

# **Generalized quotients and holographic duals for 5d S-fold SCFTs**

Based on arXiv:2211.13243 with O. Bergman, H. Kim, C. Uhlemann

# A different corner of the F-theory/IIB landscape

## 6d AdS Solutions of IIB supergravity

- Dual to 5d superconformal field theories (SCFTs)
- Near-horizon of IIB 5-branes webs
- They have non-trivial axio-dilaton
- 5-branes sources have been successfully incorporated
- Solution with perturbative 7-branes are known (D7, O7)
- They enlarge the landscape of IIB vacua

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- Solving a (personal) “longstanding” challenge:  $AdS_6$  **F-theory solutions!**
- New construction of 5d SCFTs
- Generically do not have a low-energy gauge theory description, holography key to access observables

# Outline

1. Holographic Setup
2. S-folds of brane webs
3. Back to the near-horizon limit
4. Observables: Free Energy, Central Charges, Twisted Indices

# Holographic Setup

# AdS holographic dual solutions

The strategy adopted to classify these solutions is the standard one

- Solve the supersymmetric equations of motion of IIB, with ansatz

$$AdS_6 \times_w M_4$$



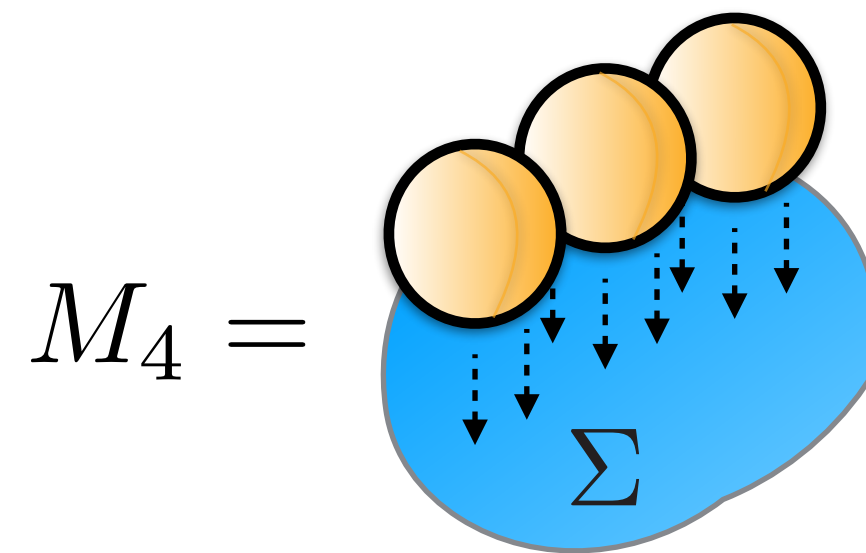
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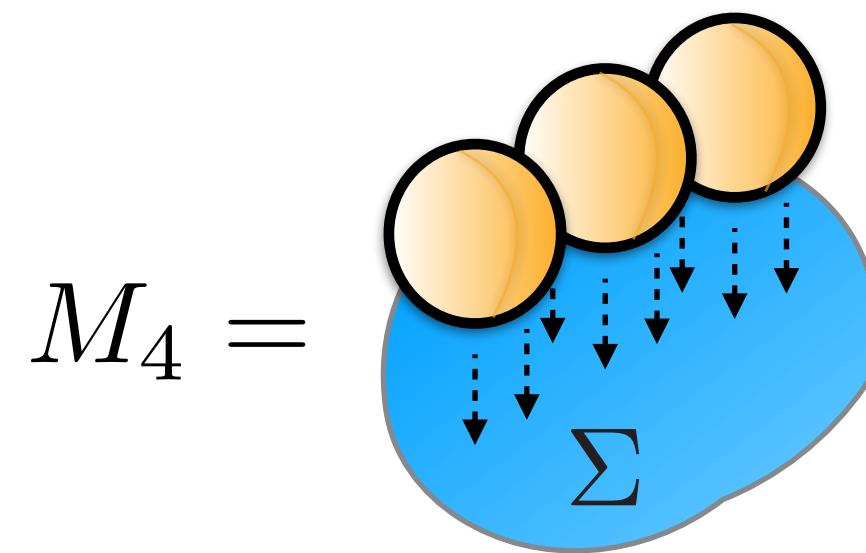
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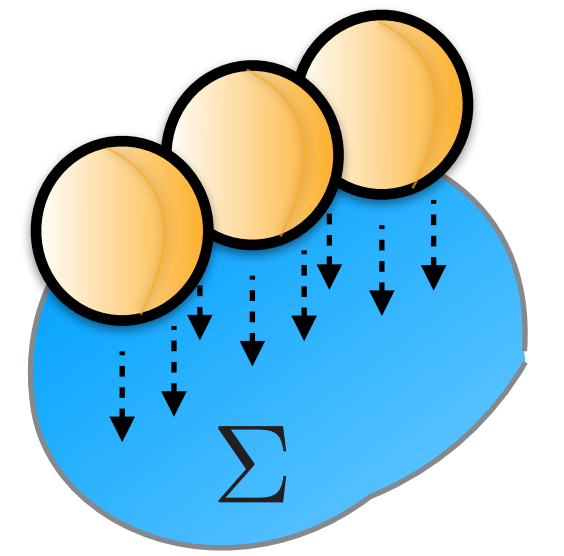
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- Inputting into the equations that  $\Sigma$  is a complex surfaces, leads to a more explicit classification

[D'hoker, Gutperle, Karch, Uhlemann 17-21] [Legrarnandi, Nunez 21]

# AdS holographic dual solutions

$$AdS_6 \times_w M_4$$



The metric is fully determined by two **holomorphic functions**  $\mathcal{A}_\pm(w)$  of the Disc coordinates  $w$  and derivatives  $\partial\mathcal{A}_\pm, \bar{\partial}\bar{\mathcal{A}}_\pm$

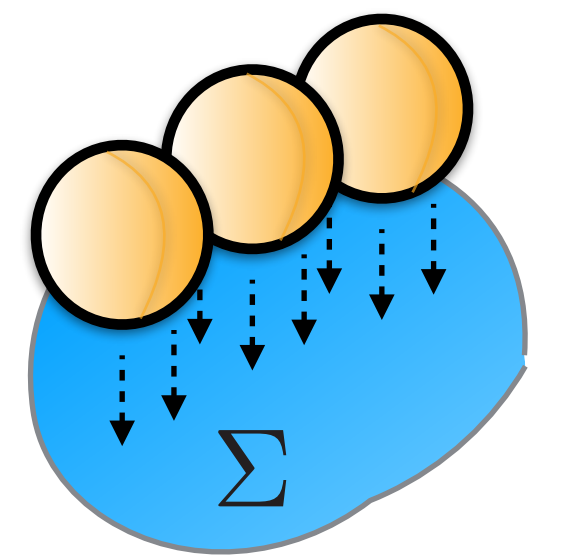
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$$ds^2 = f_6 ds_{AdS_6}^2 + f_2^2 ds_{S^2}^2 + 4\rho^2 |dw|^2$$

$$f_6(\mathcal{A}_\pm, \bar{\mathcal{A}}_\pm), \quad f_2(\mathcal{A}_\pm, \bar{\mathcal{A}}_\pm), \quad \rho(\mathcal{A}_\pm, \bar{\mathcal{A}}_\pm)$$

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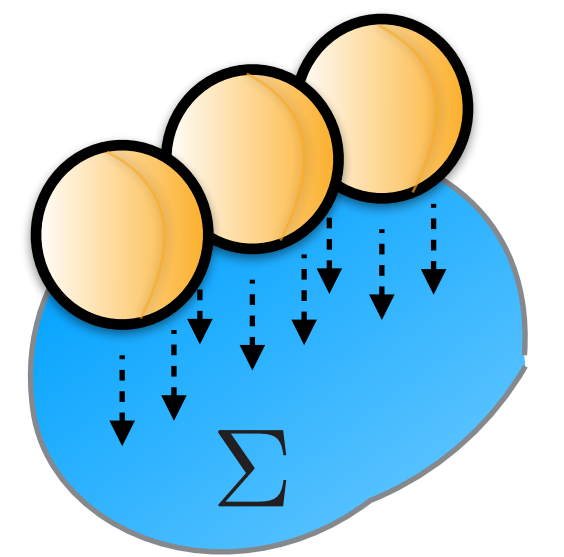
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The rest of the fields are also fully determined by  $\mathcal{A}_\pm(w)$  and their derivatives

$$\mathcal{F}_3(\mathcal{A}_\pm, \bar{\mathcal{A}}_\pm) = H_3 + iF_3 \quad \tau(\mathcal{A}_\pm, \bar{\mathcal{A}}_\pm) \quad F_5 = 0$$

