

# Generalized Symmetries and Frozen Singularities

Ling Lin

University of Oxford



Mathematical  
Institute

Based on

- 2203.03644 with M. Cvetič, M. Dierigl, H.Y. Zhang
- w.i.p. with M. Cvetič, M. Dierigl, E. Torres, H.Y. Zhang

**Strings & Geometry '23**  
**Philadelphia, Mar 08, 2023**

# Motivation

# Motivation

- Two main themes of this workshop: Swampland and Generalized Symmetries; intersection: geometric engineering / string compactification.

# Motivation

- Two main themes of this workshop: Swampland and Generalized Symmetries; intersection: geometric engineering / string compactification.
- One (of many, cf Michele's talk) uncharted territories: “frozen” singularities.

# Motivation

- Two main themes of this workshop: Swampland and Generalized Symmetries; intersection: geometric engineering / string compactification.
- One (of many, cf Michele's talk) uncharted territories: “frozen” singularities.
- Involve data beyond “pure geometry”.

# Motivation

- Two main themes of this workshop: Swampland and Generalized Symmetries; intersection: geometric engineering / string compactification.
- One (of many, cf Michele's talk) uncharted territories: “frozen” singularities.
- Involve data beyond “pure geometry”.
- More challenging to understand
  - resulting landscape of effective field theories (coupled to gravity);
  - spectrum of extended operators and their interplay.

# Motivation

- Two main themes of this workshop: Swampland and Generalized Symmetries; intersection: geometric engineering / string compactification.
- One (of many, cf Michele's talk) uncharted territories: “frozen” singularities.
- Involve data beyond “pure geometry”.
- More challenging to understand
  - resulting landscape of effective field theories (coupled to gravity);
  - spectrum of extended operators and their interplay.
- To get started: more SUSY, less “compact” dimensions.

# Plan of the talk

1. Frozen singularities in 8d F-theory (= O7+) via junctions
2. Frozen singularities in 7d M-theory via junctions
3. Frozen singularities in 6d IIA and topological operators



# Frozen singularities in 8d F-theory

# Frozen singularities in 8d F-theory

- Perturbative description:  $O7_+$ ; only type of frozen singularities [\[Tachikawa '15\]](#), give  $\mathfrak{sp}_n$  (beyond ADE) in 8d.

# Frozen singularities in 8d F-theory

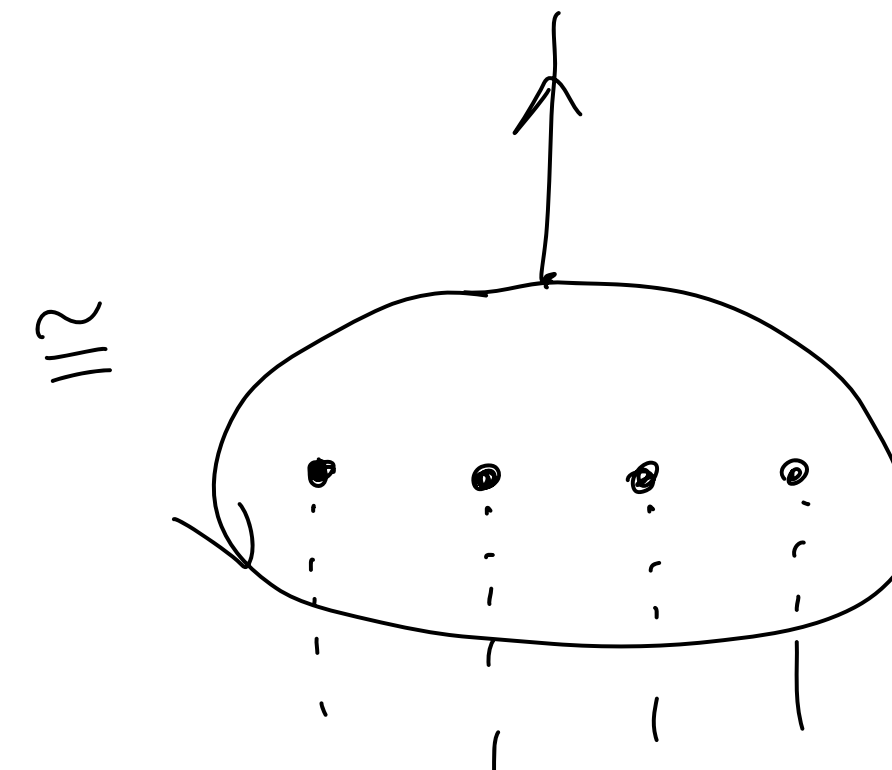
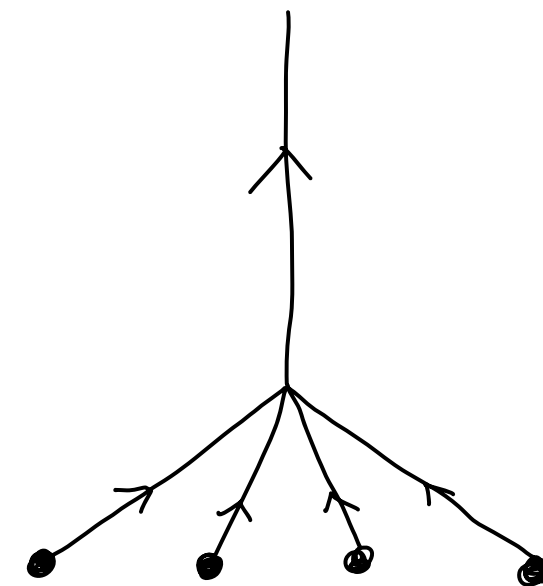
- Perturbative description:  $O7_+$ ; only type of frozen singularities [\[Tachikawa '15\]](#), give  $\mathfrak{sp}_n$  (beyond ADE) in 8d.
- Global 8d models: heterotic dual amenable to lattice embedding techniques [\[Mikhailov '98, Font/Fraiman/Graña/Núñez/Parra de Freitas '21, Cvetič/Dierigl/LL/Zhang '21 \(x2\)\]](#).

