Non-Invertible Symmetry, Holography, and Branes

Fabio Apruzzi, IB, Federico Bonetti, Sakura Schäfer-Nameki
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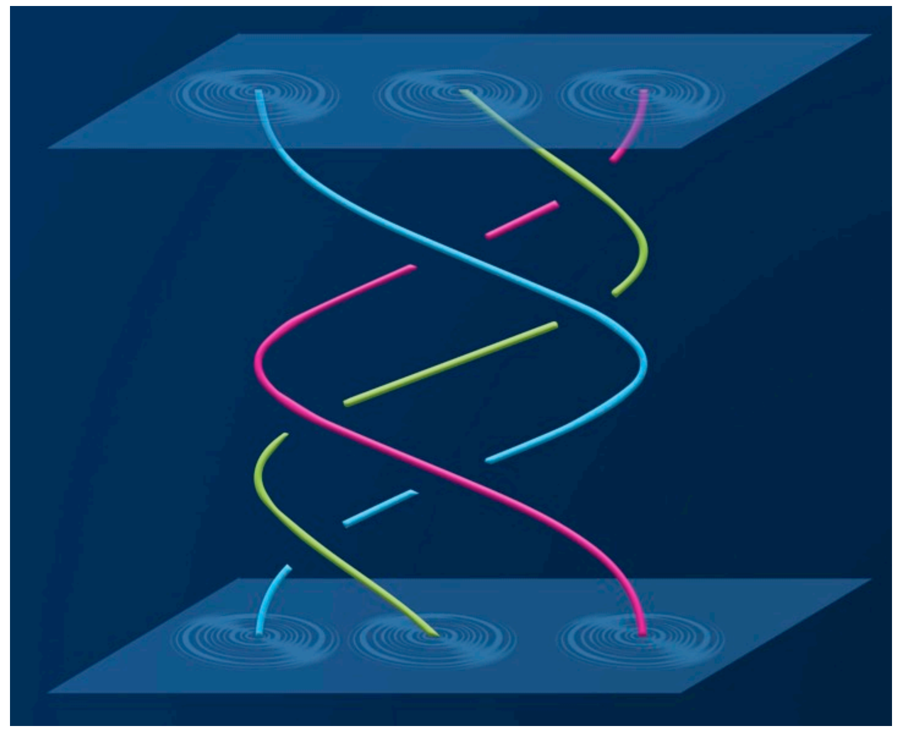
Ibrahima Bah
Johns Hopkins University



Simons Collaboration on Global Categorical Symmetry

Generalized Symmetries

The notion of Global Symmetry in Quantum Systems has seen a vast generalization in recent times

Novel perspective: global symmetries are implemented by Topological Operators in Quantum System

(Gaiotto, Kapustin, Seiberg, Willett '15)

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Intuitively in Quantum Mechanics, the topological nature of symmetry operators follows for:

$$[U, H] = 0 \xrightarrow{\text{RG Flow}} [U, \tilde{H}] = 0$$

"Constants" of motion preserved under time-evolution and Renormalization Group (RG) flow

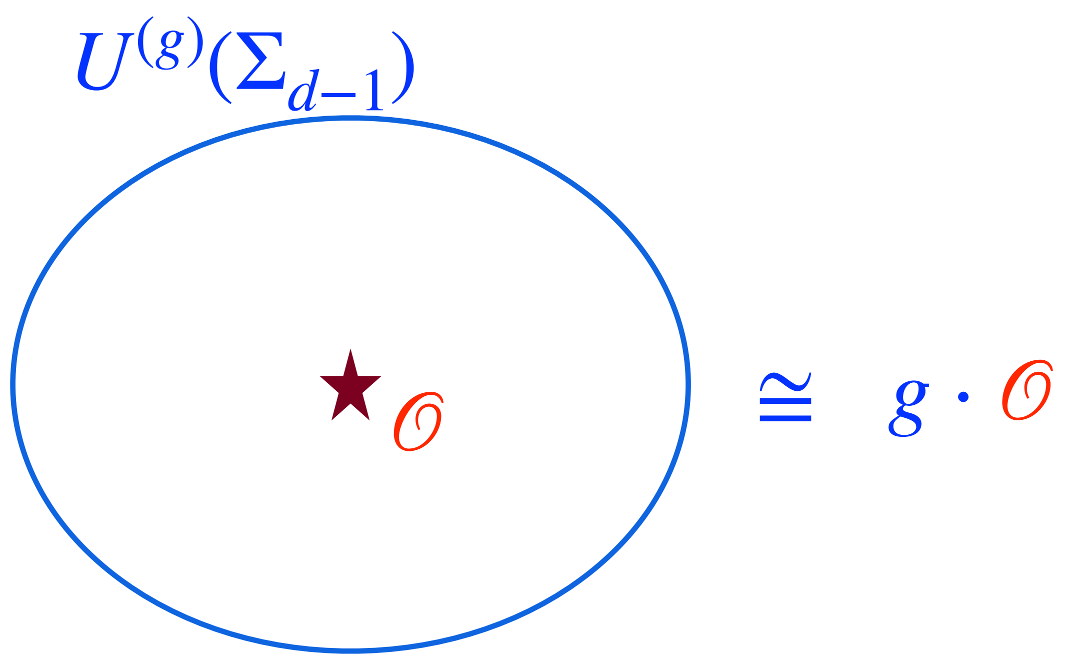
Super-selection sectors, Effective field theory, RG flows,

Anomalies and control IR phases, the Landau paradigm…

Topological Operators

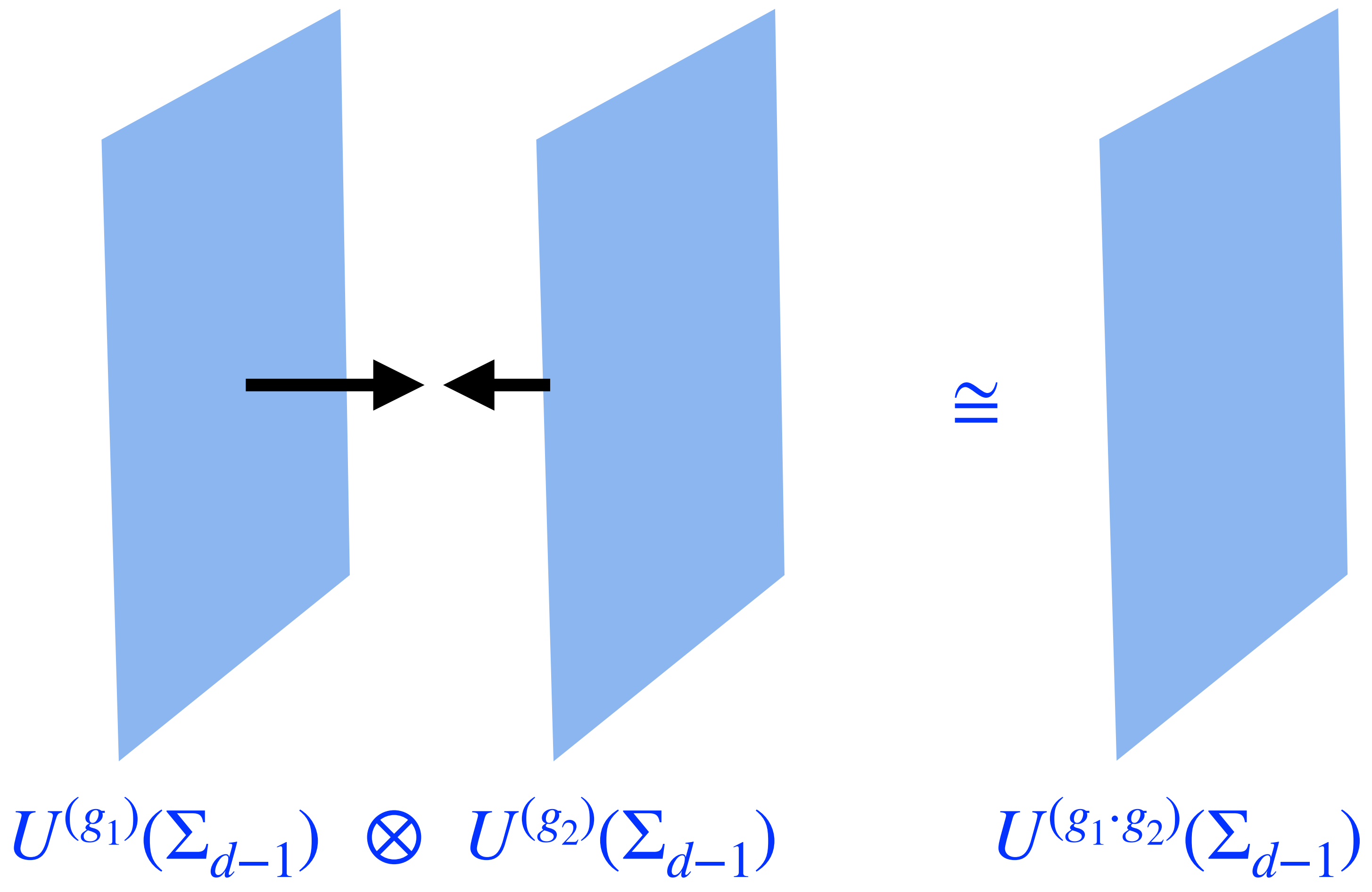
Ordinary symmetries implemented by co-dim 1 Topological Operators

Fusion rule of Top. Operators implemented by a group action



For Continuous Symmetries topological nature of operator follows from Nother's theorem

In General Topological Operators implement both continuous and discrete symmetries



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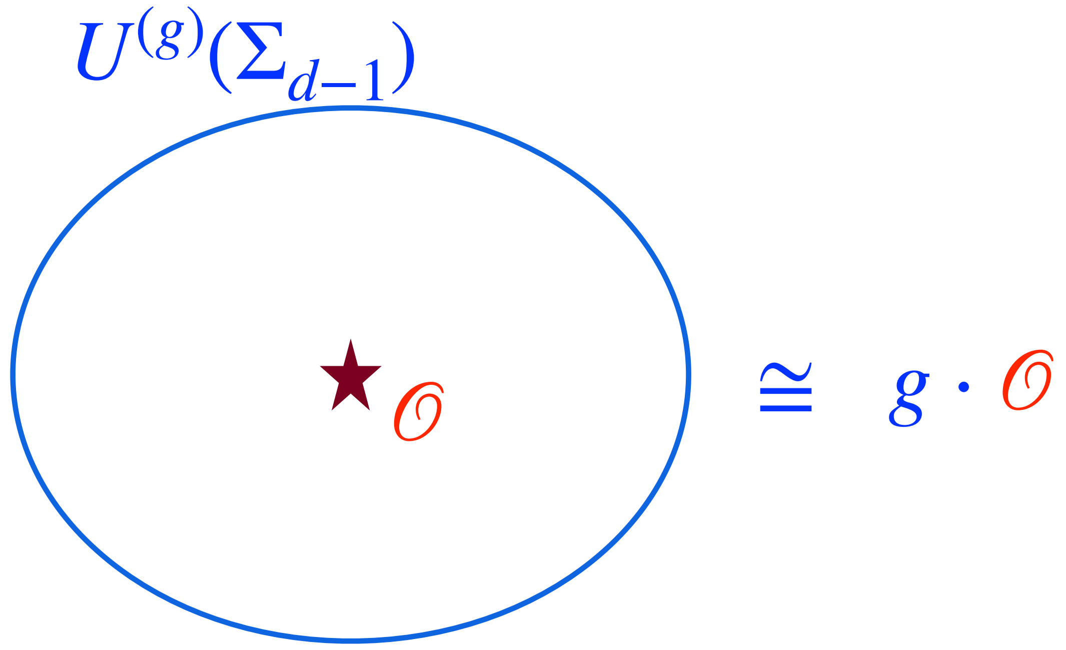

