the previous paragraph, but the intuition differs. The less liquid a bank is (i.e. higher θ), the more significant the impact of increases in *running intensity* on the failure rate. This initially makes the first effect quite strong, reaching the levels for which the failure rate has little impact on running behavior earlier. At the same time, a high level of $\theta * N$ arrived earlier, elevates *inattention risk* for low values of λ. Consequently, the decreasing part of the curve is reached earlier, although the difference is almost negligible under our calibration. In addition, as rollover concerns remain strong, the decrease is slower once the corresponding region is reached.

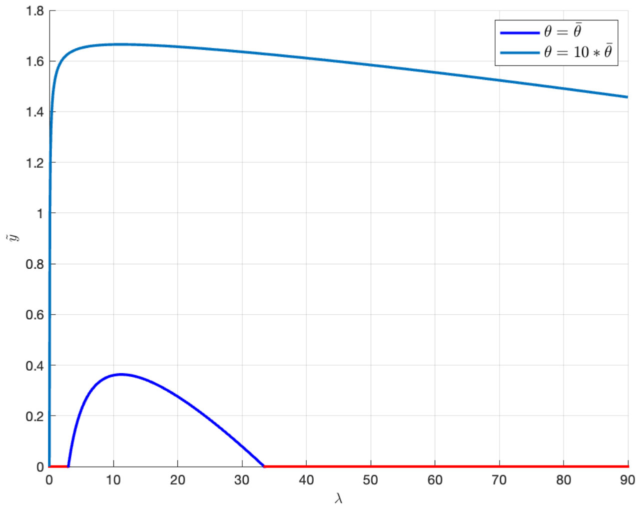

Figure 4: Equilibrium threshold $\tilde{y}$ as a function of λ for different liquidity levels. All other parameters remain as in the standard calibration. The red line corresponds to the threshold being at zero, which means that the only equilibrium possible under said parametrization is that of no runs. Both curves reach zero for λ close enough to zero.

Another possible concern is that banks may adapt their access to liquidity under distress, increasing it to hedge from an increase in *running intensity* and perhaps a higher likelihood of a run. This maps into a decrease in θ, decreasing running incentives (i.e., *ceteris paribus* the thresholds of both agents decrease) and making the first effect markedly less strong.[19] The second effect is affected indirectly and to a lesser degree, meaning our results strengthen. This is illustrated in figure 5 where we can see that increases in λ that are accompanied by decreases in θ have a more negative impact on the equilibrium threshold. In particular, even for levels of λ where the increase in *running intensity* is the dominant effect, if that is accompanied by a slight decrease in θ, the threshold will respond negatively. If the bank adapts θ so that the failure rate remains unchanged after the increase in λ, the threshold will change dramatically

[19] To be precise, decreasing θ can increase the equilibrium threshold as we already said when θ and σ are high enough. This is because of a tricky effect: a higher failure rate makes the bank less likely to fail with a considerably low asset value when the agent is inattentive if the volatility is high enough. So far, we have ignored this in our discussion since it is irrelevant for our main result and it actually strengthens it. At least for banks that suffer an increase in the running threshold when increasing λ, we can ignore this issue, as it is not present.

downwards. This channel is stronger for low values of θ (i.e., banks that are less likely to fail under a run), implying that, to resist the impact of increases in λ, fragile banks need to make a greater effort to ensure their depositors that they have access to the liquidity needed to face a run. This is because changing the failure rate has a lower impact on running decisions for high values of it.

This reinforces the decrease in exposure to *innatention risk* as the primary force driving running decisions if θ is endogenous. At least for low values of λ, where we would expect the bank to attempt increasing its liquidity to face runs that are faster and more likely. However, once increases in λ decrease the running threshold, the bank might decide to increase θ as two different forces interplay affecting its fragility. On the one hand, a running event is less likely for higher values of λ. On the other hand, if it does happen, *running intensity* is higher, and the bank is more likely to fail. The bank has to face this trade-off when deciding on adjusting θ, impacting the new equilibrium outcomes.[20]

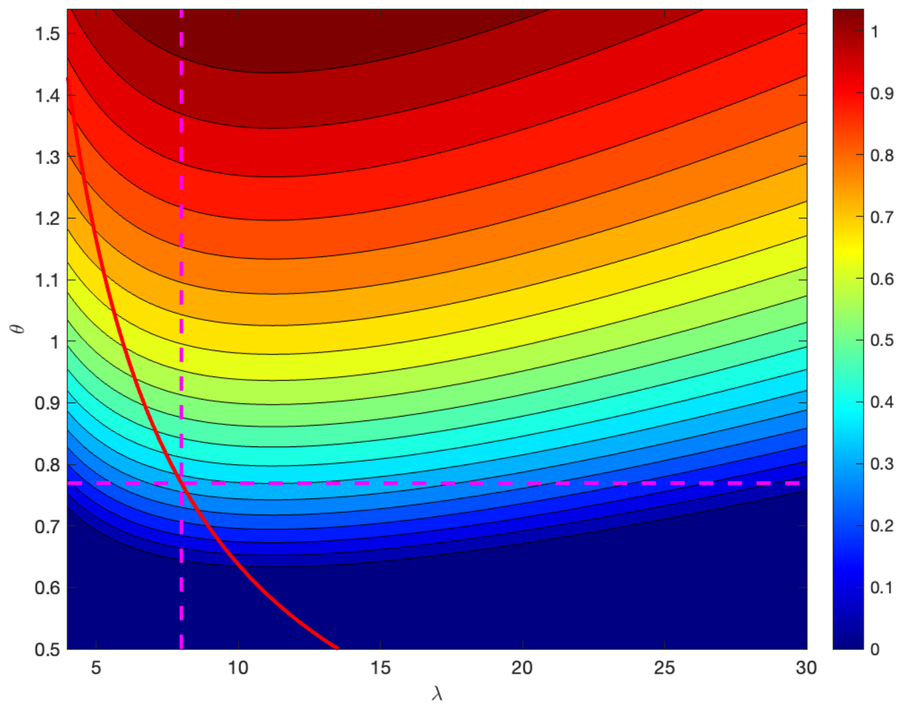

Figure 5: Equilibrium threshold $\tilde{y}$ as a function of λ and θ. All other parameters remain as in the standard calibration. The thick red line represents the pairs of λ and θ that keep the running intensity constant. The dashed magenta lines mark the values under the original calibration.

2.6 On depositors' behavior while inattentive

We have solved this model as if it were optimal for depositors to not act while inattentive even if modern withdrawal technologies allow them to do so. This assumption is motivated by the fact that, while inattentive, the only available information is the last observation of y and the

[20]Following previous comments, if volatility is high enough then for large values of λ it might even be more helpful to increase θ, lowering the access to liquidity under distress.

fact that the bank still hasn't failed. This seems to shift posterior beliefs upward over time, as the bank not failing is good news for inattentive depositors. Then, if inattentive depositors decide to run they will do so at the instant when they become inattentive.[21]

However, this behavior for the beliefs over time only holds in case the last observation of the bank's assets is low enough. On the contrary, if the last observation of y is large, the increased uncertainty over time might lead the depositor to eventually withdraw within the inattentive window. This is because knowing that the bank hasn't failed does not condition posterior beliefs much at the beginning of the inattentive window.

Making this assumption simplifies the setting considerably, as we do not have to keep track of past signals to account for withdrawal decisions made by inattentive agents, which will generally depend on their last observation. Further, although this makes attentive depositors more willing to run at any time since they cannot act at all when they become inattentive, it does not affect the main mechanisms at interplay here and, hence, the results. This is because the exposure to *inattention risk* would still be a key driver of running decisions as even if depositors are allowed to act while inattentive, their behavior would be deterministic until they receive new information and, thus, they would be exposed to others running while they are not paying attention.

2.7 Summary of the main results

In this model, raising λ increases the speed of a run (expected time until failure conditional on a running event) as more people are attentive when bad news emerge. In addition, a larger fraction of agents become aware of this during the run at a rate λdt per unit of time. At first, this results in a higher running threshold due to higher rollover concerns and, hence, a higher probability of a run and subsequent banking failure, making banks more fragile. Yet, as communication technologies improve, they may reduce the probability of a bank run as running incentives decrease when people can monitor the bank's assets more frequently. This comes from the fact that observing information about the bank more often reduces the risk of failure under inattention.

Furthermore, banks with highly volatile portfolios suffer more from improvements in communication technologies as they are more likely to face a run with a higher *running intensity* both because of their intrinsic volatility and because higher rollover concerns make the running threshold increase rapidly at first and then keep it elevated for longer. In addition, if banks increase their access to liquidity to face a higher *running intensity* due to improvements in communication technologies, the result would be a substantially reduced the frequency of running outcomes. However, less liquid banks have to make stronger efforts.

Having presented our main results related to changes in the frequency with which agents can monitor the bank and subsequently make withdrawal decisions, we extend the model in the next section to account for an increased reach of information production.

[21] We could allow depositors to make a withdrawal decision immediately when they become inattentive, proceeding with the same approach. Allowing this does not modify any of the effects at interplay in the comparative static. However, it makes both computations less cumbersome and eliminates an additional threshold to account for the latter.

3 The extensive margin

What happens if more people have access to information about a particular bank? Consider agents that do not get informed because it is too costly originally, but improvements in communication technologies make it cheaper for them to do so. Say that the share of depositors that get informed, as described in the previous model, is $1-\nu$ with $\nu \in (0,1)$. This means we have a measure ν of depositors that do not monitor the bank. The new equilibrium will be analogous to that in the benchmark model just replacing N with $(1-\nu)\cdot N$ everywhere. In the HJB equation of an attentive depositor, it looks as follows

$$\rho V^A(y_t;\tilde{y}) = r + \mu y_t V_1^U(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \delta\left[V^I(y_t;\tilde{y}) - V^U(y_t;\tilde{y})\right]$$
$$+ \theta(1-\nu)N(\lambda,\delta)I_{\{y_t\leq\tilde{y}^A\}}\left[\min\{1,\alpha y_t\} - V^A(y_t;\tilde{y})\right]$$

We want to know what happens if some of these depositors suddenly have access to new information technologies that allow them to inform themselves. Say they do with the same technology as the other $1-\nu$ creditors of the bank. This maps into decreasing ν in the previous equation, increasing the *running intensity* and, hence, the failure rate while not affecting the asset side of the bank at all.[22] Then, we know that the equilibrium threshold will surely increase, making the bank more fragile as runs are more likely and intense.

Alternatively, we can consider the full information case and think of ν determining the measure of captive depositors that just do not make any withdrawal decisions not necessarily because they do not receive information about the bank, but because their withdrawal technology does not allow them to do so. For instance, you can think of a farmer working the field during the harvest season or a person living abroad having deposits in a U.S. bank before the Digital Era. Allowing these agents to make withdrawal decisions as any other, say with their mobile phones, puts more pressure on the bank since more depositors will run when it is in distress. This results in a higher frequency of runs in equilibrium as agents will be concerned about a higher *running intensity* that makes the bank more likely to fail under a run.

Now, we consider the possibility of these new agents receiving information but with a worse technology than those getting informed initially. For simplicity, we focus on an extreme case. Say $1-\nu$ depositors can monitor the bank continuously with a supreme technology of $\lambda = \infty$ (or $\delta = 0$), while the remaining ν depositors cannot get informed so that they have $\lambda = 0$. We call them "expert" and "captive" depositors, respectively. We get the following result, which proof is in Appendix B.

Proposition. *If the only depositors able to monitor the bank can monitor its assets at all times, the only monotone stationary equilibrium exhibits no runs.*

The intuition is quite simple. If all other agents run at a particular time, you, as a depositor, benefit from waiting just a little longer to ensure that the bank is unlikely to fail in between and benefit from the rents. In other words, without *inattention risk*, depositors are always incentivized to wait a little longer and receive more rents. This result parallels the main model's, as it is the equilibrium outcome when λ is substantially large. This result not resent in most, if

[22] Typically, when introducing new technologies that allow the new $\nu - \nu'$ depositors to monitor the bank, it would take time to achieve the stationary fraction N of them monitoring at any time. For simplicity, we focus here on steady state comparisons.

not all, models of bank runs in the literature follows from the fact that the bank need not die with probability one the moment all its depositors (or those aware) run against it.[23]

We now explore what happens if the remaining ν depositors that could not get informed suddenly can do so but with the worst technology being more exposed to *inattention risk*. For intermediate values of λ, we would expect that, if ν is sufficiently large, a running equilibrium would emerge as these agents may choose to run under the threat of others running if enough of them do.

