THE CHRONICLES OF IIBORDIA PART I:

BRANES FROM THE ABYSS

Strings and Geometry workshop,
UPenn, Mar 7th 2023

Miguel Montero
IFT Madrid

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of the adventures, exploits & misfortunes

that I had the pleasure of sharing with my group of friends

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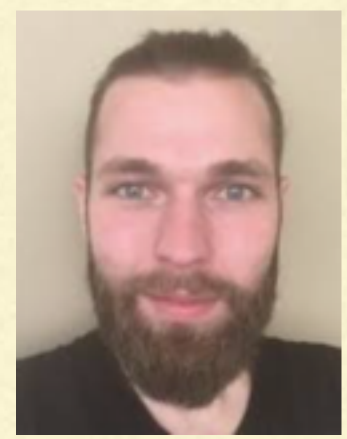

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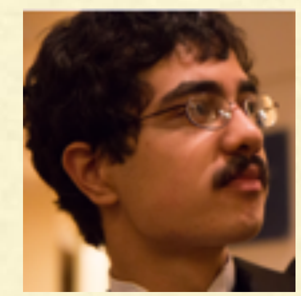

Arun
Debray

Over the course of the two-year
long voyage we undertook

into the lands of **IIBordia.**

[Based on 2302.00007 w. Markus Dierigl, J Heckman, A.Debray
and
2212.05077 w. Dierigl, Heckman, Ethan Torres]

What is IIBordia?

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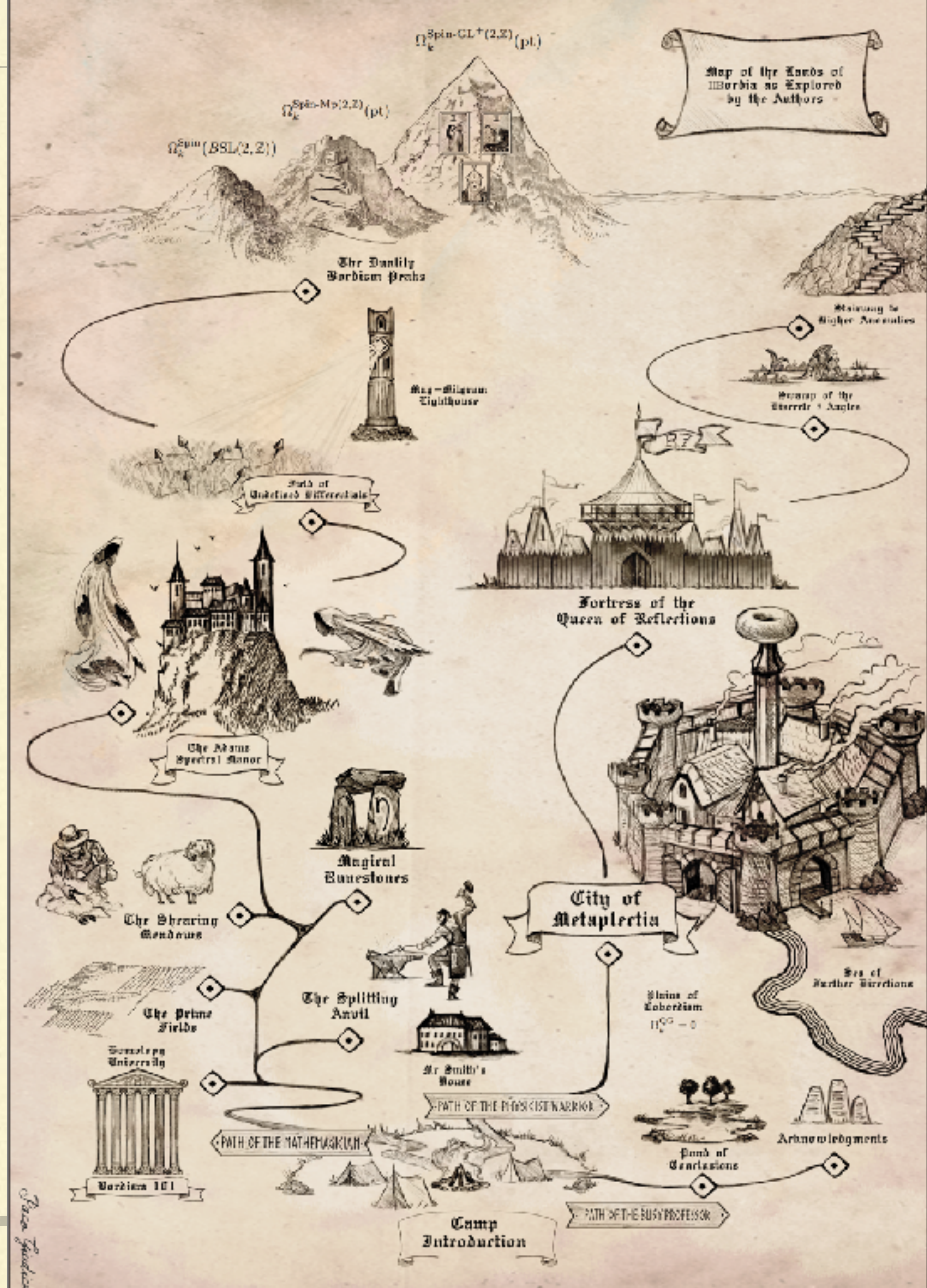
A fantastic realm inhabited by the **cobordism classes** of **type IIB** supergravity,

as well as the **branes** and **defects** that trivialize them.

There are two main **routes** leading to the far reaches of IIBordia.



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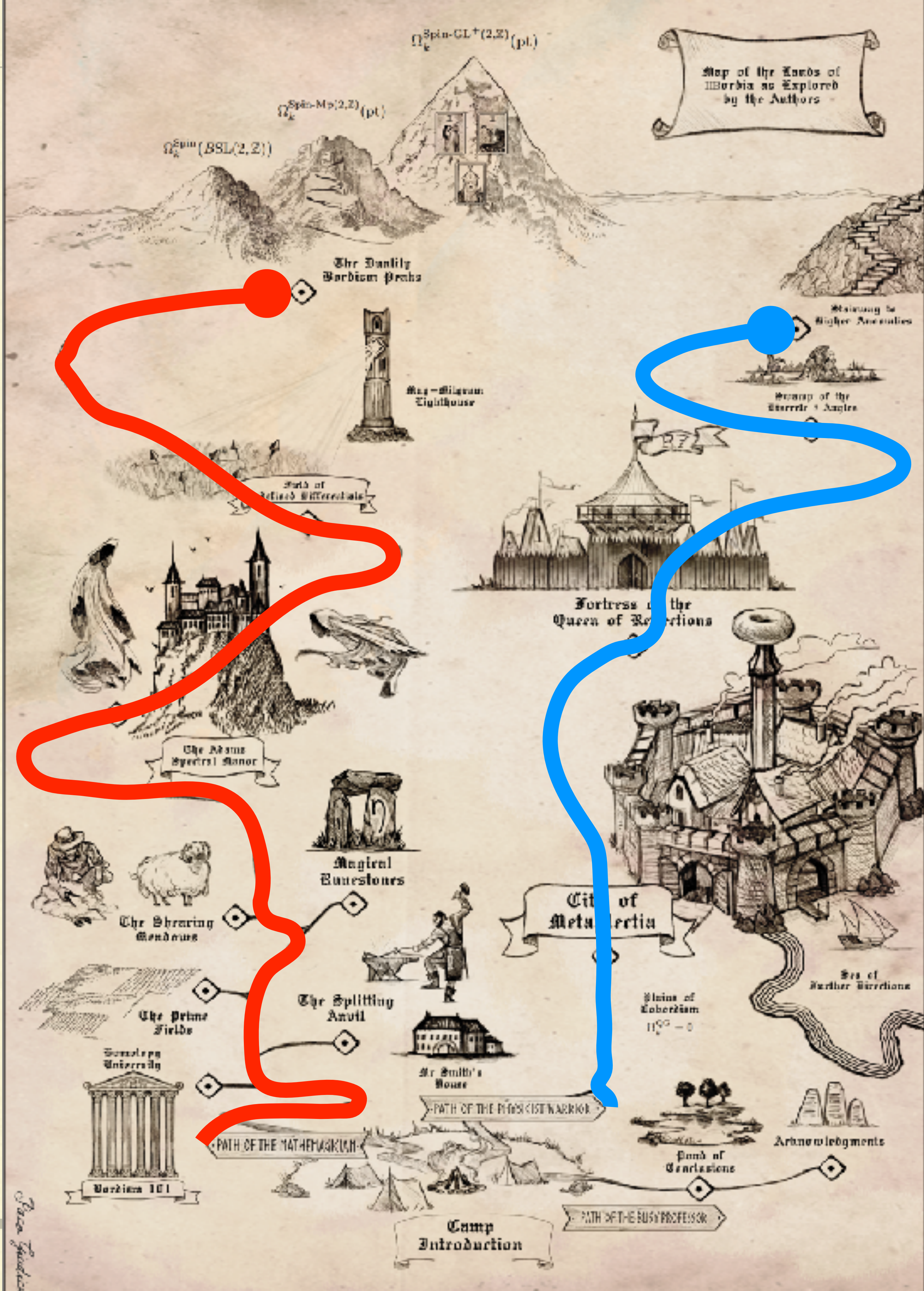

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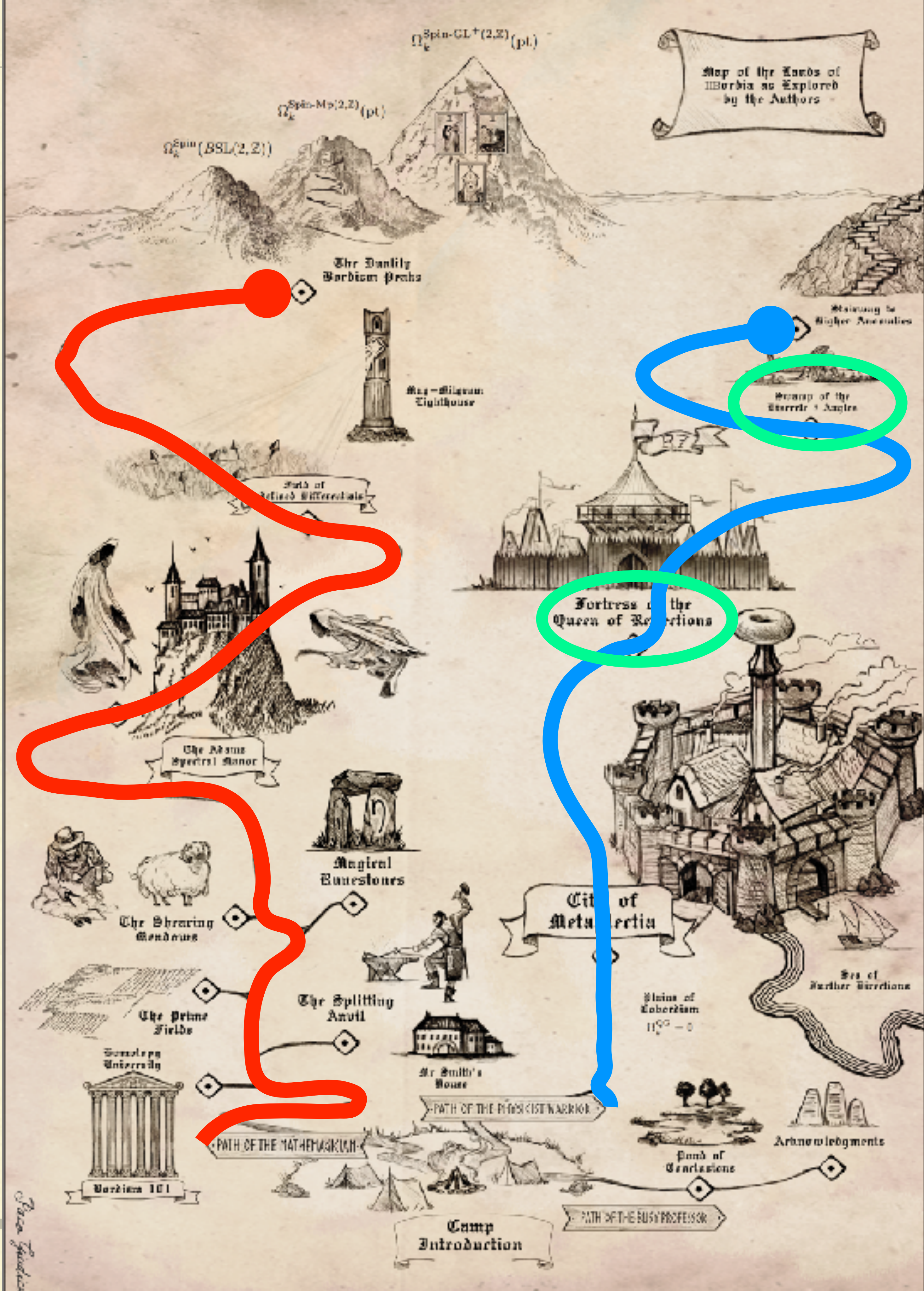

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We will recover some SUSY branes, and find a few new ones
along the way.

