$a = c$ and Emergent $\mathcal{N} = 4$ from $\mathcal{N} = 1$

Craig Lawrie (DESY)

Based on: 2106.12579 w/ M.J. Kang and J. Song
2111.12092 w/ M.J. Kang, K-H. Lee, and J. Song
2207.05764 w/ M.J. Kang, K-H. Lee, M. Sacchi, and J. Song
2210.06497 w/ M.J. Kang, K-H. Lee, and J. Song
2302.06622 w/ M.J. Kang, K-H. Lee, and J. Song

Strings and Geometry 2023

06-03-2023

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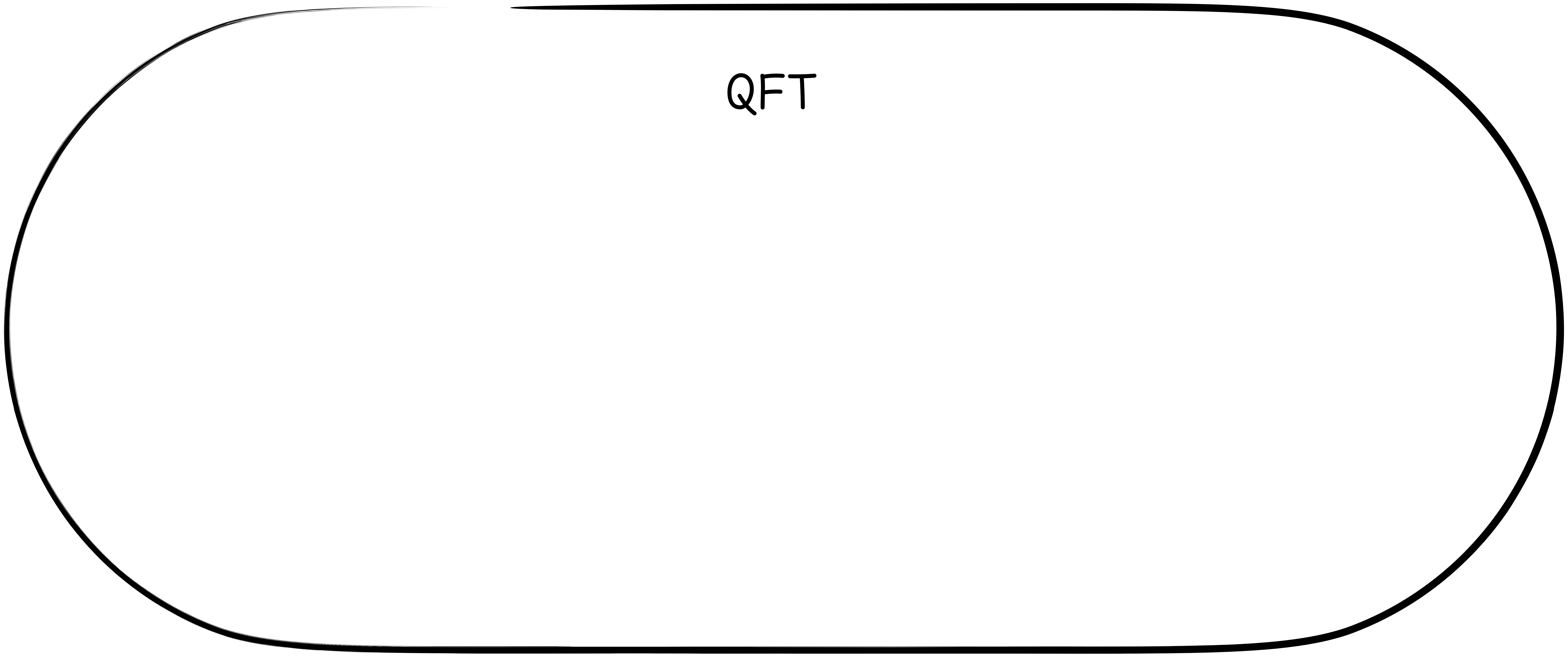
Strings and Geometry 2023

06-03-2023

Motivation



Quantum field theory is a widely applicable framework for answering diverse questions in physics

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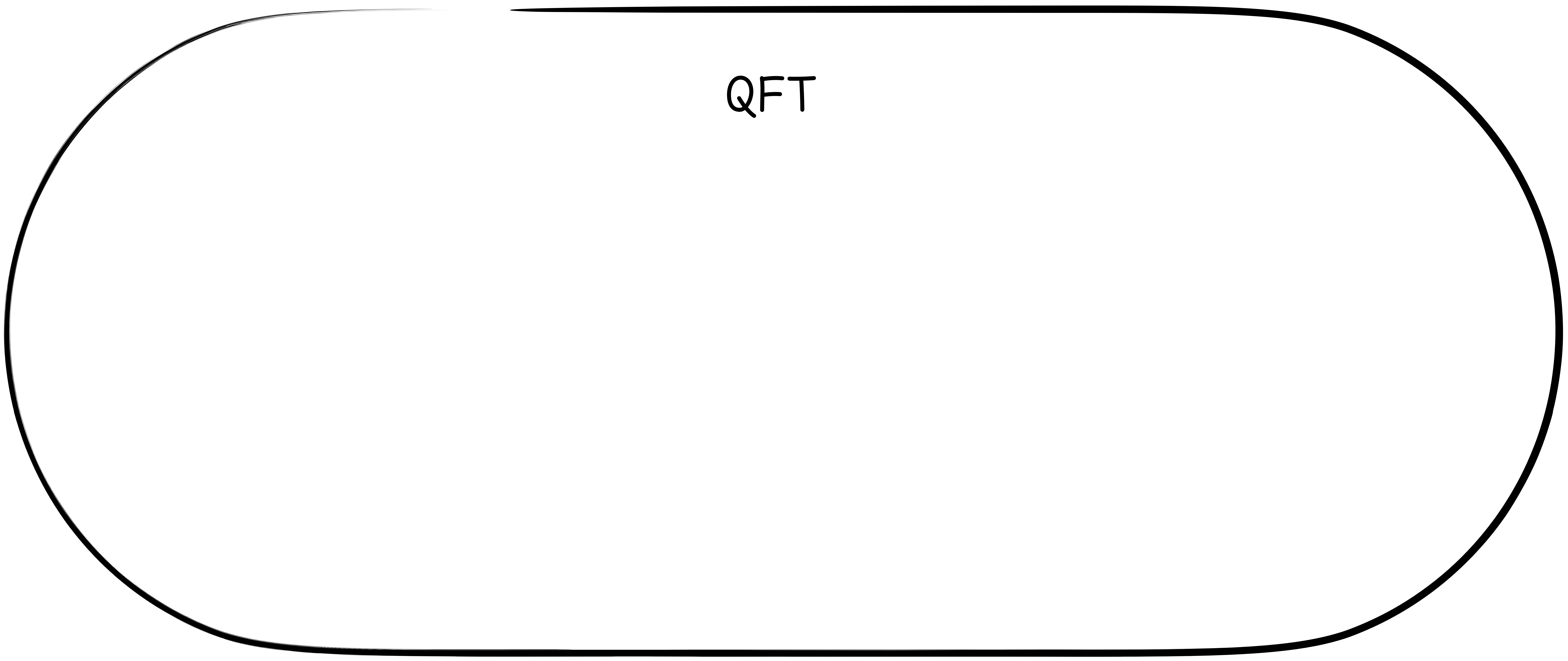

Quantum field theory is a widely applicable framework for answering diverse questions in physics

→ generally hard to study outside of the “perturbative regime”

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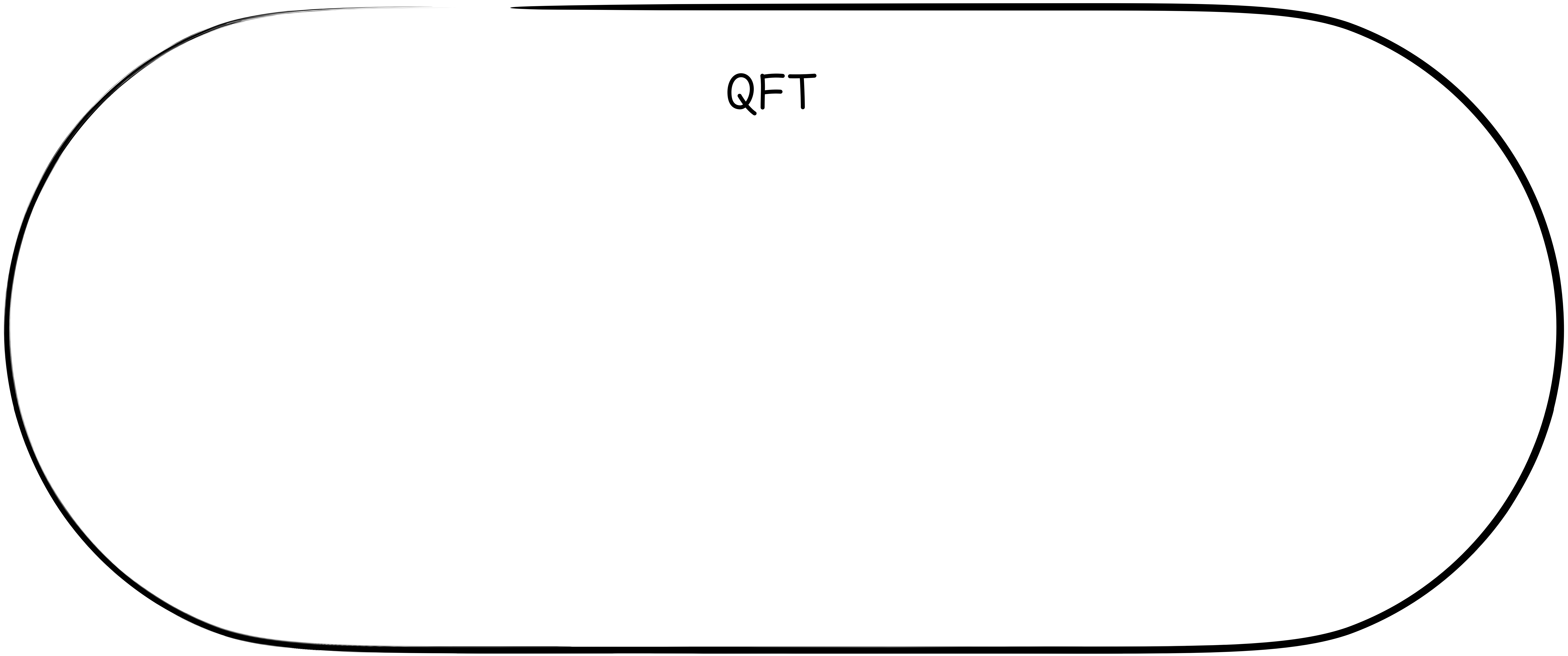

