The Chronicles of IIBordia

— The Rise of Metaplectia —

Based on:

2302.00007 and 2107.14227 with Arun Debray, Jonathan J. Heckman, and Miguel Montero

see also 2212.05077 with Jonathan J. Heckman, Miguel Montero and Ethan Torres

Strings and Geometry 2023 — March 3 - March 6

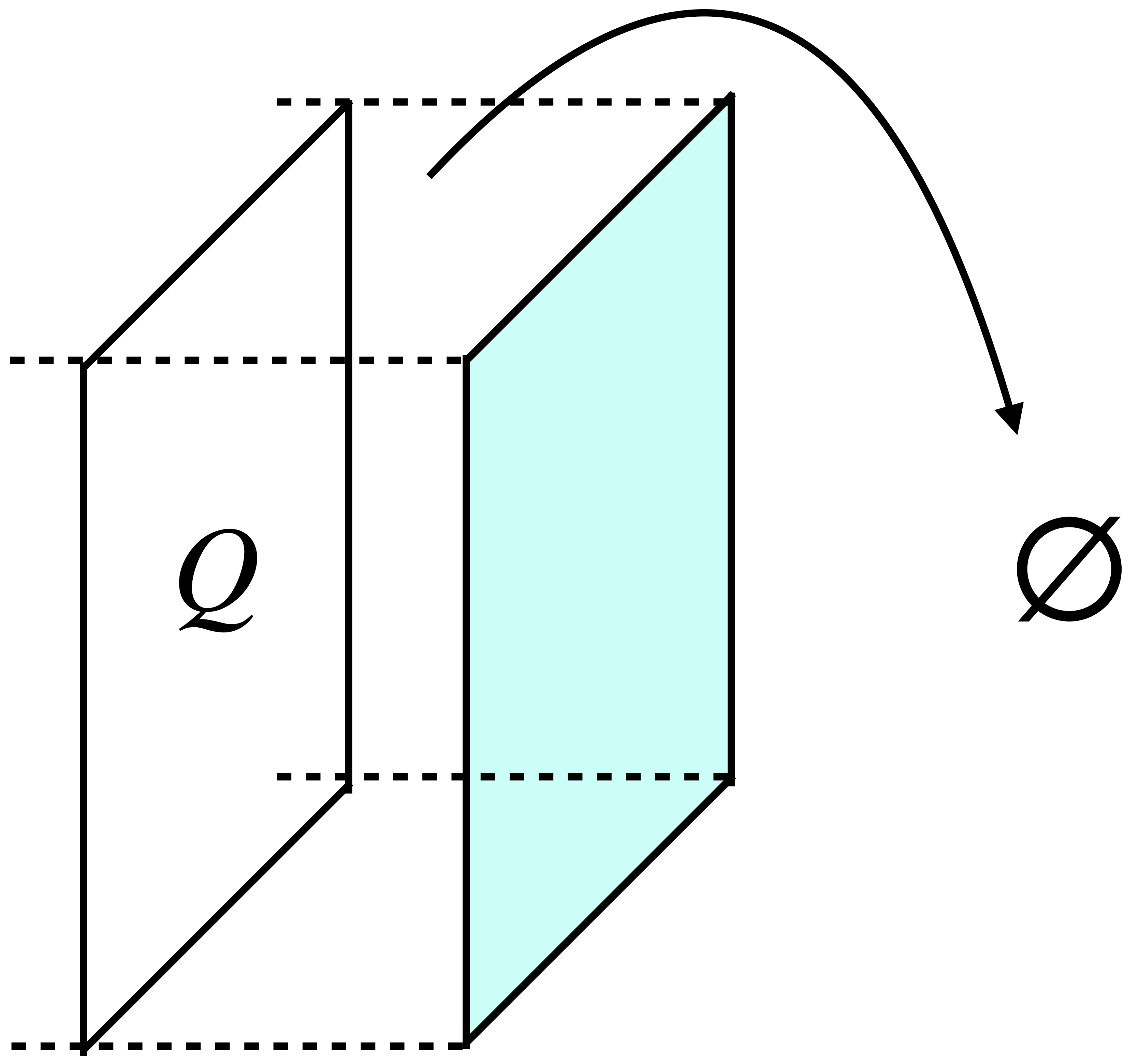
LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Markus Dierigl

All things must end (including spacetime)

Cobordism Conjecture

[McNamara, Vafa ’19], see also [Montero, Vafa ’20], [Garcia Etxebarria, Montero, Sousa, Valenzuela ’20]



Every **quantum gravity theory** must **allow for boundary condition** (deformable to nothing)

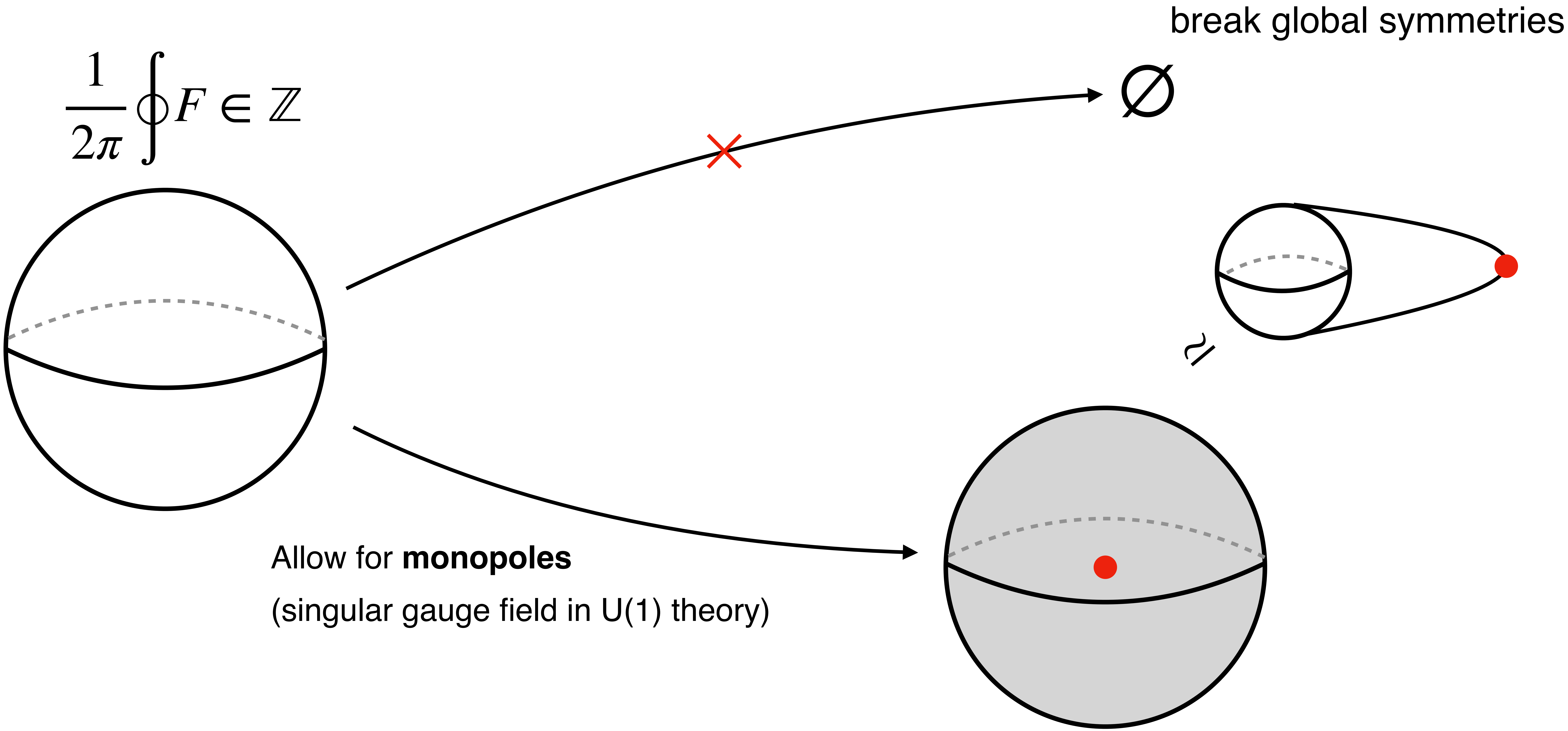
↓

No conserved global charges

obstructions

Singular objects

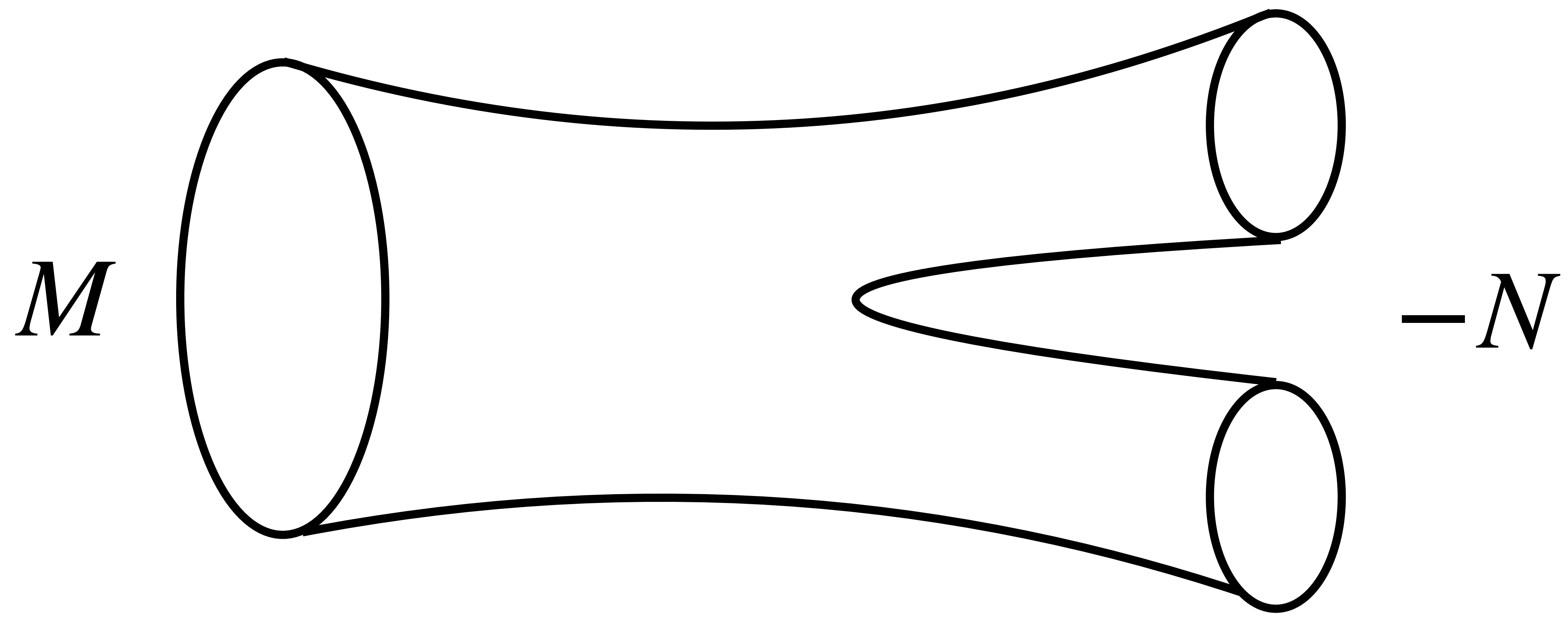
This boundary condition might require the introduction of **new objects**



