New Perspectives on Heterotic Conifold Transitions

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Based on work with James Gray and Callum Brodie

arXiv:2211.05804,23xx.xxxxx

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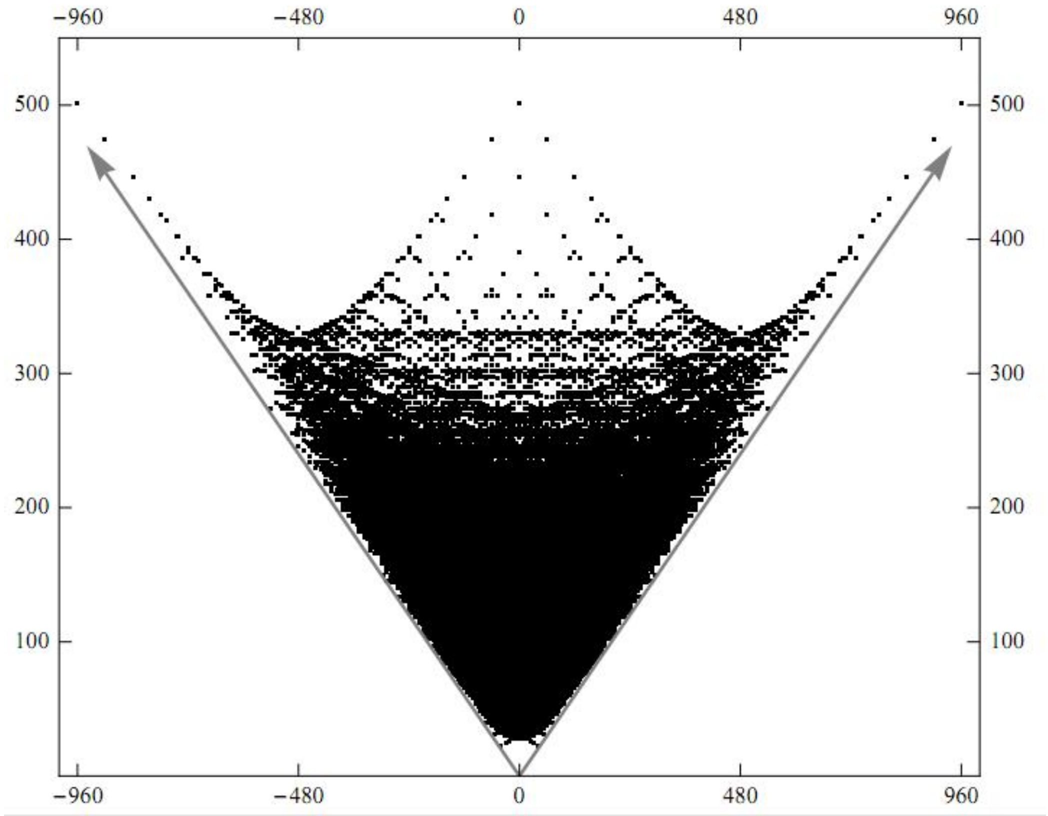
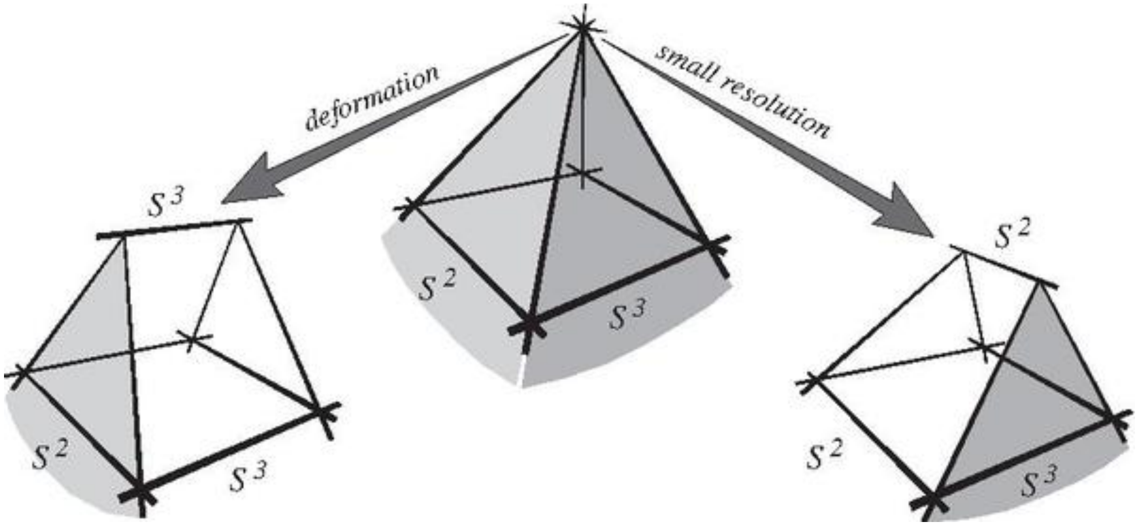
Geometric Transitions

