Sharpening the Distance Conjecture in Diverse Dimensions

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Based on 2206.04063 with Muldrow Etheredge, Ben Heidenreich, Sami Kaya, and Yue Qiu, 2208.08989
Work to appear with Muldrow Etheredge, Ben Heidenreich, Jacob McNamara, Ignacio Ruiz Garcia, Irene Valenzuela

Outline

I. A Sharpened Distance Conjecture
II. Evidence for a Sharpened Distance Conjecture
    i) Dimensional reduction
    ii) Maximal supergravity
    iii) Minimal supergravity
III. Implications of the Conjecture
    i) Asymptotic Scalar Field Potentials
    ii) Large-field inflation

A Sharpened Distance Conjecture

The Distance Conjecture

Massless scalar fields parametrize a "moduli space" of vacua.

At large distances in moduli space, a tower of particles becomes light exponentially quickly with increasing distance:

$$m_n(\phi) \sim n e^{-\lambda \phi}$$

Ooguri, Vafa '06

Sharpening the Distance Conjecture

- It has long been understood that the coefficient λ appearing in the Distance Conjecture must be order-one in Planck units
- However, the precise values that are allowed has been a subject of much discussion and debate
- We propose a novel lower bound on λ for the lightest tower in a given infinite-distance limit:

$$\boxed{\lambda \geq 1/\sqrt{d-2}}$$

- More precisely, our claim is that in a given infinite distance limit, there is always at least one tower which satisfies this bound. (There may be other towers which do not)

Etheredge, Heidenreich, Kaya, Qiu, TR ’22

Aside: The Emergent String Conjecture

- The Emergent String Conjecture will play an important role in the remainder of this talk
- This conjecture holds that any infinite-distance limit in moduli space is either a decompactification limit (accompanied by a tower of Kaluza-Klein modes) or an emergent string limit (accompanied by a tower of string oscillator modes)
- Neither our “sharpened" Distance Conjecture nor the Emergent String Conjecture implies the other, but together they offer a simple, coherent picture of infinite-distance limits in moduli space

Lee, Lerche, Weigand ‘19

Evidence for a Sharpened Distance Conjecture

Evidence for the Conjecture

- In this talk, I will sketch three lines of evidence in favor of this conjectured bound:
    - i) Dimensional reduction
    - ii) Top-down evidence from string/M-theory
    - iii) Bottom-up evidence from minimal supergravity

Evidence for the Conjecture

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Dimensional Reduction

- Many of the most well-supported Swampland conjectures are exactly preserved under dimensional reduction (cf. Heidenreich, Reece, TR ’15, TR ’21)
- E.g. Weak Gravity Conjecture:

$$e^2_{P;D} q^2 \geq \gamma_{P;D} T_P^2$$

$$\gamma_{P;D} = \gamma_{p;d} = \gamma_{P,d}\,, \quad p = P - 1\,,\ d = D - 1$$

Satisfied in *D* dimensions ⇔ satisfied in *d* dimensions!

- So, preservation under dimensional reduction is a useful tool for determining order-one factors in Swampland conjectures

Dim. Reduction of the Distance Conjecture

- Start with tower in D dimensions:

$$m \sim \exp(-\lambda_D \phi_D)$$

- After reduction to d=D-1 dimensions, find tower:

$$m \sim \exp\Big(- \lambda_D \phi_d - \frac{1}{\sqrt{(d-1)(d-2)}} \rho \Big)$$

$$\sim \exp\left(- \left(\lambda_D^2 + \frac{1}{(d-1)(d-2)} \right)^{1/2} \phi_d' \right)$$

$$\equiv \exp(-\lambda_d \phi_d')$$

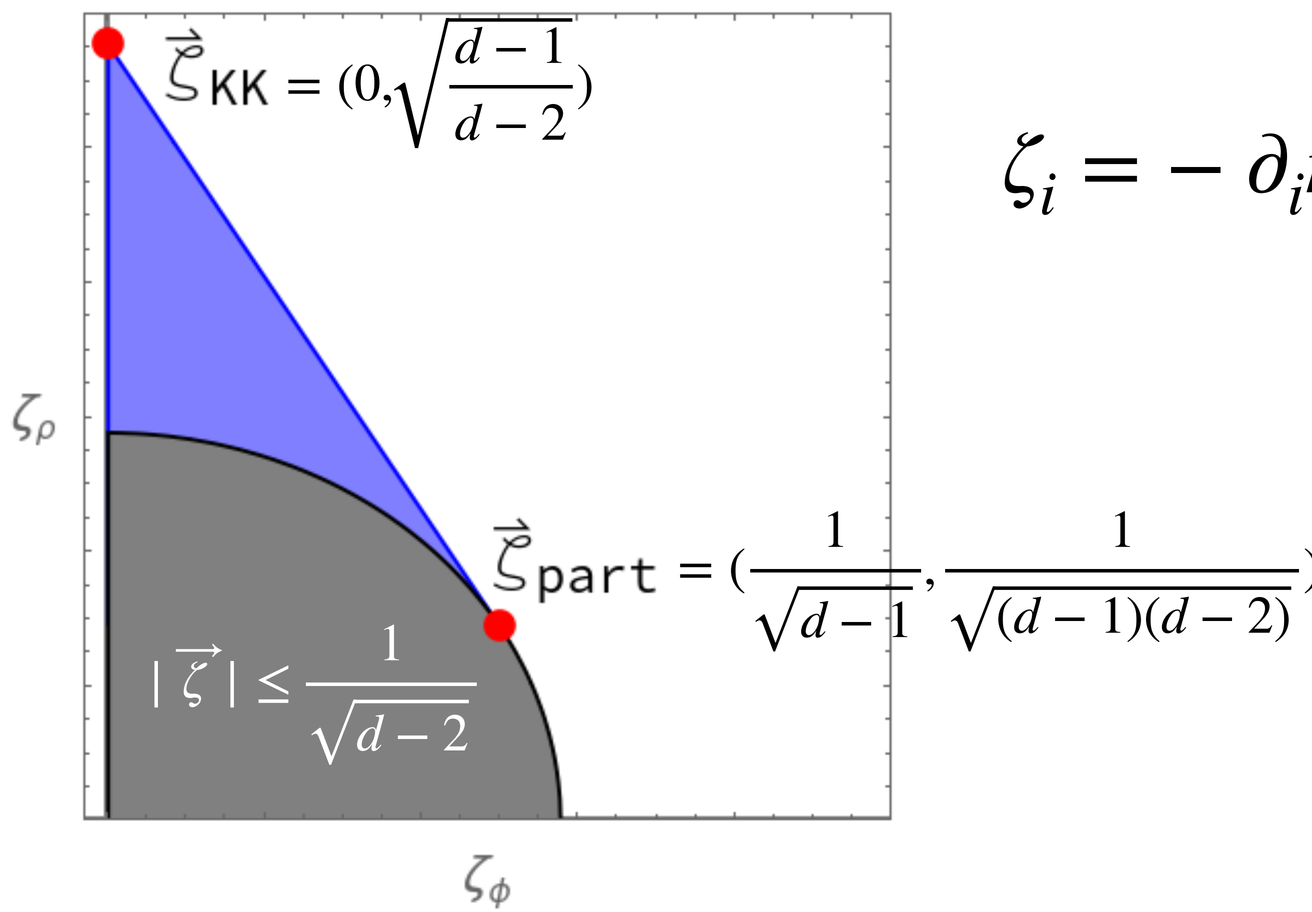
- Solved by $\lambda_D = 1/\sqrt{D-2}$, $\lambda_d = 1/\sqrt{d-2}$

$\Rightarrow \lambda_d = 1/\sqrt{d-2}$ preserved under dimensional reduction!