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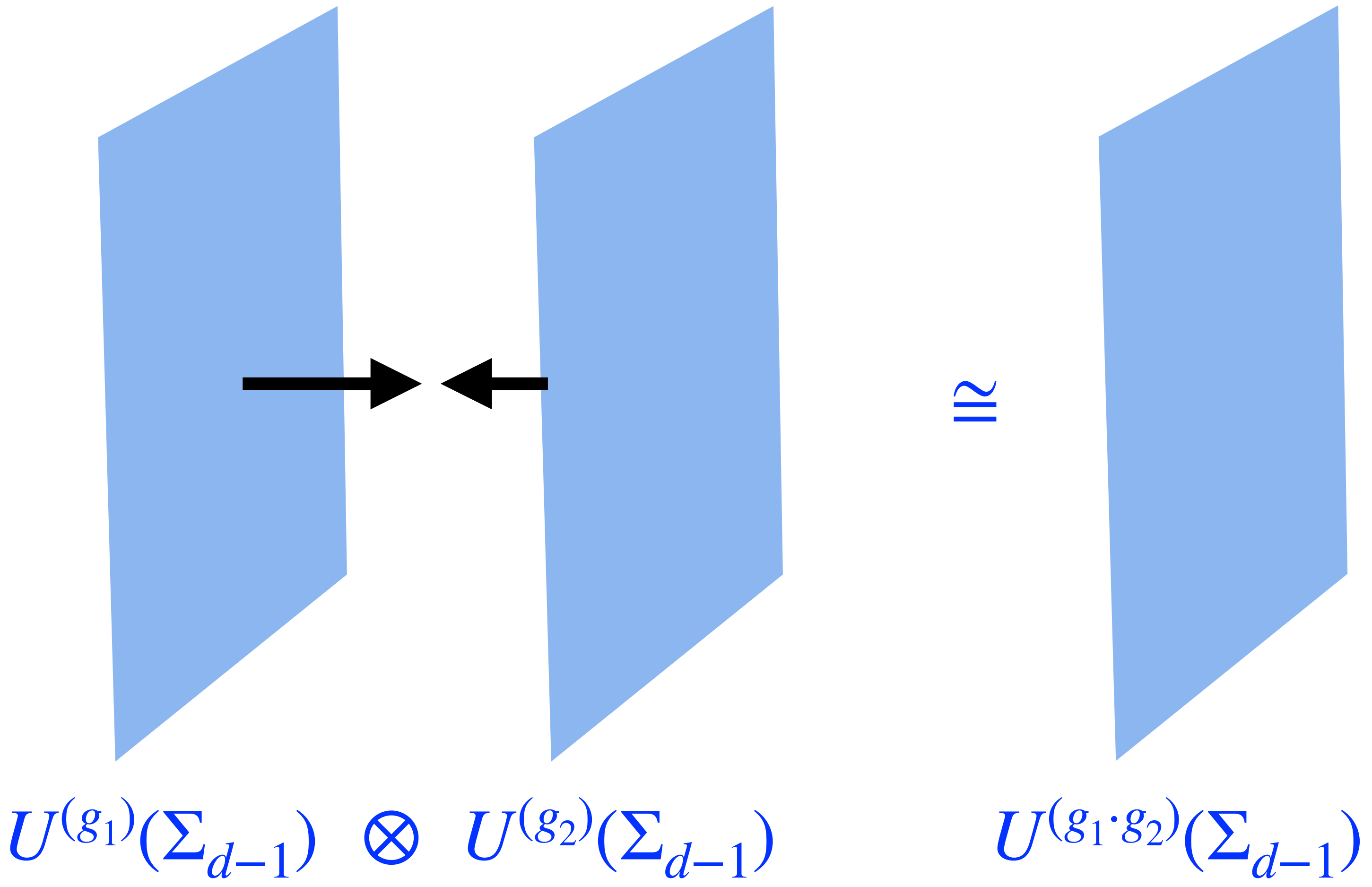
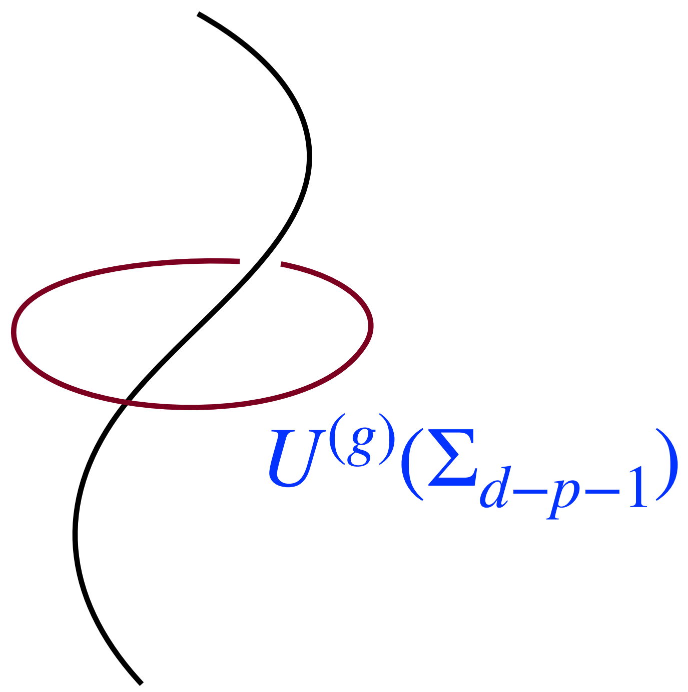


Quantum systems may also enjoy a spectrum of extended operators These can be charged under higher-form symmetries

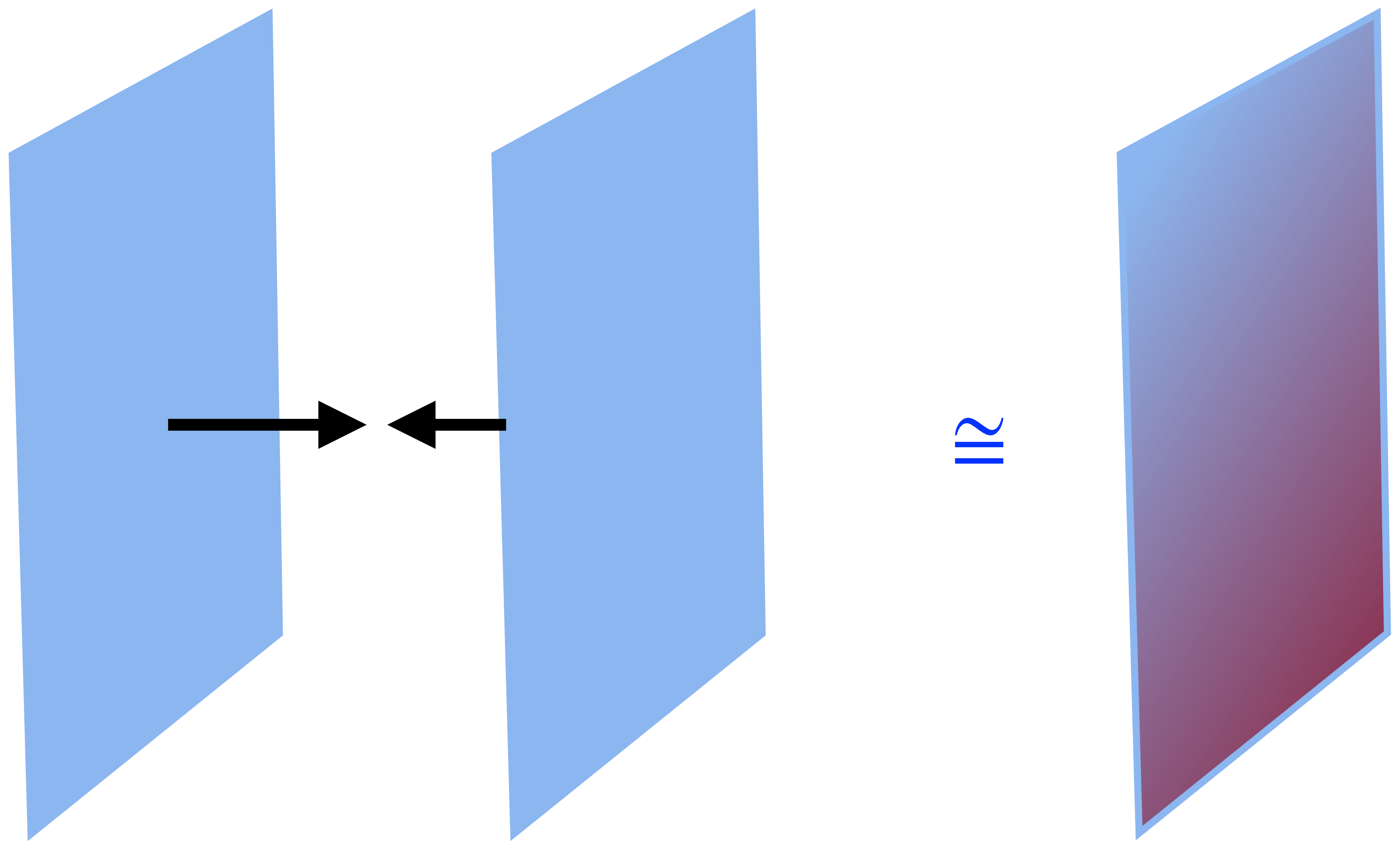


Symmetry operators supported on (d-p-1)—dimensional surfaces, $U^{(g)}(\Sigma_{d-p-1})$, which links with p-extended operators

