Progress and Puzzles in the 4D F-theory landscape

Strings and Geometry 2023
U. Penn

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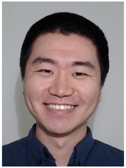
Washington (Wati) Taylor, MIT

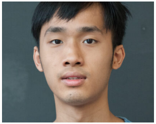
Collaborators

Based in part on recent and upcoming papers with:

Patrick Jefferson



Manki Kim



Shing Yan (Kobe) Li

Andrew Turner

Yinan Wang

Yihang Yu

Motivating questions:

What is the shape of the string landscape?

What is typical?

How does the standard model fit in?

Try to address these questions in the context of the 4D F-theory landscape

– Largest connected set of vacua with global analytic control

– Similar structure for 8 and 4 supercharges(6D, 4D):
ubiquitous multiple rigid gauge factors in almost all vacua

More specifically, in this context:

What is the relative abundance of geometric features such as rigid gauge factors E_6, E_7, E_8, and remainder cohomology $H^{rem}_{2,2}$?

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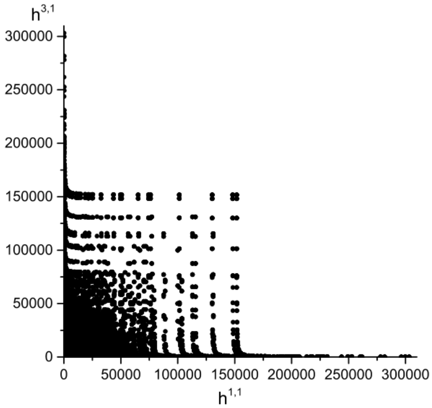
1. The shape of the 4D F-theory landscape

Elliptic CY4 over a base threefold B



[CY4's: Kreuzer/Skarke '97; almost all elliptic Gray/Haupt/Lukas, Anderson/Gao/Gray/Lee, Huang/WT]

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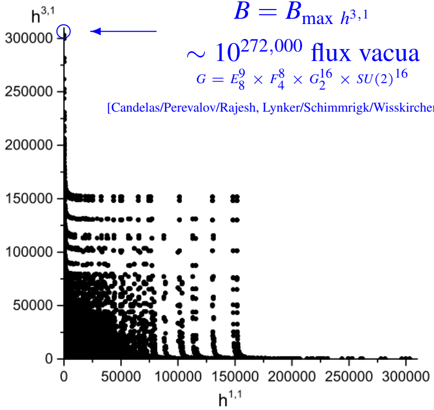

$B = B_{\text{max } h^{3,1}}$

$\sim 10^{272,000}$ flux vacua

$G = E_8^9 \times F_4^8 \times G_2^{16} \times SU(2)^{16}$

[Candelas/Perevalov/Rajesh, Lynker/Schimmrigk/Wisskirchen, WT/Wang]

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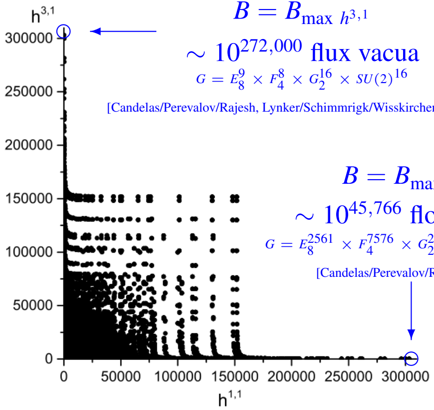

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$B = B_{\max\ h^{1,1}}$

$\sim 10^{45,766}$ flop phases

$G = E_8^{2561} \times F_4^{7576} \times G_2^{20168} \times SU(2)^{30200}$

[Candelas/Perevalov/Rajesh, Wang]