Bordisms

These **deformation classes** of manifolds with a given structure (Spin, gauge fields, orientation) are given by **bordism groups**

$[M] \sim [N]$ if there is Y such that $\partial Y = [M] \sqcup -[N]$



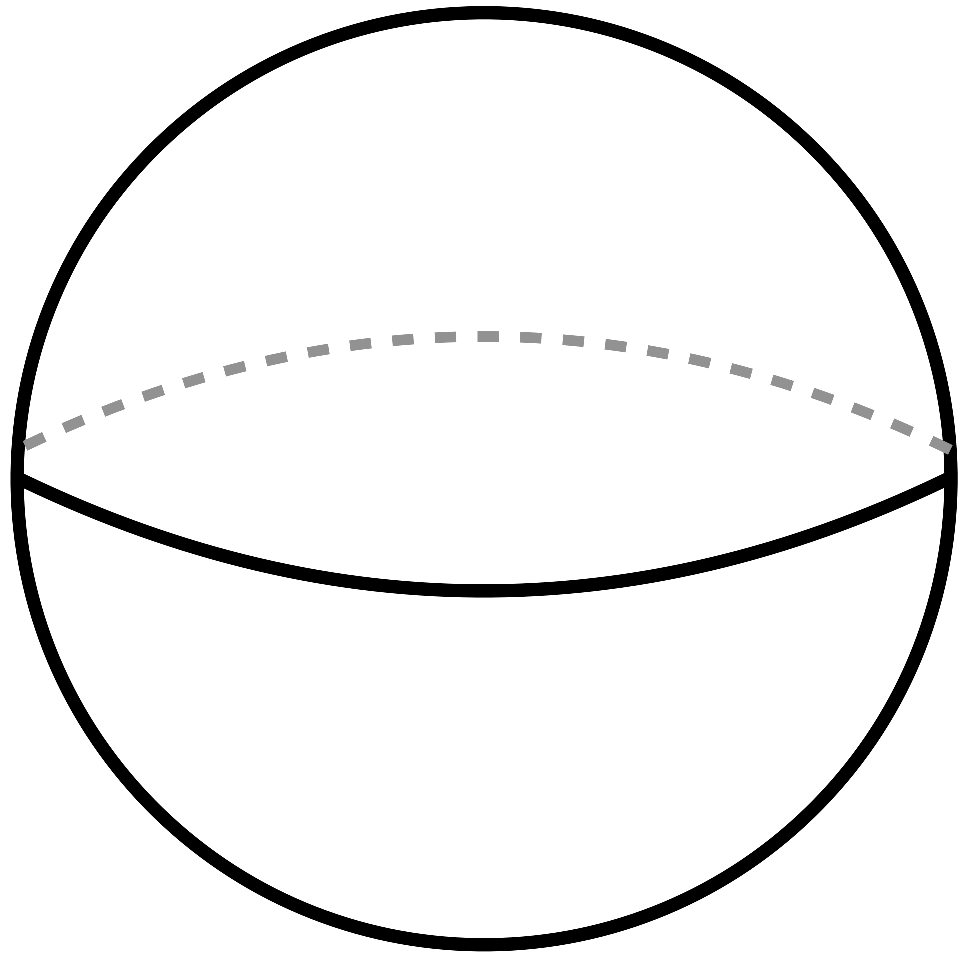
and all structures extend to Y

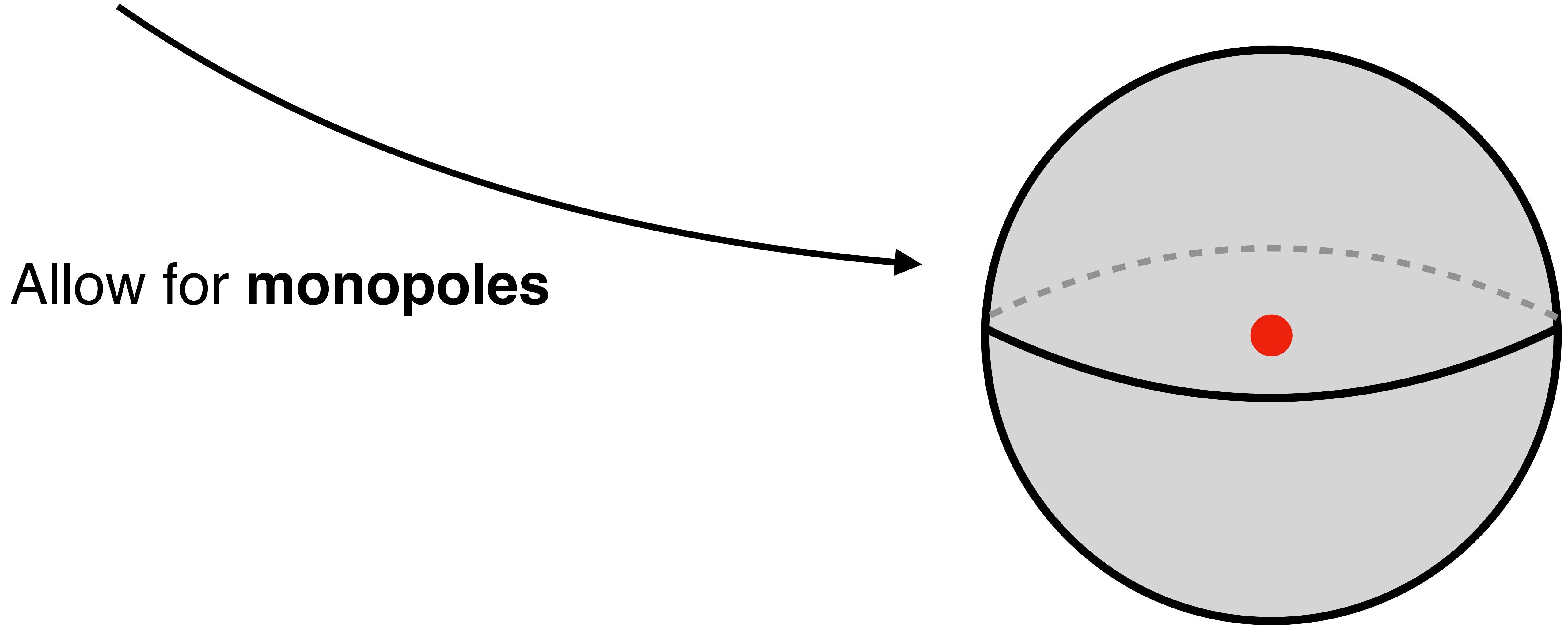
Test in example

$$\frac{1}{2\pi}\oint F = n \in \mathbb{Z}$$

$$\Omega_2^{Spin}\big(BU(1)\big) \supset \mathbb{Z}$$

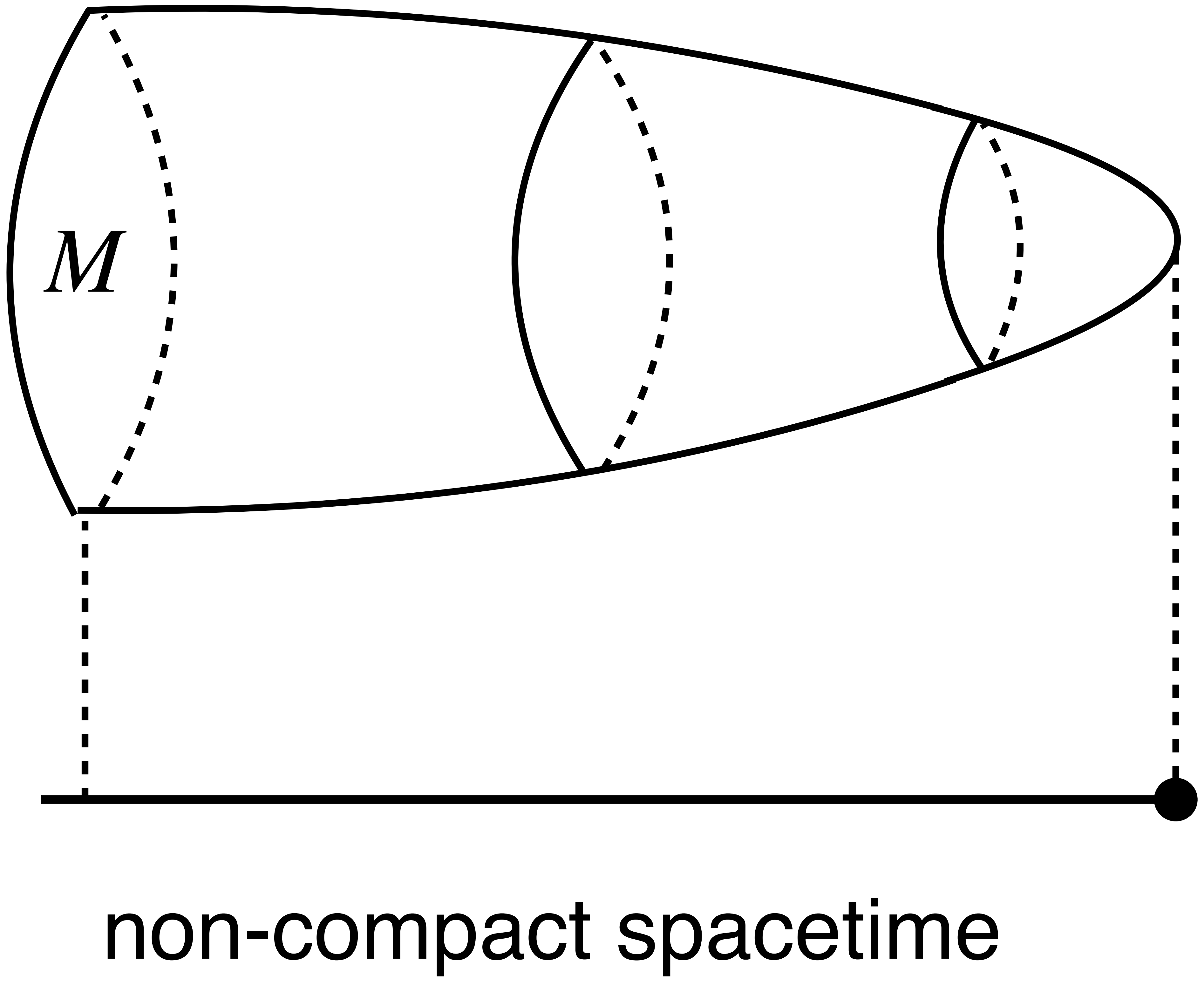
generated by $[S^2_n]$





Bordisms in our setup

In our case M appears as **compactification manifold** in type IIB supergravity ($\dim M \leq 9$)