# Frozen singularities in 8d F-theory

- Perturbative description:  $O7_+$ ; only type of frozen singularities [\[Tachikawa '15\]](#), give  $\mathfrak{sp}_n$  (beyond ADE) in 8d.
- Global 8d models: heterotic dual amenable to lattice embedding techniques [\[Mikhailov '98, Font/Fraiman/Graña/Núñez/Parra de Freitas '21, Cvetič/Dierigl/LL/Zhang '21 \(x2\)\]](#).
- Locally: string junctions encode gauge dynamics and center symmetries, well-known for  $[p,q]$ -7-branes, agrees with geometry under M-/F-duality [\[Cvetič/Dierigl/LL/Zhang '21, Hüber/Morrison/Schäfer-Nameki/Wang '22\]](#)



vs

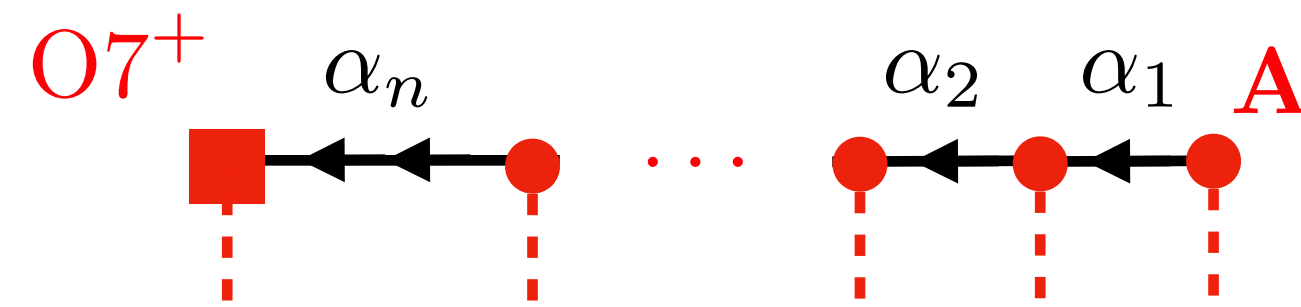


# String junctions on $O7+$

- $O7+$  has same monodromy as an  $\mathfrak{so}_{16}$ -stack, but no root junctions [\[Witten '97\]](#).

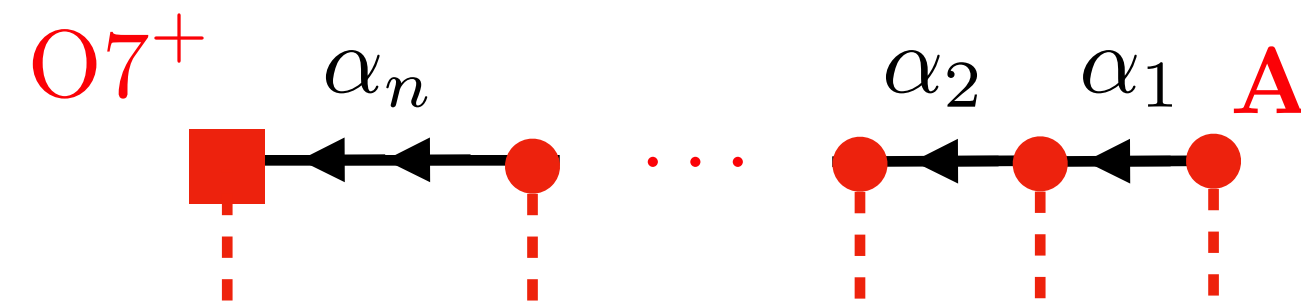
# String junctions on $O7^+$

- $O7^+$  has same monodromy as an  $\mathfrak{so}_{16}$ -stack, but no root junctions [Witten '97].
- Additionally: only  $(p,q)$ -strings with *even*  $p$  and  $q$  can end on it [Imamura '99, Bergman/Gimon/Sugimoto '01], necessary to obtain long root for  $\mathfrak{sp}_n$ .



# String junctions on $O7+$

- $O7+$  has same monodromy as an  $\mathfrak{so}_{16}$ -stack, but no root junctions [Witten '97].
- Additionally: only  $(p,q)$ -strings with *even*  $p$  and  $q$  can end on it [Imamura '99, Bergman/Gimon/Sugimoto '01], necessary to obtain long root for  $\mathfrak{sp}_n$ .



- From extended junctions (with asymptotic  $(p,q)$ -charge), also obtain  $\mathbb{Z}_2$  1-form and 5-form symmetries.

