Strings and Geometry 2023
University of Pennsylvania
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4d SCFTs in different guises: **a 6d reveal**

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Based on arXiv:2203.08829[Jacques Distler, MJK, Craig Lawrie]
2212.11983 [J. Distler, Grant Elliot, MJK, C. Lawrie]

When do two QFTs describe the same physics?

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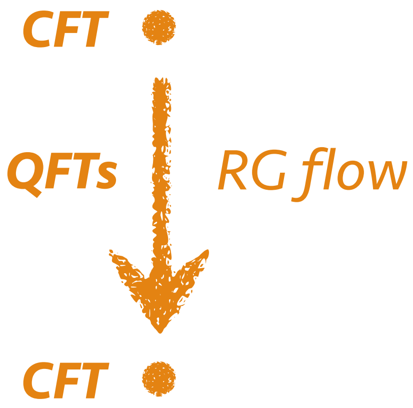

- If they have the same local operator spectrum and all n-point correlation functions

That is not tangible?!

Let’s add conformal symmetry — exists at the fixed points of RG flows between QFTs

*When do two **CFTs** describe the same physics?*

- If they have the same conformal data $\{\Delta_i \,,\, \lambda_{ijk}\}$

  2-pt correlations *3-pt correlations*

  Still not tangible!

- If they have the same "invariants"?

  What is the minimal set of invariants that can distinguish any pair of CFTs?

- Higher-dimensional origins help!

Example: $\mathcal{N} = 4$ SYM

- Lagrangian:

$$\mathscr{L} = \frac{\mathrm{Im}[\tau]}{4\pi}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi_i D^\mu\phi_i + \frac{1}{2}\left[\phi_i, \phi_j\right]\left[\phi_i, \phi_j\right]\right) - \frac{\mathrm{Re}[\tau]}{4\pi}F_{\mu\nu} \star F^{\mu\nu} + \mathscr{L}_{fermions}$$

with complexified coupling $\tau = \dfrac{\theta}{2\pi} + \dfrac{4\pi i}{g^2}$.

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- Are the theories $\left(SU(2),\tau\right)$ and $\left(SO(3),-1/\tau\right)$ the same?
  - They have different vector boson spectrum

Example: $\mathcal{N} = 4$ SYM

➡ Lagrangian: (all fields in adjoint rep. of gauge group G)

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- One theory is at weak coupling, while the other is at strong coupling
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Prove this using higher-dimensional origin!

Top-down perspectives give insights via "geometrization"

Compactifications and Geometry

Higher-dimensional theory $\mathbb{R}^D \ (D > d)$

Compactifying space

Geometric & topological data

Made "manifest"

Geometric-engineering

Lower-dimensional theory $\mathbb{R}^d \times M_{D-d}$

Ex) Montonen-Olive $SL(2,\mathbb{Z})$ and $\mathcal{N} = 4$ super Yang—Mills

Complex coupling

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \Rightarrow SL(2,\mathbb{Z})$$

Montonen-Olive $SL(2,\mathbb{Z})$ and $\mathcal{N}=4$ SYM

$$\begin{array}{ccc} \text{6d (2,0) SCFT of type } \mathfrak{g} & & S_{\mathfrak{g}} \\ \big\downarrow & \text{Compactify on } T^2 & \\ \text{4d super Yang—Mills} & & S_{\mathfrak{g}}\langle T^2 \rangle \end{array}$$

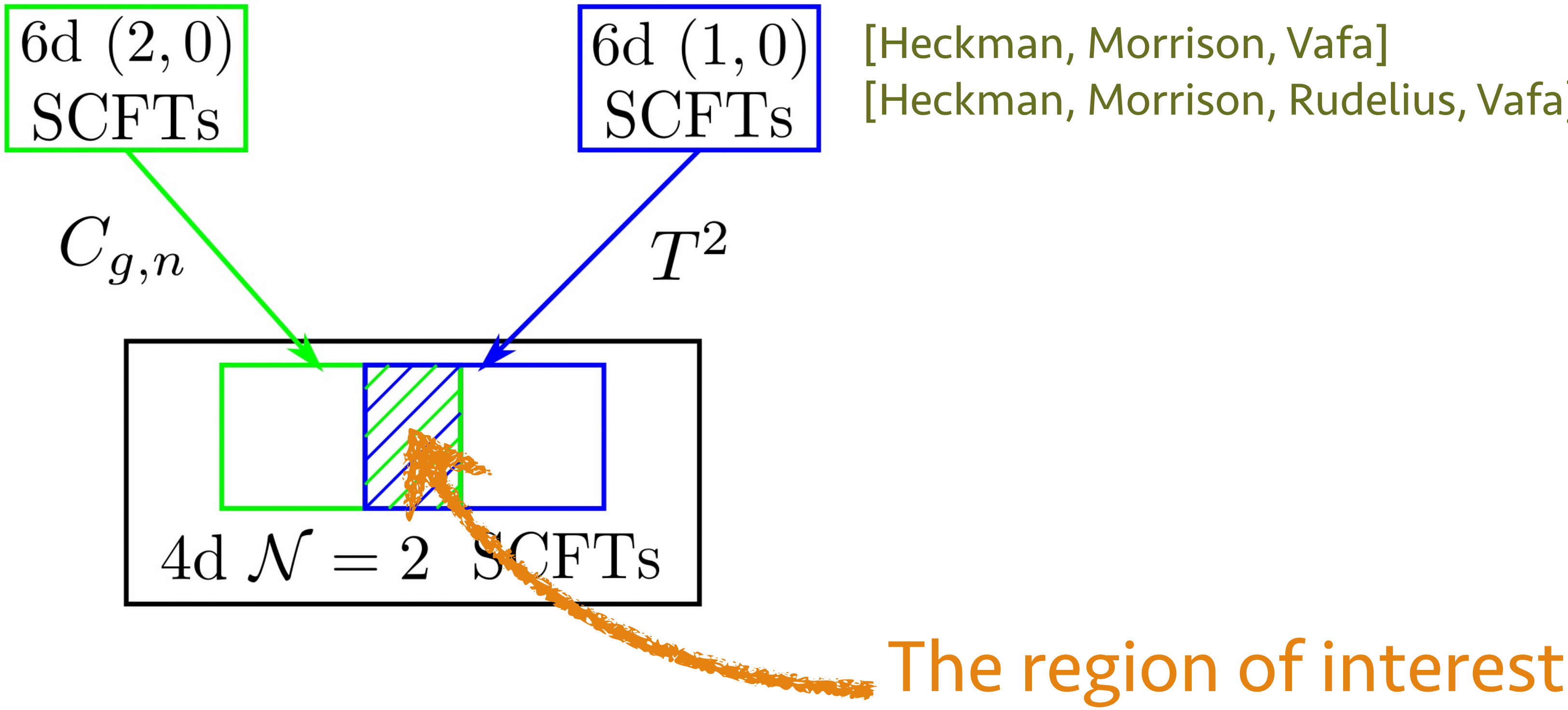
Coupling constant τ $=$ Torus complex structure τ

$SL(2,\mathbb{Z})$ self-duality group

Geometrization of $SL(2,\mathbb{Z})$

Our interest today

[Witten][Gaiotto]
[Gaiotto, Moore, Neitzke]



[Heckman, Morrison, Vafa]
[Heckman, Morrison, Rudelius, Vafa]

[Florent, MJK, Lawrie]

Similarly:
6d (2,0) to 4d $\mathcal{N} = 2$ SCFT (class $\mathcal{S}$)

A twisted compactification on a punctured Riemann surface

of punctures $Y_1, Y_2, \cdots, Y_n$

$$S_{\mathfrak{g}} \quad \langle C_{g,n} \rangle \quad \{Y_1, Y_2, \cdots, Y_n\}$$

6d (2,0) SCFT
of type $\mathfrak{g}$

||

Type IIB on an orbifold $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$
[Witten]

n-punctured, genus g
Riemann surface

puncture data =
codim-2 defects
on 6d (2,0) SCFT

[Gaiotto][Gaiotto,Moore,Neitzke]

Another way:
6d (1,0) to 4d $\mathcal{N} = 2$ SCFT

F-theory

Geometric-engineering $\big\downarrow$ Elliptically-fibered Non-compact CY_3

- *The curves C_i has negative-definite intersection matrix: $A^{ij} = C^i \cdot C^j < 0$.*
- *The singular fiber above the points where any two distinct C_i intersect is minimal.*

[Heckman,Morrison,Vafa]
[Heckman,Morrison,Rudelius,Vafa]

6d (1,0) SCFT with minimal conformal matter (G, G) $\quad : \mathcal{T}_{\mathfrak{g}}\{Y_1, Y_2\}$

