# New tools for analyzing non-perturbative superpotentials in 4D string vacua

**Patrick Jefferson (MIT)**

**Based on [2211.00210] and w.i.p. with Manki Kim (MIT)**

# Motivation

## Kähler moduli stabilization in type IIB on CY3 orientifold

[Kachru Kallosh Linde Trivedi `03]

- $W = W_0 + W_{n.p.}$ where
  $W_{n.p.} = \sum_{\hat{D}} A_{\hat{D}}(z,\tau) e^{-\frac{2\pi}{c_{\hat{D}}} T_{\hat{D}}}$ [Witten, `96] [Gukov Vafa Witten `99]

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- $Re(T_{\hat{D}_i}) \approx \frac{\#}{2\pi} log(W_0^{-1})$

  [Demirtas Kim McAllister Moritz `19]
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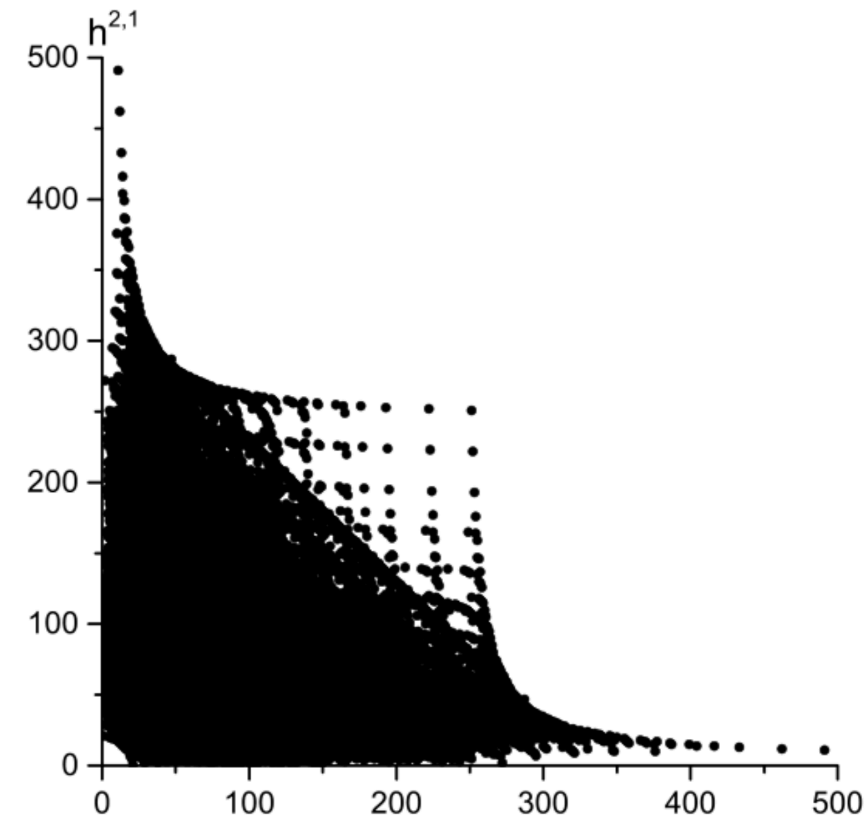
- $h^{2,1}$ small $\implies$ $h^{1,1}$ large




(taken from [Wang, `15])

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- $z, \tau$ dependence is open problem, but

$$\dim J_{\bar{D}} = \sum_{p+q=3} h^{p,q}(\bar{D}) = 2(h^{3,0} + h^{2,1}) = 0 \implies A_{\bar{D}} = \text{ const.}$$

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indicates understanding Hodge structure of $\bar{D}$ important!

# Summary of results

- Combinatorial formulae for $h^{p,q}(\bar{D})$, focusing on rigid case, $h^{\bullet,0}(\bar{D}) = (1,0,0,0)$ (sufficient to generate $W_{n.p.}$)

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    - Hodge numbers $h^{p,q}(\hat{D}) = h_+^{p,q}(\varphi_I(\hat{D})), h_-^{p,q}(\varphi_I(\hat{D}))$

    - CY3/$\mathbb{Z}_2$ divisors: $\hat{D}$, seven brane, and anticanonical divisor

    - # of O3 planes (terminal $\mathbb{Z}_2$ singularities) [Collinucci `09,`10]

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    - CY3/$\mathbb{Z}_2$ divisors: $\hat{D}$, seven brane, and anticanonical divisor
    - # of O3 planes (terminal $\mathbb{Z}_2$ singularities) [Collinucci `09,`10]
- Does not require an explicit construction of F-theory CY4 uplift

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- Hodge numbers from stratification of CY3 (toric hypersurface), combined with action of orientifold involution

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- Dependence on $\hat{D}$, $S_i$, $-K_B$ and $N_{O3}$ fixed using conjectured expression for Euler characteristic:

$$2(h^{1,1} - h^{2,1}) = \chi = \int_{\bar{D}} c_3 + 2N_{O3}$$

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[PJ Kim, to appear]

- Cross check: Hodge-Deligne numbers in known F-theory uplifts, and heuristic general arguments in local models

# Plan for talk

A. Sketch derivation of Hodge number formulae

B. Cross checks: local and global geometric models

C. Conclusions and future directions

# Setting the stage

## Type IIB on coordinate flip CY3 orientifold

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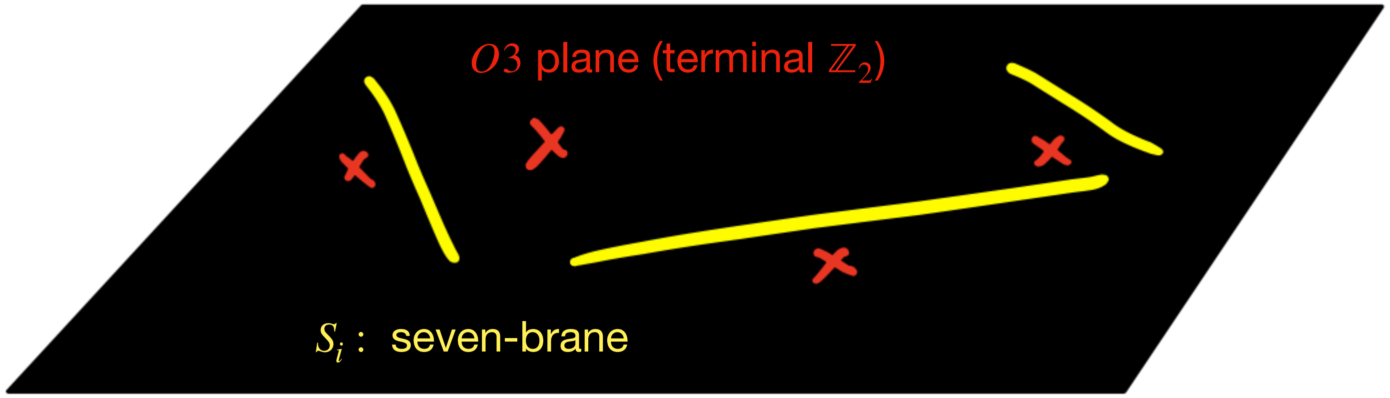
\begin{array}{ccc}
B & \subset & \hat{V}_4 \\
\downarrow & & \downarrow \varphi_I \\
\text{CY3} & \subset & V_4
\end{array}
$$