Following the same steps as with the previous model, we guess and verify that each type of agent will run if and only if the most recent observed signal is below some threshold $\tilde{y}^i$, where $i \in \{\lambda, \infty\}$ indexes the type. Trivially, $\tilde{y}^\lambda > \tilde{y}^\infty$ since the agents with $\lambda = \infty$ do not face any *inattention risk*. This additional source of risk makes uninformed agents less likely to keep their deposits in the bank. Let $\gamma = \{\tilde{y}^\lambda, \tilde{y}^\infty\}$. Then, at any point in time, the value function of a least informed or captive agent who is monitoring and chooses a threshold $\bar{y} > 0$ in the continuation region (i.e., when the most recent signal is above the threshold) is given by

$$\rho V^A(y_t;\gamma) = r + \mu y_t V_1^A(y_t;\gamma) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\gamma) + \delta\left[V^I(y_t;\gamma) - V^A(y_t;\gamma)\right] \\ + \theta\left((1-\nu)I_{\{y_t \le \tilde{y}^\infty\}} + \nu N(\lambda,\delta) I_{\{y_t \le \tilde{y}^\lambda\}}\right)\left[\min\{1, \alpha y_t\} - V^A(y_t;\gamma)\right]$$

which is C^2 and satisfies $\lim_{y \to \bar{y}+} V^A(y;\gamma) = 1$. Note that there is only one key difference in this differential equation concerning the one for the model with homogeneous depositors. The term $\left[\min\{1, \alpha y_t\} - V^A(y_t;\gamma)\right]$ that reflects the potential loss under the bank's failure is now multiplied by a different failure rate, which increases for lower values of the bank's assets. This is because of two reasons. First, not all depositors run simultaneously because they have different running incentives. Second, and in contrast with captive depositors, all those depositors who continuously monitor the bank will run simultaneously and only when the bank is facing severe difficulties. In this lower region, the failure rate is proportional to the weighted average of the fraction of expert and captive agents that monitor the bank at any time, as those are the ones that will put the bank in the greatest distress. This reflects the fact that, once $\nu > 0$, decreasing it leads to a lower *running intensity* when the bank starts facing distress and but increases it more so when the bank is in the most danger of failure.

Analogously, the value function of an agent with $\lambda = \infty$ who chooses a threshold $\bar{y} > 0$ is, in the continuation region, given by

$$\rho V^\infty(y_t;\gamma) = r + \mu y_t V_1^\infty(y_t;\gamma) + \frac{\sigma^2}{2} y_t^2 V_{11}^\infty(y_t;\gamma) \\ + \theta\left((1-\nu)I_{\{y_t \le \tilde{y}^\infty\}} + \nu N(\lambda,\delta) I_{\{y_t \le \tilde{y}^\lambda\}}\right)\left[\min\{1, \alpha y_t\} - V^\infty(y_t;\gamma)\right]$$

which is C^2 and satisfies $\lim_{y \to \bar{y}^+} V(y;\gamma) = 1$. The main difference with the previous expression is that the term reflecting the risk of becoming inattentive is not present in this differential equation.

The value function within the inattentive window for the least informed depositor is analogous and depends on λ as usual. We solve for the equilibrium and prove all the results for this

[23] Although here it relies on the instantaneous failure probability to be zero, in a scenario where the bank might instantaneously fail or not if all others run, we expect this result to hold as long as the failure probability in the first periods of the run is low enough. If the former is large, then the running equilibrium emerges as in Diamond and Dybvig (1983).

extension in Appendix B. Before presenting the main findings of this extension, we show some equilibrium features that will be useful for the analysis.

3.1 Equilibrium characterization

In this section, we will discuss the main features of the equilibrium that might emerge in this economy with two types of agents. As in the main model, we can show the following:

Proposition. *There is no asymmetric monotone equilibrium.*

Interestingly, we can show that as long as the depositors with the worst technology to get informed decide to run, the others will do so too for values of y close enough to zero since the expected loss under failure becomes then significant. This is summarized in the following proposition.

Proposition. *An equilibrium characterized by thresholds $\tilde{y}^{\lambda} > 0$ and $\tilde{y}^{\infty} = 0$, with $\tilde{y}^{\lambda}$ determined as in the primary model with failure rate $\theta \nu N(\lambda, \delta)$, does not exist.*

Then, we can focus on an equilibrium where both types of agents run. An illustration of such an equilibrium can be found in figure 3.1. The main difference with that in 2.3 is that here, two different shaded areas mark the different *running intensities* that a bank faces during a run. The light red region represents that only those with the worse technology run. A measure $\nu \cdot N$ of them suddenly withdraws their money from the bank as soon as the bank's assets touch $\tilde{y}^{\lambda}$, and a fraction λdt per unit of time of the remaining depositors become aware of the bank's situation and withdraws their deposits during the run. Moreover, as soon as the bank's assets touch $\tilde{y}^{\infty}$, all those who continuously monitor the bank cash out their deposits, putting the bank in quite a fragile situation reflected in a higher failure rate in the dark red region.

Concerning the simulations, one key difference emerges. The bank that fails does recover deposits at some point during the run. This is because its portfolio improves enough so that depositors continuously monitoring the bank have incentives to deposit back their money. However, the bank's assets deteriorate again and eventually it fails.

3.2 Numerical illustration

We are now ready to answer the question that motivated studying the extended model. We want to know how equilibrium outcomes change when a fraction ν of captive depositors start monitoring the bank with an inferior technology (i.e., $\lambda > 0$). We show the resulting equilibrium thresholds in figure 7 for various values of ν using our standard calibration. We can see that making the captive depositors monitor the bank's assets pushes equilibrium thresholds upwards. In this case, where the other $1 - \nu$ depositors monitor the bank continuously, it may generate an additional equilibrium that exhibits runs. However, for this to happen, the measure of captive depositors that start monitoring the bank must be large enough so that the threat of them running against it is sufficiently meaningful for others to run.

In addition, we observe that the impact on $\tilde{y}^{\lambda}$ is non-monotonic in the sense that it is not necessarily true that banks with a higher share of captive depositors will suffer a higher increase

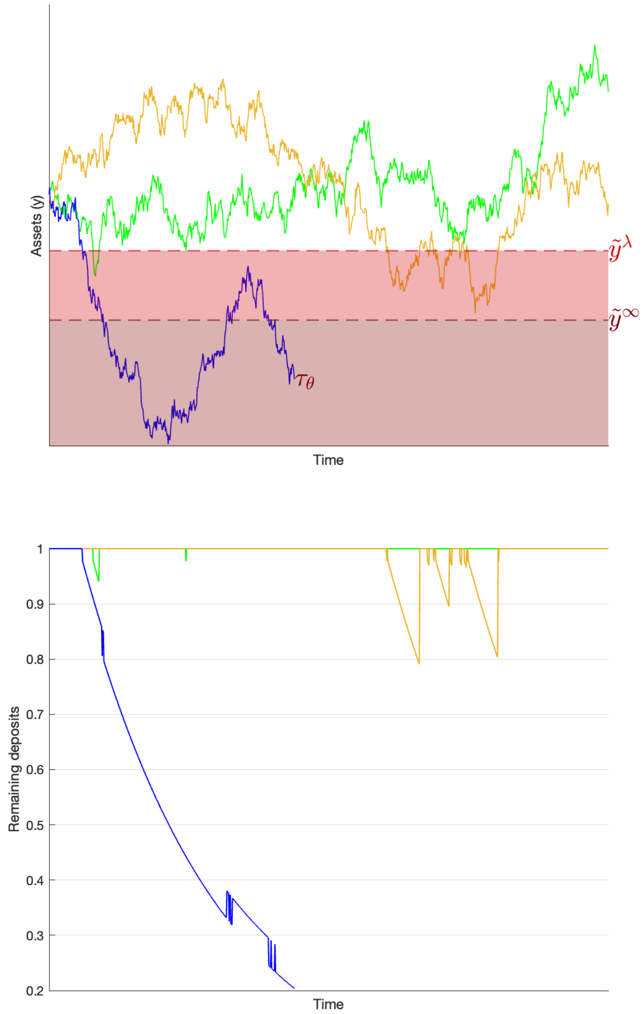

Figure 6: Simulated paths for the bank's assets and the corresponding equilibrium outcomes in an economy with two types of depositors, varying in their degree of exposure to *inattention risk*. Agent's thresholds are denoted according to the frequency with which they exit inattention $i \in \{\infty, \lambda\}$. Shaded areas represent the regions where the bank might fail at a random time that we denote τ_θ as it depends on the running intensity and the availability of credit lines θ.

in the likelihood of a run when all of them start monitoring it.[24] To understand why this is the case note that the experts will also run in equilibrium. Then, while banks with a large share of captive depositors will suffer from a higher *running intensity*, $\nu \cdot N$, at the beginning of the run, they benefit from a lower *running intensity*, $1 - \nu + \nu \cdot N$, in the region where all attentive agents run as there will be less of them. Further, when banks have a large fraction of captive depositors monitoring, the experts will not want to risk waiting too much in the event of a run, running earlier to avoid the risk of failure amid a higher *running intensity* from the latter.[25]

[24]Note that looking at this threshold tells us for which values of its portfolio we will have a run on the bank as these agents (at least those of them who are attentive) are the first to run. We omit presenting a plot for the other one as their qualitative behavior concerning ν, except for that shown in the second plot of figure 7, is identical.

[25]Another way to think about this is that captive depositors with a weaker technology will have less of a need

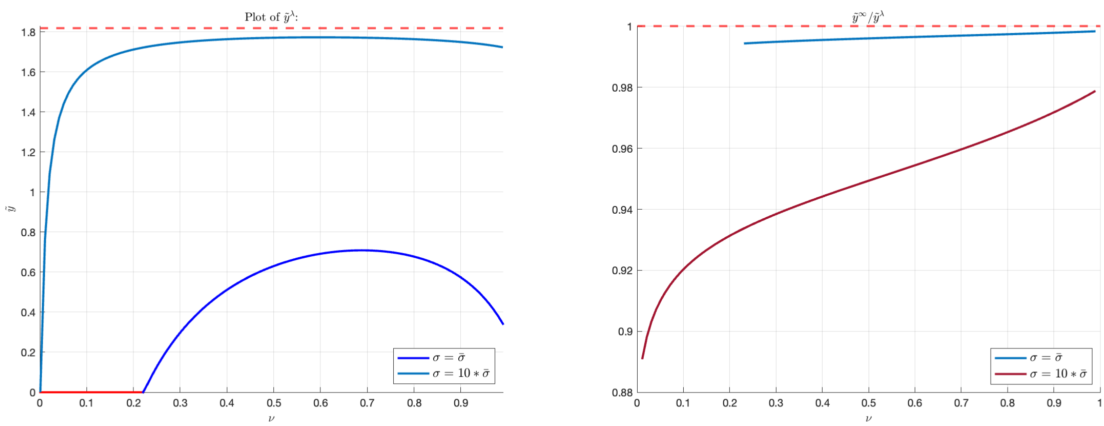

Figure 7: Equilibrium outcomes for $\nu \in (0, 1)$ at two volatility levels when depositors have different technologies to get informed. Sll other parameters are as in the standard callibration. The red line marks that the only equilibrium is that of no runs for the corresponding case.

This translates to the region where only captive depositors run being small. As a consequence, for large enough values of ν, banks with a larger share of captive depositors face a smaller increase in the risk of failure under a run, resulting in a smaller $\tilde{y}^{\lambda}$ and, hence, lower likelihood of a run. As long as ν is not too large, we obtain the opposite; a higher *running intensity* from those who run first leads to a higher likelihood of a run.

