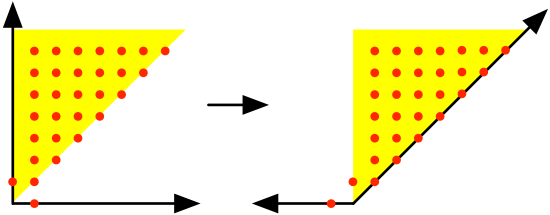
Swampland and Geometry in 5d

Ben Heidenreich
(UMass Amherst)



Alim, BH, Rudelius, 2108.08309
Gendler, BH, McAllister, Moritz, Rudelius, 2212.10573
BH, Rudelius, 2304.xxxxx

String and Geometry 2023 — Mar. 7, 2023

The tower / sublattice WGC

Arkani-Hamed, Motl, Nicolis, Vafa '06
BH, Reece, Rudelius '15, '16, 17', '19
Andriolo, Junghans, Noumi, Shiu '18

"For every U(1) gauge field, there exists a **superextremal** charged particle, i.e., one with charge-to-mass ratio:

$$\left|\frac{\vec{Q}}{m}\right| \geq \left|\frac{\vec{Q}}{m}\right|_{\text{ext BH}}$$"

(In this talk, superextremal **never** means self-repulsive)

The tower / sublattice WGC

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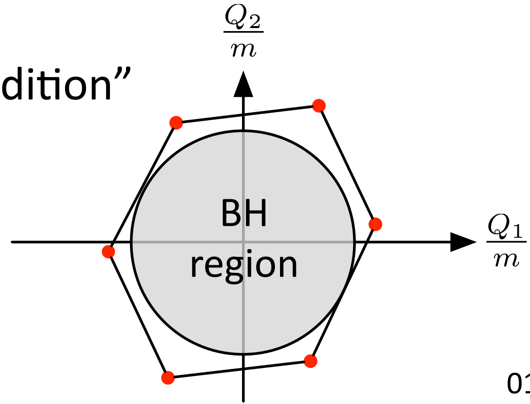
Andriolo, Junghans, Noumi, Shiu ‘18

For every $Q \in \Gamma_Q$, $\exists\, k \in \mathbb{N}$ s.t. there is a superext. multiparticle state of charge kQ

“Convex Hull Condition”

Cheung, Remmen ‘14



The tower / sublattice WGC

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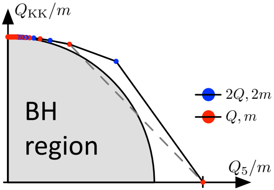
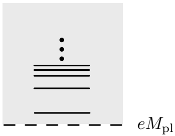
For every $Q \in \Gamma_Q$, $\exists\, k \in \mathbb{N}$ s.t. there is a superext. **single-particle** state of charge kQ



- Required to preserve WGC upon compactification
- Related to the emergence of weak (gauge) coupling at low energies

Harlow '15, BH, Reece Rudelius '17, '18, Grimm, Palti, Valenzuela '18



- Plays nicely with the Distance Conjecture

Oguri, Vafa '06

The tower / sublattice WGC

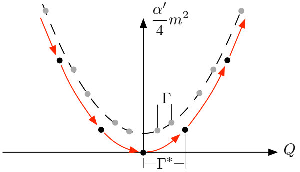
Arkani-Hamed, Motl, Nicolis, Vafa '06
BH, Reece, Rudelius '15, '16, '17, '19
Andriolo, Junghans, Noumi, Shiu '18

There exists $k \in \mathbb{N}$ s.t. $\forall Q \in \Gamma_Q$, there is a superext. single-particle state of charge kQ

- A **theorem** in elec. NSNS sector of tree-level ST

Sketch: Arkani-Hamed, Motl Nicolis, Vafa '06 → (Modular invariance) → BH, Reece Rudelius '16 Montero, Shiu, Soler '16 → (Superextremality) → BH, Lotito, 23xx.xxxxx (2x)

- **coarseness** *k* "never too large" (known *k>1* exs are all orbifolds)



- **Strongest** form of WGC without known counterexs (in *d*≥5; *renormalized* version ok in 4d)

The tower / sublattice WGC

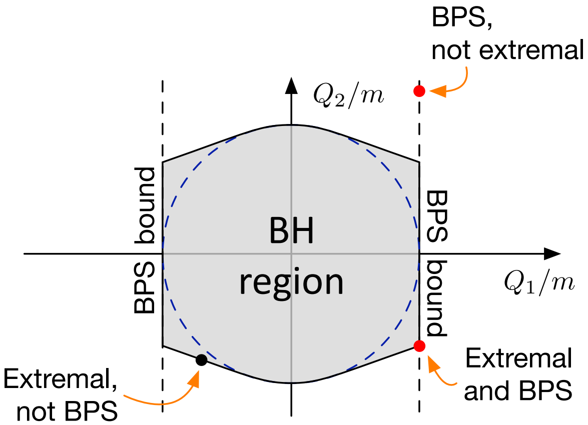
Arkani-Hamed, Motl, Nicolis, Vafa '06
BH, Reece, Rudelius '15, '16, '17, '19
Andriolo, Junghans, Noumi, Shiu '18

Most existing evidence is **perturabative**

Are these strong forms "just" special properties of weak coupling limits?