Defines a p-form symmetry

Non-Invertible Symmetry

General fusion rule of Topological Operators is NOT group-like



$$U^i(\Sigma) \otimes U^j(\Sigma) \cong \sum N_k^{ij}\, U^k(\Sigma)$$

...Implies non-invertible symmetry generators

$$U^i(\Sigma) \oplus U^i(\Sigma)^\dagger \ncong I$$

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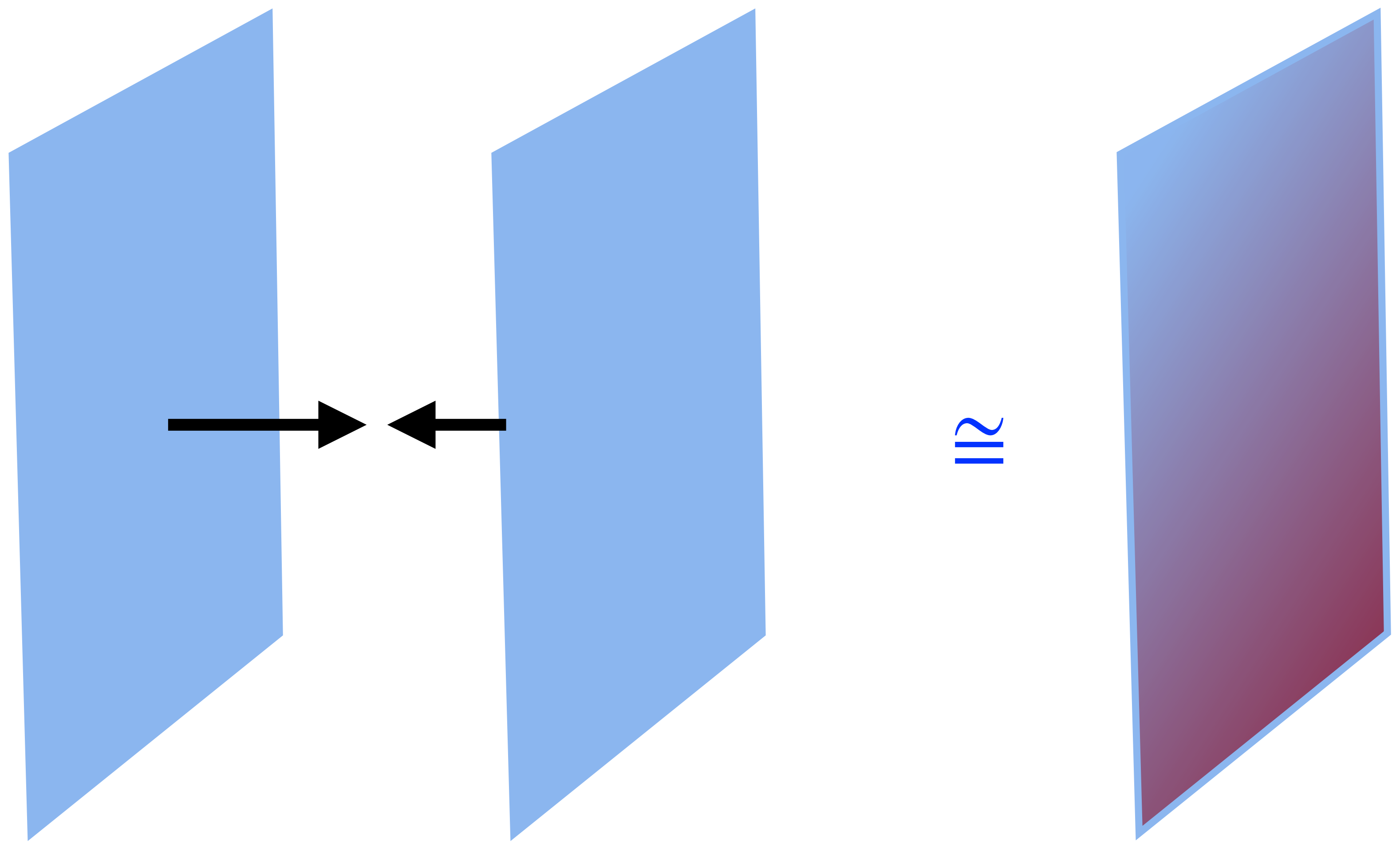

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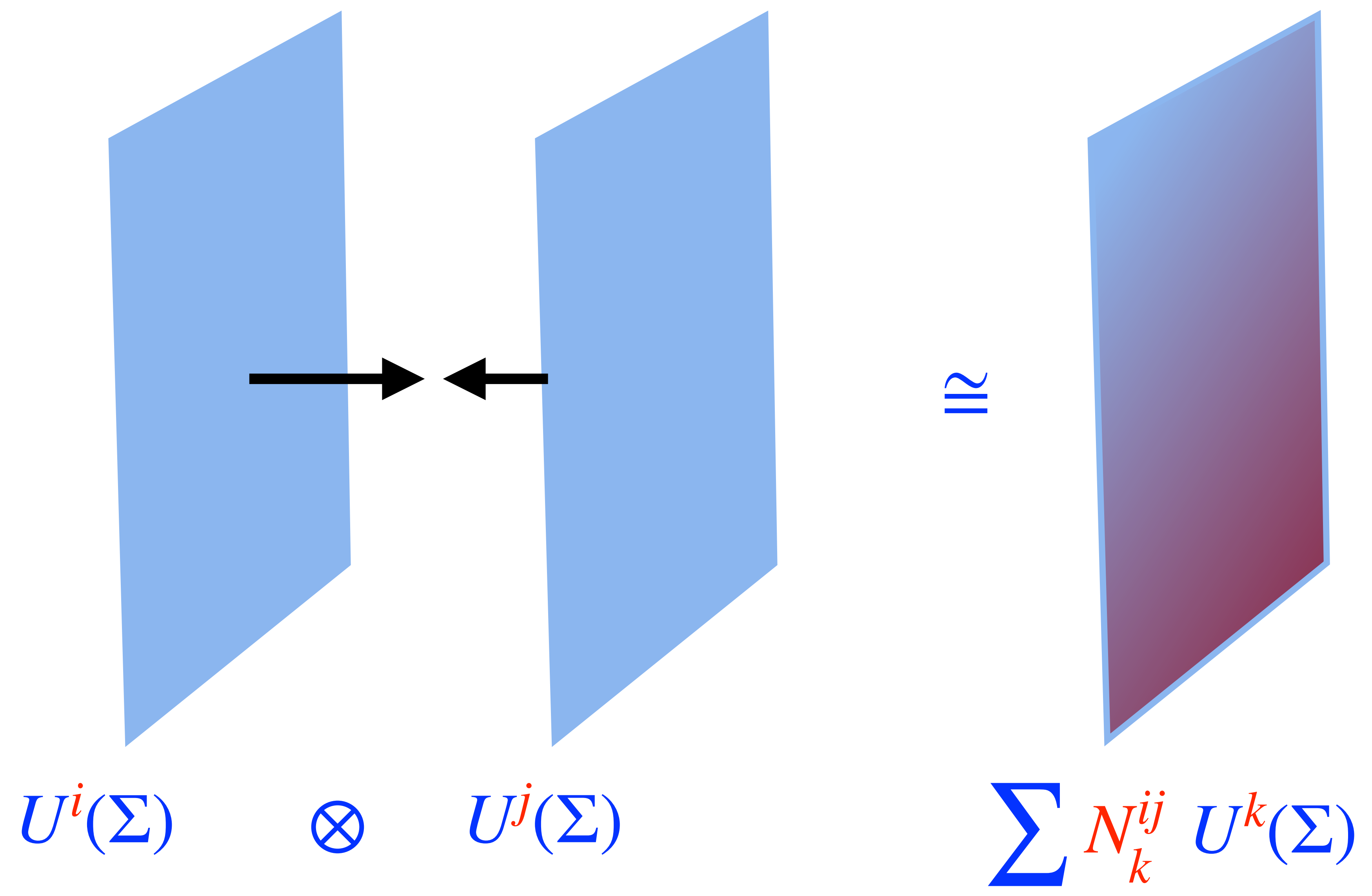

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Recent Surprise: Non-Invertible Symmetries exist for $D \geq 4$, for bread-and-butter theories such as QED and QCD (Tachikawa '97; Choi, Cordova, Hsin, Lam, Shao '21; Kaidi, Ohmori, Zheng '21)

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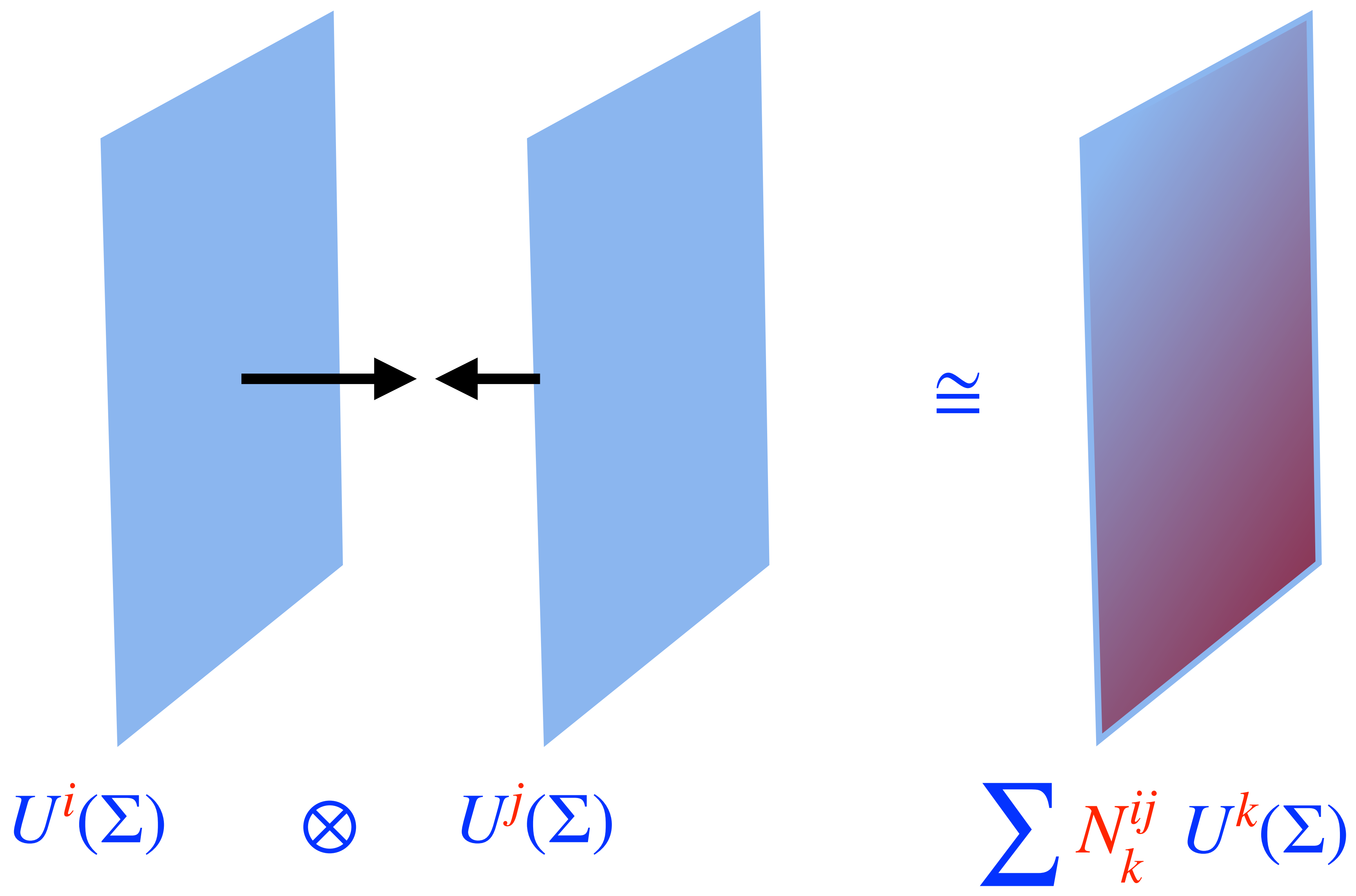