Topic

This talk is an approximation* to the question

$$\boxed{\Omega_k^{IIB} = 0} \qquad \text{for } 1 < k \leq 9 \quad ?$$

For $k = 1$ see Miguel's talk

and **if not**, **what are the defects** we need to include **that break** the associated **global symmetry**

*(only spacetime manifold and duality bundle as well as their mixing,
but no RR-fluxes, …)

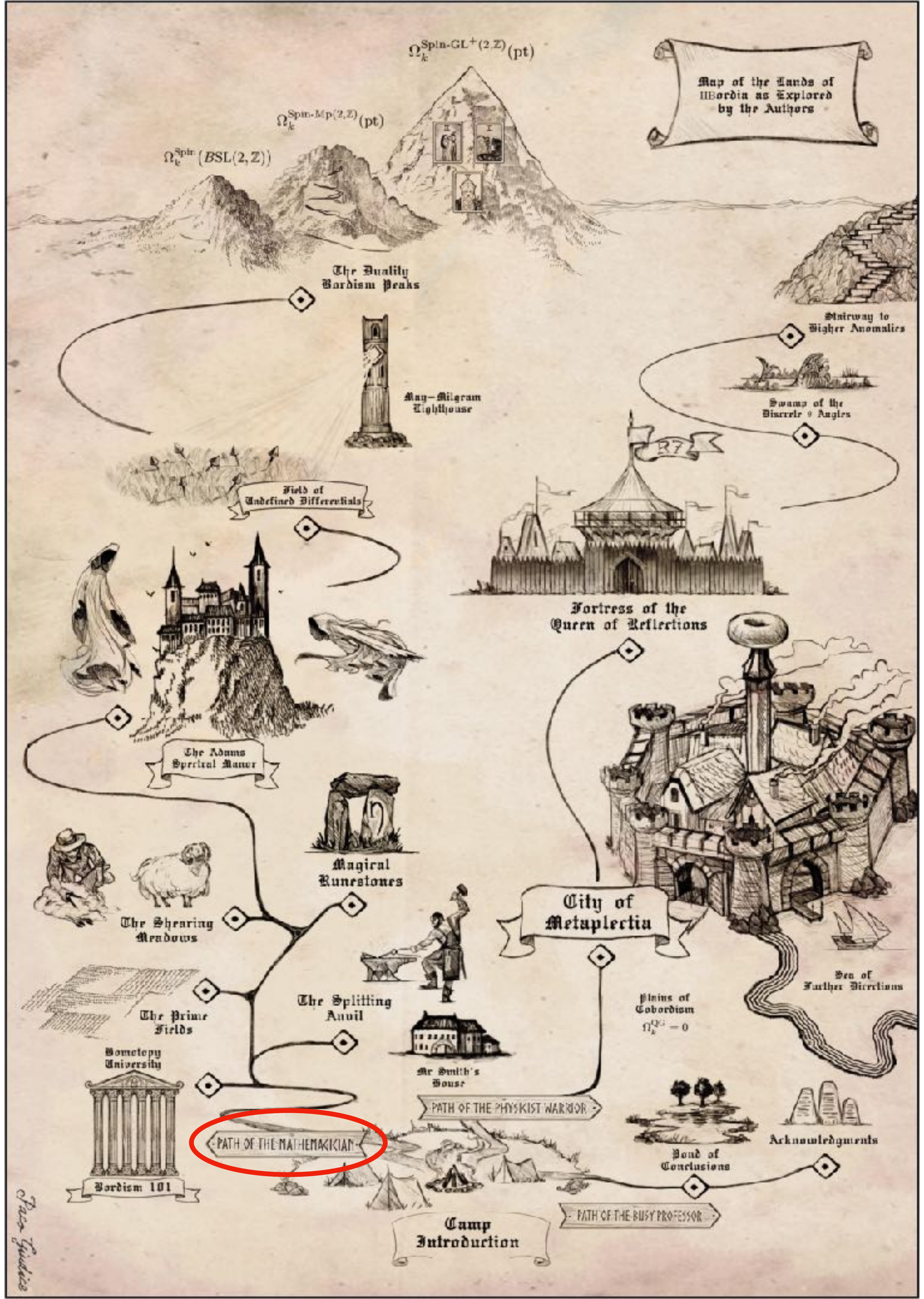
Bordism groups

So the groups we are interested in are:

$$\Omega_k^{Spin}\left(BSL(2,\mathbb{Z})\right)$$

$$\Omega_k^{Spin-Mp(2,\mathbb{Z})}(pt)$$

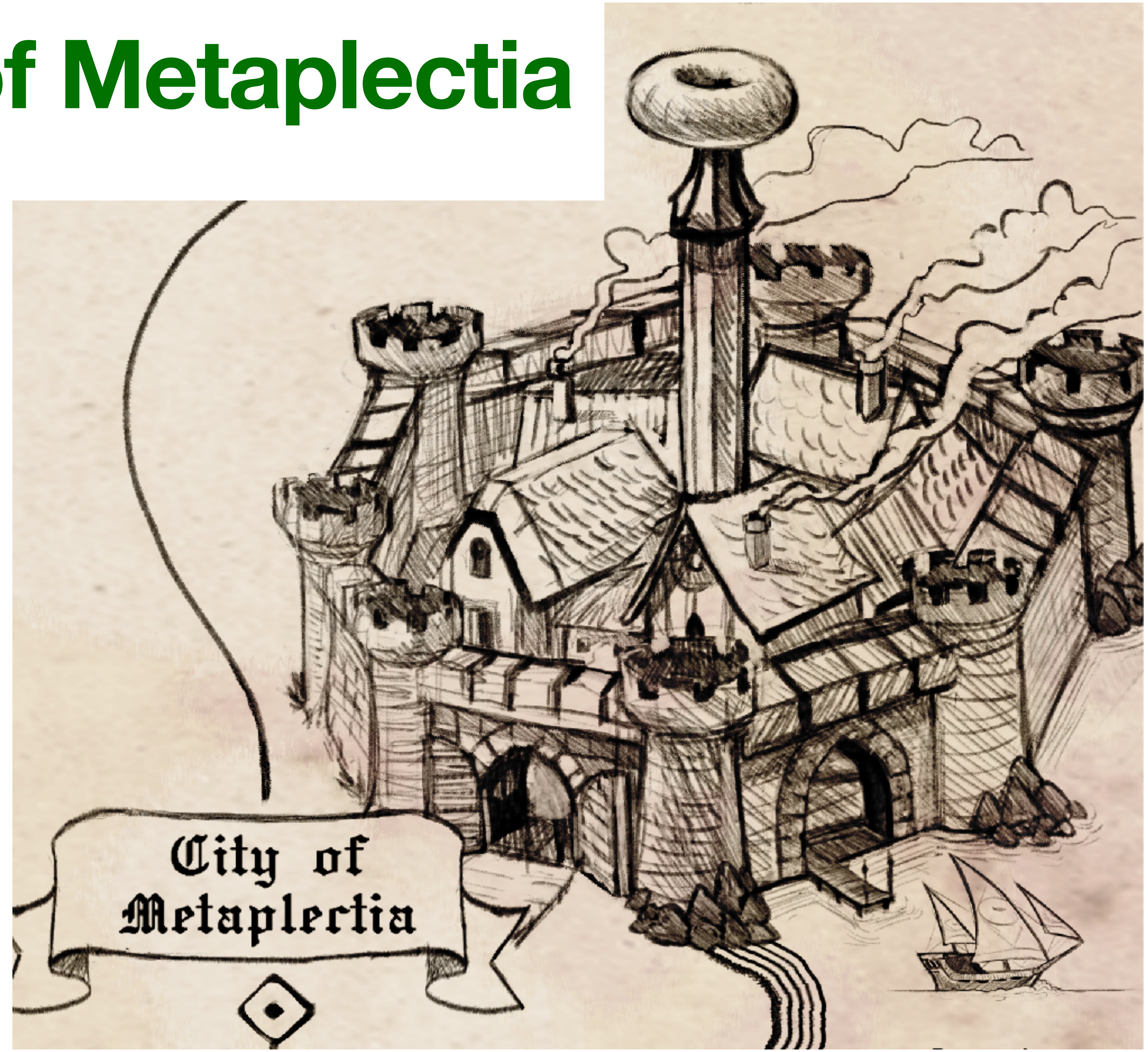
$$\Omega_k^{Spin-GL^+(2,\mathbb{Z})}(pt)$$



$\Omega_k^{\text{Spin-GL}^+(2,\mathbb{Z})}(\text{pt})$
$\Omega_k^{\text{Spin-Mp}(2,\mathbb{Z})}(\text{pt})$
$\Omega_k^{\text{Spin}}(BSL(2,\mathbb{Z}))$
Map of the Lands of Bordia as Explored by the Authors
The Duality Bordism Peaks
Stairway to Higher Anomalies
May–Milgram Lighthouse
Swamp of the Discrete θ Angles
Field of Undefined Differentials
Fortress of the Queen of Reflections
The Adams Spectral Manor
Magical Runestones
City of Metaplectia
The Shearing Meadows
Sea of Further Directions
Plains of Cobordism
$\Omega_k^{QG} = 0$
The Prime Fields
The Splitting Anvil
Homotopy University
Mr Smith's House
PATH OF THE PHYSKIST WARRIOR
PATH OF THE MATHEMAGICIAN
Bordism 101
Pond of Conclusions
Acknowledgments
PATH OF THE BUSY PROFESSOR
Camp Introduction

d	$\Omega_d^{\mathrm{Spin}}(B\mathrm{SL}(2,\mathbb{Z}))$	$\Omega_d^{\mathrm{Spin\text{-}Mp}(2,\mathbb{Z})}$	$\Omega_d^{\mathrm{Spin\text{-}GL}^+(2,\mathbb{Z})}$
	§4	§5	§6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