# Classification of 8d/9d N=1 SUGRA

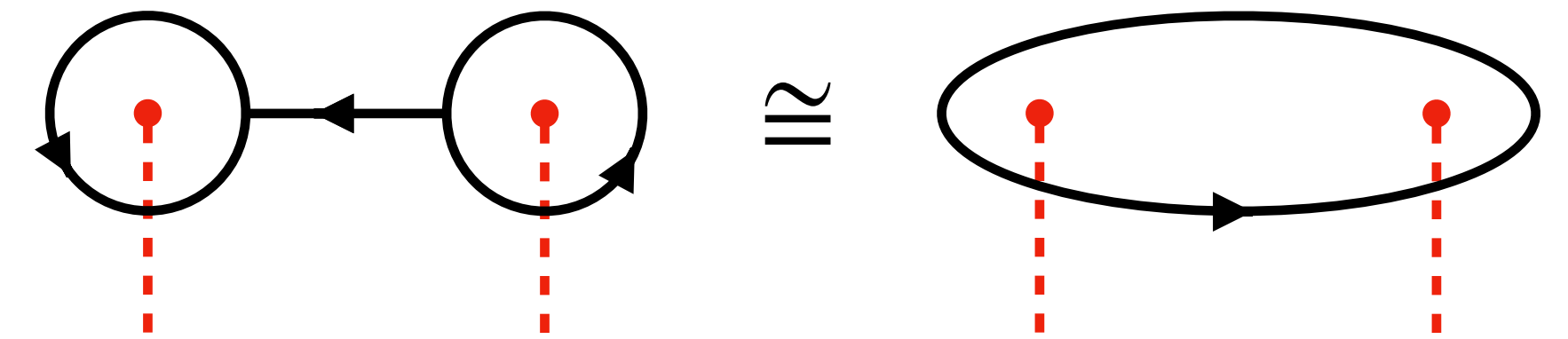
- Global models by “gluing” (w/ trivial total monodromy, no asymp. (p,q)-charges).



# Classification of 8d/9d N=1 SUGRA

- Global models by “gluing” (w/ trivial total monodromy, no asymp. (p,q)-charges).

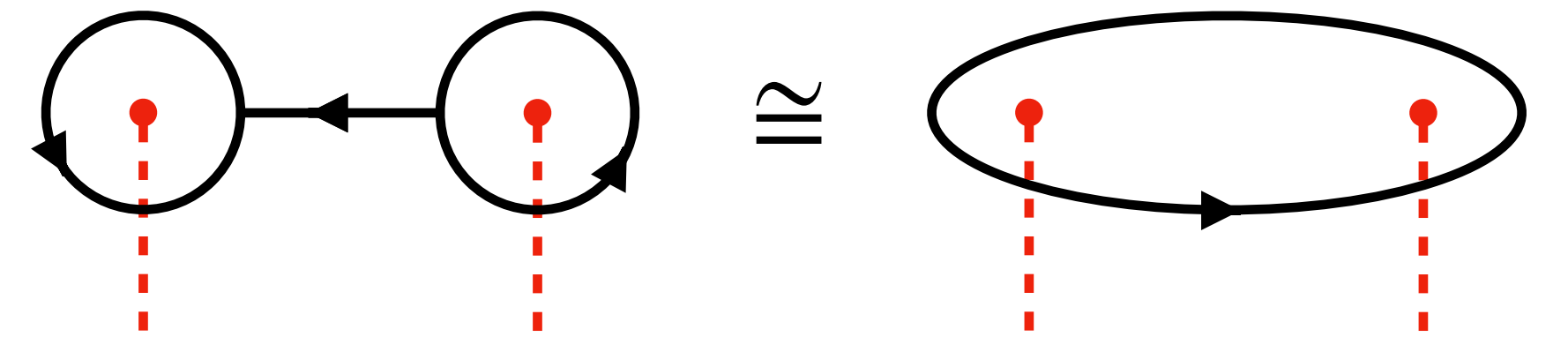
- Upshot: gauge *group* from “global null junctions”.



# Classification of 8d/9d N=1 SUGRA

- Global models by “gluing” (w/ trivial total monodromy, no asymp. (p,q)-charges).

- Upshot: gauge *group* from “global null junctions”.