Plan of the talk

Cobordism Conjecture

Cobordism groups of IIB

New IIB branes

Conclusions

Basic idea of the cobordism conjecture: **[Jake's talk]**

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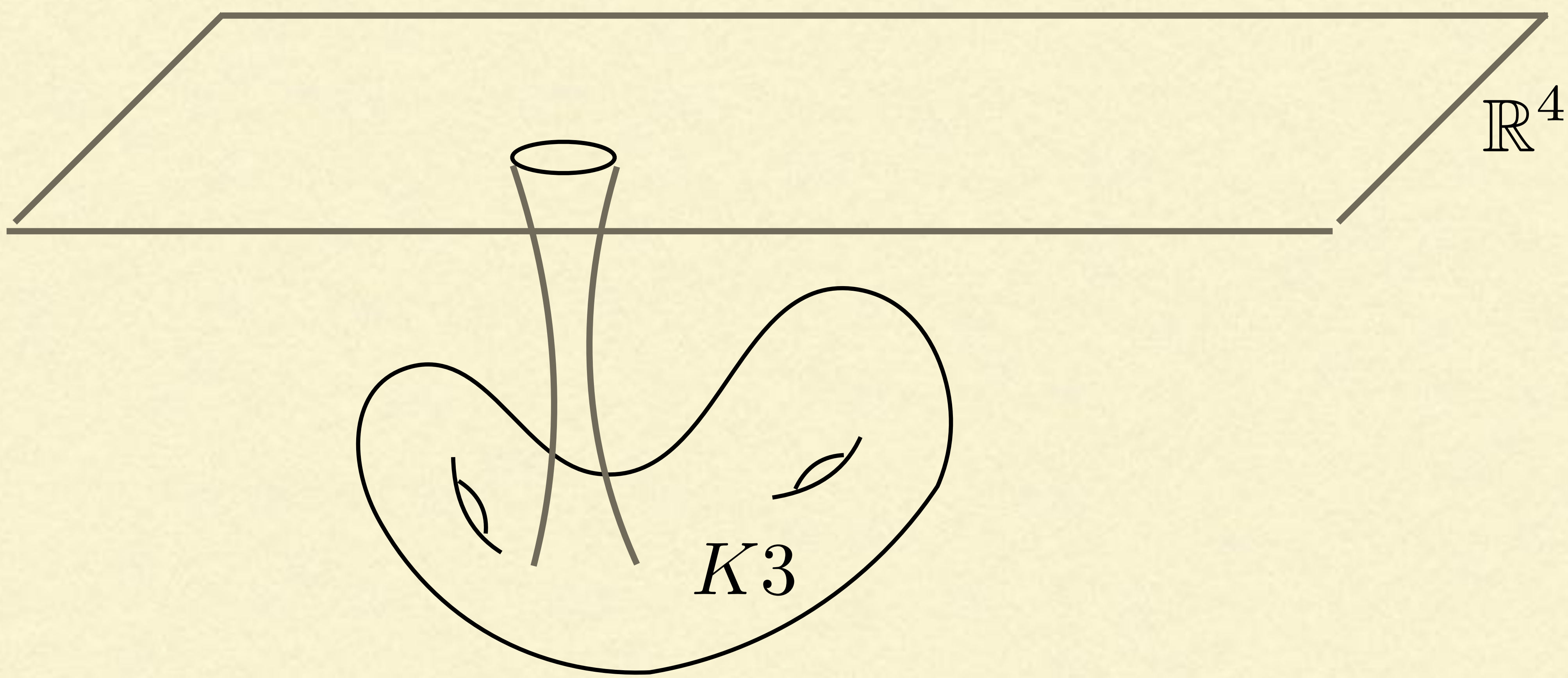

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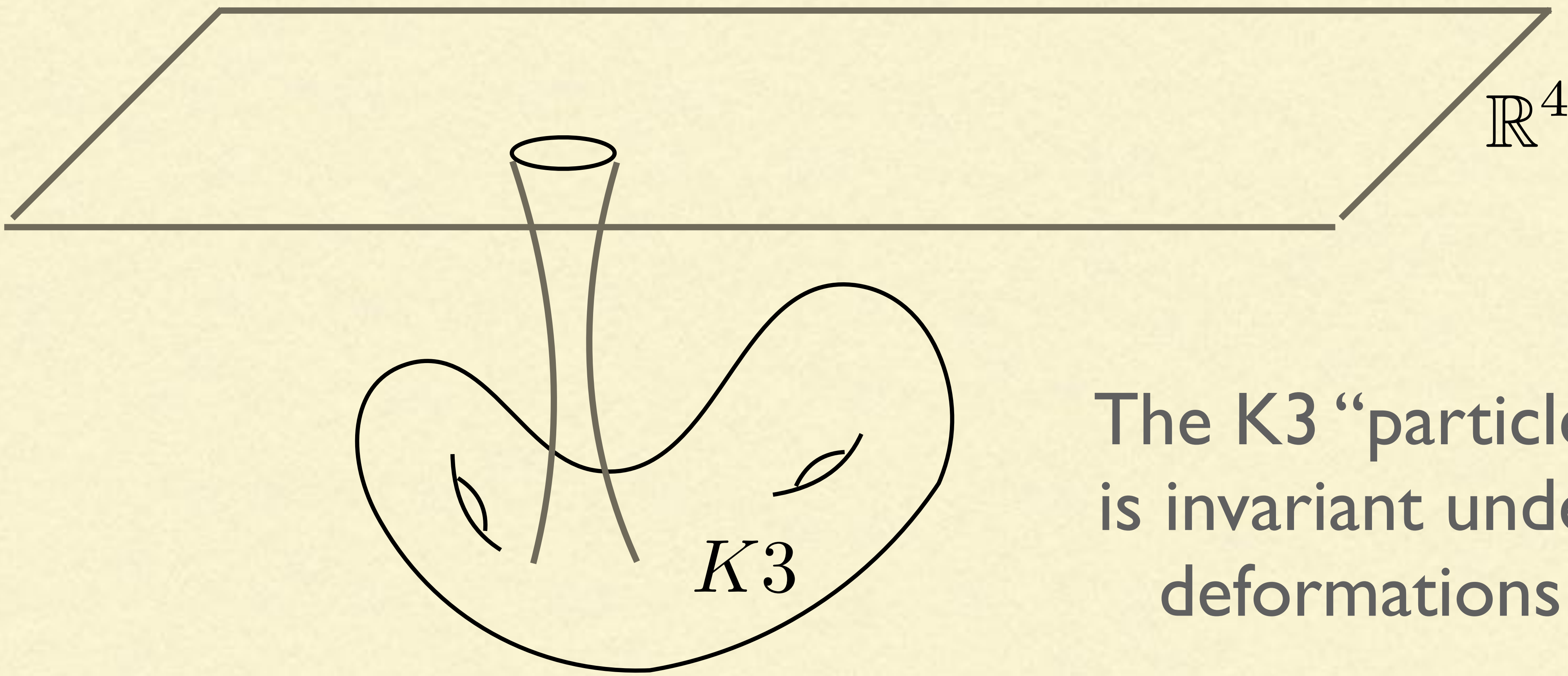

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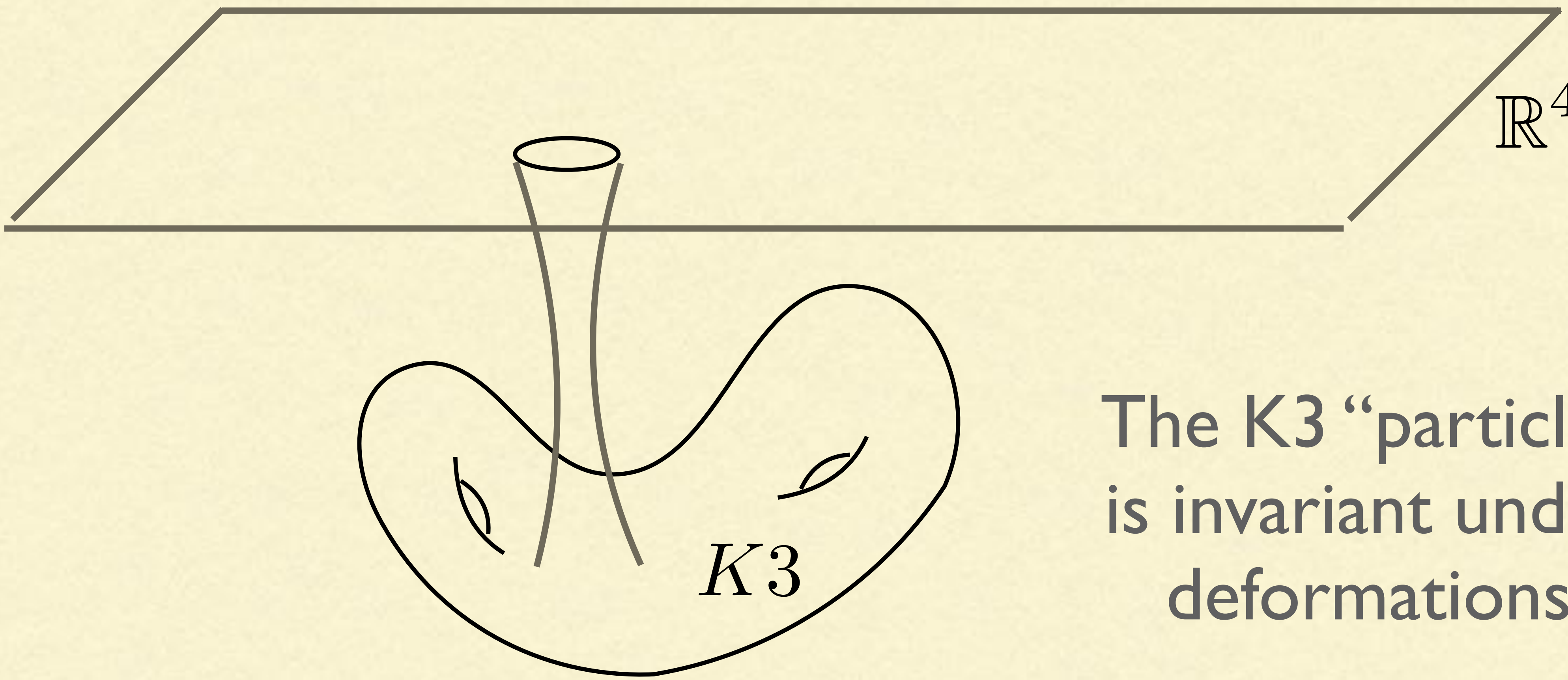

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But in QG, topology is **dynamical…**

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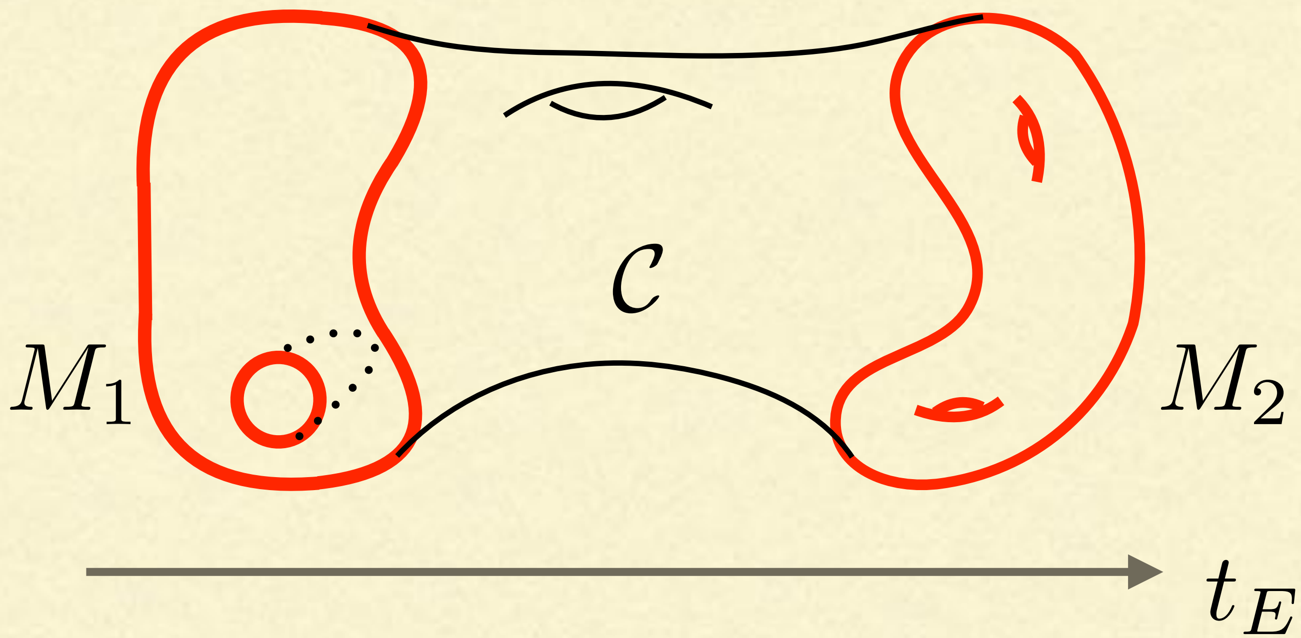