Higgsed with $\{Y_1, Y_2\}$ nilpotent orbits

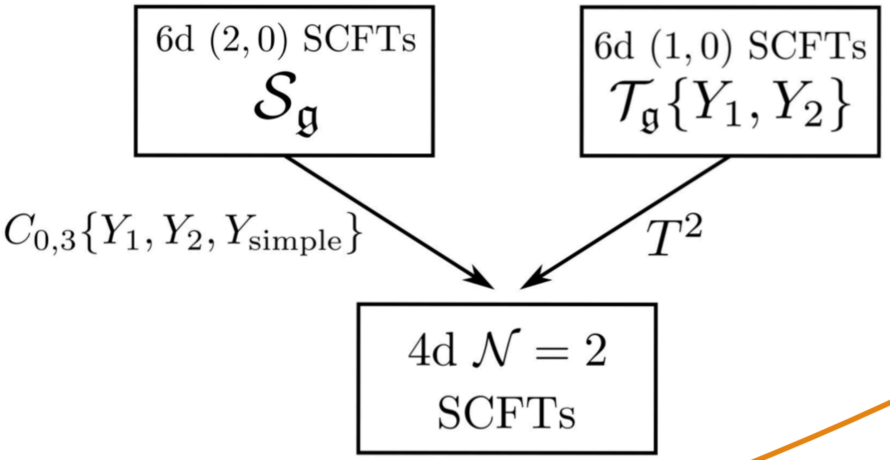
The world volume of an M5-brane probing $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$

[del Zotto,Heckman,Tomasiello,Vafa]

T^2 $\big\downarrow$

4d $\mathcal{N} = 2$ SCFT $\quad : \mathcal{T}_{\mathfrak{g}}\{Y_1, Y_2\}\langle T^2 \rangle$

Two different geometric ways to build 4d $\mathcal{N}=2$ SCFTs:



✓ They can give rise to the same 4d $\mathcal{N}=2$ SCFTs.

$$\mathcal{S}_{\mathfrak{g}}\langle C_{0,3}\rangle\{Y_1, Y_2, Y_{\text{simple}}\} = \mathcal{T}_{\mathfrak{g}}\{Y_1, Y_2\}\langle T^2\rangle$$

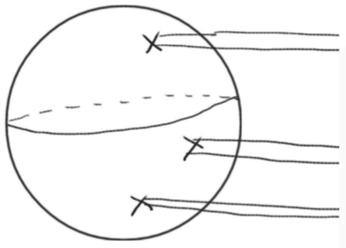
[Ohmori, Shimizu, Tachikawa, Yonekura]
[Del Zotto, Vafa, Xie][Baume, MJK, Lawrie]

Interacting SCFTs arising from 6d (2,0) SCFTs after subtracting any free hypers

✓ The Higgs branch flows of 4d $\mathcal{N}=2$ SCFTs and their parent 6d (1,0) SCFTs are isomorphic.

$$\begin{array}{ccc} \mathcal{S}_{\mathfrak{g}}\langle C_{0,3}\rangle\{Y_1, Y_2, Y_{\text{simple}}\} & \rightarrow & \mathcal{S}_{\mathfrak{g}}\langle C_{0,3}\rangle\{\widetilde{Y}_1, Y_2, Y_{\text{simple}}\} \\ \parallel & & \parallel \\ \mathcal{T}_{\mathfrak{g}}\{Y_1, Y_2\}\langle T^2\rangle & \rightarrow & \mathcal{T}_{\mathfrak{g}}\{\widetilde{Y}_1, Y_2\}\langle T^2\rangle \end{array}$$

[Baume, MJK, Lawrie]



Three-punctured sphere

cf) "tinkertoy" theories

[Chalcataca, Distler, Trim, Zhu]

4d $\mathcal{N} = 2$ "conventional" invariants

- Central charges a and c
  (or equivalently, the number of vectors and hypers)

$$\langle T_{\mu\nu} \rangle = \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} E_4 + \cdots \qquad \begin{cases} n_v = 4(2a - c) \\ n_h = -4(4a - 5c) \end{cases}$$

- The flavor symmetry group + the flavor central charges

$$\langle J^i_\mu(x) J^j_\nu(0) \rangle = \frac{3k_i}{4\pi^4} \delta^{ij} \frac{x^2 \eta_{\mu\nu} - 2x_\mu x_\nu}{x^8}$$

- Coulomb branch operator spectrum
- Schur Index and Hall—Littlewood Index (up to computable order)

4d $\mathcal{N} = 2$ "conventional" invariants

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- ➡ Coulomb branch operator spectrum
- ➡ Schur Index and Hall—Littlewood Index

Would these invariants uniquely identify the theory?

In low-ranks, these data (or even subsets) suffices to uniquely characterize the theory.

Example: Minahan–Nemeschansky

➡ A rank-one 4d $\mathcal{N} = 2$ SCFT with Coulomb branch generators

$$\Delta = 6 \quad \text{and} \quad (a, c) = \left(\frac{95}{24}, \frac{31}{6}\right)$$

is unique. It must be the $(E_8)_{12}$ Minahan–Nemeschansky theory.

[Argyres, Lotito, Lu, Martone]

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$\Longrightarrow$ *They are all isomorphic*

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- *There are a multitude of distinct realizations of this SCFT in class S*

$\Longrightarrow$ *They are all isomorphic*

- For higher rank cases, these "conventional invariants" are not sufficient [Distler,MJK,Lawrie] [Distler, Elliott]

$\Longrightarrow$ *Use higher-dimensional perspective*

6d (1,0) “conventional” invariants

- ➡ Anomaly polynomial

$$I_8 = \frac{\alpha}{24} c_2(R)^2 + \frac{\beta}{24} c_2(R) p_1(T) + \frac{\gamma}{24} p_1(T)^2 + \frac{\delta}{24} p_2(T) \\ + \sum_a \mathrm{Tr} F_a^2 \left(\kappa_a p_1(T) + \nu_a c_2(R) + \sum_b \rho_{ab} \mathrm{Tr} F_b^2 \right) + \sum_a \mu_a \mathrm{Tr} F_a^4$$

- ➡ The continuous flavor symmetry group
- ➡ The discrete flavor symmetry group

When Higgsing $(\mathfrak{so}_{4k}, \mathfrak{so}_{4k})$ conformal matter by two very-even nilpotent orbits of $\mathfrak{so}_{4k}$, the two SCFTs appear to differ by a discrete $\mathbb{Z}_2$ symmetry.

[Distler, MJK, Lawrie]

Use the 6d (1,0) perspective to answer when two 4d class S theories are isomorphic

Which theories to consider?

[Distler, Elliot, MJK, Lawrie]

Consider pairs of class S theories where

- ➡ the **genus** of the Riemann surface is the same ,
- ➡ the **6d (2,0) origin** is the same
- ➡ **all but two of the n punctures** are the same.

When are these theories isomorphic?

[Distler, Elliot, MJK, Lawrie]

$$S_{\mathfrak{g}}\langle C_{g,n}\rangle\{O_1, O_2, \cdots, O_n\}$$

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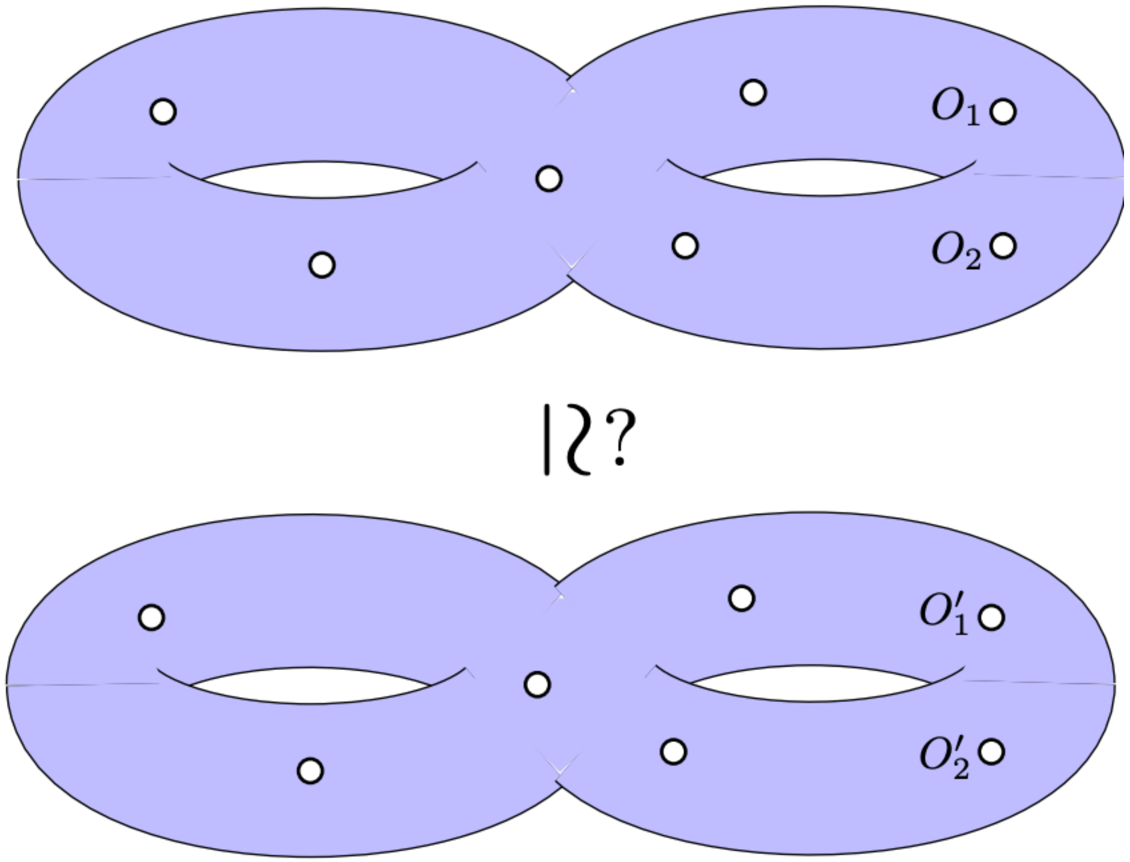


More restrictions

[Distler, Elliot, MJK, Lawrie]

➡ The pair of theories are evidently not isomorphic if they possess different conventional invariants.