Another, perhaps more intuitive, way to interpret the results in plot 7 is to think about changing the composition of a bank's depositors, decreasing the share of those that monitor it with an inferior technology when ν decreases. At the beginning, starting from the right, increasing the fraction of experts increases the frequency of runs as it raises the fraction of agents that will be aware of a run and withdraw only some time after it starts. However, once the measure of experts is large enough, we see that increasing it results in a decreased frequency of runs up to the point that they are eliminated. In this case, the previous effect becomes small since the latter will wait considerably longer to run when the *running intensity* of captive agents is small. Precisely for that reason, now the fact that increasing the share of experts decreases the *running intensity* of those that run first becomes the dominant effect, resulting in a decreased running threshold. The idea is that increasing the share of experts makes depositors in general less concerned about the running behavior of those more prone to run.[26]

In general, however, we do not expect that increasing the share of experts in a bank's portfolio will eliminate runs, as these may not have such a superior technology. Yet, the same intuition follows. In a way, that of the main model applies here: at first, increasing the access to information for the average depositor raises rollover concerns and, hence, the likelihood of a run, but eventually, the decreased exposure to *inattention risk* for the average depositor dominates, decreasing the equilibrium threshold.

Increasing the fraction of informed agents almost mechanically increases the *running intensity* in the region where the bank is most vulnerable. At the same time, it decreases it in the one where only the joint depositors are aware of the bank's health run. This is because fewer

to hedge from the experts when there are few of them.

[26]Recall that these agents are the ones that originate the run in the first place in the particular case considered here.

of them will run in the first region while the share of experts rises. The distance between both thresholds also increases as the experts face less risk from waiting during a run since captive depositors put less pressure on the bank. Further, the running thresholds do not respond monotonically to increases in the share of experts in the bank's portfolio. It is unclear then how a bank's health will be impacted by this.

Both thresholds increase for banks with a small share of experts, suggesting that the bank may be worse off as the region facing the largest withdrawals becomes more extensive and intense and the likelihood of a run increases. Yet, since *running intensity* decreases at the beginning of the run, the result is not obvious and requires defining a proper metric to evaluate whether the bank's health deteriorates. For instance, we could compute the probability of bank failure in the following year, which is directly linked to the likelihood of a run as well as the amount of withdrawals in it.

The difference for banks with a large share of experts in their portfolio is that the running thresholds decrease when more depositors can monitor it frequently. Eventually, this leads to stopping runs, which is better for the bank. However, at first, runs become less likely with decreased withdrawals from captive depositors and a greater expected time until experts secure their deposits in a run. Still, the latter puts much more pressure on the bank. As it takes a much severe situation for the experts to retire their deposits, this seems to suggest that bank's health improves but, again, we need a proper metric to evaluate this statement.

Concerning the frequency of information arrival of captive depositors, we can see from figure 8 that the numerical results go in the same direction as with the main model.[27] While at first the threat of others with inferior technologies running does not concern any particular creditor enough to run, when their *running intensity* becomes high enough, an equilibrium with runs emerges. Further increases in the frequency of information arrival raise running incentives through a higher *running intensity*. However, once λ is large enough, a decreased exposure to *inattention risk* becomes the dominant effect driving down the running threshold of both types of agents.

Additionally, this extended model allows us to note that, although both thresholds seem to behave similarly, that of the experts decreases slightly later than that of common depositors. This is because it is affected by a decreased exposure to *inattention risk* only through the latter's behavior but does face a higher *running intensity* from them directly. This means their running incentives will only decrease if those with a worse technology diminish their willingness to run enough to counteract the latter. Moreover, both thresholds seem to converge fast as λ increases. The reason is trivial; since more information arrives to the common depositor, a higher fraction of those are monitoring the bank at any time. This increases *running intensity* from them, making it riskier for the experts to wait too long when the run starts, making them withdraw their deposits earlier.

Since increasing λ raises the share of captive depositors that withdraw their deposits during a run and it makes experts thereby run earlier, its effect on *running intensity* is unequivocally negative for the bank, increasing deposit outflows at any time of a run.

[27]We compare here steady state outcomes as well. The same caveats apply.

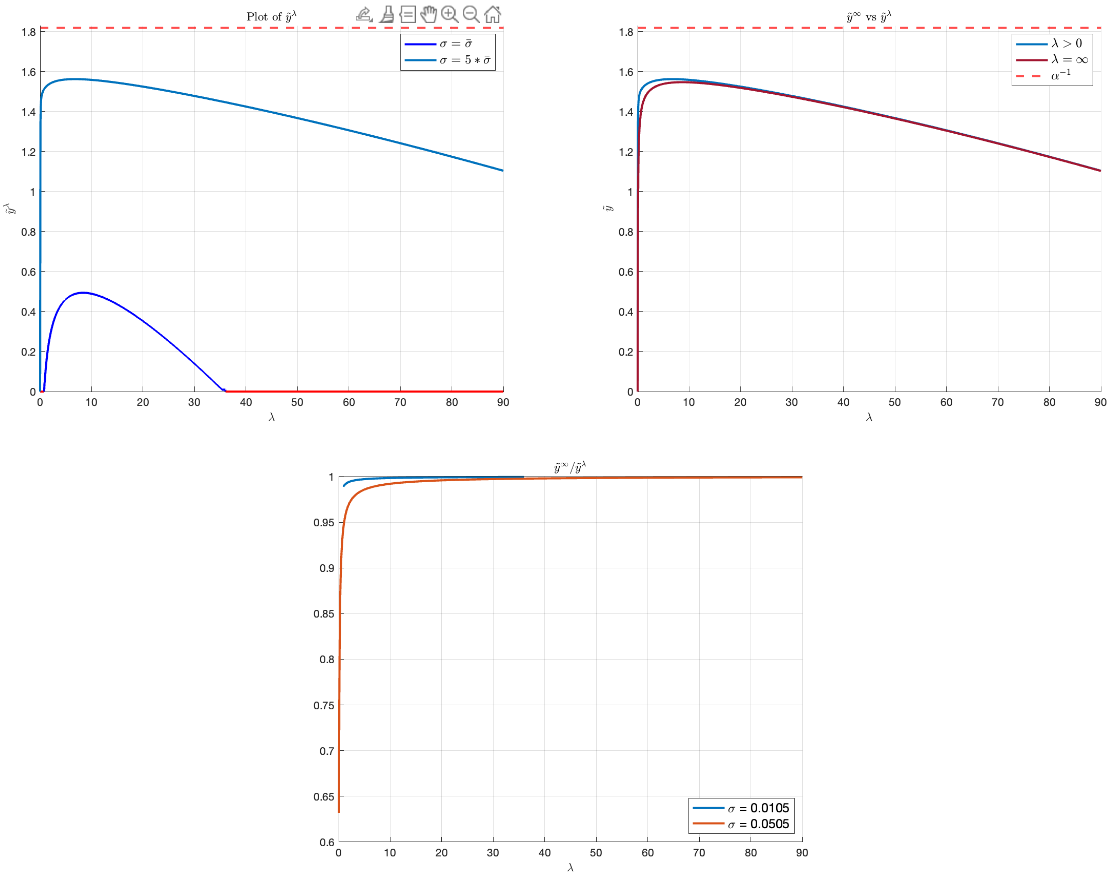

Figure 8: Equilibrium thresholds as a function of λ for the standard calibration and $\nu = 95\%$.

4 Discussion

In this section we discuss the role of social media on information dissemination and how it can change our understanding of the main results. We first explain how social media distinguishes from other communication technologies and then we discuss which specific mechanisms it brings into the model.

4.1 What is special about social media?

Social Media distinguishes in the role of information dissemination for at least four reasons. First of all, due to the ease of accessing news. Getting informed through social media does not require access to a computer or finding a particular webpage to access news; just picking up your phone occasionally, you can access information from all news providers you follow in one location. Further, people get into social media to consume other media productions or get news about their friends and come across all sorts of posts, such as those of the media providers they or their friends follow. Second, social media interactions as a response to media coverage help spread the news to other users who further like or share and help disseminate information even further. Sharing makes available information more reliable (Allcott and Gentzkow, 2017), resulting in another channel through which social media spurs information dissemination. Likewise, people might share to verify the truthfulness of the information received. Third, in social media, anyone can spread news to a broad audience, resulting both in a higher reach of word of mouth and potentially in an increased noise in the information available as discussed earlier.[28]

At last, social media (and perhaps internet blogs) also facilitates coordinated actions across depositors.[29] This factor is also present in the general discussion about the topic, and it is not obvious how it can affect our understanding of modern bank runs. It is unclear why depositors who know a bank is in distress would spread the word about it, except to those close to them, before withdrawing. Yet, since withdrawal decisions are instantaneous nowadays, this becomes irrelevant. Additionally, altruistic behavior or other interests could motivate individuals to share news publicly once they secure their deposits. The latter seems to have been relevant in the sequence of runs on Silicon Valley Bank, First Republic Bank, and Signature Bank on March 2023 (Cookson et al., 2023; Rose, 2023).

Furthermore, as with any coordination device, social media makes it easier for depositors to affect the behavior of others by releasing possibly incorrect information about the bank's health. On the one hand, this implies that people with wrong information can spread it to others quickly, making the bank more fragile. This suggests that any financial institution should diversify their creditors even more not only to restrict the possibility of common liquidity shocks but to avoid collusive outcomes where agents coordinate into withdrawing their deposits, either through individual depositors sharing their concerns with many others or because they receive correlated (bad) signals.[30] Diversification helps making less likely a scenario where all depositors receive the same negative signal about the bank as well as one in which some receive it and share it with everybody else. Yet, to the extent that everyone has access to everybody else's social media posts or the same information, diversification does not help to avoid collusive outcomes.

On the other hand, depositors can intentionally share wrong information to influence others. For instance, it may be useful to send sound signals about the bank's well-being before withdrawing to hedge from others if the depositor believes the bank's assets are not so valuable. This gives you time to keep earning rents until taking your deposit out of the bank and diminishes the risk of bank failure under inattention. This makes receiving bad information more reliable, making the bank more fragile as people will react more to bad signals.

We think this line of research is highly relevant as it seems that introducing better coordination devices makes banks more fragile to people's behavior. We leave it for future research.

4.2 Social Media: frequency and precision

We discuss here which new insights we can obtain when incorporating into the main model noisy signals about the bank's health. This allows us to study the additional effect of introducing new information technologies well discussed in the media, an increase in the noise agents receive. This introduces two new effects. On the one hand, extra noise makes it harder for agents to infer the true value of the bank's assets, which makes them less likely to run as they have an incentive to wait to gather more information before making a decision. This resembles waiting

[28] Note, for instance, that in general, there are rules that regulate how financial advisors can behave, but in social media, anyone can give advice to a big audience with almost no restrictions at all.

[29] Few is written on the role of social media as a coordination device, but an interesting reference is Enikolopov et al. (2020). In the paper, the authors show that the penetration of social media caused more protest activities in Russia in 2011 through a higher probability of a protest in a given city and a higher number of protesters. Their results suggest that the main driver is that social media reduced coordination costs rather than spreading critical government information.

[30] Certainly, there is a greater concern of coordinated attacks similar to the GameStop event (Malz, 2021).

to double-check the news received on social media or the Internet, potentially with alternative sources, to make sure that the information is somewhat correct. Another way to think about it is to wait to see if the original source keeps posting news on the same track, potentially correcting itself.[31] On the other hand, higher uncertainty about the bank's health makes agents more likely to run as, for any signal, it is more likely that the bank's assets are at a low level. This resembles running on the bank because the likelihood of a significantly low return under failure is high even if the news channels report minor concerns. [32]

Note that a higher uncertainty level increases both the value of learning and the risk of considerably low payments under failure. Which effect dominates will typically depend on N and, hence, λ. We expect the second one to dominate for large values of N, increasing the frequency of runs in equilibrium. This is because the risk of receiving a low payoff if waiting a little longer than others becomes substantial. However, when N is small, the agent knows that the bank is likely to not fail in the close future, diminishing the importance of an increase in the expected losses under failure and, hence, increasing the relative value of waiting to receive better information.

Furthermore, this addition brings the possibility of "Fake News" creating bank runs. That is, the possibility of a bank run triggered by inaccurate signals received by depositors making them believe that the bank is insolvent when it is not. Likewise, it can temporarily sustain a bank that suffers from bad fundamentals, avoiding a run on it.