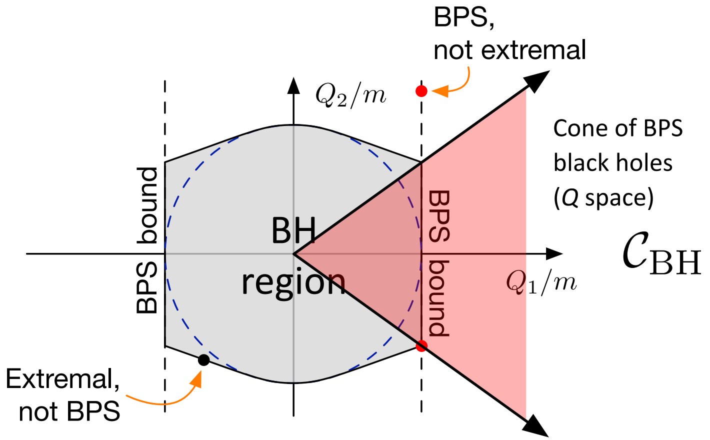
I'll argue that they **are not**.

The T/sLWGC for BPS particles



Infinite towers of BPS particles **required** in *Q* directions where BPS = Extremal

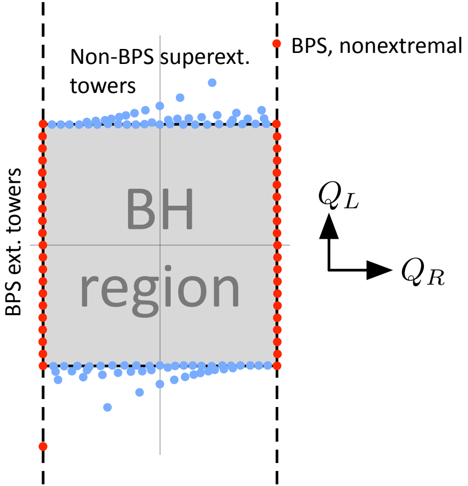
The T/sLWGC for BPS particles



Infinite towers of BPS particles **required** in *Q* directions where BPS = Extremal

The T/sLWGC for BPS particles

Het ST on S^1:

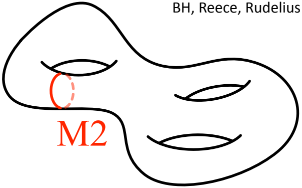


Infinite towers of BPS particles **not required** in *Q* directions where BPS ≠ Extremal (See Timo's talk)

T/sLWGC is linked to geometry

BH, Reece, Rudelius '15

M theory on CY3
↳ 5d *N=1* SUGRA EFT



M2s on hol. curves ⟶ BPS particles
counted by Gopakumar-Vafa invariants

Goal: determine $\mathcal{C}_{\mathrm{BH}}$, compare with GVs
strenuous nonpert/geometric test
of T/sLWGC

5d *N=1* SUGRA

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\mathfrak{g}_{ij}(\phi)\partial\phi^i\cdot\partial\phi^j\right) - \frac{1}{2g_5^2}\int a_{IJ}(\phi)F^I\wedge\star F^J$$
$$+\frac{1}{6(2\pi)^2}\int C_{IJK}A^I\wedge F^J\wedge F^K,$$

$$g_5^2 = (2\pi)^{4/3}(2\kappa_5^2)^{1/3}$$
$$I = 0, 1, \ldots, n$$
$$i = 1, \ldots, n$$

$$\mathcal{F}[Y] = \frac{1}{6}C_{IJK}Y^IY^JY^K$$

CY3 intersection #s

Exact away from phase transitions

5d *N=1* SUGRA

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\mathfrak{g}_{ij}(\phi)\partial\phi^i\cdot\partial\phi^j\right) - \frac{1}{2g_5^2}\int a_{IJ}(\phi)F^I\wedge\star F^J$$
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CY3 intersection #s

$$\mathcal{F}[Y(\phi)] = 1 \qquad \mathfrak{g}_{ij}(\phi) = a_{IJ}(\phi)\partial_iY^I\partial_jY^J$$
$$a_{IJ} = \mathcal{F}_{IJ} - \mathcal{F}_I\mathcal{F}_J \qquad C_{IJK} = \mathcal{F}_{IJK}$$

$$\mathcal{F}_I = \mathcal{F}_{,I}, \quad \mathcal{F}_{IJ} = \mathcal{F}_{,IJ}, \quad \ldots$$