➡ Consider pairs with the subset of conventional invariants are the same:

- The central charges a and c
- The flavor symmetry algebras and levels
- The graded Coulomb branch dimensions
- The Higgs branch dimension

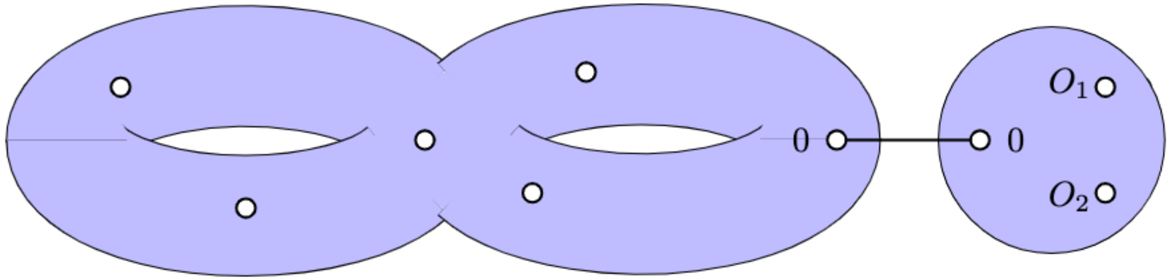
4d SCFTs with the same invariants

[Distler, Elliot, MJK, Lawrie]

For any class S theory, the complex structure moduli of the punctured Riemann surface parametrize exactly-marginal deformations of the SCFT.

Take a degeneration limit:

$$S_{\mathfrak{g}}\langle C_{g,n}\rangle\{O_1, O_2, \cdots, O_n\}$$



$\wr ?$

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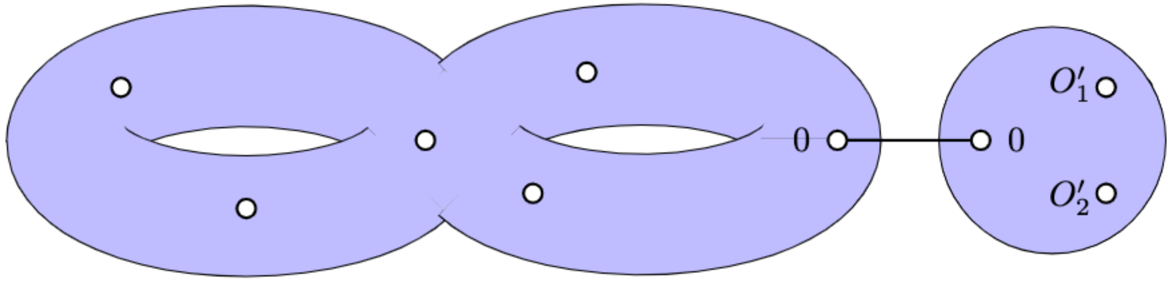
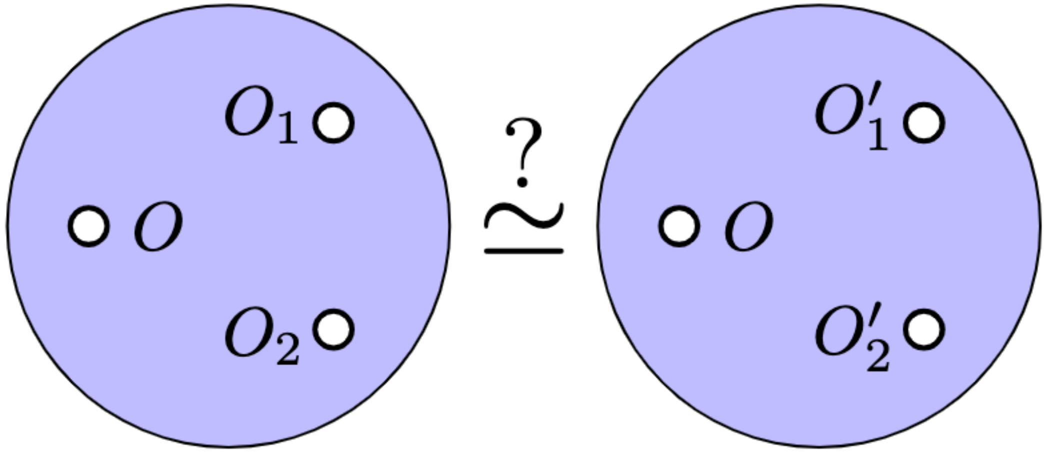


4d SCFTs with the same invariants

[Distler, Elliot, MJK, Lawrie]

Reduces further down to:



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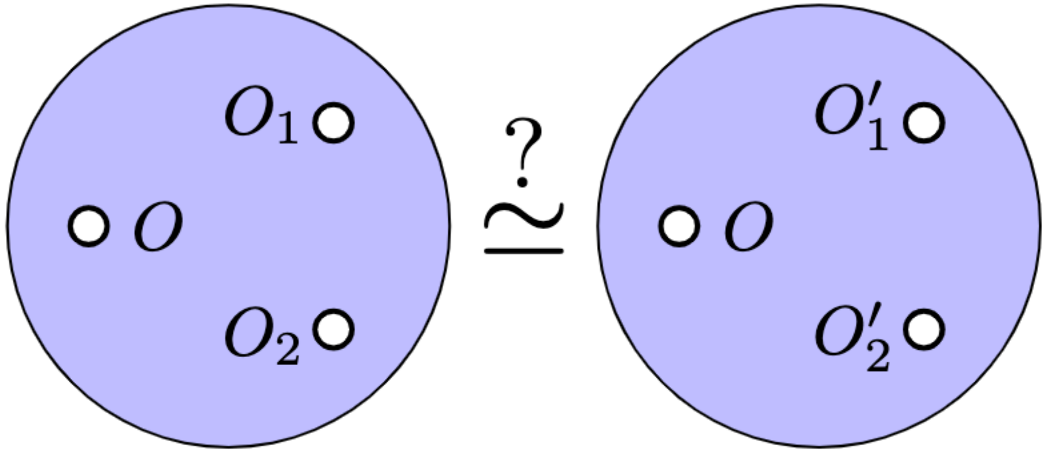
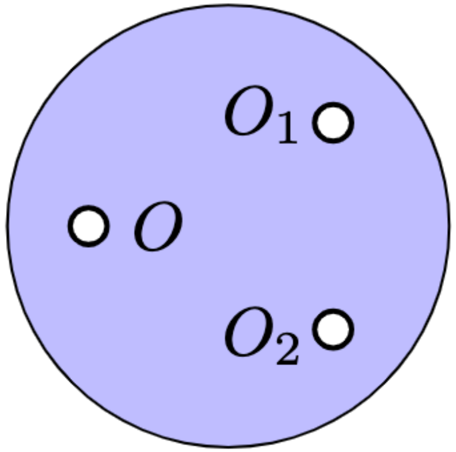
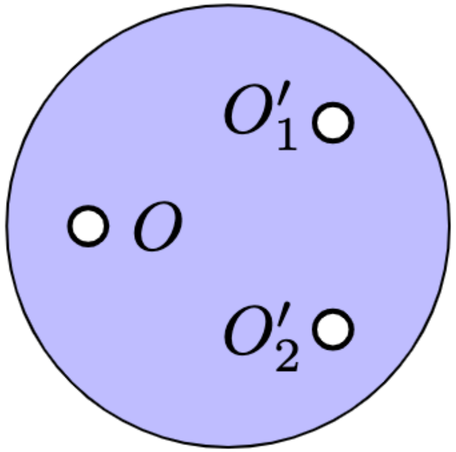


Construct suitable pairs (O_1, O_2) and (O'_1, O'_2) such that the resulting 4d SCFTs have all the same conventional invariants.