A model of this sort allows us to study how the introduction of social media and the Internet can affect the banking system's stability through increased production of information that comes with higher noise. This is relevant as these technologies, by decreasing the entry cost to journalism and the production of news, as well as facilitating information sharing, have increased the amount of information available to the general public. At the same time this could have led to a decreased quality in its production because of an accelerated journalism race that leads to less fact-checking and mistakes, together with a less specialized or expert journalist population.

Our intuition suggests that introducing changes in the precision of the information available can have opposite effects to those considered in the main model. However, the size of this effects and the exact moment in which one dominates the other may be substantially different to the former. We describe how to introduce these features into the main model in Appendix C. We leave solving the model and doing a proper comparative static for future work.

5 Conclusion

Recent runs in the United States have spurred discussions over the impact of the introduction of social media and instant withdrawal technologies on the banking sector's stability. The little academic research on the topic and the general discussion about it in the media suggest that the

[31] An example of the latter can be found in: https://www.nytimes.com/2023/10/23/pageoneplus/editors-note-gaza-hospital-coverage.html

[32] With heterogeneous signals, an additional effect is present: it is more likely that the bank is facing severe difficulties without the depositor knowing this when the signals are noisy. This is because you know that many agents may receive bad enough signals that would make them run even if your own information suggests that the bank's portfolio is in a good condition. We abstract from this as dealing with Global Games features in this model is complicated and most of the discussion in the media focuses on the fact that depositors may receive wrong signals rather than different ones. We believe the latter follows from the fact that social media allowing for faster information dissemination implies that this heterogeneity is getting small.

latter made banks more fragile, exacerbating the intensity of runs and perhaps making them more likely because of the possibility of spreading panic quickly.

We argue that there is an additional channel through which the introduction and substantial penetration of social media, together with other improvements in communication and withdrawal technologies (i.e., cellphones, news websites, blogs, apps, etc.), can affect the likelihood of runs. If the latter allows people to get informed more frequently, then exposure to the risk of bank failure under inattention decreases, increasing the value of keeping your deposits in the bank. While these technological advancements increase the number of depositors withdrawing their money during a bank run, once this severity reaches a certain point, further increases have little effect on withdrawal behavior, as the recovery value imposes a natural limit. Additionally, larger withdrawals during a run amplifies the importance of minimizing the exposure to inattention risk. As a result, when technological progress in communication reaches a sufficient level, further advancements lead to a decreased frequency of bank runs.

Moreover, suppose the introduction of social media allows more depositors to get informed about the bank's well-being. In that case, the fear of faster withdrawals elevates rollover concerns, increasing the likelihood of runs. However, to the extent that social media allows more and more depositors to monitor the bank's assets as if they were experts, we obtain the same results. More withdrawals from experts in case of a run initially elevate rollover concerns, resulting in a higher frequency of runs. Eventually, as the share of expert depositors increases, the decreased exposure to the risk of failure under inattention for the average depositor leads to a lower frequency of runs, potentially eliminating them.

Our results suggest that the introduction of improved communication and withdrawal technologies, such as social media and mobile banking apps, does not necessarily undermine financial stability. Although it increases the severity of runs, it could be a blessing in the limit, reducing the frequency of them to a minimum. According to these results, in the time series, we should observe, all else equal, an increasing frequency of runs with the introduction of new communication technologies, followed by a decline as these technologies mature and further progress is made.

Furthermore, considering the effects on people outside the financial system, the results could be even more promising. Specifically, if agents who do not allocate their money into the banking sector have incentives to do so once they can monitor the value of banks assets more frequently, the result would be a decreased cost of financing, increasing welfare. The latter still has not been explored.

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A Appendix: main model

Here, we solve the main model and provide all related proofs for the propositions stated in the text.

A.1 Solution

To solve this differential equation, let's express it differently. For $\alpha^{-1} < \tilde{y}$ we have

$$\rho V^A(y_t;\tilde{y}) - \delta\left[V^I(y_t;\tilde{y}) - V^A(y_t;\tilde{y})\right] = \begin{cases} r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) & \text{for } y_t > \tilde{y} \\ r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \theta N(\lambda,\delta)\left[1 - V^A(y_t;\tilde{y})\right] & \text{for } \frac{1}{\alpha} < y_t \leq \tilde{y} \\ r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \theta N(\lambda,\delta)\left[\alpha y_t - V^A(y_t;\tilde{y})\right] & \text{for } 0 < y_t \leq \frac{1}{\alpha} \end{cases}$$

For $\alpha^{-1} \geq \tilde{y}$ we have

$$\rho V^A(y_t;\tilde{y}) - \delta\left[V^I(y_t;\tilde{y}) - V^A(y_t;\tilde{y})\right] = \begin{cases} r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) & \text{for } y_t > \tilde{y} \\ r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \theta N(\lambda,\delta)\left[\alpha y_t - V^A(y_t;\tilde{y})\right] & \text{for } 0 < y_t \leq \tilde{y} \end{cases}$$

The remaining conditions to impose are optimality (smooth-pasting) and symmetry. Further, we must characterize V^I. The way we do it is noticing that the value of a depositor that just became inattentive at time t is identical to that of an agent who at time t becomes inhibited from withdrawing for $\Delta \sim exp(\delta)$ periods but does observe y in the meantime. That is, $V^I(y_t)$, the value after 0 periods of inattention, is identical for a depositor who will not monitor the bank's assets during the inattention period and one that does but is unable to withdraw. The reason is that, at time t, they both have the same information to forecast future outcomes, which do not differ among them.[33] Then, we can compute V^I as the value of an agent that cannot withdraw but does observe y during the inattentive period and use it to evaluate the payoff of exiting the attention window. Said function will satisfy the following HJB equation as it is C^2:

$$\begin{aligned} \rho V^I(y_t;\tilde{y}) = r + \mu y_t V_1^I(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^I(y_t;\tilde{y}) + \theta N(\lambda,\delta) I_{\{y_t \leq \tilde{y}\}}\left[\min\{1, \alpha y_t\} - V^I(y_t;\tilde{y})\right] \\ + \lambda\left[V^A(y_t;\tilde{y}) - V^I(y_t;\tilde{y})\right] \end{aligned}$$

For any $\tilde{y}$, in the continuation region, we can subtract the HJB equations that characterize the value functions of the uninformed both when they can monitor and when they are not to get

$$(\rho + \lambda + \delta)\Gamma = \mu y_t \Gamma_1 + \frac{\sigma^2}{2} y_t^2 \Gamma_{11} - \theta N(\lambda,\delta) I_{\{y_t \leq \tilde{y}\}} \Gamma$$

where $\Gamma \equiv V^I(y;\tilde{y}) - V^A(y;\tilde{y})$. With appropriate border conditions,[34] we get that

$$\Gamma(y_t;\tilde{y}) = \begin{cases} A_1 y_t^{\alpha_1} & \text{for } y_t > \tilde{y} \\ A_2 y_t^{\alpha_2} + A_3 y_t^{\alpha_3} & \text{for } 0 < y_t \leq \tilde{y} \end{cases}$$

[33] Of course the value after T periods of inattention will not be the same for an agent that is able to observe y but not withdraw, this is because it can forecast future outcomes using more updated information. The important thing is that, under our assumptions, we do not care about such value but that at the instant of becoming inattentive, in which case they do coincide.

[34] We use that $\lim_{y\to\infty} \Gamma(y) = 0$. We need an extra border condition, which we will obtain from solving for Γ in the withdrawal region. The issue is that this function will depend on the underlying case considered for $\tilde{y}$, so we will leave it for later.

where α_1 is the negative root of $\rho+\lambda+\delta = \mu\alpha_1 + \frac{\sigma^2}{2}\alpha_1(\alpha_1-1)$. The other roots are given by α_2 and α_3 which are respectively the positive and negative roots of $\rho+\lambda+\delta = \mu\alpha_i + \frac{\sigma^2}{2}\alpha_i(\alpha_i-1) - \theta N(\lambda,\delta)$ for $i = 2,3$. The constants A_i are the solution to the following system of equations:

$$\begin{aligned} A_1(\tilde{y})^{\alpha_1} &= A_2(\tilde{y})^{\alpha_2} + A_3(\tilde{y})^{\alpha_3} \\ A_1\alpha_1(\tilde{y})^{\alpha_1-1} &= A_2\alpha_2(\tilde{y})^{\alpha_2-1} + A_3\alpha_3(\tilde{y})^{\alpha_3-1} \end{aligned} \tag{2}$$

With an expression for Γ we can now solve for V^A from

$$\begin{aligned} \rho V^A(y_t;\tilde{y}) &= r + \mu y_t V_1^A(y_t;\tilde{y}) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\tilde{y}) + \delta\Gamma(y_t;\tilde{y}) \\ &+ \theta\left(\nu I_{\{y_t \leq \tilde{y}^I\}} + (1-\nu)N(\lambda,\delta)I_{\{y_t\leq\tilde{y}\}}\right)\left[\min\{1,\alpha y_t\} - V^A(y_t;\tilde{y})\right] \end{aligned}$$

Since Γ is a polynomial, we can solve this differential equation with a polynomial of equal exponents as a particular solution. We solve for the equilibrium threshold in the following, considering each case separately.

A.1.1 Case $\tilde{y} \leq \alpha^{-1}$

We begin by characterizing the value function of an uninformed agent in the withdrawal region when she cannot observe y. We do this to obtain an additional border condition for Γ. This solves the following differential equation for any $y \in (0,\bar{y}]$

$$\rho V^I = \begin{cases} r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \lambda[1-V^I] & \text{for } y > \tilde{y} \\ r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \theta N(\lambda,\delta)\left[\alpha y - V^I\right] + \lambda[1-V^I] & \text{for } 0 < y \leq \tilde{y} \end{cases}$$

The solution to this differential equation is given by[35]

$$V^I(y) = \begin{cases} D_1 y^{\iota_1} + D_2 y^{\iota_2} + \frac{r+\lambda}{\rho+\lambda} & \text{for } y > \tilde{y} \\ D_3 y^{\iota_3} + \frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu} y + \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)} & \text{for } 0 < y \leq \tilde{y} \end{cases}$$

where ι_1 and ι_2 are the positive and negative roots respectively of $\rho + \lambda = \mu\iota_i + \frac{\sigma^2}{2}\iota_i(\iota_i - 1)$, and ι_3 is the positive root of $\rho+\lambda = \mu\iota_i + \frac{\sigma^2}{2}\iota_i(\iota_i-1) - \theta N(\lambda,\delta)$. The constants D_i and A_i are jointly determined by the system (2) and the following equations:

$$\begin{aligned} &D_1(\tilde{y})^{\iota_1} + D_2(\tilde{y})^{\iota_2} + \frac{r+\lambda}{\rho+\lambda} = D_3(\tilde{y})^{\iota_3} + \frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}\tilde{y} + \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)} \\ &D_1\iota_1(\tilde{y})^{\iota_1-1} + D_2\iota_2(\tilde{y})^{\iota_2-1} = D_3\iota_3(\tilde{y})^{\iota_3-1} + \frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu} \\ &V^I(\bar{y}) = 1 + \lim_{y\to\bar{y}}\Gamma(y) \\ &V_1^I(\bar{y}) = \lim_{y\to\bar{y}}\Gamma_1(y) + \underbrace{V_1^A(\bar{y})}_{=0} \end{aligned}$$

This gives the final linear system of six equations in six unknowns that we must solve as a function of $\tilde{y}$ and $\bar{y}$. We now turn to the value function of an uninformed agent who is monitoring. In the inaction region it will be given by[36]

$$V^A(y;\tilde{y}) = \begin{cases} B_1 y^{\beta_2} + C_1 y^{\alpha_1} + C_2 & \text{for } y_t > \tilde{y} \\ B_2 y^{\beta_3} + B_3 y^{\beta_4} + C_3 y^{\alpha_2} + C_4 y^{\alpha_3} + C_5 y + C_6 & \text{for } 0 < y_t \leq \tilde{y} \end{cases}$$

[35] We already imposed that $\lim_{y\to 0} V^I(y)$ has to be finite, eliminating the term with a negative root (i.e., $D_4 = 0$).