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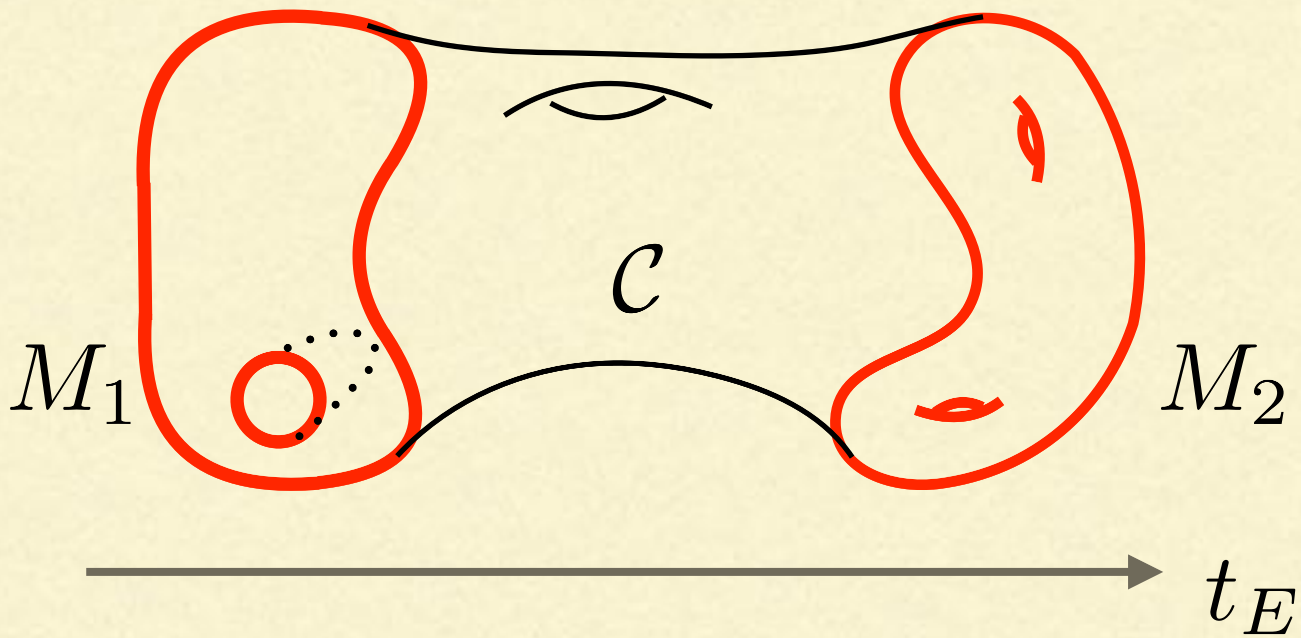

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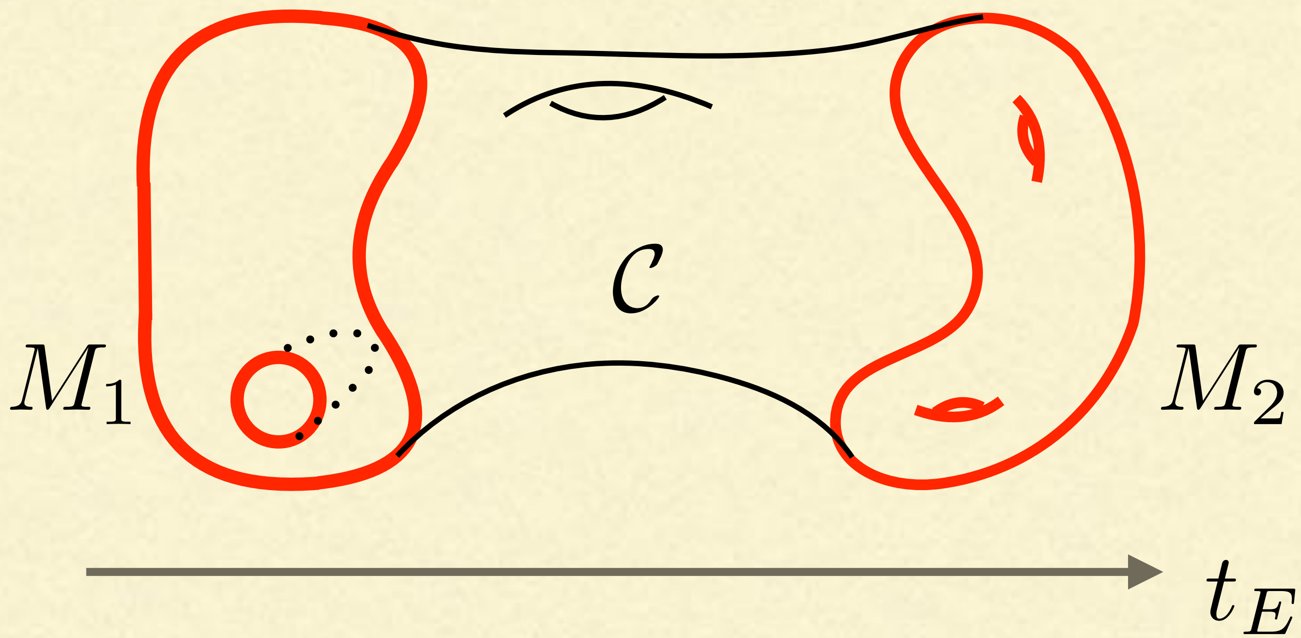

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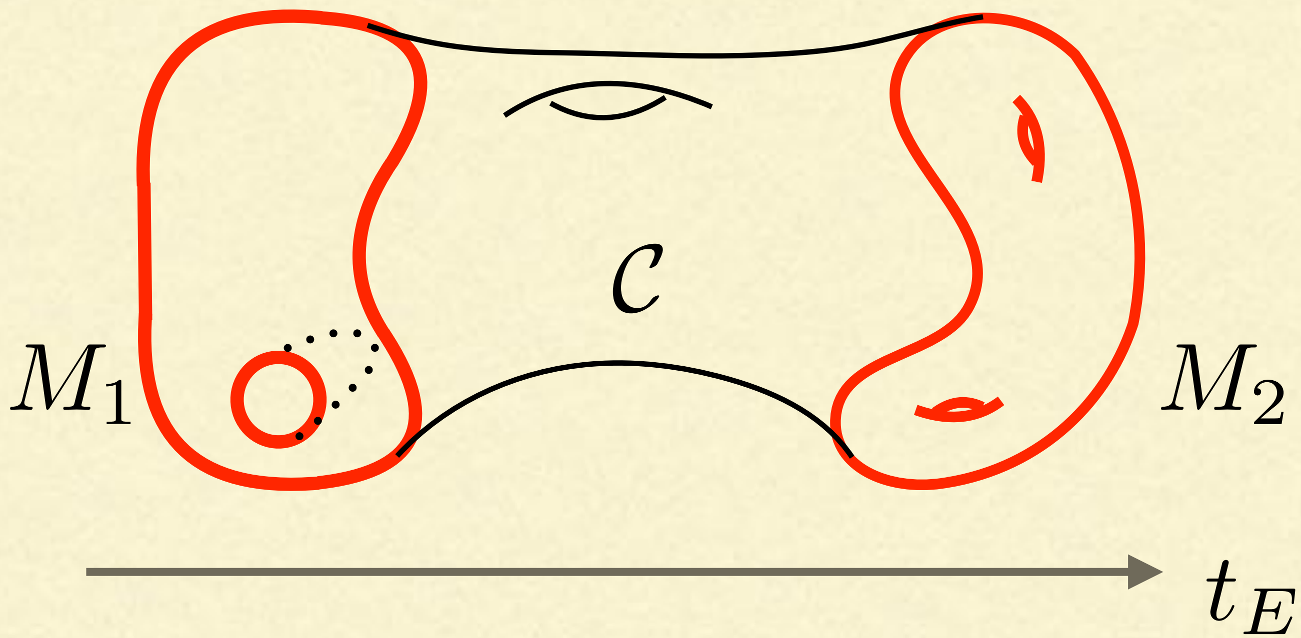

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e.g. $\Omega_4^{\mathrm{Spin}} = \mathbb{Z}$ generated by [K3]

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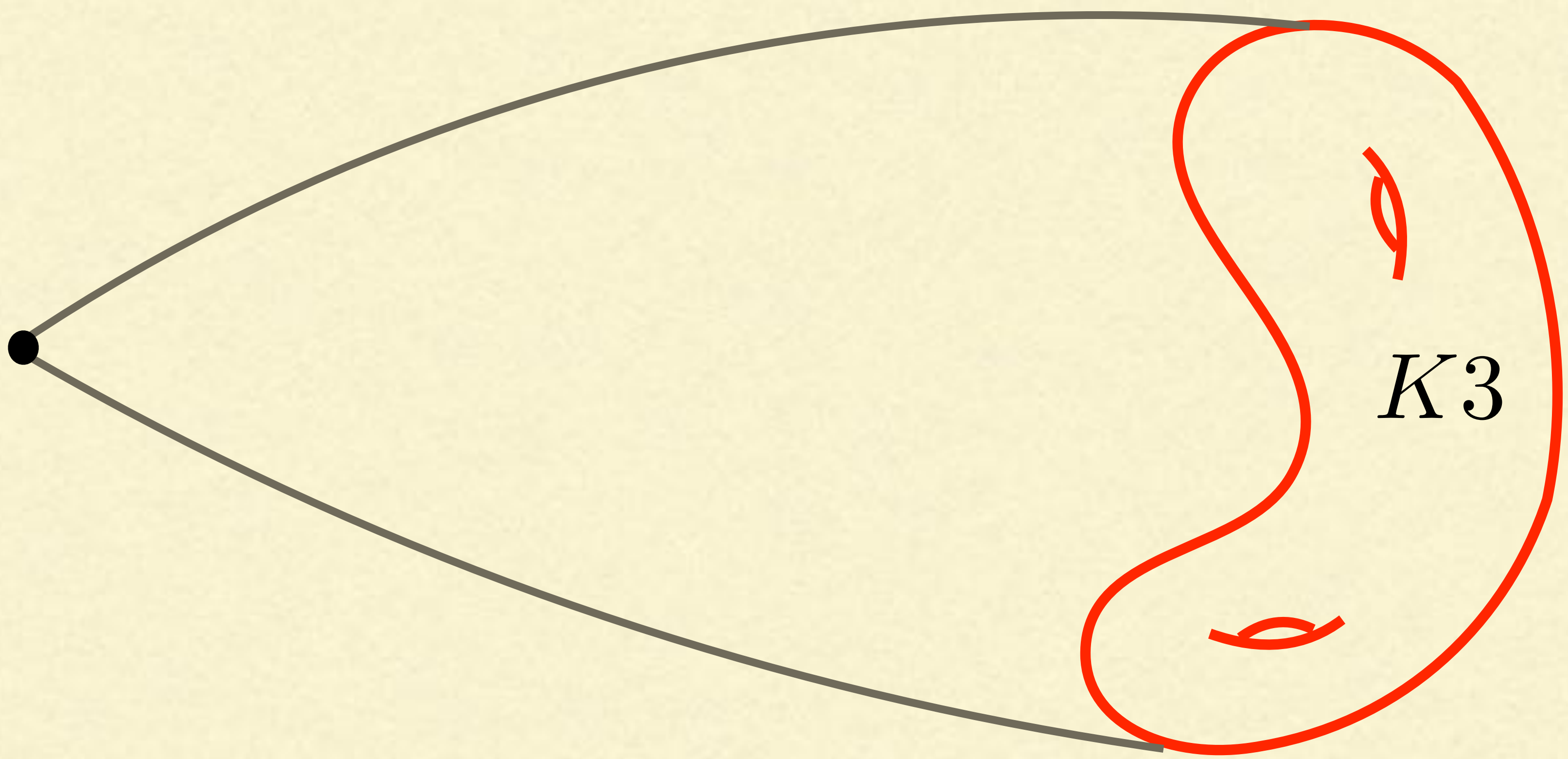

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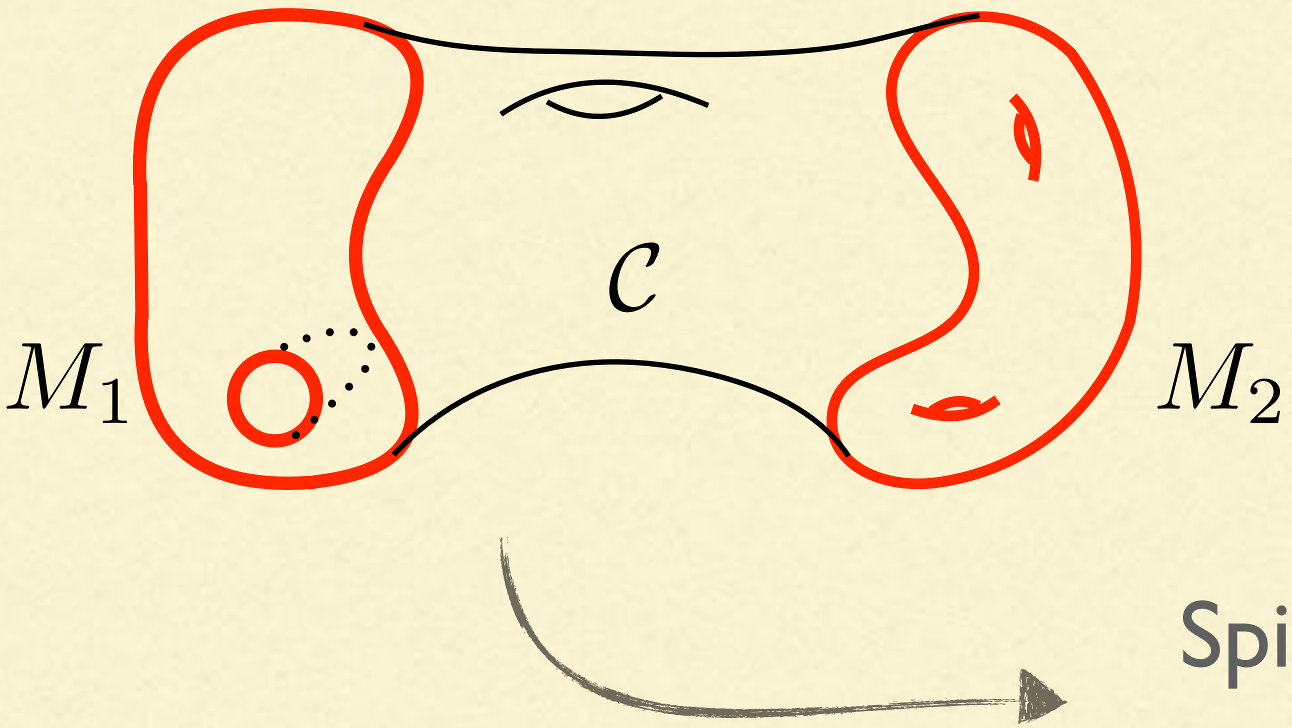
Killing the global symmetry forces the introduction of **defects**

Cobordisms can be decorated with additional **structure** (spin, gauge bundle…)

One just demands that M1, M2 have the required structure, and that this extends smoothly to C.