The Rise of Metaplectia

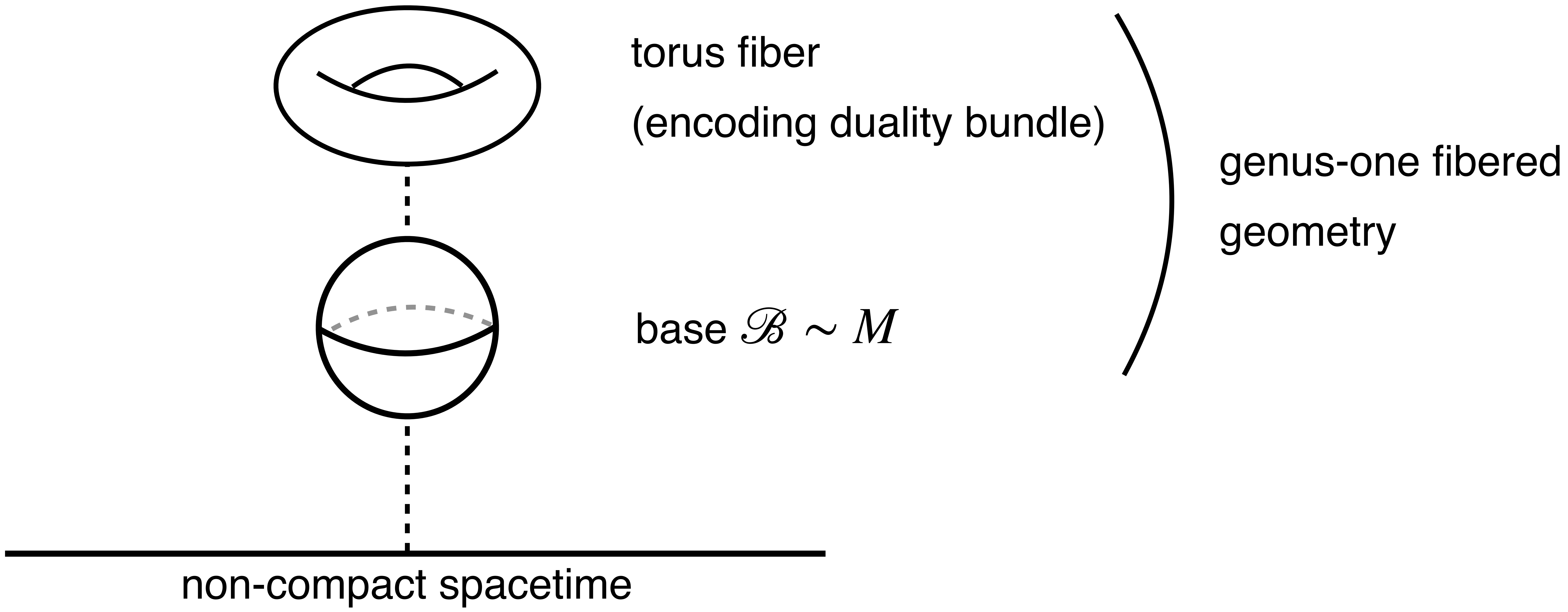


Spin-Mp(2,Z) manifolds

$Spin - Mp(2,\mathbb{Z})$ structure $\sim$ $Spin$ structure on **torus fibration**

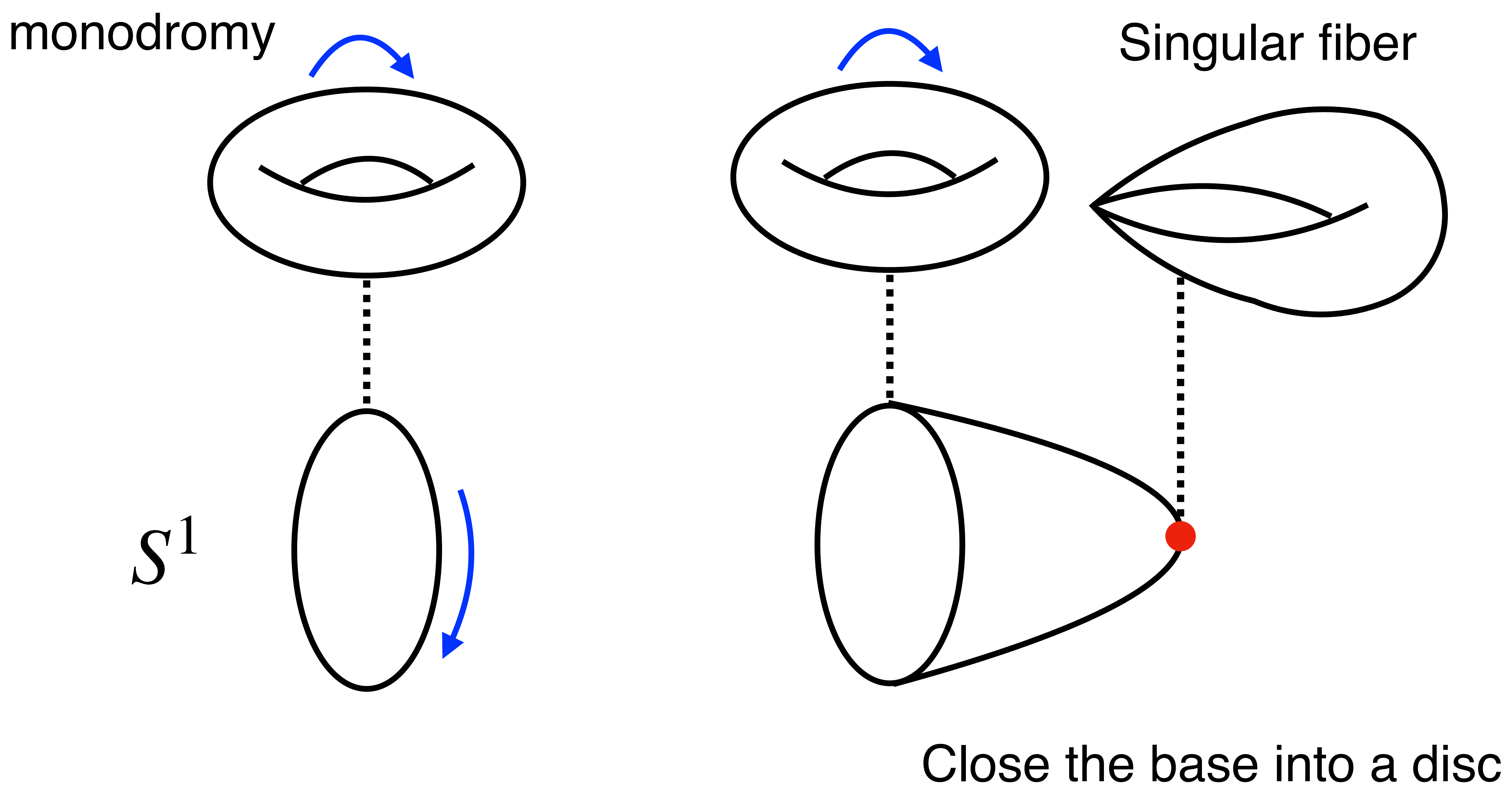
➔ F-theory [Vafa '96], [Morrison, Vafa '96]

(Type IIB with varying axio-dilaton encoded in an auxiliary torus)



$$\dim_{\mathbb{R}} \mathscr{B} = 1$$

$$\Omega_1^{Spin-Mp(2,\mathbb{Z})}(pt) = \mathbb{Z}_{24} = \mathrm{Ab}\big(Mp(2,\mathbb{Z})\big)$$



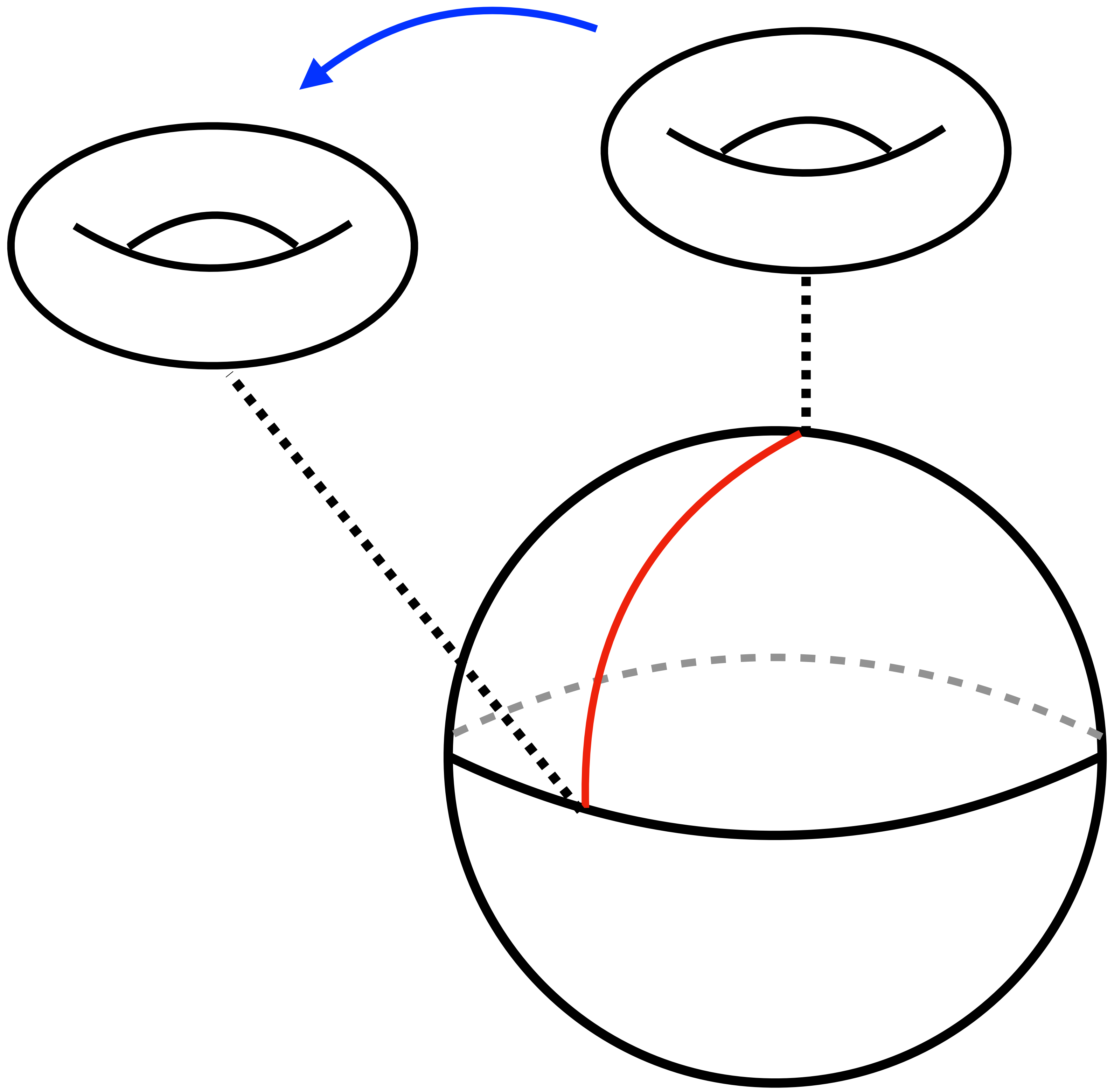
→ **defects are [p,q]-7-brane stacks**

all associated global symmetries broken after inclusion of 7-branes

See also [MD, Heckman '20]