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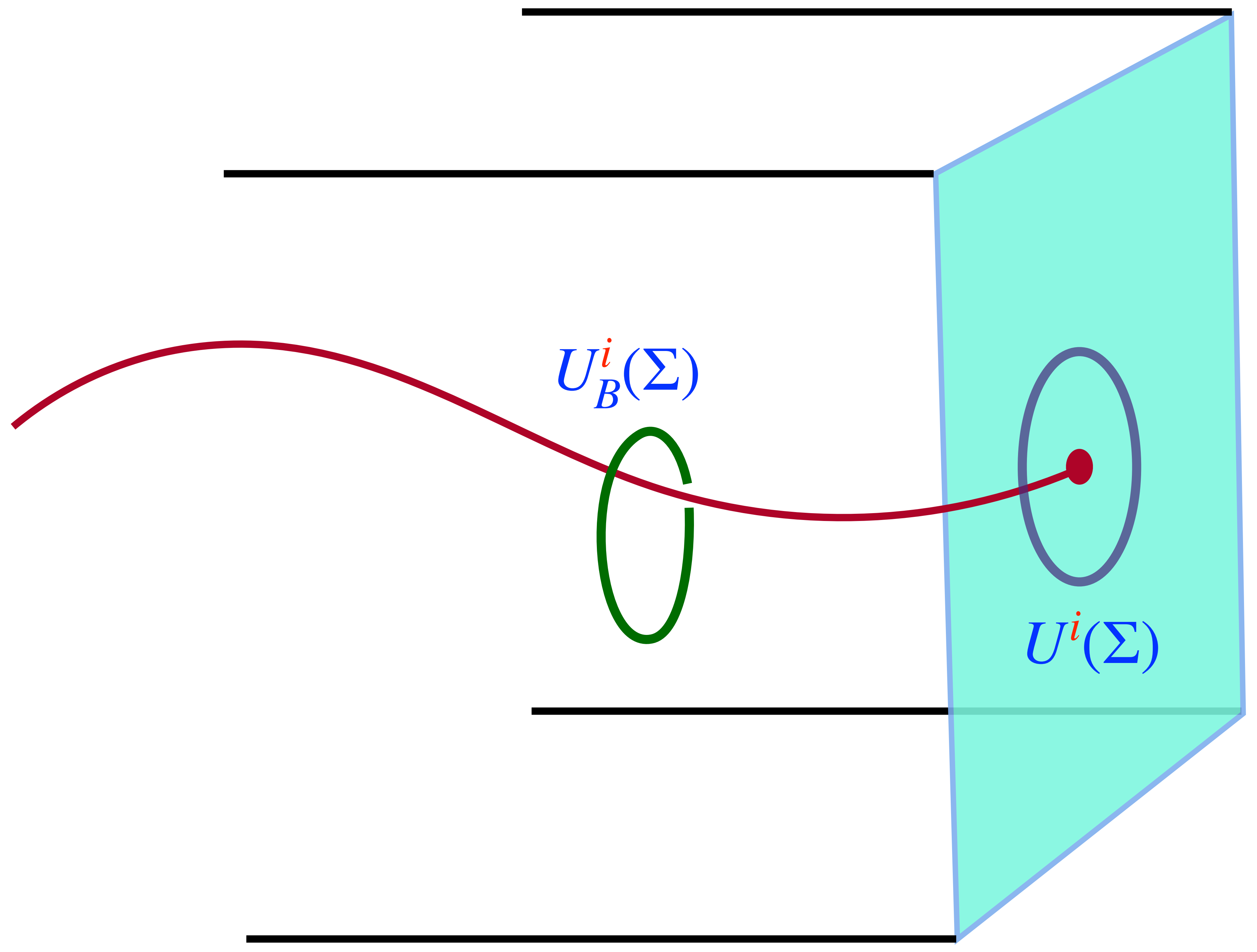
Since then... There has been a large and growing literature of examples of non-invertible symmetries in simple QFTs... New methods... Various generalizations... Categorifications (Bhardwaj, Schafer-Nameki...) with QFT methods

... From Strings and Holography

Interest: Where do topological and non-invertible symmetries come from in holography?



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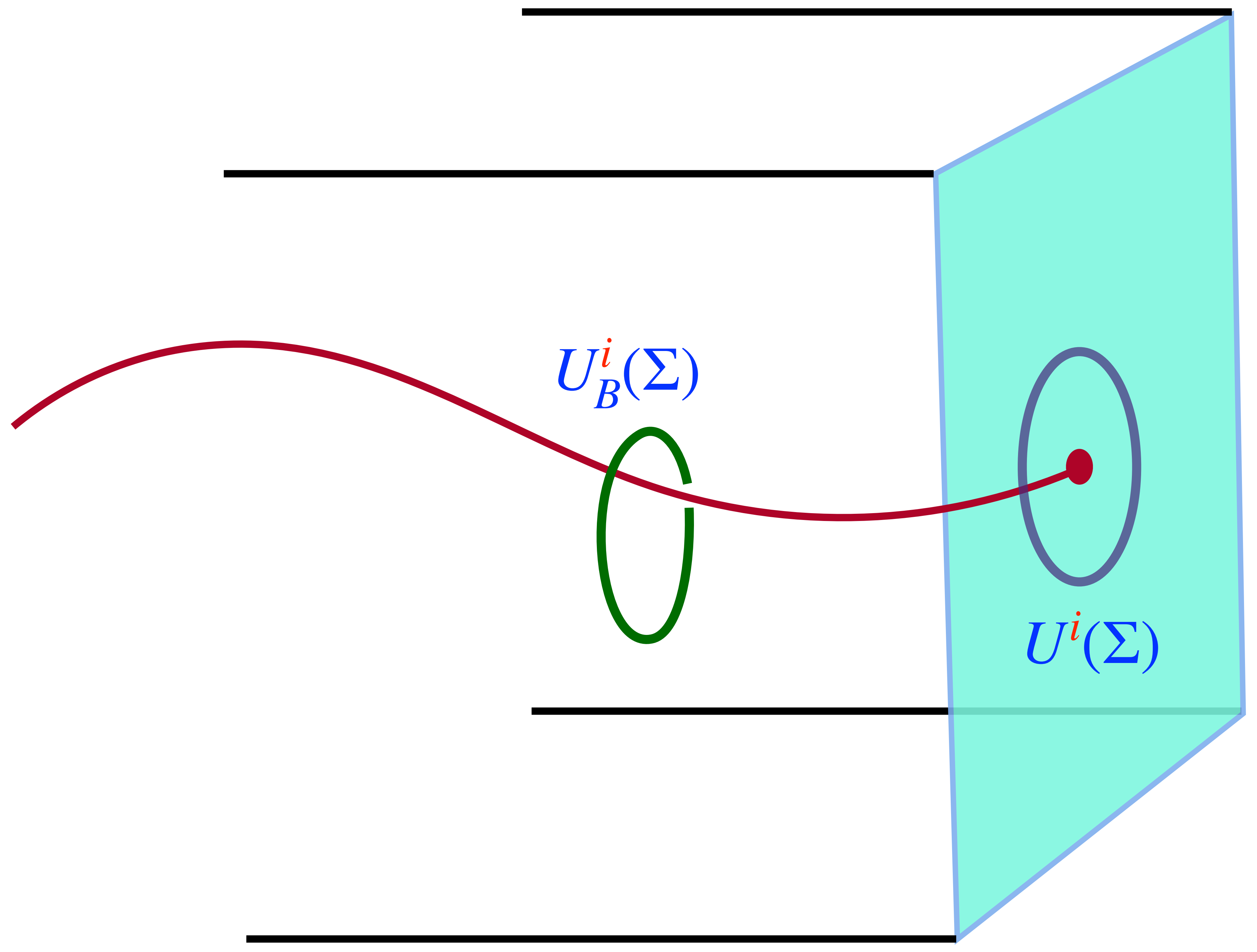

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Bottom-Up: Given effective SUGRA in AdS, what is the construction of $U^i_B(\Sigma)$?

Top-Down: Given a $AdS \times X$ in string theory or M-theory, what stringy states $U^i_B(\Sigma)$?