[36] I use the fact that $\lim_{y\to\infty} V^A(y;\tilde{y}) = r/\rho$ as a border condition already.

where β_2 is the negative root of $\rho = \mu\beta_2 + \frac{\sigma^2}{2}\beta_2(\beta_2 - 1)$, β_3 and β_4 are respectively the positive and negative roots of $\rho + \theta N(\lambda, \delta) = \mu\beta_i + \frac{\sigma^2}{2}\beta_i(\beta_i - 1)$ for $i = 3, 4$. The constants B_i and C_i are the solution to the following system of equations:

$$
\begin{aligned}
&C_1 = -\delta A_1(\lambda + \delta)^{-1} \\
&C_2 = \frac{r}{\rho} \\
&C_3 = -\delta A_2(\lambda + \delta)^{-1} \\
&C_4 = -\delta A_3(\lambda + \delta)^{-1} \\
&C_5 = \frac{\theta N(\lambda, \delta)\alpha}{\rho + \theta N(\lambda, \delta) - \mu} \\
&C_6 = \frac{r}{\rho + \theta N(\lambda, \delta)} \\
&B_1(\tilde{y})^{\beta_2} + C_1(\tilde{y})^{\alpha_1} + C_2 = B_2(\tilde{y})^{\beta_3} + B_3(\tilde{y})^{\beta_4} + C_3(\tilde{y})^{\alpha_2} + C_4(\tilde{y})^{\alpha_3} + C_5\tilde{y} + C_6 \\
&B_1\beta_2(\tilde{y})^{\beta_2 - 1} + C_1\alpha_1(\tilde{y})^{\alpha_1 - 1} = B_2\beta_3(\tilde{y})^{\beta_3 - 1} + B_3\beta_4(\tilde{y})^{\beta_4 - 1} + C_3\alpha_2(\tilde{y})^{\alpha_2 - 1} + C_4\alpha_3(\tilde{y})^{\alpha_3 - 1} + C_5
\end{aligned}
$$

together with $\lim_{y \to \bar{y}^+} V^A(y; \tilde{y}) = 1$ and the *smooth-pasting* condition $V_1^U(\bar{y}) = 0$. Note that these two introduce another constant to obtain: the individual threshold or best response of the agent to $\tilde{y}$. Finally, imposing symmetry so that $\bar{y} = \tilde{y}$ we get that the last two conditions are equivalent to

$$
\begin{aligned}
1 &= B_1(\tilde{y})^{\beta_2} + C_1(\tilde{y})^{\alpha_1} + \frac{r}{\rho} \\
0 &= B_1\beta_2(\tilde{y})^{\beta_2 - 1} + C_1\alpha_1(\tilde{y})^{\alpha_1 - 1}
\end{aligned}
$$

After solving for A_1, we use these last two equations to obtain the following equilibrium threshold

$$
\tilde{y} = \frac{(\alpha_1 - \iota_3)\left(\frac{r}{\rho} - 1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2 - \alpha_1} - \iota_3\left(1 - \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}\right)}{(1 - \iota_3)\frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}} \tag{3}
$$

which exists as long as the numerator is strictly negative.[37] Having solved for the equilibrium threshold, we can recover the value of A_1 and with it, all the remaining integration constants.

A.1.2 Case $\tilde{y} > \alpha^{-1}$

We proceed the same way as before, characterizing V^I in the withdrawal region. This solves the following differential equation for any $y \in (0, \bar{y}]$

$$
\rho V^I = \begin{cases} r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \lambda[1 - V^I] & \text{for } \; y > \tilde{y} \\ r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \theta N(\lambda, \delta)\left[1 - V^I\right] + \lambda[1 - V^I] & \text{for } \; \alpha^{-1} < y \le \tilde{y} \\ r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \theta N(\lambda, \delta)\left[\alpha y - V^I\right] + \lambda[1 - V^I] & \text{for } \; 0 < y \le \alpha^{-1} \end{cases}
$$

The solution to this differential equation is given by[38]

$$
V^I(y) = \begin{cases} D_1 y^{\iota_1} + D_2 y^{\iota_2} + \frac{r+\lambda}{\rho+\lambda} & \text{for } \; y > \tilde{y} \\ D_3 y^{\iota_3} + D_3 y^{\iota_4} + \frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)} & \text{for } \; \alpha^{-1} < y \le \tilde{y} \\ D_5 y^{\iota_3} + \frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu} y + \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)} & \text{for } \; y \le \alpha^{-1} \end{cases}
$$

[37] The denominator is always strictly negative as long as it is well-defined.

[38] We already imposed that $\lim_{y \to 0} V^I(y)$ has to be finite, eliminating the term with a negative root (i.e., $D_6 = 0$).

where ι_1 and ι_2 are the positive and negative roots respectively of $\rho+\lambda=\mu\iota_i+\frac{\sigma^2}{2}\iota_i(\iota_i-1)$, and ι_3 and ι_4 are the respective roots of $\rho+\lambda=\mu\iota_i+\frac{\sigma^2}{2}\iota_i(\iota_i-1)-\theta N(\lambda,\delta)$. In this case the constants D_i and A_i are jointly determined by the system (2) and the following equations:

$$D_1(\tilde{y})^{\iota_1}+D_2(\tilde{y})^{\iota_2}+\frac{r+\lambda}{\rho+\lambda}=D_3(\tilde{y})^{\iota_3}+D_4(\tilde{y})^{\iota_4}+\frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}$$
$$D_1\iota_1(\tilde{y})^{\iota_1-1}+D_2\iota_2(\tilde{y})^{\iota_2-1}=D_3\iota_3(\tilde{y})^{\iota_3-1}+D_4\iota_4(\tilde{y})^{\iota_4-1}$$
$$D_3(\alpha)^{-\iota_3}+D_4(\alpha)^{-\iota_4}+\frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}=D_5(\alpha)^{-\iota_3}+\frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}\alpha^{-1}+\frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}$$
$$D_3\iota_3(\alpha)^{-\iota_3+1}+D_4\iota_4(\alpha)^{-\iota_4+1}=D_5\iota_3(\alpha)^{-\iota_3+1}+\frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}$$
$$V^I(\bar{y})=1+\lim_{y\to\bar{y}}\Gamma(y)$$
$$V_1^I(\bar{y})=\lim_{y\to\bar{y}}\Gamma_1(y)+\underbrace{V_1^A(\bar{y})}_{=0}$$

This gives the final linear system of eight equations in eight unknowns that we need to solve as a function of $\tilde{y}$ and $\bar{y}$. We now turn to the value function of an uninformed agent who is monitoring. In the inaction region it will be given by[39]

$$V^A(y;\tilde{y})=\begin{cases}B_1y^{\beta_2}+C_1y^{\alpha_1}+C_2 & \text{for } y_t>\tilde{y}\\ B_2y^{\beta_3}+B_3y^{\beta_4}+C_3y^{\alpha_2}+C_4y^{\alpha_3}+C_5 & \text{for } \alpha^{-1}<y_t\le\tilde{y}\\ B_5y^{\beta_3}+B_6y^{\beta_4}+C_3y^{\alpha_2}+C_4y^{\alpha_3}+C_6y+C_7 & \text{for } 0<y_t\le\alpha^{-1}\end{cases}$$

where β_2 is the negative root of $\rho=\mu\beta_2+\frac{\sigma^2}{2}\beta_2(\beta_2-1)$, β_3 and β_4 are respectively the positive and negative roots of $\rho+\theta N(\lambda,\delta)=\mu\beta_i+\frac{\sigma^2}{2}\beta_i(\beta_i-1)$ for $i=3,4$. The constants B_i and C_i are the solution to the following system of equations:

$$C_1=-\delta A_1(\lambda+\delta)^{-1}$$
$$C_2=\frac{r}{\rho}$$
$$C_3=-\delta A_2(\lambda+\delta)^{-1}$$
$$C_4=-\delta A_3(\lambda+\delta)^{-1}$$
$$C_5=\frac{r+\theta N(\lambda,\delta)}{\rho+\theta N(\lambda,\delta)}$$
$$C_6=\frac{\theta N(\lambda,\delta)\alpha}{\rho+\theta N(\lambda,\delta)-\mu}$$
$$C_7=\frac{r}{\rho+\theta N(\lambda,\delta)}$$
$$B_1(\tilde{y})^{\beta_2}+C_1(\tilde{y})^{\alpha_1}+C_2=B_2(\tilde{y})^{\beta_3}+B_3(\tilde{y})^{\beta_4}+C_3(\tilde{y})^{\alpha_2}+C_4(\tilde{y})^{\alpha_3}+C_5$$
$$B_1\beta_2(\tilde{y})^{\beta_2-1}+C_1\alpha_1(\tilde{y})^{\alpha_1-1}=B_2\beta_3(\tilde{y})^{\beta_3-1}+B_3\beta_4(\tilde{y})^{\beta_4-1}+C_3\alpha_2(\tilde{y})^{\alpha_2-1}+C_4\alpha_3(\tilde{y})^{\alpha_3-1}$$
$$B_2(\alpha)^{-\beta_3}+B_3(\alpha)^{-\beta_4}+C_5=B_5(\alpha)^{-\beta_3}+B_6(\alpha)^{-\beta_4}+C_6\alpha^{-1}+C_7$$
$$B_2\beta_3(\alpha)^{-\beta_3+1}+B_3\beta_4(\alpha)^{-\beta_4+1}=B_5\beta_3(\alpha)^{-\beta_3+1}+B_6\beta_4(\alpha)^{-\beta_4+1}+C_6$$

together with $\lim_{y\to\bar{y}^+}V^A(y;\tilde{y})=1$ and the *smooth-pasting* condition $V_1^U(\bar{y})=0$. Finally, imposing symmetry so that $\bar{y}=\tilde{y}$ we get that the last two conditions are equivalent to

$$1=B_1(\tilde{y})^{\beta_2}+C_1(\tilde{y})^{\alpha_1}+\frac{r}{\rho}$$
$$0=B_1\beta_2(\tilde{y})^{\beta_2-1}+C_1\alpha_1(\tilde{y})^{\alpha_1-1}$$

After solving for A_1, we use these last two equations to obtain the following equilibrium threshold

$$\alpha\cdot\tilde{y}=\left[\frac{(\alpha_1-\iota_3)\left(\frac{r}{\rho}-1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2-\alpha_1}-\iota_3\left(1-\frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}\right)}{\iota_3\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}+(1-\iota_3)\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}}\right]^{\frac{1}{\iota_4}} \qquad (4)$$

[39] I use the fact that $\lim_{y\to\infty}V^A(y;\tilde{y})=r/\rho$ as a border condition already.

which exists since both the numerator and denominator are strictly positive. Having solved for the equilibrium threshold, we can recover the value of A_1 and with it, all the remaining integration constants.