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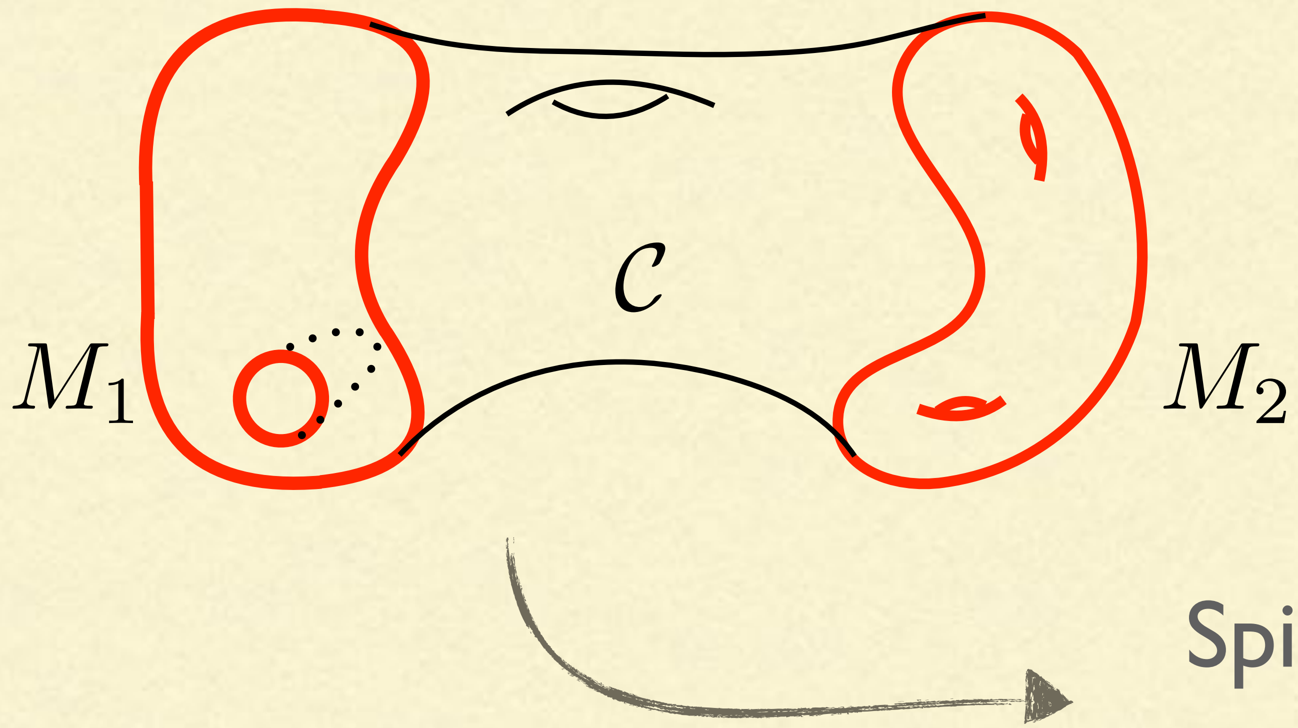

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So let us now then **compute** $\Omega_*^{\mathrm{IIB+Duality}}$

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[Tachikawa-Yonekura '18]

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Because M-theory includes reflections

↓

$GL(2,\mathbb{Z})$

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$SL(2,\mathbb{Z})$ ↓ (Because M-theory includes reflections) ↓ $GL(2,\mathbb{Z})$

$\mathrm{Mp}(2,\mathbb{Z})$ ↓ $\mathrm{Pin}^+(GL(2,\mathbb{Z}))$

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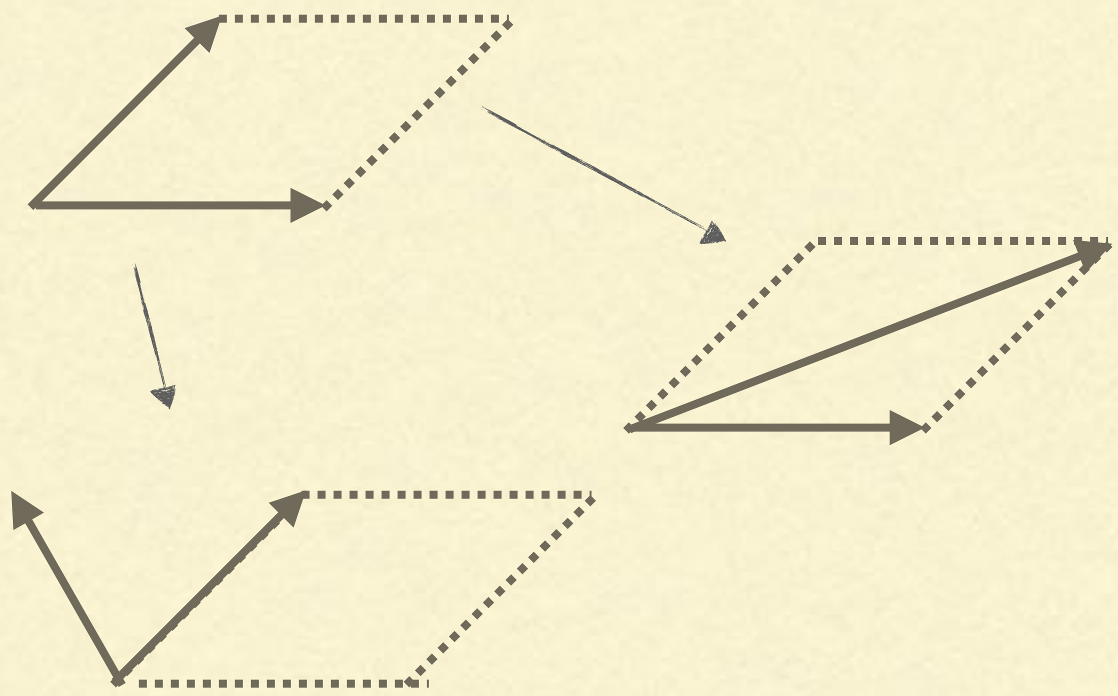

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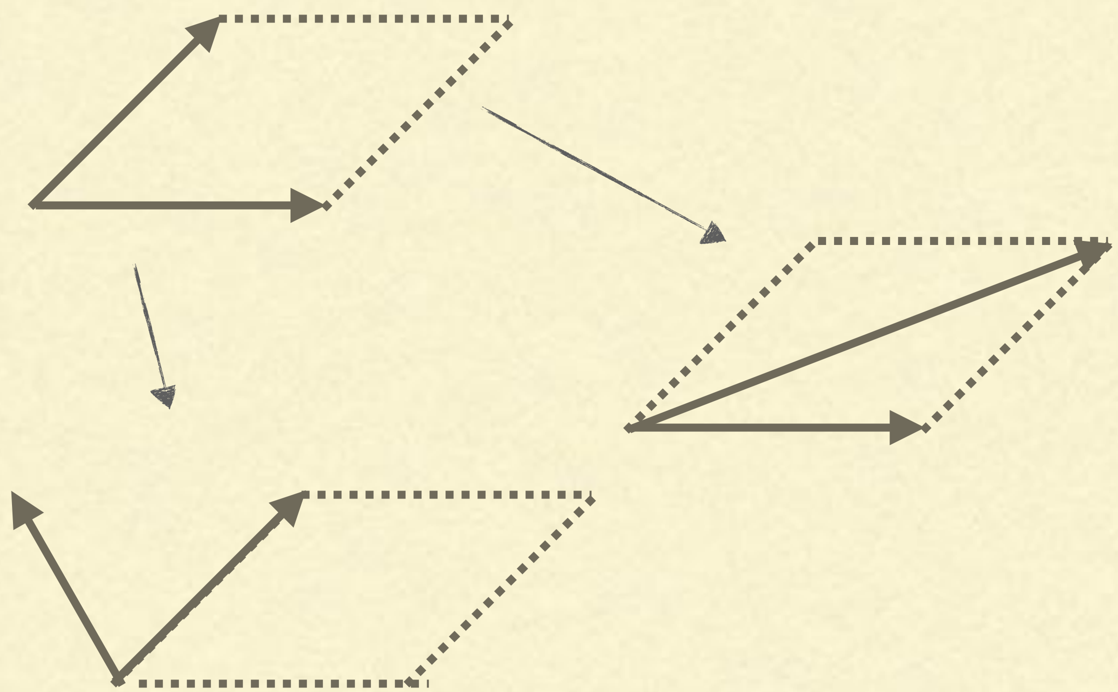

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Taking into account fermions:

$$\frac{\mathrm{Spin}\times \mathrm{Mp}(2,\mathbb{Z})}{\mathbb{Z}_2}$$

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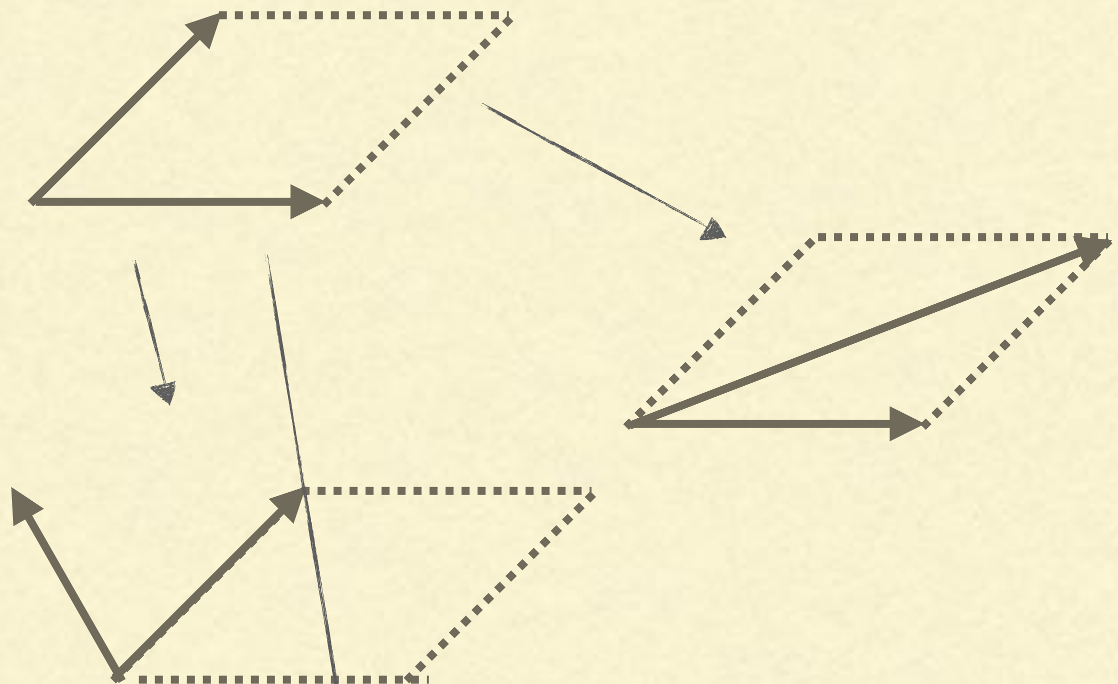

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Taking into account **reflections**: $$\frac{\text{Spin}\times \text{GL}^+(2,\mathbb{Z})}{\mathbb{Z}_2}$$

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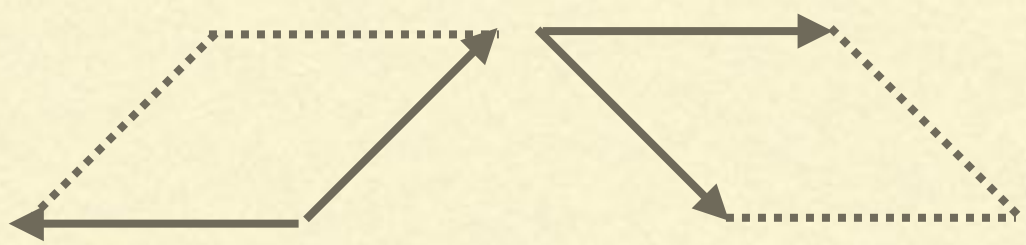

$(-1)^{F_L}$ and Ω symmetries of perturbative IIB

We computed **all three** approximations!

d	$\Omega_d^{\text{Spin}}\big(BSL(2,\mathbb{Z})\big)$	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

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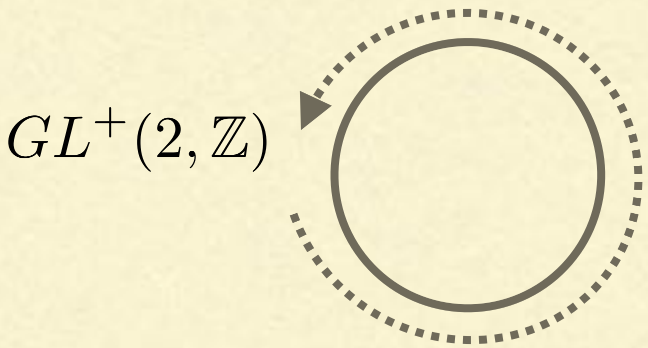

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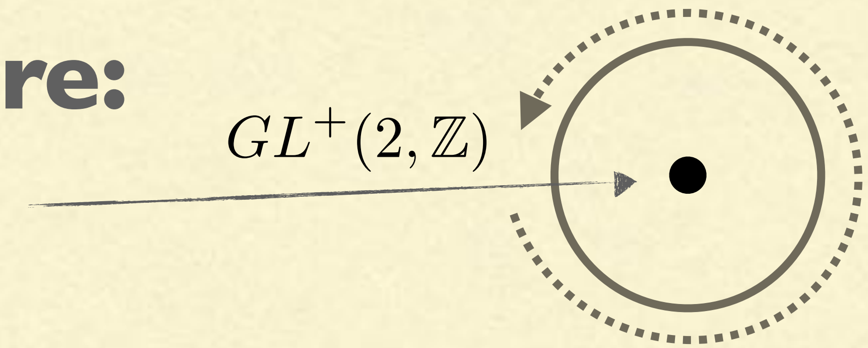
Cobordism conjecture:
Must have 7-branes



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Class of a circle with U holonomy

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Class of a circle with U holonomy

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Since S,U generate SL(2,Z), we have **all ordinary F-theory 7-branes**

Class of a circle with S holonomy

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But … $\Omega_1^{\text{Spin-Mp}(2,\mathbb{Z})} = \mathbb{Z}_3 \oplus \mathbb{Z}_8$ is not the **true** duality cob group group of IIB sugra.

