4d $\mathcal{N} = 2$ SCFTs, Higgs Branch RG Flows and 2d VOAs

Based on joint work with G. Elliot (UT), M. J. Kang (CalTech) and C. Lawrie (DESY)

Zoo of 4d $\mathcal{N} = 2$ QFTs/SCFTs:

- Lagrangian Theories
- Class-S
- T^2 compactifications of 6d (1,0) SCFTs
- Other F-theory constructions
- ...

Elaborate Structure

Nontrivial isomorphisms between different constructions (see M.J. Kang's talk).

S-dualities: Best understood in class-S:

Conformal manifold $\sim \mathcal{M}_{g,n}$.

Each pants-decomposition of $C_{g,n} \Longrightarrow$ realization as a gauging (with $3g - 3 + n$ simple factors) of isolated SCFT.

RG flows between UV and IR SCFTs

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Triggered by VEV for superconformal primary (scalar) in $\hat{B}_R$ multiplet.

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  RG invariant

  or change in computable ways under RG

How to study?

6d tensor branch F-theory Geometry (geometric transitions)

2d VOA (Beem et al: every 4d $\mathcal{N} = 2$ SCFT $\Rightarrow$ 2d VOA)

Changes in well-defined way under certain class of RG flows

Schur Operators

Conformal primaries with $\Delta - j_1 - j_2 - 2R = r + j_1 - j_2 = 0$

Superconformal primary in $\hat{B}_R$

$Q\tilde{Q}$ of primary in $\hat{C}_{R(j_1,j_2)}$

$\tilde{Q}$ of primary in $D_{R(0,j_2)}$

Q of primary in $\overline{D}_{R(j_1,0)}$

Important special cases:

$\hat{B}_R$: coordinate ring of Higgs branch

$2\hat{B}_{1/2}$: free hypermultiplet. $\hat{B}_1$: moment map

$D_{0(0,0)} + \overline{D}_{0(0,0)}$: free vector

$\hat{C}_{0(0,0)}$: stress tensor multiplet

Constructing the VOA

Work in $x_1 = x_2 = 0$ plane; let $z = x_3 + ix_4$.

Can find $\mathbb{Q} = Q + \tilde{S}$ such that

$$\mathbb{Q}^2 = 0$$
$$\{\mathbb{Q}, O_{\text{Schur}}(0)] = 0$$
$$[\mathbb{Q}, L_{\pm 1,0}] = 0 \quad \text{for } L_{-1} = P_{+\dot{+}},\ L_0 = \frac{1}{2}(D+M),\ L_1 = K^{\dot{+}+}$$

$\overline{L}_{\pm 1,0}$ not $\mathbb{Q}$-closed. But twisted versions

$$\begin{aligned} \hat{L}_{-1} &= \overline{L}_{-1} + R^- \\ \hat{L}_1 &= \overline{L}_1 - R^+ \\ \hat{L}_0 &= \overline{L}_0 - R \\ Z &= r + M^{\perp} \end{aligned}$$

are $\{\mathbb{Q}, \cdot\ \}$. Restricting to $\hat{L}_0 = Z = 0$ subspace $\Rightarrow$ Schur operators.

$$O(z,\bar{z}) = e^{zL_{-1} + \bar{z}\hat{L}_{-1}} O(0) e^{-zL_{-1} - \bar{z}\hat{L}_{-1}}$$

is independent of $\bar{z}$ on the level of $\mathbb{Q}$-cohomology.

Dumb Examples

Free hypermultiplet $(2\hat{B}_{1/2})$ Schur ops = two complex scalars $\phi, \tilde{\phi}$

$$\gamma(z) = [\phi(z,\bar{z}) + \bar{z}\tilde{\phi}(z,\bar{z})]_{\mathbb{Q}}$$
$$\beta(z) = [\tilde{\phi}(z,\bar{z}) - \bar{z}\phi(z,\bar{z})]_{\mathbb{Q}}$$

Spin-1/2 $\beta\gamma$-system:

$$\gamma(z)\beta(w) \sim \frac{1}{z-w}, \quad \beta(z)\gamma(w) \sim -\frac{1}{z-w}, \quad T = -\tfrac{1}{2}\beta\partial\gamma + \tfrac{1}{2}(\partial\beta)\gamma, \quad c_{2d} = -1$$

Free vector $(D_{0(0,0)} + \overline{D}_{0(0,0)})$ Schur ops = photini $\Rightarrow$ Spin-1 bc-system, $c_{2d} = -2$

$\hat{C}_{0(0,0)}$: Schur op = R-symmetry current $\Rightarrow T(z)$, $c_{2d} = -12c_{4d}$

$\hat{B}_1$: Schur op = moment map for hyperKähler isometry $\Rightarrow J^a(z)$, $k_{2d} = -\frac{1}{2}k_{4d}$

Gauging G**:** Lagrangian field theory: free vectors in $\mathfrak{g}$, free hypers in R.