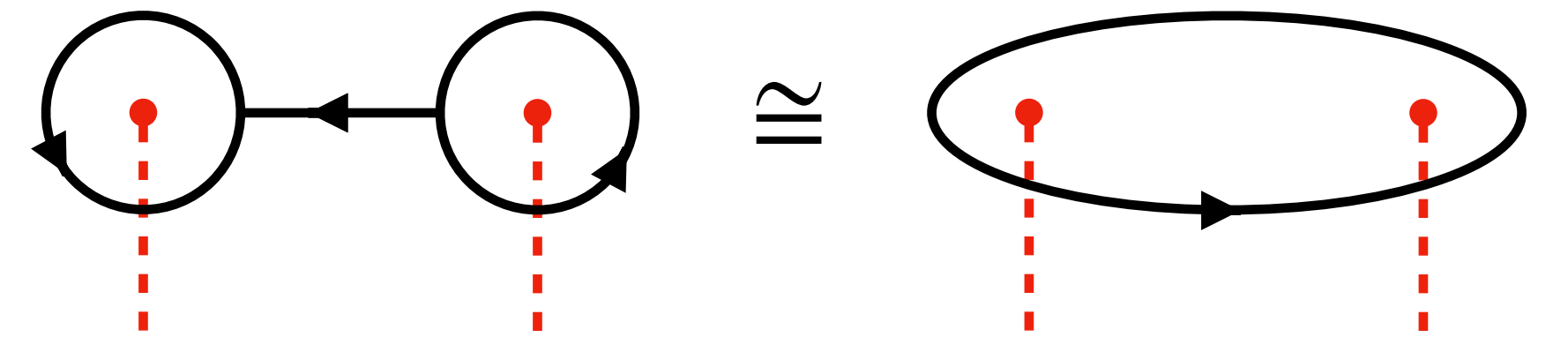


- Facilitates classification of 8d  $\mathcal{N} = 1$  supergravity, including (a posteriori) new branch of moduli space [(Montero/Parra De Freitas '22; cf. Miguel's talk)].

# Classification of 8d/9d N=1 SUGRA

- Global models by “gluing” (w/ trivial total monodromy, no asymp. (p,q)-charges).

- Upshot: gauge *group* from “global null junctions”.



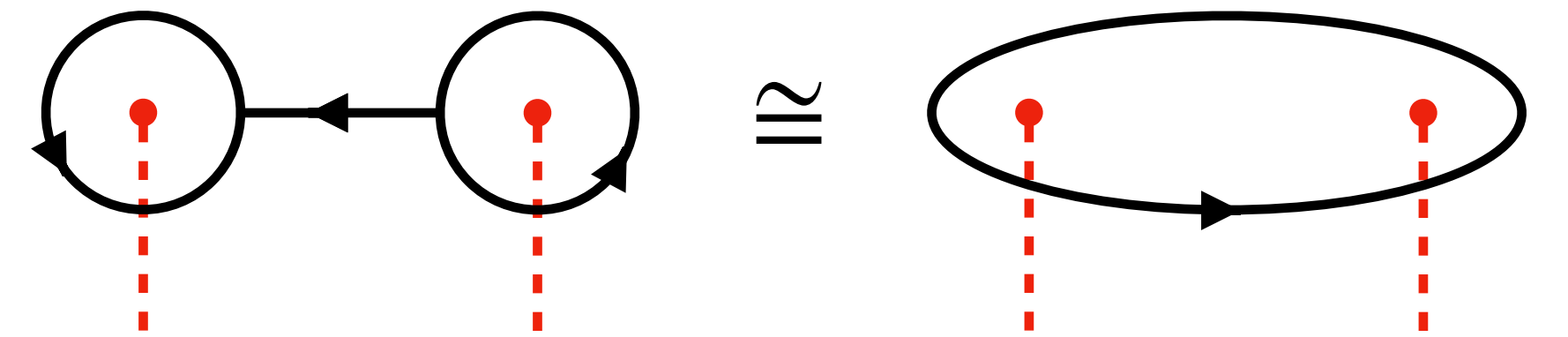
- Facilitates classification of 8d  $\mathcal{N} = 1$  supergravity, including (a posteriori) new branch of moduli space [\[\(Montero/Parra De Freitas '22; cf. Miguel's talk\)\]](#).

- Upon forming *affine* stacks  $\hat{E}_n$ , junctions now describe 9d  $\mathcal{N} = 1$ ! (natural generalization of [\[Lee/\(Lerche\)/Weigand '21\]](#) for F-theory on non-frozen K3).

# Classification of 8d/9d N=1 SUGRA

- Global models by “gluing” (w/ trivial total monodromy, no asymp. (p,q)-charges).

- Upshot: gauge *group* from “global null junctions”.



- Facilitates classification of 8d  $\mathcal{N} = 1$  supergravity, including (a posteriori) new branch of moduli space [[Montero/Parra De Freitas '22](#); cf. [Miguel's talk](#)].

- Upon forming *affine* stacks  $\hat{E}_n$ , junctions now describe 9d  $\mathcal{N} = 1$ ! (natural generalization of [[Lee/\(Lerche\)/Weigand '21](#)] for F-theory on non-frozen K3).

- Classifies 9d  $\mathcal{N} = 1$  gauge groups; also includes new moduli branch.

# Frozen singularities in 7d M-theory

# Frozen singularities in 7d M-theory

- M-theory on  $\mathbb{C}^2/\Gamma$  engineer 7d  $\mathfrak{g}_{\text{ADE}}$  SYM. Generalized symmetries from  $H_2^{\text{rel}}(\mathbb{C}^2/\Gamma) \cong H_1(S^3/\Gamma)$  [Morrison/Schäfer-Nameki/Willett, Albertini/del Zotto/Garcia-Etxebarria/Hosseini '20].