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What is going on?

Where did the F-theory 7-branes go?

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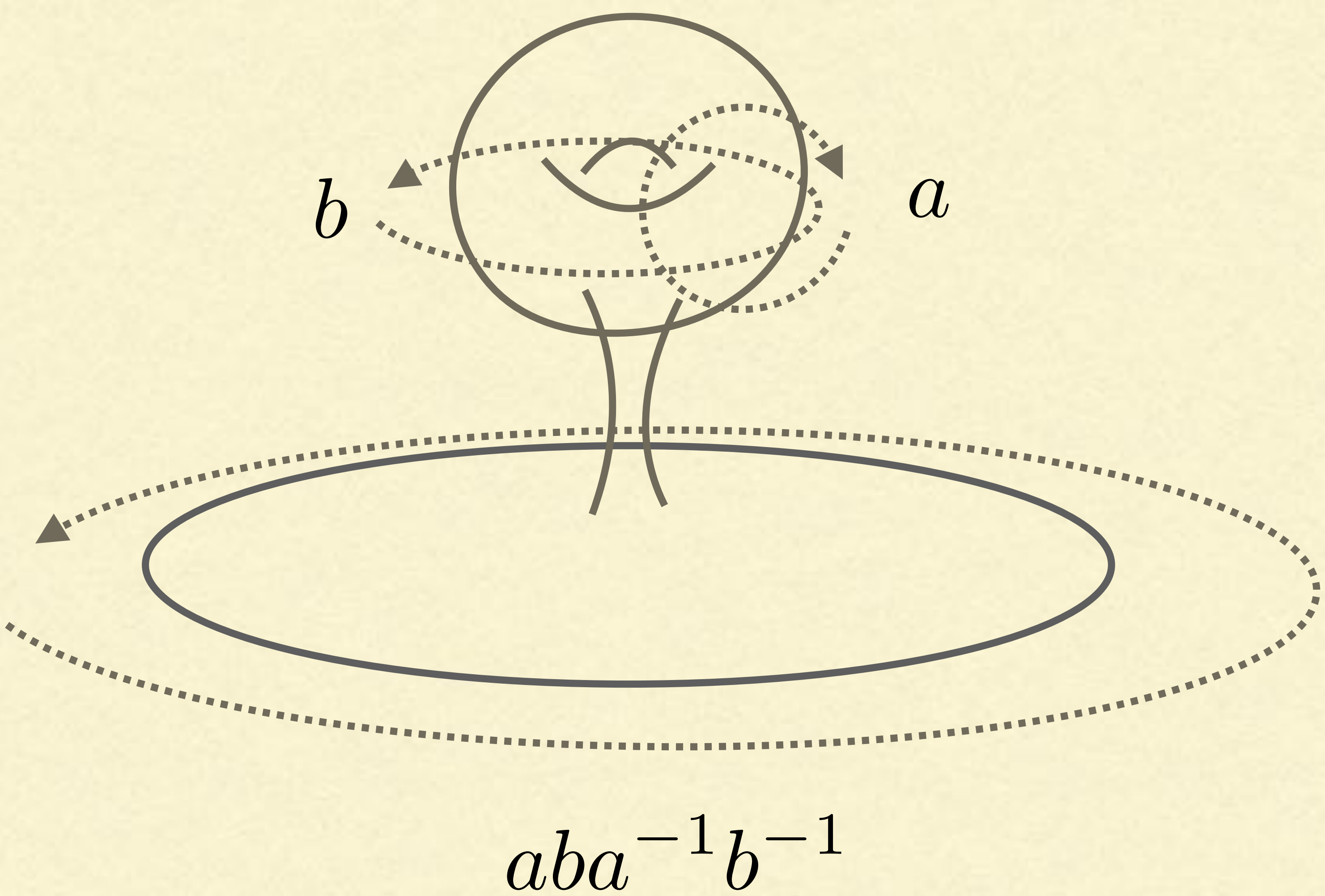

Basic construction in McNamara '21: Torus with holonomies allows one to construct **commutators**

This construction allows us to give a new perspective on a problem that was left open in **[McNamara-Vafa '19]**

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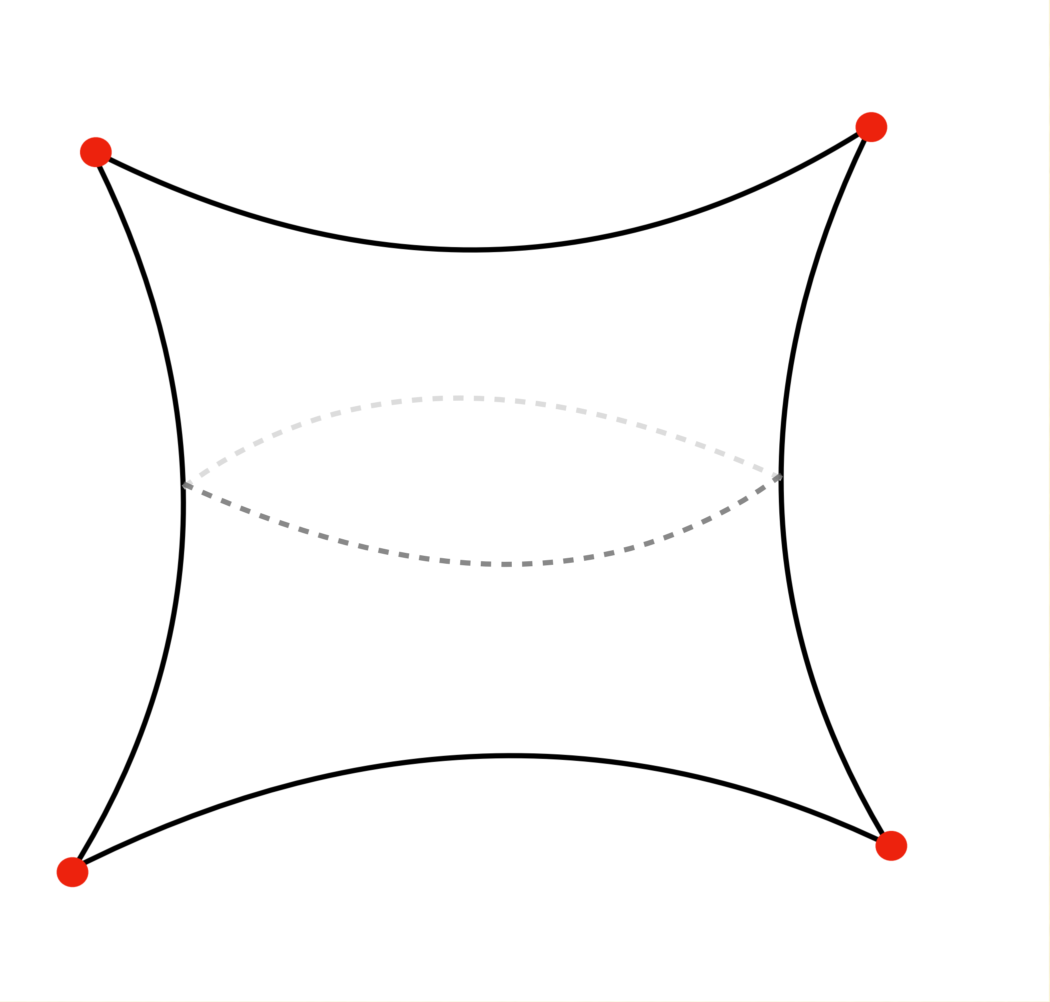

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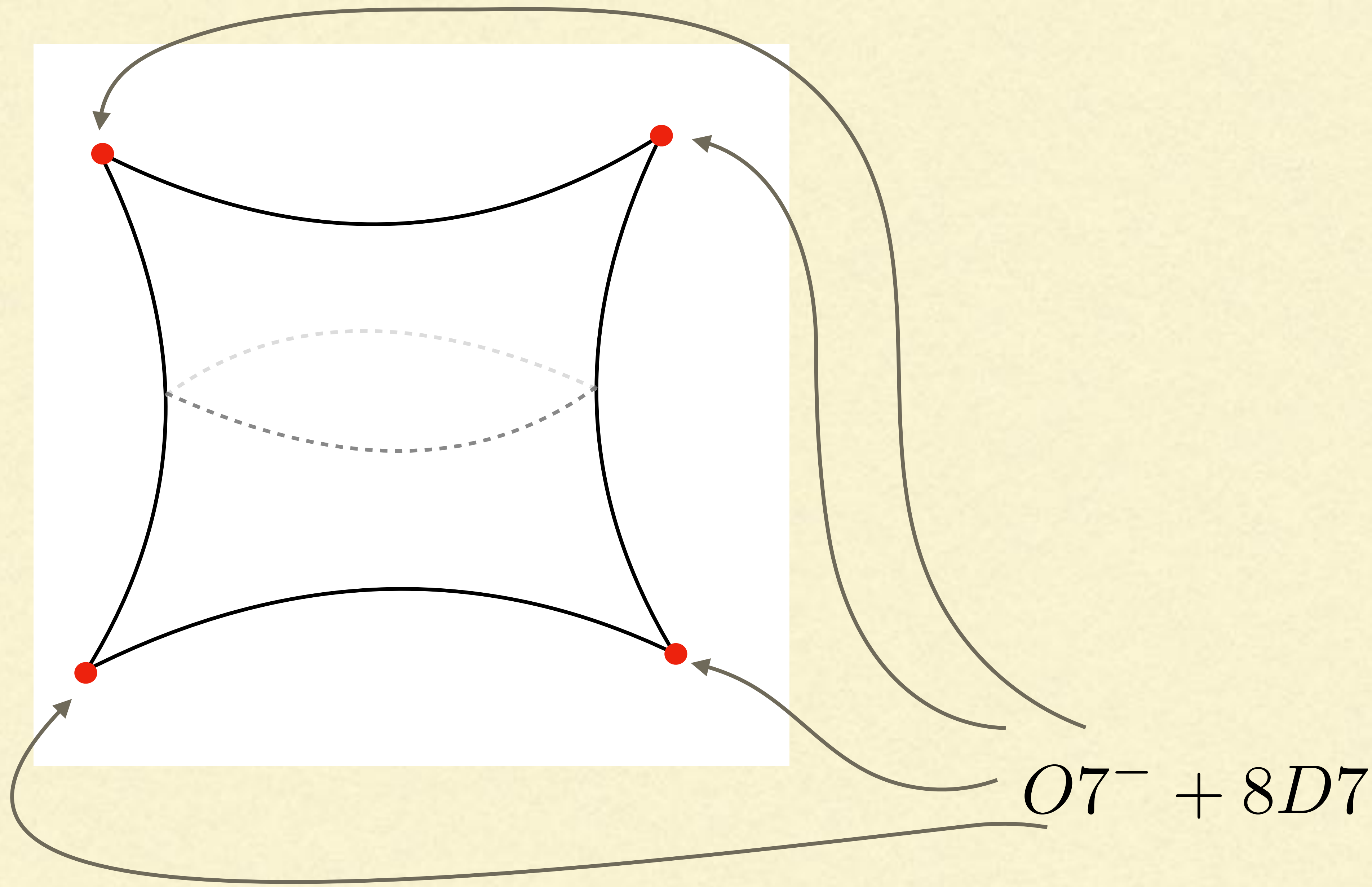