$\dim_{\mathbb{R}} \mathscr{B} = 3$

$$\Omega_3^{Spin-Mp(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_3$$



$L_n^3 = S^3/\mathbb{Z}_n$

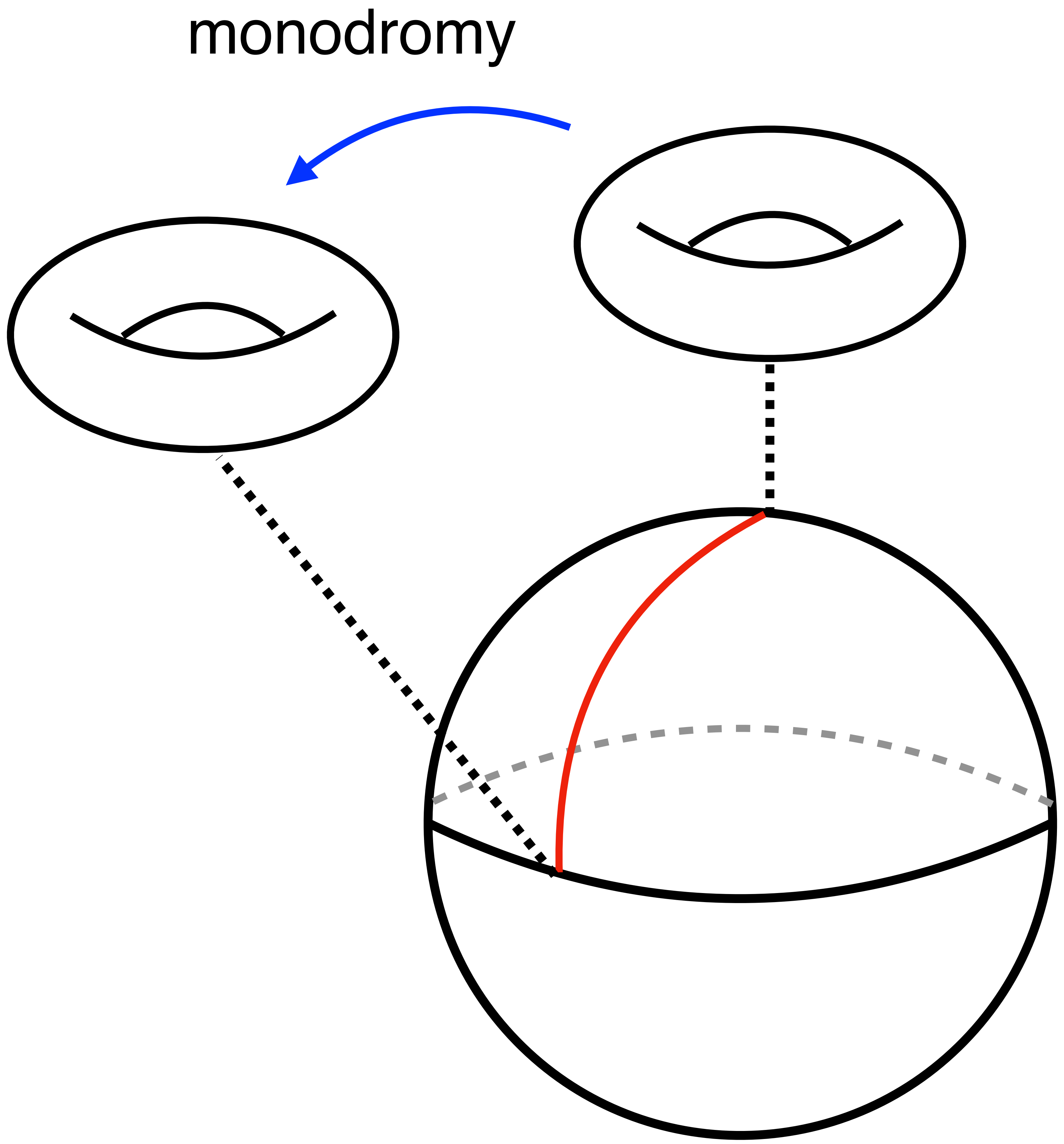
Transition function around non-trivial (torsion) 1-cycles

Use Hopf fibration:

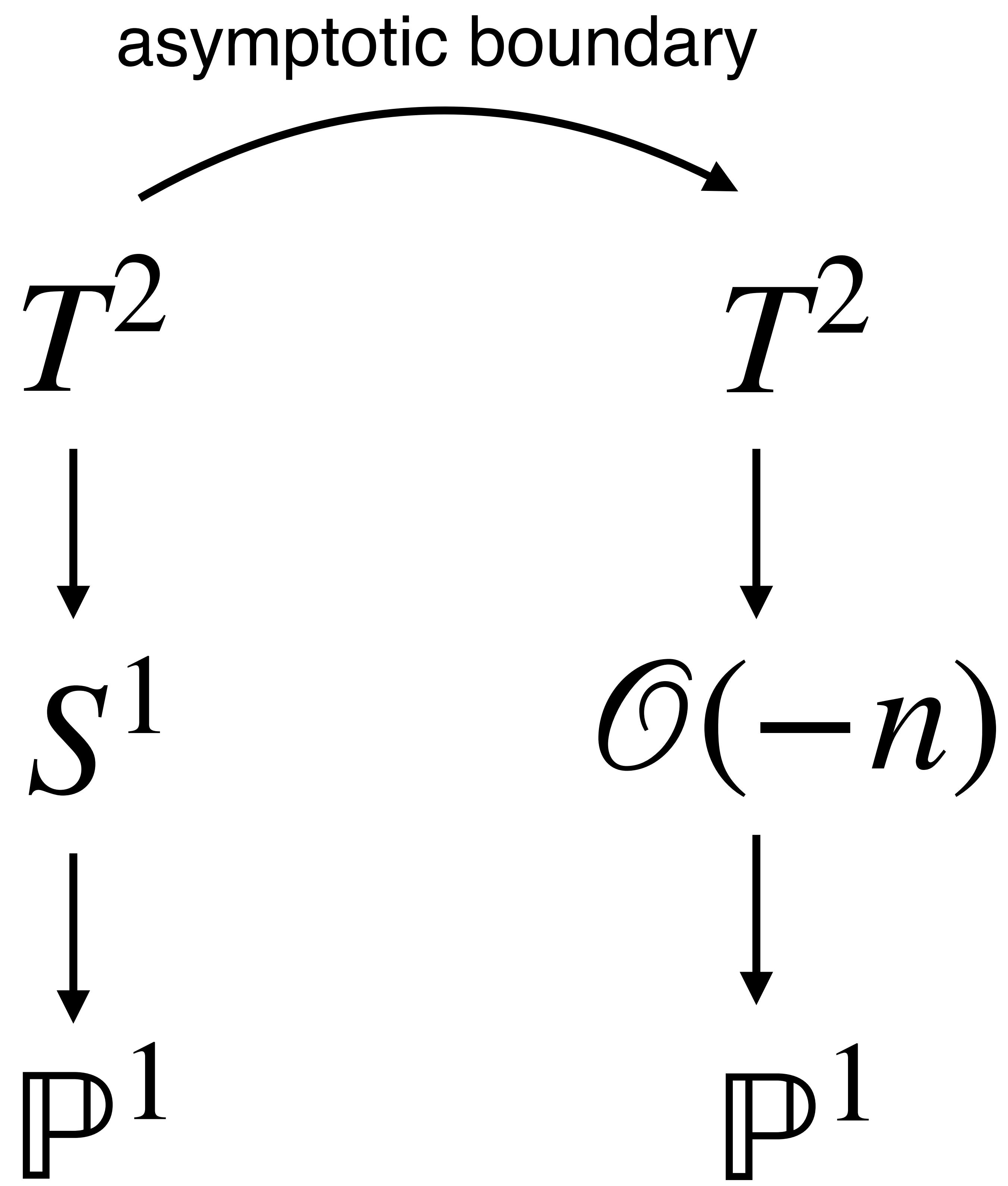
$$\begin{array}{ccc} S^1 & \hookrightarrow & S^3 \\ & & \downarrow \\ & & \mathbb{P}^1 \end{array}$$

$\dim_{\mathbb{R}} \mathscr{B} = 3$

$$\Omega_3^{Spin-Mp(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_3$$



$L_n^3 = S^3/\mathbb{Z}_n$



Non-Higgsable clusters

[Morrison, Taylor '12]

$$\begin{array}{c} T^2 \\ \downarrow \\ \mathcal{O}(-n) \\ \downarrow \\ \mathbb{P}^1 \end{array}$$