5d *N=1* SUGRA

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\mathfrak{g}_{ij}(\phi)\partial\phi^i\cdot\partial\phi^j\right) - \frac{1}{2g_5^2}\int a_{IJ}(\phi)F^I\wedge\star F^J$$
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$$g_5^2 = (2\pi)^{4/3}(2\kappa_5^2)^{1/3}$$
$$I = 0, 1, \ldots, n$$
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BPS particle bound

$$m(\phi) \geq \frac{g_5}{\sqrt{2}\kappa_5}|\zeta(\phi)| = \frac{g_5}{\sqrt{2}\kappa_5}|q_I Y^I(\phi)|$$

BPS string bound ($\tilde{g}_5 = 2\pi/g_5$)

$$T(\phi) \geq \frac{\tilde{g}_5}{\sqrt{2}\kappa_5}|\tilde{\zeta}(\phi)| = \frac{\tilde{g}_5}{\sqrt{2}\kappa_5}|\tilde{q}^I\mathcal{F}_I(\phi)|$$

5d *N=1* SUGRA

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\mathfrak{g}_{ij}(\phi)\partial\phi^i\cdot\partial\phi^j\right) - \frac{1}{2g_5^2}\int a_{IJ}(\phi)F^I\wedge\star F^J$$
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$$g_5^2 = (2\pi)^{4/3}(2\kappa_5^2)^{1/3}$$
$$I = 0, 1, \ldots, n$$
$$i = 1, \ldots, n$$

Rather than taking a slice $\mathcal{F}[Y] = 1$,
can use “homogeneous” (projective) coords:

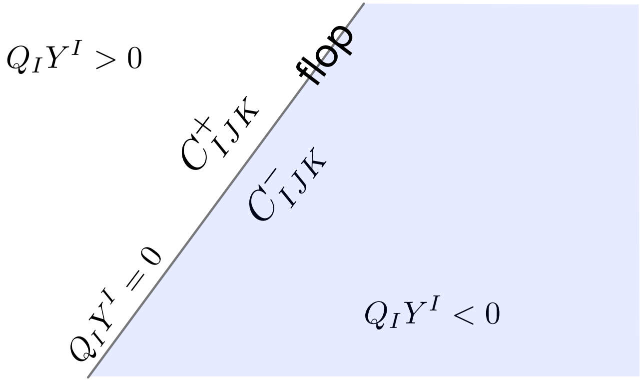
$$Y^I \cong \lambda Y^I$$

(overall volume lives in a hypermultiplet)

Phase trans: Flops and Weyl flops

N charge Q_I hypers become massless

$$C^+_{IJK} = C^-_{IJK} + N Q_I Q_J Q_K$$ "Flop"



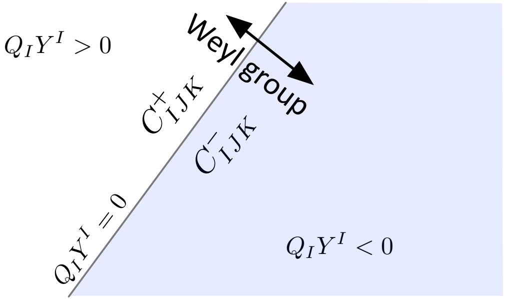
$\mathcal{K} \equiv$ Kähler cone extended via flops

Phase trans: Flops and Weyl flops

N charge Q_I ***vectors*** become massless

$$C^+_{IJK} = C^-_{IJK} + NQ_IQ_JQ_K$$ "Weyl Flop"

Corresponds to nonabelian enhancement, e.g., to, *su(2)* ⟶ gauge redundant



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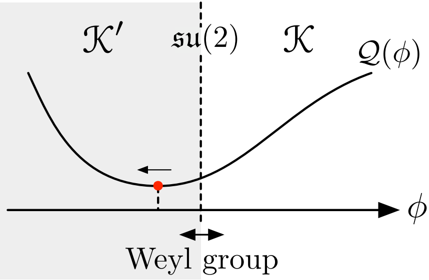
…but attractor point can hide in the Weyl group image!