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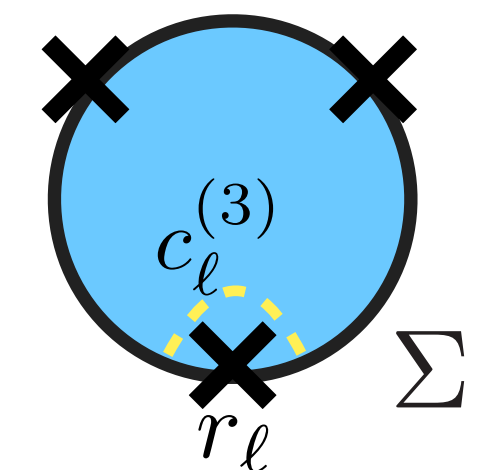
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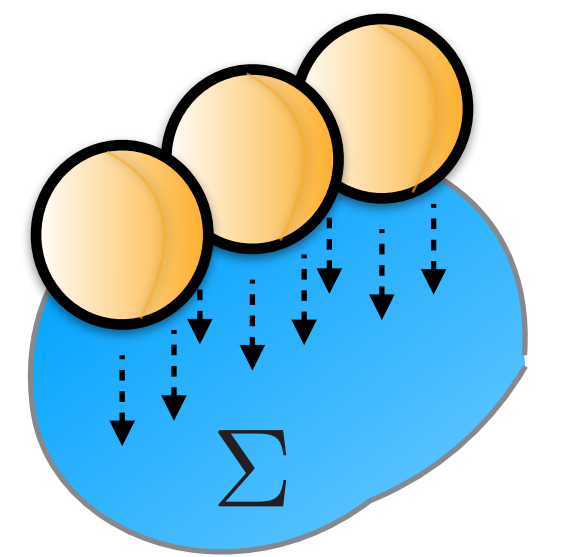
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The differential  $\partial\mathcal{A}_\pm$  have poles at the boundary of  $\Sigma$ , generating 5-brane charge

$$\text{Res}_{w=r_\ell}(\partial_w \mathcal{A}_\pm) = \frac{3}{4}\alpha'(\pm q_\ell + ip_\ell) \quad \Rightarrow \quad \mathcal{F}_3 \stackrel{w=r_\ell}{\sim} (\pm q_\ell + ip_\ell) \text{vol}(c_\ell^{(3)}) \quad c_\ell^{(3)} \cong S^3$$



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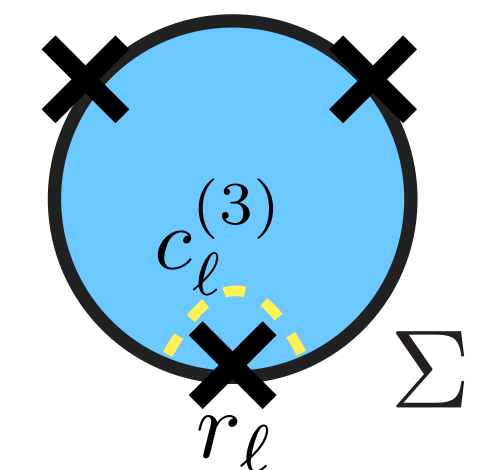
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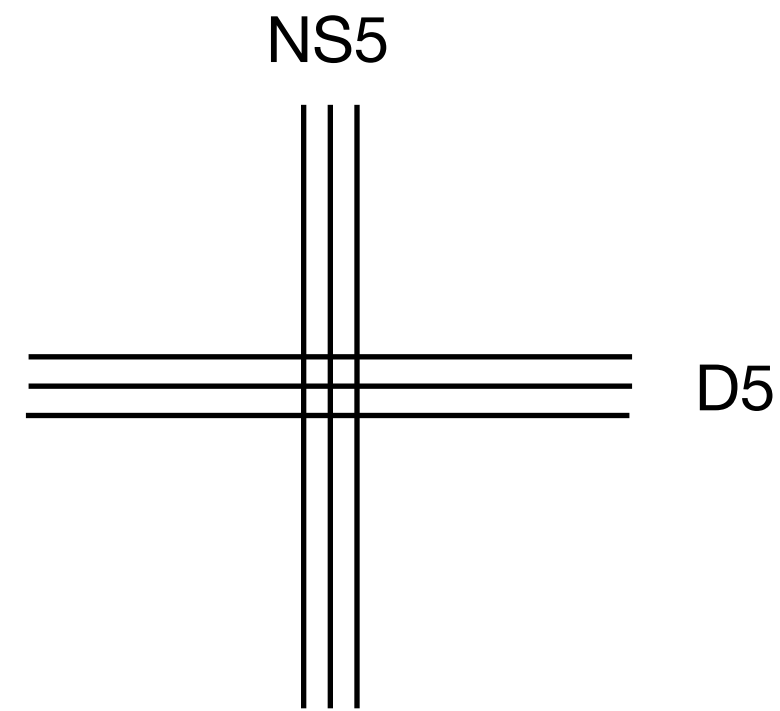