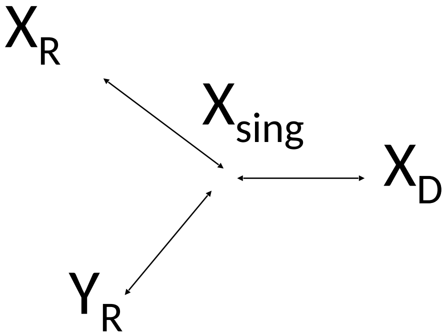
- Connect (most) Known CY manifolds
- All CY manifolds? (Reid)
- All SU(3) Structure manifolds??



Geometric Transitions

- Topology changing transitions: Conifolds and Flops





- Geometrically these transitions are often well understood.
- They are also well understood field-theoretically in some contexts (i.e. Type II).

- In heterotic string theory things are more difficult. The theory has a Bianchi Identity:

$$dH = -\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

Three-form

Negative sources from gauge and positive sources from gravitational sectors

- Chern class condition:

$$[\mathrm{tr} R \wedge R] = [\mathrm{tr} F \wedge F] \Rightarrow c_2(\Omega_X) = c_2(V)$$

- With 5-branes: $c_2(\Omega_X) = c_2(V) + [C]$

- This means that we cannot set the gauge fields to zero.

  Need to understand the change in the bundle during a geometric transition in addition to the CY geometry itself. This has been a stumbling block for decades.

  Motivation:

  Try and understand what happens to the gauge fields in heterotic string theory during a certain type of geometric transition (a conifold).

  See also Collins, Gukov, Picard, Yau math.DG/2102.11170 and Candelas, de la Ossa, He, Szendroi hep-th/0706.3134

Disclaimer: It is very hard to maintain good control of the N=1 theory in these singular limits.

We will begin with geometry: Find minimal modification of bundles that can traverse transition. Look for convincing structure in the resulting linked compactifications.

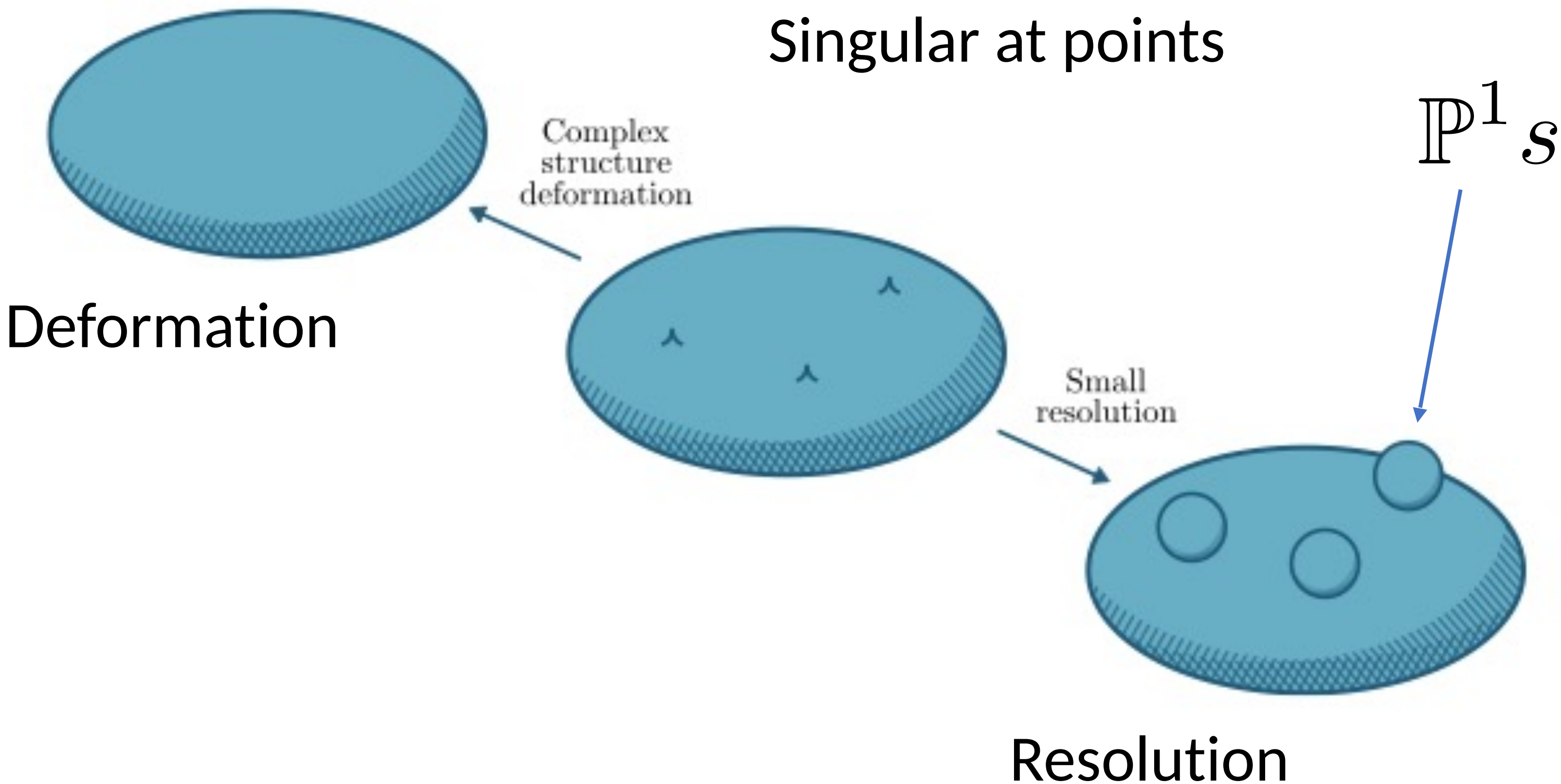
We'll give a proposal that results in new dualities between compactifications. That is, topologically distinct compactifications of the heterotic string that give rise to precisely the same four-dimensional physics.