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→ add new symmetries to make questions tractable

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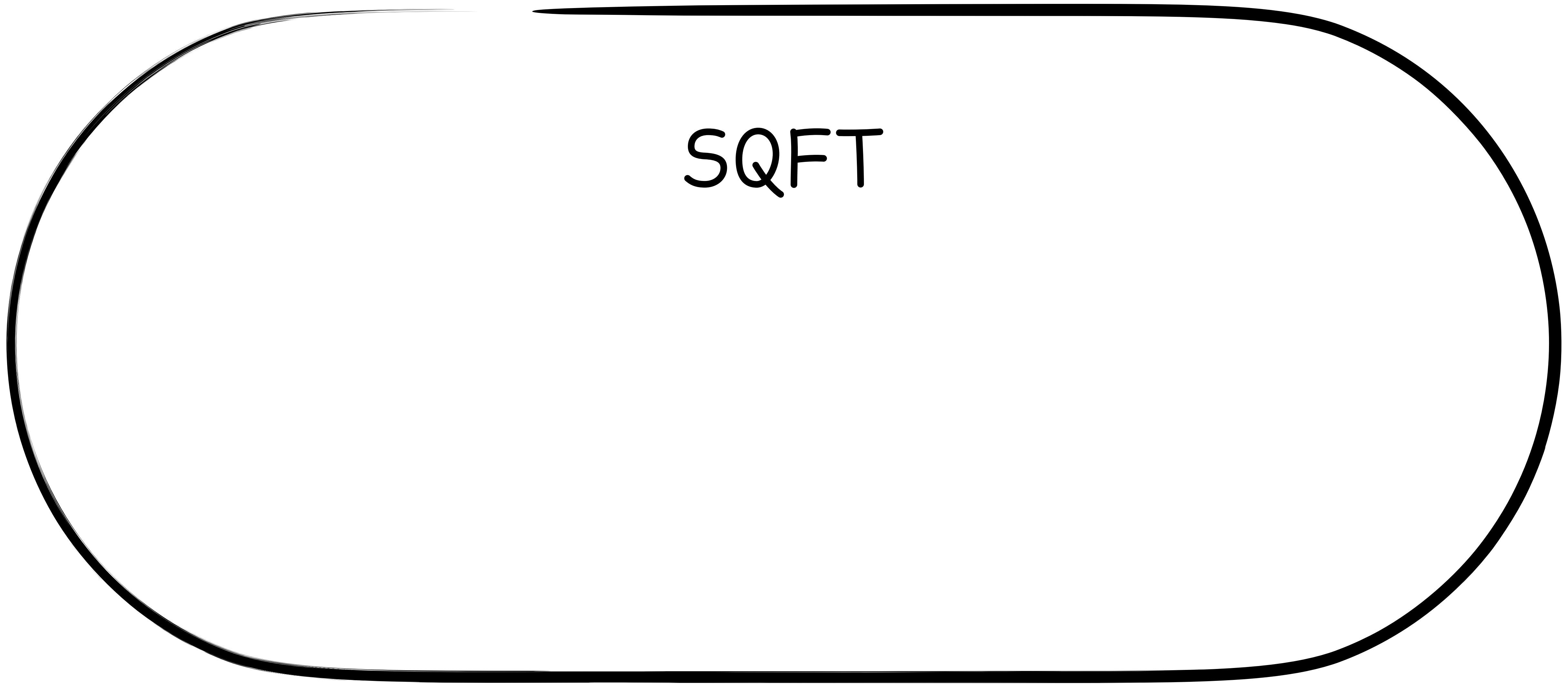

Supersymmetry $\Longrightarrow$ certain quantities are protected from quantum corrections

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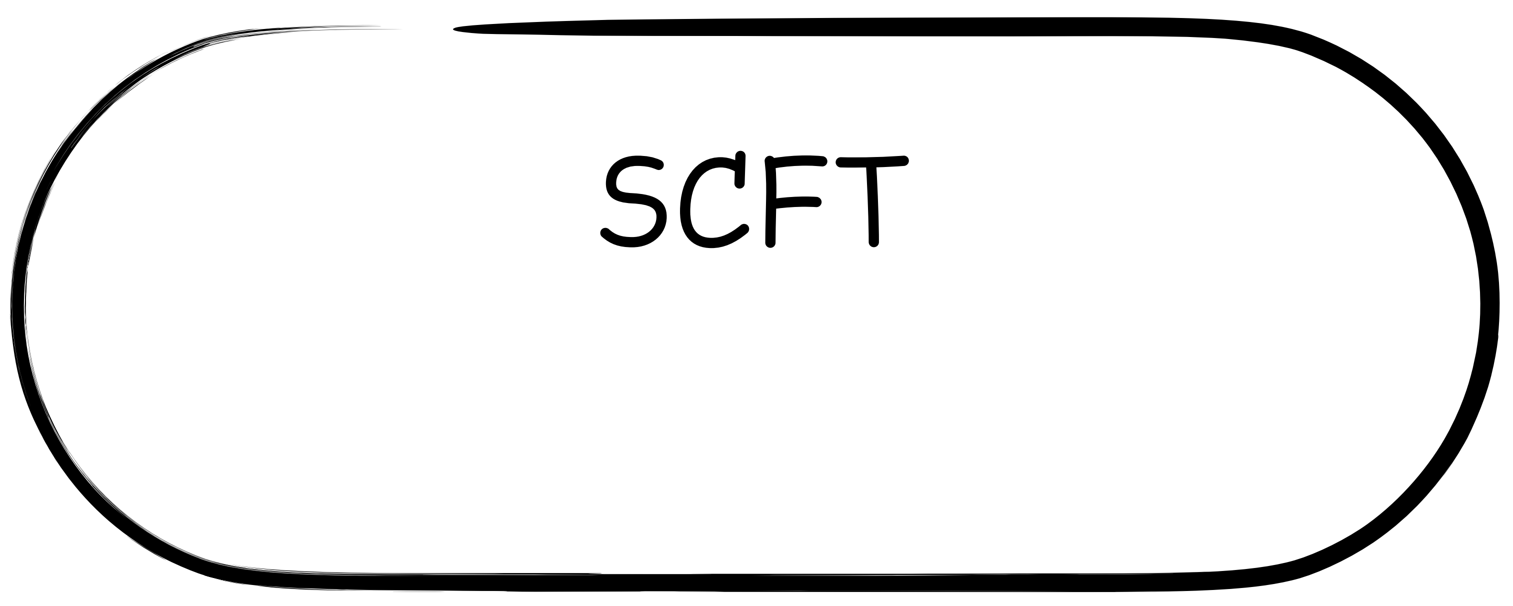

