Cobordism and a Modified Gauss Law

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Based on:

- **1909.10355 with Cumrun Vafa**
- **2212.00039 with Matthew Reece**

Motivating Example: D-Branes in NS-NS Flux

Textbook Example: The D3-brane worldvolume supports a dynamical $U(1)$ gauge field, under which the ends of D1-strings are magnetically charged particles.

Suppose the D3-brane worldvolume is $\mathbb{R}_t \times M^3$, and we turn on k units of NS-NS flux through M^3

$$\int_{M^3} H_3 = k.$$

Then we have a **modified Gauss Law:** there must be exactly k D1-strings ending at points in M^3. [Maldacena, Moore, Seiberg '01]

Due to **anomaly inflow:** the $U(1)$ gauge field is charged under B_2.

Goal Today: Replace everything in sight with the spacetime manifold.

Overview

Dictionary:

Magnetic Flux $\leftrightarrow$ Geometric Flux

Monopoles $\leftrightarrow$ Cobordism defects

NS-NS flux $\leftrightarrow$ Normal bundle

Outline:

1. Review the Swampland Cobordism Conjecture
2. Examples: Nelson-Barr Models and F-theory
3. Connection to Adams Spectral Sequence

Swampland Cobordism Conjecture

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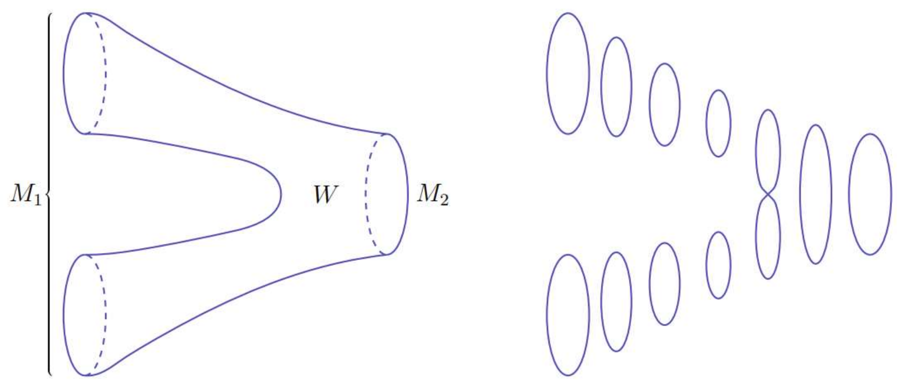
Topological Charges and Cobordism

Topological charges of the spacetime manifold are classified by **cobordism**.



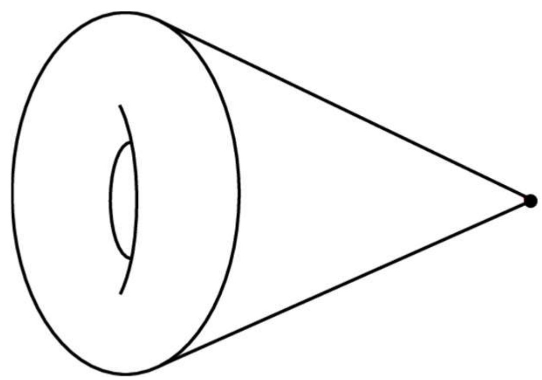
Fix structure $\mathcal{X}$ (orientation, spin structure, etc.), study k-dimensional $\mathcal{X}$-manifolds up to $\mathcal{X}$-preserving topology changes. Set of equivalence classes forms an abelian group $\Omega_k^{\mathcal{X}}$ under disjoint union.

Defines a **global symmetry** of semiclassical gravity.

Swampland Cobordism Conjecture

Cobordism Conjecture: Any UV complete theory of quantum gravity must have trivial cobordism groups, $\Omega_k^{\mathrm{QG}} = 0$, once all effects and objects are included.

Suppose the semiclassical cobordism groups are nontrivial, $\Omega_k^{\mathcal{X}} \neq 0$. Then the UV complete theory must include **defects** that trivialize the cobordism classes.



Examples: Orientifold planes, etc. See also Miguel and Markus's talks.