- New computational power

- There appears to be an "embedding" of the heterotic moduli spaces as h11 increases (new perspectives on heterotic moduli spaces, model building).

Conifold transitions

- At the level of geometry we have schematically:



- In an example:

Smooth quintic

$$X_D = \left[\begin{array}{c|c} \mathbb{P}^4 & 5 \end{array} \right]$$

Nodal quintic

Deform complex structure

$$X_0 = \left[\begin{array}{c|c} \mathbb{P}^4 & 5 \end{array} \right]$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array} \right]$$

- To see this write the resolution side defining relations:

$$l_0 x_0 + l_1 x_1 = 0 \qquad q_0 x_0 + q_1 x_1 = 0$$

in matrix form:

$$\begin{pmatrix} l_0 & l_1 \\ q_0 & q_1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = 0$$

We have a solution for the $\mathbb{P}^1$ coordinates iff

$$l_0 q_1 - l_1 q_0 = 0$$

  - The solution is a full $\mathbb{P}^1$ iff
$$l_0 = l_1 = q_0 = q_1 = 0$$
  - The solution is points in $\mathbb{P}^1$ otherwise.

How do the tangent bundles change?

- Topology change:

$$ch_2(X_R) = ch_2(X_D) + [\mathbb{P}^1]$$
$$h^{1,1}(X_R) = h^{1,1}(X_D) + 1$$
$$h^{2,1}(X_R) = h^{2,1}(X_D) - \Delta$$