Dim. Reduction of the Distance Conjecture

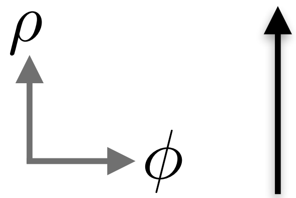
- Also have KK modes:

$$m_{\mathrm{KK}} \sim \exp(-\sqrt{(d-1)/(d-2)}\rho)$$



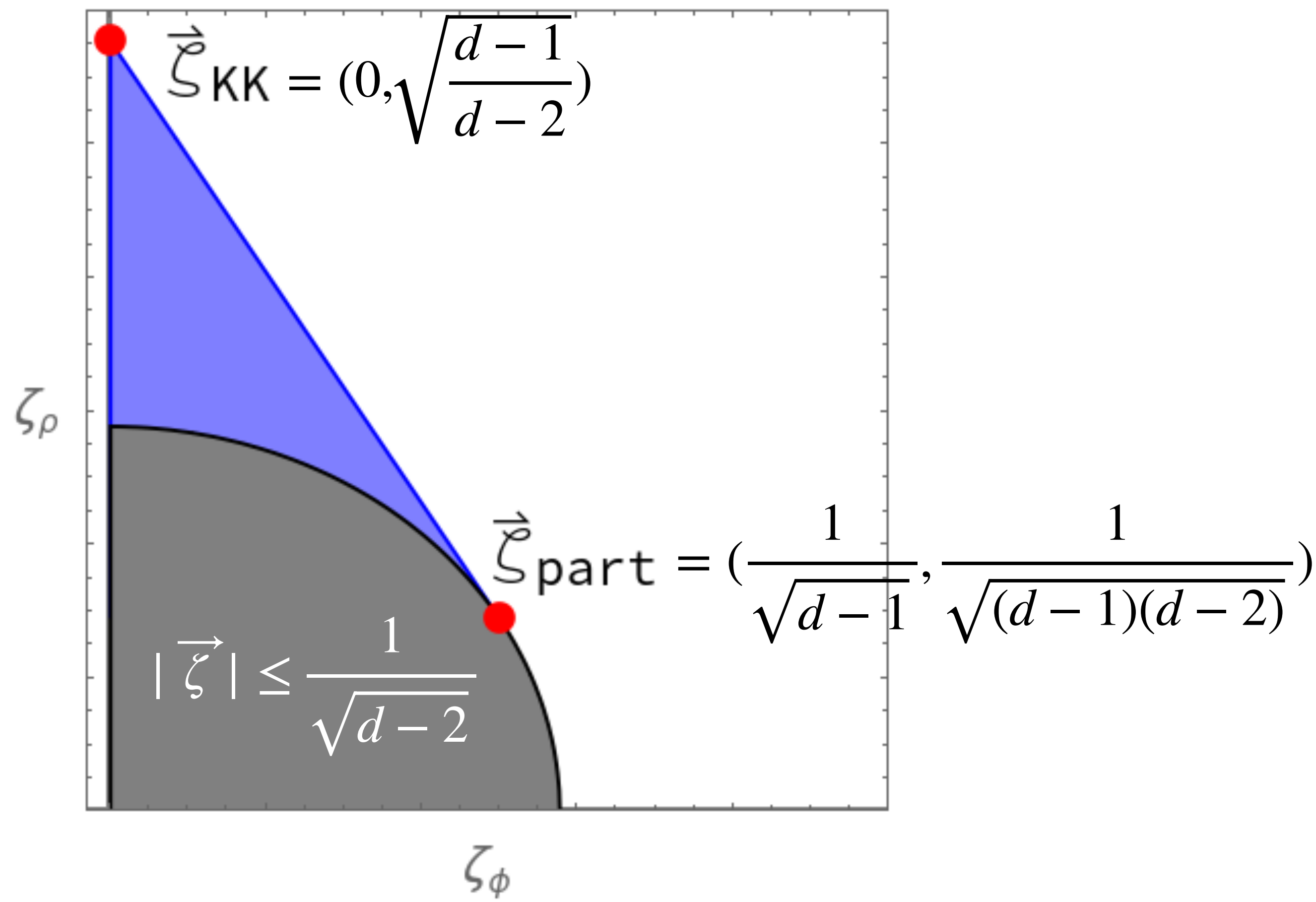
$$\zeta_i = -\partial_i m/m$$

Dim. Reduction of the Distance Conjecture