A.2 Proofs

Proposition. *There is no asymmetric equilibrium under monotone strategies.*

Proof. We prove it by contradiction following Lemma 4 in He and Xiong (2012). Suppose that there exists an asymmetric monotone equilibrium. Then, there exist at least two groups of depositors who use two different monotone strategies with thresholds $\tilde{y}_1 < \tilde{y}_2$. Let γ be the set containing all equilibrium thresholds. Let i denote the type of depositor that uses threshold $\tilde{y}_i$, with corresponding value when being attentive $V^{A,i}(y;\gamma)$. At the corresponding thresholds, we must have

$$V^{A,1}\left(\tilde{y}_1;\gamma\right) = V^{A,2}\left(\tilde{y}_2;\gamma\right) = 1$$

and then

$$V^{A,1}\left(\tilde{y}_2\right) = V^{A,2}\left(\tilde{y}_1\right) = 1$$

since both creditors are free to switch between these two strategies. Then, for all $y \in [\tilde{y}_1, \tilde{y}_2]$, we must have $V^{A,1}(y) = V^{A,2}(y) = 1$. Otherwise the threshold strategies cannot be optimal. This implies that each creditor is indifferent between choosing any threshold in the set $[\tilde{y}_1, \tilde{y}_1]$. Denote by $\zeta(y)$ the measure of creditors who use a threshold lower than $y \in [\tilde{y}_1, \tilde{y}_2]$. Then, $V^{A,i}$ has to satisfy the following HJB equation for all y in this region:

$$\begin{aligned}\rho V^{A,i}(y) = r + \mu y V_1^{A,i} + \frac{\sigma^2}{2} y^2 V_{11}^{A,i} + \delta\left[V^{I,i}(y;\gamma) - V^{A,i}(y;\gamma)\right] \\ + \theta N(\lambda,\delta)\zeta(y)\left[\min\{1,\alpha y\} - V^{A,i}(y;\gamma)\right]\end{aligned}$$

Since $V^{A,i}(y) = 1$ for any $y \in [\tilde{y}_1, \tilde{y}_2]$, we have

$$\rho = r + \delta\left[V^{I,i}(y;\gamma) - 1\right] + \theta N(\lambda,\delta)\zeta(y)\left[\min\{1,\alpha y\} - 1\right]$$

Since $\zeta(y)$ and $\min\{1,\alpha y\}$ are non-decreasing in y and $V^{I,i}(y;\gamma)$ is monotonically increasing in y this equation cannot hold for all $y \in [\tilde{y}_1, \tilde{y}_2]$ since $r > \rho$ by assumption. ■

Proposition. *An equilibrium threshold $\tilde{y} > 0$ exists, and it is the unique stationary equilibrium under monotone strategies if and only if*

$$r < \frac{\rho + \theta N(\lambda,\delta) + \frac{\delta+\lambda}{\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))}{\left(1 + \frac{\delta+\lambda}{\rho\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))\right)} \in (0, \rho + \theta N(\lambda,\delta))$$

Proof. The function

$$W(\bar{y}) \equiv V^A(\bar{y};\bar{y}) = \begin{cases} \frac{r}{\rho} - \frac{\delta}{\lambda+\delta}\frac{\beta_2-\alpha_1}{\beta_2}\frac{1}{\alpha_1-\iota_3}\left(\iota_3\left(1 - \frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}\right) + (1-\iota_3)\frac{\theta N(\lambda,\delta)\alpha}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}\bar{y}\right) & \text{for } \bar{y} \le \alpha^{-1} \\ \frac{r}{\rho} - \frac{\delta}{\lambda+\delta}\frac{\beta_2-\alpha_1}{\beta_2}\frac{1}{\alpha_1-\iota_3}\left(\iota_3\left(1 - \frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}\right) + (\alpha\bar{y})^{\iota_4}\left(\iota_3\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)} + (1-\iota_3)\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}\right)\right) & \text{for } \alpha^{-1} < \end{cases}$$

is continuous and strictly increasing over $(0,\infty)$. Then, as long as $W(0) < 1$ and $W(\infty) > 1$, there exists a unique $\tilde{y}$ such that $W(\tilde{y}) = 1$. Then, we need

$$W(0) = \frac{r}{\rho} - \frac{\delta}{\lambda+\delta}\frac{\beta_2-\alpha_1}{\beta_2}\frac{1}{\alpha_1-\iota_3}\iota_3\left(1-\frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}\right) < 1$$
$$W(\infty) = \frac{r}{\rho} - \frac{\delta}{\lambda+\delta}\frac{\beta_2-\alpha_1}{\beta_2}\frac{1}{\alpha_1-\iota_3}\iota_3\left(1-\frac{r+\lambda+\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)}\right) > 1$$

The second condition holds since we have $r > \rho$. As for the first inequality, we can reduce it to

$$\frac{\rho+\theta N(\lambda,\delta)+\frac{\delta+\lambda}{\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))}{\left(1+\frac{\delta+\lambda}{\rho\delta}\frac{\iota_3-\alpha_1}{\beta_2-\alpha_1}\frac{-\beta_2}{\iota_3}(\rho+\lambda+\theta N(\lambda,\delta))\right)} > r$$

which determines a strictly positive upper bound for the interest rate r. ■

Proposition. *The effect of changes in λ on the equilibrium running threshold is non-monotonic. For low enough values of λ, $\tilde{y}$ increases with λ. For high enough values of λ, $\tilde{y}$ decreases with λ.*

Proof. We begin with the case where $\tilde{y} \leq \alpha^{-1}$. An equilibrium exists as long as the expression for this threshold is strictly positive. Recall that $r < \rho+\theta N(\lambda,\delta)$ is required for the equilibrium threshold to be strictly positive. Then, it must be strictly increasing, and below α^{-1} for small values of λ since we have $r > \rho$ and the expression for the threshold is continuous.

As for the second part, it is also true that the equilibrium threshold for the first case becomes negative for large enough values of λ, implying that it must be decreasing and below α^{-1}. This follows from checking whether the condition for existence in equation (**??**) holds for large values of λ. Considering the case where $\tilde{y} > \alpha^{-1}$, we can see from equation (4) that The threshold's expression will decrease for large values of λ, making ι_4 more negative and the fraction inside the brackets arbitrarily large.

This concludes the proof showing that when λ is small enough the equilibrium threshold lies on $(0,\alpha^{-1})$ and is increasing in λ, whereas for large values of λ it is decreasing and eventually becomes below α^{-1}. ■

Proposition. *Keeping N fixed, the equilibrium threshold decreases with λ. Furthermore, $\lim_{\lambda\to\infty}\tilde{y} = -\infty$.*

Proof. To be completed. ■

Proposition. *The equilibrium threshold increases with N for low values of N. In addition, $\lim_{N\to\infty}\tilde{y} \propto \alpha^{-1}$.*

Proof. We know the expression for the equilibrium threshold becomes negative if $\theta N + \rho < r$, implying that it must be increasing in N for low enough values of it. To see the behavior in the limit we have to consider both possible expressions for the threshold. If we consider that for $\tilde{y} < \alpha^{-1}$, we obtain the following

$$\lim_{N\to\infty}\tilde{y} = \lim_{N\to\infty}\alpha^{-1}\cdot\frac{\left(\frac{\alpha_1}{\iota_3}-1\right)\left(\frac{r}{\rho}-1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2-\alpha_1}-\left(1-\frac{r+\lambda}{\rho+\lambda+\theta N(\lambda,\delta)}\right)}{\left(\frac{1}{\iota_3}-1\right)\frac{\theta N(\lambda,\delta)}{\rho+\lambda+\theta N(\lambda,\delta)-\mu}}$$
$$= \alpha^{-1}\cdot\left[\left(\frac{r}{\rho}-1\right)\frac{\lambda+\delta}{\delta}\frac{\beta_2}{\beta_2-\alpha_1}+1\right] < \alpha^{-1}$$

which, given our existence proof, implies that the limiting equilibrium threshold exists and is below the recovery value α^{-1}. ■

B Appendix: increasing the reach of information

Here, we present the proofs and derivations of the model in this extension with $\nu > 0$.

Proposition. *If the only depositors able to monitor the bank can monitor its assets at all times, the only monotone stationary equilibrium exhibits no runs.*

Proof. To be completed. ■

B.1 Case with $\nu > 0$ and two types of positive λ

We can characterize the value of becoming inattentive as before. In this case, the HJB equation to solve for is

$$\begin{aligned}\rho V^I(y_t;\gamma) &= r + \mu y_t V_1^I(y_t;\gamma) + \frac{\sigma^2}{2} y_t^2 V_{11}^I(y_t;\gamma) + \lambda \left[V^A(y_t;\gamma) - V^I(y_t;\gamma)\right] \\ &+ \theta \left((1-\nu) I_{\{y_t \le \tilde{y}^\infty\}} + \nu N(\lambda,\delta) I_{\{y_t \le \tilde{y}^\lambda\}}\right) \left[\min\{1, \alpha y_t\} - V^I(y_t;\gamma)\right]\end{aligned}$$

For any thresholds in the continuation region, we can subtract the HJB equations that characterize the value functions of the least informed both when they can monitor and when they are not to get

$$(\rho + \lambda + \delta)\Gamma = \mu y_t \Gamma_1 + \frac{\sigma^2}{2} y_t^2 \Gamma_{11} - \theta \left((1-\nu) I_{\{y_t \le \tilde{y}^\infty\}} + \nu N(\lambda,\delta) I_{\{y_t \le \tilde{y}^\lambda\}}\right) \Gamma$$

where $\Gamma \equiv V^I(y;\gamma) - V^A(y;\gamma)$. With appropriate border conditions,[40] we get that

$$\Gamma(y_t;\gamma) = \begin{cases} A_1 y_t^{\alpha_1} & \text{for } y_t > \tilde{y}^\lambda \\ A_2 y_t^{\alpha_2} + A_3 y_t^{\alpha_3} & \text{for } \tilde{y}^\infty < y_t \le \tilde{y}^\lambda \\ A_4 y_t^{\alpha_4} + A_5 y_t^{\alpha_5} & \text{for } y_t \le \tilde{y}^\infty \end{cases}$$

where α_1 was defined earlier, the pair (α_2, α_3) are respectively the positive and negative roots of $\rho + \lambda + \delta = \mu \alpha_i + \frac{\sigma^2}{2}\alpha_i(\alpha_i - 1) - \theta\left(\nu N(\lambda,\delta)\right)$, and the pair (α_4, α_5) are those of $\rho + \lambda + \delta = \mu \alpha_i + \frac{\sigma^2}{2}\alpha_i(\alpha_i - 1) - \theta\left((1-\nu) + \nu N(\lambda,\delta)\right)$. The constants A_i are the solution to the following system of equations:

$$\begin{aligned} A_5(\tilde{y}^\infty)^{\alpha_5} + A_4(\tilde{y}^\infty)^{\alpha_4} &= A_2(\tilde{y}^\infty)^{\alpha_2} + A_3(\tilde{y}^\infty)^{\alpha_3} \\ A_5\alpha_5(\tilde{y}^\infty)^{\alpha_5-1} + A_4\alpha_4(\tilde{y}^\infty)^{\alpha_4-1} &= A_3\alpha_3(\tilde{y}^\infty)^{\alpha_3-1} + A_2\alpha_2(\tilde{y}^\infty)^{\alpha_2-1} \\ A_1(\tilde{y}^\lambda)^{\alpha_1} &= A_2(\tilde{y}^\lambda)^{\alpha_2} + A_3(\tilde{y}^\lambda)^{\alpha_3} \\ A_1\alpha_1(\tilde{y}^\lambda)^{\alpha_1-1} &= A_2\alpha_2(\tilde{y}^\lambda)^{\alpha_2-1} + A_3\alpha_3(\tilde{y}^\lambda)^{\alpha_3-1} \end{aligned} \tag{5}$$

[40] We use that $\lim_{y\to\infty}\Gamma(y) = 0$. We need an extra border condition, which we will obtain from solving for γ in the withdrawal region. The issue is that this function will depend on the underlying cause considered so that we will leave it for later.

Now solve for V^A from

$$\rho V^A(y_t;\gamma) = r + \mu y_t V_1^A(y_t;\gamma) + \frac{\sigma^2}{2} y_t^2 V_{11}^A(y_t;\gamma) + \delta\Gamma(y_t) + \theta\left(\nu I_{\{y_t \le \tilde{y}^I\}} + (1-\nu)N(\lambda,\delta) I_{\{y_t \le \tilde{y}\}}\right)\left[\min\{1,\alpha y_t\} - V^A(y_t;\gamma)\right]$$

Since Γ is a polynomial, we can solve this differential equation with a polynomial of equal exponents as a particular solution. We solve for the equilibrium thresholds considering only the case of both thresholds below the recovery value. The rest are analogous.