All non-trivial information are captured by  $\Sigma$



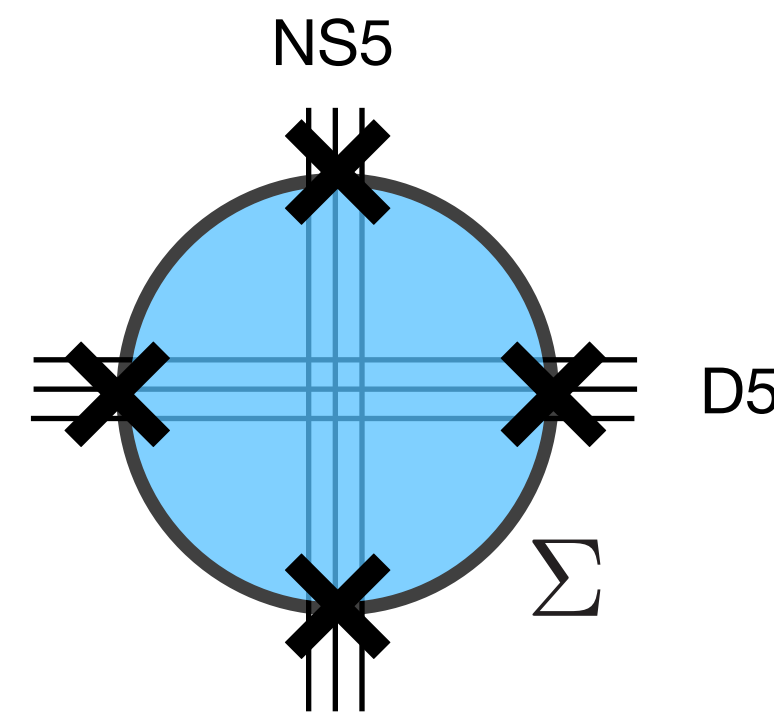
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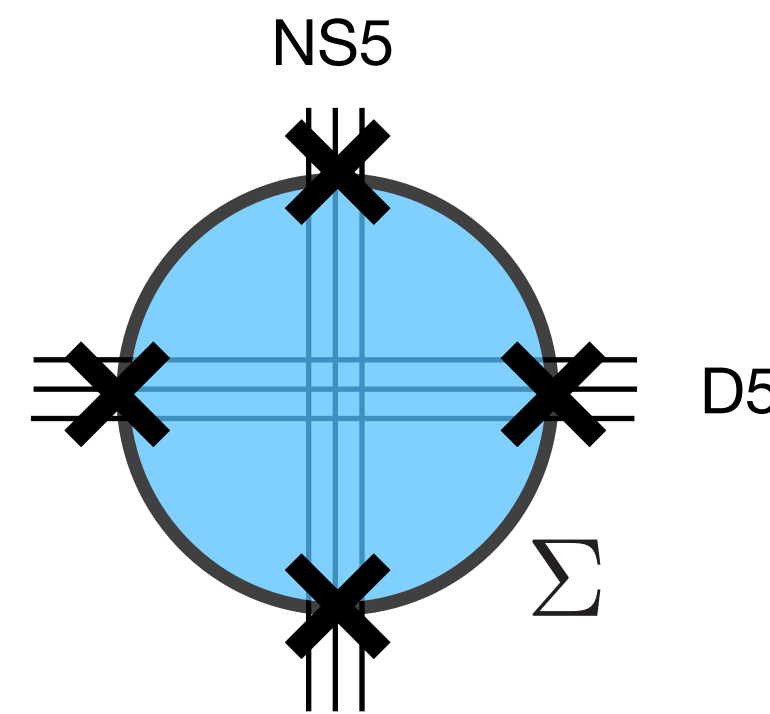
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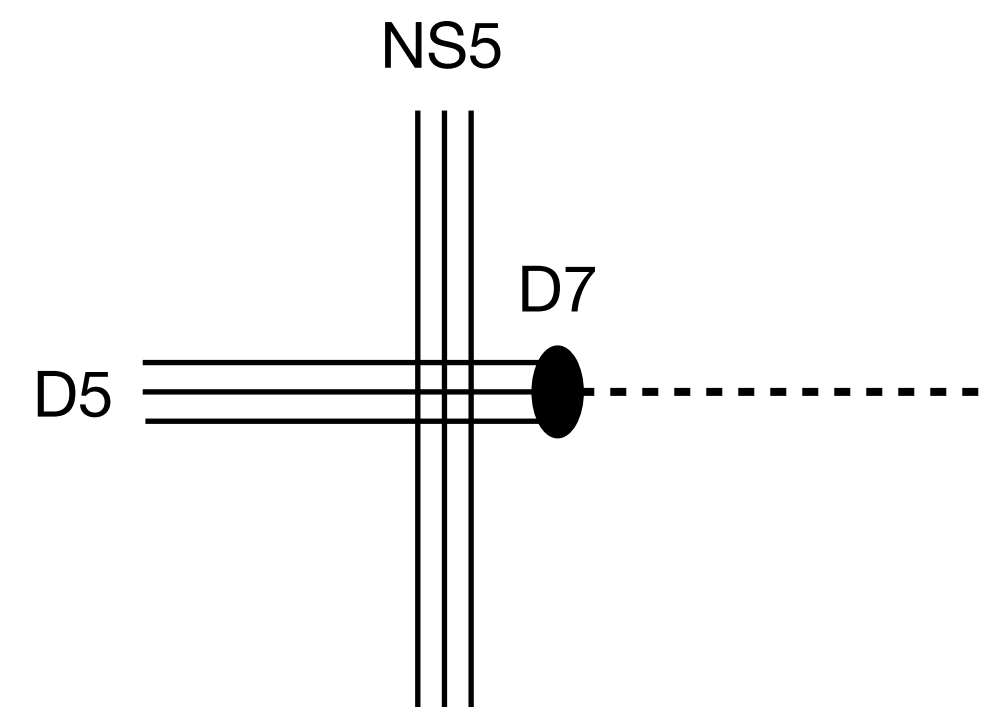
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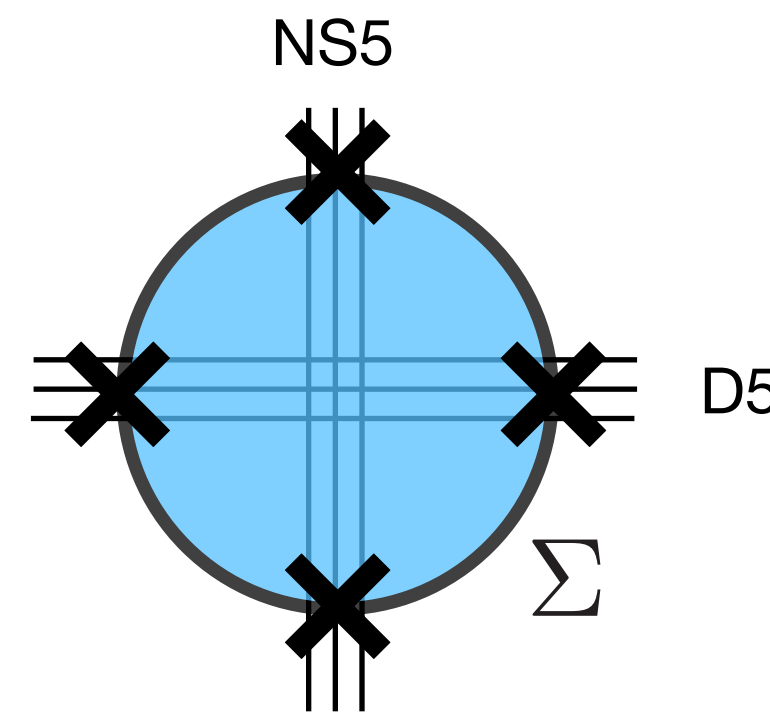
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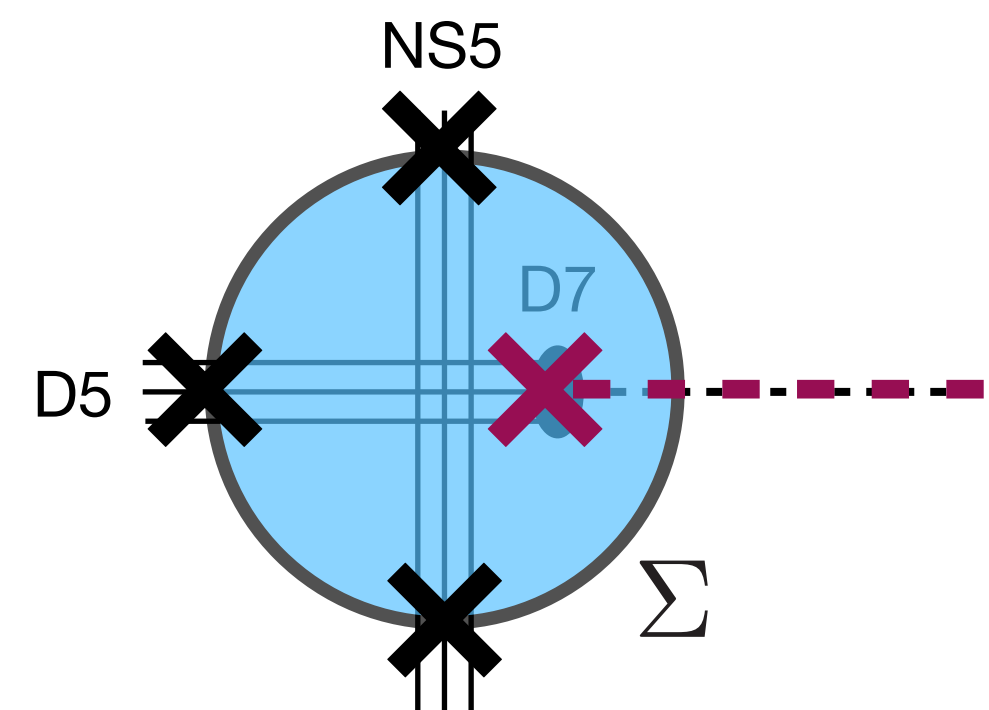
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# **$SL(2, \mathbb{Z})$ action on the disc**

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$$SL(2, \mathbb{R}) : \tau \mapsto \frac{a\tau + b}{c\tau + d} \quad SU(1, 1) : \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix} \mapsto \begin{pmatrix} u & -v \\ -\bar{v} & \bar{u} \end{pmatrix} \begin{pmatrix} \mathcal{A}_+ \\ \mathcal{A}_- \end{pmatrix}, \quad u = \frac{1}{2}(a + ib - ic + d), \quad v = \frac{1}{2}(-a + ib + ic + d)$$

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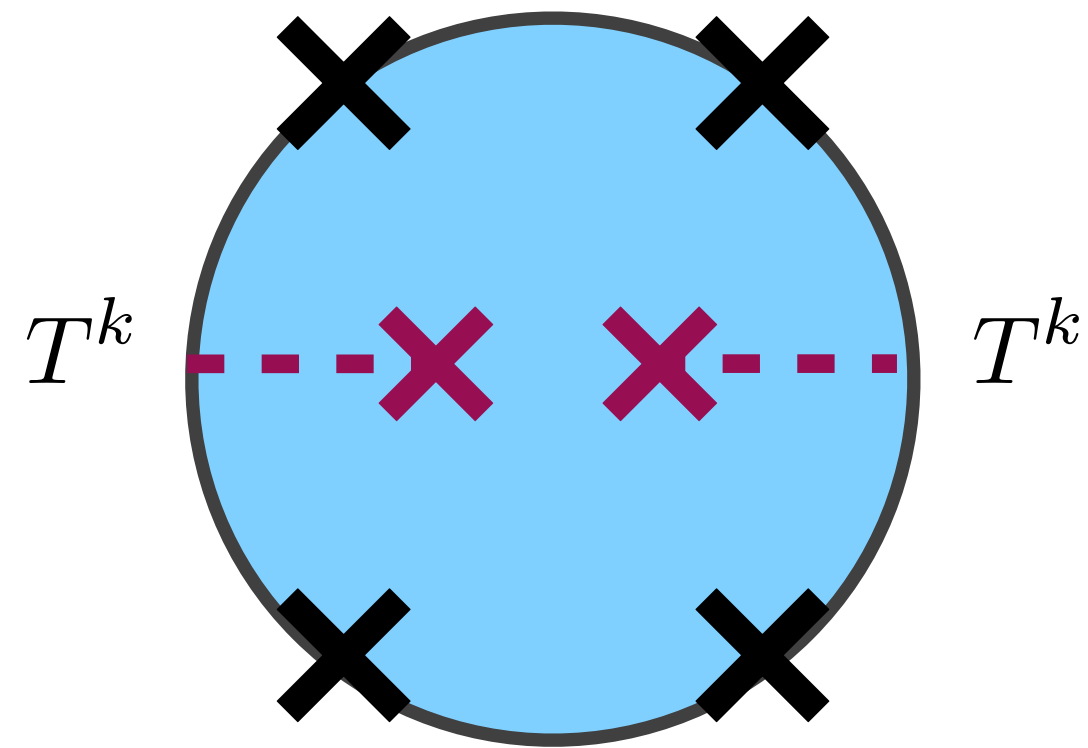
For a D7 brane close to  $w = w_i$  the metric takes the form of a flat D7 solution

$$ds^2 \sim \left( ds_{\text{AdS}_6}^2 + \frac{1}{9} ds_{S^2}^2 \right) + \text{Im}(\mathcal{H}) |dw|^2, \quad \tau \sim \mathcal{H} \sim -\frac{in_i^2}{2\pi} \ln|w - w_i| \quad \Leftrightarrow \quad M_{n_i[1,0]} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & n_i \\ 0 & 1 \end{pmatrix} = T^{n_i}$$

# Solutions with O7-planes

[Uhlemann 19]