4d SCFTs with the same invariants

➡ Consider class S theories with different punctured spheres





$$O_1 \xrightarrow{\mathfrak{f}_k} O_1' \,, \qquad O_2' \xrightarrow{\mathfrak{f}_k} O_2$$

$$\mathfrak{f}(O_1) = \mathfrak{f}_k \oplus \mathfrak{f}(O_1') \,, \qquad \mathfrak{f}(O_2') = \mathfrak{f}_k \oplus \mathfrak{f}(O_2)$$

➡ They have the same conventional invariants. [Distler, Elliot]

[Distler, Elliot, MJK, Lawrie]

4d SCFTs with the same invariants

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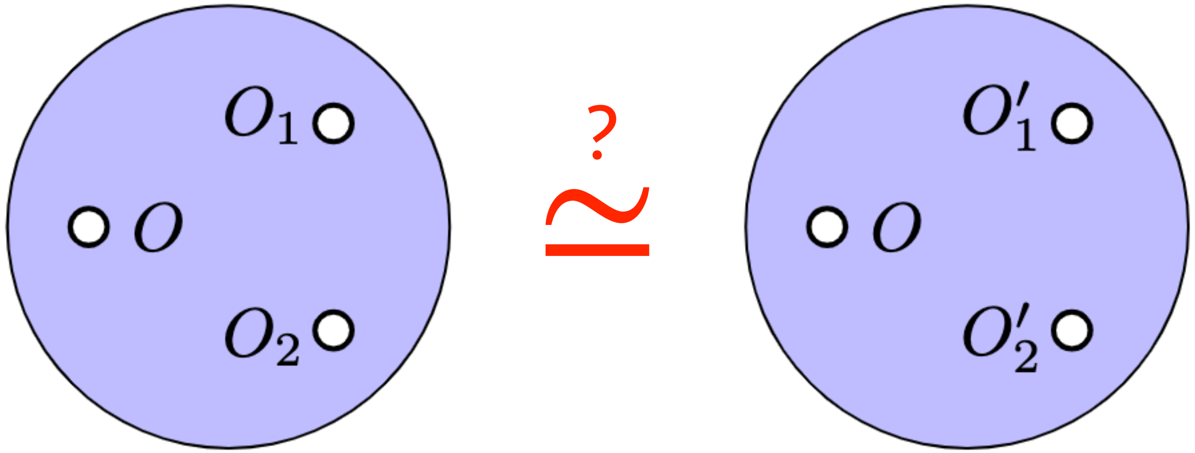

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Not sufficient to decide that they are identical

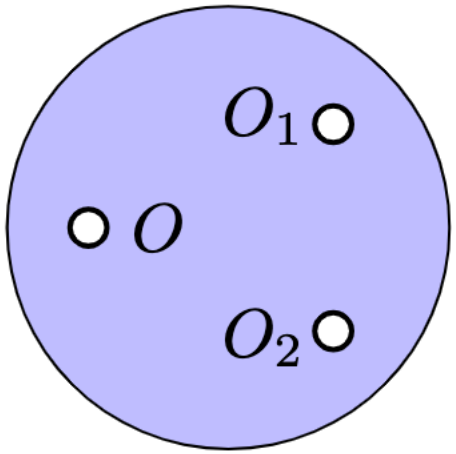
A 6d rephrase

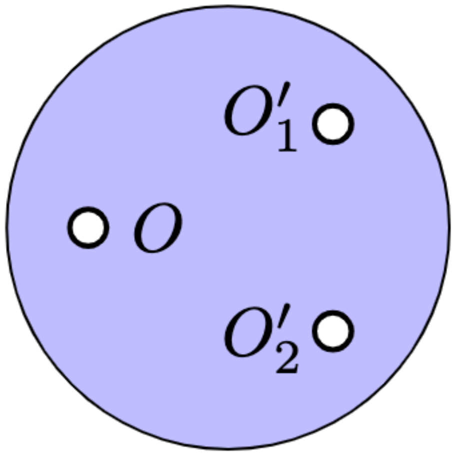
[Distler, Elliot, MJK, Lawrie]

➡ Consider their 6d (1,0) parent theories.

$$\mathscr{T}_{\mathfrak{g},n-2}(O_1, O_2) \qquad\qquad \mathscr{T}_{\mathfrak{g},n-2}(O_1', O_2')$$

$$\Big\downarrow T^2 \qquad\qquad\qquad\qquad \Big\downarrow T^2$$





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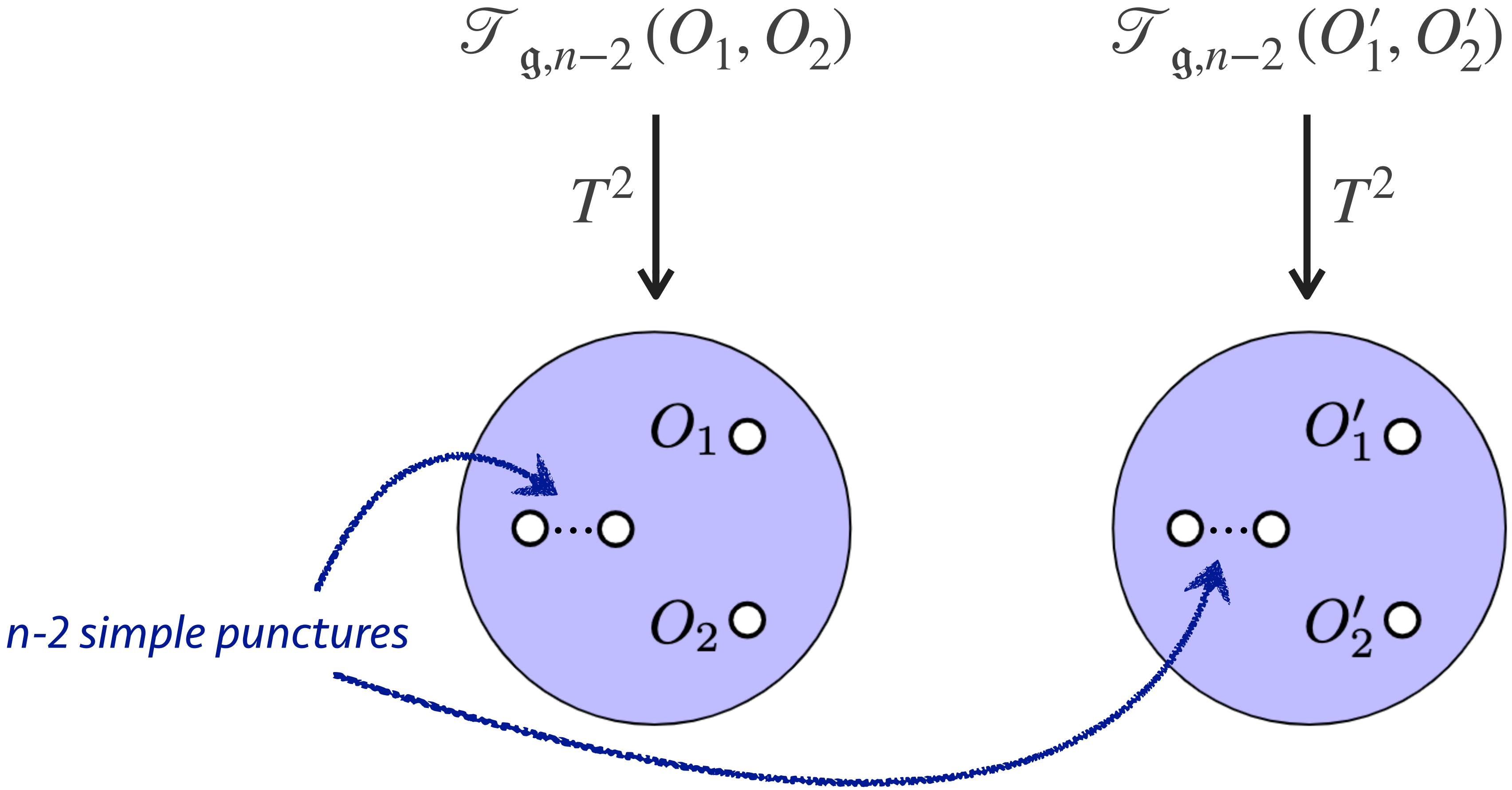
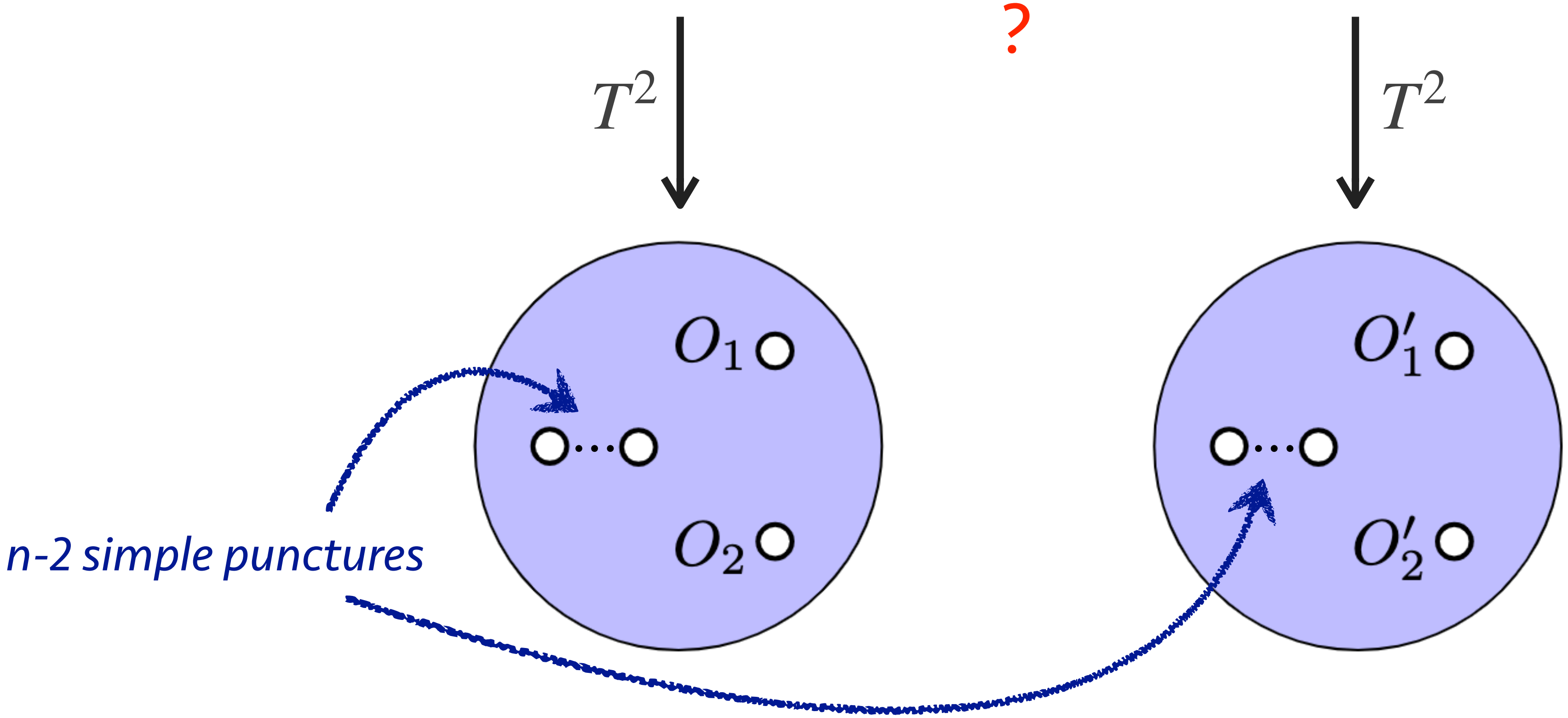