# Frozen singularities in 7d M-theory

- M-theory on  $\mathbb{C}^2/\Gamma$  engineer 7d  $\mathfrak{g}_{\text{ADE}}$  SYM. Generalized symmetries from  $H_2^{\text{rel}}(\mathbb{C}^2/\Gamma) \cong H_1(S^3/\Gamma)$  [Morrison/Schäfer-Nameki/Willett, Albertini/del Zotto/Garcia-Etxebarria/Hosseini '20].
- If  $\int_{S^3/\Gamma} C_3 = \frac{n}{d} \equiv r \neq 0 \pmod{\mathbb{Z}}$ , instead have “frozen”  $\mathfrak{h}_r$ : [de Boer et al, Atiyah/Witten '01]

$\mathfrak{g}$	$\mathfrak{so}_{2k+8}$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
$\frac{n}{d}$	$\frac{1}{2}$	$\frac{1}{2}, \frac{2}{3}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
$\mathfrak{h}$	$\mathfrak{sp}_k$	$\mathfrak{su}_3, \emptyset$	$\mathfrak{so}_7, \mathfrak{su}_2, \emptyset$	$\mathfrak{f}_4, \mathfrak{g}_2, \mathfrak{su}_2, \emptyset, \emptyset$

# Frozen singularities in 7d M-theory

- M-theory on  $\mathbb{C}^2/\Gamma$  engineer 7d  $\mathfrak{g}_{\text{ADE}}$  SYM. Generalized symmetries from  $H_2^{\text{rel}}(\mathbb{C}^2/\Gamma) \cong H_1(S^3/\Gamma)$  [Morrison/Schäfer-Nameki/Willett, Albertini/del Zotto/Garcia-Etxebarria/Hosseini '20].

- If  $\int_{S^3/\Gamma} C_3 = \frac{n}{d} \equiv r \neq 0 \pmod{\mathbb{Z}}$ , instead have “frozen”  $\mathfrak{h}_r$ : [de Boer et al, Atiyah/Witten '01]

$\mathfrak{g}$	$\mathfrak{so}_{2k+8}$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
$\frac{n}{d}$	$\frac{1}{2}$	$\frac{1}{2}, \frac{2}{3}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
$\mathfrak{h}$	$\mathfrak{sp}_k$	$\mathfrak{su}_3, \emptyset$	$\mathfrak{so}_7, \mathfrak{su}_2, \emptyset$	$\mathfrak{f}_4, \mathfrak{g}_2, \mathfrak{su}_2, \emptyset, \emptyset$

- Global: M-theory on “frozen K3” dual to heterotic on  $T^3$  with “triples”, resulting 7d landscape from lattice techniques [(Font)/Fraiman/(Graña/Núñez)/Parra De Freitas '21(x3), '22].



# Frozen singularities in 7d M-theory

- M-theory on  $\mathbb{C}^2/\Gamma$  engineer 7d  $\mathfrak{g}_{\text{ADE}}$  SYM. Generalized symmetries from  $H_2^{\text{rel}}(\mathbb{C}^2/\Gamma) \cong H_1(S^3/\Gamma)$  [Morrison/Schäfer-Nameki/Willett, Albertini/del Zotto/Garcia-Etxebarria/Hosseini '20].

- If  $\int_{S^3/\Gamma} C_3 = \frac{n}{d} \equiv r \neq 0 \pmod{\mathbb{Z}}$ , instead have “frozen”  $\mathfrak{h}_r$ : [de Boer et al, Atiyah/Witten '01]

$\mathfrak{g}$	$\mathfrak{so}_{2k+8}$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
$\frac{n}{d}$	$\frac{1}{2}$	$\frac{1}{2}, \frac{2}{3}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
$\mathfrak{h}$	$\mathfrak{sp}_k$	$\mathfrak{su}_3, \emptyset$	$\mathfrak{so}_7, \mathfrak{su}_2, \emptyset$	$\mathfrak{f}_4, \mathfrak{g}_2, \mathfrak{su}_2, \emptyset, \emptyset$

- Global: M-theory on “frozen K3” dual to heterotic on  $T^3$  with “triples”, resulting 7d landscape from lattice techniques [(Font)/Fraiman/(Graña/Núñez)/Parra De Freitas '21(x3), '22].
- Local: Microscopic description? “Geometric” origin of generalized symmetries?

# Frozen singularities in 7d M-theory

- Embed  $(\mathfrak{g}, r = \frac{n}{d})$  into elliptic fibration  $Y$  with singular central fiber of  $\mathfrak{g}$ -type.

# Frozen singularities in 7d M-theory

- Embed  $(\mathfrak{g}, r = \frac{n}{d})$  into elliptic fibration  $Y$  with singular central fiber of  $\mathfrak{g}$ -type.
- T-duality  $\Rightarrow$  F-theory on  $(Y^d \times S^1)/\mathbb{Z}_d$ , aka IIB “shift-orientifold” [\[Tachikawa '15\]](#), with  $Y^d$  a “ $d$ -fold cover” of  $Y$ .

# Frozen singularities in 7d M-theory

- Embed  $(\mathfrak{g}, r = \frac{n}{d})$  into elliptic fibration  $Y$  with singular central fiber of  $\mathfrak{g}$ -type.
- T-duality  $\Rightarrow$  F-theory on  $(Y^d \times S^1)/\mathbb{Z}_d$ , aka IIB “shift-orientifold” [\[Tachikawa '15\]](#), with  $Y^d$  a “ $d$ -fold cover” of  $Y$ .
- (Subgroup of)  $\mathbb{Z}_d$  acts as outer automorphism on  $\mathfrak{g}(Y^d) \Rightarrow \mathfrak{h}_r$ .