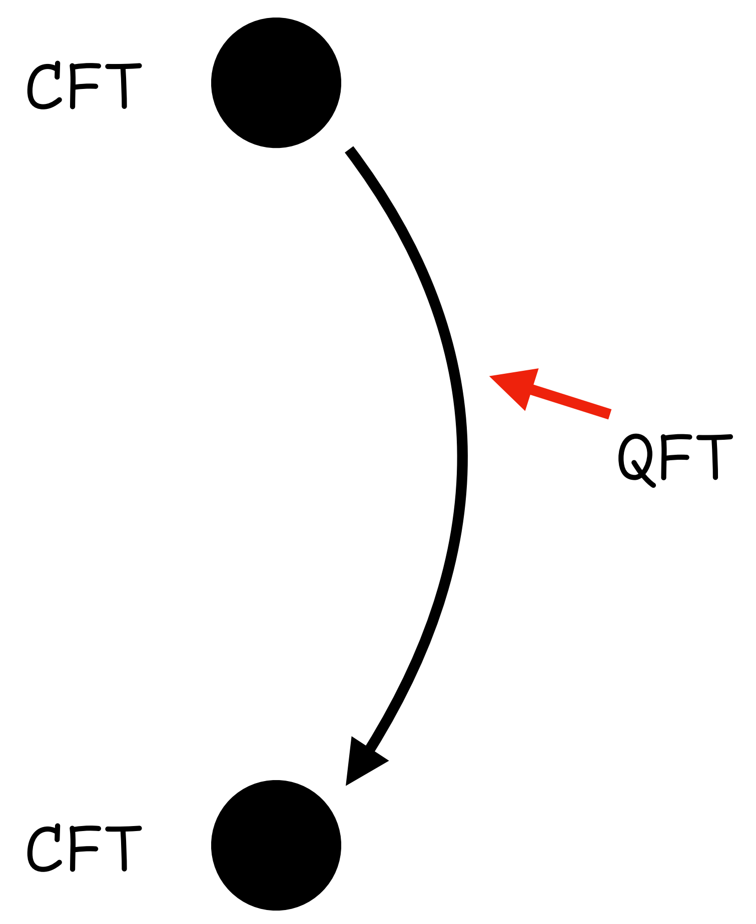

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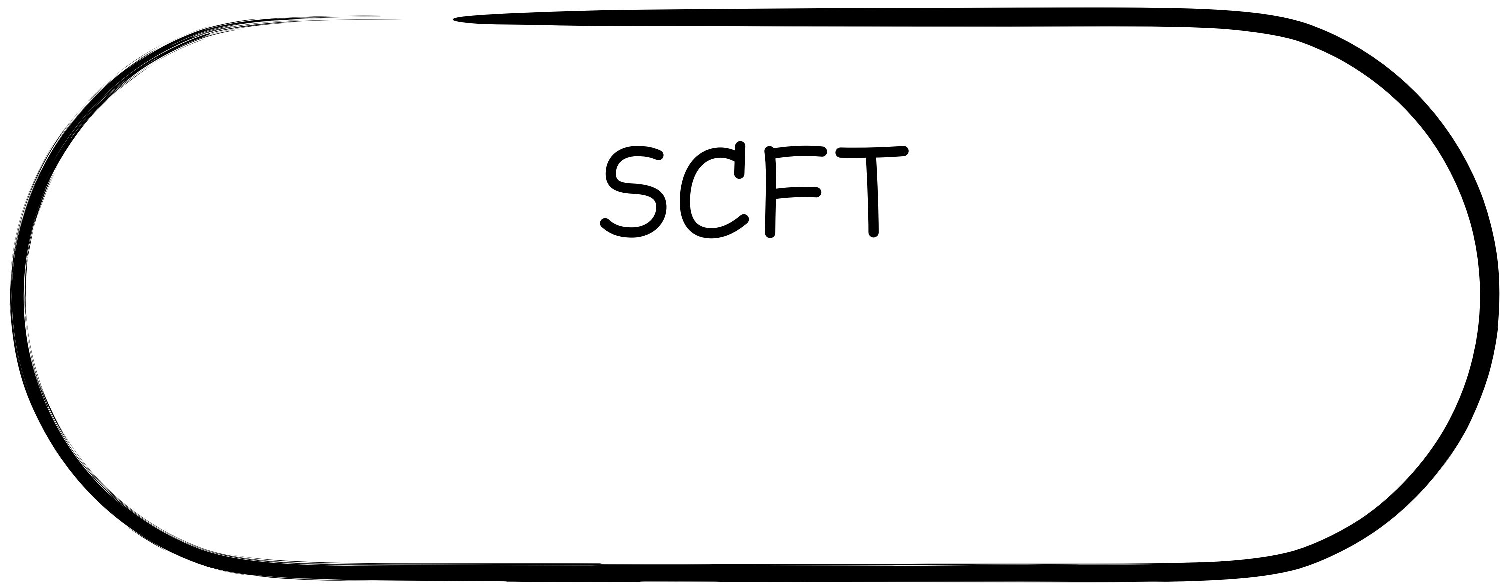


Conformal symmetry $\Longrightarrow$ exists at the fixed points of renormalization group flows between QFTs

Supersymmetry $\Longrightarrow$ certain quantities are protected from quantum corrections

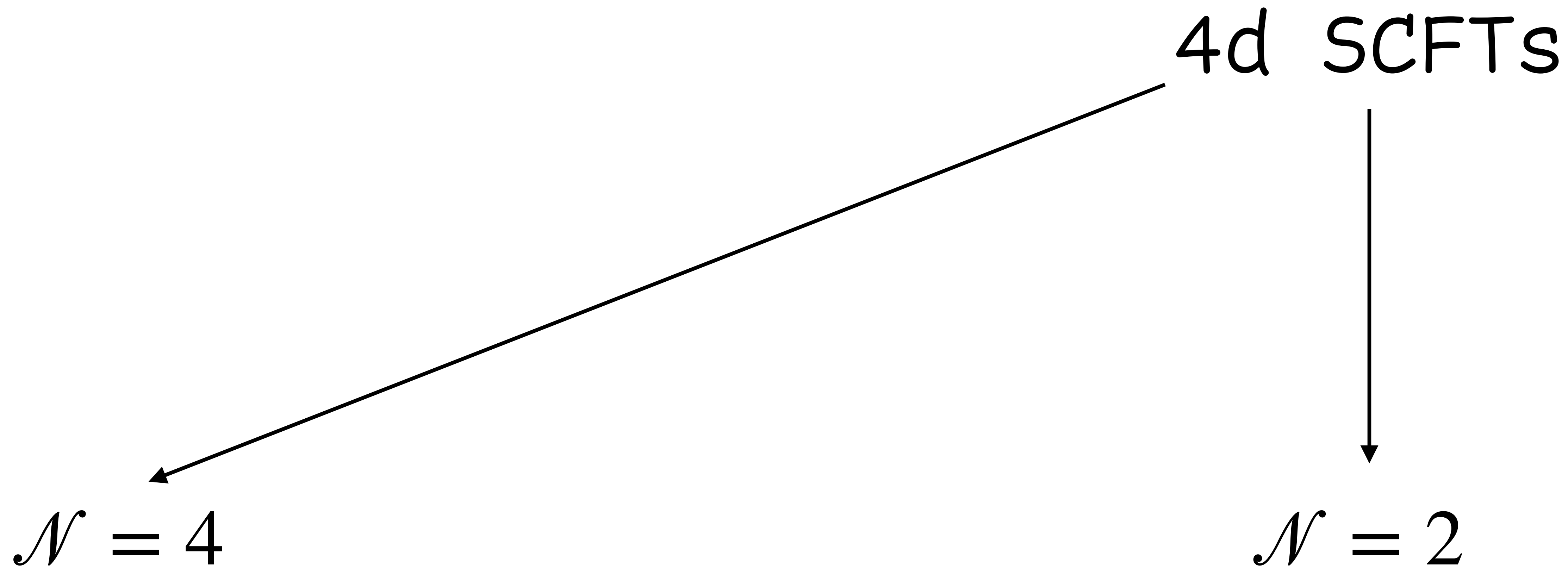
We study this subspace of the space of all QFTs

→ what can we understand via stringy/geometric techniques?

Strings, Geometry, and the Landscape of 4d SCFTs

4d SCFTs

Strings, Geometry, and the Landscape of 4d SCFTs



Strings, Geometry, and the Landscape of 4d SCFTs

4d SCFTs

$\mathcal{N} = 4$

$\mathbb{C}^2/\Gamma$ orbifolds

...

$\mathcal{N} = 2$

Type IIB on Calabi–Yau threefolds

class $\mathcal{S}$

Coulomb branch complex geometry

...

many, many geometric approaches; see recent reviews [Akhond, Arias-Tamargo, Mininno, Sun, Sun, Wang, Xu] [Argyres, Heckman, Intriligator, Martone]

Strings, Geometry, and the Landscape of 4d SCFTs

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Coulomb branch complex geometry

...

$\mathcal{N} = 1$

can we use some geometry here?

many, many geometric approaches; see recent reviews [Akhond, Arias-Tamargo, Mininno, Sun, Sun, Wang, Xu] [Argyres, Heckman, Intriligator, Martone]

Central Charges

Conformal symmetry becomes anomalous when the CFT is placed in an arbitrary background

central charges

$$\langle T^{\mu}_{\mu} \rangle = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$$

stress-energy tensor

Weyl tensor

Euler density

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Euler density

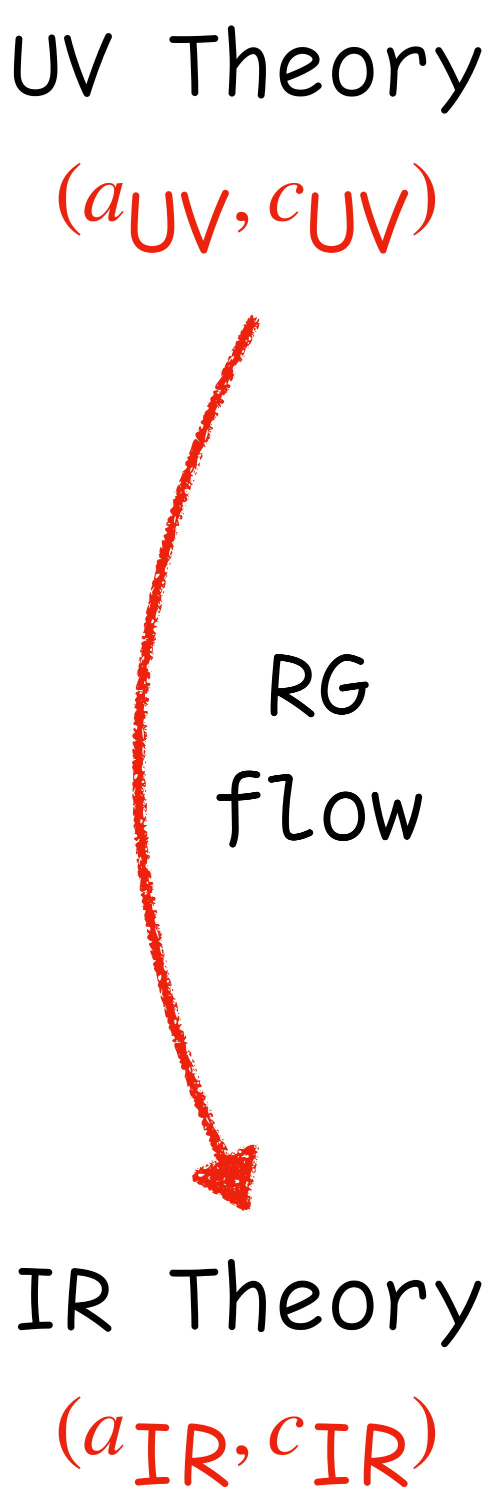
the central charges are “conventional invariants” of the SCFT

see talks

Kang, today
Distler, Thursday

The a-theorem

[Komargodski, Schwimmer]



a monotonically decreases along any renormalization group flow

there is no equivalent statement for c

in this sense, a measures the "degrees of freedom" of the theory

$\mathcal{N} = 1$ from $\mathcal{N} = 2$?

can we use the (geometric?) constructions for $\mathcal{N} = 2$ SCFTs
to learn about (a subsector of) the $\mathcal{N} = 1$ landscape?