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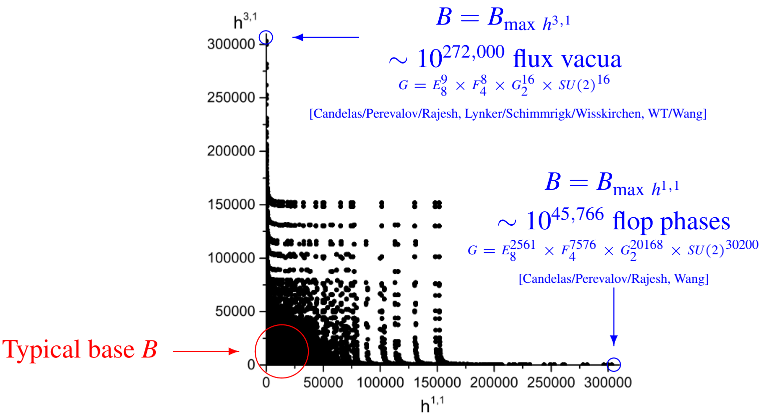

[CY4's: Kreuzer/Skarke '97; almost all elliptic Gray/Haupt/Lukas, Anderson/Gao/Gray/Lee, Huang/WT]

What measure to use?

– Standard (Ashok-Denef-Douglas) flux story $\rightarrow$ max $h^{3,1}$ dominates

– Simply counting geometries suggests max $h^{1,1}$ dominates

These approaches suggest that if the supersymmetric 4D F-theory landscape is a good guide, the SM may arise in a rigid E_8 factor.

But this is difficult to arrange. [Tian/Wang]

In this talk, we consider the broader set of allowed threefold bases B

– Start with toric B (work w/ Wang, Yu, main focus today), then consider non-toric (project w/ Kim, Li)

Motivation:

– Get a better picture of the full landscape

– Investigate prevalence of, e.g., E_6, E_7 (fluxes: $E_6, E_7 \rightarrow G_{\mathrm{SM}}$ [Li/WT])

– We really don't know the cosmological measure; e.g., finite versus infinite multiverse influences statistics.

– Maybe most of the flux vacua are inconsistent.

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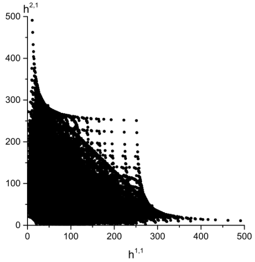
Recall 6D story. Similar landscape:



– All 61,539 toric bases explicitly constructed [Morrison/WT]

– All 162,404 semi-toric bases explicitly constructed [Martini/WT]

– All (including non-toric) bases giving $h^{2,1}(X) \geq 150$ constructed explicitly; 2640 beyond toric + semi-toric [WT/Wang]

– Toric bases a good representative sample at large Hodge numbers

– No crucial physics seems to require non-toric structure

Classifying bases for elliptic CY4's

– Start with toric threefolds B, although non-toric structure may be physically relevant

– Previous work considered flop phases (triangulations) as geometrically distinct; various lower bounds (10^{750}, 10^{3000}, $10^{44,000}$) [Long/Halverson/Sung, Wang/WT, Wang]

We will consider different triangulations as equivalent,
i.e. consider only polytope data

Justification: simplifies analysis + much of physics (e.g. rigid gauge group) independent of triangulation. Exploring more general independence of e.g. $H_{2,2}^{\text{vert}}(X, \mathbb{Z})$ ([w.i.p. w/ Jefferson, Kim], following [Jefferson/WT/Turner])

Problem: identify all integer 3D polytopes Δ such that the dual

$$\nabla_6 = \{m : m \cdot v \geq -6 \; \forall v \in \Delta\}$$

contains the origin as an interior point.

(Note: includes mild orbifold singularities in base, ok for F-theory)

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Finding all base polytopes (ongoing work w/ Wang, Yu)

Computationally challenging: Far too many polytopes to enumerate, currently running different codes on clusters in China and at MIT to estimate statistical shape of landscape. Note: infinite representations for each polytope.

Approaches:

1) Pick v points in a box of size r, estimate statistics

– Good for small number of vertices v

2) Monte Carlo: start from e.g. $\mathbb{P}^3$, add or subtract one vertex at a time randomly, put in canonical form, repeat.

– Needs cutoff k for distance to next vertex

– Global thermalization?