# Setting the stage

## Type IIB on coordinate flip CY3 orientifold

- Example: orientifold of quintic threefold $\mathbb{P}^4[5]$ (at $\mathbb{Z}_2$ symmetric locus)

$$\begin{array}{ccc} B & \subset & \hat{V}_4 \\ \downarrow & & \downarrow \ \varphi_I \\ \text{CY3} & \subset & V_4 \end{array}$$

| | $\hat{x}_1$ | $\hat{x}_2$ | $\hat{x}_3$ | $\hat{x}_4$ | $\hat{x}_5$ | $\hat{P}$ |
|---|---|---|---|---|---|---|
| $U(1)$ | 2 | 1 | 1 | 1 | 1 | $-5$ |

$$\downarrow \ \varphi_I$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $P$ |
|---|---|---|---|---|---|---|
| $U(1)$ | 1 | 1 | 1 | 1 | 1 | $-5$ |

$$I : x_1 \to -x_1$$

Fixed loci:

$$O7 : \{x_1 = 0\}$$

$$O3 : \{x_2 = x_3 = x_4 = x_5 = 0\}$$

# Setting the stage

## F-theory uplift (admitting global Sen limit)

$$
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\text{ECY4} & & \\
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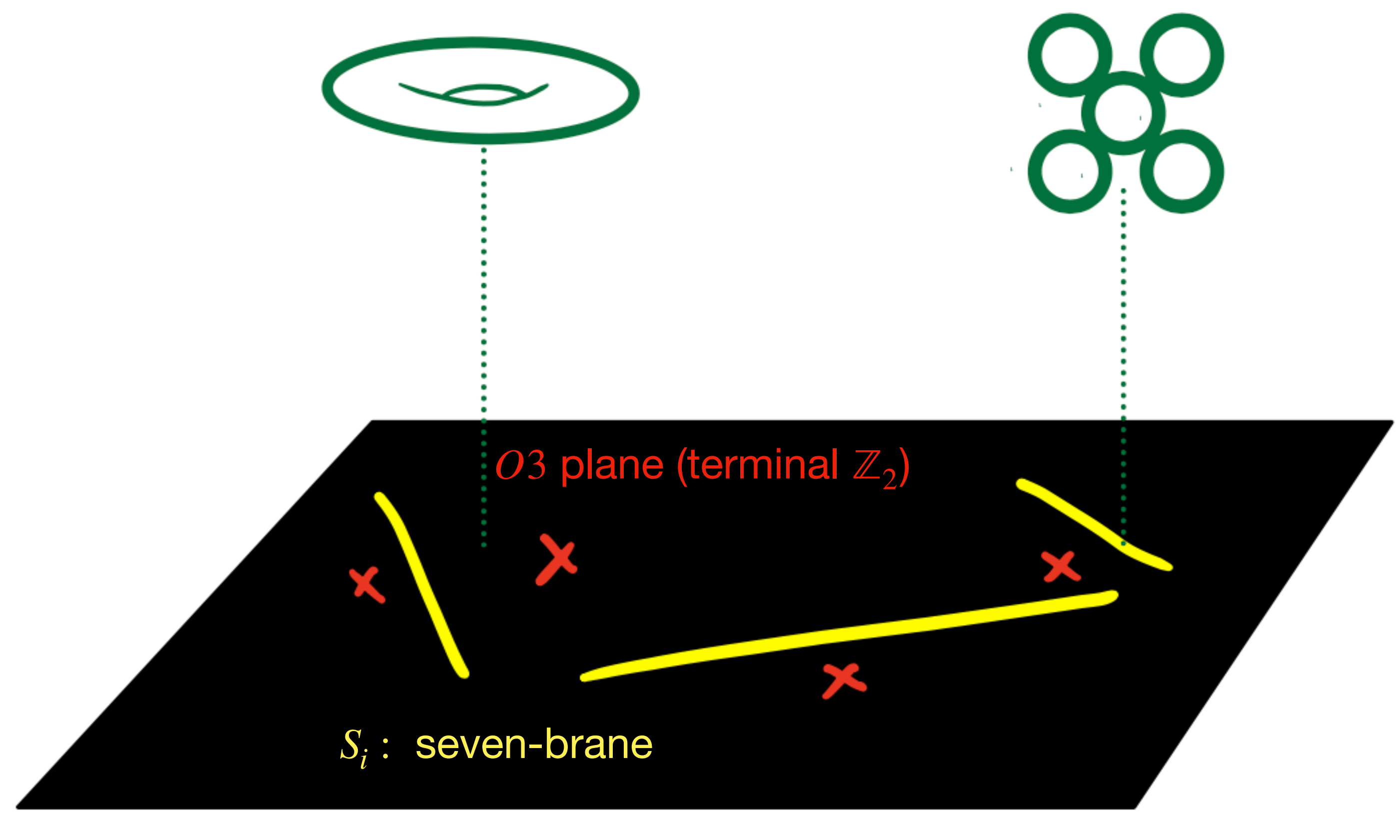


[Collinucci `08]

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- Example: F-theory uplift of orientifold of quintic threefold $\mathbb{P}^4[5]$

[Collinucci `08]

$$\begin{array}{ccc} \text{ECY4} & & \\ \downarrow & & \\ B & \subset & \hat{V}_4 \\ \downarrow & & \downarrow \varphi_I \\ \text{CY3} & \subset & V_4 \end{array}$$

$$Y^2 = X^3 + fXZ + gZ^6$$

$$L = \frac{1}{2}[\hat{x}_1] = [\hat{x}_{i\neq 1}]$$

| | $\hat{x}_1$ | $\hat{x}_2$ | $\hat{x}_3$ | $\hat{x}_4$ | $\hat{x}_5$ | $X$ | $Y$ | $Z$ | $\hat{P}$ | $\hat{P}'$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $U(1)_1$ | 2 | 1 | 1 | 1 | 1 | 2 | 3 | 0 | $-5$ | $-6$ |
| $U(1)_2$ | 0 | 0 | 0 | 0 | 0 | 2 | 3 | 1 | 0 | $-6$ |