$$Q = \oint \frac{dz}{2\pi i}\, c_a(z)J^a(z) + \tfrac{1}{2} f^{ab}{}_c b^c(z)c_a(z)c_b(z)$$

$$Q^2 = 0 \Longleftrightarrow \beta = 0$$

Higgs Branch RG Flows

Relating VOA_{IR} to VOA_{UV}

1) "Improve" T_{UV}: $T = T_{\text{UV}} + \frac{1}{2}\alpha \cdot \partial H$

$\vec{\alpha} \in \Lambda_{\text{(co)root}}$, $\vec{H}(z) \in \text{Cartan}$

2) Conformal weights shift:

$$\vec{H}(z)\phi_i(w) = \frac{\vec{w}_i \phi}{z - w}$$

$$T_{\text{UV}}(z)\phi_i(w) = \frac{h_i^{\text{UV}}\phi_i}{(z-w)^2} + \frac{\partial\phi_i}{z-w}$$

$$h_i = h_i^{\text{UV}} - \frac{1}{2}\alpha \cdot w$$

3) Need to decouple all the generators with $h \leq \frac{1}{2}$

For each generator with $h \leq 0$, introduce spin-$(1-h)$ bc ghost system.

$h = \frac{1}{2}$ become free half-hypers; introduce spin-$\frac{1}{2}$ bc ghost system for each hyper.

4) Introduce Q_{BRST}. $\text{VOA}_{\text{IR}} = [\text{improved VOA}_{\text{UV}} \otimes \text{ghosts}]_{Q_{\text{BRST}}}$

Properties of $\mathrm{VOA}_{\mathrm{IR}}$

$$c_{2d}^{\mathrm{IR}} = c_{2d}^{\mathrm{UV}} - 3\,|\,\alpha\,|^2 k_{2d} + c_{\mathrm{ghost}}$$

$$\dim \mathscr{H}^{\mathrm{IR}} = \dim \mathscr{H}^{\mathrm{UV}} - \#(bc\text{-ghosts})$$

In terms of 4d quantities:

$$n_v = 4(2a - c)$$

$$n_h = 4(5c - 4a)$$

$$\delta \dim \mathscr{H} = 24(\delta c - \delta a)$$

we can write

$$3\delta n_v + \delta \dim \mathscr{H} = \frac{3}{2}\,|\,\alpha\,|^2 k_{4d} + c_{\mathrm{ghost}}$$

Vanilla case

$\mathfrak{f}_k =$ simple factor in 4d flavour symmetry at level k

$\alpha = \lambda =$ highest root of $\mathfrak{f}$

After improvement, $h(J_\lambda) = 0$.

$h(J_{\beta_i}) = h(J_{\lambda-\beta_i}) = 1/2$ for $i = 1,\ldots,\check{h}(\mathfrak{f}) - 2$.

$$J_{\beta_i}(z)J_{\lambda-\beta_j}(w) = \frac{\delta_{ij}J_\lambda}{z-w}$$

All other Schur operators have $h \geq 1$.

$$Q = \oint \frac{dz}{2\pi i}\, c(z)(J_\lambda(z) - a) + \sum_{i=1}^{\check{h}(\mathfrak{f})-2} c^i(z)J_{\beta_i}(z), \qquad a \in \mathbb{C}^*$$

$$\left.\begin{aligned} c_{\text{ghost}} &= (-2) + (\check{h}(\mathfrak{f}) - 2)(1) \\ &= \check{h}(\mathfrak{f}) - 4 \\ \delta \dim \mathscr{H} &= \check{h}(\mathfrak{f}) - 1 \end{aligned}\right\} \implies \delta n_v = k - 1$$

A More Complicated Example

$(A_3 + A_2 + A_1)$ puncture in E_7 Class-S theory

$\mathfrak{f}_k = \mathfrak{su}(2)_{224}$. As before, let λ = positive root.

$\hat{B}_1 \quad j = 1 \qquad h = 0, 1, 2$

$\hat{B}_2 \quad j = 2 \qquad h = 0, 1, 2, 3, 4$

$\qquad j = 4 \qquad h = -2, -1, 0, 1, \ldots, 6$

$\hat{B}_3 \quad j = 3 \qquad h = 0, 1, \ldots, 6$

$c_{\text{ghost}} = 4(-2) + 1(-26) + 1(-74) = -108$

$\delta \dim \mathscr{H} = 6$

Generically, need to turn on VEVs for all 4 dimension-0 operators.

Putting these together, $\delta n_v = 186 \Rightarrow (A_3 + A_2 + A_1) \rightarrow E_7(a_5)$.

Beyond Counting on Our Fingers

I have shown you how the VOA predicts δn_v and $\delta \dim \mathscr{H}$ under Higgs branch RG flow.

But the VOA contains (infinitely) more information.

Together with geometrical information (e.g. from 6d tensor branch geometry) can say *much more*.

Thanks for listening!