We begin by characterizing the value function of the least informed agent in the withdrawal region, who cannot observe y. This solves the following differential equation for any $y \in (0,\bar{y}]$

$$\rho V^I = \begin{cases} r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \lambda[1 - V^I] & \text{for } y > \tilde{y}^\lambda \\ r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \theta\nu N(\lambda,\delta)\left[\alpha y - V^I\right] + \lambda[1-V^I] & \text{for } \tilde{y}^\infty < y \le \tilde{y}^\lambda \\ r + \mu y V_1^I + \frac{\sigma^2}{2} y^2 V_{11}^I + \theta\left(1-\nu+\nu N(\lambda,\delta)\right)\left[\alpha y - V^I\right] + \lambda[1-V^I] & \text{for } y \le \tilde{y}^\infty \end{cases}$$

The solution to this differential equation is given by[41]

$$V^I(y) = \begin{cases} D_1 y^{\iota_1} + D_2 y^{\iota_2} + \frac{r+\lambda}{\rho+\lambda} & \text{for } y > \tilde{y} \\ D_3 y^{\iota_3} + D_4 y^{\iota_4} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\lambda+\theta\nu N(\lambda,\delta)-\mu} y + \frac{r+\lambda}{\rho+\lambda+\theta\nu N(\lambda,\delta)} & \text{for } \tilde{y}^I < y \le \tilde{y} \\ D_5 y^{\iota_5} + \frac{\theta(1-\nu+\nu N(\lambda,\delta))\alpha}{\rho+\lambda+\theta(1-\nu+\nu N(\lambda,\delta))-\mu} y + \frac{r+\lambda}{\rho+\lambda+\theta(1-\nu+\nu N(\lambda,\delta))} & \text{for } y \le \tilde{y}^I \end{cases}$$

where ι_1 and ι_2 are the positive and negative roots respectively of $\rho+\lambda = \mu\iota_i + \frac{\sigma^2}{2}\iota_i(\iota_i - 1)$, ι_3 and ι_4 are respectively the positive and negative roots of $\rho+\lambda = \mu\iota_i + \frac{\sigma^2}{2}\iota_i(\iota_i-1) - \theta\nu N(\lambda,\delta)$ for $i=3,4$. Finally, ι_5 is the positive root of $\rho+\lambda = \mu\iota_i + \frac{\sigma^2}{2}\iota_i(\iota_i-1) - \theta\left(1-\nu+\nu N(\lambda,\delta)\right)$. The constants D_i and A_i are jointly determined by the system (5) and the following equations:

$$D_1(\tilde{y}^\lambda)^{\iota_1} + D_2(\tilde{y}^\lambda)^{\iota_2} + \frac{r+\lambda}{\rho+\lambda} = D_3(\tilde{y}^\lambda)^{\iota_3} + D_4(\tilde{y}^\lambda)^{\iota_4} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\lambda+\theta\nu N(\lambda,\delta)-\mu}\tilde{y}^\lambda + \frac{r+\lambda}{\rho+\lambda+\theta\nu N(\lambda,\delta)}$$

$$D_1\iota_1(\tilde{y}^\lambda)^{\iota_1-1} + D_2\iota_2(\tilde{y}^\lambda)^{\iota_2-1} = D_3\iota_3(\tilde{y}^\lambda)^{\iota_3-1} + D_4\iota_4(\tilde{y}^\lambda)^{\iota_4-1} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\lambda+\theta\nu N(\lambda,\delta)-\mu}$$

$$D_5(\tilde{y}^\infty)^{\iota_5} + \frac{\theta\left(1-\nu+\nu N(\lambda,\delta)\right)\alpha}{\rho+\lambda+\theta\left(1-\nu+\nu N(\lambda,\delta)\right)-\mu}\tilde{y}^\infty + \frac{r+\lambda}{\rho+\lambda+\theta\left(1-\nu+\nu N(\lambda,\delta)\right)} = D_3(\tilde{y}^\infty)^{\iota_3} + D_4(\tilde{y}^\infty)^{\iota_4} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\lambda+\theta\nu N(\lambda,\delta)-\mu}$$

$$D_5\iota_5(\tilde{y}^\infty)^{\iota_5-1} + \frac{\theta\left(1-\nu+\nu N(\lambda,\delta)\right)\alpha}{\rho+\lambda+\theta\left(1-\nu+\nu N(\lambda,\delta)\right)-\mu} = D_3\iota_3(\tilde{y}^\infty)^{\iota_3-1} + D_4\iota_4(\tilde{y}^\infty)^{\iota_4-1} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\lambda+\theta\nu N(\lambda,\delta)-\mu}$$

$$\lim_{y\to\bar{y}} V^I(y) = 1 + \Gamma(\bar{y})$$

$$V_1^I(\bar{y}) = \Gamma_1(\bar{y}) + \underbrace{V_1^A(\bar{y})}_{=0}$$

This gives the final linear system of ten equations in ten unknowns that we must solve as a function of γ and $\bar{y}$. We now turn to the value function of a least informed agent who is monitoring. In the inaction region it will be given by[42]

$$V^A(y;\gamma) = \begin{cases} B_1 y^{\beta_2} + C_1 y^{\alpha_1} + C_2 & \text{for } y_t > \tilde{y}^\lambda \\ B_2 y^{\beta_3} + B_3 y^{\beta_4} + C_3 y^{\alpha_2} + C_4 y^{\alpha_3} + C_5 y + C_6 & \text{for } \tilde{y}^\infty < y_t \le \tilde{y}^\lambda \\ B_5 y^{\beta_5} + B_6 y^{\beta_6} + C_7 y^{\alpha_4} + C_8 y^{\alpha_5} + C_9 y + C_{10} & \text{for } y_t \le \tilde{y}^\infty \end{cases}$$

where β_2 is the negative root of $\rho = \mu\beta_2 + \frac{\sigma^2}{2}\beta_2(\beta_2-1)$, β_3 and β_4 are respectively the positive and negative roots of $\rho + \theta\nu N(\lambda,\delta) = \mu\beta_i + \frac{\sigma^2}{2}\beta_i(\beta_i-1)$ for $i=3,4$. Finally, β_5 and β_6 are

[41] I already imposed that $\lim_{y\to 0} V^I(y)$ has to be finite, eliminating the term with a negative root (i.e., $D_6 = 0$).

[42] We use the fact that $\lim_{y\to\infty} V^A(y;\gamma) = r/\rho$ as a border condition already.

those of $\rho + \theta(\nu N(\lambda, \delta) + 1 - \nu) = \mu\beta_i + \frac{\sigma^2}{2}\beta_i(\beta_i - 1)$. The constants B_i and C_i are the solution to the following system of equations:

$$C_1 = -\delta A_1(\lambda + \delta)^{-1}$$
$$C_2 = \frac{r}{\rho}$$
$$C_3 = -\delta A_2(\lambda + \delta)^{-1}$$
$$C_4 = -\delta A_3(\lambda + \delta)^{-1}$$
$$C_5 = \frac{\theta\nu N(\lambda, \delta)\alpha}{\rho + \theta\nu N(\lambda, \delta) - \mu}$$
$$C_6 = \frac{r}{\rho + \theta\nu N(\lambda, \delta)}$$
$$C_7 = -\delta A_4(\lambda + \delta)^{-1}$$
$$C_8 = -\delta A_5(\lambda + \delta)^{-1}$$
$$C_9 = \frac{\theta\left(1 - \nu + \nu N(\lambda, \delta)\right)\alpha}{\rho + \theta\left(1 - \nu + \nu N(\lambda, \delta)\right) - \mu}$$
$$C_{10} = \frac{r}{\rho + \theta\left(1 - \nu + \nu N(\lambda, \delta)\right)}$$
$$B_1(\tilde{y}^\lambda)^{\beta_2} + C_1(\tilde{y}^\lambda)^{\alpha_1} + C_2 = B_2(\tilde{y}^\lambda)^{\beta_3} + B_3(\tilde{y}^\lambda)^{\beta_4} + C_3(\tilde{y}^\lambda)^{\alpha_2} + C_4(\tilde{y}^\lambda)^{\alpha_3} + C_5\tilde{y} + C_6$$
$$B_1\beta_2(\tilde{y}^\lambda)^{\beta_2-1} + C_1\alpha_1(\tilde{y}^\lambda)^{\alpha_1-1} = B_2\beta_3(\tilde{y}^\lambda)^{\beta_3-1} + B_3\beta_4(\tilde{y}^\lambda)^{\beta_4-1} + C_3\alpha_2(\tilde{y}^\lambda)^{\alpha_2-1} + C_4\alpha_3(\tilde{y}^\lambda)^{\alpha_3-1} + C_5$$
$$B_4(\tilde{y}^\infty)^{\beta_5} + B_5(\tilde{y}^\infty)^{\beta_6} + C_7(\tilde{y}^\infty)^{\alpha_4} + C_8(\tilde{y}^\infty)^{\alpha_5} + C_9\tilde{y}^\infty + C_{10} = B_2(\tilde{y}^\infty)^{\beta_3} + B_3(\tilde{y}^\infty)^{\beta_4} + C_3(\tilde{y}^\infty)^{\alpha_2} + C_4(\tilde{y}^\infty)^{\alpha_3} + C_5\tilde{y}^I + C_6$$
$$B_4\beta_5(\tilde{y}^\infty)^{\beta_5-1} + B_5\beta_6(\tilde{y}^I)^{\beta_6-1} + C_7\alpha_4(\tilde{y}^\infty)^{\alpha_4-1} + C_8\alpha_5(\tilde{y}^\infty)^{\alpha_5-1} + C_9 = B_2\beta_3(\tilde{y}^\infty)^{\beta_3-1} + B_3\beta_4(\tilde{y}^\infty)^{\beta_4-1} + C_3\alpha_2(\tilde{y}^\infty)^{\alpha_2-1} + C_4$$

together with $\lim_{y\to\bar{y}^+} V^A(y;\gamma) = 1$ and the *smooth-pasting* condition $V_1^A(\bar{y}) = 0$. Finally, imposing symmetry so that $\bar{y} = \tilde{y}^\lambda$ we get that the last two conditions are equivalent to

$$1 = B_1(\tilde{y}^\lambda)^{\beta_2} + C_1(\tilde{y}^\lambda)^{\alpha_1} + \frac{r}{\rho}$$
$$0 = B_1\beta_2(\tilde{y}^\lambda)^{\beta_2-1} + C_1\alpha_1(\tilde{y}^\lambda)^{\alpha_1-1}$$

It remains to solve for the value of informed agents in their continuation region so that we can finally characterize the equilibrium (as long as it is true that $\alpha^{-1} \geq \tilde{y} > \tilde{y}^\infty$). The solution to the corresponding differential equation is of the form[43]

$$V^I(y) = \begin{cases} E_2 y^{\beta_2} + \frac{r}{\rho} & \text{for } \ y > \tilde{y} \\ E_3 y^{\beta_3} + E_4 y^{\beta_4} + \frac{\theta(1-\nu)N(\lambda,\delta)\alpha}{\rho+\theta\nu N(\lambda,\delta)-\mu} y + \frac{r}{\rho+\theta\nu N(\lambda,\delta)} & \text{for } \ \tilde{y}^\infty < y \leq \tilde{y} \\ E_5 y^{\beta_5} + E_6 y^{\beta_6} + \frac{\theta(1-\nu+\nu N(\lambda,\delta))\alpha}{\rho+\theta(1-\nu+\nu N(\lambda,\delta))-\mu} y + \frac{r}{\rho+\theta(1-\nu+\nu N(\lambda,\delta))} & \text{for } \ y \leq \tilde{y}^\infty \end{cases}$$

where the constants E_i and $\bar{y}^\infty$ solve[44]

$$E_2(\tilde{y}^\lambda)^{\beta_2} + \frac{r}{\rho} = E_3(\tilde{y}^\lambda)^{\beta_3} + E_4(\tilde{y}^\lambda)^{\beta_4} + \frac{\theta\nu N(\lambda, \delta)\alpha}{\rho + \theta\nu N(\lambda, \delta) - \mu}\tilde{y} + \frac{r}{\rho + \theta\nu N(\lambda, \delta)}$$
$$E_5(\tilde{y}^\infty)^{\beta_5} + E_6(\tilde{y}^\infty)^{\beta_6} + \frac{\theta\left(1 - \nu + \nu N(\lambda, \delta)\right)\alpha}{\rho + \theta\left(1 - \nu + \nu N(\lambda, \delta)\right) - \mu}\tilde{y}^\infty + \frac{r}{\rho + \theta\left(1 - \nu + \nu N(\lambda, \delta)\right)} = E_3(\tilde{y}^\infty)^{\beta_3} + E_4(\tilde{y}^\infty)^{\beta_4} + \frac{\theta\nu N(\lambda, \delta)\alpha}{\rho + \theta\nu N(\lambda, \delta) - \mu}$$
$$E_2\beta_2(\tilde{y}^\lambda)^{\beta_2-1} = E_3\beta_3(\tilde{y}^\lambda)^{\beta_3-1} + E_4\beta_4(\tilde{y}^\lambda)^{\beta_4-1} + \frac{\theta\nu N(\lambda, \delta)\alpha}{\rho + \theta(1 - \nu)N(\lambda, \delta) - \mu}$$
$$E_5\beta_5(\tilde{y}^\infty)^{\beta_5-1} + E_6\beta_6(\tilde{y}^\infty)^{\beta_6-1} + \frac{\theta\left(1 - \nu + \nu N(\lambda, \delta)\right)\alpha}{\rho + \theta\left(1 - \nu + \nu N(\lambda, \delta)\right) - \mu} = E_3\beta_3(\tilde{y}^\infty)^{\beta_3-1} + E_4\beta_4(\tilde{y}^\infty)^{\beta_4-1} + \frac{\theta\nu N(\lambda, \delta)\alpha}{\rho + \theta\nu N(\lambda, \delta) - \mu}$$
$$\lim_{y\to\bar{y}^\infty} V^I(y) = 1$$
$$V_1^I(\bar{y}^\infty) = 0$$

[43]Again, I already imposed that $\lim_{y\to\infty} V^I(y) = r/\rho$, eliminating the term with a positive root (i.e. $E_1 = 0$).