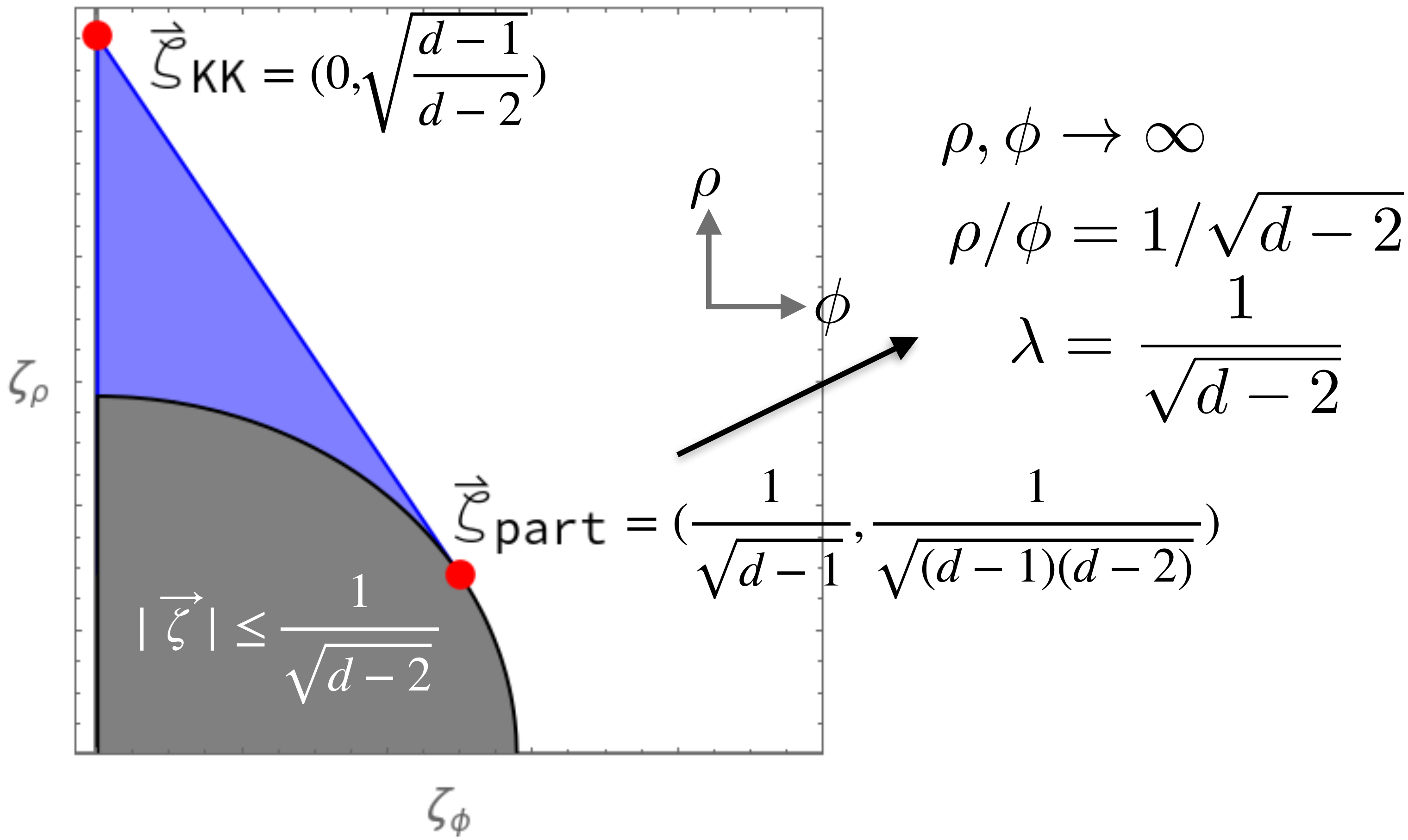


$$\rho \to \infty$$

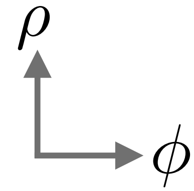
$$\lambda = \sqrt{\frac{d-1}{d-2}}$$



Dim. Reduction of the Distance Conjecture



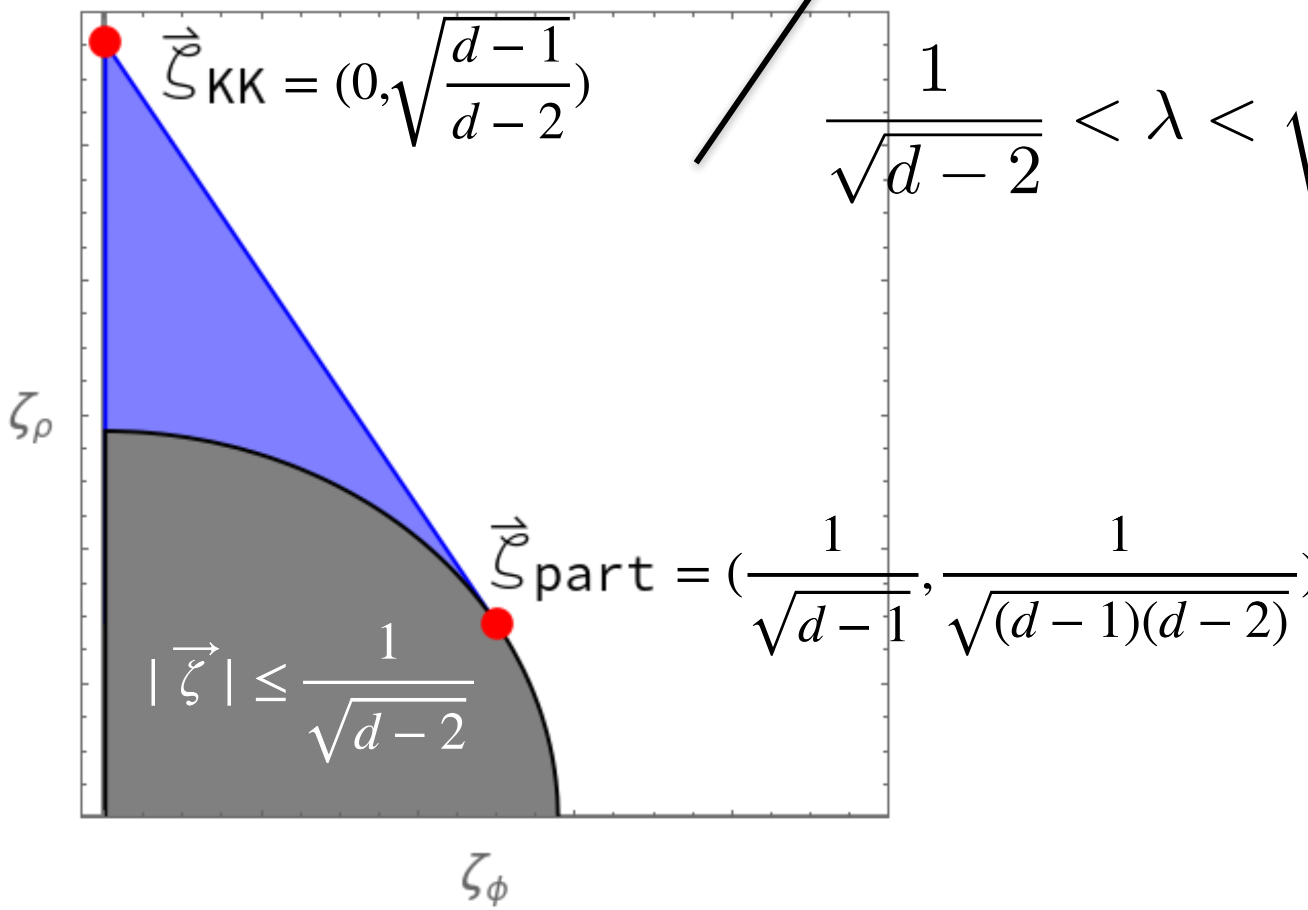
Dim. Reduction of the Distance Conjecture