$g$	$Y$	$\theta_g/\frac{\pi}{6}$	alg.	$r = \frac{n}{d}$	$g^d$	$Y^d$	$\theta_{g^d}/\frac{\pi}{6}$	alg.	outer	fixed = $\mathfrak{h}_r$
$\begin{pmatrix} -1 & -k \\ 0 & -1 \end{pmatrix}$	$I_k^*$	$6-k$	$\mathfrak{so}(2k+8)$	$\frac{1}{2}$	$\begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}$	$I_{2k}$	$12-2k$	$\mathfrak{su}(2k)$	$\mathbb{Z}_2$	$\mathfrak{sp}(2k)$
$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	$IV^*$	4	$\mathfrak{e}_6$	$\frac{1}{2}$	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$IV$	8	$\mathfrak{su}(3)$	*	$\mathfrak{su}(3)$
$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	$IV^*$	4	$\mathfrak{e}_6$	$\frac{1}{3}, \frac{2}{3}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$I_0$	12	*	*	*
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$III^*$	3	$\mathfrak{e}_7$	$\frac{1}{2}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$I_0^*$	6	$\mathfrak{so}(8)$	$\mathbb{Z}_2$	$\mathfrak{so}(7)$
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$III^*$	3	$\mathfrak{e}_7$	$\frac{1}{3}, \frac{2}{3}$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$III$	9	$\mathfrak{su}(2)$	*	$\mathfrak{su}(2)$
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$III^*$	3	$\mathfrak{e}_7$	$\frac{1}{4}, \frac{3}{4}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$I_0$	12	*	*	*
$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	$II^*$	2	$\mathfrak{e}_8$	$\frac{1}{2}$	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	$IV^*$	4	$\mathfrak{e}_6$	$\mathbb{Z}_2$	$\mathfrak{f}_4$
$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	$II^*$	2	$\mathfrak{e}_8$	$\frac{1}{3}, \frac{2}{3}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$I_0^*$	6	$\mathfrak{so}(8)$	$\mathbb{Z}_3$	$\mathfrak{g}_2$
$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	$II^*$	2	$\mathfrak{e}_8$	$\frac{1}{4}, \frac{3}{4}$	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$IV$	8	$\mathfrak{su}(3)$	$\mathbb{Z}_2$	$\mathfrak{su}(2)$
$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$II^*$	2	$\mathfrak{e}_8$	$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	$II$	10	*	*	*
$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	$II^*$	2	$\mathfrak{e}_8$	$\frac{1}{6}, \frac{5}{6}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$I_0$	12	*	*	*

[taken from Tachikawa '15]

# Frozen singularities in 7d M-theory via junctions

- Observation:  $\#_{7\text{-branes}}(Y^d) = d \#_{7\text{-branes}}(Y) + 12(d - 1)$ .

# Frozen singularities in 7d M-theory via junctions

- Observation:  $\#_{7\text{-branes}}(Y^d) = d \#_{7\text{-branes}}(Y) + 12(d - 1)$ .
- Fact: dP9-fibration has twelve 7-branes.

# Frozen singularities in 7d M-theory via junctions

- Observation:  $\#_{7\text{-branes}}(Y^d) = d \#_{7\text{-branes}}(Y) + 12(d - 1)$ .
- Fact: dP9-fibration has twelve 7-branes.
- No coincidence:  $d$  copies of 7-branes for  $Y \xrightarrow{\text{Hanany-Witten}} Y^d + (d - 1) \times \hat{E}_9$ .



# Frozen singularities in 7d M-theory via junctions

- Observation:  $\#_{7\text{-branes}}(Y^d) = d \#_{7\text{-branes}}(Y) + 12(d - 1)$ .
- Fact: dP9-fibration has twelve 7-branes.
- No coincidence:  $d$  copies of 7-branes for  $Y \xrightarrow{\text{Hanany-Witten}} Y^d + (d - 1) \times \hat{E}_9$ .
- Example:  $(A^6BC)(A^6BC) \rightarrow A^{12}BCBC = A^4(A^8BCBC)$ .

# Frozen singularities in 7d M-theory via junctions

- Example:  $(A^6BC)(A^6BC) \rightarrow A^{12}BCBC = A^4(A^8BCBC)$ .

# Frozen singularities in 7d M-theory via junctions

- Example:  $(A^6BC)(A^6BC) \rightarrow A^{12}BCBC = A^4(A^8BCBC)$ .
- “Freezing mechanism”: starting from  $\mathfrak{g} = \mathfrak{so}_{12}$  root junctions, only certain linear combinations  $\{\beta_i\}$  can be “detached” from  $\hat{E}_9$  after rearranging.

# Frozen singularities in 7d M-theory via junctions

- Example:  $(A^6BC)(A^6BC) \rightarrow A^{12}BCBC = A^4(A^8BCBC)$ .
- “Freezing mechanism”: starting from  $\mathfrak{g} = \mathfrak{so}_{12}$  root junctions, only certain linear combinations  $\{\beta_i\}$  can be “detached” from  $\hat{E}_9$  after rearranging.
- This is a subspace in root lattice of  $\mathfrak{g}(Y^2) = \mathfrak{su}_4$ .