- As bundles on the resolution manifold

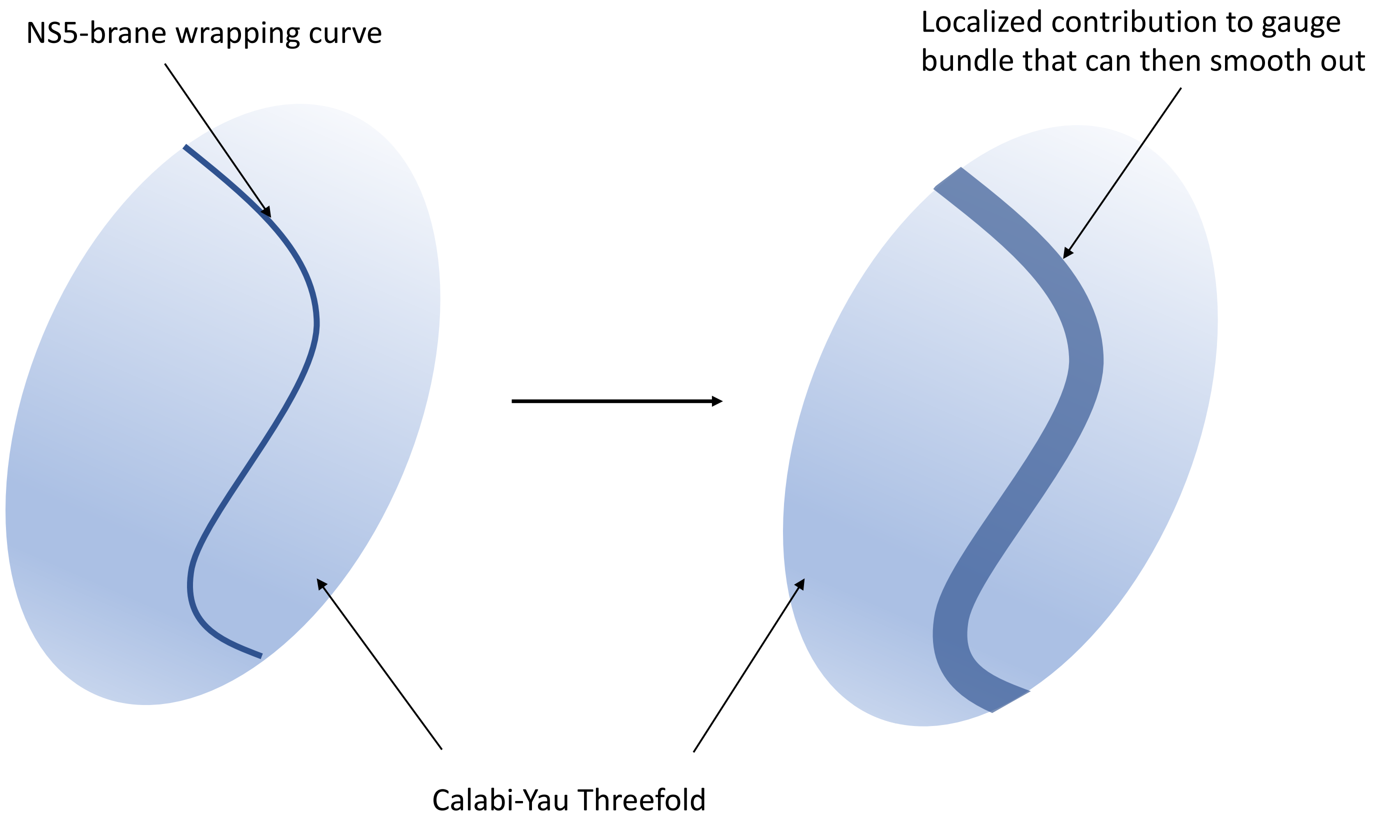
$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1{}_s}(-2) \to 0$$

Small contraction: $f : X_R \to X_0$

- This looks familiar…

Small Instanton Transitions (SITs)

- SITs are the process whereby a five-brane is "dissolved" into the gauge bundle:



- How do we describe small instanton transitions mathematically?:
  - By brute force recombining resolutions of sheaves over curves and bundles
  - Or by using (deformations of) Hecke transforms:

Aspinwall, Donagi hep-th/9806094 and Ovrut, Pantev, Park hep-th/0001133

$$0 \to V \to V_0 \to F_c \to 0$$

Sheaf supported on curve (representing five-brane) [F_c]

Starting bundle into which five-brane will be absorbed [V_0]

Combined sheaf which can be smoothed into final gauge bundle. [V]

- Rank changing version: $0 \to V \to V_0 \oplus \mathcal{O} \to F_c \to 0$

Conifolds as Small Instanton Transitions

- We will work on the resolution side of the conifold (so we can see the $\mathbb{P}^1$ s explicitly).
- We will see a transition between bundles here but it isn't really linked to the geometry. That only happens in the singular limit.

Hecke Transform for the gravitational sector:

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1 s}(-2) \to 0$$

You can view $f^*\Omega_{X_0}$ as being created when a curve like instanton is absorbed onto Ω_{X_R} . Just like an SIT!