How to find $m_{\text{ext}}(Q)$? (assume spherical symmetry)

see, e.g., Harlow, BH, Reece, Rudelius '22 for review

Find all solutions to PDE

$$\mathcal{Q}^2(\phi) = \mathfrak{g}^{ij} W_{,i}(\phi) W_{,j}(\phi) + \frac{1}{3} W(\phi)^2$$

such that gradient flow

$$\frac{d\phi^i}{d\tau} = -\mathfrak{g}^{ij} W_{,j}$$

starting at $\phi^i = \phi^i_\infty$ satisfies

$$W(\phi) > 0 \quad \forall \quad \tau > 0$$

Then: $$\boxed{m_{\text{ext}}(Q) = \frac{g_5}{\sqrt{2}\kappa_5} \inf_{\text{“good” } W(\phi)} W(\phi_\infty)}$$

How to find $m_{\text{ext}}(Q)$? (assume spherical symmetry)

see, e.g., Harlow, BH, Reece, Rudelius '22 for review

BPS case: $W(\phi) = Q_I Y^I / \mathcal{F}^{1/3}$

Closed-form solution to gradient flow:

$$T_I = T_I^{\infty} + z \frac{Q_I}{\mathcal{F}^{1/3}}$$

in terms of "dual coords" $T_I = \mathcal{F}_I / \mathcal{F}$

How to find $m_{\rm ext}(Q)$? (assume spherical symmetry)

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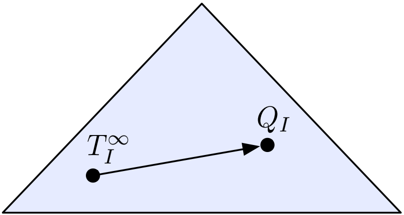
BPS black holes exist when flow satisfies

$$Q_I Y^I > 0 \quad \forall \quad z > 0$$

Note: dual coordinate map $\mathscr{T} : Y^I \to T_I$ is invertible as a consequence of convexity of extended Kähler cone $\mathcal{K}$, positivity of a_{IJ}

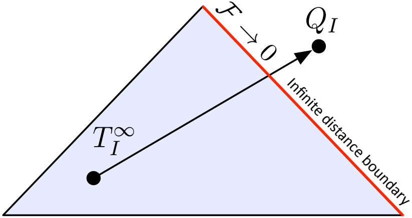
The good, the bad, ...and the indeterminate

$$T_I = T_I^\infty + z\frac{Q_I}{\mathcal{F}^{1/3}}$$



$\mathcal{T} = \mathscr{T}(\mathcal{K})$
cone of dual coordinates

"**Good** flow"
BPS black hole

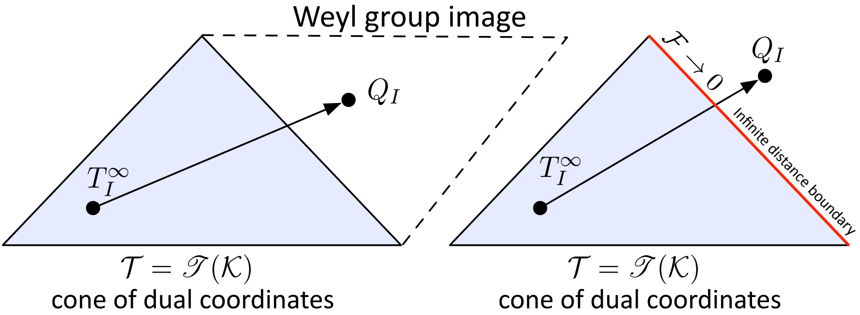


$\mathcal{T} = \mathscr{T}(\mathcal{K})$
cone of dual coordinates

"**Bad** flow"
No BPS black hole

The good, the bad, …and the indeterminate

$$T_I = T_I^\infty + z\frac{Q_I}{\mathcal{F}^{1/3}}$$

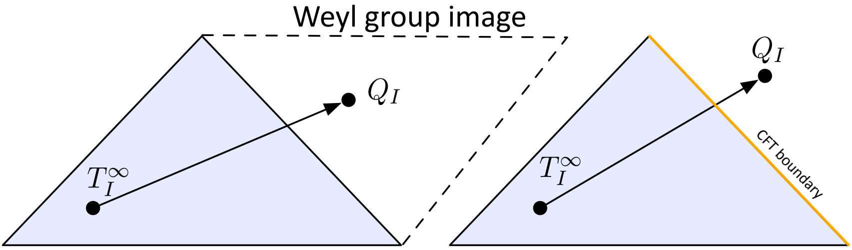


Also a **good** flow

"**Bad** flow"

The good, the bad, …and the indeterminate

$$T_I = T_I^{\infty} + z\frac{Q_I}{\mathcal{F}^{1/3}}$$



$\mathcal{T} = \mathscr{T}(\mathcal{K})$
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cone of dual coordinates

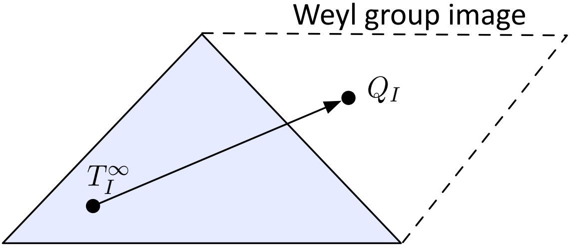
Also a **good** flow

Indeterminate flow

(need to understand BHs
in CFT coupled to gravity)