# Frozen singularities in 7d M-theory via junctions

- Example:  $(A^6BC)(A^6BC) \rightarrow A^{12}BCBC = A^4(A^8BCBC)$ .
- “Freezing mechanism”: starting from  $\mathfrak{g} = \mathfrak{so}_{12}$  root junctions, only certain linear combinations  $\{\beta_i\}$  can be “detached” from  $\hat{E}_9$  after rearranging.
- This is a subspace in root lattice of  $\mathfrak{g}(Y^2) = \mathfrak{su}_4$ .
- Precisely those that survive the subsequent outer-automorphism quotient  $\mathfrak{su}_4 \rightarrow \mathfrak{sp}_2 = \mathfrak{h}_{1/2}$  [Bonora/Savelli '10, Grassi/Halverson/(Long/Shaneson/(Tian) '13+'18, Heckman/Lawrie/Rochais/Zhang/Zoccarato '20)].

# Frozen singularities in 7d M-theory via junctions

- Geometric interpretation in M-theory:  $\{\beta_i\}$  = linear combination of  $\mathfrak{g}$ -roots = 2-cycles in  $Y$  that can be wrapped by M2-branes after freezing.

# Frozen singularities in 7d M-theory via junctions

- Geometric interpretation in M-theory:  $\{\beta_i\}$  = linear combination of  $\mathfrak{g}$ -roots = 2-cycles in  $Y$  that can be wrapped by M2-branes after freezing.
- Has a surprisingly simple characterization (see Hao's 1-minute explanation).

# Frozen singularities in 7d M-theory via junctions

- Geometric interpretation in M-theory:  $\{\beta_i\}$  = linear combination of  $\mathfrak{g}$ -roots = 2-cycles in  $Y$  that can be wrapped by M2-branes after freezing.
- Has a surprisingly simple characterization (see Hao's 1-minute explanation).
- Also gives a handle on center symmetries via extended junctions in  $Y^d$  that are invariant under outer automorphism.



# Frozen singularities in 7d M-theory via junctions

- Geometric interpretation in M-theory:  $\{\beta_i\}$  = linear combination of  $\mathfrak{g}$ -roots = 2-cycles in  $Y$  that can be wrapped by M2-branes after freezing.
- Has a surprisingly simple characterization (see Hao's 1-minute explanation).
- Also gives a handle on center symmetries via extended junctions in  $Y^d$  that are invariant under outer automorphism.
- In particular:  $(e_8, 1/4)$  with  $\mathfrak{h} = \mathfrak{su}_2$  has  $\mathbb{Z}_2$  even though  $H_1(S^3/\Gamma_{e_8}) = 0$ .

# Frozen IIA compactifications and topological point-operators

# Frozen IIA compactifications and topological point-operators

- IIA on  $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$  gives 6d (1,1)  $\mathfrak{g}$  gauge theory; experiences rank reduction similar to “freezing” if  $\int_{\lambda} C_1 \neq 0 \pmod{\mathbb{Z}}$  for  $\lambda \in H_1(S^3/\Gamma)$  [de Boer et al '01].

# Frozen IIA compactifications and topological point-operators

- IIA on  $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$  gives 6d (1,1)  $\mathfrak{g}$  gauge theory; experiences rank reduction similar to “freezing” if  $\int_{\lambda} C_1 \neq 0 \pmod{\mathbb{Z}}$  for  $\lambda \in H_1(S^3/\Gamma)$  [de Boer et al '01].
- To relate to a configuration with  $C_3$ -holonomy ( $\rightsquigarrow$  frozen M-theory), need to take “double T-dual” (hard).

# Frozen IIA compactifications and topological point-operators

- IIA on  $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$  gives 6d (1,1)  $\mathfrak{g}$  gauge theory; experiences rank reduction similar to “freezing” if  $\int_{\lambda} C_1 \neq 0 \pmod{\mathbb{Z}}$  for  $\lambda \in H_1(S^3/\Gamma)$  [de Boer et al '01].
- To relate to a configuration with  $C_3$ -holonomy ( $\rightsquigarrow$  frozen M-theory), need to take “double T-dual” (hard).
- In view of stringy realization of symmetry operators [Apruzzi/Bah/Bonetti/Schäfer-Nameki, Garcia-Etxebarria (holographic); Heckman/Hübner/Torres/Zhang (geo. eng.) '22]: topological point-operators from D0-branes on  $\lambda \in H_1(S^3/\Gamma)$ .

# Frozen IIA compactifications and topological point-operators

- IIA on  $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$  gives 6d (1,1)  $\mathfrak{g}$  gauge theory; experiences rank reduction similar to “freezing” if  $\int_{\lambda} C_1 \neq 0 \pmod{\mathbb{Z}}$  for  $\lambda \in H_1(S^3/\Gamma)$  [de Boer et al '01].
- To relate to a configuration with  $C_3$ -holonomy ( $\rightsquigarrow$  frozen M-theory), need to take “double T-dual” (hard).
- In view of stringy realization of symmetry operators [Apruzzi/Bah/Bonetti/Schäfer-Nameki, Garcia-Etxebarria (holographic); Heckman/Hübner/Torres/Zhang (geo. eng.) '22]: topological point-operators from D0-branes on  $\lambda \in H_1(S^3/\Gamma)$ .
- Vevs of these operators:  $\exp(2\pi i \int_{\lambda} C_1)$ .