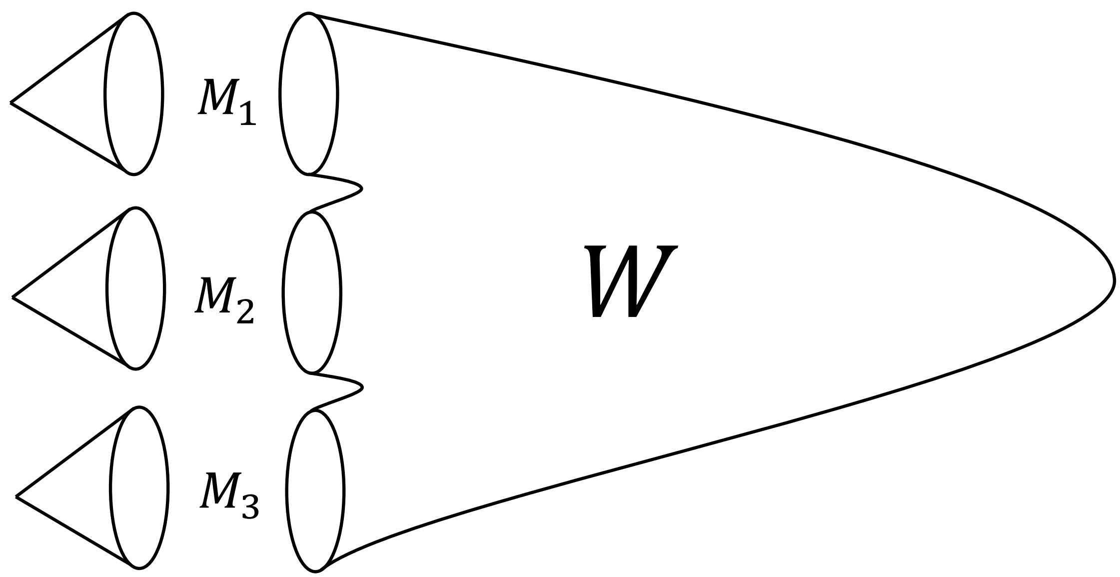
Gauge Charge of Cobordism Defects

Defects carry a **topological gauge charge** valued in Ω_k^{χ}. Can be detected inside a black brane by measuring the cobordism class of the horizon manifold.

Gauss Law: Net number of cobordism defects transverse to closed slice must vanish.

$$\sum_i [M_i] = [\sqcup_i M_i] = [\partial W] = 0.$$

Hidden Assumption: Normal bundle is trivial.



Example: Nelson-Barr Models

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Review: Nelson-Barr Models

Strong CP Problem: Our universe has order-one CP violation, yet $\bar{\theta}_{QCD} \lesssim 10^{-10}$.

Nelson-Barr Models: Promote CP or parity to exact symmetry, spontaneously broken at low energies. [Nelson '84, Barr '84]

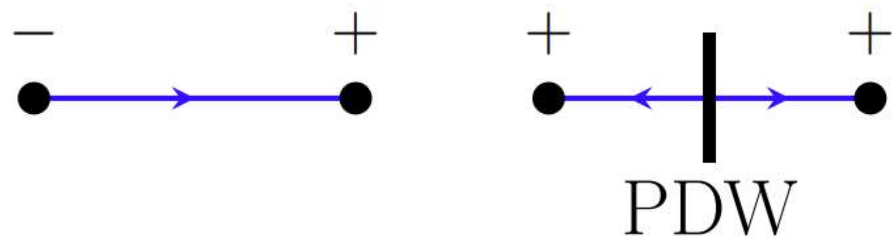
In terms of cobordism:

- Low-energy theory is chiral, depends on orientation ($\mathcal{X} = SO$).
- High-energy theory makes sense on non-orientable manifolds ($\mathcal{X} = O$).

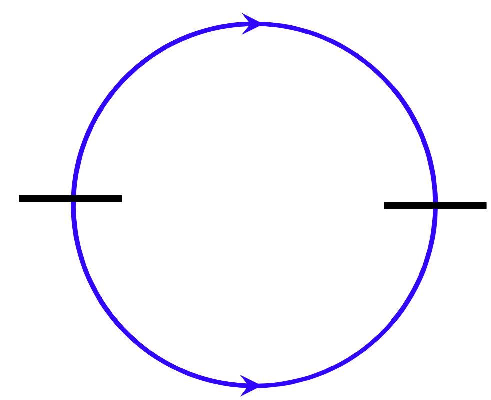
Nelson-Barr models contain a cobordism defect, much discussed in pheno community: the **parity domain wall (PDW)**.