The good, the bad, …and the indeterminate

$$T_I = T_I^\infty + z\frac{Q_I}{\mathcal{F}^{1/3}}$$



$\mathcal{T} = \mathscr{T}(\mathcal{K})$
cone of dual coordinates

<u>At least:</u>

$\mathcal{C}_{\mathrm{BH}} \supseteq \mathrm{Vis}(\mathcal{T})$

Weyl-extended region "visible" from $\mathcal{T}$

Moduli space reconstruction

Gendler, BH, McAllister, Moritz, Rudelius '22

For a given CY3, map out **all possible flops** to construct $\mathcal{K}$, $\mathcal{T} = \mathscr{T}(\mathcal{K})$

…then find Weyl boundaries, determine $\mathrm{Vis}(\mathcal{T})$

Moduli space reconstruction

Gendler, BH, McAllister, Moritz, Rudelius '22

For a given CY3, map out **all possible flops** to construct $\mathcal{K}$, $\mathcal{T} = \mathscr{T}(\mathcal{K})$

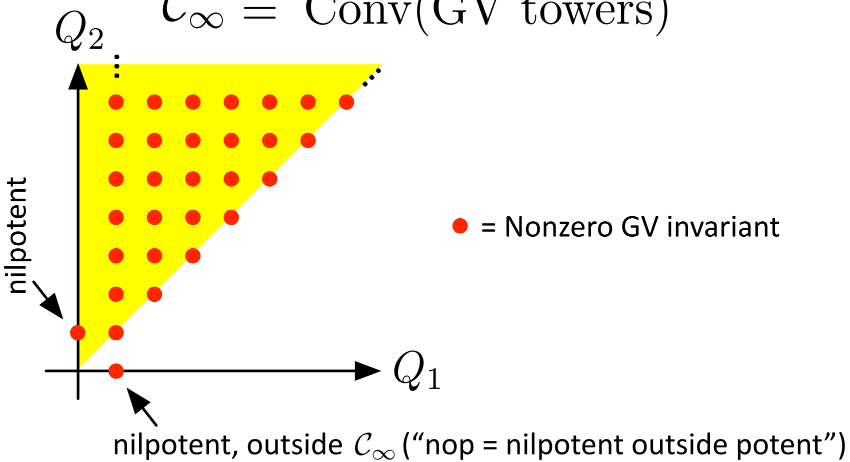
...then find Weyl boundaries, determine $\mathrm{Vis}(\mathcal{T})$

Non-trivial problem when we wish to repeat for a large number of CY3s!

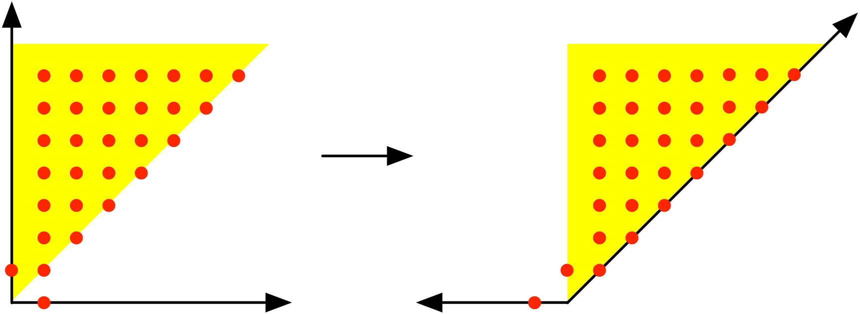
We'll reconstruct everything using only prepotential / genus 0 GV invs of a single phase.

Infinity cone & nilpotent curves

$$\mathcal{C}_\infty \equiv \overline{\mathrm{Conv(GV\ towers)}}$$



Flopping a nop curve



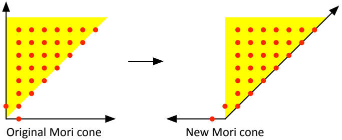
Original Mori cone New Mori cone

We simply replace $Q_I \to -Q_I$ for nop curve (leaving GV inv alone)

New Mori cone = Span(non-zero GVs)*

*With caveat to be discussed

Flopping a nop curve



$\mathrm{GV}_0 = N_H - N_V$ (# hypers – # vectors)