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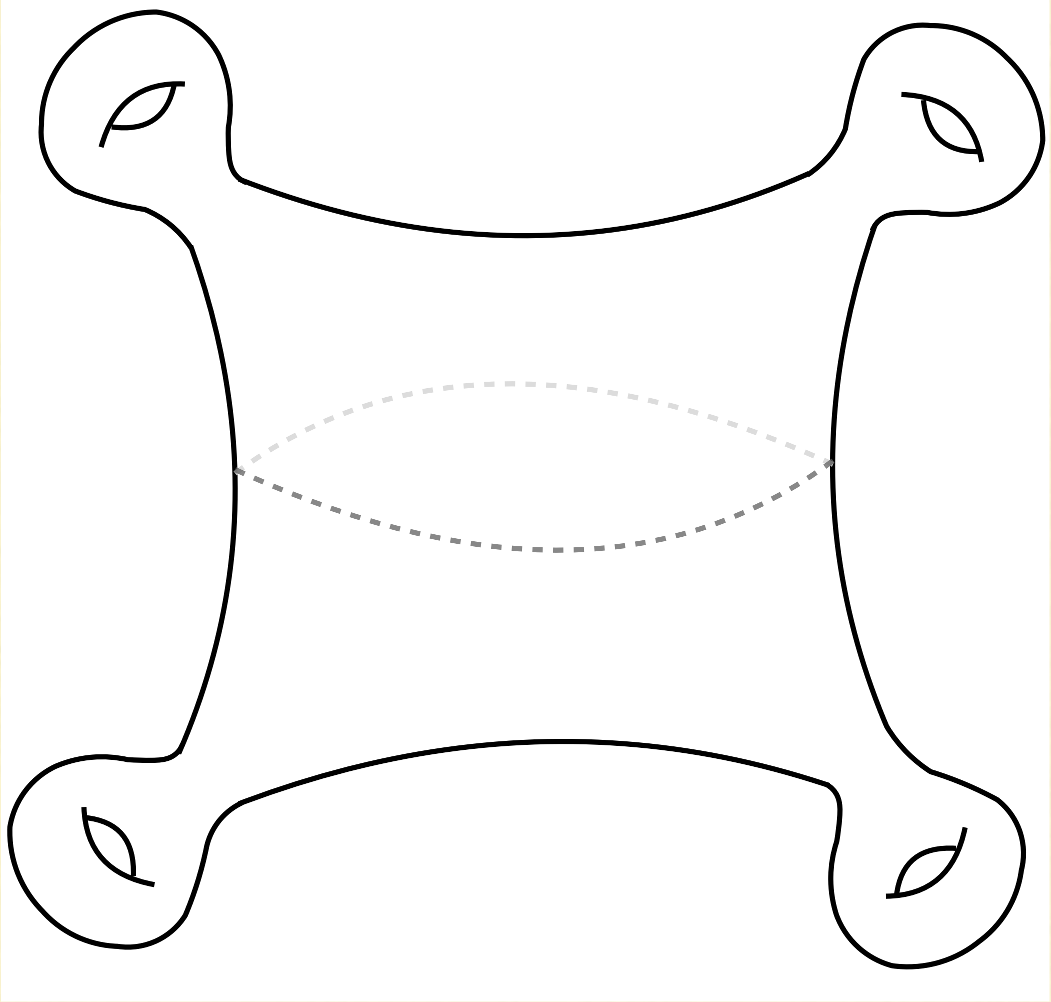

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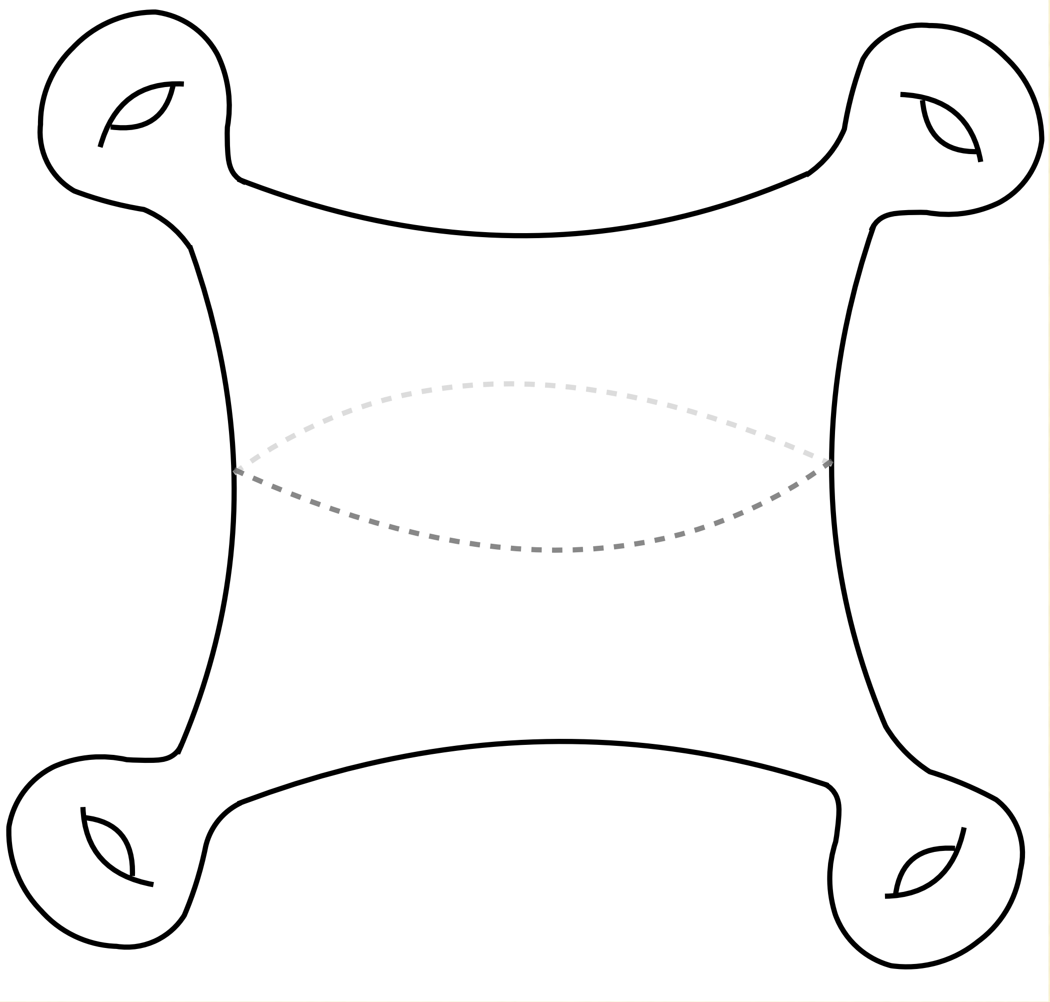

But…

$$\Omega_2^{\text{Spin-Mp}(2,\mathbb{Z})} = 0$$

Ok fine. But what are the Z2's in

$$\Omega_1^{\text{Spin-GL}^+(2,\mathbb{Z})} = 2\mathbb{Z}_2$$

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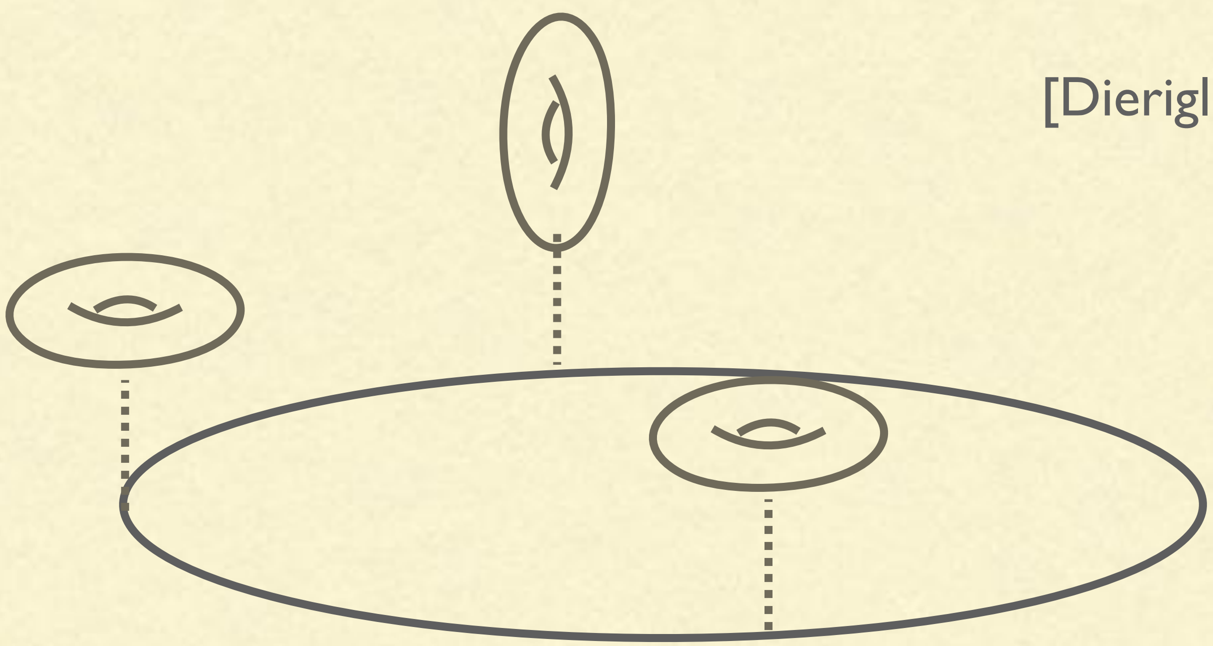
[Dierigl-Heckman-MM-Torres, 2212.05077]



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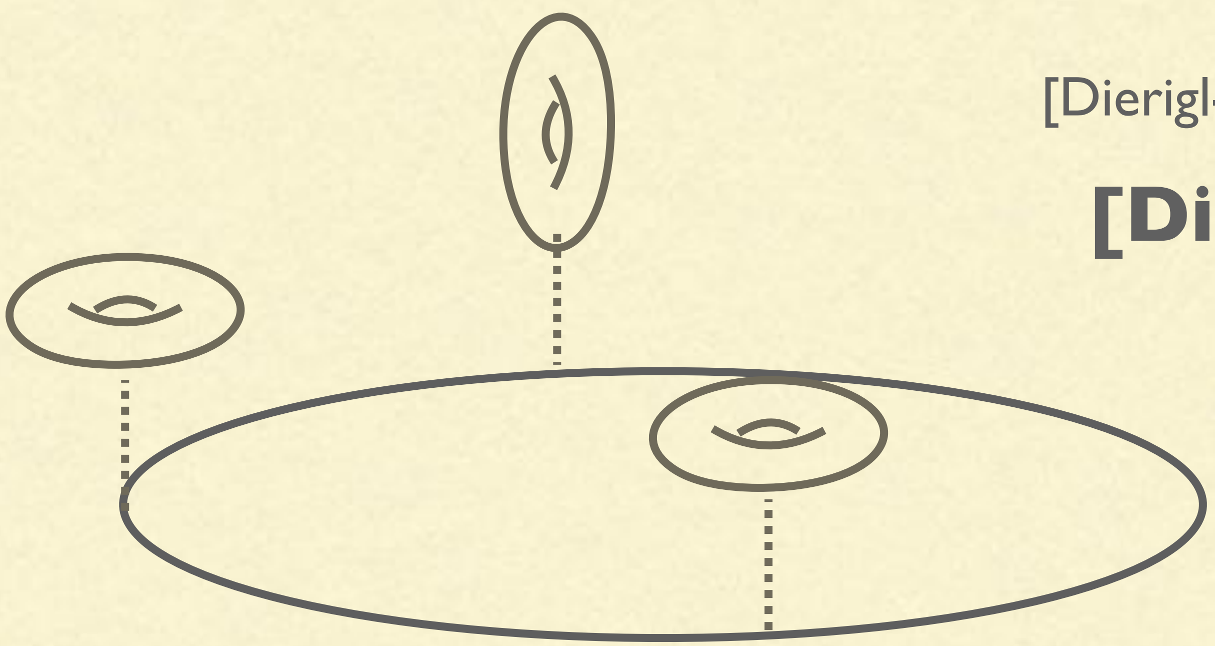
[Distler-Freed-Moore 0906.0795]



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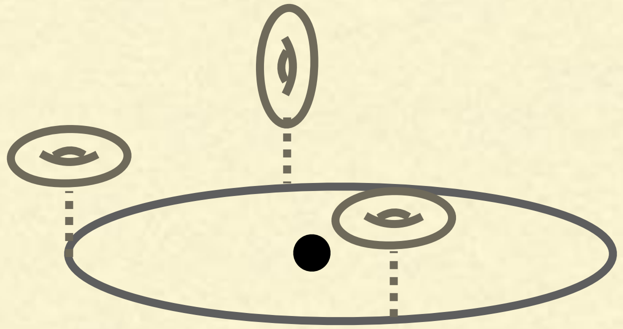