$$\rho, \phi \to \infty$$

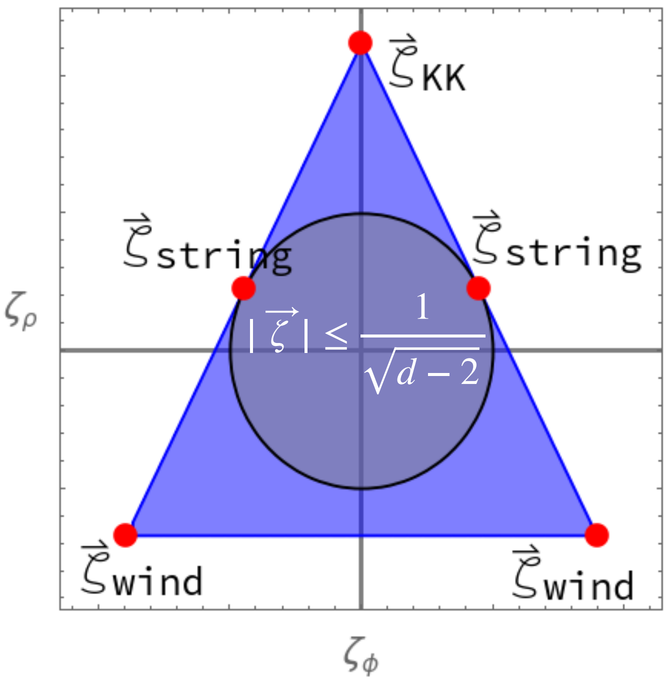
$$\rho/\phi > 1/\sqrt{d-2}$$

$$\frac{1}{\sqrt{d-2}} < \lambda < \sqrt{\frac{d-1}{d-2}}$$

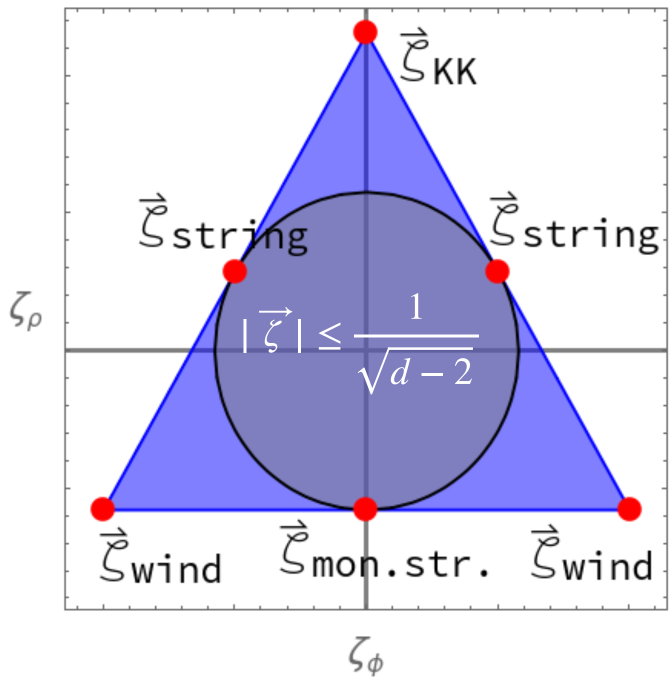


Dim. Reduction of the Distance Conjecture

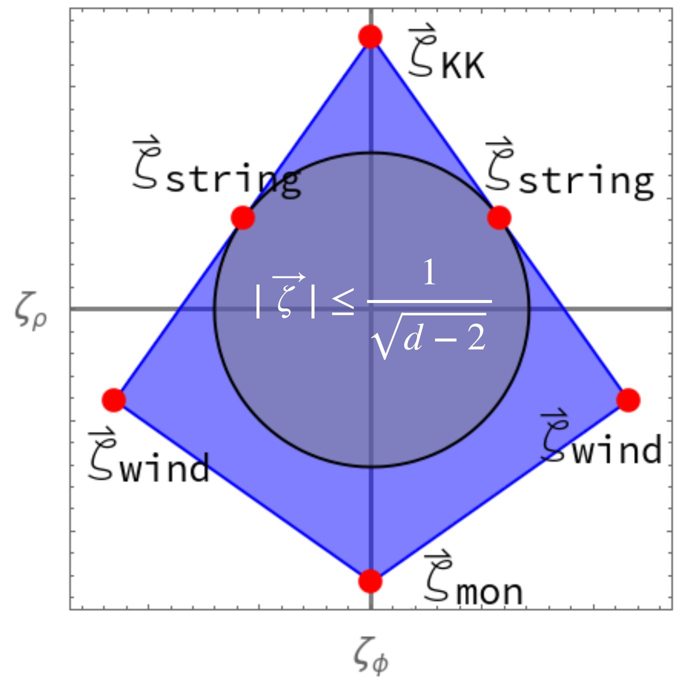
- If the tower in D dimensions is a tower of string oscillator modes, then we also have winding modes (plus KK monopole strings in $d = 5$, KK monopoles in $d = 4$):



$d > 5$



$d = 5$



$d = 4$

Evidence for the Conjecture

- In this talk, I will sketch three lines of evidence in favor of this conjectured bound:
  - i) Dimensional reduction
  - **ii) Top-down evidence from string/M-theory**
  - iii) Bottom-up evidence from minimal supergravity

Top-Down Evidence: 10d string theory

- Weak coupling limits $\phi \to \infty$ of Type IIA, IIB, Type I, heterotic string theory:

$$T \sim \exp(-2\phi/\sqrt{8})$$

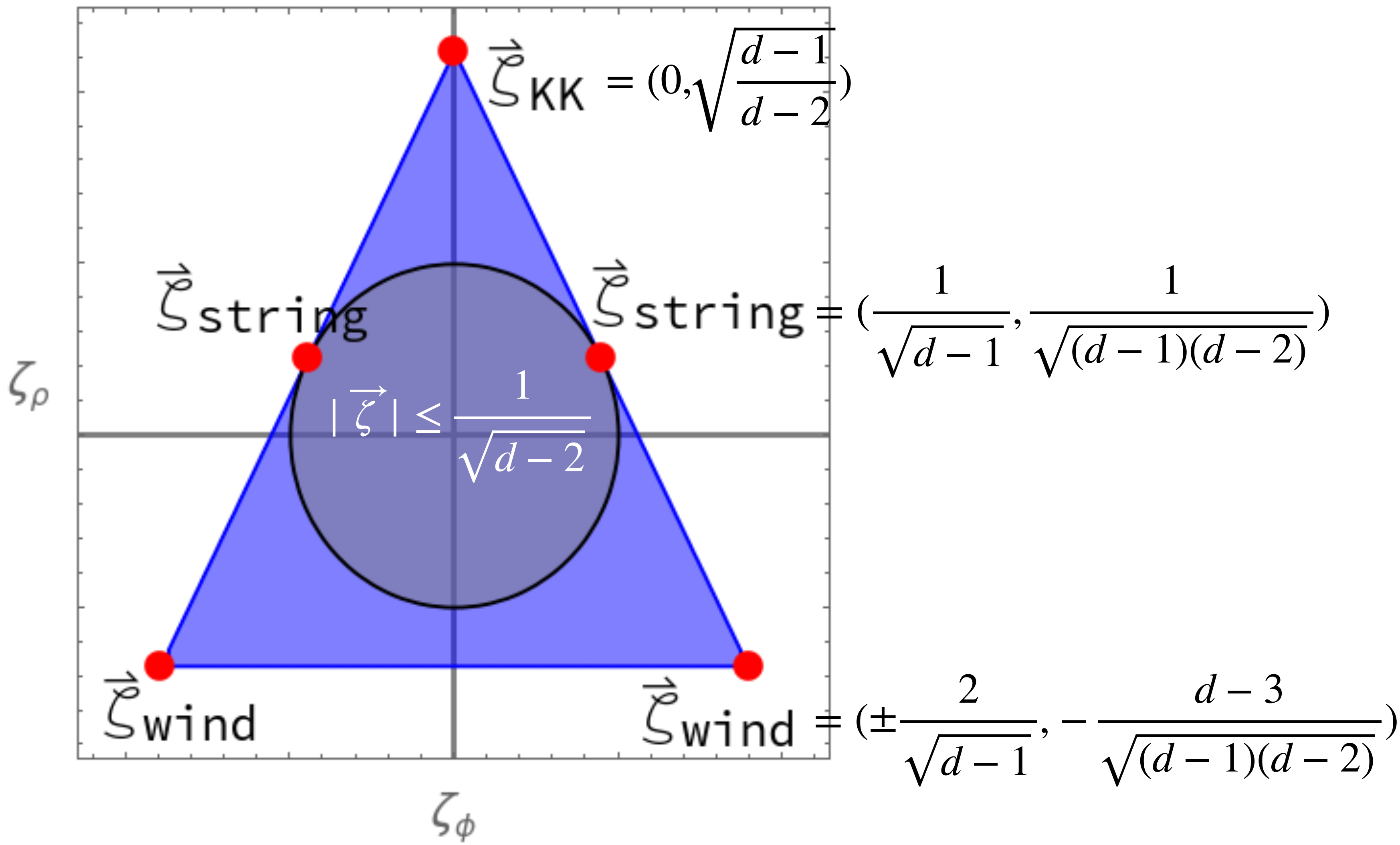
- Get string oscillator modes:

$$m \sim M_{\text{string}} \sim \sqrt{T} \sim \exp(-\phi/\sqrt{8})$$

- Saturates bound $\lambda \geq 1/\sqrt{d-2}$!
- By S-duality, find same scaling in strong coupling limits for Type IIB, Type I, SO(32) heterotic
- By duality with M-theory, strong coupling limits $\phi \to -\infty$ of Type IIA, E_8 x E_8 heterotic give KK tower:

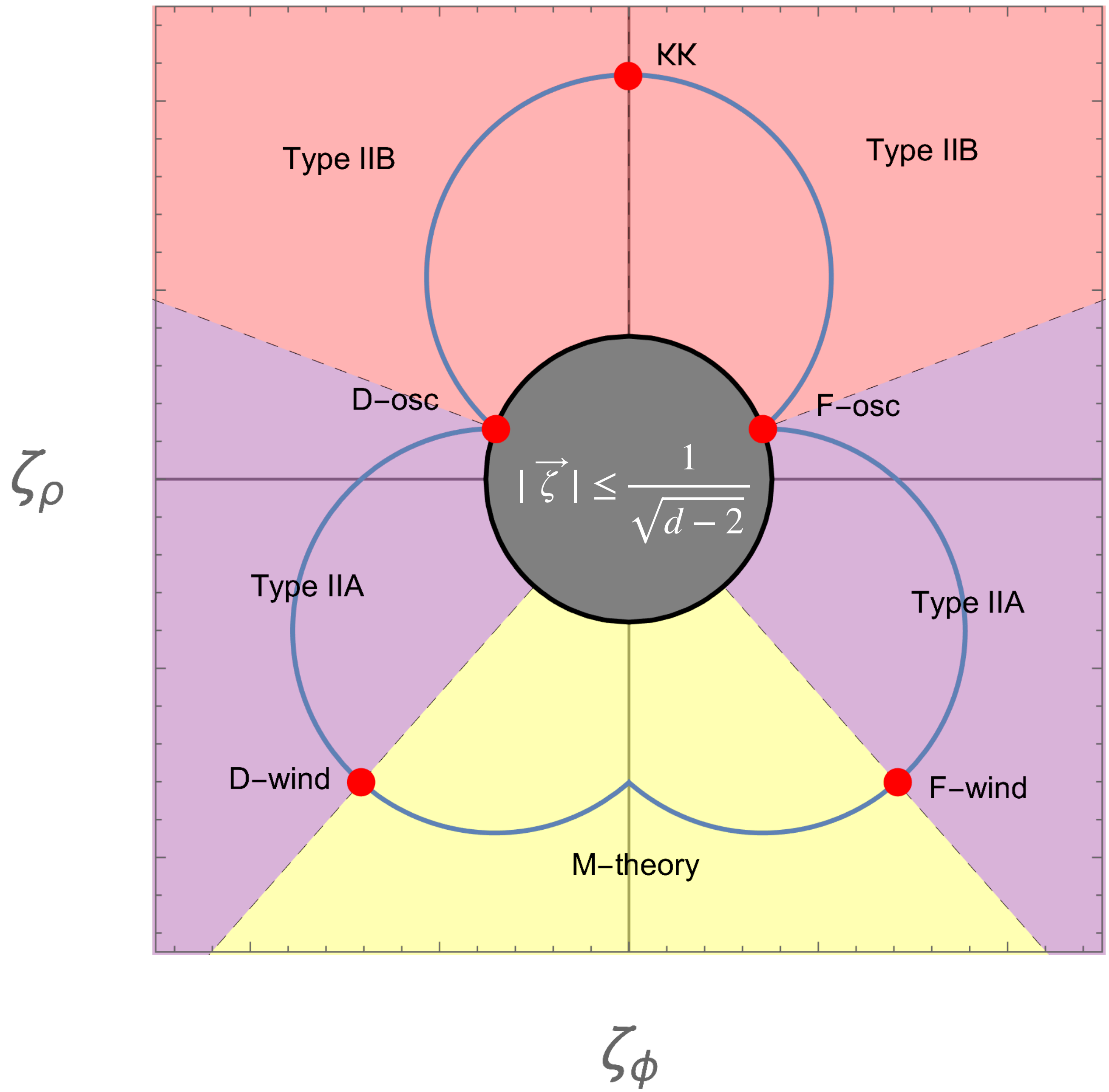
$$m_{\text{KK}} \sim \exp(-|\phi|\sqrt{9/8})$$

Top-Down Evidence: 9d string theory



$d = 9$

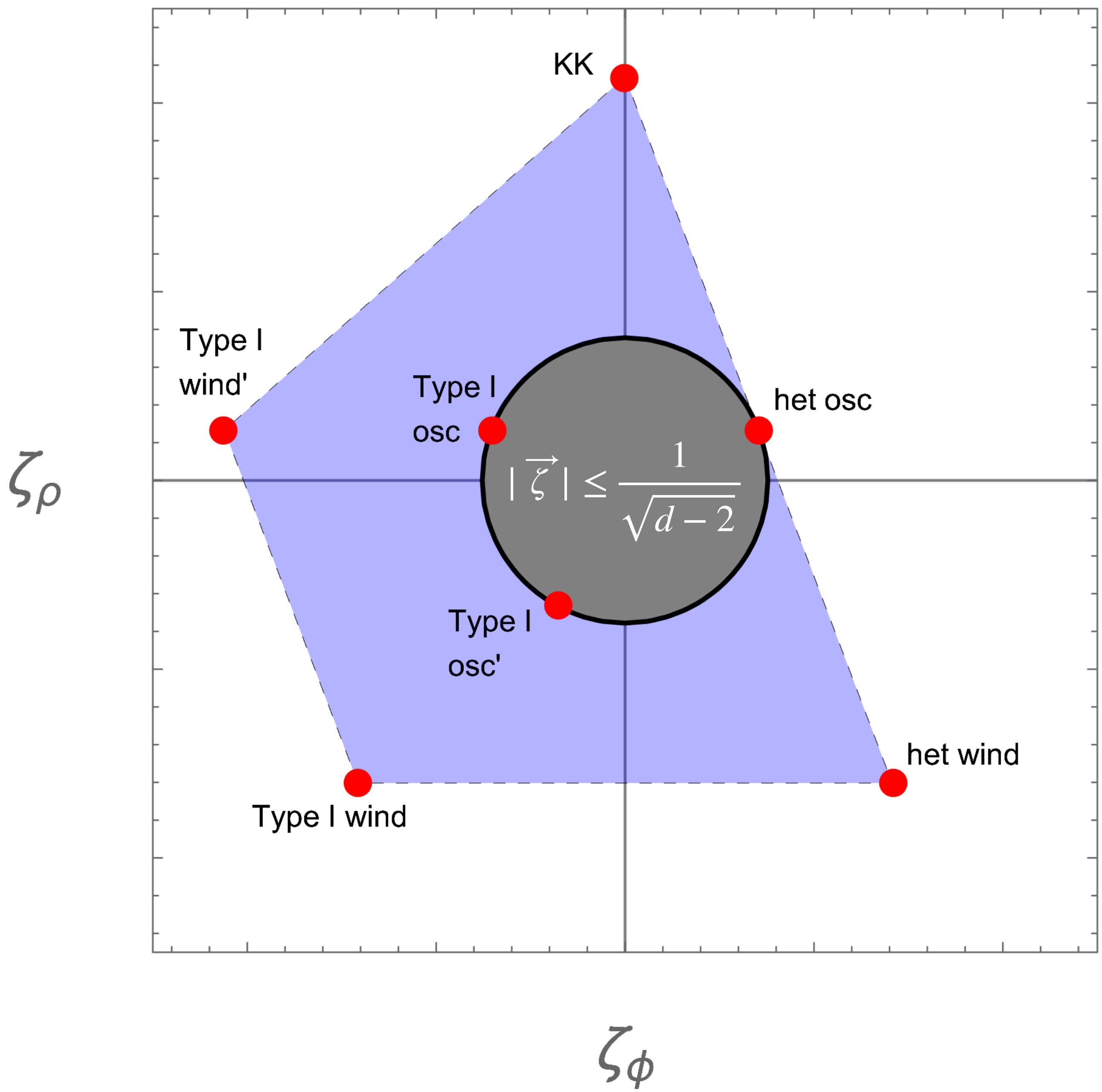
Top-Down Evidence: IIB in 9d



Top-Down Evidence: Maximal Supergravity in $d \geq 4$

- Through similar process, can check Distance Conjecture in all limits of moduli space for maximal supergravity in $d \geq 4$ dimensions (M-theory compactified on T^{11-d})
- Find (after significant effort!) that the bound $\lambda \geq 1/\sqrt{d-2}$ is saturated in certain limits and satisfied in all limits
- Limits that saturate the bound are always emergent string limits, which feature a tower of string oscillator modes with $\lambda = 1/\sqrt{d-2}$