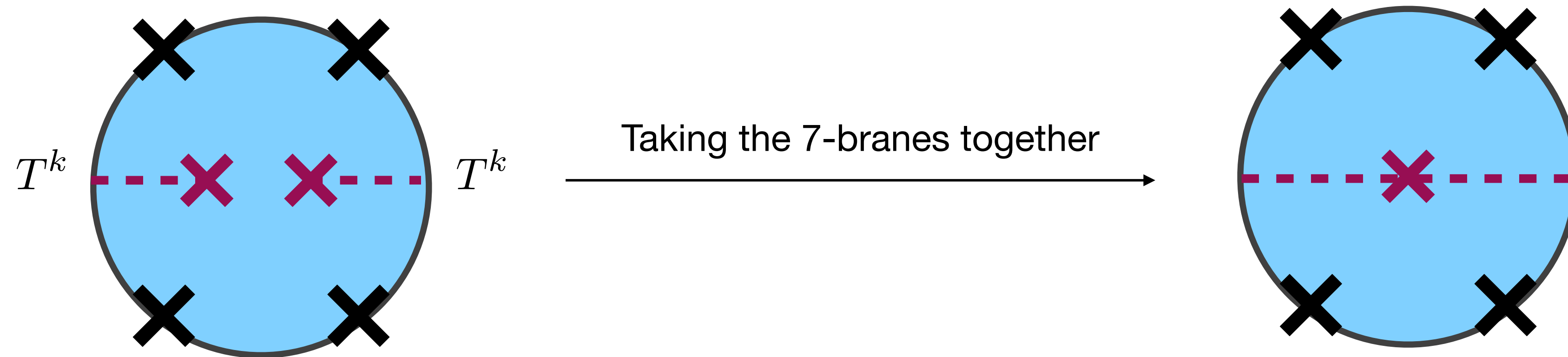
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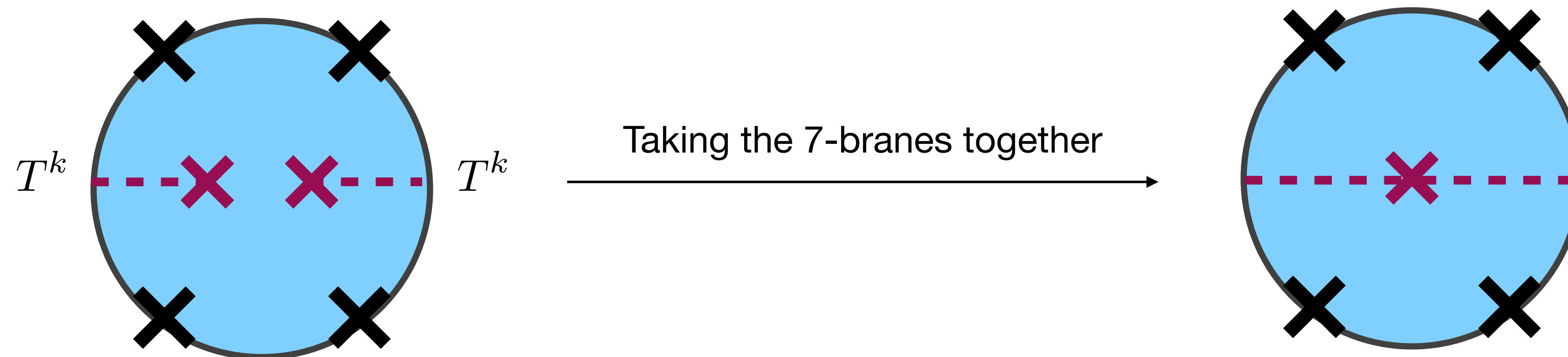
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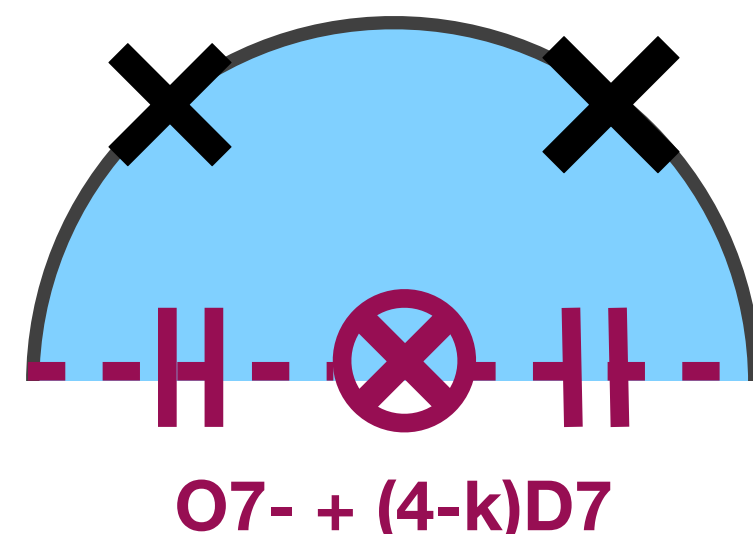
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A rotation by  $\pi$  together with the action of  $-1_2 \in SL(2, \mathbb{Z})$  on  $\mathcal{A}_{\pm}$  leaves the solution invariant. We can quotient by this combined  $\mathbb{Z}_2$ -action and we get



$$M_{O7- + (4-k)D7} = -1_2 T^k = -T^k$$

 = also a  $\mathbb{Z}_2$ -action fixed point



*Notice that the quotient involves a symmetry of the web and a  $SL(2, \mathbb{Z})$  transformation*

*Does this remind you of anything?*

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**S-folds**

[Garcia-Etxebarria, Regalado 15] [Aharony-Tachikawa 15] [FA, Giacomelli, Schäfer-Nameki 20] [Heckman, Lawrie, Lin, Zhang, Zoccarato 22] [Assel, Tomasiello 18]

**S-folds quotient of 5d SCFTs**

# Quotients of 5d SCFTs

[J.Tian, Y-N. Wang 21]

[B. Acharya, N. Lambert, M. Najjar, E.E. Svanes, J. Tian 21]

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Quotient of 5d SCFTs have been studied in the context of M-theory on Calabi-Yau threefold and IIB branes webs. In this talk I focus on the latter. Strategy:

- Quotient by a  $\mathbb{Z}_n$  symmetry of Web-Plane
- Combined with  $\mathbb{Z}_n \subset SL(2, \mathbb{Z})$   $n = 2, 3, 4, 6$  transformation of 5-brane charges

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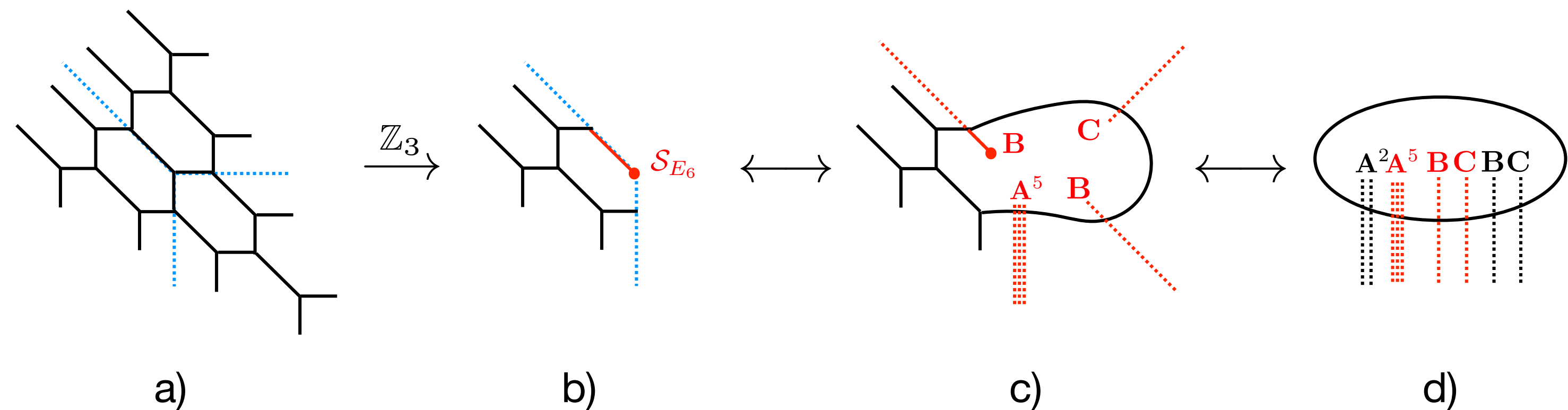
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This procedure leads to a fixed point in the web interpreted as 7-branes.

Examples:  $\mathbb{Z}_3$  quotient of T4



# Action of the quotient on the prepotential

[H. Kim, S. Kim, K. Lee 21]

The Coulomb branch prepotential for the quotient theory is given by

$$\mathcal{F}_{\mathcal{T}/\mathbb{Z}_n} = \frac{1}{n} \mathcal{F}_{\mathcal{T}}|_{\phi_{S(i)}=\phi_i}$$

The  $1/n$  factor comes from the fact that the total area of the compact faces of the webs corresponds to the first derivative of the prepotential wrt the coulomb branch scalars. The coulomb branch scalars are identified under quotient.