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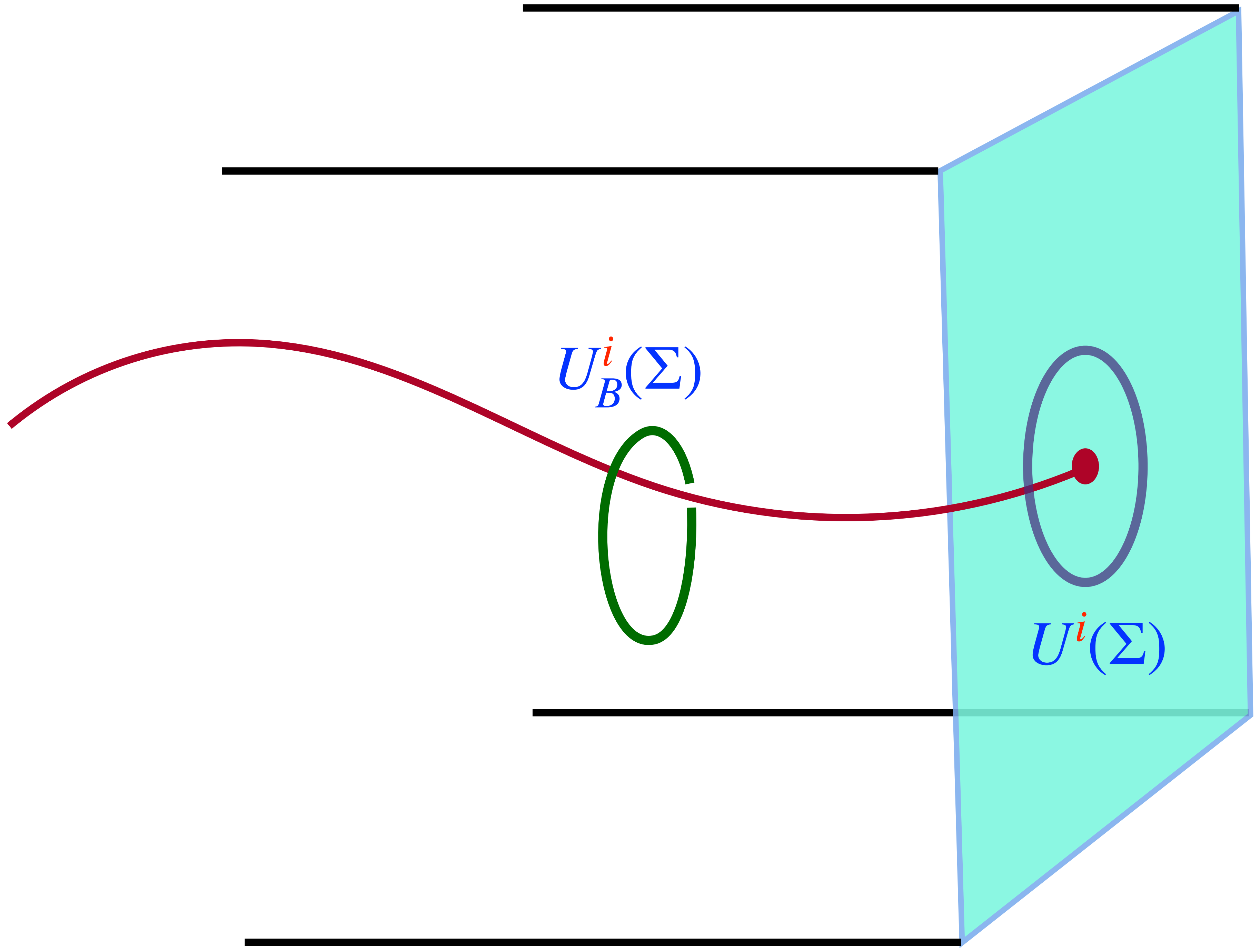

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Duals of topological operators obtained from Gauss Law constraints of bulk gauge theory

Top-Down: Given a $AdS \times X$ in string theory or M-theory, what stringy states $U^i_B(\Sigma)$?

D-branes suitably reduced on X describe duals of topological operators

Gauss Law and Symmetry Generators

Global Symmetries in the boundary extend to gauge symmetries in the bulk

Consider a bulk $U(1)$ p-form gauge symmetry with gauge transformation $A \to A + d\lambda_p$

(Apruzzi, IB, Bonetti, Schäfer-Nameki '22; Belov, Moore '04)

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In Hamiltonian quantization of gauge theories there are Gauss Law constraints
When a time direction is fixed, the A_t is non-dynamical

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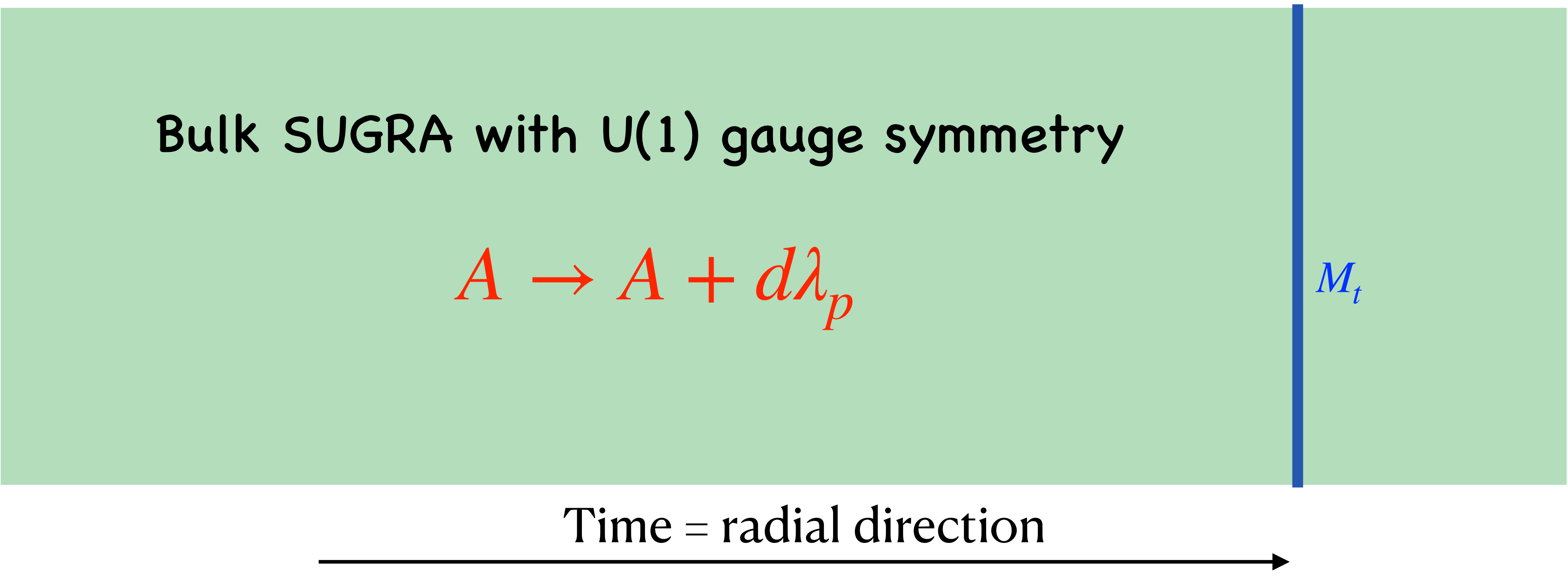


The Gauss Law $\mathcal{G}$ is imposed as a constraint

On constant time slices M_t classically

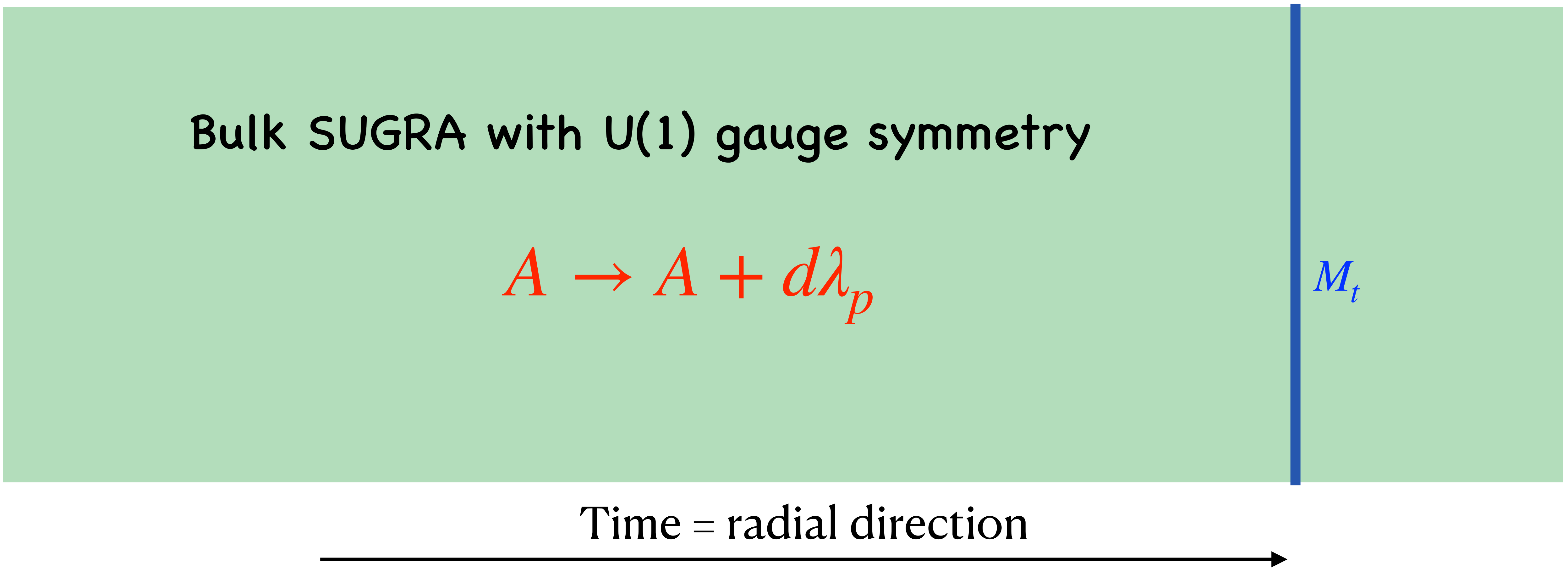
Quantum mechanically it generates gauge transformations on M_t

$$e^{i \int_{M_t} \lambda_p \wedge \mathcal{G}} |\Psi >$$

(Apruzzi, IB, Bonetti, Schäfer-Nameki '22; Belov, Moore '04)

Page Charge and Symmetry Generators

(Apruzzi, IB, Bonetti, Schäfer-Nameki ’22)



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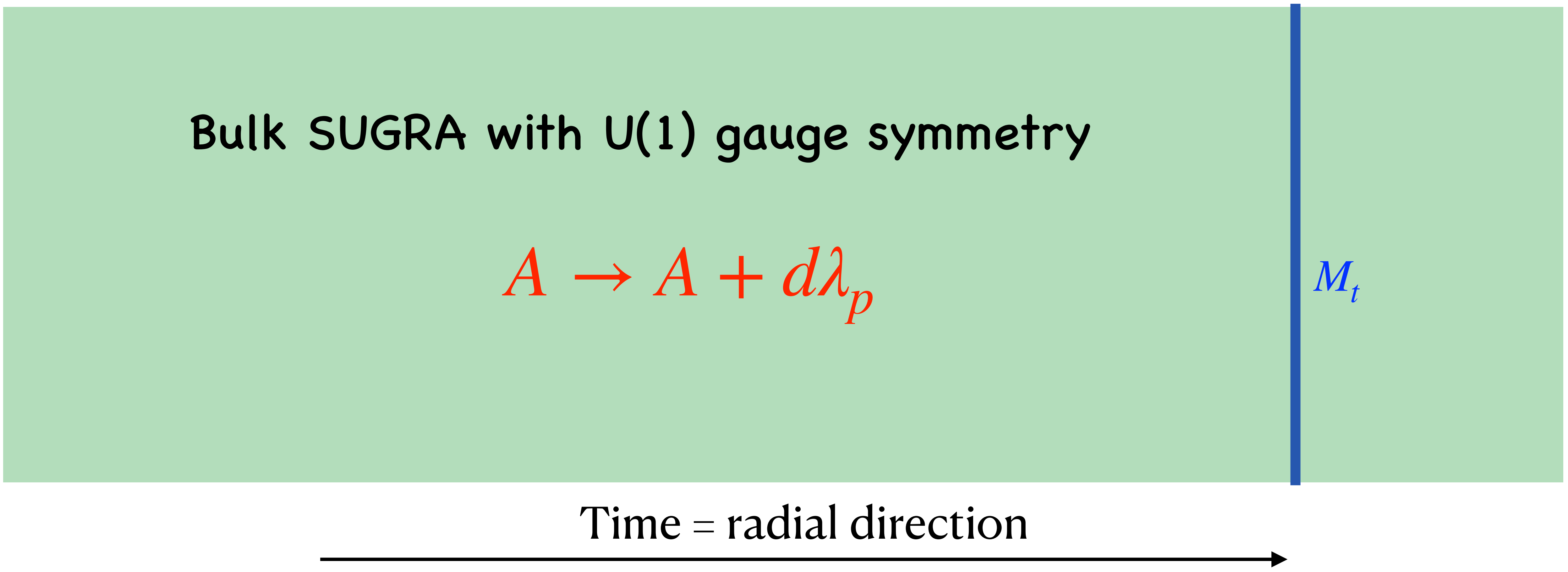