A 6d rephrase

[Distler, Elliot, MJK, Lawrie]

➡ When do the interacting sectors of two 6d (1,0) SCFTs match?

$$\mathscr{T}_{\mathfrak{g},n-2}(O_1, O_2) \underset{?}{\simeq} \mathscr{T}_{\mathfrak{g},n-2}(O_1', O_2')$$



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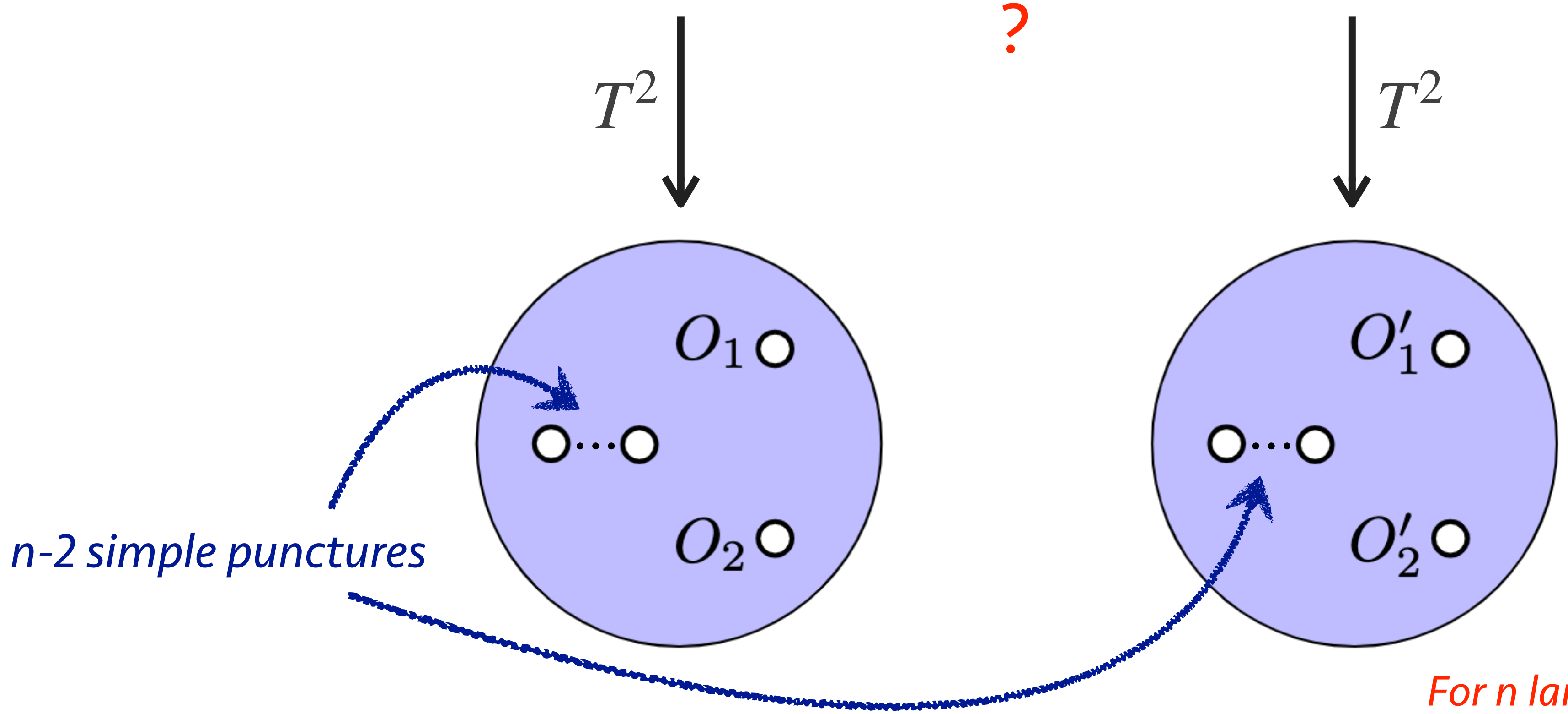

For n large, they are not isomorphic

Distinguishing 6d (1,0) SCFTs

[Distler, Elliot, MJK, Lawrie]

➡ Considering when n is large,

$$\mathcal{T}_{\mathfrak{e}_7,N}(A_3,(A_3+A_1)')$$
$$\overset{\mathfrak{su}_2}{2}\,1\,\underset{1}{\overset{\mathfrak{e}_7}{8}}\,\overbrace{1\,\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{so}_7}{3}\,\overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{8}\cdots 1\,\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{so}_7}{3}\,\overset{\mathfrak{su}_2}{2}\,1}^{N-2}\,\underset{1}{\overset{\mathfrak{e}_7}{8}}\,1\,2$$
$$\mathfrak{f}=(\mathfrak{su}_2)_{12}^{\oplus 2}\oplus(\mathfrak{su}_2)_{24}\oplus(\mathfrak{su}_2)_{13}\oplus(\mathfrak{so}_7)_{16}$$

$(A_3+A_1)'\to A_3+2A_1$ $\qquad$ $A_3\to(A_3+A_1)''$

$$\mathcal{T}_{\mathfrak{e}_7,N}(A_3,A_3+2A_1)$$
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$$\mathfrak{f}=(\mathfrak{su}_2)_{12}\oplus(\mathfrak{su}_2)_{24}\oplus(\mathfrak{su}_2)_{13}\oplus(\mathfrak{so}_7)_{16}$$

$$\mathcal{T}_{\mathfrak{e}_7,N}((A_3+A_1)'',(A_3+A_1)')$$
$$\overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{7}\,\overbrace{1\,\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{so}_7}{3}\,\overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{8}\cdots 1\,\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{so}_7}{3}\,\overset{\mathfrak{su}_2}{2}\,1}^{N-2}\,\underset{1}{\overset{\mathfrak{e}_7}{8}}\,1\,2$$
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they do not match. Their curve configurations are different.
In fact, their anomaly polynomial is different.