$\Downarrow$

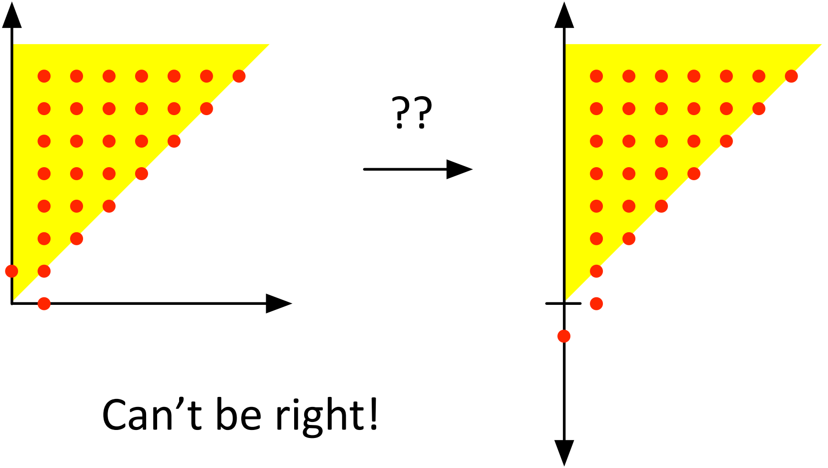
Can't always distinguish Weyl flop ($N_V > 0$) from std flop ($N_V = 0$)

But $C'_{IJK} = C_{IJK} - (N_H - N_V) Q_I Q_J Q_K$ can still be tracked

In fact, "$\mathcal{N} = 4$" flops with $N_H = N_V$ go completely unnoticed!

…so we might misidentify the Mori cone (in a harmless way)

Flopping a nilpotent curve in $\mathcal{C}_\infty$

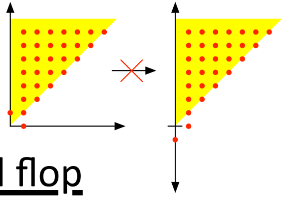


Can't be right!

Flopping a nilpotent curve in $\mathcal{C}_\infty$

Wall crossing must occur!

Only possible if there's a tensionless string: must be Weyl flop ('t Hooft-Polyakov monopole string)



Alim, BH, Rudelius '21

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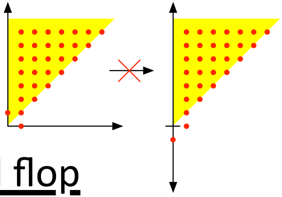
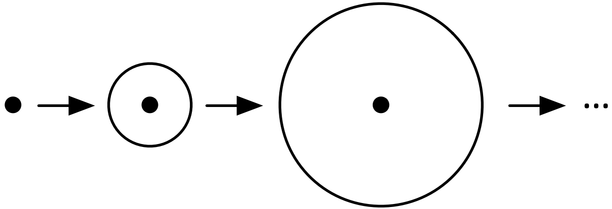


Alim, BH, Rudelius '21

Maybe something like:



string carrying diffuse elec charge

Flopping a nilpotent curve in $\mathcal{C}_\infty$

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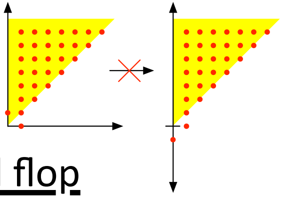

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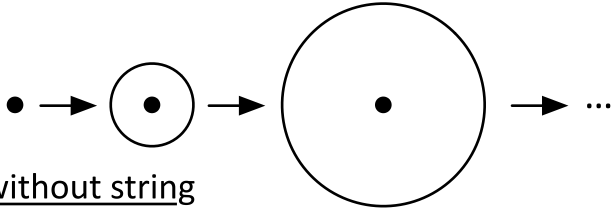


string carrying diffuse elec charge

Can't happen without string because 4d wall crossing always involves magnetic charge

Stable/unstable Weyl flops

Weyl flops associated to nop curves are “stable”
No wall crossing in the GV invariants

Nipotent gens of $\mathcal{C}_\infty$ give “unstable” Weyl flops.
Wall crossing **occurs** in the GV invariants

$$\mathcal{K}_{\rm hyp} \equiv \bigcup_{w\in\mathcal{W}_{\rm stable}} w(\mathcal{K})$$

“Hyperextended Kähler cone”

No GV wall crossing within $\mathcal{K}_{\rm hyp}$

Relating $\mathcal{K}_{\rm hyp}$ with $\mathcal{C}_\infty$

By construction, at every boundary of $\mathcal{K}_{\rm hyp}$,
either a nilpotent curve in $\mathcal{C}_\infty$ flops
or an inf. tower of BPS particles become massless
(clearly lying within $\mathcal{C}_\infty$ as well.)

(In the latter case, the boundary is either
(1) at infinite distance or (2) a CFT boundary.)