# Setting the stage

## Vertical divisor of F-theory uplift

$$
\begin{array}{ccccc}
\bar{D} & \subset & \text{ECY4} & & \\
\downarrow & & \downarrow & & \\
\hat{D} & \subset & B & \subset & \hat{V}_4 \\
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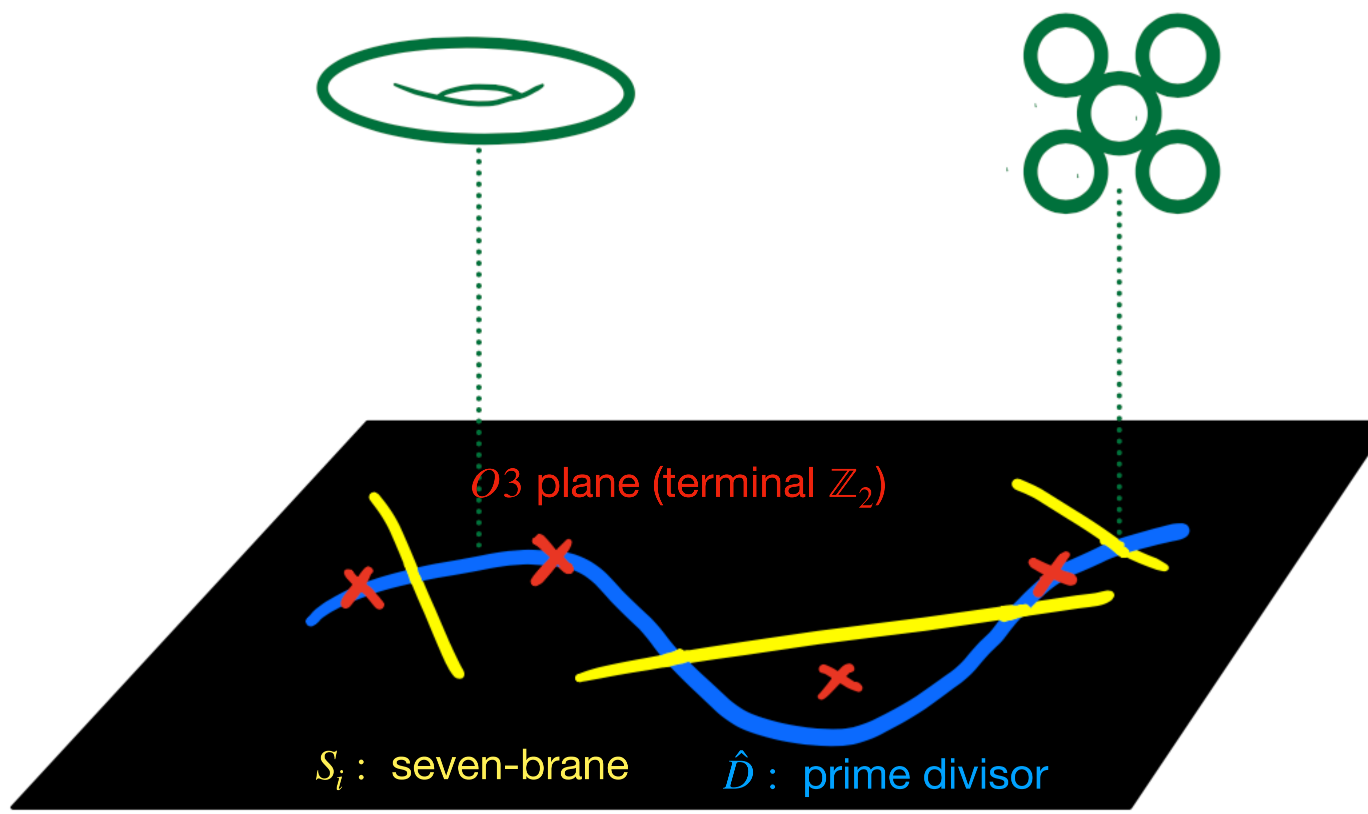
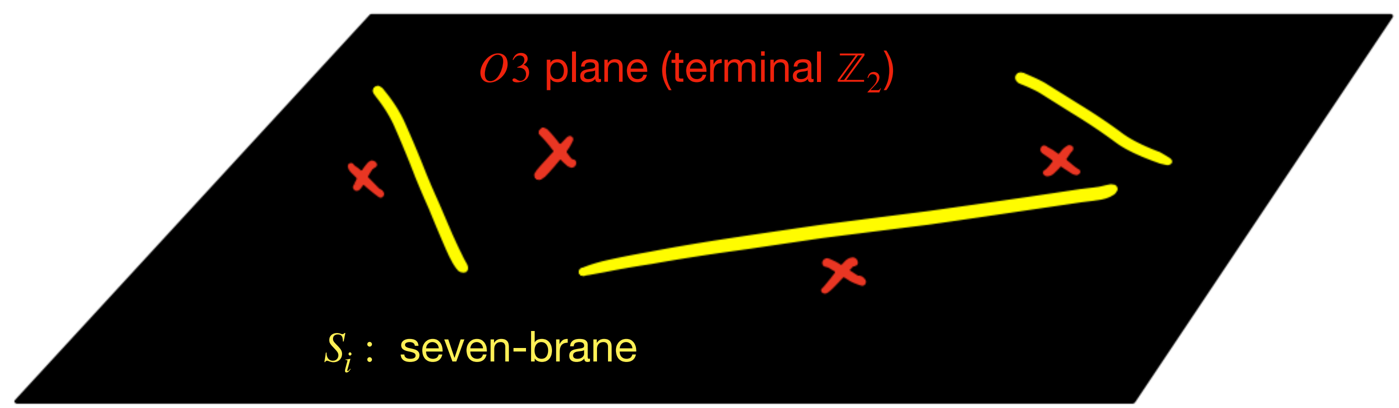


# Step 1: Hodge numbers $h_{\pm}^{p,q}(\varphi(\hat{D}))$

- Key data: 1) refinement map $\varphi_I$ and 2) stratification (e.g., $\mathbb{P}^1 = \mathbb{C}^\times \amalg \mathrm{pt} \amalg \mathrm{pt}$)

$$\begin{array}{ccccc} \hat{D} & \subset & B & \subset & \hat{V}_4 \\ & & \downarrow & & \downarrow \\ & & \mathrm{CY3} & \subset & V_4 \end{array}$$