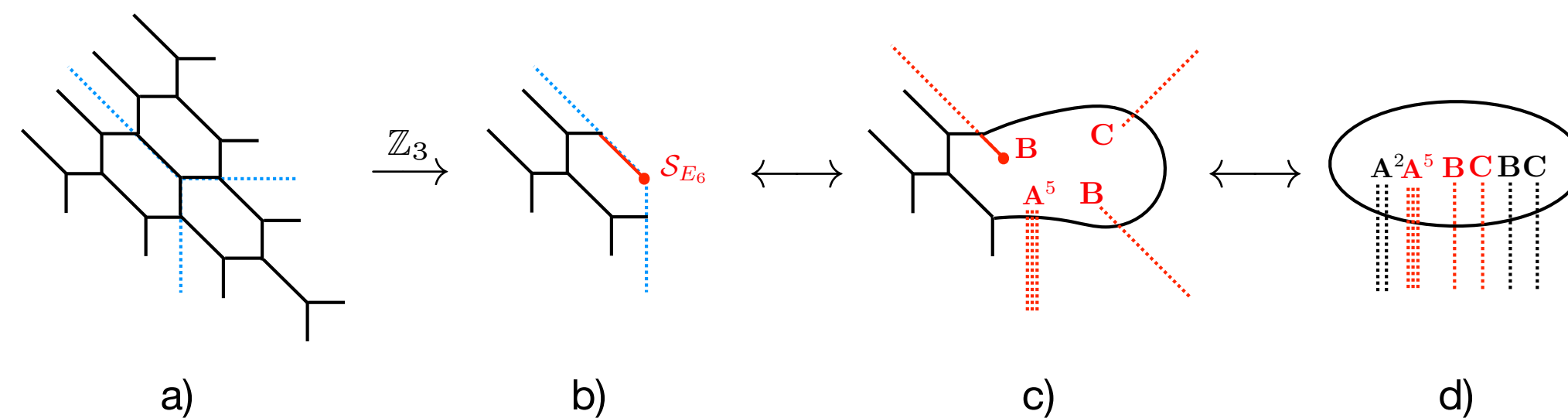
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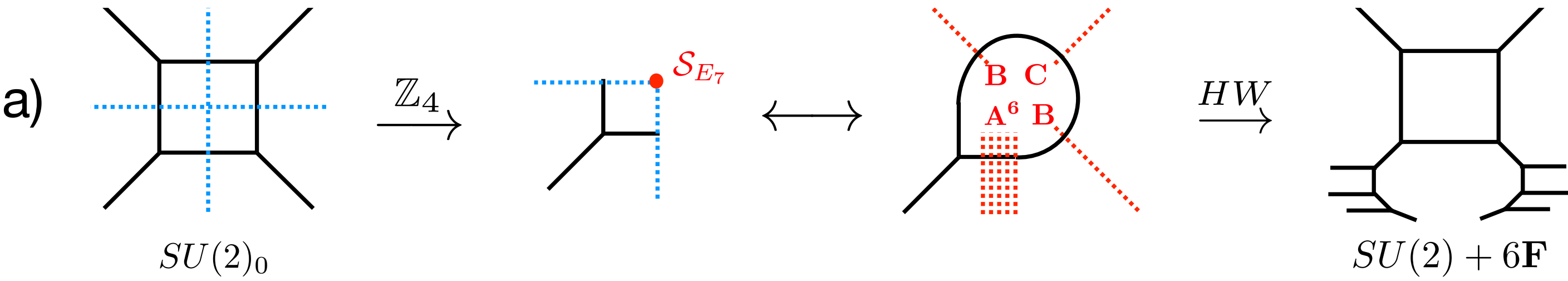
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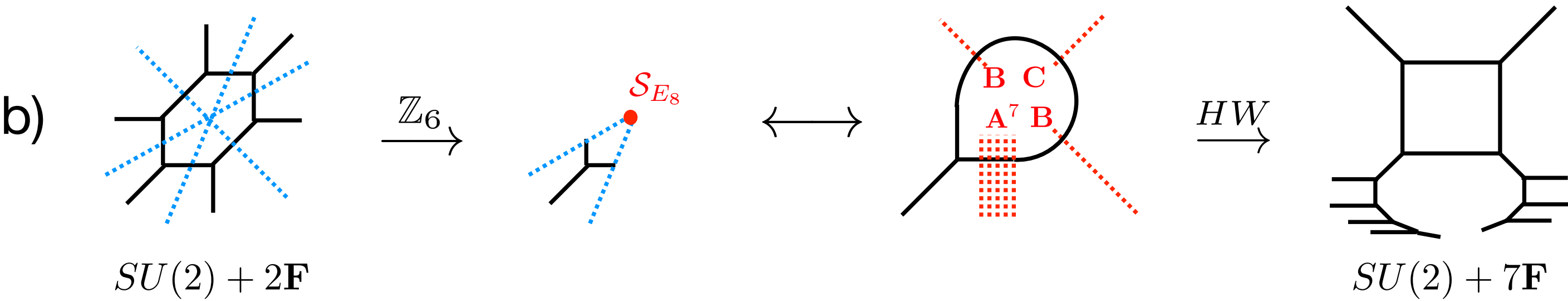
$$6\mathcal{F}_{T_4} = \sum_{i=1}^3 5\phi_i^3 - 3 \sum_{i<j}^3 (\phi_i^2 \phi_j + \phi_i \phi_j^2) + 6\phi_1 \phi_2 \phi_3 \longrightarrow 6\mathcal{F}_{T_4/\mathbb{Z}_3} = \frac{1}{3} \times 6\mathcal{F}_{T_4}|_{\phi_1=\phi_2=\phi_3} = \phi_1^3$$

# Other examples

[H. Kim, S. Kim, K. Lee 21]



$$6\mathcal{F}_{SU(2)_0} = 8\phi^3 \xrightarrow{\mathbb{Z}_4} 6\mathcal{F}_{SU(2)_0/\mathbb{Z}_4} = \frac{1}{4} \times 6\mathcal{F}_{SU(2)_0} = 2\phi^3$$



$$6\mathcal{F}_{SU(2)+2\mathbf{F}} = 6\phi^3 \xrightarrow{\mathbb{Z}_6} 6\mathcal{F}_{SU(2)+2\mathbf{F}/\mathbb{Z}_6} = \frac{1}{6} \times 6\mathcal{F}_{SU(2)+2\mathbf{F}} = \phi^3$$



# Generalized Quotients

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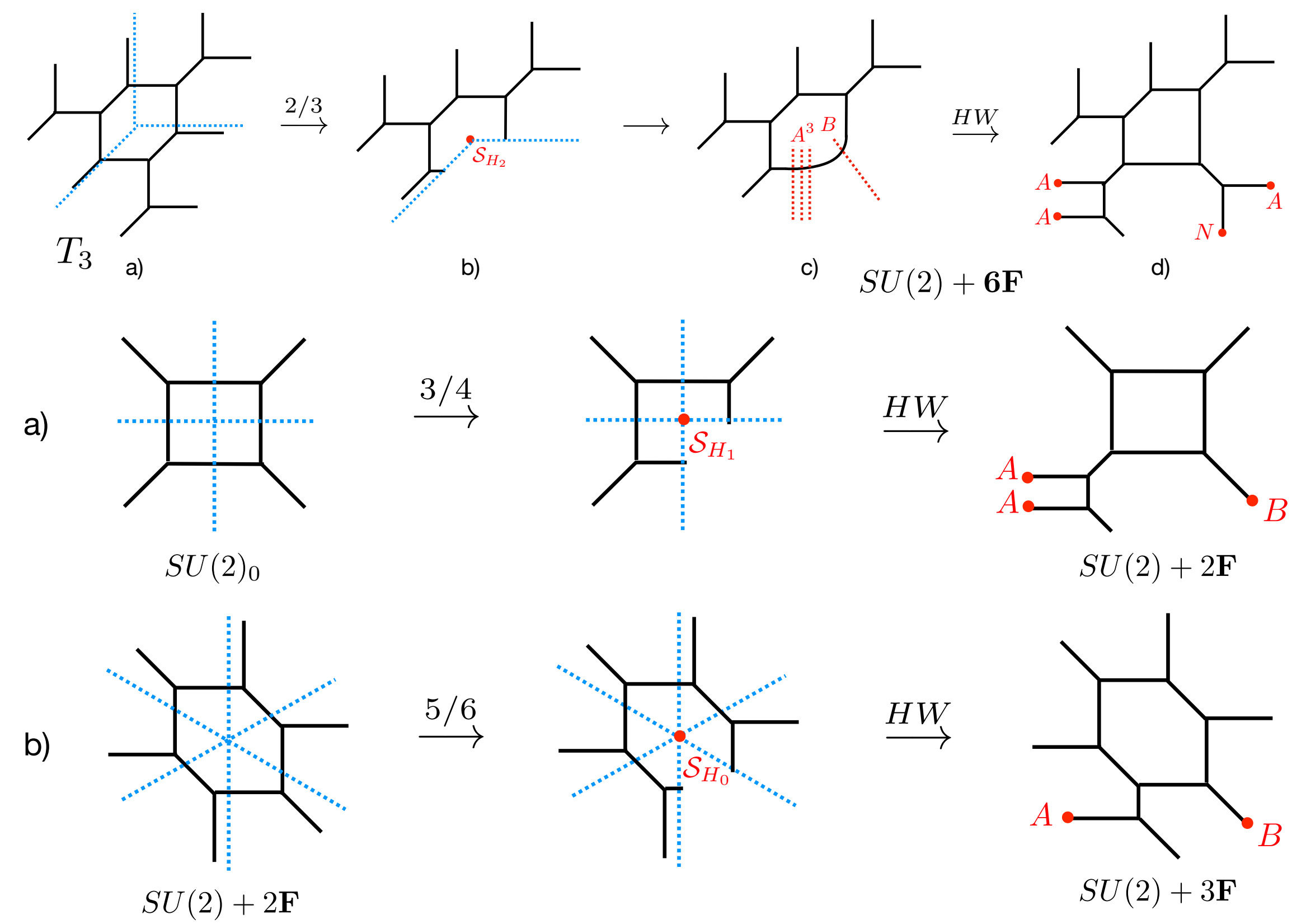
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$$6\mathcal{F}_{T_3} = 3\phi^3 \xrightarrow{2/3} \frac{1}{3} \left( 3(\phi^{(1)})^3 + 3(\phi^{(2)})^3 \right) = 2\phi^3 = 6\mathcal{F}_{SU(2)+6\mathbf{F}} ,$$