[44]This is not the best notation, here $\bar{y}^\infty$ represents the optimal boundary for an informed agent given γ.

Finally, we have fully characterized the equilibrium in this economy by imposing symmetry so that $\bar{y}^\infty = \tilde{y}^\infty$. Imposing symmetry, we can write the last two conditions as

$$\begin{aligned} 1 &= E_3(\tilde{y}^\infty)^{\beta_3} + E_4(\tilde{y}^\infty)^{\beta_4} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\theta\nu N(\lambda,\delta)-\mu}\tilde{y}^\infty + \frac{r}{\rho+\theta\nu N(\lambda,\delta)} \\ 0 &= E_3\beta_3(\tilde{y}^\infty)^{\beta_3-1} + E_4\beta_4(\tilde{y}^\infty)^{\beta_4-1} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\theta\nu N(\lambda,\delta)-\mu} \end{aligned}$$

Since the only non-linear part of the system of equations that we have to solve is concerning γ, we can solve for the equilibrium by reducing the number of equations to two using matrix notation and then solve these numerically. It turns out that this is numerically unstable, so we proceed by solving the system of equations directly, finding E_3, E_4, B_1, and A_1 as a function of γ so that we can find the equilibrium thresholds solving the following two equations:

$$\begin{aligned} 1 &= E_3(\tilde{y}^\infty)^{\beta_3} + E_4(\tilde{y}^\infty)^{\beta_4} + \frac{\theta\nu N(\lambda,\delta)\alpha}{\rho+\theta\nu N(\lambda,\delta)-\mu}\tilde{y}^\infty + \frac{r}{\rho+\theta\nu N(\lambda,\delta)} \\ 1 &= B_1(\tilde{y}^\lambda)^{\beta_2} - \frac{\delta}{\lambda+\delta}A_1(\tilde{y}^\lambda)^{\alpha_1} + \frac{r}{\rho} \end{aligned}$$

The same can be done for the cases where $\tilde{y}^\infty > 1/\alpha$ and $\alpha^{-1} \in [\tilde{y}^\infty, \tilde{y})$. Numerical results suggest that for large values of σ, the case considered here will not be an equilibrium.

B.1.1 Proofs

Proposition. *There is no asymmetric monotone equilibrium.*

Proof. To be completed ■

Proposition. *An equilibrium characterized by thresholds $\tilde{y}^\lambda > 0$ and $\tilde{y}^\infty = 0$, with $\tilde{y}^\lambda$ determined as in the main model with failure rate $\theta\nu N(\lambda,\delta)$, does not exist.*

Proof. For such an equilibrium to exist, we need

$$\begin{aligned} \frac{r}{\rho} - \frac{\delta}{\lambda+\delta}\frac{\beta_2-\alpha_1}{\beta_2}\frac{1}{\alpha_1-\iota_3}\iota_3\left(1-\frac{r+\lambda}{\rho+\lambda+\theta\nu N(\lambda,\delta)}\right) &< 1 \\ \frac{r}{\rho+\theta\nu N(\lambda,\delta)} &\geq 1 \end{aligned}$$

The first inequality is obtained in the existence proof for the main model. The steps are identical, using a failure rate of $\theta\nu N(\lambda,\delta)$. The second part is equivalent to $V^\infty(0;\{\tilde{y}^\lambda,0\}) \geq 1$. Both inequalities cannot hold simultaneously as the second term of the first one is non-negative with $r \leq \rho + \theta\nu N(\lambda,\delta)$. ■

Proposition. *If a monotone equilibrium characterized by a vector of thresholds $\gamma = \{\tilde{y}, \tilde{y}^I\} \in \mathbb{R}^2_{++}$ exists, then it is unique among the monotone equilibria.*

Proof. To be completed ■

C Appendix: adding noise to the main model

Introducing noise into this continuous time model is not an easy endeavor as computing posteriors with continuous observations is generally quite complicated. However, we can make some assumptions that allow us to use results from the optimal filtering literature (Øksendal, 2003; Liptser and Shiryayev, 1977). First, we assume that all attentive depositors receive the same signal every period. This assumption allows us to avoid having to deal with beliefs over other's beliefs, which depend on all past signals observed by them.[45] Second, we assume that depositors are updated with the information they missed when they become attentive. This, together with the assumption of signal homogeneity, helps to simplify the computation of posterior beliefs as everybody that is monitoring the banks asset's has the same information about it.

Our idea is that agents observe a noisy signal, a, of the true process y in every period. For instance, we can think that a evolves continuously as follows:

$$a_t = y_t + \sigma_\varepsilon \varepsilon_t$$

where ε_t is a one-dimensional white noise independent of the process for y, this means that uninformed agents every period observe the accurate signal with some noise that has zero mean. Note that this type of signal makes past observations crucial for these agents; they help to estimate the true value of y at t and in all subsequent periods. Intuitively, an agent knows that each observation carries some noise but that observations are correlated as paths are continuous, so that past information can help infer today's true value and in consequence that for the following periods.

In addition, we assume that agents learn the truth at an exogenous Poisson rate φ. This reflects the idea that some signals that agents receive are of higher quality, meaning that they are more precise. For example, information circulating on social media about a bank facing difficulties, whether from other users or amateur journalists, may suggest trouble, but its accuracy is uncertain. However, when a major news outlet reports on the issue, the information is perceived as much more reliable. Furthermore, even when major news companies report on developing situations, there can still be initial inaccuracies as discussed earlier. Over time, these outlets typically release more detailed and accurate updates, confirming the events as they unfold. Additionally, the mere fact that major news sources continue to cover the story tends to increase the perceived veracity of the information reported at the outset.

We now focus then on the uncertainty that attentive depositors face until they observe the true value of y again. Time t will then reflect the length of the noisy period and information at $t = 0$ is the last verified value of the bank's assets. In a clear abuse of notation we denote the latter as y_0 here. Note that observing y_t is analogous to observing W_t,[46] so we will focus on the agent observing a noisy signal of W_t instead of y_t. Then, we modify our setting slightly, assuming that the agent observes a noisy signal $a_t = W_t + \sigma_\varepsilon \varepsilon_t$, with $\sigma_\varepsilon > 0$ and ε_t being a white

[45] Note how complicated this gets when there is learning in several periods since an agent has to use all information up to time T to infer all the signals that others received previous to T to know the current distribution of beliefs.

[46] There is a one-to-one mapping between y_t and W_t given y_0 since

$$y_t = y_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

noise independent of the process for W_t. It will prove helpful then to focus on $Z_t = \int_0^t a_s ds$, which follows

$$dZ_t = W_t dt + \sigma_\varepsilon d\tilde{W}_t \quad \text{with } Z_0 = 0$$

where $\tilde{W}_t$ is a one-dimensional Brownian motion independent of the process for W. Note that observing the signals one can back down the value of Z_t and conversely we can obtain the signals out of observing Z_t each period.

Since attentive depositors are up to date with the past information available, it is as if they have observed all past signals. Then, it turns out that a transformation of Z_t will be the state variable of a depositor when it is monitoring, as it is a sufficient statistic of all past information. This result follows from Theorem 4.1 in Olofsson (2018) which states that[47]

$$W_t \mid \mathcal{F}_t \sim \mathcal{N}(m_t, \theta_t)$$

where $[m_t, \theta_t]$ is the unique pair of continuous processes that solves

$$\begin{aligned}
\frac{d\theta_t}{dt} &= 1 - \left(\frac{\theta_t}{\sigma_\varepsilon}\right)^2 \\
\theta_0 &= 0 \\
m_0 &= 0 \\
dm_t &= \frac{\theta_t}{\sigma_\varepsilon^2}\left(dZ_t - m_t dt\right)
\end{aligned}$$

where $\mathcal{F}_t$ is the $\sigma-$algebra generated by observing Z up to time t. This gives $\theta_t = \sigma_\varepsilon tanh(t/\sigma_\varepsilon)$, which is non-negative, strictly increasing with time and bounded at σ_ε. Moreover, it determines that m_t, which depends on all past values of Z_t, is a sufficient statistic to predict W_t. Note that m_t depends on a weighted average of all past signals a_t and updates upwards for signals above the prior mean. Further, more weight is given to recent observations.

This way, the only sufficient information to predict y_t and any future values is contained in m_t and the last observed value y_0. The same holds for a given individual's future values of m_t. We can now approach the problem of characterizing the value functions and corresponding equilibrium policies. The same kind of guess and verify approach will be implemented. In particular, we look for an equilibrium in monotone strategies where depositors will withdraw their deposits when given the opportunity if $m_t \leq \bar{m}_t(y_0)$.

Given a path for other's symmetric withdrawal decisions $\{\bar{m}_t\}_{t\in\mathbb{R}_+}$, at each t the HJB equation of a monitoring depositor will be given by

$$\begin{aligned}
\rho V^A(m_t, t; y_0) = r + V_2^A(m_t, t; y_0) &+ \frac{\theta_t^2}{2\sigma_\varepsilon^2} V_{11}^A(m_t, t; y_0) + \delta \left[V^I(m_t, t; y_0) - V^A(m_t, t; y_0)\right] \\
&+ \varphi \left[\mathbb{E}[V^A(0, 0; y_t) \mid m_t, y_0] - V^A(m_t, t; y_0)\right] \\
&+ \theta N(\lambda, \delta) I_{\{m_t \leq \bar{m}_t\}} \left[\mathbb{E}\left[\min\{1, \alpha y_t\} \mid m_t, y_0\right] - V^A(m_t, t; y_0)\right]
\end{aligned}$$

where the term $W_t - m_t$ of the drift disappears as it is expected to be zero conditional on m_t. Compared to the main model this differential equation differs in the presence of uncertainty from the payment under failure, which is reflected both in the term related to the failure rate and in the drift that reflects the expected evolution of beliefs, and in the term that reflects that uncertainty may be resolved at a frequency φ.

[47] The authors refer to Liptser and Shiryayev (1977) which proves it for a more general case.

Let $\mu_t(m_t) = \log(\alpha y_0) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma m_t$, then

$$\log(\alpha y_t) \mid_{m_t} \sim \mathcal{N}\left(\mu_t(m_t), \sigma^2\theta_t\right)$$

so that

$$\mathbb{E}_{m_t}\left[\min\{1, \alpha y_t\}\right] = \Phi\left(\frac{\mu_t(m_t)}{\sigma\sqrt{\theta_t}}\right) + \Phi\left(-\frac{\mu_t(m_t) + \sigma^2\theta_t}{\sigma\sqrt{\theta_t}}\right) e^{\mu_t(m_t) + \frac{\sigma^2\theta_t}{2}}$$

Note that this expectation in the last term of the differential equation eliminates the kink that we had earlier, as the agent will not be able to determine whether $\alpha \cdot y > 1$ or not. However, it makes solving for V^A quite complicated due to the high non-linearities. To find an equilibrium we would have to solve for the corresponding differential equations with appropriate boundary conditions, which include *value-matching* and *smooth-pasting*.