3) Construct all minimal dual ∇_6, sample in corresponding boxes

Have implemented 1, 2, match in appropriate regime; working on 3

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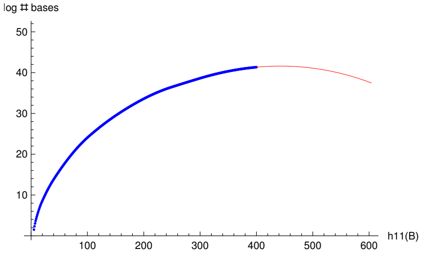
Monte Carlo results (*tentative conclusions*)

– Slows down as $h^{1,1}(B)$ increases; current results for $h^{1,1}(B) \leq 400$

– Verified results converge well as cutoff $k = 1, 2, \ldots$ (use $k = 2$)



– Peak seems to be around $h^{1,1}(B) = 430$, $N(430) \sim 10^{41}$; vertices: $v \sim 45$.

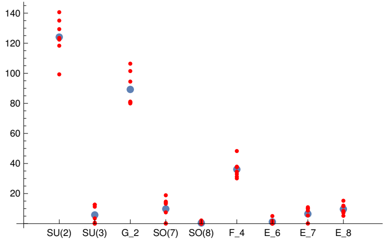
– Total number of bases (flop equivalence classes) $N \sim 10^{43}$.

– Thermalization may only be local: significant fluctuations in N'/N; investigating possible hidden dominant regions

Gauge group factors (*tentative conclusions*)

Typical gauge group near peak:

$$G \sim SU(2)^{124} \times SU(3)^6 \times G_2^{89} \times SO(11)^{10} \times F_4^{36} \times E_6 \times E_7^6 \times E_8^{10}$$

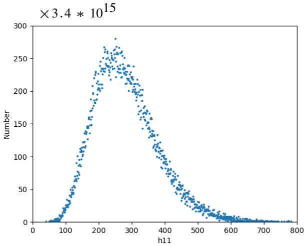


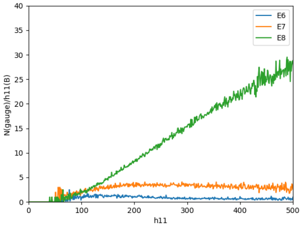
– Plenty of room for SM from E_7 or E_8 + dark matter

– $SU(2), G_2, F_4, E_8$ grow roughly linearly in $h^{1,1}(B)$; at 400, $N(E_7) \sim N(E_8)$

Alternative approach: fix # vertices (*tentative conclusions*)

Example: $v = 10$, box size 19^3





– Match prediction for $v = 10, h^{1,1} = 100$ to MC ($\sim 10^{14}$)

– Linear growth of $E_8, SU(2), G_2, F_4$ natural: boundaries at distance 6

Toric threefold bases: comments

– Typical base seems to have $h^{1,1}(B) \sim 400, h^{1,1}(X) \sim 1000$

– Possible that E_7 factors are present on typical base

– Further work needed to understand thermalization, "stretched" cases

– Number of bases (flop equivalent) $\sim 10^{43} \ll$ # triangulations, fluxes

Beyond toric: hypersurfaces/CI (w.i.p. Kim, Li)

Consider a hypersurface $B \subset A_4$ or complete intersection

For any given toric A, finite number of $0 < B < -K_A$; can compute structure of base B using various methods

Question: place explicit finite bounds? (known finite B [Di Cerbo/Svaldi])

of A unbounded (e.g., $\mathbb{P}^1$ bundle over any B w/ arbitrary twist)

Motivation:

Middle cohomology decomposes into [Greene/Morrison/Plesser, Braun/Watari]

$$H^4(X) = H^{2,2}_{\text{vert}}(X) \oplus H^4_{\text{hor}}(X) \oplus H^4_{\text{rem}}(X) ,$$

Remainder flux needed for hypercharge breaking $SU(5), E_6, E_7 \to G_{\text{SM}}$

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Standard Model constructions in F-theory

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– Many E_8 factors but no clear flux breaking to SM + matter [Tian/Wang, Li/WT]

– E_6, E_7 factors rarer but common on "typical" bases; flux bk $\rightarrow G_{\rm SM}$ [Li/WT]

Puzzles and open questions:

– Can we say enough about the measure to determine what is really "typical"?

– Is there something wrong with most expected flux vacua?

– Are most base flops essentially irrelevant?

– Does a typical threefold base B give remainder cycles?

– Can we make sense of strongly coupled matter ($\sim$ SC matter) for e.g. E_8?

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