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$$
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 & & \amalg_m Z_{\hat{\Delta}^{(m)}} \times E_{\hat{\sigma}^{(4-m)}} & & \amalg_n T_{\hat{\sigma}^{(n)}} \times E_{\hat{\sigma}^{(n)}} \\
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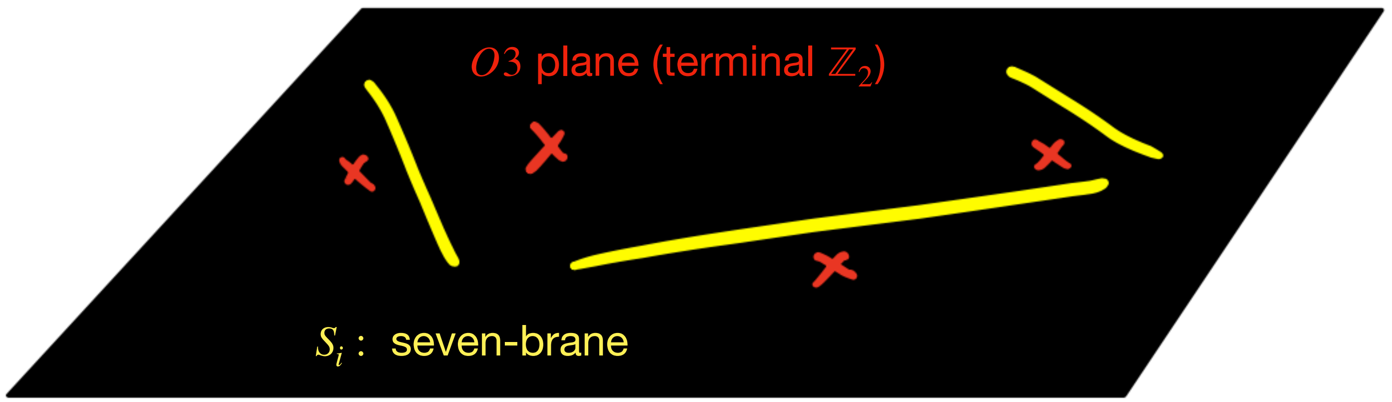

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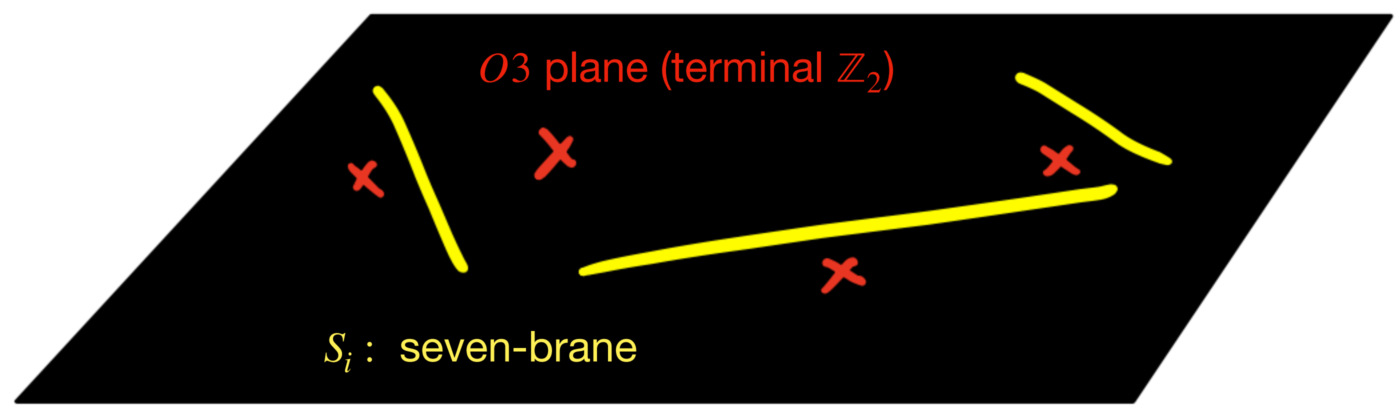


- $h^{p,q}$ from Hodge-Deligne numbers $e^{p,q} = \sum_k (-1)^k h^{p,q}(H_c^k)$
- HD characters $e = \sum_{p,q} x^p \bar{x}^q e^{p,q}$ additive over $\amalg$, multiplicative over $\times$

[Danilov Khovanskii, `87]

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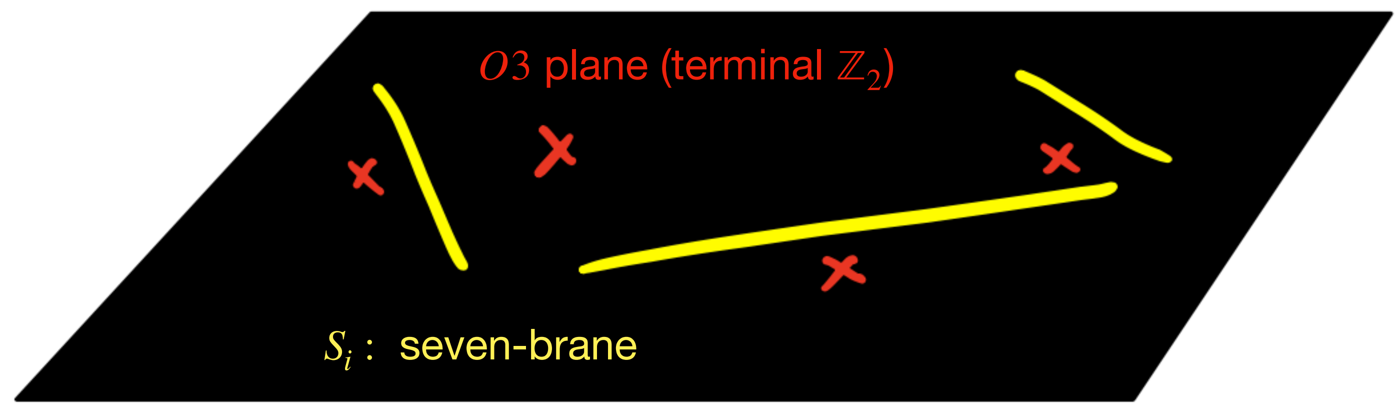