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one approach: consider $\mathcal{N} = 2$ SCFT with flavor symmetry G and ($\mathcal{N} = 1$)-gauge G

$\mathcal{N} = 1$ from $\mathcal{N} = 2$?

focus on the class $\mathcal{S}$ construction

can we use the (geometric?) constructions for $\mathcal{N}=2$ SCFTs
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one approach: consider $\mathcal{N}=2$ SCFT with flavor symmetry G and $(\mathcal{N}=1)$-gauge G

Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

$$\mathcal{S}_{\mathfrak{g}}\langle C_{g,n}\rangle\{\cdots\}$$

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the 6d $(2,0)$ SCFT
of type $\mathfrak{g}$

String theory: Type IIB on an orbifold $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$
[Witten]

$\mathfrak{g}$ of type ADE:
simple, simply-laced Lie algebra

Class $\mathcal{S}$

[Gaiotto], [Gaiotto, Moore, Nietzke]

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the 6d $(2,0)$ SCFT
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twisted compactification on
an n-punctured genus g Riemann surface

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data describing punctures =
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$= 4\text{d } \mathcal{N}=2$ SCFT

twisted compactification on
an n-punctured genus g Riemann surface

Complicated physical features
(e.g. S-dualities)
captured by the geometry of $C_{g,n}$

Class $\mathcal{S}$ Building Blocks for $\mathcal{N} = 1$ SCFTs

Class $\mathscr{S}$ Building Blocks for $\mathcal{N} = 1$ SCFTs

Some known 4d $\mathcal{N} = 2$ SCFTs:

1) $\mathscr{D}_p(G)$ – non-Lagrangian

[Xie], [del Zotto, Cecotti]
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fractional Coulomb branch scaling dimensions
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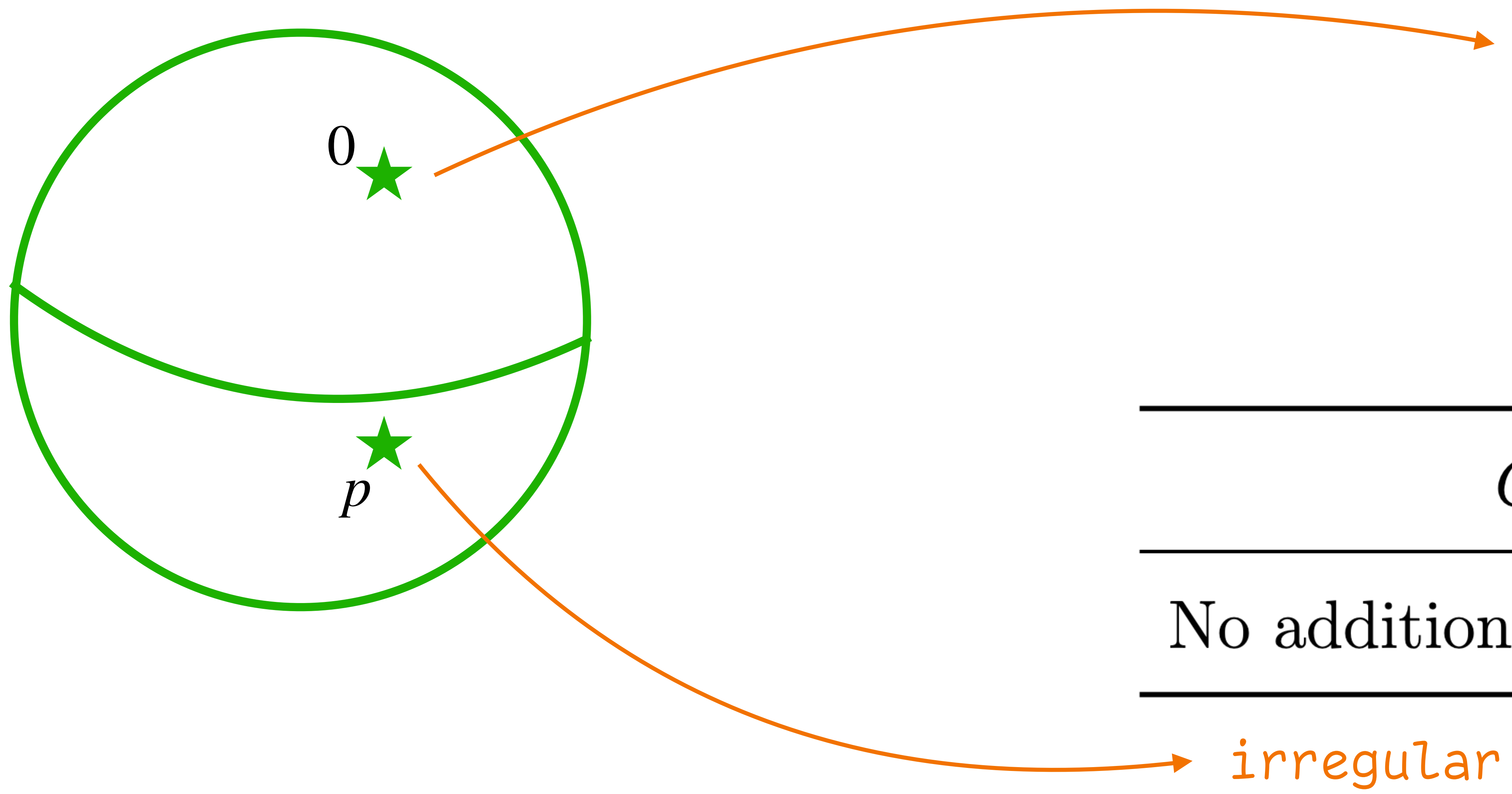

G	$SU(N)$	$SO(2N)$	E_6	E_7	E_8
No additional symmetry	$(p, N) = 1$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 3\mathbb{Z}_{>0}$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 30\mathbb{Z}_{>0}$

Class $\mathcal{S}$ Building Blocks for $\mathcal{N} = 1$ SCFTs

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1) $\mathcal{D}_p(G)$ – non-Lagrangian [Xie], [del Zotto, Cecotti] [del Zotto, Cecotti, Giacomelli], [Xie, Wang]

2) (G, G) conformal matter [del Zotto, Heckman, Tomasiello, Vafa]

a strongly-coupled generalization of an $SU(\ell) \times SU(\ell)$ bifundamental hypermultiplet

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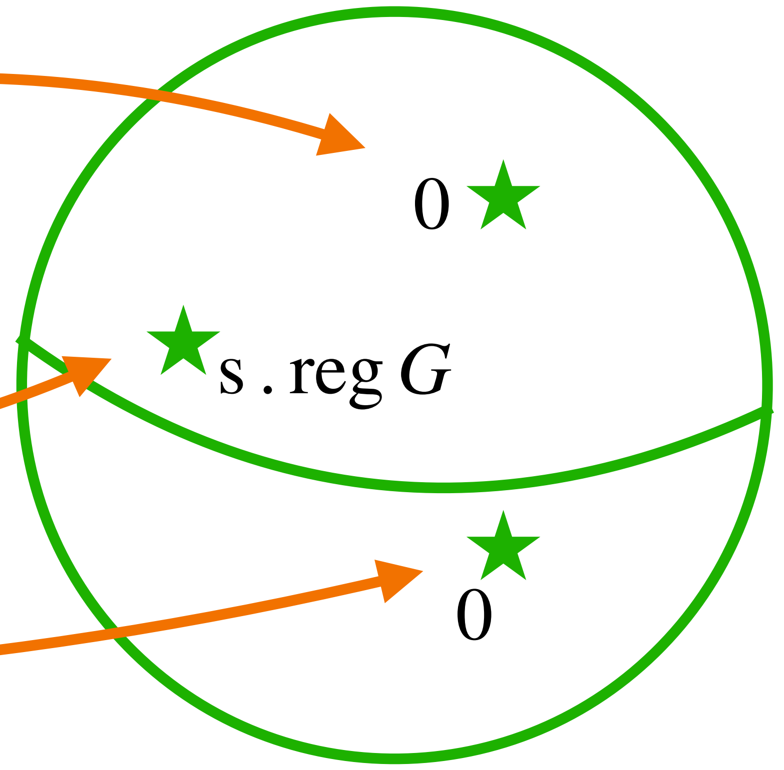
Class $\mathscr{S}$:

regular maximal punctures each with flavor symmetry G

subregular puncture



[Ohmori, Shimizu, Tachikawa, Yonekura]
[del Zotto, Vafa, Xie]
[Baume, Kang, CL]

Class $\mathcal{S}$ Building Blocks for $\mathcal{N} = 1$ SCFTs

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2) (G, G) conformal matter [del Zotto, Heckman, Tomasiello, Vafa]

→ can we construct new 4d SCFTs using these strongly-coupled theories as building blocks?

consider $\mathcal{N}=2$ or $\mathcal{N}=1$ gauging of all G flavor symmetries

Class $\mathscr{S}$ Building Blocks for $\mathcal{N} = 1$ SCFTs

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→ can we construct new 4d SCFTs using these strongly-coupled theories as building blocks?

consider $\mathcal{N} = 2$ or $\mathcal{N} = 1$ gauging of all G flavor symmetries

→ for $\mathcal{N} = 2$ there is an ADE classification $\widehat{\Gamma}(G)$ [Kang, CL, Song '20]