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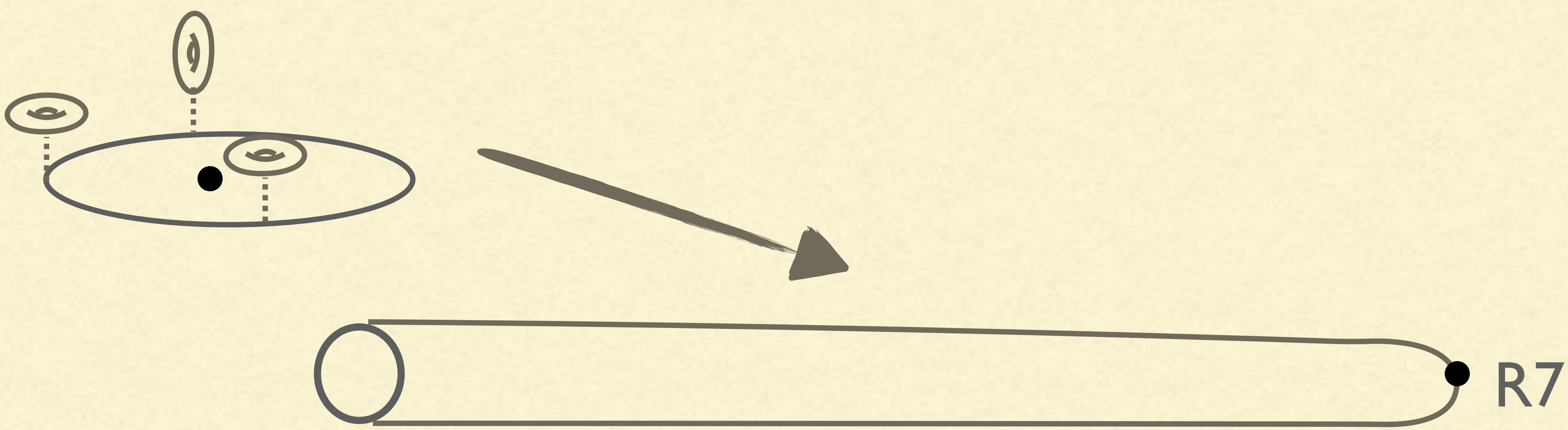
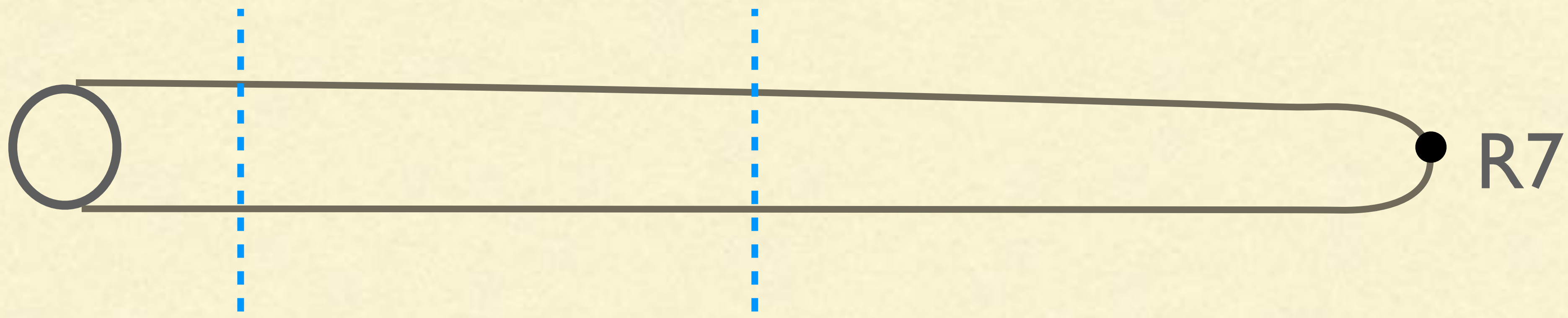
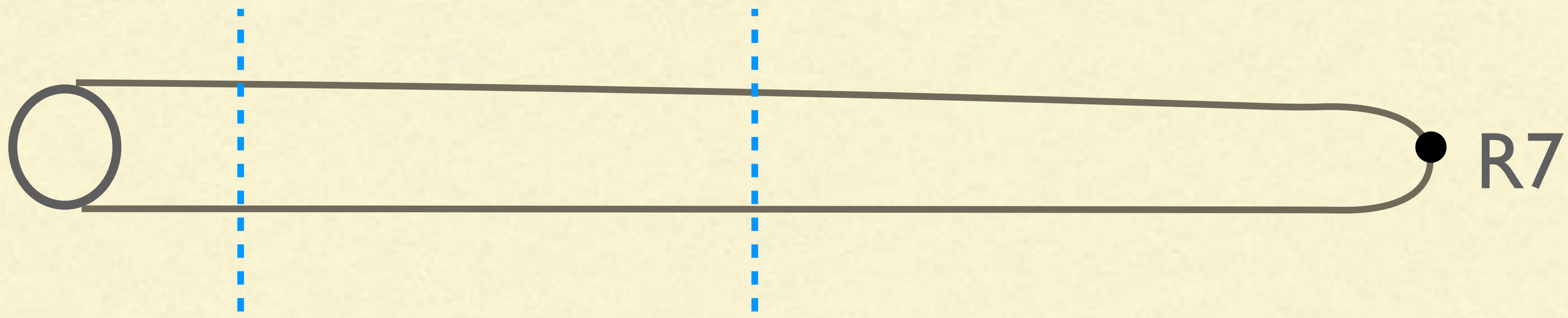




Far away from the R7 (e.g. between the blue lines), the configuration is just like IIB on a circle with holonomy



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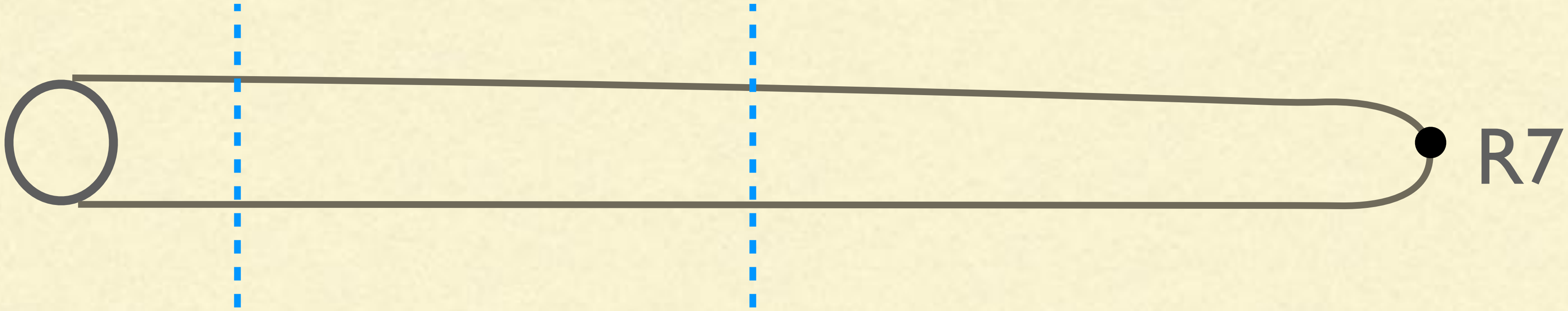
There is a **one-loop** Chern-Simons term

$$\frac{A_R \wedge \left(7p_1^2 - 4p_2\right)}{11520}$$

$$p_1 = -\frac{1}{8\pi^2}\mathrm{tr}(R^2), \ldots$$



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By **anomaly inflow**, this is the anomaly of the worldvolume theory of the R7-brane

In other words, the anomaly polynomial of the R7 brane is

$$\frac{F_R \wedge \left(7p_1^2 - 4p_2\right)}{11520} + \dots$$

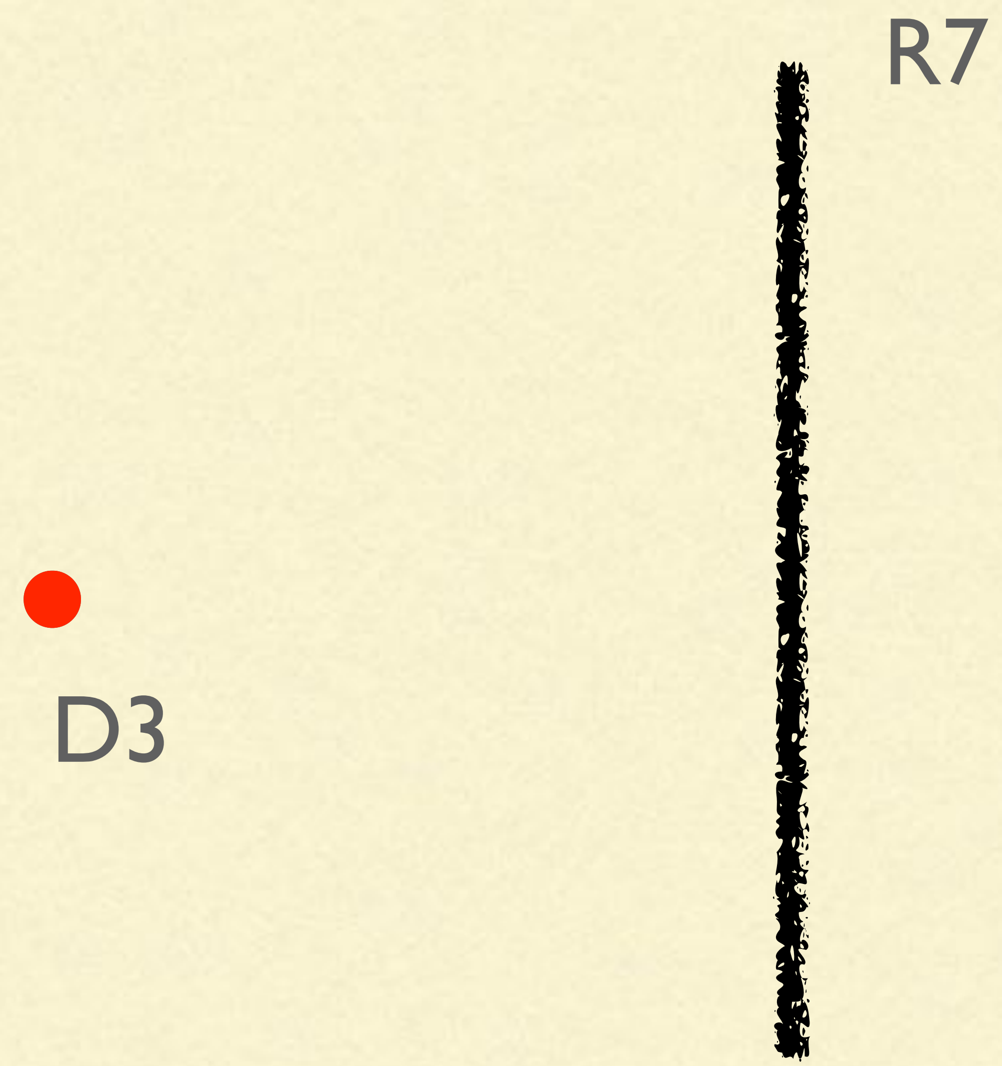
This is the anomaly polynomial of **one** Dirac fermion of R-charge 1/2.

That the coefficients **come out correctly quantized** is nontrivial. For instance,

$$\frac{F_R \wedge \left(7p_1^2 - 4p_2\right)}{35000}$$

would not have corresponded to any sensible worldvolume theory.

It also acts as an **Alice string** for e.g. D3 branes:



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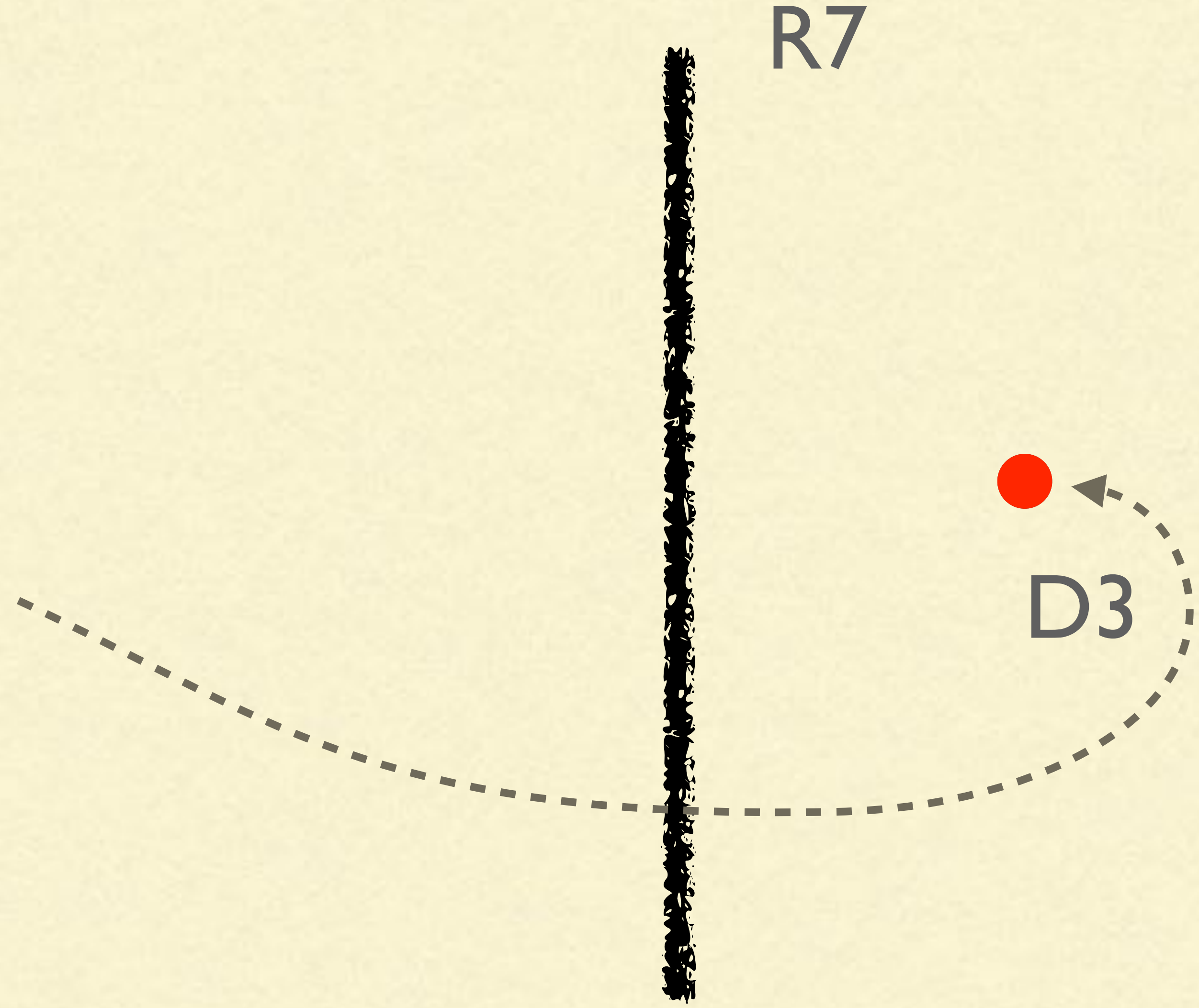