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ Consider n=4 with following punctures:

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3,(A_3+A_1)')$

$$\begin{array}{ccccc} & & 1 & & \\ \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{8} & 1 & 2 \\ & & 1 & & \end{array}$$

$\mathfrak{f} = (\mathfrak{su}_2)_{12}^{\oplus 2} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

$(A_3+A_1)' \to A_3+2A_1$

$A_3 \to (A_3+A_1)''$

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3, A_3+2A_1)$

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$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

$\mathcal{T}_{\mathfrak{e}_7,2}((A_3+A_1)'', (A_3+A_1)')$

$$\begin{array}{ccccc} \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{7} & 1 & 2 \\ & & 1 & & \end{array}$$

$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ Consider n=4 with following punctures:

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3,(A_3+A_1)')$

$$\begin{array}{ccccc} & & 1 & & \\ \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{8} & 1 & 2 \\ & & 1 & & \end{array}$$

$\mathfrak{f}=(\mathfrak{su}_2)_{12}^{\oplus 2}\oplus(\mathfrak{su}_2)_{24}\oplus(\mathfrak{su}_2)_{13}\oplus(\mathfrak{so}_7)_{16}$

$(A_3+A_1)'\to A_3+2A_1$ $\qquad$ $A_3\to(A_3+A_1)''$

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3,A_3+2A_1)$

$$\begin{array}{ccccc} \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{7} & 1 & 2 \\ & & 1 & & \end{array}$$

$\mathfrak{f}=(\mathfrak{su}_2)_{12}\oplus(\mathfrak{su}_2)_{24}\oplus(\mathfrak{su}_2)_{13}\oplus(\mathfrak{so}_7)_{16}$

$\simeq$

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their tensor branch descriptions match. These are isomorphic!

A 6d reveal

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Notice the $\mathbb{Z}_2$ automorphism

$(A_3+A_1)' \to A_3+2A_1$ | $A_3 \to (A_3+A_1)''$

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3, A_3+2A_1)$

$$\begin{array}{ccccc} \overset{\mathfrak{su}_2}{2} & 1 & \overset{\mathfrak{e}_7}{7} & 1 & 2 \\ & & 1 & & \end{array}$$

$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

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$\mathcal{T}_{\mathfrak{e}_7,2}((A_3+A_1)'',(A_3+A_1)')$

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A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ Consider n=4 with following punctures:

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3, (A_3+A_1)')$

$$\begin{array}{c} \phantom{\overset{\mathfrak{su}_2}{2}}\,\,1 \\ \overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{8}\,1\,2 \\ \phantom{\overset{\mathfrak{su}_2}{2}}\,\,1 \end{array}$$

$\mathfrak{f} = (\mathfrak{su}_2)_{12}^{\oplus 2} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

Notice the $\mathbb{Z}_2$ automorphism between two Higgsings

$(A_3+A_1)' \to A_3+2A_1$ $\langle \phi \rangle$

$\langle \phi' \rangle$ $A_3 \to (A_3+A_1)''$

$\mathcal{T}_{\mathfrak{e}_7,2}(A_3, A_3+2A_1)$

$$\begin{array}{c} \overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{7}\,1\,2 \\ 1 \end{array}$$

$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$

$\simeq$

$\mathcal{T}_{\mathfrak{e}_7,2}((A_3+A_1)'', (A_3+A_1)')$

$$\begin{array}{c} \overset{\mathfrak{su}_2}{2}\,1\,\overset{\mathfrak{e}_7}{7}\,1\,2 \\ 1 \end{array}$$

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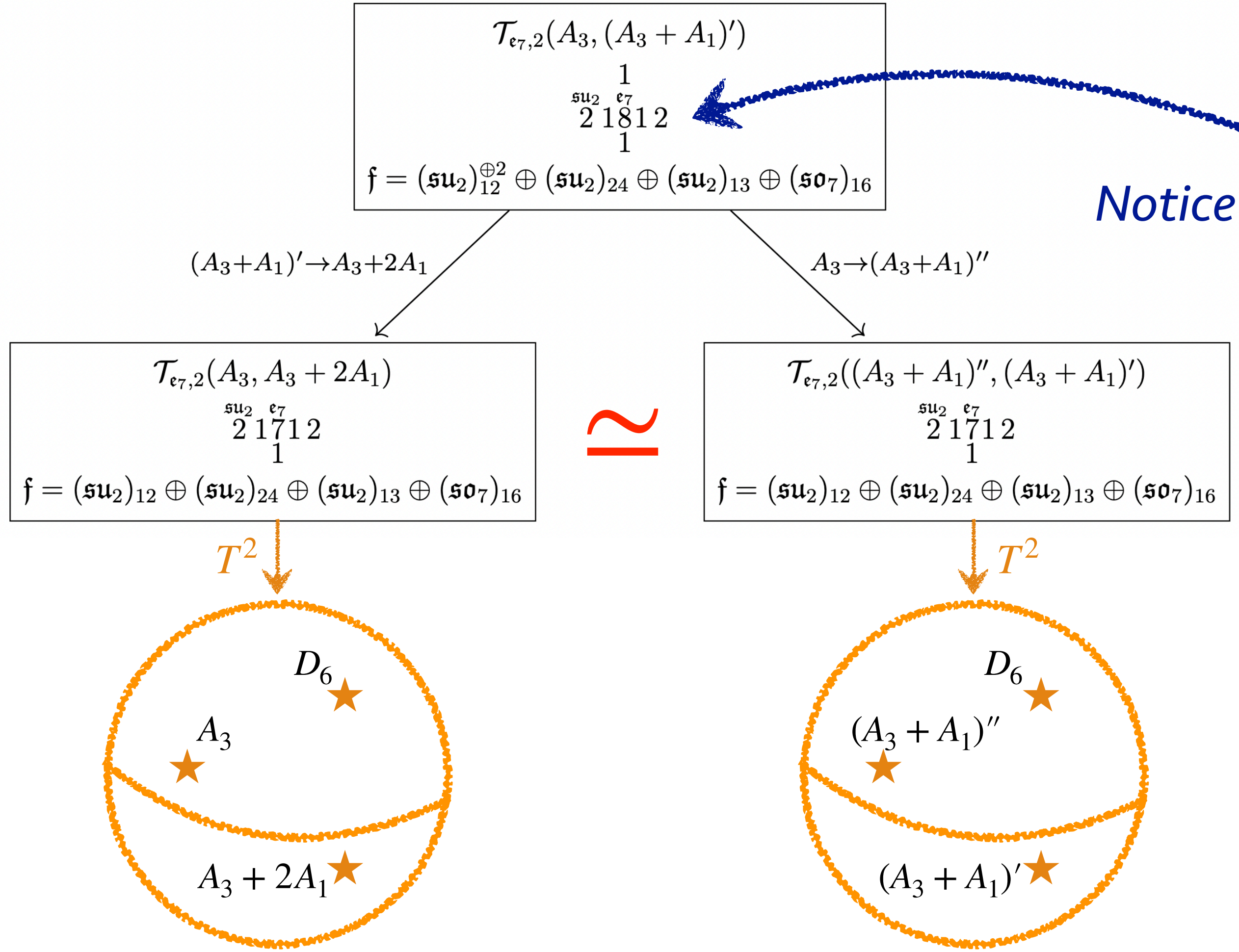
A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ Consider n=4 with following punctures:



Notice the $\mathbb{Z}_2$ automorphism

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

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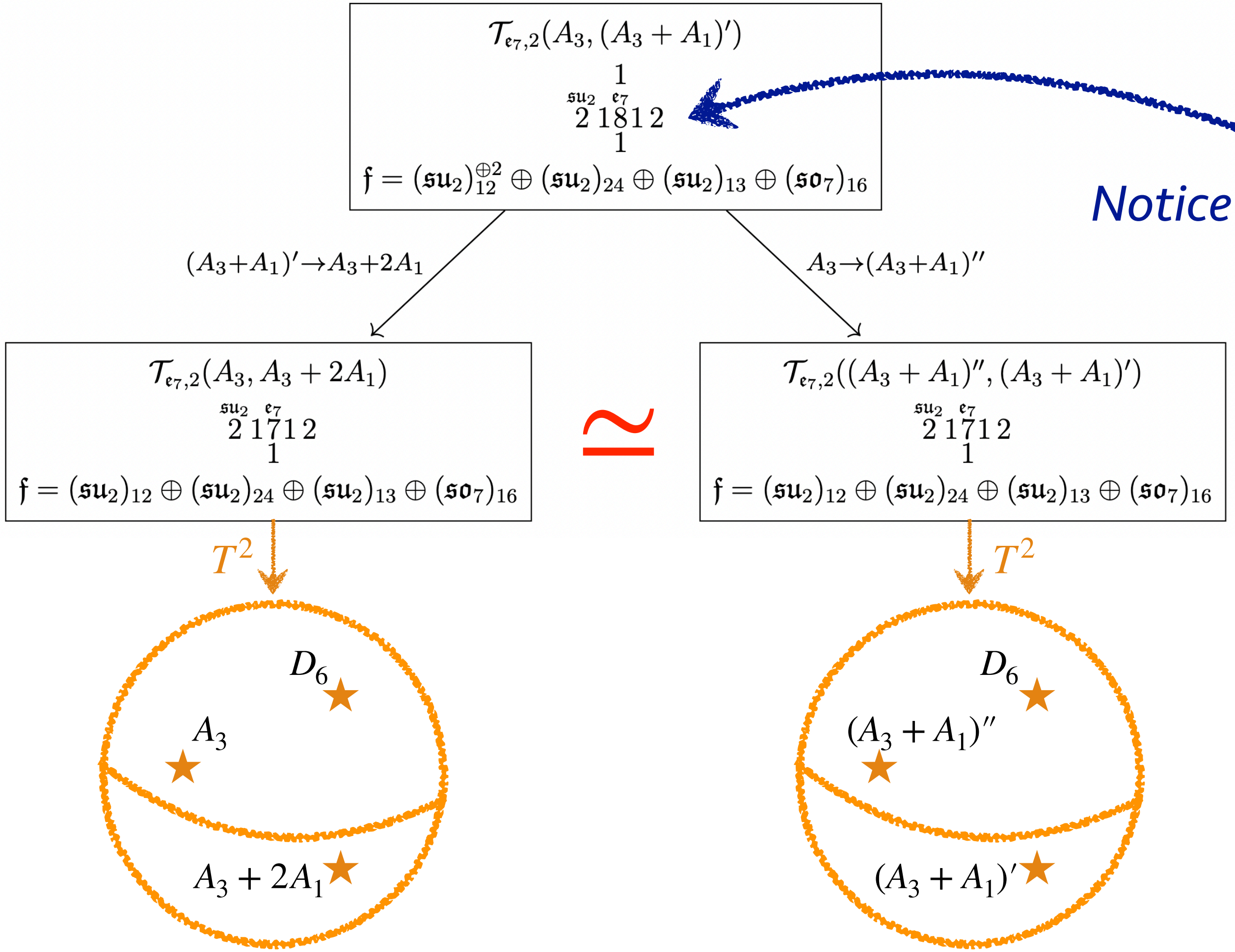

Notice the $\mathbb{Z}_2$ automorphism

Recall:
The Higgs branch flows of 4d class S theories and their 6d (1,0) parents are isomorphic.

[Baume, MJK, Lawrie]

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ Consider n=4 with following punctures:

$$\mathcal{T}_{\mathfrak{e}_7,2}(A_3,(A_3+A_1)')$$

$$\begin{array}{c} 1 \\ \overset{\mathfrak{su}_2}{2}\;1\;\overset{\mathfrak{e}_7}{8}\;1\;2 \\ 1 \end{array}$$

$$\mathfrak{f} = (\mathfrak{su}_2)_{12}^{\oplus 2} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$$

Notice the $\mathbb{Z}_2$ automorphism

$(A_3+A_1)' \to A_3+2A_1$

$A_3 \to (A_3+A_1)''$

$$\mathcal{T}_{\mathfrak{e}_7,2}(A_3, A_3+2A_1)$$

$$\begin{array}{c} \overset{\mathfrak{su}_2}{2}\;1\;\overset{\mathfrak{e}_7}{7}\;1\;2 \\ 1 \end{array}$$

$$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$$

$\simeq$

$$\mathcal{T}_{\mathfrak{e}_7,2}((A_3+A_1)'',(A_3+A_1)')$$

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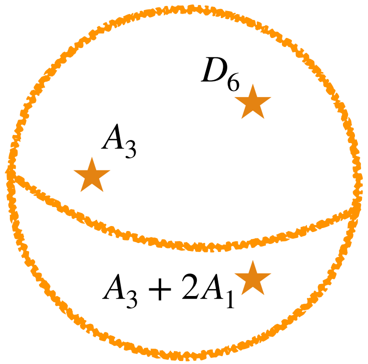
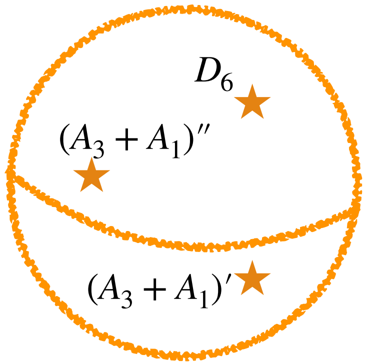
$$\mathfrak{f} = (\mathfrak{su}_2)_{12} \oplus (\mathfrak{su}_2)_{24} \oplus (\mathfrak{su}_2)_{13} \oplus (\mathfrak{so}_7)_{16}$$

T^2

T^2



$\simeq$



Recall:
The Higgs branch flows of 4d class S theories and their 6d (1,0) parents are isomorphic.

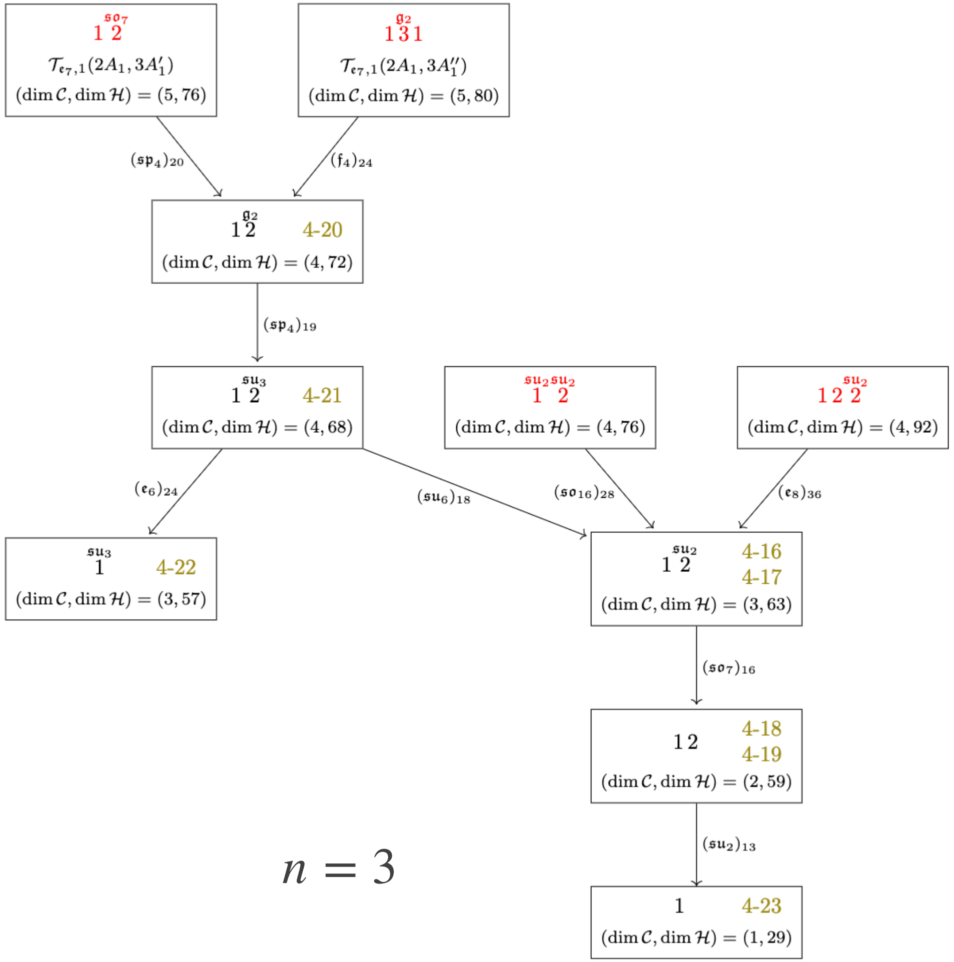
[Baume, MJK, Lawrie]

We proved isomorphisms of 4d $\mathcal{N}=2$ SCFTs of class S using their parent 6d (1,0) SCFTs.

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

➡ We exhaustively worked out for type $(\mathfrak{e}_7, \mathfrak{e}_7)$ theories.