- Equivariant Hodge numbers $h^{p,q}_{\pm}(\varphi(\hat{D}))$ from toric data

- Cohomology splits into eigenspaces of $I$, $h^{p,q}(\hat{D}) = h^{p,q}_{+}(\varphi_I(\hat{D}))$

# Step 2: Anatomy of $\bar{D}$

- We construct $\bar{D}$ using Weierstrass model defined by line bundle $\mathscr{O}(-K_B)\,|_{\hat{D}}$

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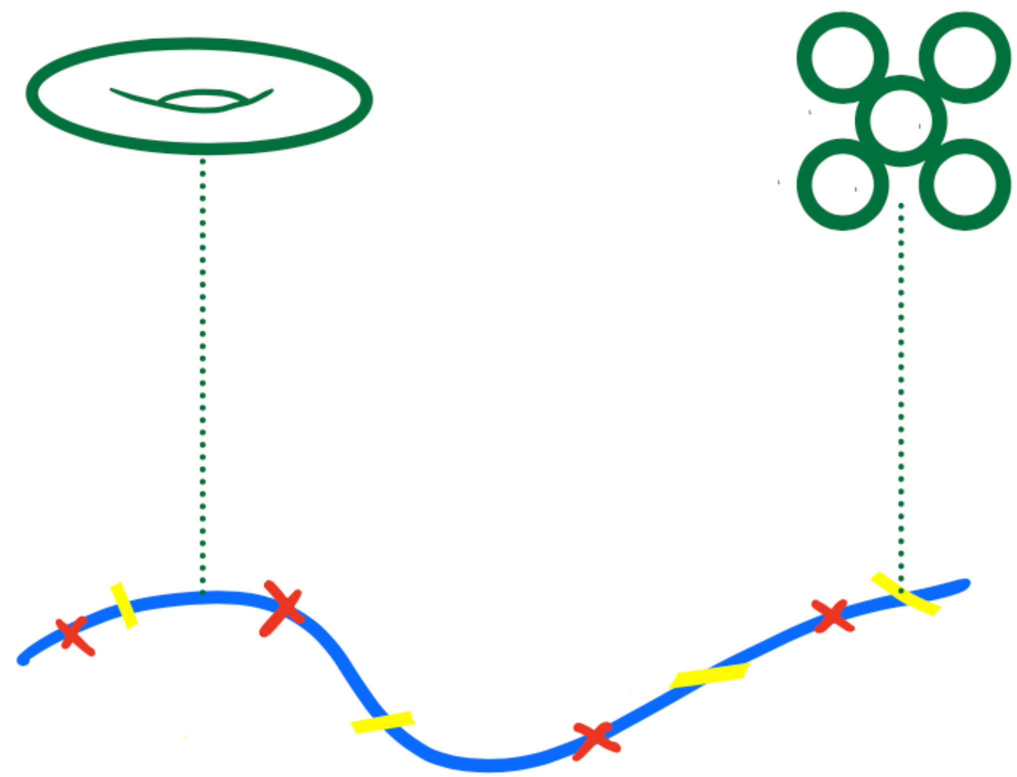

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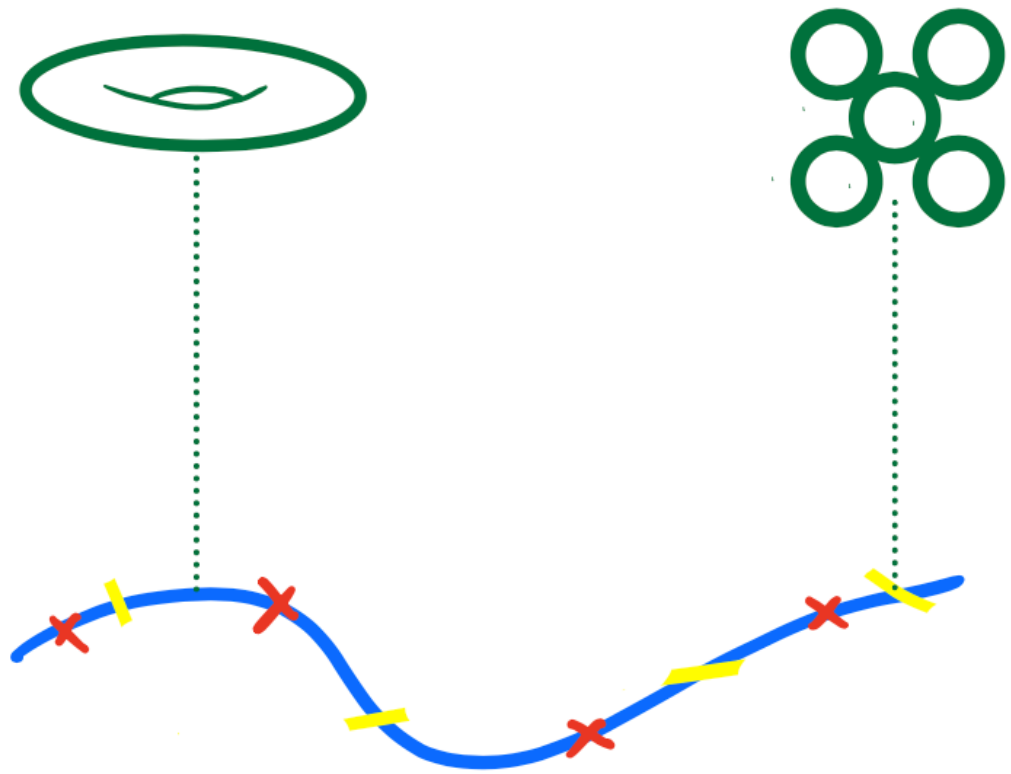

- We then classify $\bar{D}$ according to Iitaka dimension of $\mathcal{O}(-K_B)\,|_{\hat{D}}$

# Step 2: Anatomy of $\bar{D}$

- Iitaka dimension, $\dim \mathcal{O}(-K_B)|_{\hat{D}} = -\infty, 0, \ldots, 2 =$ dim $\hat{D}$
- Characterizes dimension of subspace of $\hat{D}$ over which fibration $\bar{D}$ varies nontrivially, computed as dimension of Newton polytope of $-K_B|_{\hat{D}}$