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# Generalized Quotients

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In order to better define these generalized quotients we need

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- The symmetry can generically be a map between different copies
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Example:  $\mathbb{Z}_3$  action on 2 copies of  $T_5$

$$\phi_1^{(1)} \rightarrow \phi_3^{(2)} \rightarrow \phi_2^{(2)} , \quad \phi_2^{(1)} \rightarrow \phi_1^{(2)} \rightarrow \phi_3^{(1)} , \quad \phi_4^{(1)} \rightarrow \phi_6^{(1)} \rightarrow \phi_5^{(2)} , \quad \phi_5^{(1)} \rightarrow \phi_4^{(2)} \rightarrow \phi_6^{(2)}$$

# Prepotential for Generalized Quotients

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The general prepotential formula follows from this definition

$$\mathcal{F}((\mathcal{T})^{n-1}/\mathbb{Z}_n) = \frac{1}{n} \sum_{a=1}^{n-1} \mathcal{F}(\mathcal{T}(\phi^{(a)})) \Big|_{\phi_{S(i)}^{(a')} = \phi_i^{(a)}}$$

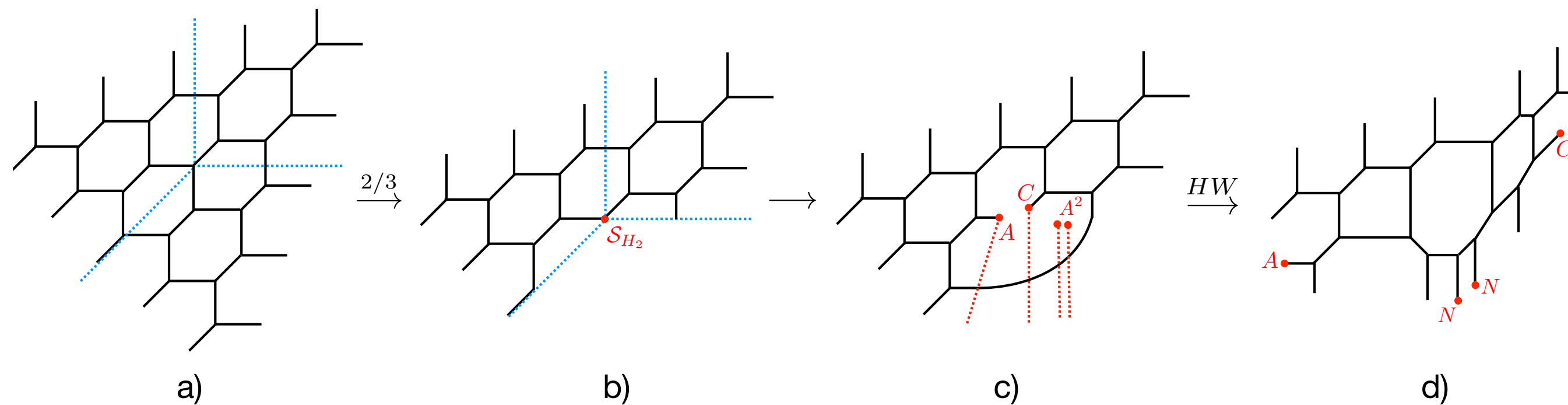
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For  $T_5$  this leads to



$$6\mathcal{F}_{(T_5)^2/\mathbb{Z}_3} = \frac{1}{3} \left( 6\mathcal{F}_{T_5}(\phi^{(1)}) + 6\mathcal{F}_{T_5}(\phi^{(2)}) \right) = 5\phi_1^3 + 5\phi_2^3 + 4\phi_3^3 - 6\phi_4(\phi_1^2 + \phi_2^2) = 6\mathcal{F}_{SU(4)_0+10\mathbf{F}}$$



# Back to Holography

# 6d AdS solutions with F-theory branes

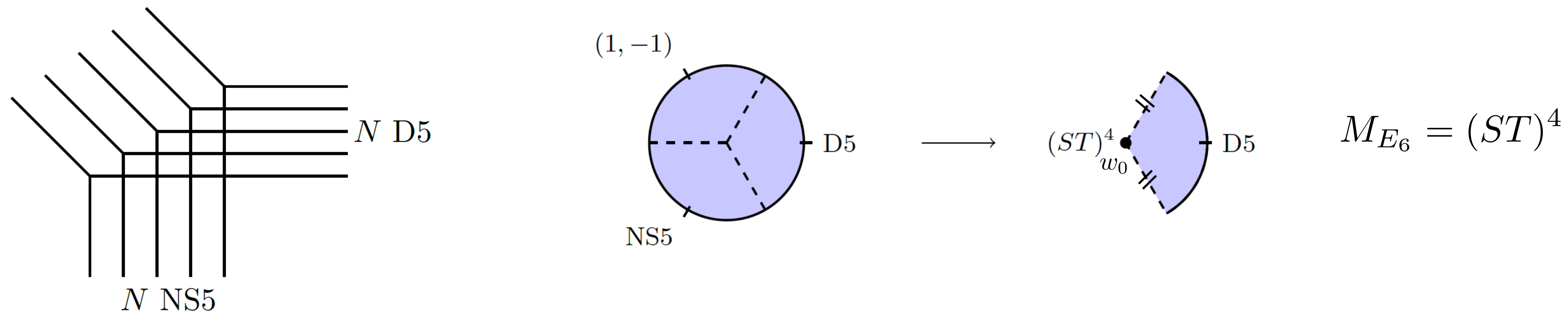
For generating new near-horizon solutions we proceed like with the O7. We quotient by a symmetry of the solution, which involve a symmetry of the disc  $\Sigma$  and  $\mathbb{Z}_n \subset SL(2, \mathbb{Z})$ -action  $n = 2, 3, 4, 6$ . This leads to a fixed point.

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$\mathbb{Z}_3$  - quotient of the solution dual to  $T_N$  parent theory

[FA, Bergman, Kim, Uhlemann 22]



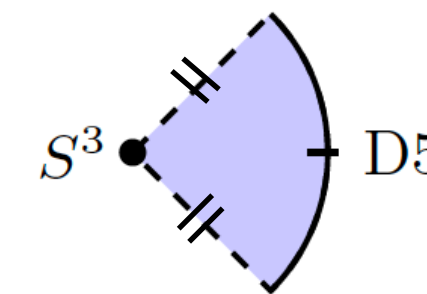
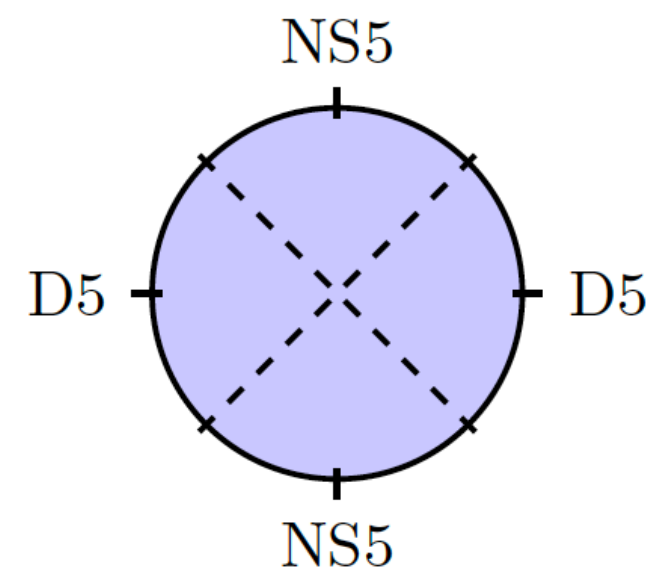
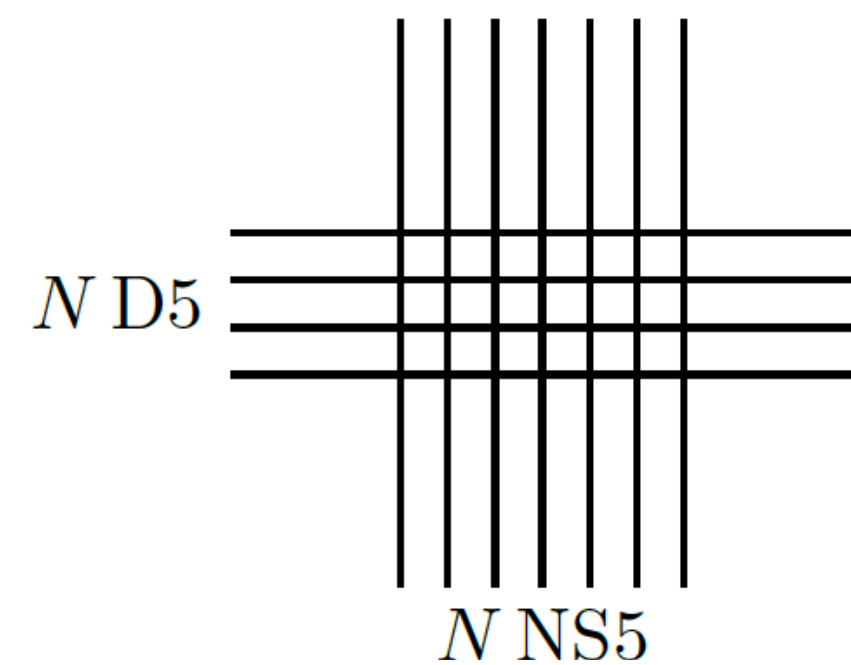
The fixed point is the  $E_6$  7-brane, with  $4\pi/3$  deficit angle and fixed axio-dilation