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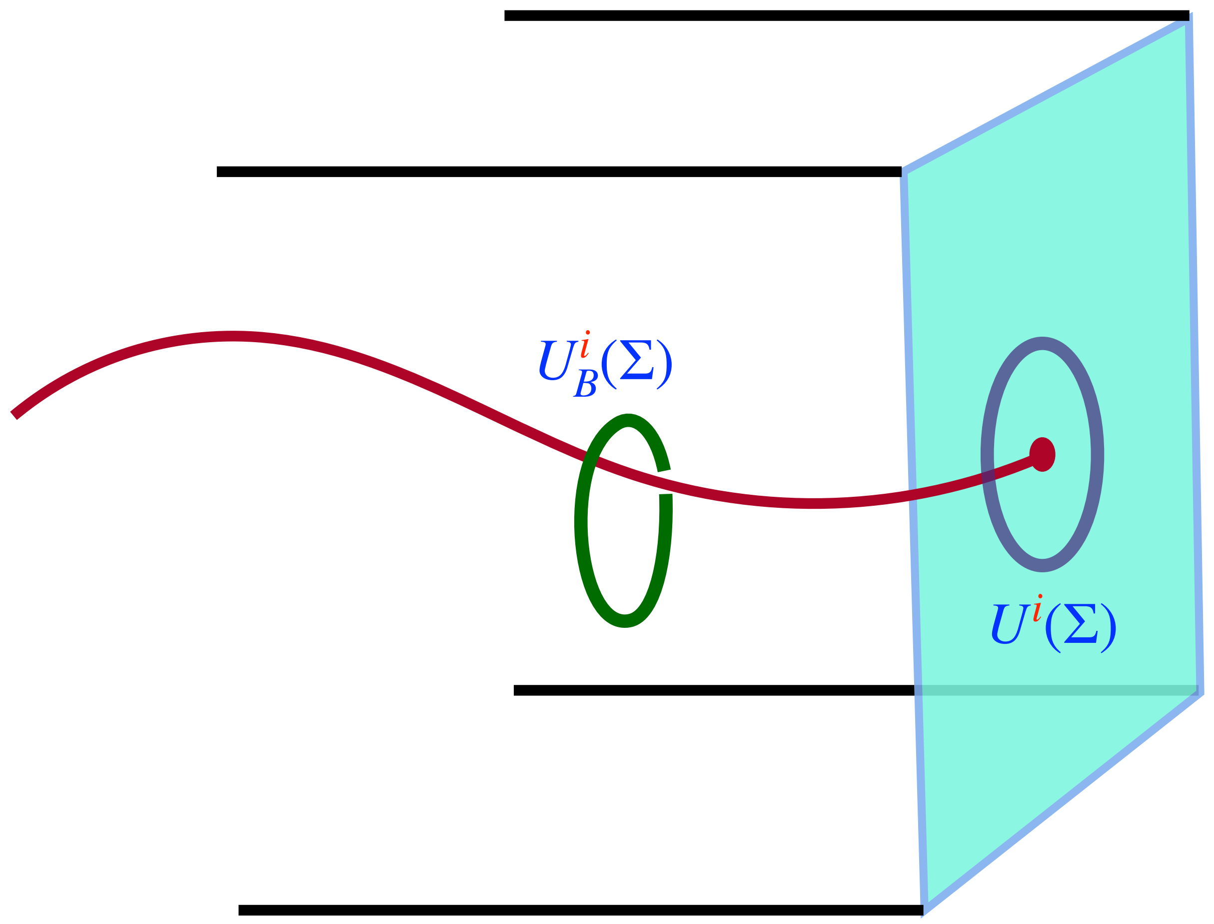

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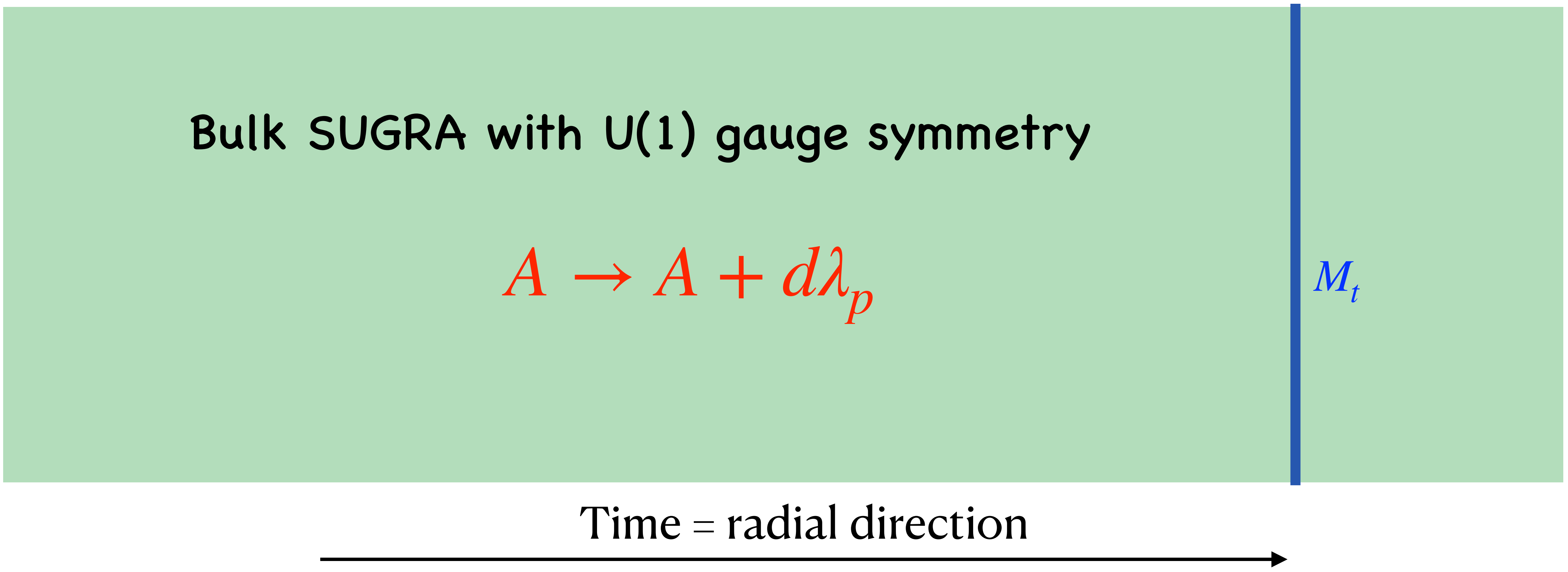

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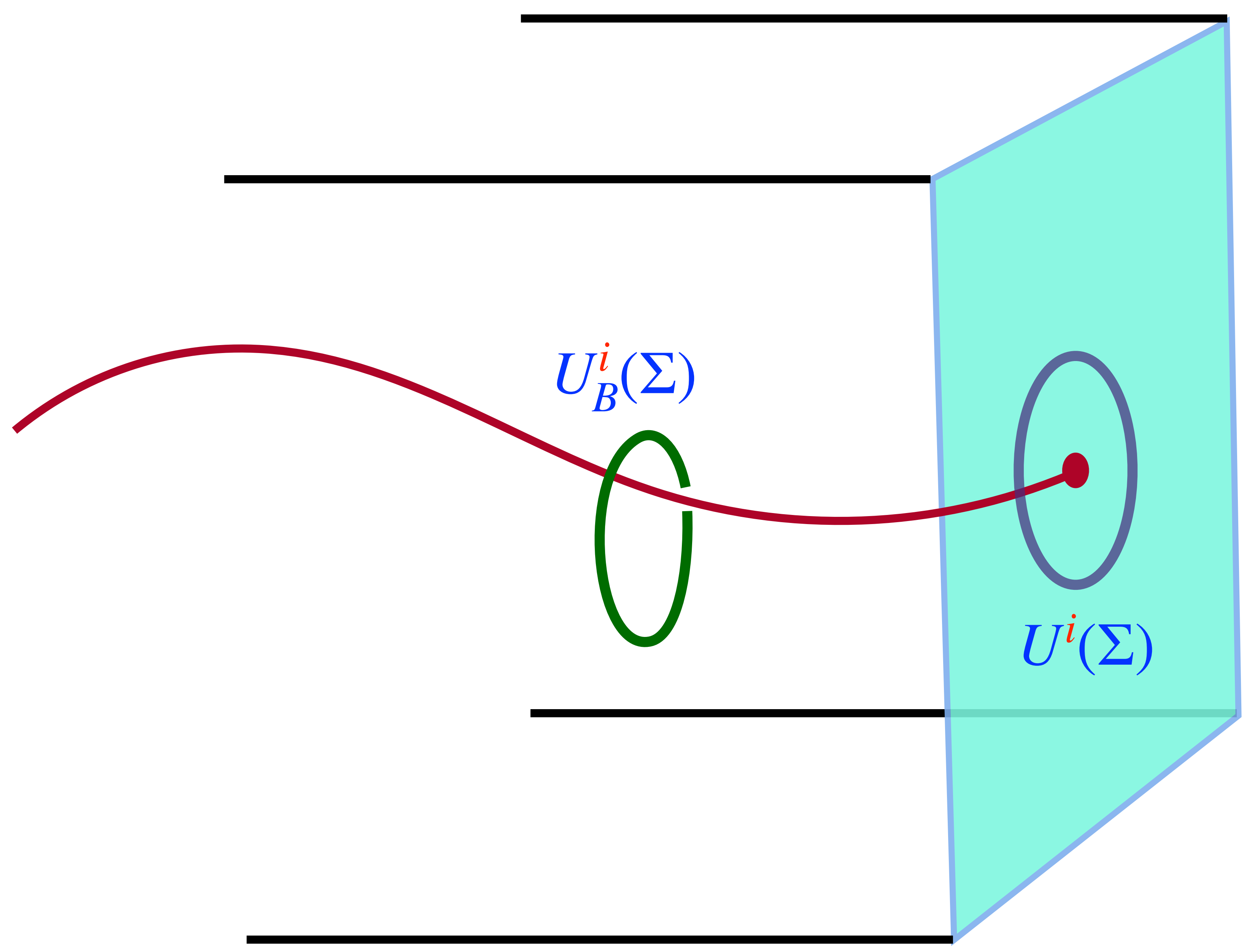
The Page charge is not always well-defined as an operator — Suitable improvements exist by adding fields with topological action that live on Σ and couple to bulk fields

$$U_\alpha(\Sigma_{d-p-1}) = \int \mathcal{D}[a] e^{i\int_{\Sigma_{d-p-1}} [\alpha P + \mathcal{L}(a, ..)]}$$