An $\mathcal{N} = 1$ Classification Problem

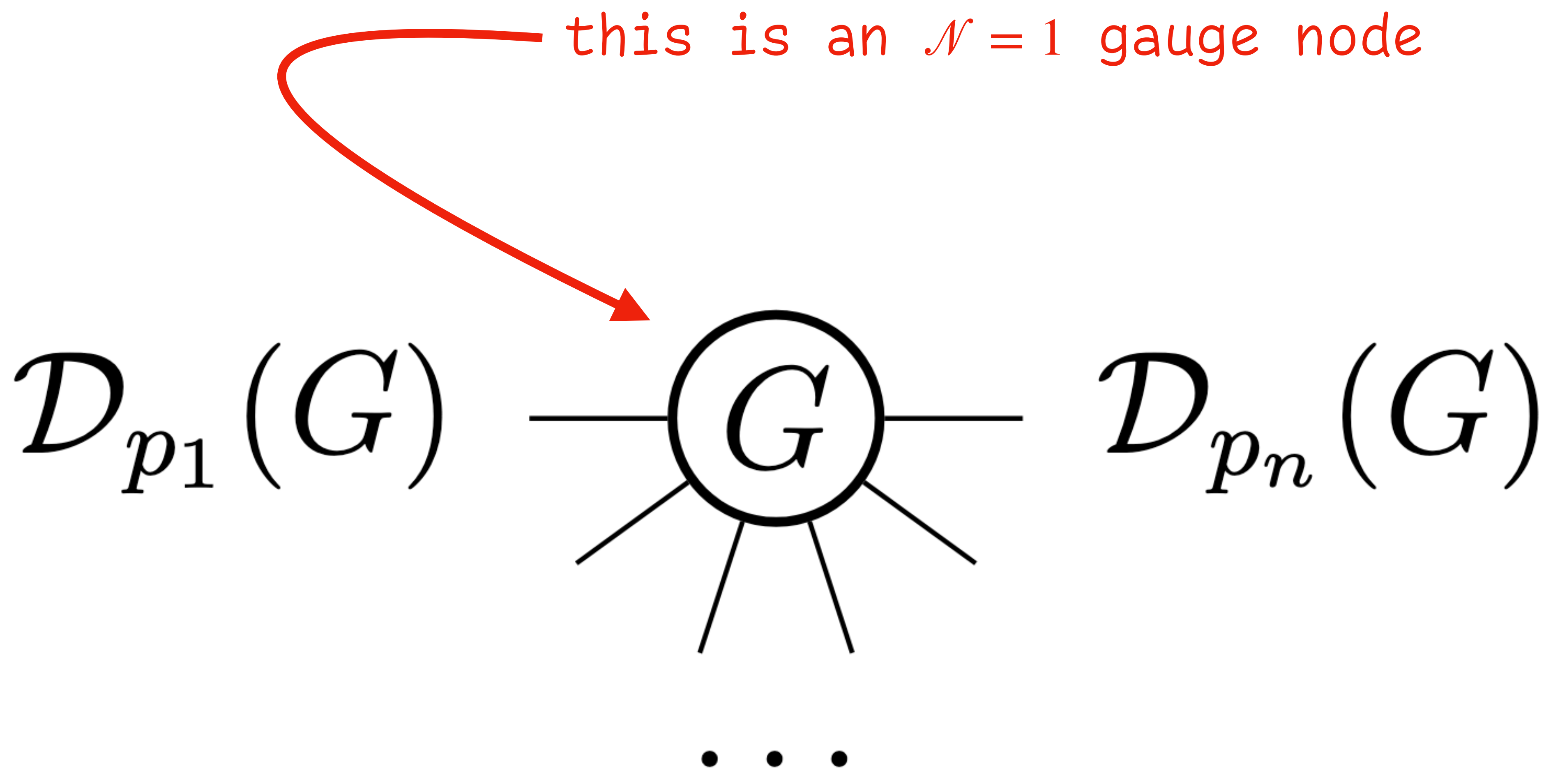
how can we gauge together all G flavor symmetries
of a collection of $\mathcal{D}_p(G)$ such that the result
flows in the infrared to an $\mathcal{N} = 1$ SCFT?

[Kang, CL, Lee, Song '21]

An $\mathcal{N} = 1$ Classification Problem

[Kang, CL, Lee, Song '21]

First focus on cases without conformal matter



For an asymptotically-free gauge coupling

$$\sum_{i=1}^{n} \frac{1}{p_i} > n - 3$$

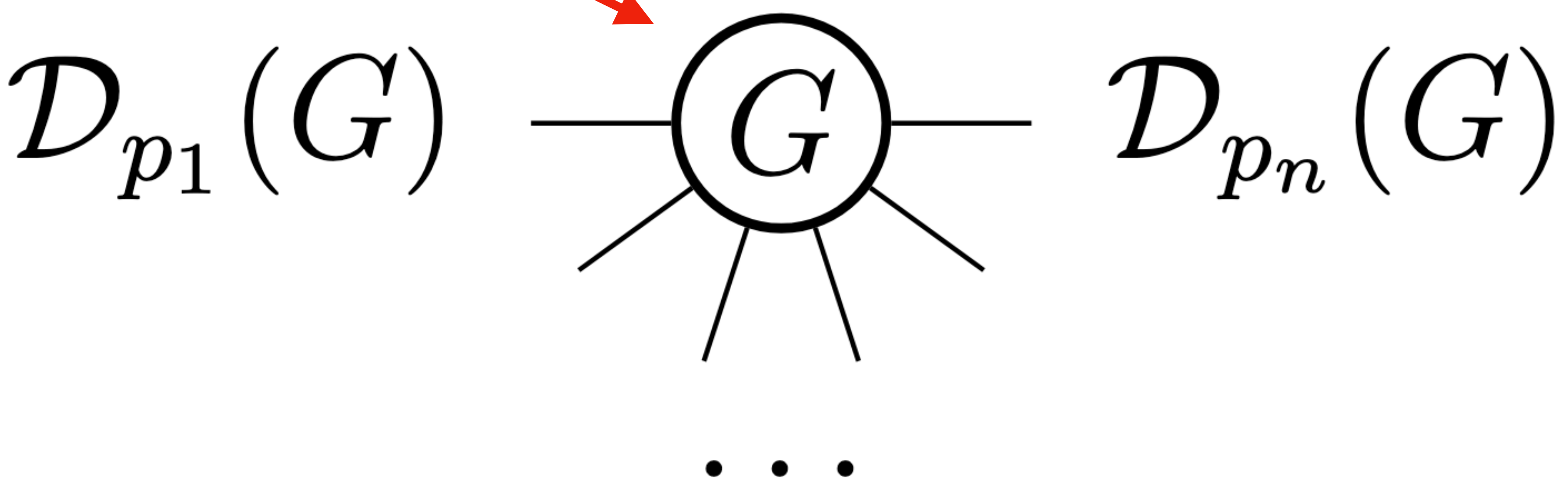
equality implies conformal gauging

An $\mathcal{N} = 1$ Classification Problem

[Kang, CL, Lee, Song '21]

First focus on cases without conformal matter

this is an $\mathcal{N}=1$ gauge node



For an asymptotically-free gauge coupling

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all solutions

equality implies conformal gauging

p_1	p_2	p_3	p_4	p_5
1	1	1	1	p_5
1	1	1	p_4	p_5
1	1	p_3	p_4	p_5
1	2	2	p_4	p_5
1	2	3	≤ 6	p_5
1	2	3	7	≤ 41
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p_1	p_2	p_3	p_4	p_5
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Not all such gaugings flow to interacting infrared SCFTs

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[Intriligator, Wecht]

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if $\gcd(p_i, h_G^\vee) = 1$ then IR fixed point has $a = c$

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why? isn't $a = c$ a feature of $\mathcal{N} \geq 3$ SUSY?

$\mathcal{N} = 1$: Adding Matter

[Kang, CL, Lee, Song '21]

With $\mathcal{N} = 1$ gauging we can also add one or two adjoint chiral multiplets while preserving $a = c$

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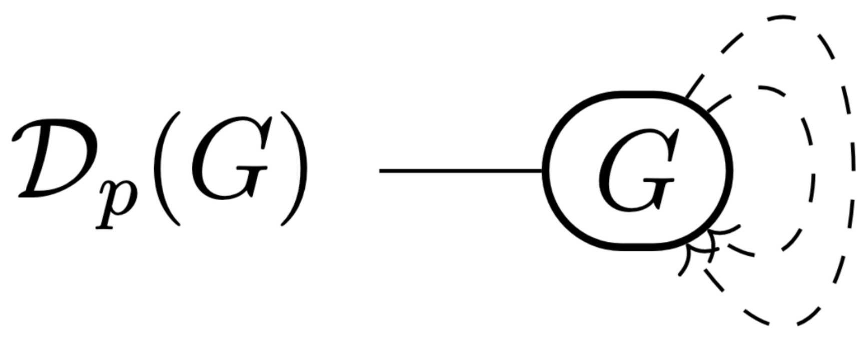
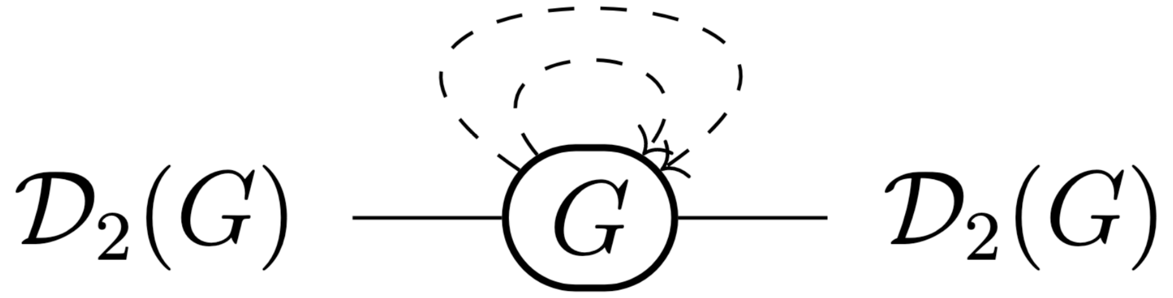
only two options

SCFTs living on the conformal manifold of $\mathcal{N} = 2$ $\widehat{\Gamma}(G)$ theories