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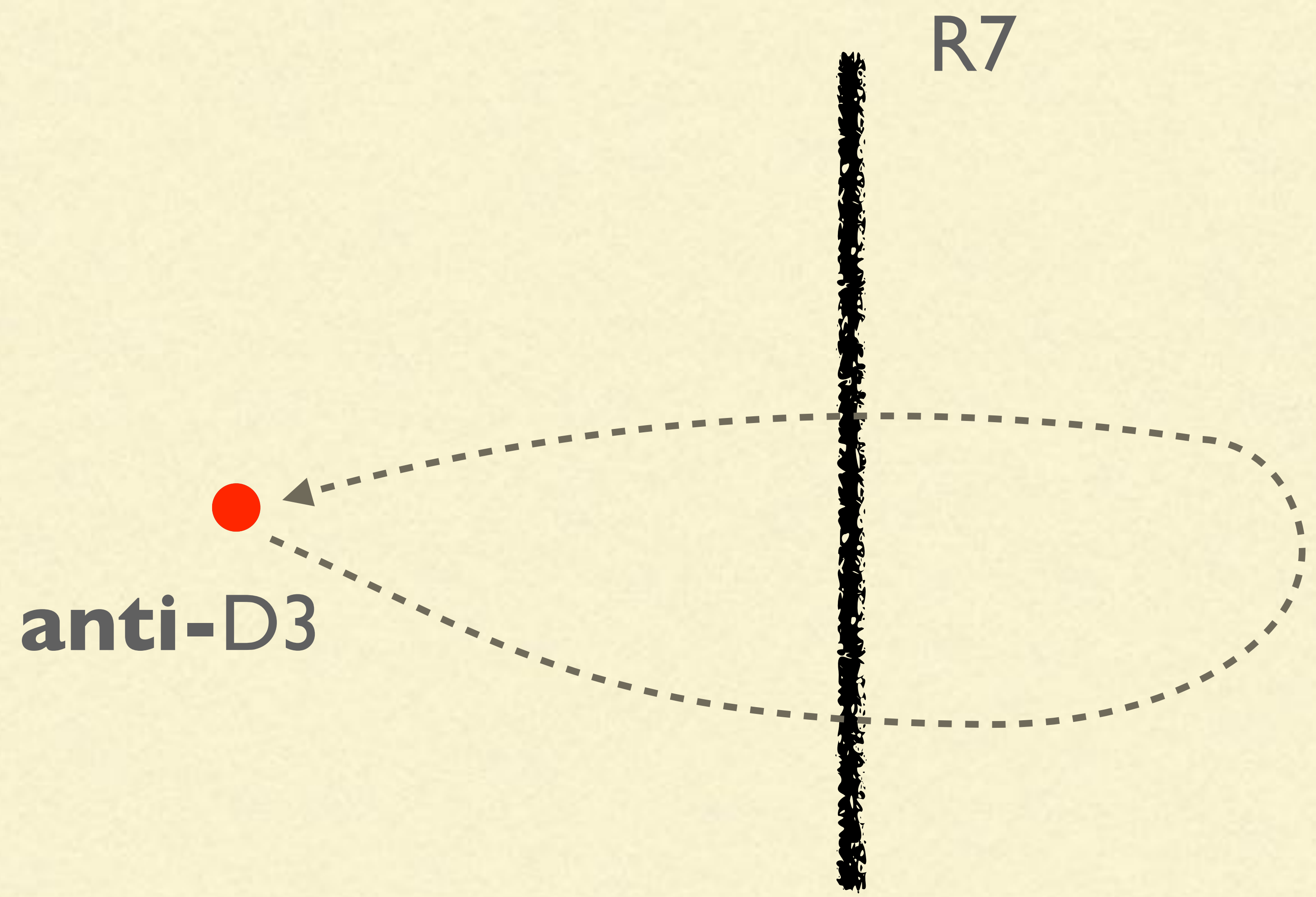

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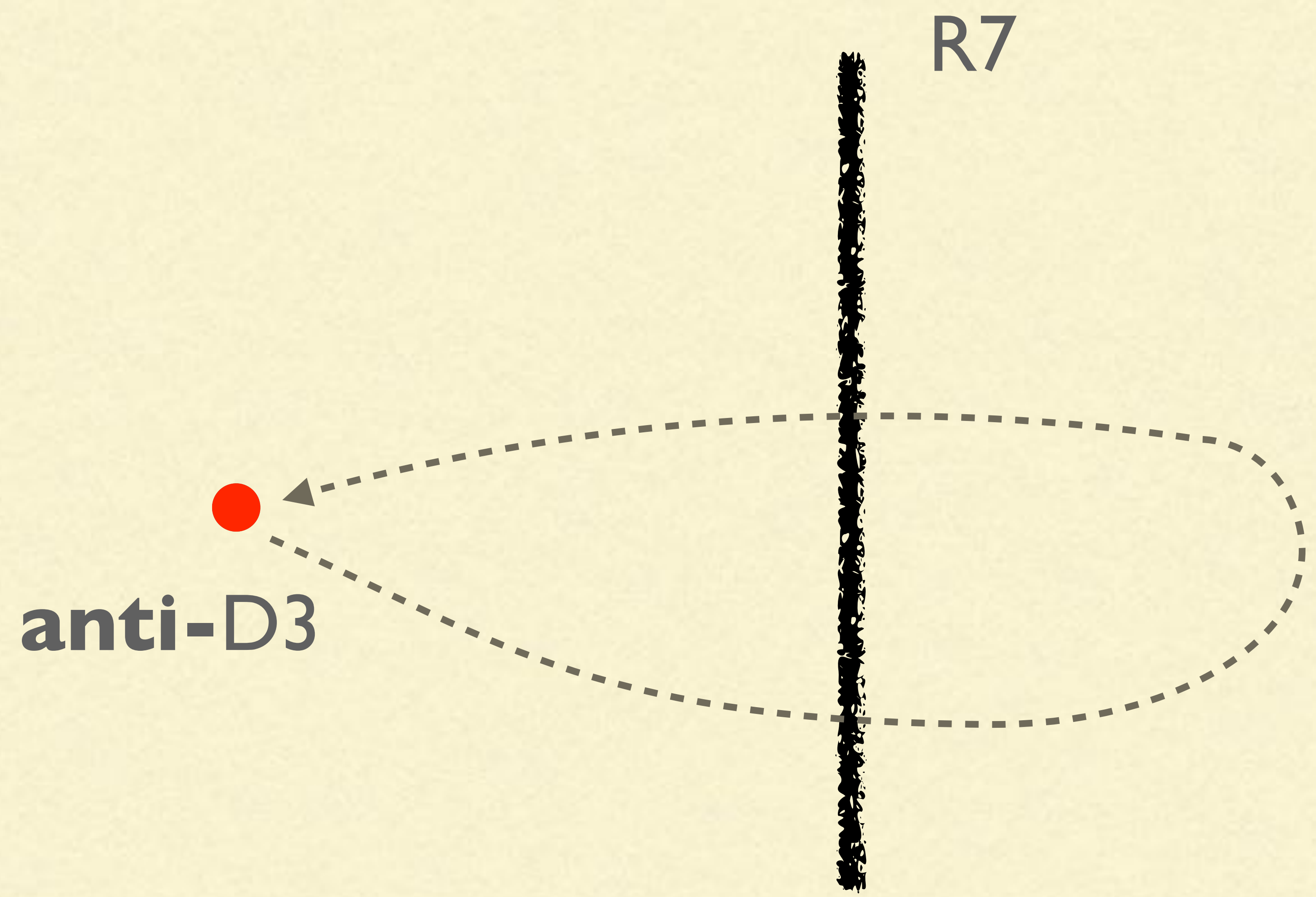

This means that there is a **massless 3-form field** on the R7 that couples to D3-brane charge.

(first example of a worldvolume theory with a massless 3-form)

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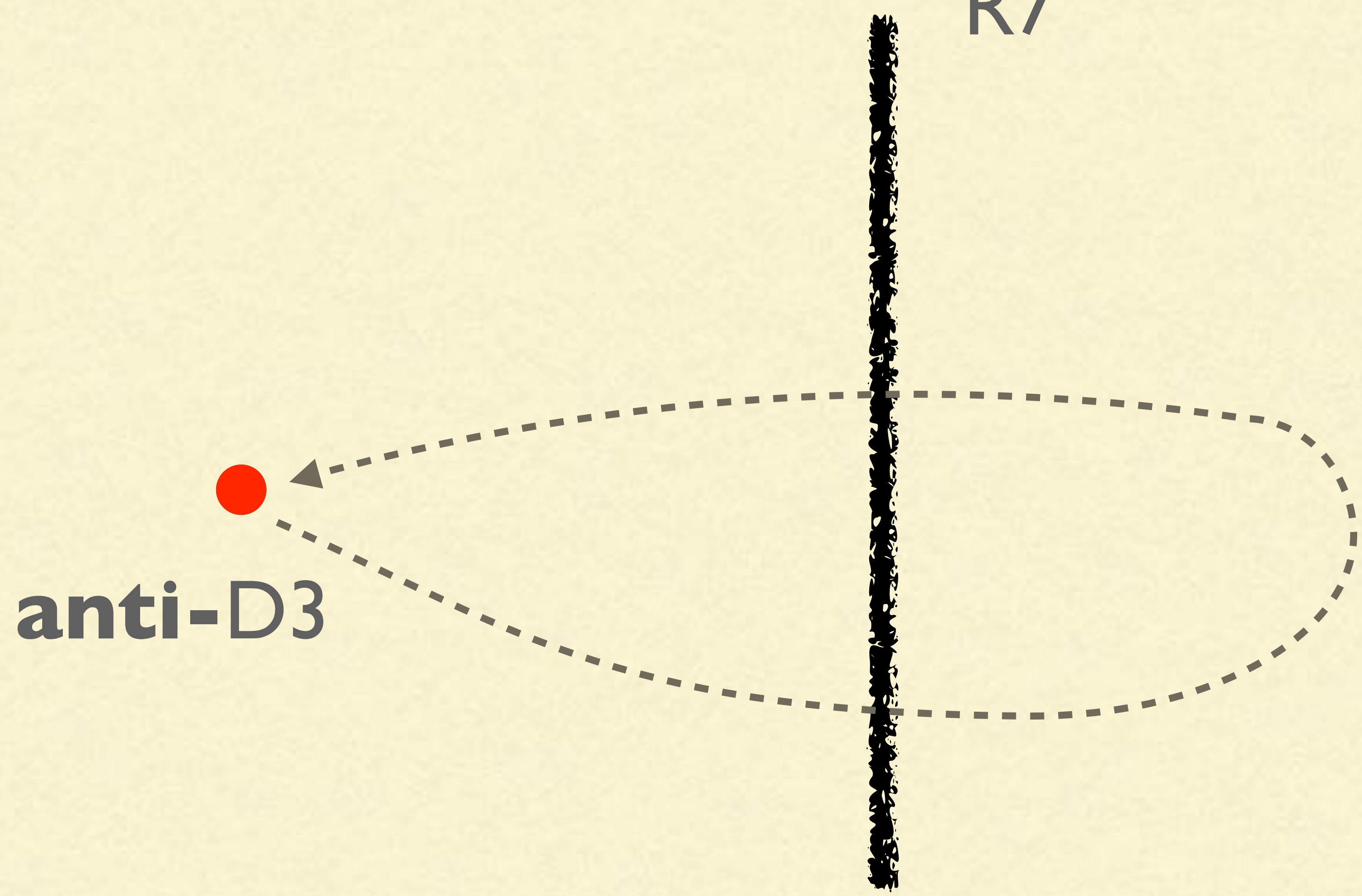
$$\int_{R7} dA_3 \wedge C_4$$



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Let us now jump to the last row

d	$\Omega_d^{\text{Spin}}(BSL(2,\mathbb{Z}))$	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

Cobordism invariants detecting the classes in this row are **topological couplings** —discrete **theta angles** of type IIB

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describes possible theta angles in 10d IIB.

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[Freed Hopkins '19]

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This illustrates how the corresponding cobordism classes are killed by **stringy** symmetries.

CONCLUSIONS

- **We have computed the cobordism groups of IIB+duality bundle**
- **Killing some classes leads to a new, nonsusy R7 brane, passes consistency checks**
- **Also classify IIB discrete theta angles; killed some of them**

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Many more branes to be uncovered by Markus after the coffee break!

Thank you!