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- Using

$$h^{p,q}(\bar{D}) \geq h_+^{p,q}(\varphi_I(\hat{D})) + h_-^{p-1,q}(\varphi_I(\hat{D})) + h_p^{p,q-1}(\varphi_I(\hat{D})) + h_+^{p-1,q-1}(\varphi_I(\hat{D}))$$

and physical assumption (fermion zero mode matching)

$$\sum_{i=0}^{3} h^{i,0}(\bar{D}) = \sum_{i=0}^{2} h_\pm^{i,0}(\varphi_I(\hat{D}))$$

we derive formulae for $h^{p,q}(\bar{D})$ in each case, except $h^{2,1}(\bar{D})$

[PJ Kim `22]

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- Using Weierstrass model:

$$\chi \stackrel{?}{=} \int_{\bar{D}} c_3 = \int_{\hat{D}} 12\Big(c_1(\hat{D}) \cdot (-K_B|_{\hat{D}}) + 6(-K_B|_{\hat{D}})^2 + \sum_i 6((-K_B|_{\hat{D}}) \cdot S_i - 2S_i^2)\Big)$$

[Esole Jefferson Kang `17]
[Jefferson Turner, `22]

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- Using Weierstrass model (with $L = -K_B|_{\hat{D}}$)

$$\chi \stackrel{?}{=} \int_{\bar{D}} c_3 = \int_{\hat{D}} 12\left(c_1(\hat{D}) \cdot L + 6L^2 + \sum_i 6(L \cdot S_i - 2S_i^2)\right)$$

[Esole PJ Kang `17]
[PJ Turner, `22]

- Formula must be corrected in presence of O3 planes (terminal $\mathbb{Z}_2$ singularities)
- Conjecture: $\chi = \int_{\bar{D}} c_3 + 2N_{O3}$, passes numerous checks

# Sample result

**Case:** $\dim \mathcal{O}(-K_B)|_{\hat{D}} = 2$

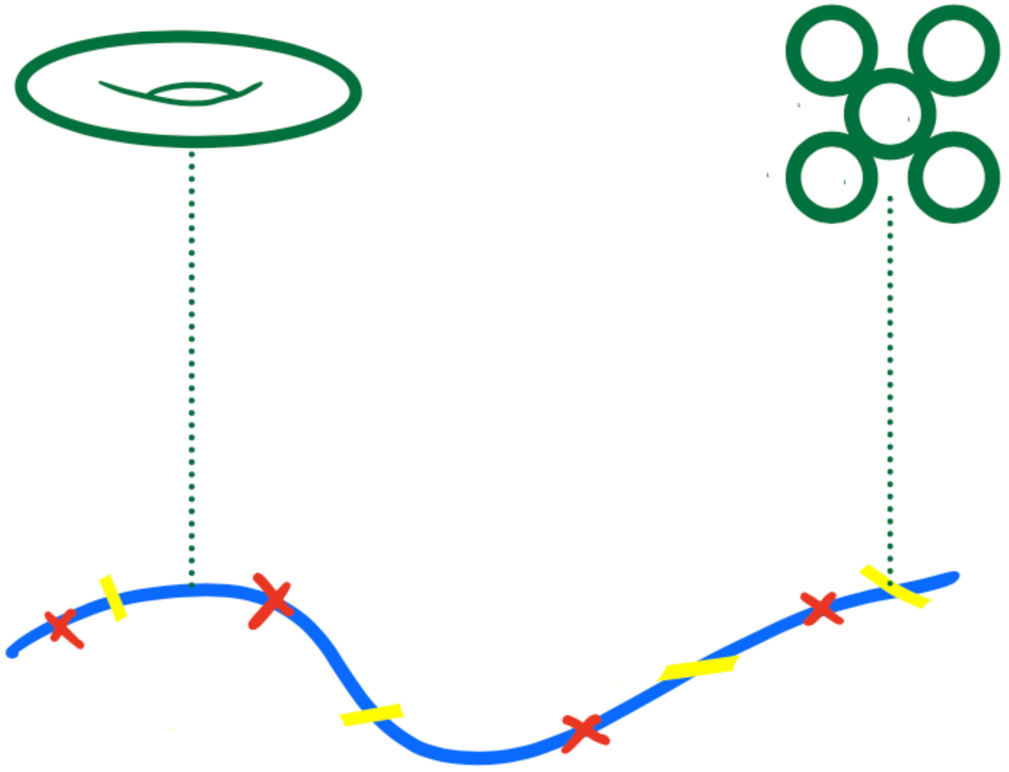
- $h^{\bullet,0}(\bar{D}) = h_-^{\bullet-1,0}(\varphi_I(\hat{D})) + h_+^{\bullet,0}(\varphi_I(\hat{D}))$
- $h^{1,1}(\bar{D}) = h^{1,1}(\hat{D}) + 1 + 4N_{SO(8)}$, where $h^{1,1}(\hat{D}) = h_+^{1,1}(\varphi_I(\hat{D}))$
- $h^{2,1}(\bar{D}) = 2 + h^{1,1}(\hat{D}) + 4N_{SO(8)} - N_{O3} - \frac{1}{2}\int_{\bar{D}} c_3$, for rigid case
- $h^{p,q}(\hat{D}) = h_+^{p,q}(\varphi_I(\hat{D})), h_-^{p,q}(\varphi_I(\hat{D}))$ expressed in terms of stratification using toric data

# Two local models

- We use local geometry to give heuristic argument for $\chi$.
- First model:

$$\begin{array}{c} \bar{D} \\ \downarrow \\ \hat{D} \\ \parallel \\ \mathbb{P}_{[1,1,2]} \end{array}$$