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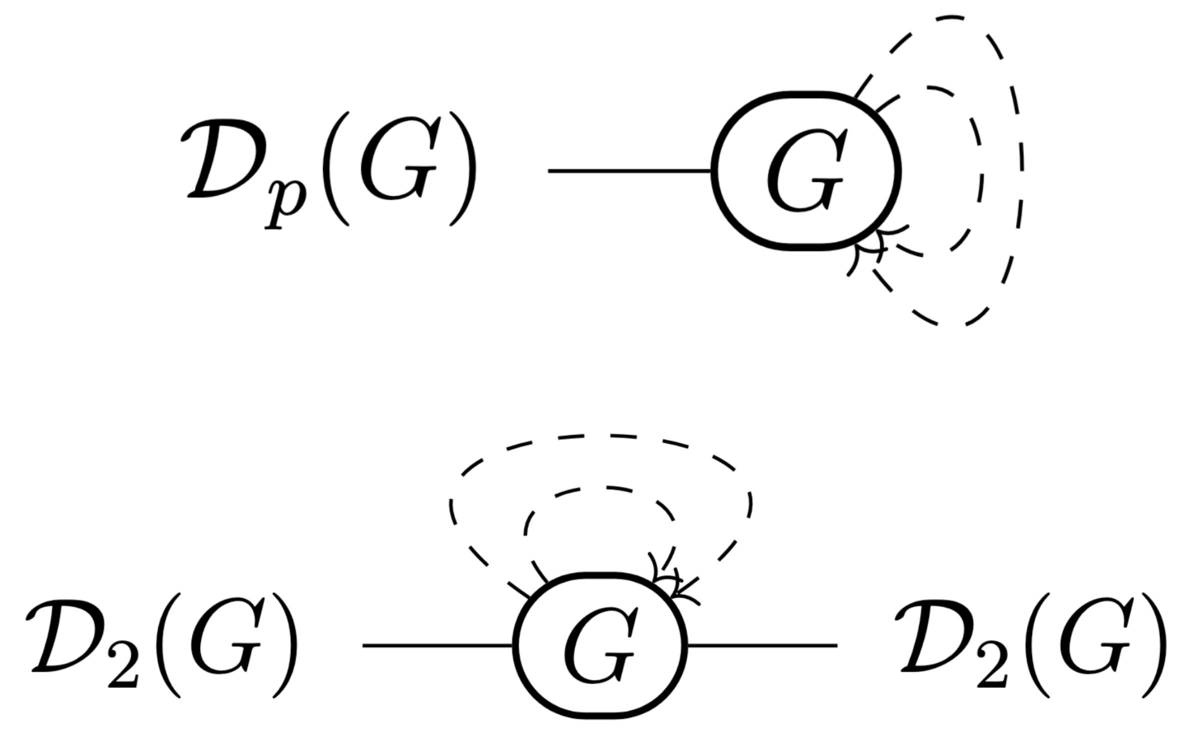

SCFTs living on the conformal manifold of $\mathcal{N} = 2$ $\widehat{\Gamma}(G)$ theories

also three adjoint chiral multiplet + zero $\mathscr{D}_p(G)s$ $\longrightarrow$ conformal manifold of $\mathcal{N} = 4$ SYM

SUSY and a/c

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SUSY and a/c

[Hofman, Maldacena]
[Hofman, Li, Meltzer, Poland, Rejon-Barrera]

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free scalar field

free vector field

SUSY and a/c

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$$\mathcal{N} = 1 : \quad \frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$$

$$\mathcal{N} = 2 : \quad \frac{1}{2} \leq \frac{a}{c} \leq \frac{5}{4}$$

$$\mathcal{N} = 3,4 : \quad a = c$$

free scalar field (lower bound, $\mathcal{N} = 0$)

free vector field (upper bound, $\mathcal{N} = 0$)

free chiral multiplet (lower bound, $\mathcal{N} = 1$)

free $\mathcal{N} = 1$ vector (upper bound, $\mathcal{N} = 1$)

free hypermultiplet (lower bound, $\mathcal{N} = 2$)

free $\mathcal{N} = 2$ vector (upper bound, $\mathcal{N} = 2$)

Why $a = c$?

Holography: if 4d SCFT has $AdS_5 \times X_5$ dual then $a = c \sim O(N^2)$ to leading order in a large N limit

The subleading terms are

$$c - a = \rho N + \sigma$$

open string contributions i.e. branes

closed string contributions $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

if $a = c$ at finite N then ρ and σ must conspire to cancel!

Why $a = c$?

$c - a$ controls many interesting quantities in a CFT

- Cardy limit of superconformal index: $I \to \exp\left(\frac{16\pi^2}{3\beta}(c-a)\right)$ [di Pietro, Komargodski]
- Entropy-viscosity ratio bound: $\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \cdots\right)$ [Kovtun, Son, Starinet] [Katz, Petrov] [Buchel, Myres, Sinha]
- Mixed current-gravitational anomaly [Anselmi, Freedman, Grisaru, Johansen]
- Single trace higher spin gap for large N [Edelstein, Maldacena, Zhiboedov]

The Story Thus Far

We have constructed a broad collection of truly $\mathcal{N}=1$ and $\mathcal{N}=2$ SCFTs with exactly $a=c$

[Kang, CL, Song ‘20]
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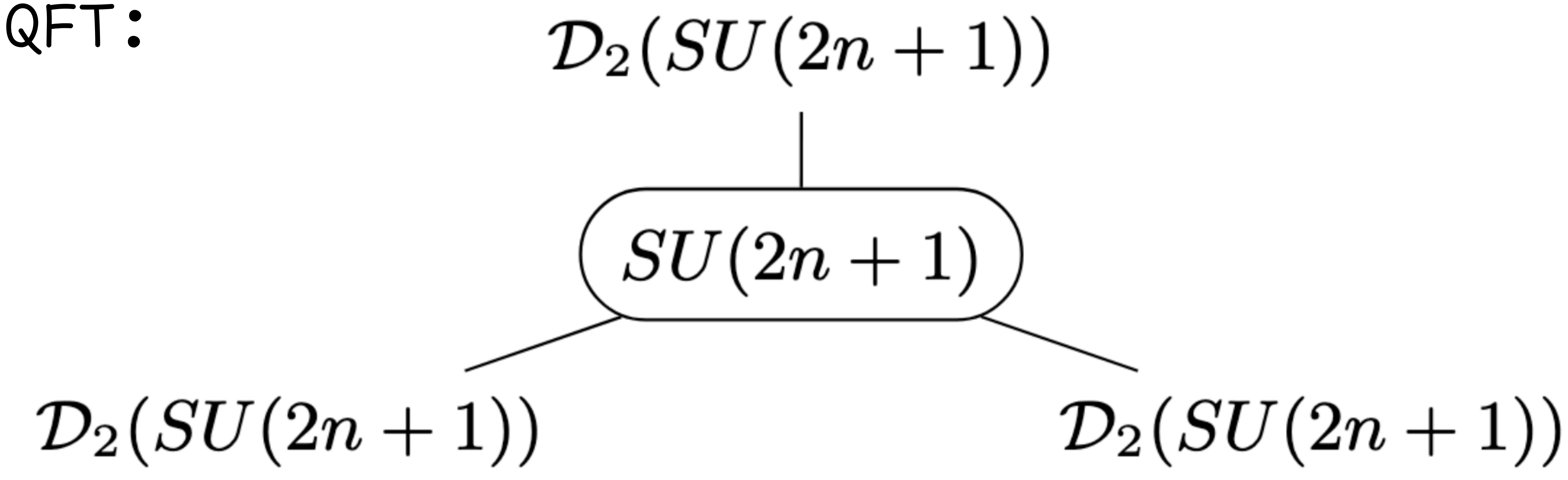
can we push the connection to $\mathcal{N}=4$ further?

A non-Lagrangian $\mathcal{N}=1$ dual to $\mathcal{N}=4$ SYM

[Kang, CL, Lee, Song '23]

Consider the $\mathcal{N}=1$ QFT:



Flows to infrared CFT with $a=c$ since $\gcd(2,2n+1)=1$

let's work out the central charges!

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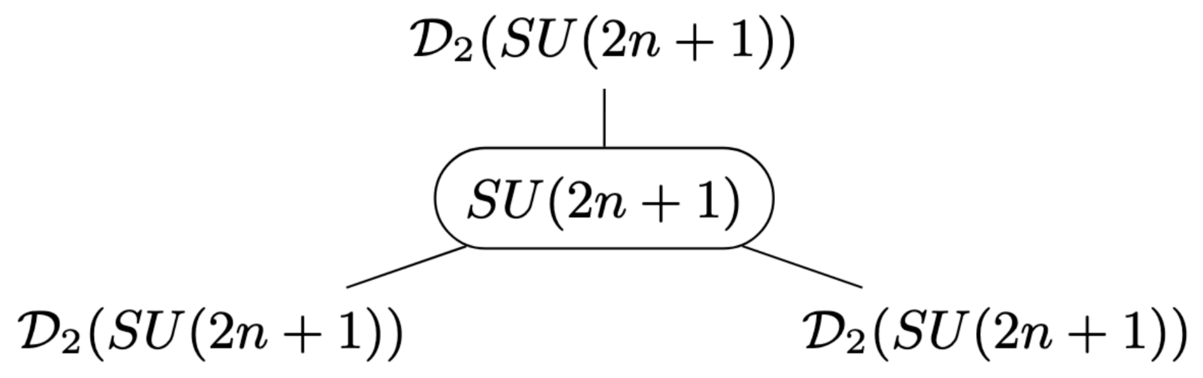

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Gauging breaks the R-symmetry of each $\mathscr{D}_2(G)$:

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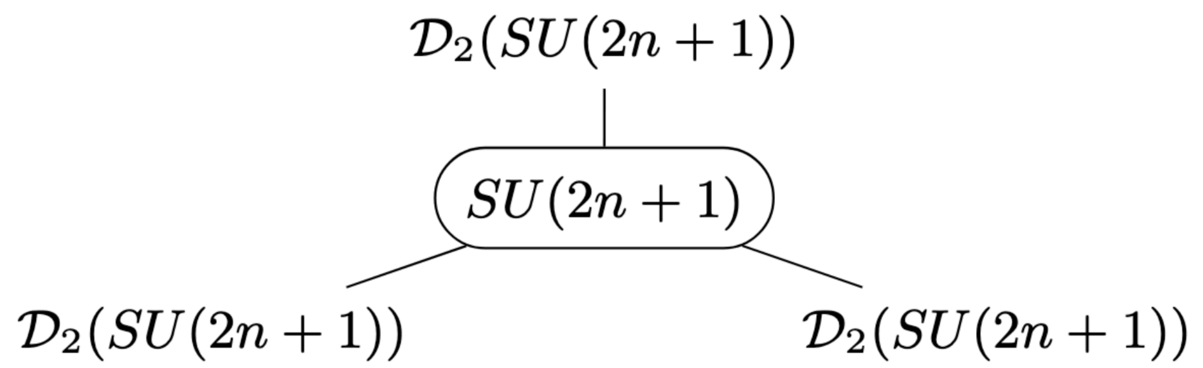