Top-Down Evidence: 9d SO(32) heterotic



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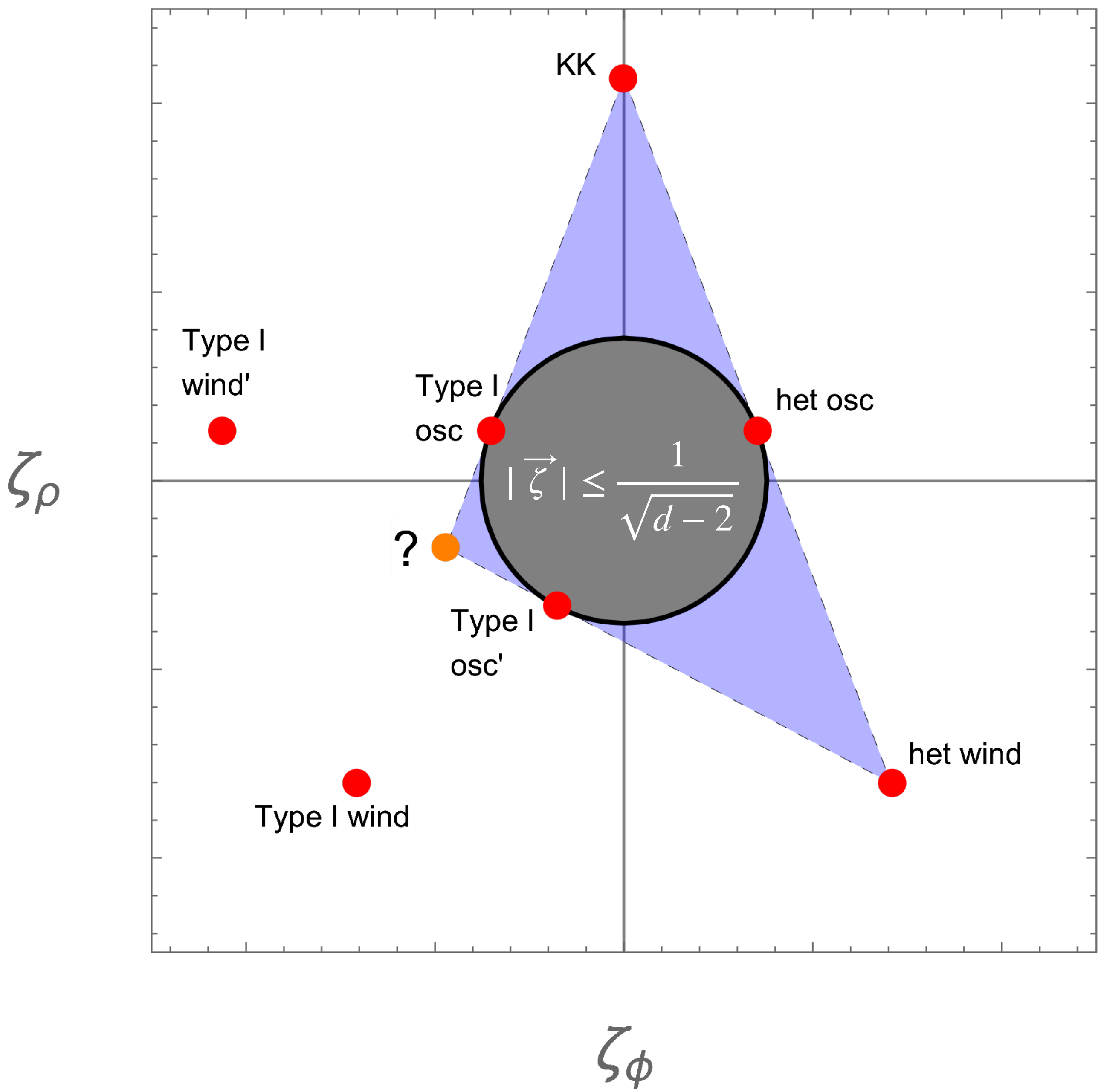
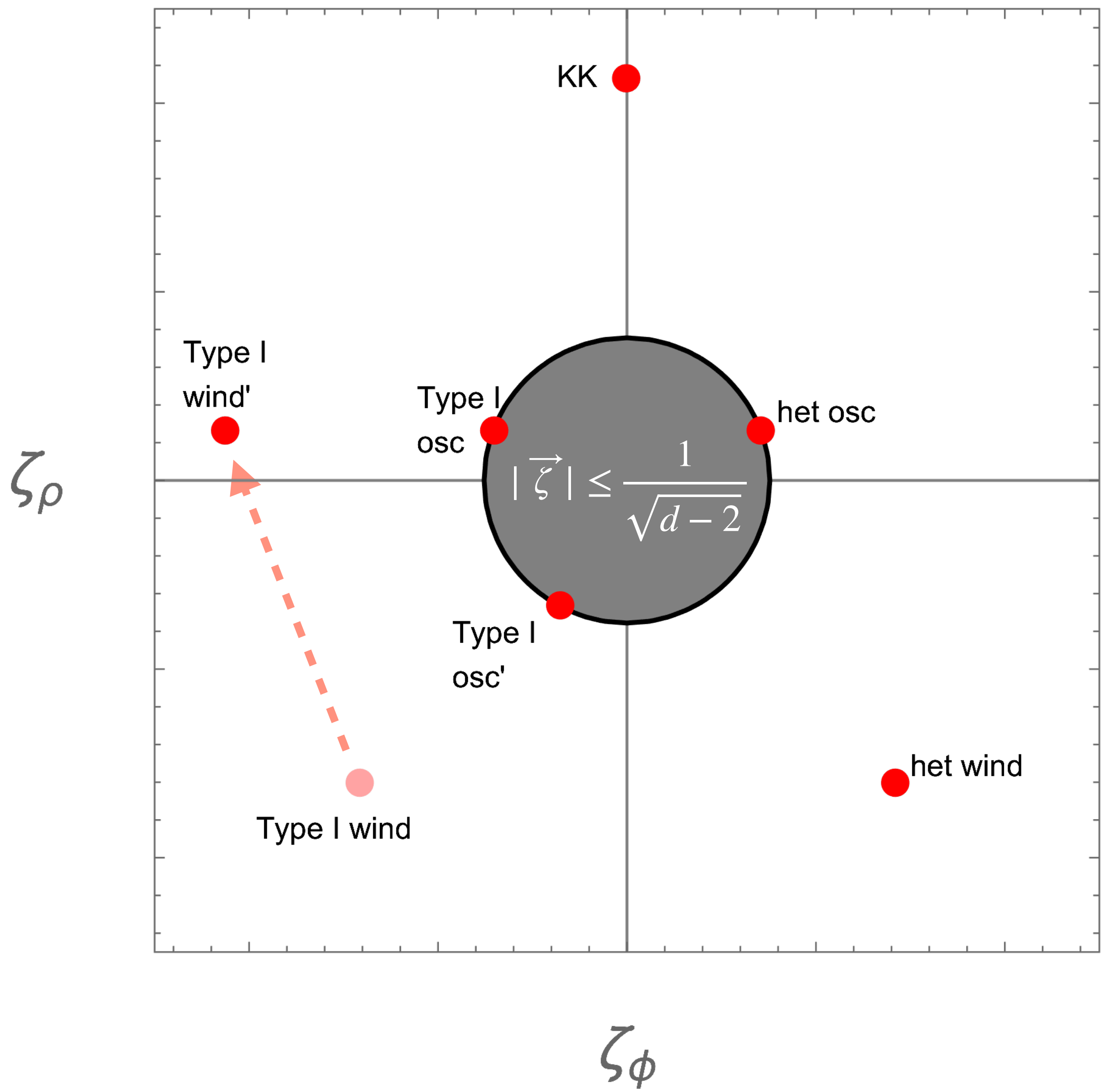