Neither can occur in the interior of $\mathcal{K}_{\rm hyp}$

Therefore: $\boxed{\mathcal{K}_{\rm hyp} = \mathcal{C}_\infty^\vee}$

Relating $\mathcal{K}_{\mathrm{hyp}}$ with $\mathcal{C}_\infty$

$$\boxed{\mathcal{K}_{\mathrm{hyp}} = \mathcal{C}_\infty^\vee}$$

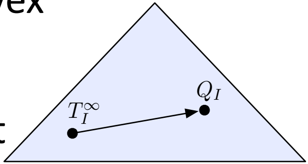
This implies that $\mathcal{K}_{\mathrm{hyp}}$ has nice properties analogous to $\mathcal{K}$, e.g., it is **convex**

These nice properties don't persist when we "overextend" $\mathcal{K}$ via unstable Weyl flops

The cone of BPS black holes

Whether $\mathcal{C}_{\rm BH}$ includes all of $\mathcal{T}_{\rm hyp} = \mathscr{T}(\mathcal{K}_{\rm hyp})$ depends on whether $\mathcal{T}_{\rm hyp}$ is convex

Can show that $\mathcal{T}$ convex
so is $\mathcal{T}_{\rm hyp}$ in exs, but no proof yet



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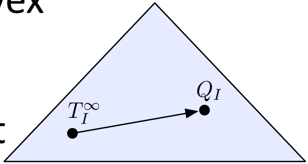

Regardless, b/c GVs don't wall-cross within $\mathcal{K}_{\rm hyp}$ we predict infinite GV towers everywhere within

$$\widehat{\mathcal{C}}_{\rm BH} \equiv \bigcup_{t \in \mathcal{T}_{\rm hyp}} \mathcal{C}_{\rm BH}(t)$$

which includes $\mathcal{T}_{\rm hyp}$ because $t \in \mathcal{C}_{\rm BH}(t)$

The cone of BPS black holes

We predict infinite GV towers everywhere within

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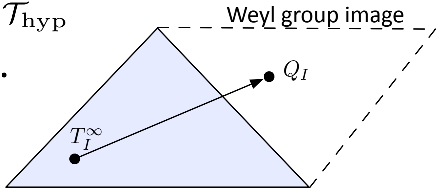
which includes $\mathcal{T}_{\rm hyp}$.



In fact, this includes a (possibly) bigger region:

$$\mathrm{Vis}(\mathcal{T}_{\rm hyp}) \equiv \bigcup_{t \in \mathcal{T}_{\rm hyp}} \mathrm{Vis}(t)$$

Checking the (sub)lattice WGC

For each CY3 in our search, we should

1. Find $\mathcal{C}_\infty$, hence $\mathcal{K}_{\rm hyp} = \mathcal{C}_\infty^\vee$

2. Determine C_{IJK} for each phase therein by flopping curves

3. Compute $\mathcal{T}_{\rm hyp} = \mathscr{T}(\mathcal{K}_{\rm hyp})$ and/or $\overbrace{\mathrm{Vis}(\mathcal{T}_{\rm hyp})}^{\text{Not yet automated!}}$

4. Check whether any GVs vanish within $\mathcal{T}_{\rm hyp}$ and/or $\mathrm{Vis}(\mathcal{T}_{\rm hyp})$, up to a specified cutoff deg.

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(If **none** vanish, then the **lattice** WGC is satisfied; in all our examples so far, it is!)

Symmetric flops

Sometimes flopped phase isomorphic to original, e.g., via a reflection

$$Y^I \to Y^I - 2\frac{Q_J Y^J}{\tilde{Q}^K Q_K}\tilde{Q}^I$$

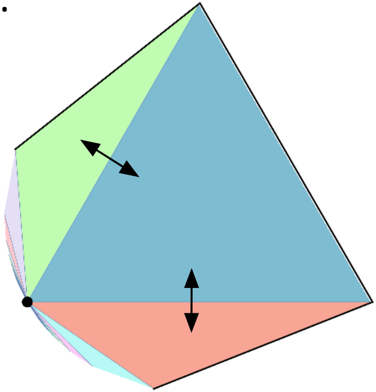
where “symmetric” flop lies at $Q_I Y^I = \tilde{Q}^I T_I = 0$

e.g., Weyl flops are **always** symmetric

We can improve the **efficiency** of our search by restricting to fund. domain $\mathcal{F}_G$ for these symms *G*

Symmetric flops

For example:



Base figure: N. Gendler

Symmetric flops

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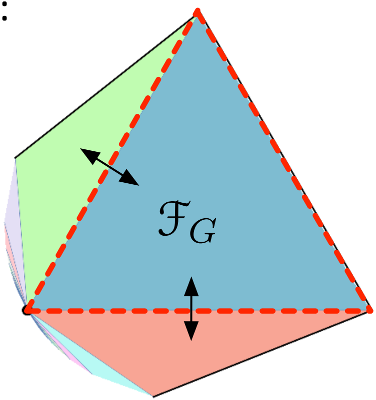

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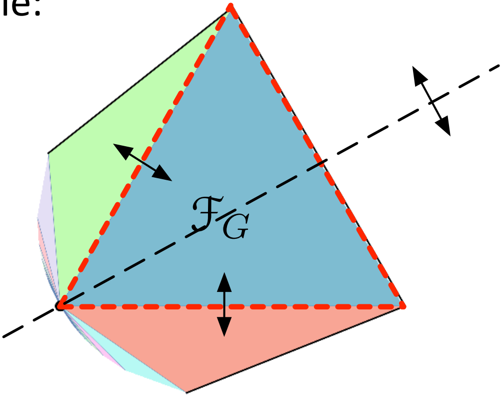