$$\tau(w_0) = e^{2\pi i/3}$$

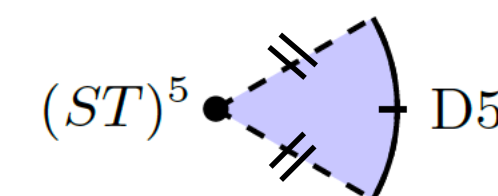
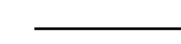
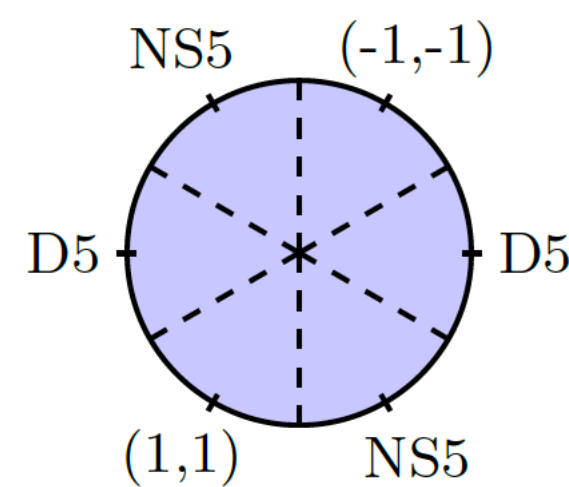
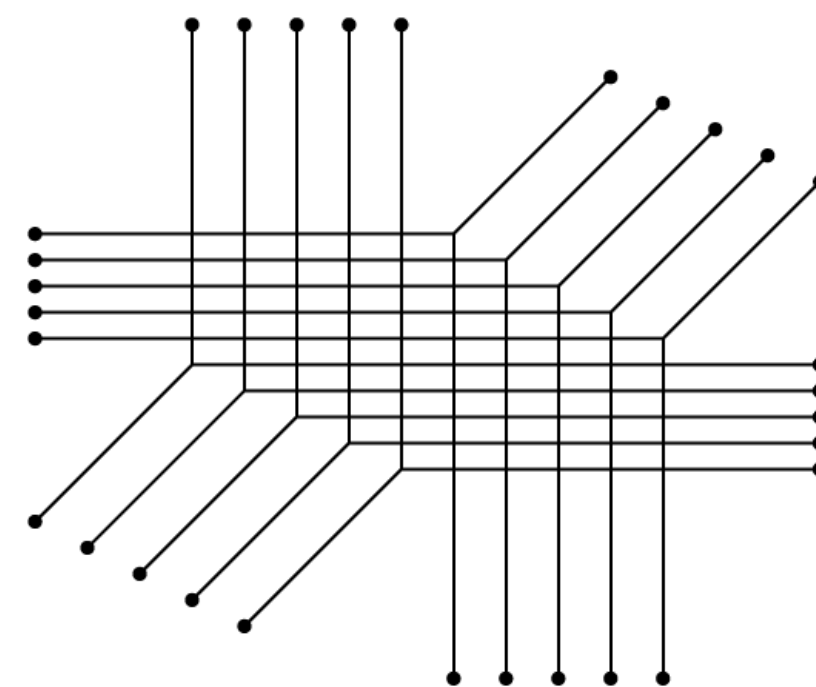
# 6d AdS F-theory solutions

[FA, Bergman, Kim, Uhlemann 22]

Other examples include  $E_7, E_8$  7-branes singularities with deficit angles  $3\pi/2, 5\pi/3$



$$M_{E_7} = S^3$$

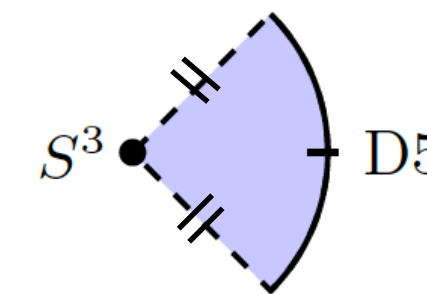
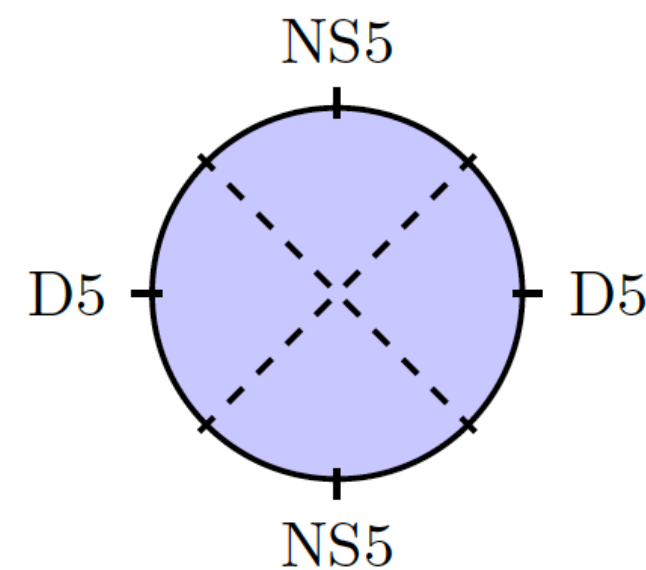
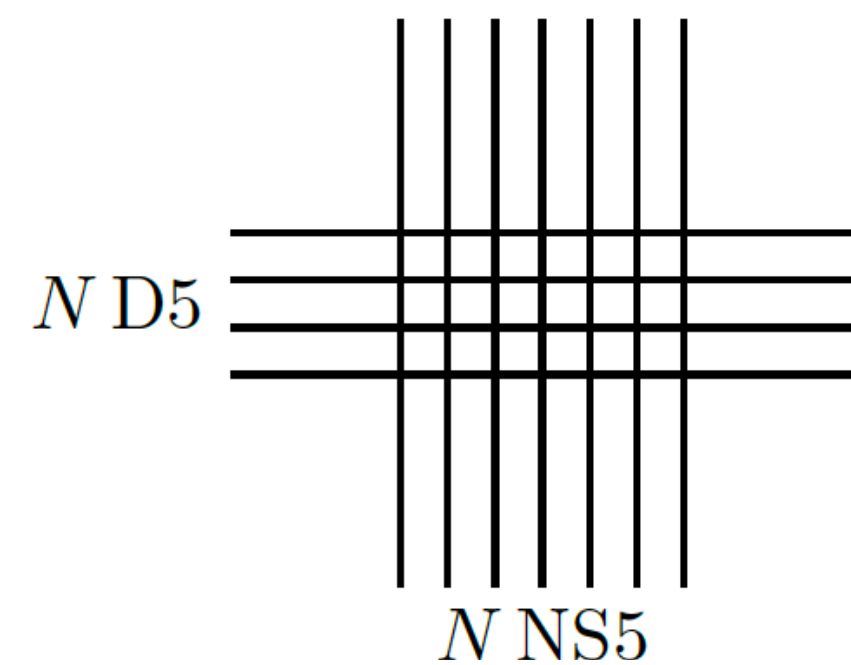