Parity Domain Walls as Cobordism Defects

Radiates nontrivial cobordism class: $2\,[\mathrm{pt}] \in \Omega_0^{SO}$. Could detect PDW inside black domain wall!



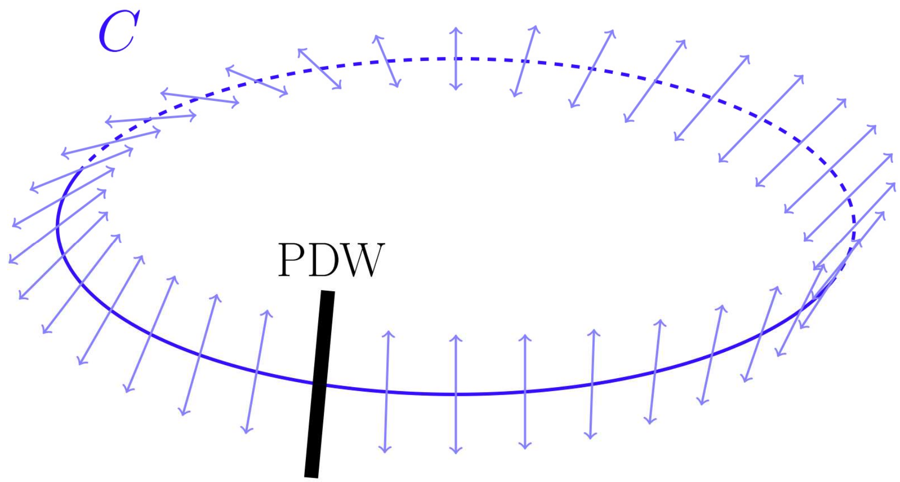
Gauss Law: Every 1-manifold is orientable, so net number of PDWs on abstract 1-manifold must vanish mod 2.



Pheno Lesson: PDWs are exactly stable. Big problem for cosmology!

Modified Gauss Law

Modified Gauss Law: Odd number of PDWs transverse to circle C with non-orientable normal bundle:



Anomaly Inflow: The cobordism class of a point in an ambient oriented manifold suffers an ambiguity: it flips sign under a normal reflection, $[\mathrm{pt}] \to -[\mathrm{pt}]$.

Example: F-theory

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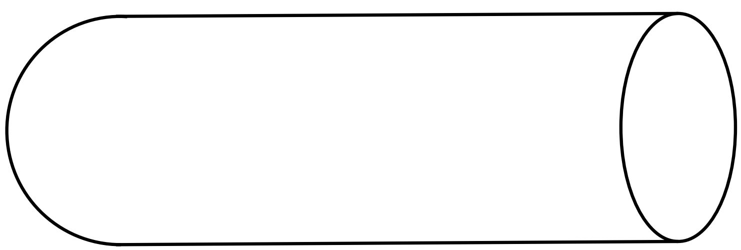
Type IIB and F-theory

Two descriptions (see Miguel and Markus's talks for more refined version):

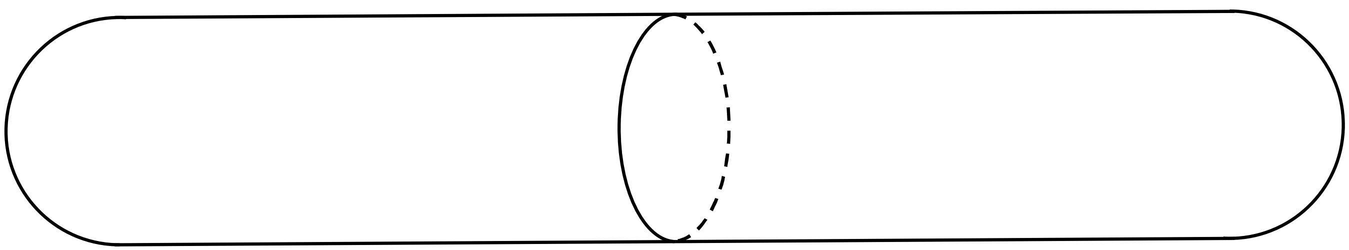
- Perturbative Type IIB requires a spin structure ($\mathcal{X} = Spin$).
- Non-perturbatively, F-theory base requires spin^c structure ($\mathcal{X} = Spin^c$).