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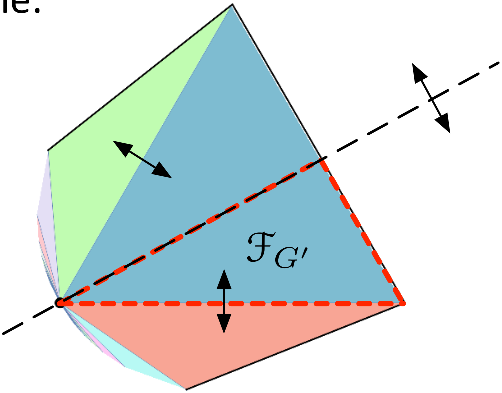

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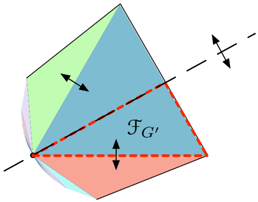


Base figure: N. Gendler

As this example illustrates can easily get **infinitely** many phases when multiple such reflection symmetries (not commuting) are present

Brodie, Constantin, Lukas, Ruehle '21

So we really **need** to restrict to the fundamental domain to make problem computable

Some examples

CY hypersurface in toric variety:

x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	-1	1	1
1	1	1	2	-1	0

$q_1 \backslash q_2$	0	1	2	3	4	5
0	*	56	-272	3240	-58432	1303840
1	20	2635	2760	-45440	1001340	-26330880
2	0	5040	541930	933760	-18770880	490600080
3	0	190	2973660	277421695	563282580	-11813767700
4	0	-40	2454600	2644224240	208000930200	470459159880
5	0	3	67980	5829698942	2855250958116	193028959075965
6	0	0	-14960	3084577280	11119027471400	3465883673329200
7	0	0	3420	75341270	14592676836440	19950547779012810
8	0	0	-760	-13884400	5711374027440	45586693863580200
9	0	0	100	2767590	132960571500	42020108300555745
10	0	0	-6	-783664	-21741657848	13122339863069280

$\mathcal{T}_{hyp}$

$\mathcal{C}_\infty$

nop curve flops to non-toric phase

Some examples

CY hypersurface in toric variety:

$$\left[\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \hline 0 & 1 & 1 & 0 & -2 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

q_1 \ q_2	0	1	2	3	4	5	6	7	8
0	*	177	177	186	177	177	186	177	177
1	−2	178	20291	317172	2998628	21195310	123413576	622393836	2806637500
2	0	3	−177	332040	73458379	3048964748	67638465983	1034258133329	12232084778113
3	0	5	−708	44790	794368	3122149716	710345698242	46445530268176	1663087069097865
4	0	7	−1068	75225	−4468169	243105088	54329854510	46884487081241	10524250865224651
5	0	9	−1448	110271	−7157586	396368217	−27580928924	2382035587157	1540781601550297
6	0	11	−1880	157734	−11253268	676476353	−48092153649	2530899579921	−241894701950815
7	0	13	−2412	231979	−18701330	1241479305	−87415077360	4793679740747	−439028227820944
8	0	15	−3122	356005	−32878062	2432078638	−172868371620	10041154974639	−797065258455869

$\mathcal{T}_{\text{hyp}}$

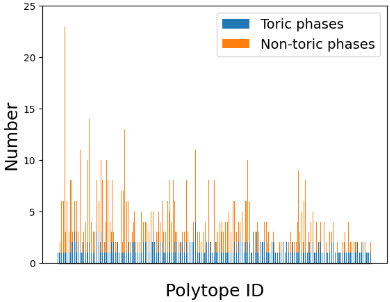
$\text{Vis}(\mathcal{T}_{\text{hyp}})$

$\mathcal{C}_\infty$ is entire Mori cone

Unstable Weyl flop

Results of scan

Looked at every Kreuzer-Skarke CY3 w/ $h^{1,1} \leq 4$
plus certain favorable ones w/ $h^{1,1} = 5$



(c) $h^{1,1} = 4$

Our algorithm reconstructs **many** non-toric phases (an increasing fraction of all phases for larger $h^{1,1}$)

From our 2062 seed geometries, found **no** counterexamples to the **lattice** WGC!

Summary and Future Dirs

We found compelling evidence for the **lattice** WGC for BPS particles in our data set

Surprising that we found no counterexs to lattice WGC, given that orbifold counterexs are **known to exist** BH, Reece, Rudelius, '16

Perhaps a hint of an underlying principle??

Stay tuned for more on geometry & swampland!

e.g., BH, Rudelius, 2304.xxxxx on WGC for BPS **strings**