$$M_{E_8} = (ST)^5$$

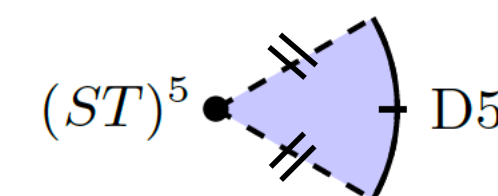
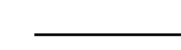
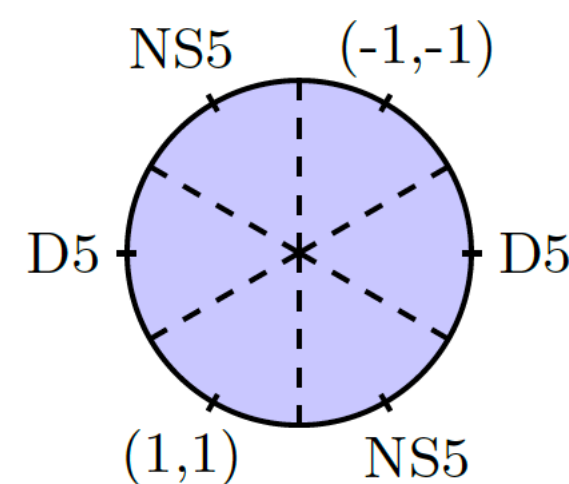
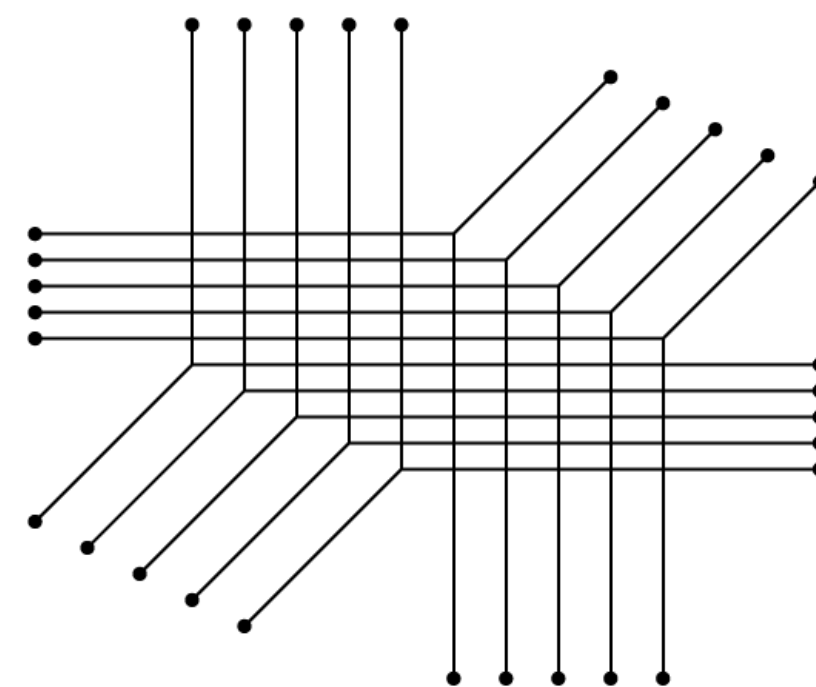
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$$M_{E_8} = (ST)^5$$

Can be seen as **F-theory solutions:**  $AdS_6 \times_w (M_4 \times T^2)/\mathbb{Z}_n \cong AdS_6 \times_w (S^4_{\text{punctured}} \times T^2)/\mathbb{Z}_n$

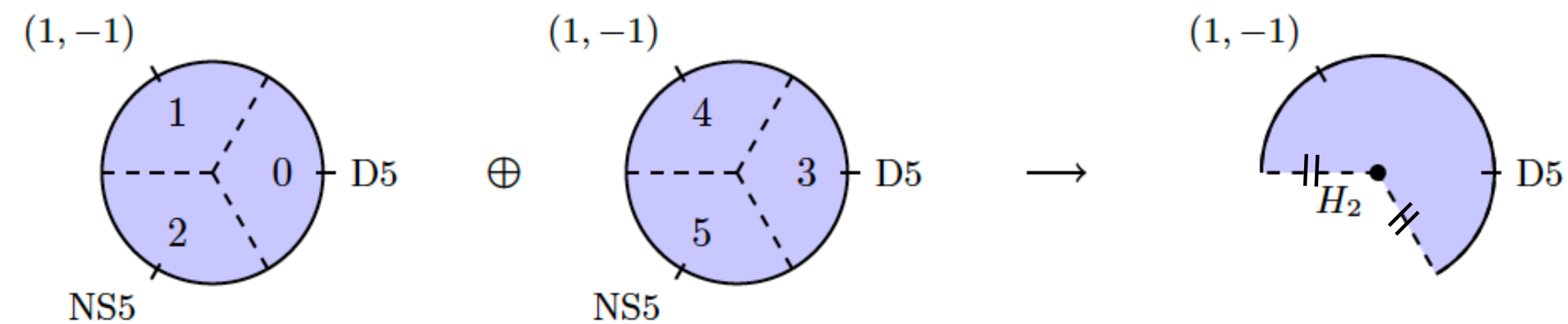
# 6d AdS solutions from generalized quotients

We can have also solutions with  $H_{0,1,2}$  7-branes singularities with deficit angles  $\pi/3, \pi/2, 2\pi/3$ , via the generalized quotients

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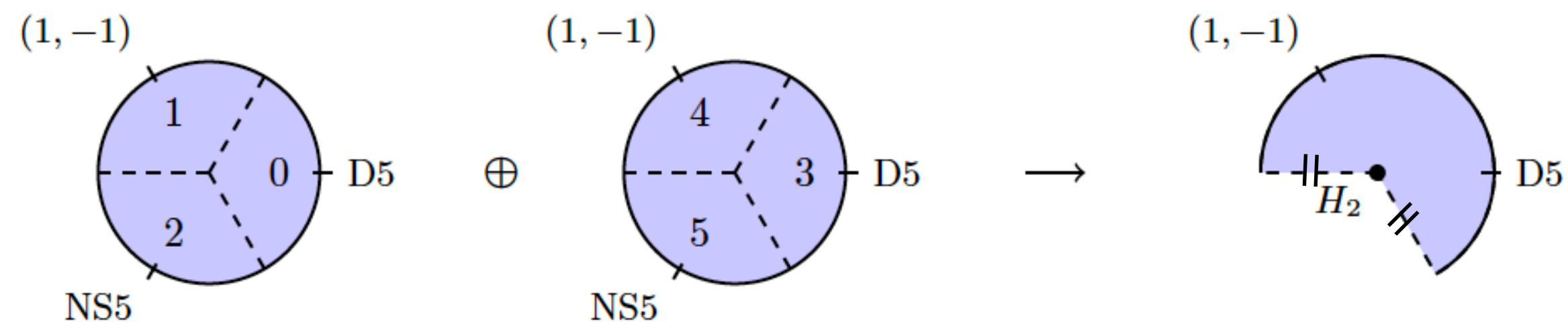
$\mathbb{Z}_3$  - quotient of 2 copies of the holographic dual  $T_N$



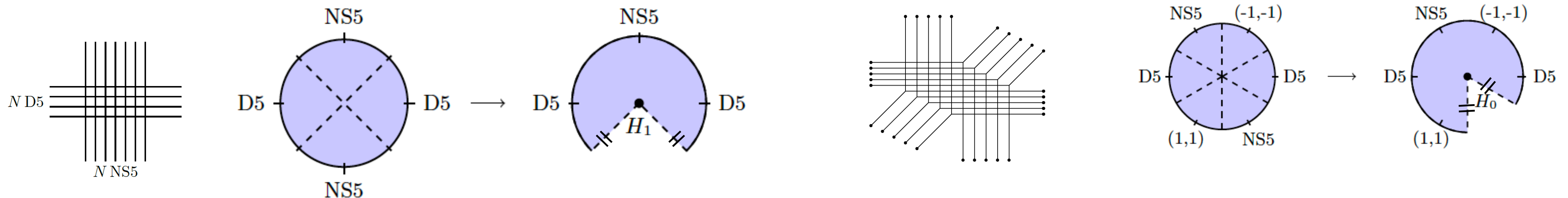
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$\mathbb{Z}_3$  - quotient of 2 copies of the holographic dual  $T_N$



$\mathbb{Z}_4$  and  $\mathbb{Z}_6$  - quotients of 3 and 5 copies of





# Observables

# Holographic observables

[FA, Bergman, Kim, Uhlemann 22]

Observables of the quotient theory are related to the parent theory at large N

- Free energy is computed by the supergravity on-shell action, and it is proportional to the volume of  $M_4$  (# of degrees of freedom)

$$F_{S^5}(T_N/\mathbb{Z}_3) = \frac{1}{3} F_{S^5}(T_N) = -\frac{9}{8\pi^2} \zeta(3) N^4 \qquad F_{S^5}[(T_N)^{2/3}] = 2F_{S^5}[T_N/\mathbb{Z}_3]$$

- Central charges appearing in stress-energy tensor and flavor symmetry current 2-point functions are related to  $F_{S^5}$ , therefore they get divided by 1/n
- Field theoretically the free energy is computed by localisation on the 5-sphere.
- The matrix model obtained from localisation takes the form of an integral over the coulomb branch involving the classical prepotential. At large N instanton contributions are suppressed.
- Even if we do not know the low-energy gauge theory description for the quotients, but we can use the **prepotential** to compute the matrix model integrals.

# Conclusion and Outlook

- We constructed 6d AdS solutions with F-theory 7-branes (via S-folding)
- We defined a generalized S-fold procedure
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  - We defined a generalized S-fold procedure
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- 
- Can we use the generalized S-folds to construct new S-fold theories in 4d?
  - Generalized symmetries of these theories from holography?

**Thank you!**