Top-Down Evidence: 9d SO(32) heterotic



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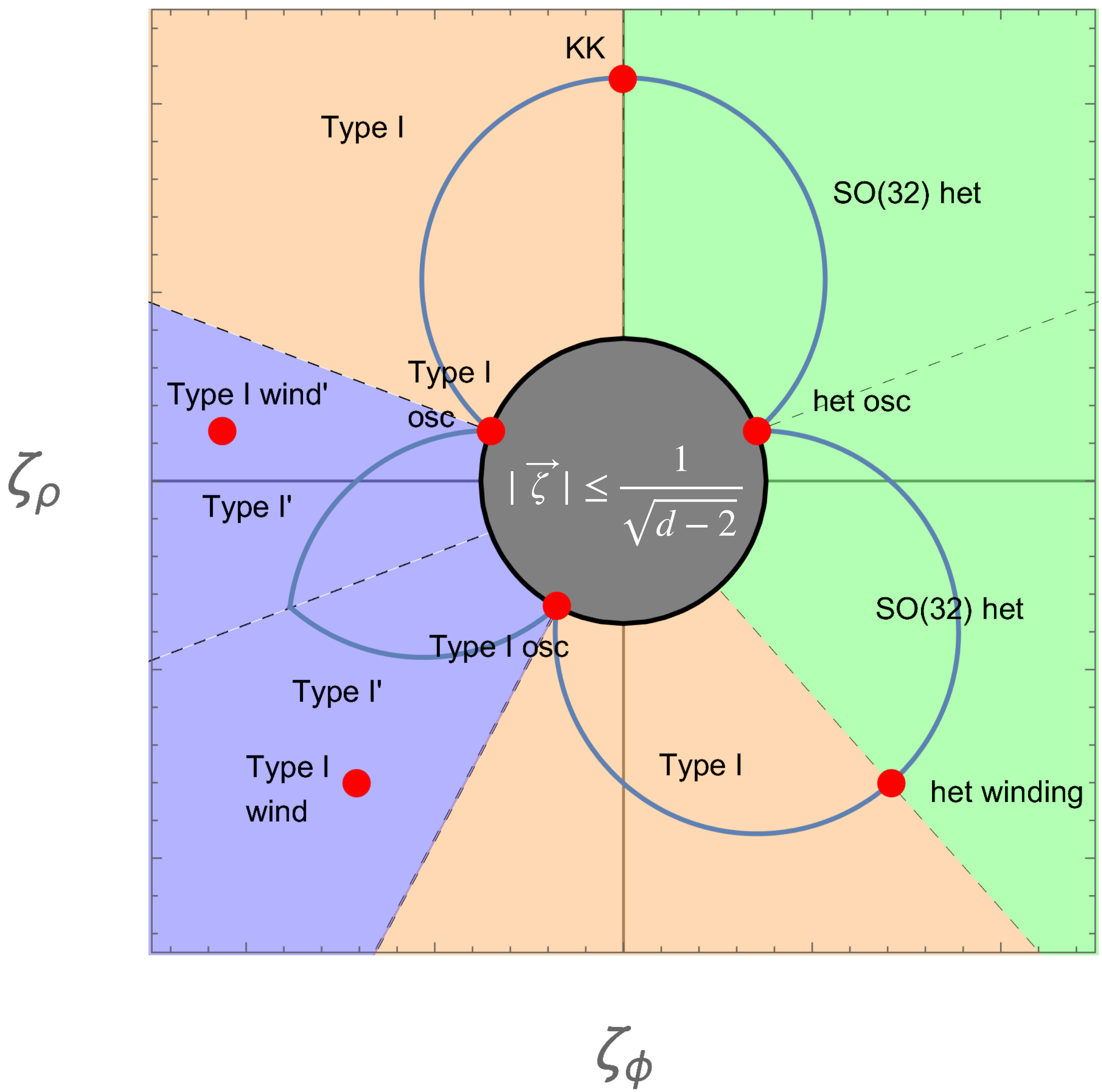
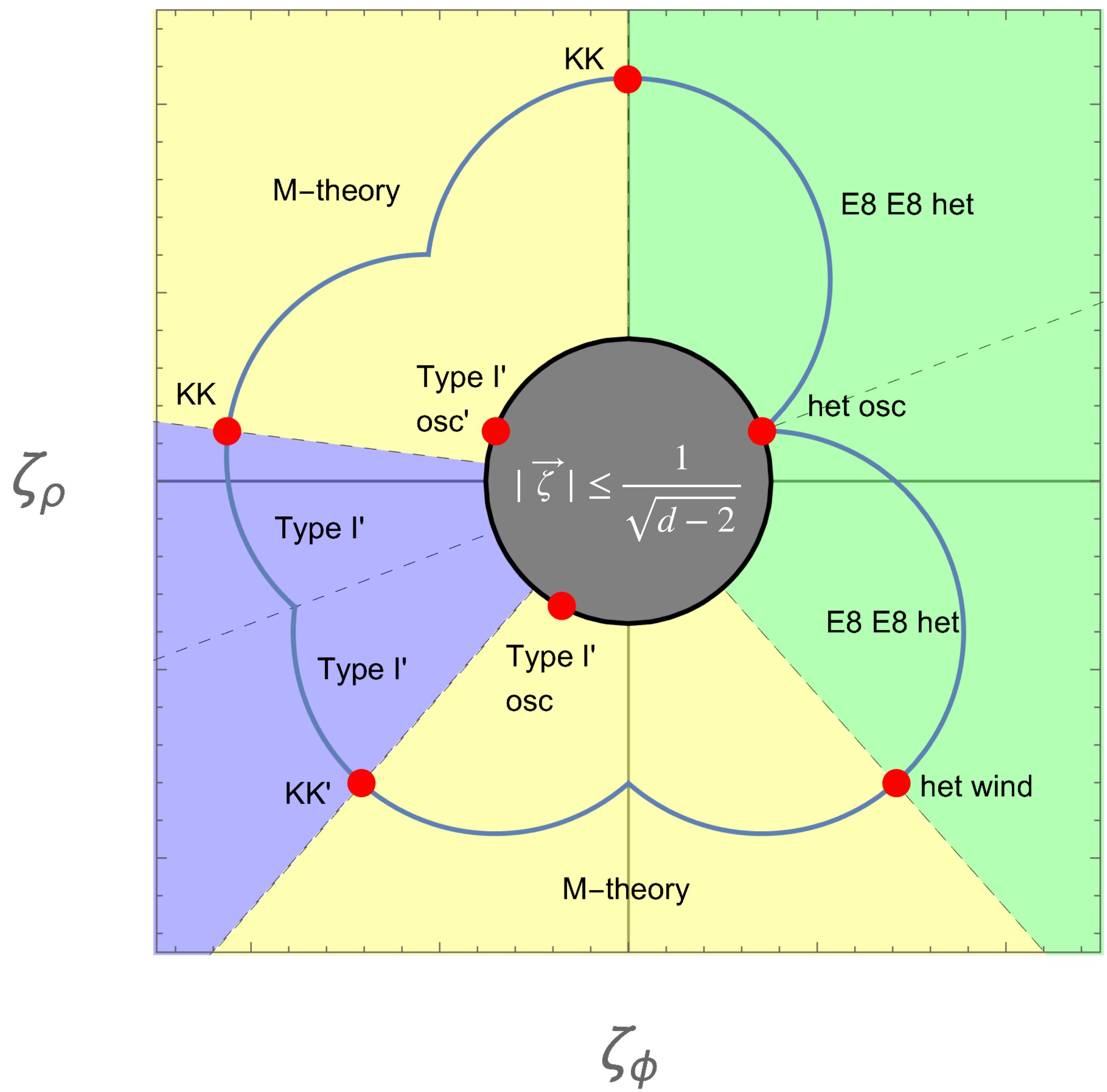


An Important Lesson!