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mixing coefficients:
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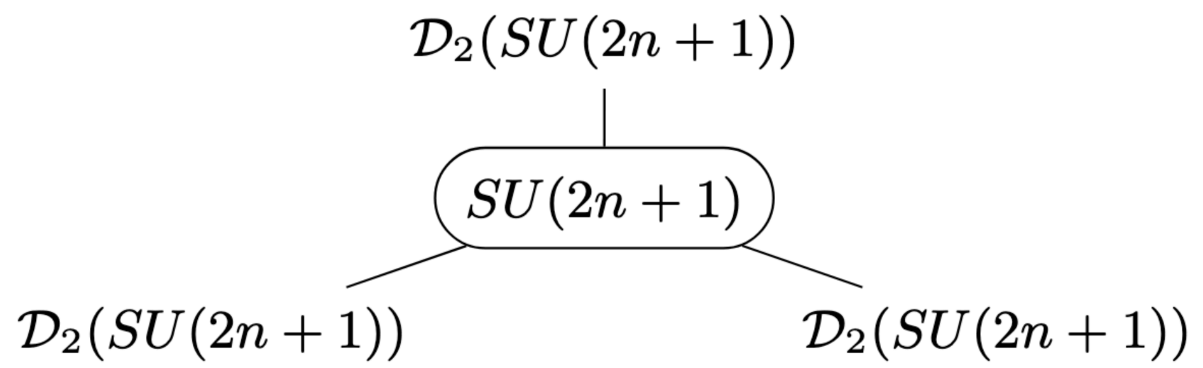

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$$a = \frac{3}{32}(3k_{RRR} - k_R)$$

UV 't Hooft anomalies

$$a = \frac{d}{32}\left(13 - 9\sum_{i=1}^{3} \epsilon_i^2(\epsilon_i + 2)\right)$$

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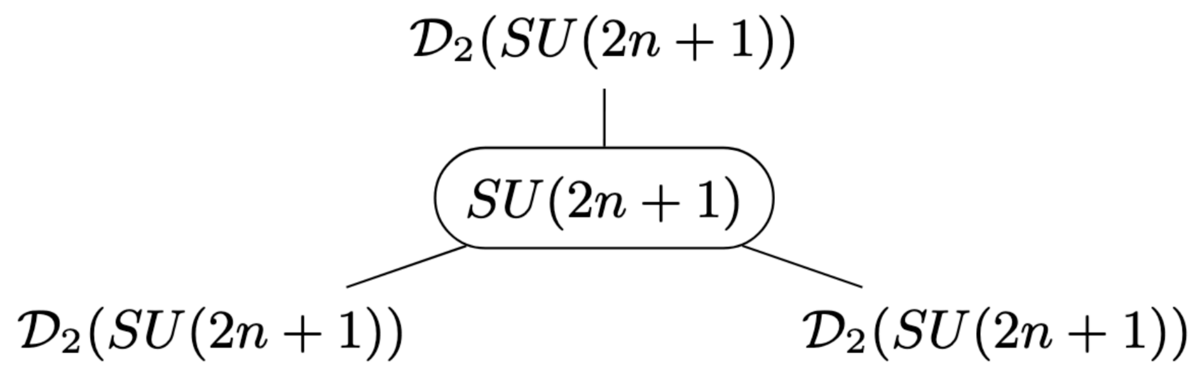

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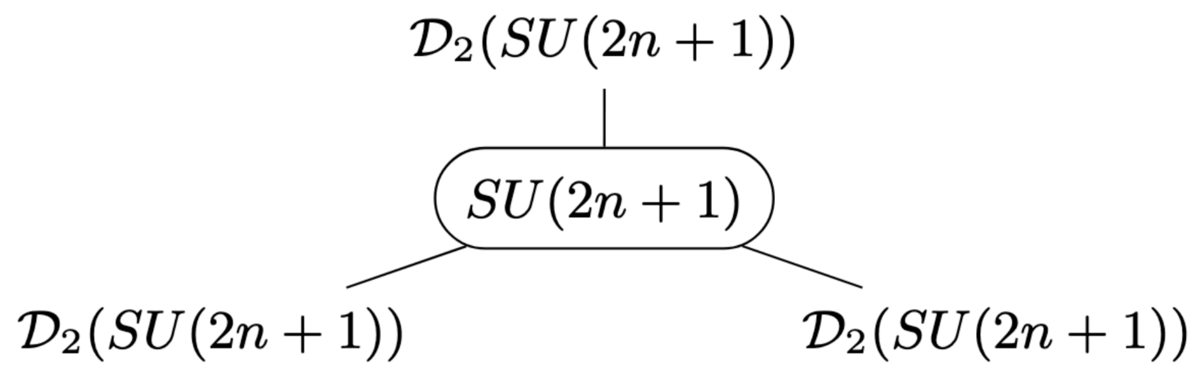

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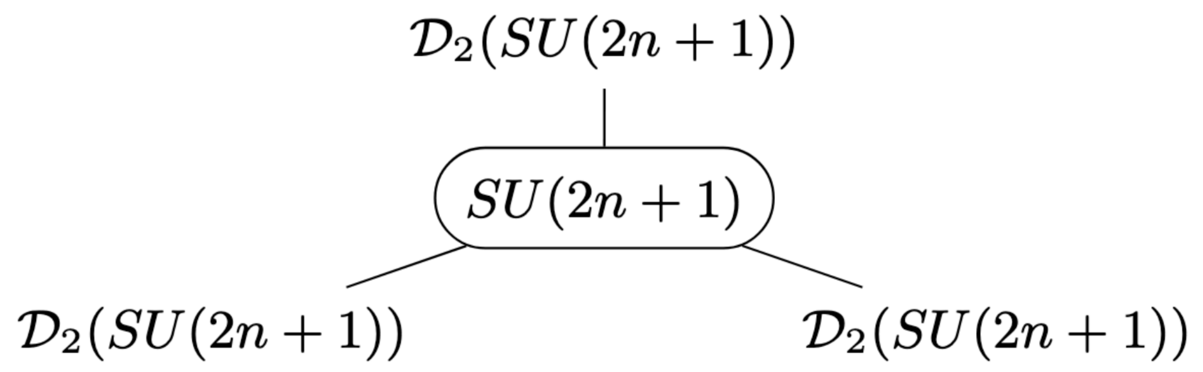

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$\mathcal{N}=4$ Casimir operator ⟶ $\mathrm{Tr}\,\phi_i^k$ ⟶ $u_i, Q^2 u_i$ ⟵ $\mathcal{D}_2(G)$ Coulomb branch operators + superdescendents

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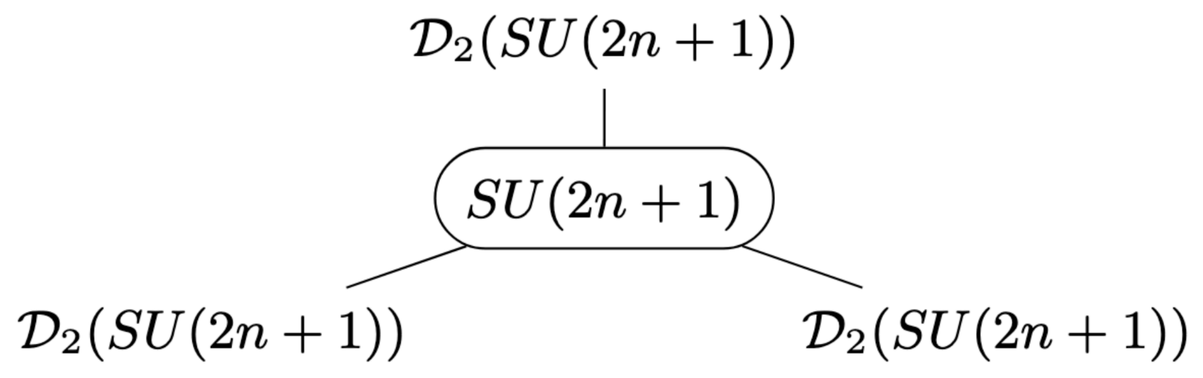

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how else can we verify this proposed duality?

Superconformal Index

The superconformal index counts certain short superconformal multiplets of an $\mathcal{N}=1$ SCFT:

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[Romelsberger]

$U(1)_R$ R-charge

$SU(2)_2$ Lorentz

flavor fugacities

$$I = \mathrm{Tr}(-1)^F t^{3(R+2j_2)} y^{2j_1} \prod_i v_i^{f_i}$$

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$\mathcal{D}_2(SU(3))$ itself has an $\mathcal{N}=1$ Lagrangian description [(Agarwal,) Maruyoshi, Song]

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$$\begin{aligned}\widehat{I}^{\mathfrak{su}_3} &\equiv (1-t^3y)(1-t^3/y)(I^{\mathfrak{su}_3}-1)\\ &= t^4\chi_{\mathbf{6}}^{\mathfrak{su}_3} - t^5\chi_{\mathbf{2}}^{\mathfrak{su}_2}\chi_{\mathbf{3}}^{\mathfrak{su}_3} + t^6(\chi_{\mathbf{10}}^{\mathfrak{su}_3} - \chi_{\mathbf{8}}^{\mathfrak{su}_3} + 1)\\ &- t^7\chi_{\mathbf{2}}^{\mathfrak{su}_2}(\chi_{\mathbf{6}}^{\mathfrak{su}_3} - \chi_{\overline{\mathbf{3}}}^{\mathfrak{su}_3}) + t^8(\chi_{\mathbf{15'}}^{\mathfrak{su}_3} - \chi_{\mathbf{15}}^{\mathfrak{su}_3} + \chi_{\overline{\mathbf{6}}}^{\mathfrak{su}_3}\\ &+2\chi_{\mathbf{3}}^{\mathfrak{su}_3}) - t^9\chi_{\mathbf{2}}^{\mathfrak{su}_2}(\chi_{\mathbf{10}}^{\mathfrak{su}_3}+1) + t^{10}\chi_{\mathbf{3}}^{\mathfrak{su}_2}\chi_{\overline{\mathbf{3}}}^{\mathfrak{su}_3}\\ &+ t^{10}(\chi_{\overline{\mathbf{21}}}^{\mathfrak{su}_3} - \chi_{\overline{\mathbf{15}}}^{\mathfrak{su}_3} + 2\chi_{\mathbf{6}}^{\mathfrak{su}_3} - 2\chi_{\overline{\mathbf{3}}}^{\mathfrak{su}_3}) + \cdots ,\end{aligned}$$

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[Kang, CL, Lee, Song '23]

We constructed a non-Lagrangian $\mathcal{N}=1$ gauge theory that flows to a point on the conformal manifold of $\mathcal{N}=4$ SYM

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can we use the powerful techniques to study maximally-supersymmetric Lagrangian theories to learn about Argyres–Douglas SCFTs?