line bundle $L = \alpha H$



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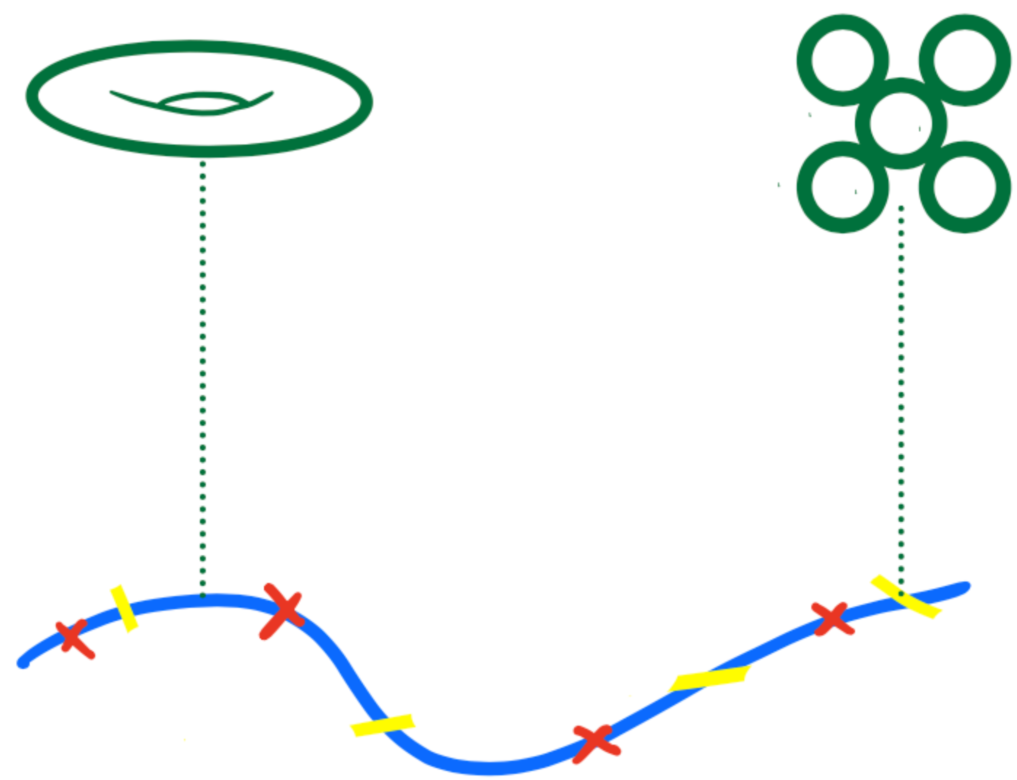


- Odd $\alpha \implies$ terminal $\mathbb{Z}_2,\ \Delta h^{2,1} = N_{O3}$

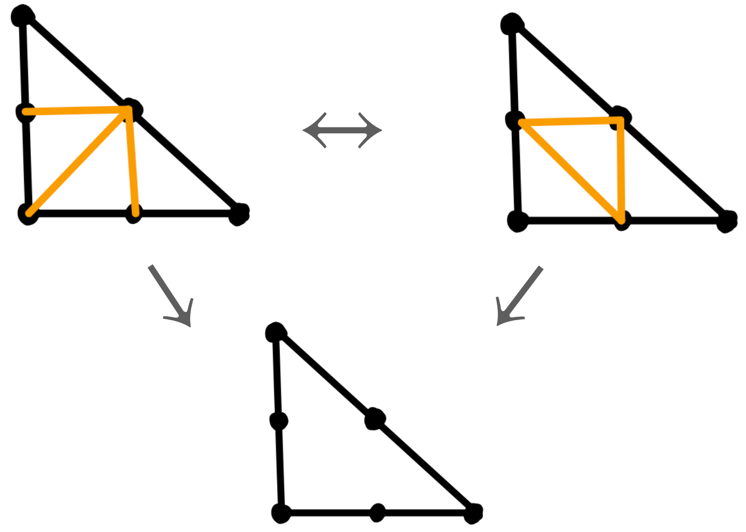
| $\alpha$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $h^{2,1}_{\text{naive}}(\bar{D})$ | 1 | 8 | 51 | 128 | 242 | 390 |
| $h^{2,1}(\bar{D})$ | 1 | 9 | 51 | 129 | 242 | 391 |

# Two local models

- Second model: local ECY4

$$
\begin{array}{ccc}
CY3^{asym} & \leftrightarrow & CY3^{sym} \\
\searrow & & \swarrow \\
\end{array}
$$

$$CY3^{sing} = \mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$$



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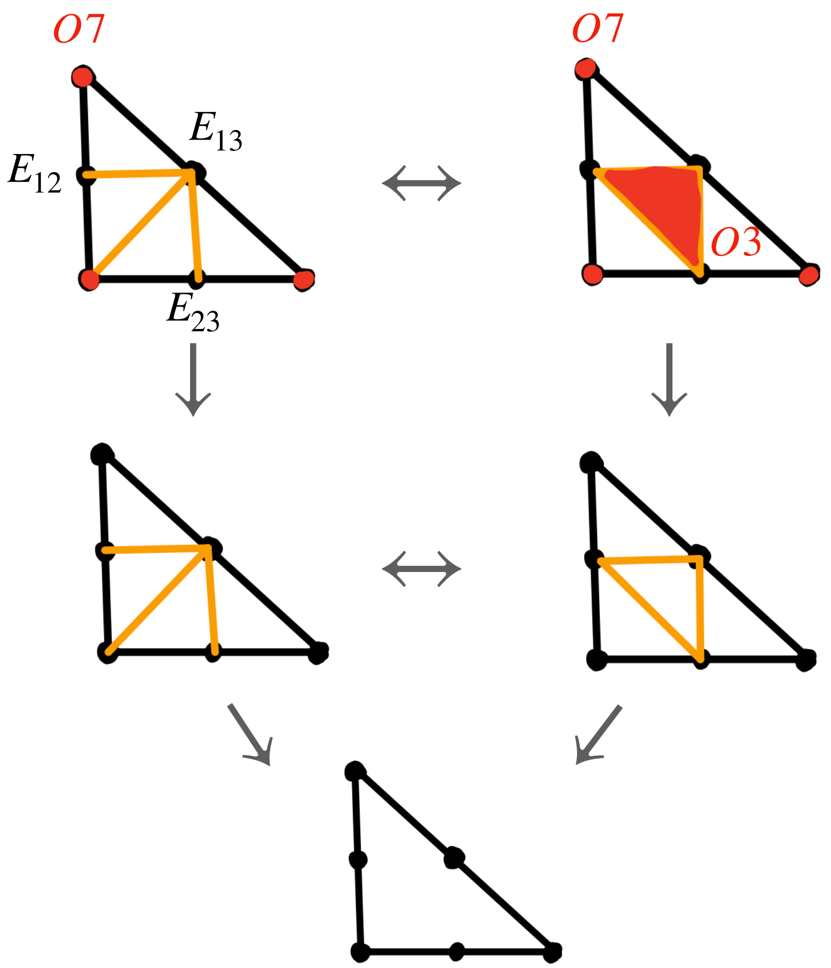
$$E_{12} \subset \quad B^{asym} \leftrightarrow B^{sym}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mathrm{CY3}^{asym} \leftrightarrow \mathrm{CY3}^{sym}$$

$$\searrow \qquad \swarrow$$

$$\mathrm{CY3}^{sing} = \mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$$