is a resolution of $(T^2 \times \mathbb{C}^2)/\mathbb{Z}_n$

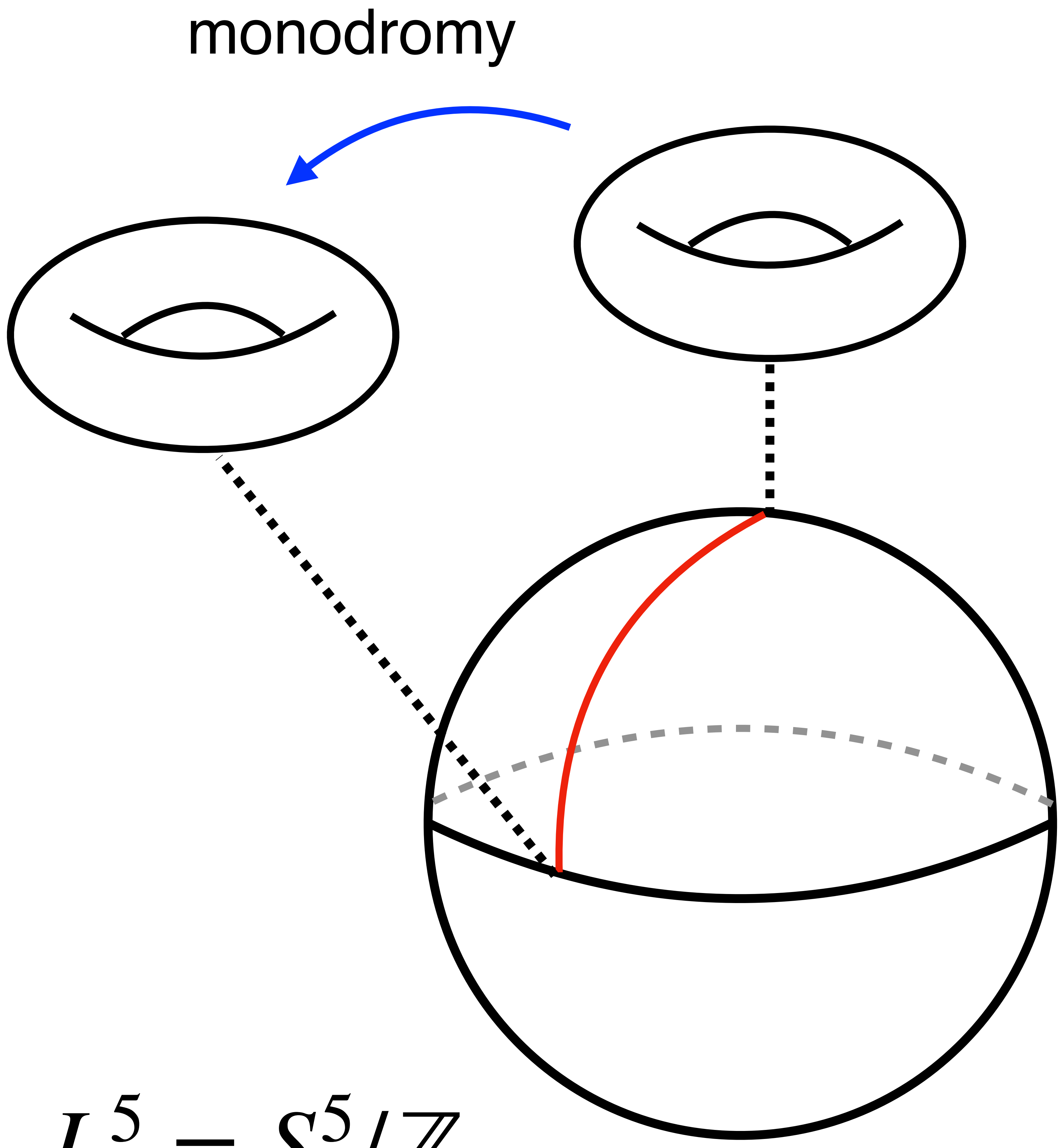
➡ **defects are non-Higgsable clusters**

Very important for the construction of **6d SCFTs**

[Heckman, Morrison, Vafa '13],…

$\dim_{\mathbb{R}} \mathscr{B} = 5$

$\Omega^{Spin-Mp(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$



$L_n^5 = S^5/\mathbb{Z}_n$

as boundary of

$$(T^2 \times \mathbb{C}^3)/\mathbb{Z}_n$$

with non-trivial duality bundle

→ defects are S-folds

[Garcia-Etxebarria, Regalado ’15]

Lead to ‘exotic’ $\mathcal{N} = 3$ theories in 4d

Spin-Off

[Debray, MD, Heckman, Montero to appear]

$$\Omega_5^{Spin-Mp(2,\mathbb{Z})}(pt) \supset \mathbb{Z}_{32} \oplus \mathbb{Z}_2$$

Both are generated by lens space

with **different** $Spin - \mathbb{Z}_8$ structure

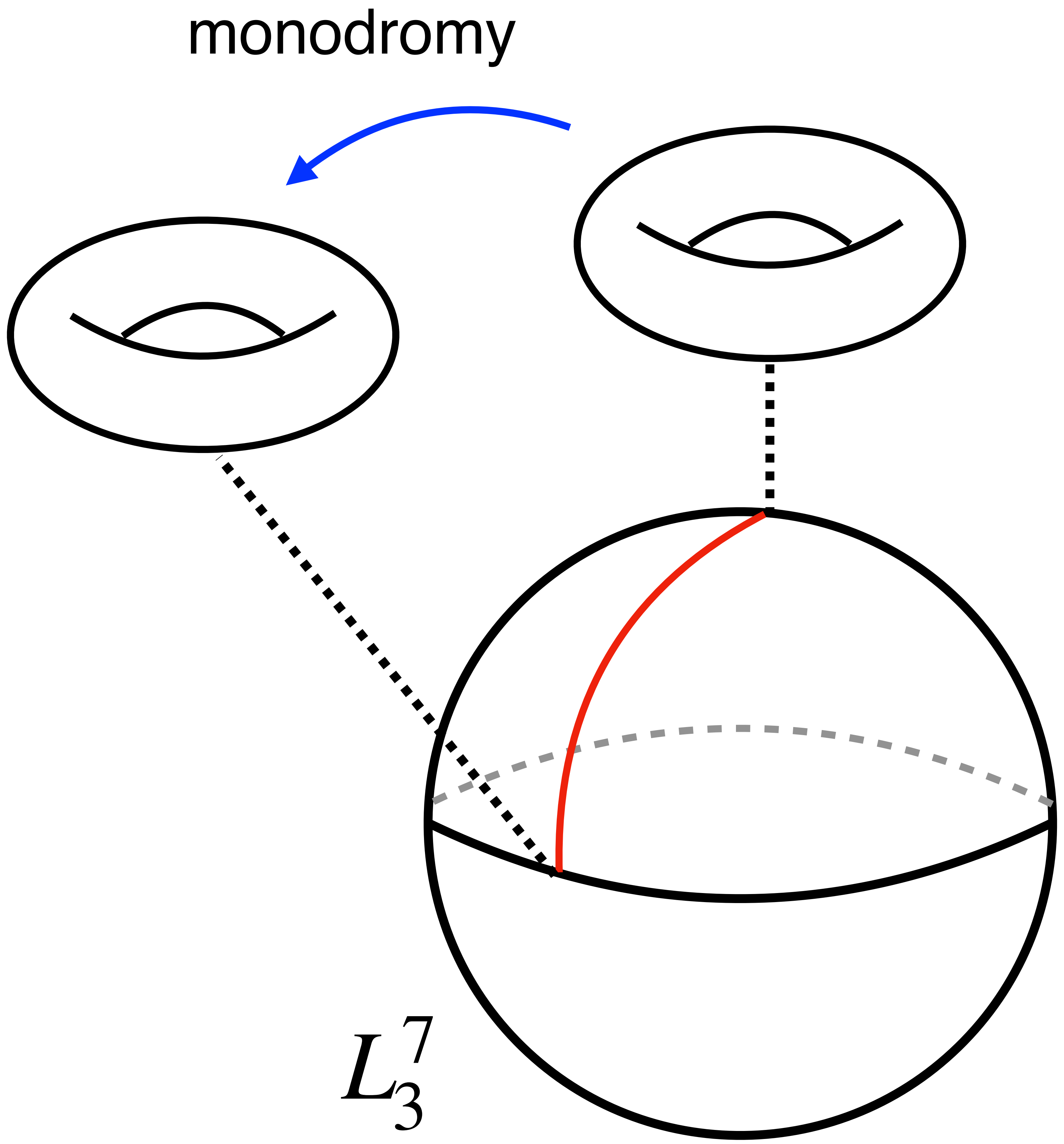
$$L_4^5 \cong S^5/\mathbb{Z}_4\,, \quad \widetilde{L}_4^5$$

$\mathcal{N} = 3$ $\qquad$ $\mathcal{N} = 1$

F-theory lift of certain M-theory configurations [Bobev, Bomans '21]

(what about central charges?)

$\dim_{\mathbb{R}} \mathscr{B} \in \{7, 9\}$



L_3^7

$L_3^9, L_4^9, \widetilde{L}_4^9$

as boundary of

$(T^2 \times \mathbb{C}^4)/\mathbb{Z}_n$ and $(T^2 \times \mathbb{C}^5)/\mathbb{Z}_n$

with non-trivial duality bundle

new type IIB backgrounds

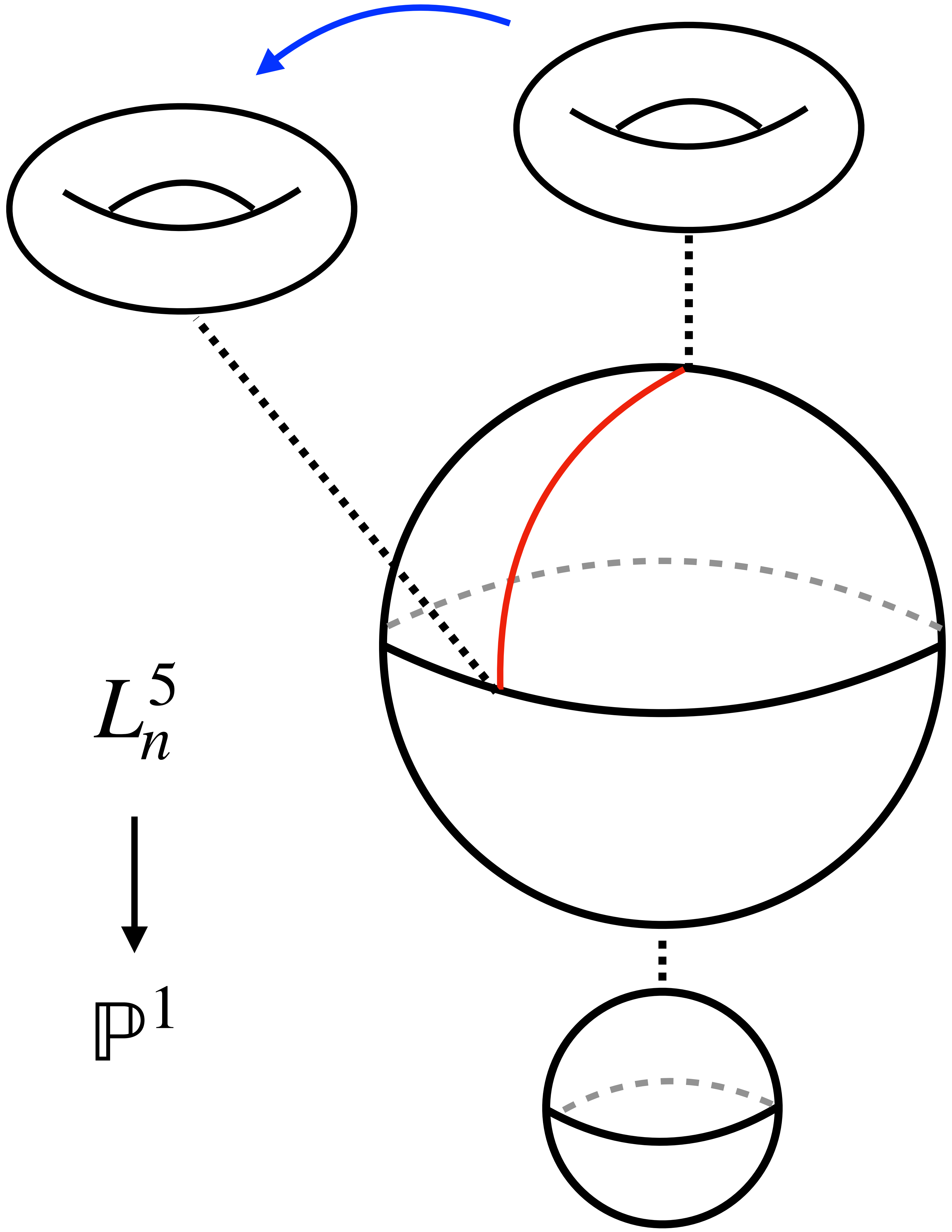
S-strings

S-instantons

Exotic (8,2) SUSY?