This, often, lead to non-invertible symmetries

From Branes

In $AdS \times X$, the dual of topological operators can be captured by branes



The Bulk Operator $U_B^i(\Sigma)$ is captured by branes wrapping internal submanifolds in X depending on the symmetry of interest

The branes are required to be stable but not necessarily calibrated

The brane is extended along M_t — constant radial slices

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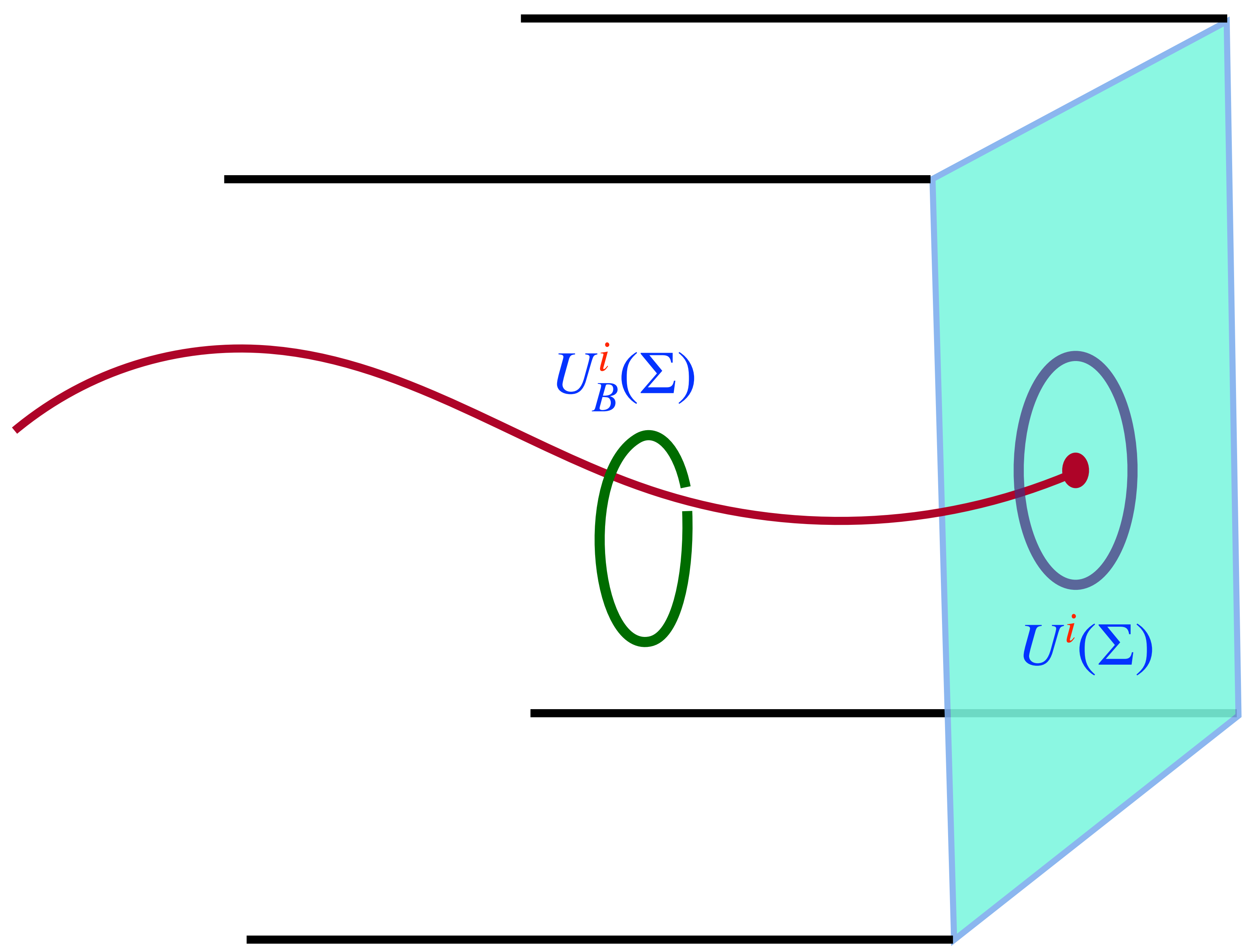

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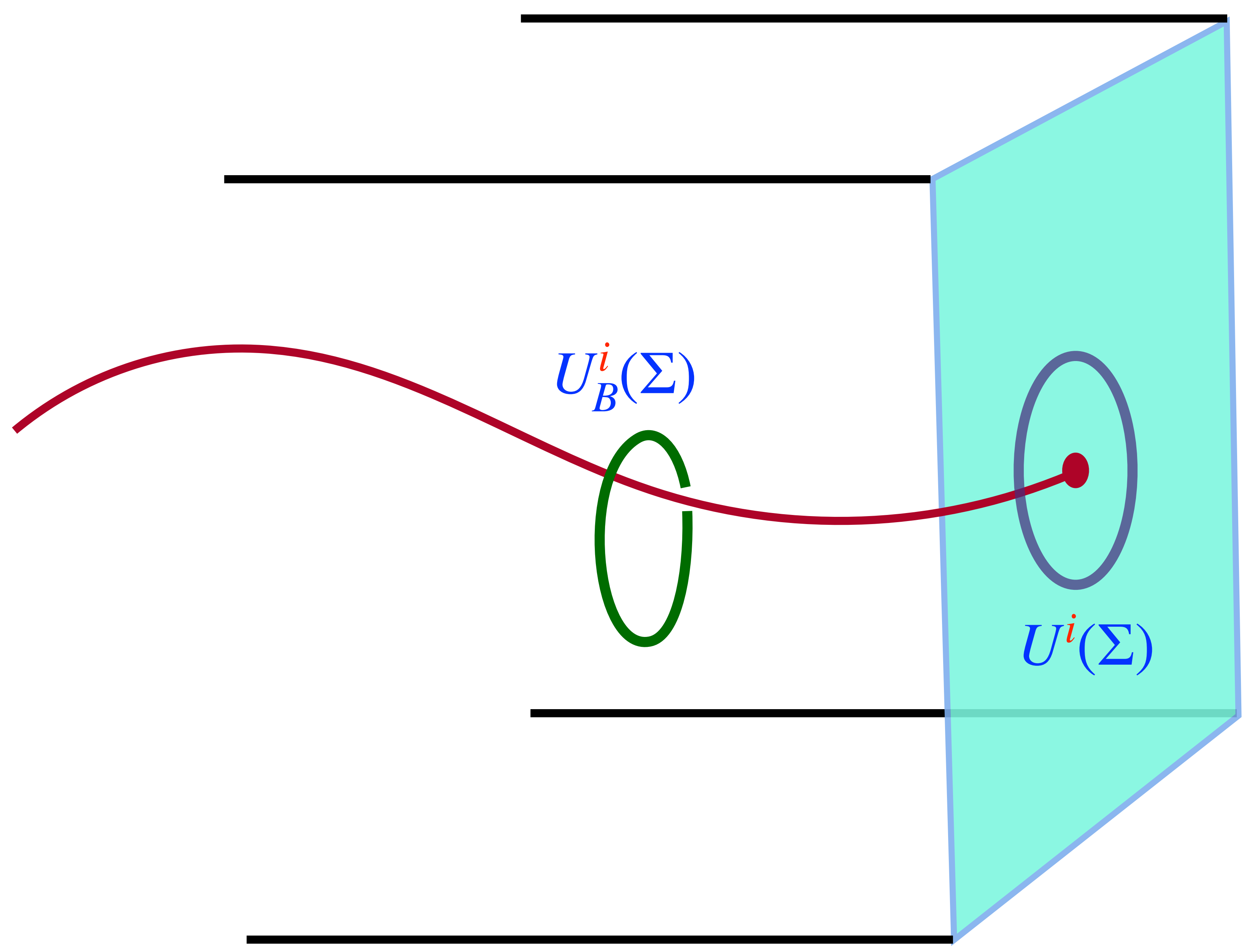