- For our example:

Nodal Cotangent bundle:

$$0 \to \mathcal{O}(0,-5) \to \Omega_{\mathbb{P}^4} \to f^*\Omega_{X_0} \to 0$$

Sheaf supported on curve:

$$0 \to \mathcal{O}(0,-5) \to \mathcal{O}(-1,-1) \oplus \mathcal{O}(-1,-4) \to \mathcal{O}(-2,0) \to \mathcal{O}_{\mathbb{P}^1{}_s}(-2) \to 0$$

The can be combined to get the Cotangent of the resolution:

$$0 \to \mathcal{O}(-1,-1) \oplus \mathcal{O}(-1,-4) \to \Omega_{\mathbb{P}^1} \oplus \Omega_{\mathbb{P}^4} \to \Omega_{X_R} \to 0$$

The line bundle $\mathcal{O}(a,b)$ is that whose first Chern class is

$$c_1(\mathcal{O}(a,b)) = aJ_{\mathbb{P}^1}|_X + bJ_{\mathbb{P}^4}|_X$$

Including a gauge bundle in the transition

- Recall the anomaly cancelation condition in heterotic string-theory:

$$c_2(\Omega_{X_R}) = c_2(V_R)$$

- How does this change during the transition?

$$c_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = c_2(V_R) + [\mathbb{P}^1 s]$$

This is how the gravitational sector changes given the transition we have seen in the cotangent bundle.

The gauge sector must change in the same way.

- A natural guess is to include the same sheaf on the two sides! So just add the curve supported sheaf

$$\mathcal{O}_{\mathbb{P}^1{}_s}(-2)$$

to both the gauge and gravitational sectors.

- Then just recombine that sheaf into the gauge bundle with a small instanton transition:

$$0 \to f^* V_0 \to V_R \to \mathcal{O}_{\mathbb{P}^1{}_s}(-2) \to 0$$

- But there is a problem…

For cases of interest this map doesn't exist!

Something more complicated happens…

Back to the structure of the conifold…

- There is a special Weil-non-Cartier divisor that appears in the nodal geometry.

  In our example:

$$l_0 = q_0 = 0 \qquad \text{inside} \qquad l_0 q_1 - l_1 q_0 = 0$$

  Curve limiting to divisor

- One can isolate special curves associated to this on either side of the transition… $[\mathcal{C}_R] + [\mathbb{P}^1 s] = [\mathcal{C}_D]$

$$\mathcal{C}_D : \quad l_0 = q_0 = 0$$

$$\mathcal{C}_R : \quad x_1 = Q_5 = 0$$

- More than at the level of classes, the skyscraper sheaves associated to these curves are related by "brane recombination":

$$0 \to \mathcal{O}_{\mathbb{P}^1{}_S}(-2) \to f^*\mathcal{O}_{\mathcal{C}_D} \otimes \mathcal{O}(-P(\mathcal{D})) \to \mathcal{O}_{\mathcal{C}_R} \to 0$$

Sheaf on exceptional $\mathbb{P}^1{}_S$

Sheaf on resolution curve

Those two combine to form the sheaf on the deformation curve (up to a twist that disappears in transitioning to the deformation manifold)

- This actually happens in the nodal limit
- In our example:

$$0 \to \mathcal{O}_{\mathbb{P}^1{}_S}(-2,0) \to f^*\mathcal{O}_{\mathcal{C}_D}(-1,0) \to \mathcal{O}_{\mathcal{C}_R} \to 0$$

The full structure of the transition

- On the next slide is the map of the transition, presented at the level of classes for clarity.

- All of the sequences of sheaves required for this process to occur exist and can be written down explicitly.

- In what follows V_s is a “spectator bundle” which goes through the transition in a trivial manner.

$$\mathrm{c}_2(\Omega_{X_R}) = \mathrm{c}_2(V_R)$$

Pair create curve supported sheaves ↓ Pair create curve supported sheaves

$$\mathrm{c}_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = \mathrm{c}_2(V_R) + [\mathbb{P}^1 s]$$

SIT in cotangent bundle ↓ SIT in gauge bundle

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_R] + [\mathbb{P}^1 s]$$

↓ “Brane” recombination

$$c_2(f^*\Omega_{X_0}) = c_2(V_s) + [\mathcal{C}_D]$$

↓ SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_D)$$

- Note that all of the steps here are standard, except the "pair creation process".

  Perhaps this should not be regarded as so non-standard however:
    - The process preserves all known conserved charges, and in such situations processes are normally expected to occur (consider brane/anti-brane creation).
    - Unlike brane/anti-brane creation supersymmetry is preserved because of the way charges enter the heterotic anomaly cancelation condition.
    - The process concerns only known objects.
    - The process can only occur in singular limits of the manifold, and so we would not expect to come across it routinely.