Topological twists

another generator for $k = 7$ given by Q_4^7:



We know how to bound L_4^5

➡ wrap defect on $\mathbb{P}^1$

The specific fibration structure induces **topological twist** that **preserves** part of **supersymmetry**

What about even k?

Many generator inherited from $\Omega_k^{Spin}(pt)$, e.g., B , $\mathbb{HP}^2$ for $k = 8$

With some interesting **refinements** in $k = 4$:

$$\Omega_4^{Spin}(pt) = \mathbb{Z}_{[K3]} \qquad \text{vs} \qquad \Omega_4^{Spin-Mp(2,\mathbb{Z})}(pt) = \mathbb{Z}_{[E]}$$

Generated by Enriques Calabi-Yau:

$$(T^2 \hookrightarrow E) \sim CY_E \sim (T^2 \times K3)/\mathbb{Z}_2$$

[Voisin '93], [Ferrara, Harvey, Strominger, Vafa '95], [Grimm '07]

Summary Spin-Mp(2,Z)

- All defects **preserve part of the supersymmetry**
- Nice interpretation in terms of known and new **F-theory backgrounds:**
  - **non-Higgsable clusters** $\longrightarrow$ **6d SCFTs**
  - **S-folds** $\longrightarrow$ $\mathcal{N} = 3$ **theories**
  - **S-strings** and **S-instantons**
  - **Topological twists**

The return of the Pin (-structure)

include elements with determinant (-1)

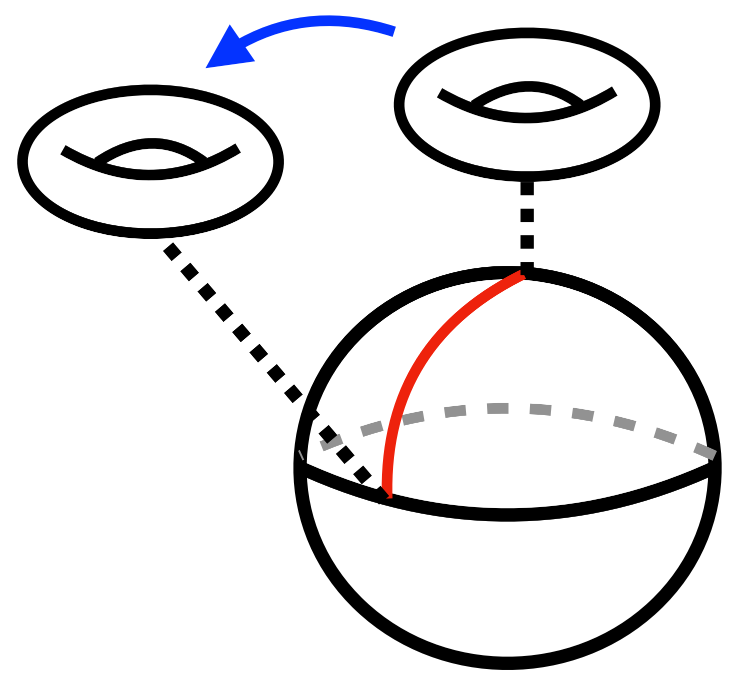
$$\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(-1)^{F_L} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\in GL(2,\mathbb{Z})$$

and their lift to $GL^+(2,\mathbb{Z})$

Orientifolds



One generator given by $S^3/\mathbb{Z}_2 \sim \mathbb{RP}^3$ with Ω monodromy

Boundary of: $\mathbb{C}^2/(\mathbb{Z}_2)_\Omega$ ➡ 'O5-plane'

equivalently given by the S-dual of the O5 with $(-1)^{F_L}$ monodromy

[Hanany, Kol '00]

Caveat: our bordism analysis does not include the RR-fields and therefore is insensitive to RR-fluxes

Another generator given by $S^3/\mathbb{Z}_2 \sim \mathbb{RP}^3$ with ΩS monodromy

Boundary of: $\mathbb{C}^2/(\mathbb{Z}_2)_{\Omega S}$

Can be generated as R7-brane and [p,q]-7-branes on collapsing (-2) curve

What about even k?

Many generator inherited from $\Omega_k^{Spin(-Mp(2,\mathbb{Z}))}(pt)$, e.g., E

Again with some **interesting refinements** (now involving reflections)

$$\Omega_8^{Spin}(pt) \supset \mathbb{Z}_{[B]} \qquad \text{vs} \qquad \Omega_4^{Spin-GL^+(2,\mathbb{Z})}(pt) \supset \mathbb{Z}_{[\frac{1}{2}B]}$$

Half a Bott/Spin(7) manifold in 8d (Dirac index is $\frac{1}{2}$), not $Spin^c$

Other classes

- similar to Spin-Mp now involving reflections
  - e.g. $\mathbb{RP}^{3+4m}$ (with different embedding of reflections)
  - non-trivial fibrations (Arcana, $\mathbb{Z}_2$ quotient of Q_4^7,…)
  - several Spin-Mp manifolds that survive (7-branes, NHC,…)
  - odd-dimensional generators can **all** be **bound by inclusion of R7-branes** and other prior defects
- some related to orientifold planes

d	$\Omega_d^{\text{Spin}}(B\text{SL}(2,\mathbb{Z}))$	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$
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8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

Duality anomalies

[Debray, MD, Heckman, Montero '21]

Potential duality(-gravity) anomalies are classified by

$$\Omega_{11}^{Spin-GL^{+}(2,\mathbb{Z})}(pt) = \mathbb{Z}_2^{\oplus 9} \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$$

Evaluate anomaly theory $\mathscr{A}$ on a set of generators shows that type IIB

is anomalous

Can be cancelled by:

- Topological Green-Schwarz (new TFT sectors)
  [Garcia-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura '17]
- Discrete Green-Schwarz (involving C_4)
  See also [MD, Oehlmann, Schimannek '22] in 6d

The beauty of generalized Green-Schwarz

[Debray, MD, Heckman, Montero '21]

All the (calculable) **anomalies canceled** if we include **additional topological coupling via quadratic refinement term** to duality and gravitational background:

$$\tilde{\mathcal{Q}}(\breve{c}_0) \text{ with } \breve{c}_0 = \left(\lambda_1 \beta(a)^2 + \lambda_2 \frac{(p_1)_3}{2}\right) \cup a + \frac{\lambda_3}{2}[(p_1)_4 - \mathcal{P}(w_2)] \cup b + \kappa \beta(b)^2 \cup b$$

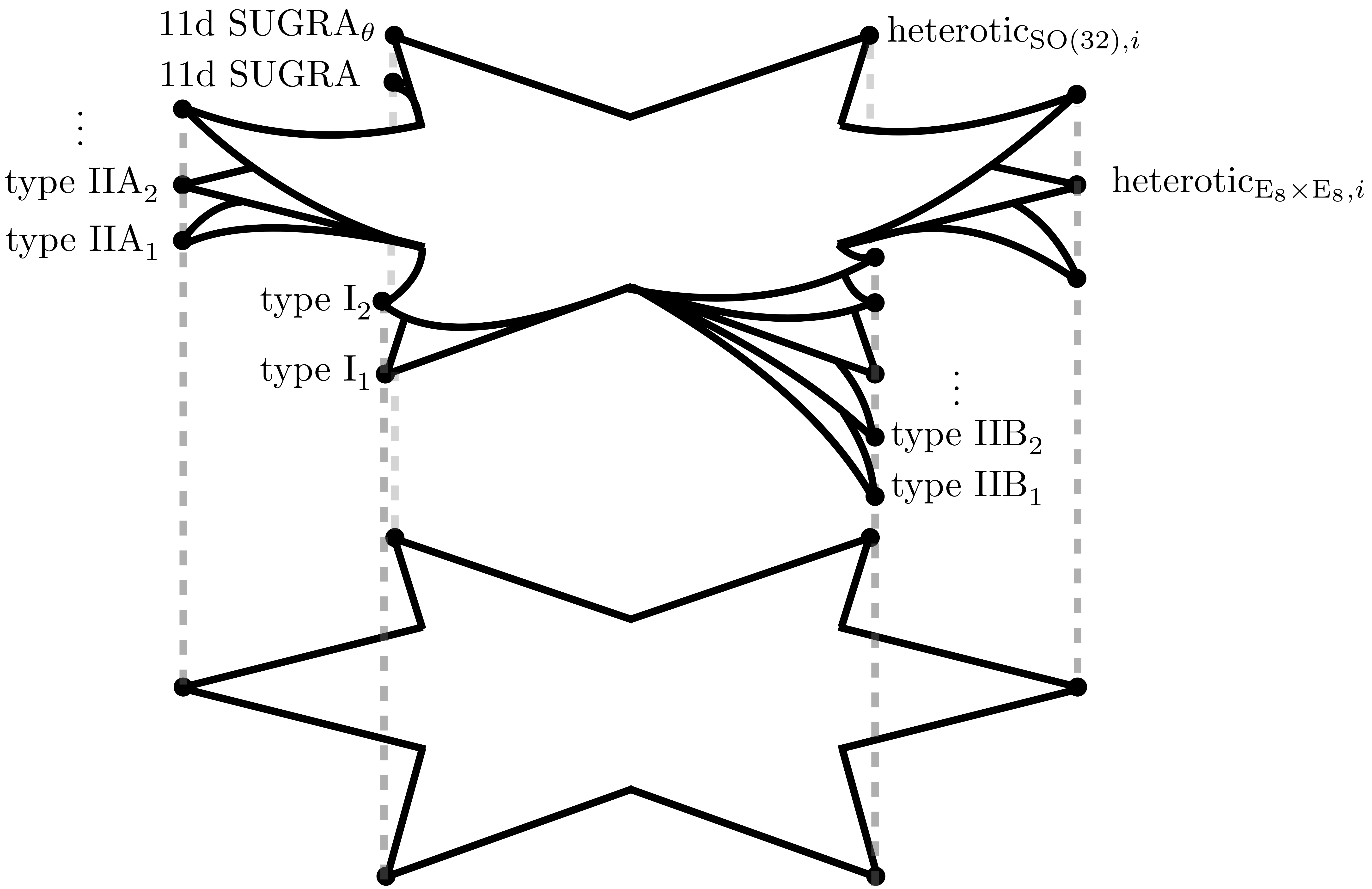
$\lambda_i \in \{-1, +1\}\,,\ \ \kappa \in \mathbb{Z} \bmod 4$ (other physical systems (S-folds) suggest $\lambda_{1,3} = 1$)

This is a coupling of the form: $(C_4, \breve{c}_0) \approx F_5 \wedge \ldots$

Discrete Landscape or topological Swampland?