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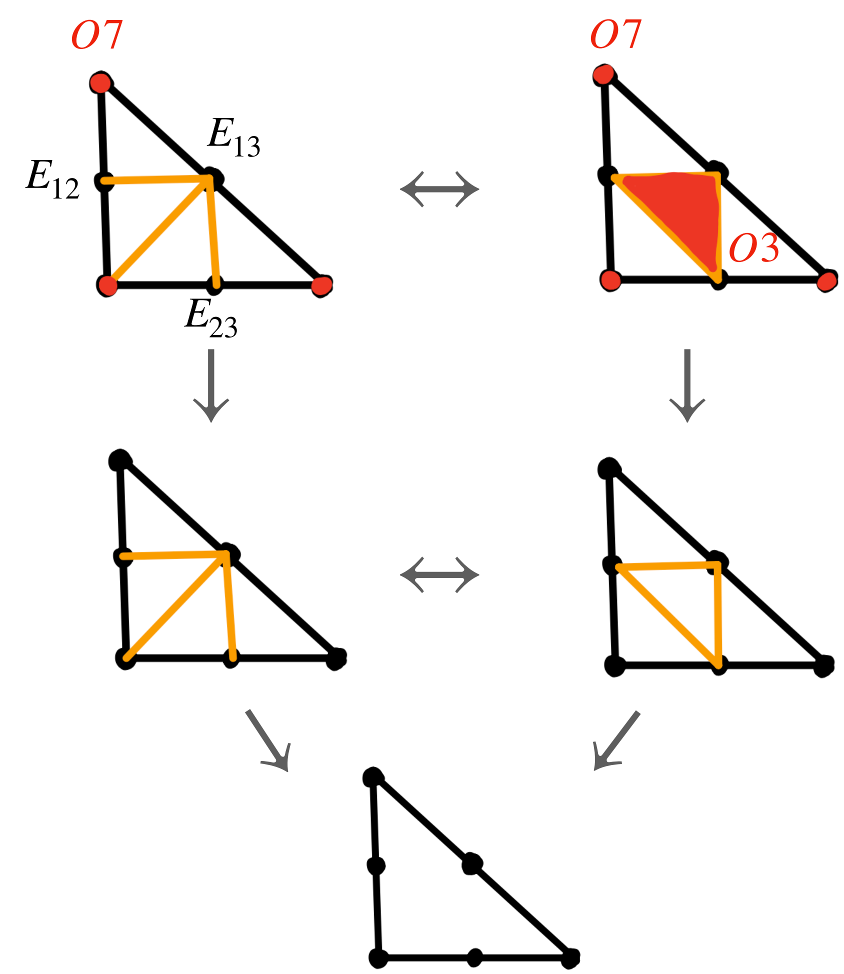
$$\begin{array}{cccc} \bar{E}_{12} \subset & \text{ECY4}^{asym} & \leftrightarrow & \text{ECY4}^{sym} \\ \downarrow & \downarrow & & \downarrow \\ E_{12} \subset & B^{asym} & \leftrightarrow & B^{sym} \\ & \downarrow & & \downarrow \\ & \text{CY3}^{asym} & \leftrightarrow & \text{CY3}^{sym} \\ & \searrow & & \swarrow \\ & \multicolumn{3}{c}{\text{CY3}^{sing} = \mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2} \end{array}$$



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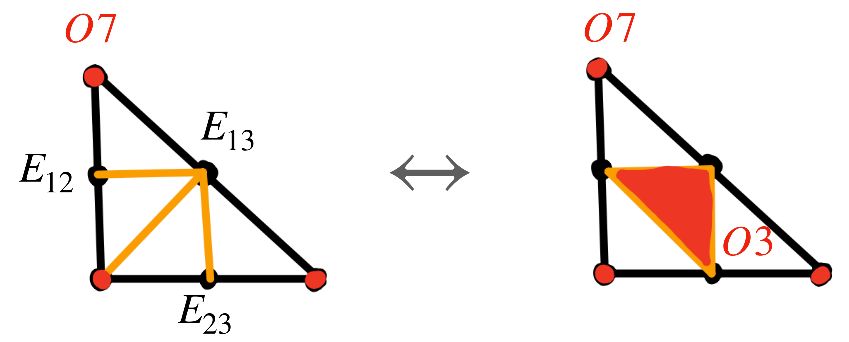


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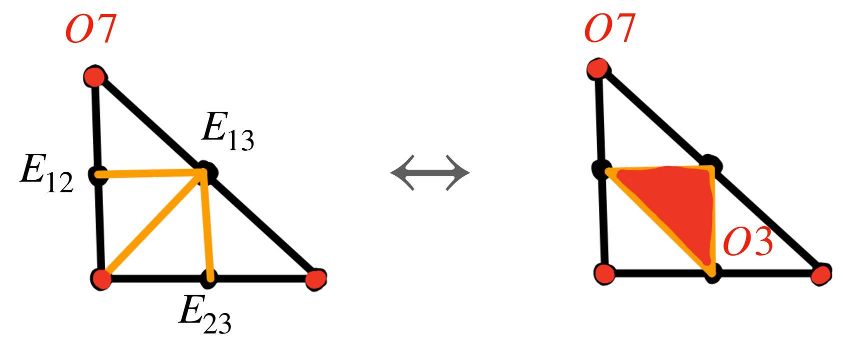


- Asymmetric phase: $\chi(\bar{E}_{12}) = \int_{\bar{E}_{12}} c_3$

- Symmetric phase: $\chi(\bar{E}_{12}) = \int_{\bar{E}_{12}} c_3 + 2N_{O3}$

# Examples

## Explicit computation of $h^{p,q}(\bar{D})$

- E.f. over orientifold quintic
- E.f. over orientifold $\mathbb{P}_{[1,1,1,6,9]}[18]$
- E.f. over orientifold $\mathbb{P}^2 \times \mathbb{P}_{[1,1,2]}$
- Mirror dual of e.f. over orientifold $\mathbb{P}_{[1,1,1,6,9]}[18]$
- Various others

# Conclusion

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- Formulae do not require explicit construction of F-theory uplift
- Immediate applications to finding divisors that generate $W_{n.p.}$
- New insights into stringy geometry in presence of orientifold singularities

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## Future directions

- W.i.p.: deriving a ``proof" of Euler characteristic formula (orbifold Euler character? Modification of Witten index for sigma models on singular targets/D3 brane tadpole?) [Dixon Harvey Vafa Witten `85] [Sethi Vafa Witten `96]

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- F-theory with more general orientifolds (terminal $\mathbb{Z}_{k>2}$ singularities?) [García-Etxebarria Regalado `16]

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- Heterotic dual dictionary
- F-theory uplifts using vex triangulations to resolve toric complete intersections

# Thank you!