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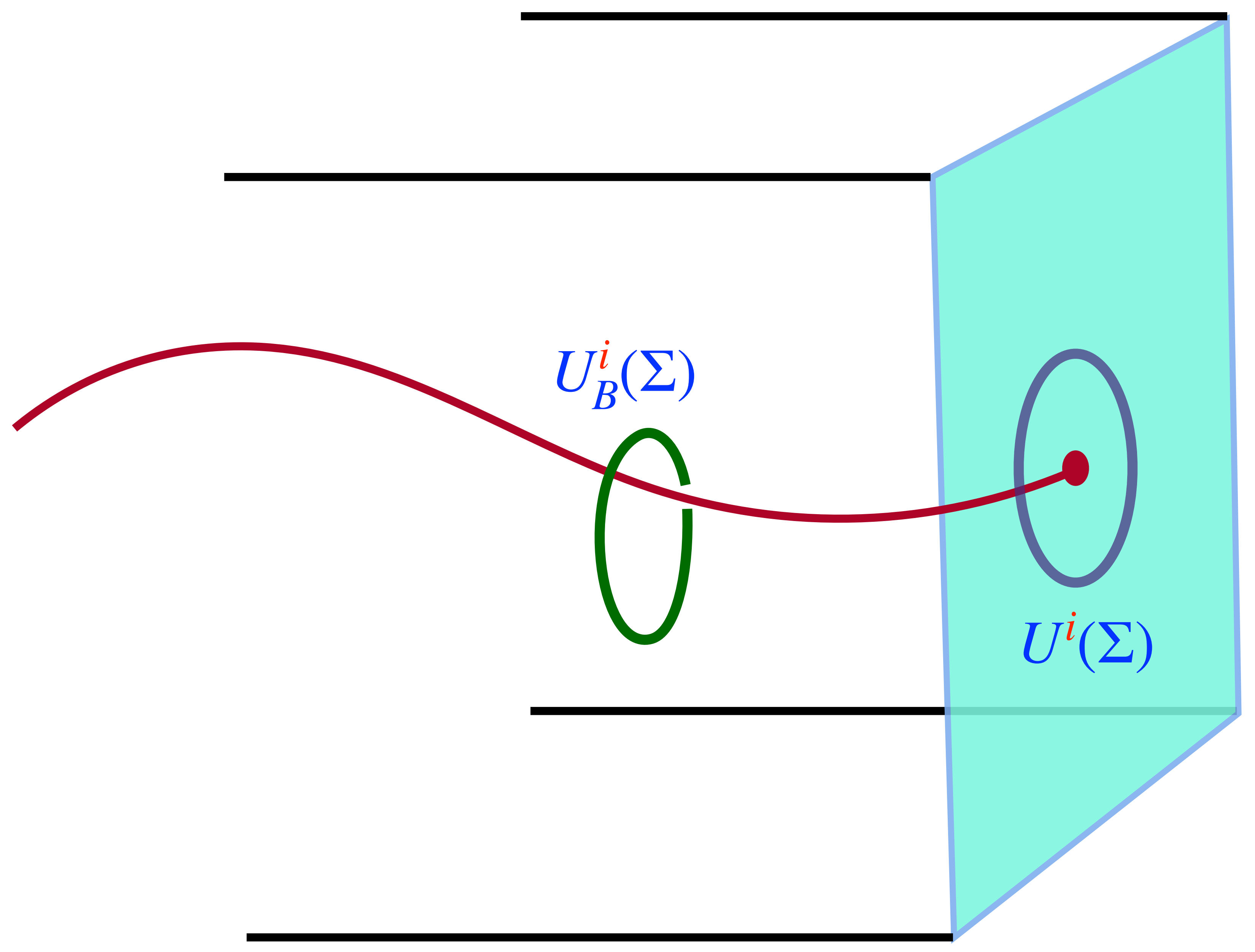
The large effective tension $T_{eff} = T_p r^p$ near the boundary decouples the local fluctuations on the brane

The residual S_{WZ} is a topological action that couples to bulk fields — leading to a topological operator

Brane Dynamics and Fusion

In $AdS \times X$, the dual of topological operators can be captured by branes



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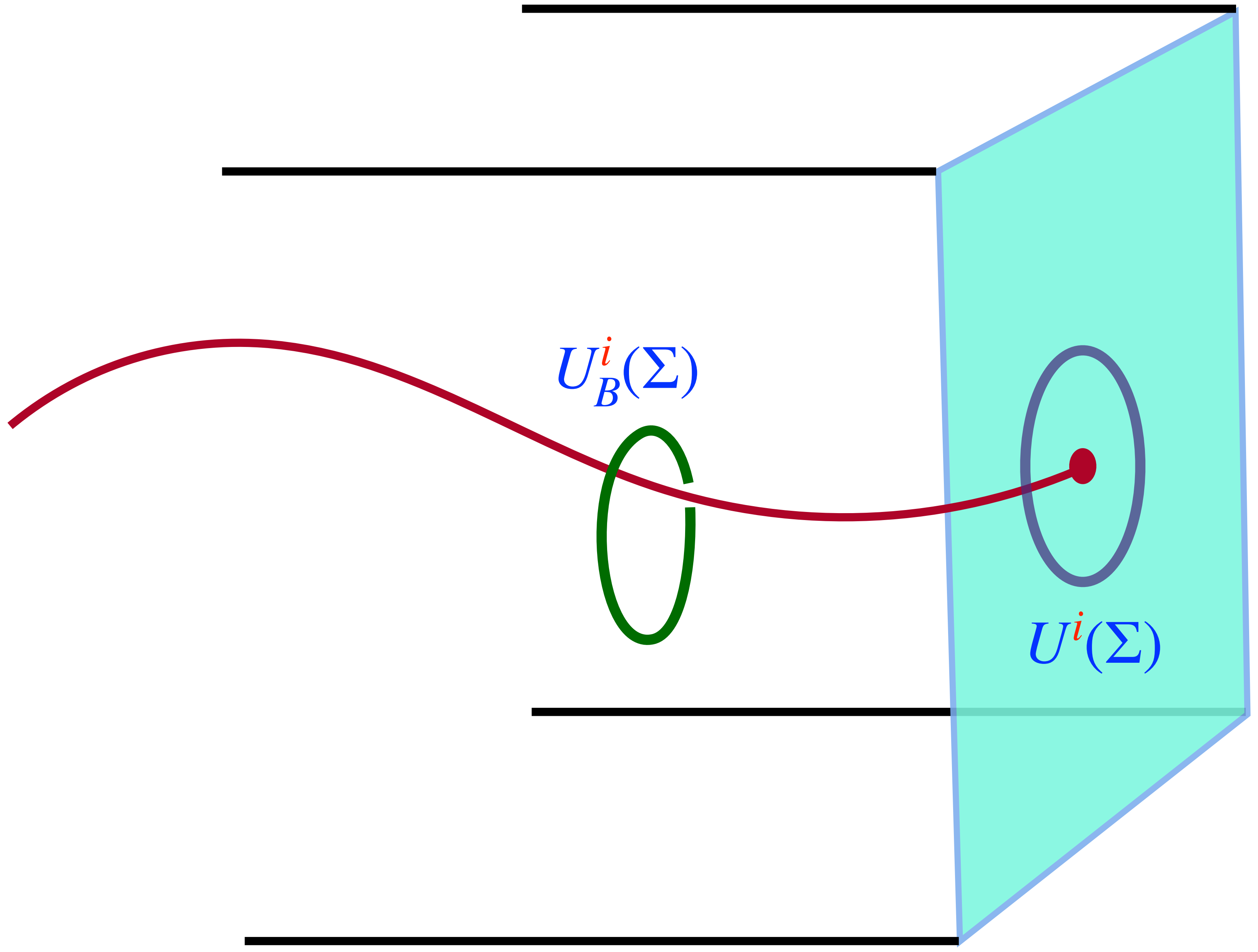

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Non-Invertible fusion from tachyon condensate and K-theory

$$U(\Sigma) \otimes U(\Sigma)^\dagger \longrightarrow D_p \otimes \bar{D}_p = \sum \text{Lower branes}$$

Example: $\mathcal{N} = 1$ SYM

A Dual of $\mathcal{N} = 1$ SYM

Consider IIB on $AdS_5 \times T^{(1,1)}$ with M units of F_3 on the 3-cycle, and N units of F_5 flux

System dual to Duality cascade of Klebanov-Strassler,

When N is a multiple of M the cascade ends with $\mathcal{N} = 1\ SU(M)$ SYM

The gauge theory admits the discrete global symmetry $\mathbb{Z}_{2M}^{(0)} \times \mathbb{Z}_M^{(1)}$

$\mathbb{Z}_{2M}^{(0)}$: Discrete 0-form symmetry from $U(1)_R$ symmetry broken down by ABJ anomaly

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There is a 't Hooft anomaly between the two inherited from the ABJ anomaly

$$\mathscr{A} = -\frac{2\pi i}{M}\int A_1 \cup \frac{\mathscr{B}(B_2)}{2}$$

(Gaiotto, Kapustin, Seiberg, Willett '19; Cordova, Dumitrescu '18)

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Gauging the 1-form symmetry breaks the 0-form symmetry… Which can be recovered as a non-invertible symmetry by stacking its symmetry generator with a 3d $U(1)_M$ Chern Simons theory — Gauge group is PSU(M)

(Hsin, Lam, Seiberg '18; Kaidi, Ohmori, Zheng '21)

Example in AdS_5

(Apruzzi, IB, Bonetti, Schäfer-Nameki '22)

Consider the action in 5D action on $W_5 = \partial M_6$ with fluxes $(g_2, F_2, h_3, f_3, f_1)$

$$S = S_{kin} + \int_{M_6} \left[N\, h_3 \wedge f_3 + F_2 \wedge g_2 \wedge g_2 - f_1 \wedge g_2 \wedge h_3 \right]$$

$$dg_2 = M h_3, \qquad df_1 = 2 M F_2$$

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$$(\, \mathbf{g}_2^{\mathbf{b}}, \mathbf{f}_1^{\mathbf{b}}, \mathbf{f}_3^{\mathbf{b}} \,) \in H^\star(W_5, \mathbb{Z})$$

Correspond to lifts of discrete gauge fields in Integral cohomology

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$$S_{Top} = \int_{W_5} \left[N\, b_2 \wedge dc_2 + 2M\, A_1 \wedge dc_3 + A_1 \wedge \mathbf{g_2^b} \wedge \mathbf{g_2^b} + \frac{1}{2} g_2 \wedge \star g_2 + \cdots \right]$$

Gauss Law and Symmetry generator

(Apruzzi, IB, Bonetti, Schäfer-Nameki '22)

We consider the symmetry generator for the discrete 0-form symmetry

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$$\exp\left(2\pi i \int_{M_4} \mathbf{g}_2^{\mathbf{b}} \wedge \mathbf{g}_2^{\mathbf{b}}\right) = \int \mathcal{D}[a] \exp\left(2\pi i \int_{M_4} M^2\, da \wedge da + 2M\, da \wedge \mathbf{g}_2^{\mathbf{b}}\right)$$

$$= \int \mathcal{D}[a] \exp\left(2\pi i\, 2M \int_{\partial M_4} \frac{M}{2} a \wedge da + a \wedge \mathbf{g}_2^{\mathbf{b}}\right)$$

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Gauss Law and Symmetry generator

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(Apruzzi, IB, Bonetti, Schäfer-Nameki '22)

We pick boundary conditions to fix global form of dual theory

$SU(M)$ gauge group: Fix $(A_1, \mathbf{g}_2^{\mathbf{b}})$ at boundary of AdS and sum over (c_2, c_3)

The dual theory admits a $\mathbb{Z}_{2M}^{(0)} \times \mathbb{Z}_M^{(1)}$ global symmetry with background fields $(A_1, \mathbf{g}_2^{\mathbf{b}})$

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$$U(\Sigma_3) \otimes U(\Sigma_3) \equiv D5(S^3) \otimes D5(S^3) \;\rightarrow\; D7(T^{(1,1)})$$

The RHS correspond to a single $D7$ brane with 2 units of WV flux — Myers' effect

Outlook

We describe how to realize novel aspects of generalized and topological symmetries from Holography

The holographic prescription provided explicit derivation of these objects and provides an opportunity for more systematic study

The String theory realization of aspects of topological symmetries brings to bear the theory of branes: K-theory as an important tools for studying generalized symmetries

[Damia, Argurio, Tizzano 22; Damia, Argurio, Garcia-Valdecasas 22; Apruzzi, IB, Bonetti, Schafer-Nameki 22; García Etxebarria 22; Heckman, Hübner, Torres, Zhang 22; Antinucci, Benini, Copetti, Galati, Rizi 22]