# Frozen compactifications = “universes” (?)

- Top. point-operators  $\rightarrow$   $(d - 1)$ -form symmetry  $\rightarrow$  “Decomposition” of theory into “universes” [[Hellerman/Henriques/Pantev/Sharpe/Ando '06](#), ...[\(Sharpe et al\)...](#), [Sharpe '22 \(review\)](#); [Komargodski/Ohmori/Roumpedakis/Seifnashri '08](#), [Meynet/Moscrop '22](#), [Vandermeulen '22](#)].

# Frozen compactifications = “universes” (?)

- Top. point-operators  $\rightarrow$   $(d - 1)$ -form symmetry  $\rightarrow$  “Decomposition” of theory into “universes” [Hellerman/Henriques/Pantev/Sharpe/Ando '06, ...(Sharpe et al)..., Sharpe '22 (review); Komargodski/Ohmori/Roumpedakis/Seifnashri '08, Meynet/Moscrop '22, Vandermeulen '22].
- Separated by infinite tension domain walls = charged objects; in IIA on  $\mathbb{C}^2/\Gamma$  given by D6s wrapping  $\Lambda \in H_2^{\text{rel}}(\mathbb{C}^2/\Gamma)$ .



# Frozen compactifications = “universes” (?)

- Top. point-operators  $\rightarrow (d - 1)$ -form symmetry  $\rightarrow$  “Decomposition” of theory into “universes” [Hellerman/Henriques/Pantev/Sharpe/Ando '06, ...(Sharpe et al)..., Sharpe '22 (review); Komargodski/Ohmori/Roumpedakis/Seifnashri '08, Meynet/Moscrop '22, Vandermeulen '22].
- Separated by infinite tension domain walls = charged objects; in IIA on  $\mathbb{C}^2/\Gamma$  given by D6s wrapping  $\Lambda \in H_2^{\text{rel}}(\mathbb{C}^2/\Gamma)$ .
- Normally, universes have same local dynamics. Here, universes labelled by different “frozen fluxes”  $\rightarrow$  counterexample (?)

# Frozen compactifications = “universes” (?)

- Top. point-operators  $\rightarrow (d - 1)$ -form symmetry  $\rightarrow$  “Decomposition” of theory into “universes” [Hellerman/Henriques/Pantev/Sharpe/Ando '06, ...(Sharpe et al)..., Sharpe '22 (review); Komargodski/Ohmori/Roumpedakis/Seifnashri '08, Meynet/Moscrop '22, Vandermeulen '22].
- Separated by infinite tension domain walls = charged objects; in IIA on  $\mathbb{C}^2/\Gamma$  given by D6s wrapping  $\Lambda \in H_2^{\text{rel}}(\mathbb{C}^2/\Gamma)$ .
- Normally, universes have same local dynamics. Here, universes labelled by different “frozen fluxes”  $\rightarrow$  counterexample (?)
- Dual (-1)-form symmetry perspective: top. spacetime-filling operators = top. sectors given by D6s wrapping  $\lambda \in H_1(S^3/\Gamma)$ ; can these modify local (SUSY) dynamics?

# Summary

# Summary

- Frozen singularities in 8d via junctions  $\rightarrow$  classification of 8d *and* 9d  $\mathcal{N} = 1$  supergravity w/ gauge group topology.

# Summary

- Frozen singularities in 8d via junctions  $\rightarrow$  classification of 8d *and* 9d  $\mathcal{N} = 1$  supergravity w/ gauge group topology.
- Junctions can also be applied to shed light on frozen singularities in 7d M-theory, incl. higher-form center symmetries.

# Summary

- Frozen singularities in 8d via junctions  $\rightarrow$  classification of 8d *and* 9d  $\mathcal{N} = 1$  supergravity w/ gauge group topology.
- Junctions can also be applied to shed light on frozen singularities in 7d M-theory, incl. higher-form center symmetries.
- In 6d IIA, find “more exotic” topological symmetry generators; (perhaps) tied to “decomposition”.

# Outlook

# Outlook

- Still lacking an inherently M-theory description for freezing, i.e., “what are consistent M2-brane states?”. Need “double T-duality” transformation of IIA.



# Outlook

- Still lacking an inherently M-theory description for freezing, i.e., “what are consistent M2-brane states?”. Need “double T-duality” transformation of IIA.
- How to understand the topological operators in 6d IIA? What happens in the M-theory uplift? (Some aspects hidden in F-theory with O7+?)

# Outlook

- Still lacking an inherently M-theory description for freezing, i.e., “what are consistent M2-brane states?”. Need “double T-duality” transformation of IIA.
- How to understand the topological operators in 6d IIA? What happens in the M-theory uplift? (Some aspects hidden in F-theory with O7+?)
- Application to symmetry aspects and supergravity landscape in lower dimensions.

# Outlook

- Still lacking an inherently M-theory description for freezing, i.e., “what are consistent M2-brane states?”. Need “double T-duality” transformation of IIA.
- How to understand the topological operators in 6d IIA? What happens in the M-theory uplift? (Some aspects hidden in F-theory with O7+?)
- Application to symmetry aspects and supergravity landscape in lower dimensions.

*Thank you!*