F-theory contains a cobordism defect relative to perturbative Type IIB: a **spin vortex**, around which fermions pick up an additional minus sign. Radiates $[S^1_{\mathrm{p}}] \in \Omega_1^{Spin}$. Realized by 12 singular fibers (deficit angle 2π). [Green, Shapere, Vafa, Yau '89]



Modified Gauss Law

Gauss Law: Elliptic fibration over a Riemann surface Σ has a multiple of 24 singular fibers. **Ex:** F-theory on $K3 \to \mathbb{P}^1$ has 24 singular fibers, comprising 2 spin vortices.



Modified Gauss Law: Consider elliptic fibration over a (-1)-curve Σ in the base. Normal bundle is non-spin. Only 12 singular fibers: a single spin vortex.

Anomaly Inflow: The cobordism class of a circle in an ambient spin manifold suffers an ambiguity: it flips under twisting the normal framing, $[S^1_{\mathrm{p}}] \to [S^1_{\mathrm{ap}}]$.

Adams Spectral Sequence

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Gauging in Cobordism

In both examples considered, the cobordism defects are measured by a **characteristic class**:

- Parity domain walls are defects in the orientation, measured by w_1.
- Spin vortices are defects in the spin structure, measured by w_2.

In both cases, the characteristic class is nontrivial in the fundamental description ($\mathcal{X} = O$ or $\mathcal{X} = Spin^c$) but vanishes on abstract manifolds of the same dimension:

- All unoriented 1-manifolds are orientable, $w_1 = 0$.
- All spin^c 2-manifolds are spin, $w_2 = 0$.

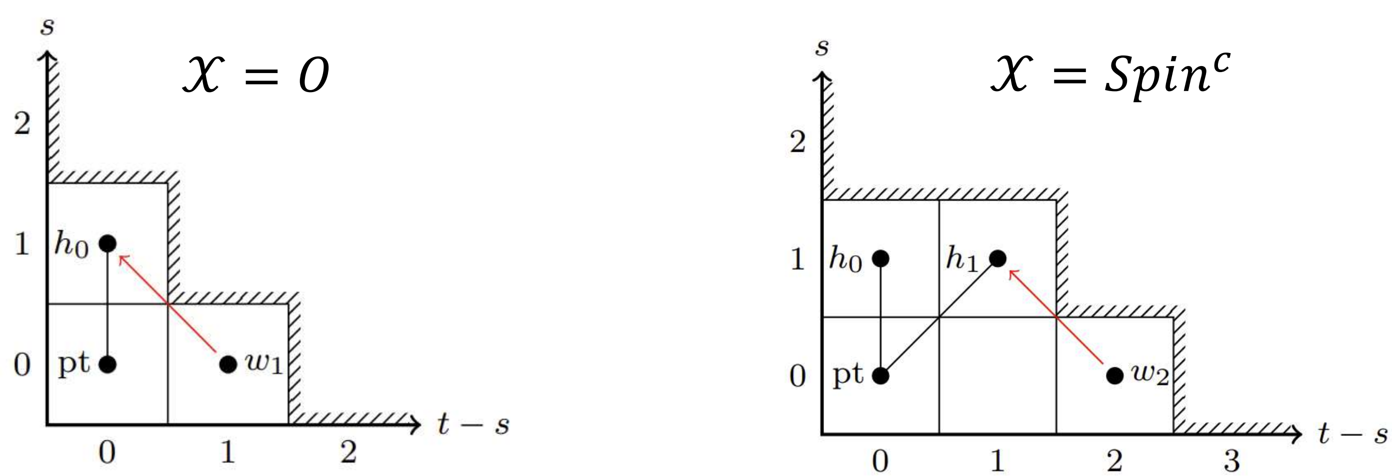
While the characteristic classes are nontrivial, they **fail to appear** in cobordism groups of the same dimension.

The Adams Spectral Sequence

Our motivating example (D-branes in NS-NS flux) is mathematically described by a **spectral sequence** (AHSS) which takes the modified Gauss Law into account.

The same is true for cobordism: our story is described by the **Adams spectral sequence,** a tool for computing cobordism groups given characteristic classes.

Both of our examples are captured by a **differential** on the E_1 page:



Thank You For Listening!