- Here is an example pair of bundles to go with our example geometry:

Deformation side bundle:

$$0 \to \mathcal{O}(-5) \to \mathcal{O}(-1)^{\oplus 5} \to V_D \to 0$$

Spectator bundle: $V_s = \hat{V}_s \oplus \mathcal{O}$ where

$$0 \to \mathcal{O}(-4) \to \mathcal{O}(-1)^{\oplus 4} \to \hat{V}_s \to 0$$

Resolution side bundle:

$$0 \to \begin{array}{c} \mathcal{O}(-1,-5) \\ \oplus \\ \mathcal{O}(0,-4) \end{array} \to \begin{array}{c} \mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \\ \oplus \\ \mathcal{O}(0,-1)^{\oplus 4} \end{array} \to V_R \to 0$$

What happens to the degrees of freedom of the theory?

Deformation side:

Kahler moduli: $h^{1,1} = 1$

Complex Structure: $h^{2,1} = 101$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 324$

Total = 426

Resolution side:

Kahler moduli: $h^{1,1} = 2$

Complex Structure: $h^{2,1} = 86$

Bundle Moduli: $h^1(\mathrm{End}_0(V)) = 338$

Total = 426

This phenomenon has appeared before…

- In (0,2) GLSMs it is possible in non-geometric phases to interchange fields in a way that leaves the theory invariant.

Calabi-Yau Defining Relations ⟷ Monad Maps

- Moving back to geometric phases results in different GLSMs with the **same target space theory** (charged/singlet matter identical).
- (0,2) Target Space Duality (Distler + Kachru, Blumenhagen…)
- Upon inspection, the two bundles I displayed earlier are TSD!

Distler and Kachru hep-th/9707198, Blumenhagen hep-th/9707198 and hep-th/9710021, Blumenhagen and Rahn 1106.4998, Anderson and Feng 1607.04628

- More generally:

  We find that any two target space duals linked by a conifold (or chain of conifolds) are bundles that are linked by our transitions.

  Any two monad bundles linked by our transition are target space dual.

Can we see why the moduli matching in target space duality works geometrically?

Moduli

- Geometrically, why do the moduli (for example) of the theory match? Let's look at the gauge singlets as an example.
- The moduli of the Hecke transform for one of the gauge bundles:

$$\mathrm{Ext}^1(V,V) = H^1(V_s \otimes V_s^\vee) \oplus \mathrm{Ext}^1(V_s, \mathcal{I}_C)$$
$$\oplus \mathrm{Ext}^1(\mathcal{I}_C, V_s) \oplus H^0(C, \mathcal{N}_C)$$

(the change in the moduli under deformation to the smooth bundle is understood)

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Stay the same on the two sides of the conifold

(the change in the moduli under deformation to the smooth bundle is understood)

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Changes in exactly the opposite way to the Hodge numbers of the manifolds

(the change in the moduli under deformation to the smooth bundle is understood)

- Generally you can prove that

$$h^0(\mathcal{C}_R, \mathcal{N}_{\mathcal{C}_R}) - h^0(\mathcal{C}_D, \mathcal{N}_{\mathcal{C}_D}) = \Delta h^{1,1} + \Delta h^{1,2}$$

so that

$$\begin{aligned} \mathrm{Ext}^1(V, V) = H^1(V_s \otimes V_s^\vee) \oplus \mathrm{Ext}^1(V_s, \mathcal{I}_C) \\ \oplus \mathrm{Ext}^1(\mathcal{I}_C, V_s) \oplus \boxed{H^0(C, \mathcal{N}_C)} \end{aligned}$$

always changes so as to keep the total number of moduli invariant.

- So we have a geometric explanation/proof of TSD moduli matching

- Can also perform whole transition/moduli matching for 5-branes
- The two theories of five-branes on Calabi-Yau manifolds, as defined using “preferred curves”, are dual.

- 5-brane limit not easily understandable in the GLSM language

Summary

- We have proposed a consistent/minimal geometric process by which compactifications of the heterotic string can undergo conifold transitions.

- Compactifications which are related by such transitions —> identical four dimensional physics.

- For certain types of compactifications (w/o five-branes) this equivalence of theories corresponds to TSD – so we have given that duality a geometric interpretation.

- More generally this duality is novel and has several potential consequences.

Further Topics

- The pair creation effect we discussed should be studied in isolation in a simpler setting (and other theories).
- Other geometric transitions? (E.g. flops)
- F-theory duals?
- Broader moduli space of heterotic compactifications? (Important for model building/searching for examples of phenomena).
- Heterotic "Wall's Data"?