- Decompactification of Type I' string theory in 9d leads to a 10d "running solution" in which the string coupling runs Aharony, Komargodski, Patir '07
- The Emergent String Conjecture therefore carries an important caveat: not every decompactification limit leads to a higher dimensional vacuum

Top-Down Evidence: 9d E8 E8 heterotic



Evidence for the Conjecture

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Aside: Strong Forms of the Weak Gravity Conjecture

The **Tower WGC** implies that any time a gauge coupling *g* vanishes in some infinite-distance limit, there is a tower of particles beginning at the mass scale

$$m \sim g M_{\mathrm{Pl};d}^{(d-2)/2}$$

The **WGC for strings** implies that any time a 2-form gauge coupling *g* vanishes in some infinite-distance limit, there is a tower of string oscillator modes beginning at the mass scale

$$m \sim \sqrt{g} M_{\mathrm{Pl};d}^{(d-2)/4}$$

Arkani-Hamed, Motl, Nicolis, Vafa '06
Heidenreich, Reece, TR '15, '16,
Andriolo, Junghans, Noumi, Shiu '18

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Supergravity in 5d controlled largely by cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- Here, Y^I are homogenous coordinates on vector multiplet moduli space, identified under simultaneous rescaling $Y^I \sim \lambda Y^I$
- Consider "straight-line" path in the space of these homogenous coordinates:

$$Y^I = Y_0^I + s Y_1^I \, , s \in [0, 1]$$

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 1: $\mathcal{F} \sim s$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\text{min}} \sim \exp(-\frac{2}{\sqrt{3}}\rho), \;\; 1/g_{\text{max}} \sim \exp(-\frac{1}{\sqrt{3}}\rho)$$

Tower WGC $\Rightarrow$

$$m_{\text{KK}} \lesssim g_{\text{min}} \sim \exp(-\frac{2}{\sqrt{3}}\rho) \sim \exp(-\sqrt{\frac{d-1}{d-2}}\rho)$$

WGC for strings $\Rightarrow$

$$m_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim 1/\sqrt{g_{\text{max}}} \sim \exp(-\frac{1}{2\sqrt{3}}\rho) \sim \exp(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho)$$

Expected scaling for decompactification limit!

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 2: $\mathcal{F} \sim s^2$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\min} \sim \exp(-\frac{1}{\sqrt{3}}\phi), \;\; 1/g_{\max} \sim \exp(-\frac{2}{\sqrt{3}}\phi)$$

Tower WGC $\Rightarrow$

$$m \lesssim g_{\min} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

WGC for strings $\Rightarrow$

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim 1/\sqrt{g_{\max}} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

Expected scaling for emergent string limit!

Bottom-Up Evidence: Minimal Supergravity in $d > 5$

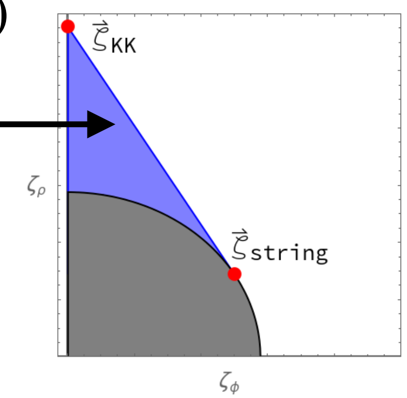
- Similar results apply to tensor multiplet moduli space in $d = 6$, moduli space in $d > 7$
- In all cases, find (assuming tower/string WGC) that infinite distance are characterized by either:
  - Charged tensionless strings with

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

  - Towers of charged particles and charged strings

$$m_{\text{KK}} \lesssim \exp(-\sqrt{\frac{d-1}{d-2}}\rho)\,, \quad M_{\text{string}} \lesssim \exp(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho)$$

  - Some intermediate regime between the two
- Fits perfectly with the sharpened Distance Conjecture and the Emergent String Conjecture



Implications of the Conjecture

Asymptotic Scalar Field Potentials

- Scalar field potentials in quantum gravity seem to fall off exponentially Dine, Seiberg ’85, Obied, Ooguri, Spodyneiko, Vafa ’18

$$V \sim \exp(-c\phi)$$

- This suggests a bound in asymptotic regimes of scalar field space of the form

$$|\nabla V| \geq c_{\min} V$$

- Like $\lambda_{\min}$, the precise value of $c_{\min}$ is often debated
- We are in a position to determine this value

Asymptotic Scalar Field Potentials

- Assume that the sharpened DC applies beyond massless moduli to scalar fields with potentials Ooguri, Vafa '05, Baume, Palti '16
- Assume the bound $\lambda \geq 1/\sqrt{d-2}$ is satisfied by either KK modes or string oscillator modes (as suggested by the Emergent String Conjecture)

$$\Rightarrow M_{\text{string}} \text{ or } m_{\text{KK}} \lesssim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

$$V \sim \exp(-c\phi) \Rightarrow H \sim \exp(-\frac{c}{2}\phi)$$

- Thus, requiring $H \lesssim M_{\text{string}}, m_{\text{KK}}$ implies Hebecker, Wrase '18

$$\boxed{c \geq \frac{2}{\sqrt{d-2}}}$$

Asymptotic Scalar Field Potentials

- In asymptotic limits of scalar field space, the bound $c \geq c_{\text{min}} = 2/\sqrt{d-2}$:
  - Is equivalent to the strong energy condition in such limits
  - Forbids accelerated expansion of the universe in such limits
  - Is exactly preserved under dimensional reduction TR '21a
  - Is satisfied in all infinite distance limits with a supersymmetric vacuum Hellerman, Kaloper, Susskind '01, TR '21b
  - Likely points to the fact that quintessence, like de Sitter space Goheer, Kleban, Susskind '03, is at best metastable

TR '21a, TR '21b, TR '22

Large Field Inflation

- The previous argument can be specialized to four dimensions, where it may have important consequences for large-field inflation
- Assume M_{string} or $m_{\text{KK}} \lesssim \exp(-|\Delta\phi|/\sqrt{2})\, M_{\text{Pl}}$
- Plugging in $H \approx 10^{-4} < m_{\text{KK}}, M_{\text{string}}$ gives

$$|\Delta\phi| \lesssim 14 M_{\text{Pl}}$$

  - $\Rightarrow m^2\phi^2$ inflation is in the swampland
  - $\Rightarrow$ Very little room for required hierarchy of scales $H \ll m_{\text{KK}} \ll M_{\text{string}}$ in controlled model Baumann, McAllister '14
- Caveat: less clear that this argument applies to axion monodromy models Silverstein, Westphal, '08, McAllister, Silverstein, Westphal '08

Summary

Main Takeaways

- **The Distance Conjecture parameter λ of the lightest tower in an infinite-distance limit is bounded as**

$$\boxed{\lambda \geq 1/\sqrt{d-2}}$$

  and saturation occurs only in emergent string limits
- Strong evidence for this bound comes from
  - Preservation under dimensional reduction
  - Examples from string theory/supergravity
  - Connection to the Emergent String Conjecture
- This bound may have important consequences for large-field inflation and scalar field potentials
  - Notably, it rules out accelerated cosmological expansion in asymptotic regions of scalar field space

Thank You