Two possibilities:

- Topological GS and θ angles in the Swampland
  Why?
- Alternative consistent UV completions
  Discrete Landscape



Conclusions

- The investigation of **bordism groups** can have **very general** and **very surprising** results for theories of quantum gravity
- Applied to **type IIB string theory** and its **duality** we find**:**
  - **(Re)discovery of various objects (7-branes, NHC, S-folds) in Spin-Mp**
  - **New backgrounds (including R7-branes) for Spin-GL**
  - **Topological terms and anomalies**

➛ **Tests and prediction of Cobordism Conjecture**

Outlook (just the beginning)

- **Investigate these new backgrounds:**
  - New S-folds [Debray, MD, Heckman, Montero, to appear]
  - Ⅎ-theory (F-theory with reflections)
- **Investigate other symmetries/dualities:**
  - T-duality
  - R-symmetries
- Track these **defects under dualities** to other string theories

The anomaly theory

[Hsieh, Tachikawa, Yonekura '20], see also [Freed, Moore, Segal '06], [Belov, Moore '06]

$$\mathcal{A}(X) = \underbrace{\eta_1^{\mathrm{RS}}(X) - 2\eta_1^{\mathrm{D}}(X) - \eta_{-3}^{\mathrm{D}}(X)}_{\text{fermions}} - \underbrace{\tfrac{1}{8}\eta_-^{\mathrm{sig}}(X) + \mathrm{Arf}(X) - \tilde{\mathcal{Q}}(\breve{c})}_{\text{4-form}}$$

- Contribution from **signature operator to index theorem** $\eta_-^{\mathrm{sig}}(X)$
- **Requires the introduction of quadratic refinement** $\tilde{\mathcal{Q}}$ of bilinear pairing in differential cohomology

$$\mathrm{Arf}(\tilde{\mathcal{Q}}) = \frac{1}{2\pi}\arg\Big(\sum_{a\in A} e^{2\pi i \tilde{\mathcal{Q}}(a)}\Big)$$

$\tilde{\mathcal{Q}}(\breve{c})$: coupling to background, e.g. $C_4 \wedge F_3 \wedge H_3$ **not** considered here

Physical assumption: there is a **canonical choice for** $\tilde{\mathcal{Q}}$

‘Miraculous’ cancellation

Remember how chiral type IIB spectrum **just right** to have **no anomalies**
[Alvarez-Gaume, Witten ’84]

Let us focus on $\mathbb{Z}_{27}$ generated by $S^{11}/\mathbb{Z}_3$

Quadratic refinement is given by: $\widetilde{Q}(n) = \frac{1}{3}n^2 = \frac{1}{3} \bmod \mathbb{Z}$

Anomaly can be cancelled this way only if:

$$\mathcal{A} - \widetilde{Q} = 0 \bmod \mathbb{Z}$$

$$\mathcal{A} \bmod \mathbb{Z} = \frac{k}{27} = \frac{1}{3}$$

→ **precisely what is realized**
(Chance 1 in 26; similar for the others)

What to expect?

More

There are **more manifolds** and every $Spin - Mp(2,\mathbb{Z})$ manifold is also a $Spin - GL^{+}(2,\mathbb{Z})$ manifold

Less

More deformations that allows the manifolds to bound

both is correct

$k = 3 + 4m$

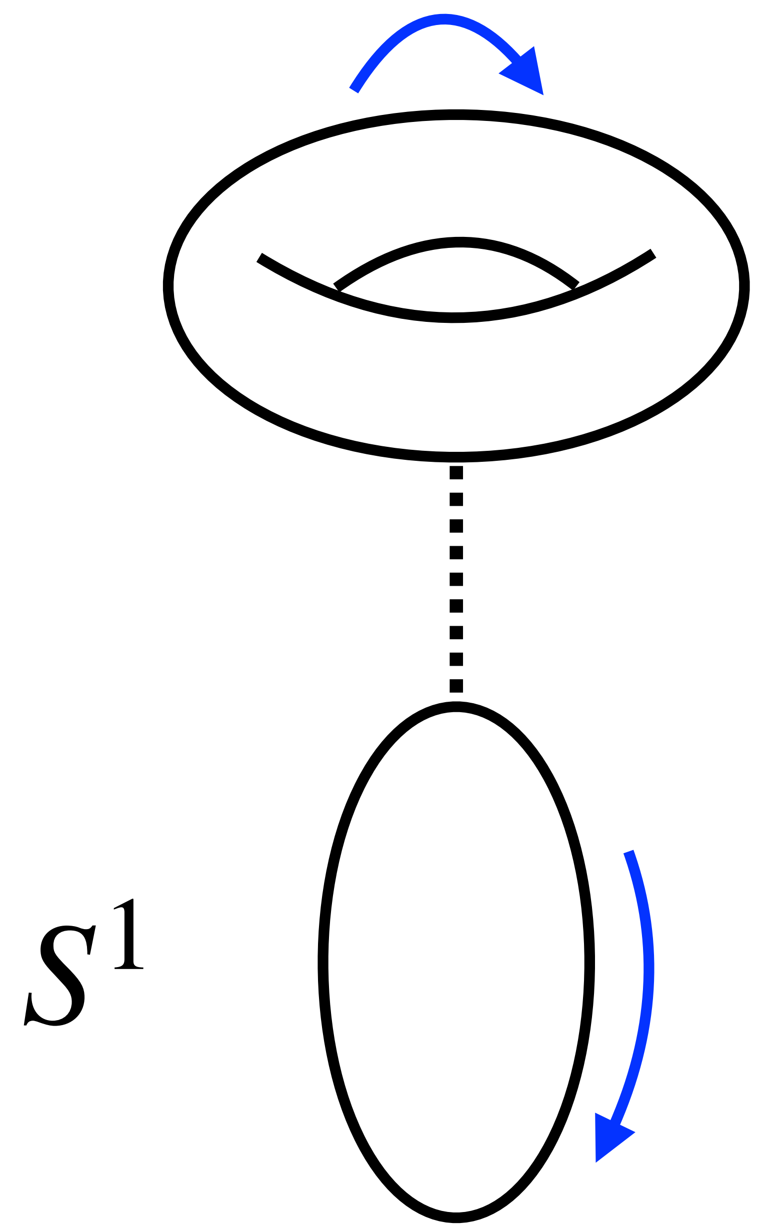
$k = 1 + 4m$

$\dim_{\mathbb{R}} \mathscr{B} = 1$

$$\Omega_1^{Spin-GL^+(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

this is Spin-Mp   this is not

monodromy involving reflection



new object: 7-brane that bounds that bounds the base circle

reflection 7-brane / R7-brane

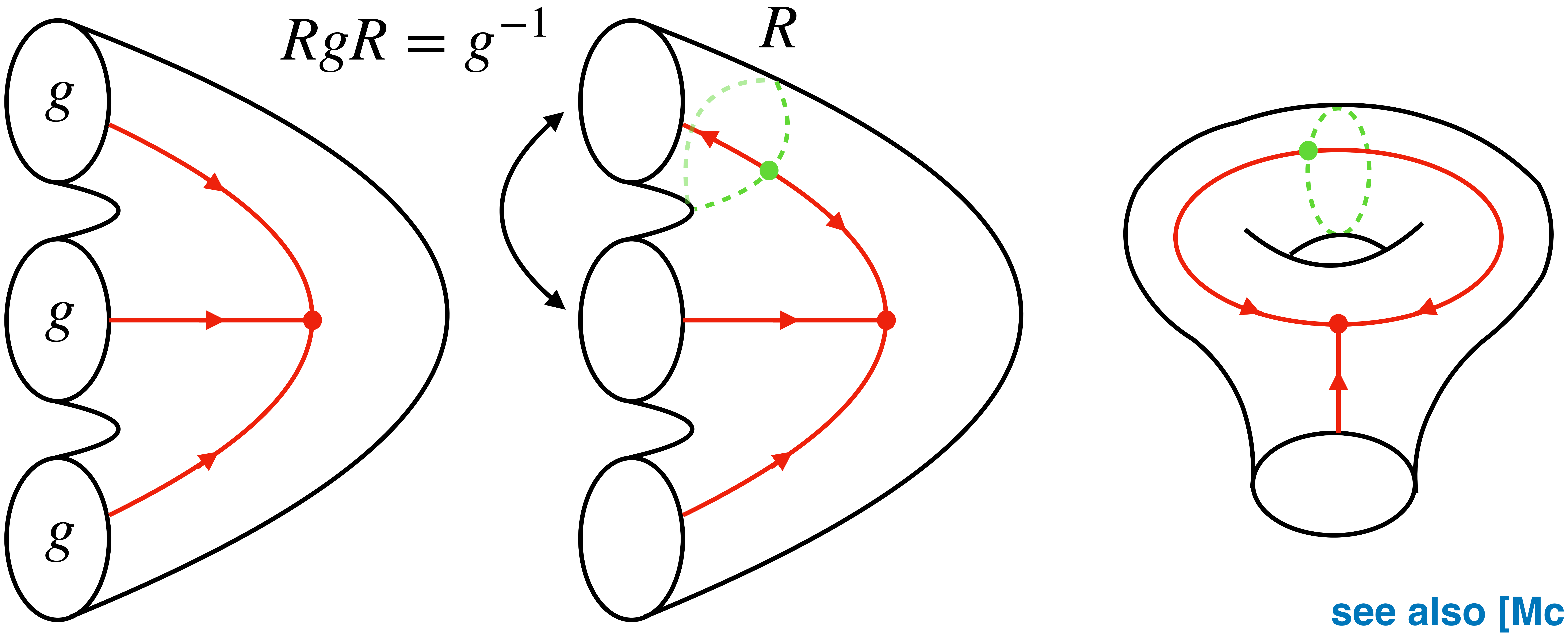
S-dual

Ω-brane (bounds Dabholkar-Park background)

$(-1)^{F_L}$-brane (bounds the asymmetric orbifold background)

Why less?

What happened to the $\mathbb{Z}_3$ brane in $Spin - Mp(2,\mathbb{Z})$?



see also [McNamara '21]

Also happens in $k = 1 + 4m$ (characteristic bundle class odd under R)