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What are the supergravity duals to $a = c$ theories?

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delicate cancellation between Kaluza–Klein modes

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do they preserve $a = c$?

[Kang, CL, Lee, Song *to appear*]

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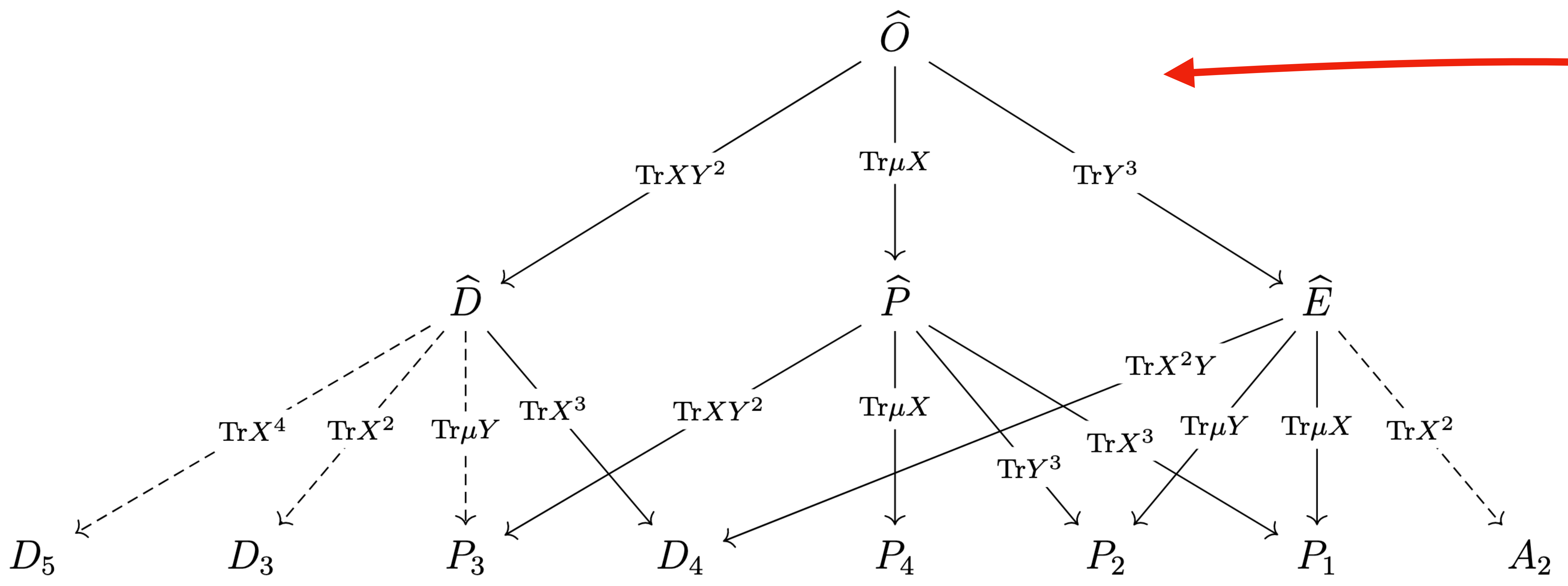
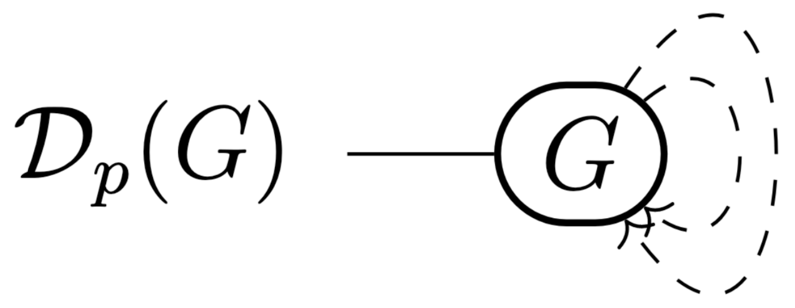


landscape of superpotential deformations of



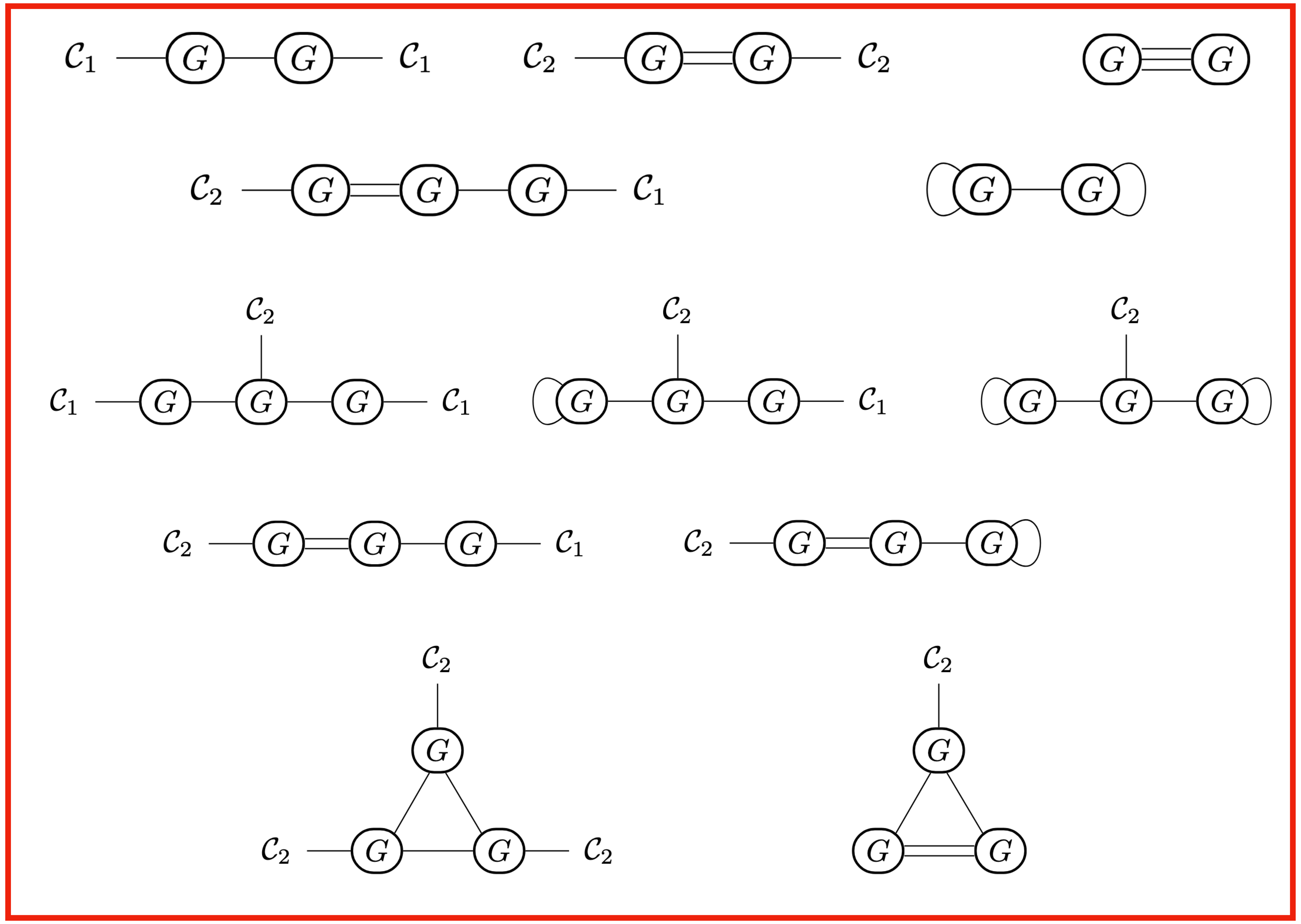
cf. adjoint SQCD [Intriligator, Wecht]

Thank you!

An $\mathcal{N} = 1$ Classification Problem

[Kang, CL, Lee, Song]

To see the ADE classification for $\mathcal{N} = 2$ we needed conformal matter



with 2 or 3 gauge nodes

Does there exist a known classification problem for which this is the answer?

Name	$\{p_i\}$	
	Asymptotically-free gauging	Conformal gauging
$\mathcal{C}_1$	$\{p \geq 2\}$ $\{p_1 \geq 2, p_2 \geq 2\}$ $\{2, 2, p \geq 2\}$ $\{2, 3, 4\}$ $\{2, 3, 5\}$	$\{2, 2, 2, 2\}$ $\{3, 3, 3\}$ $\{2, 4, 4\}$ $\{2, 3, 6\}$
$\mathcal{C}_2$	$\{p \geq 2\}$	$\{2, 2\}$