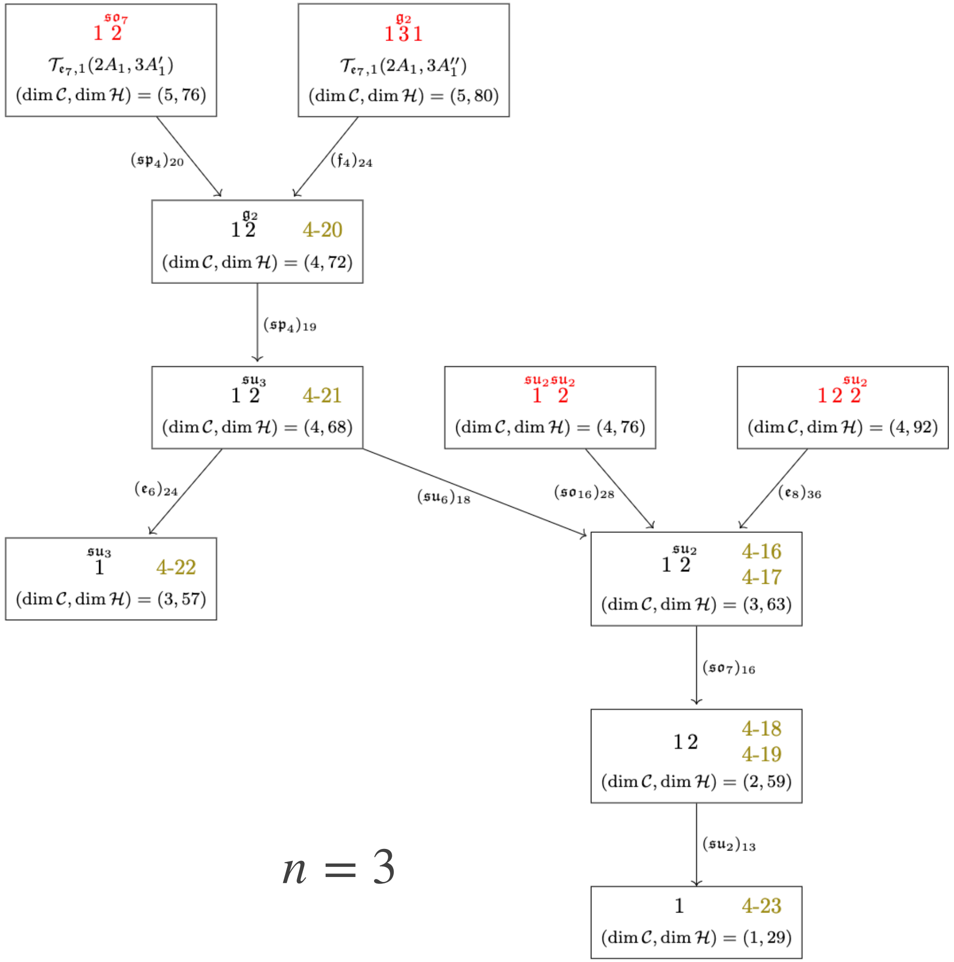


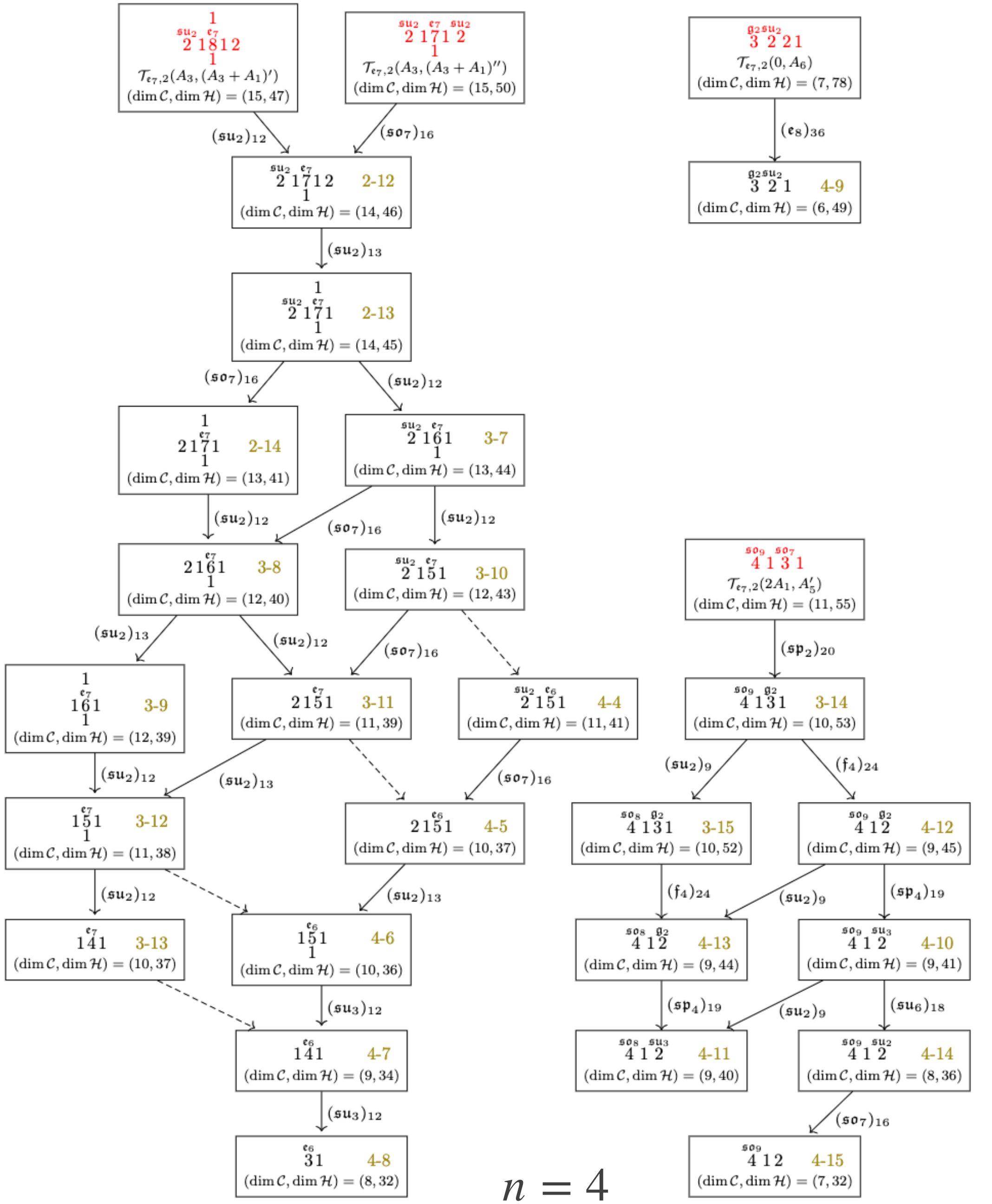
$n = 3$

A 6d reveal

[Distler, Elliot, MJK, Lawrie]

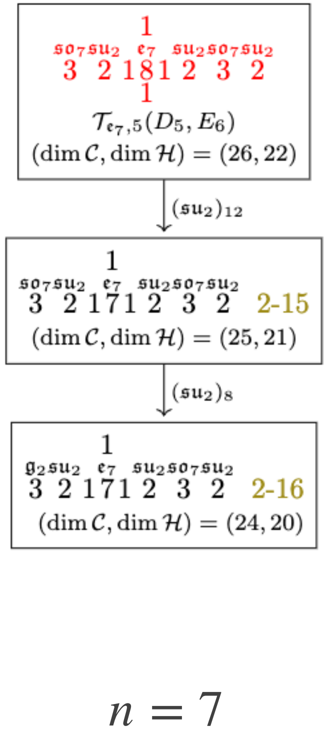
- ➡ We exhaustively worked out fc

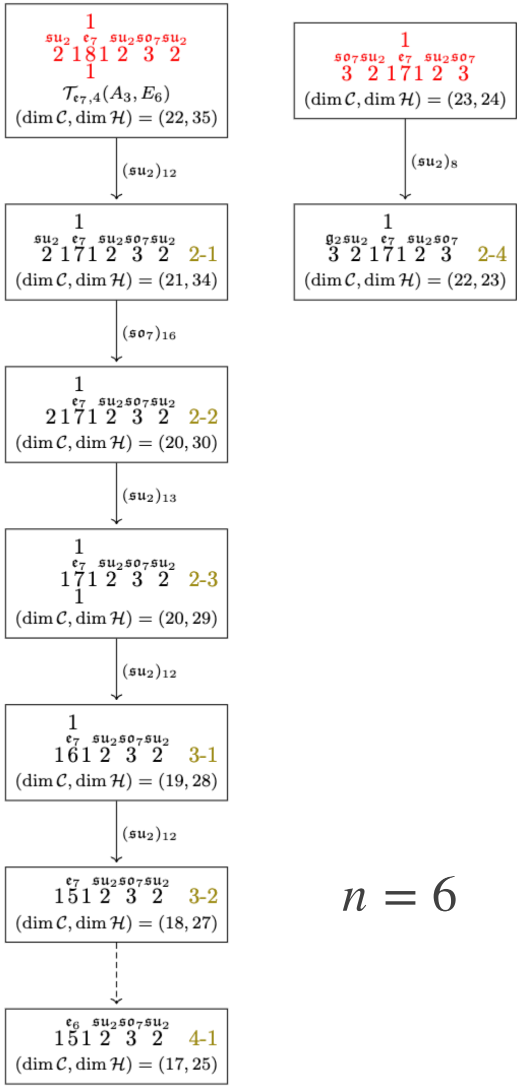


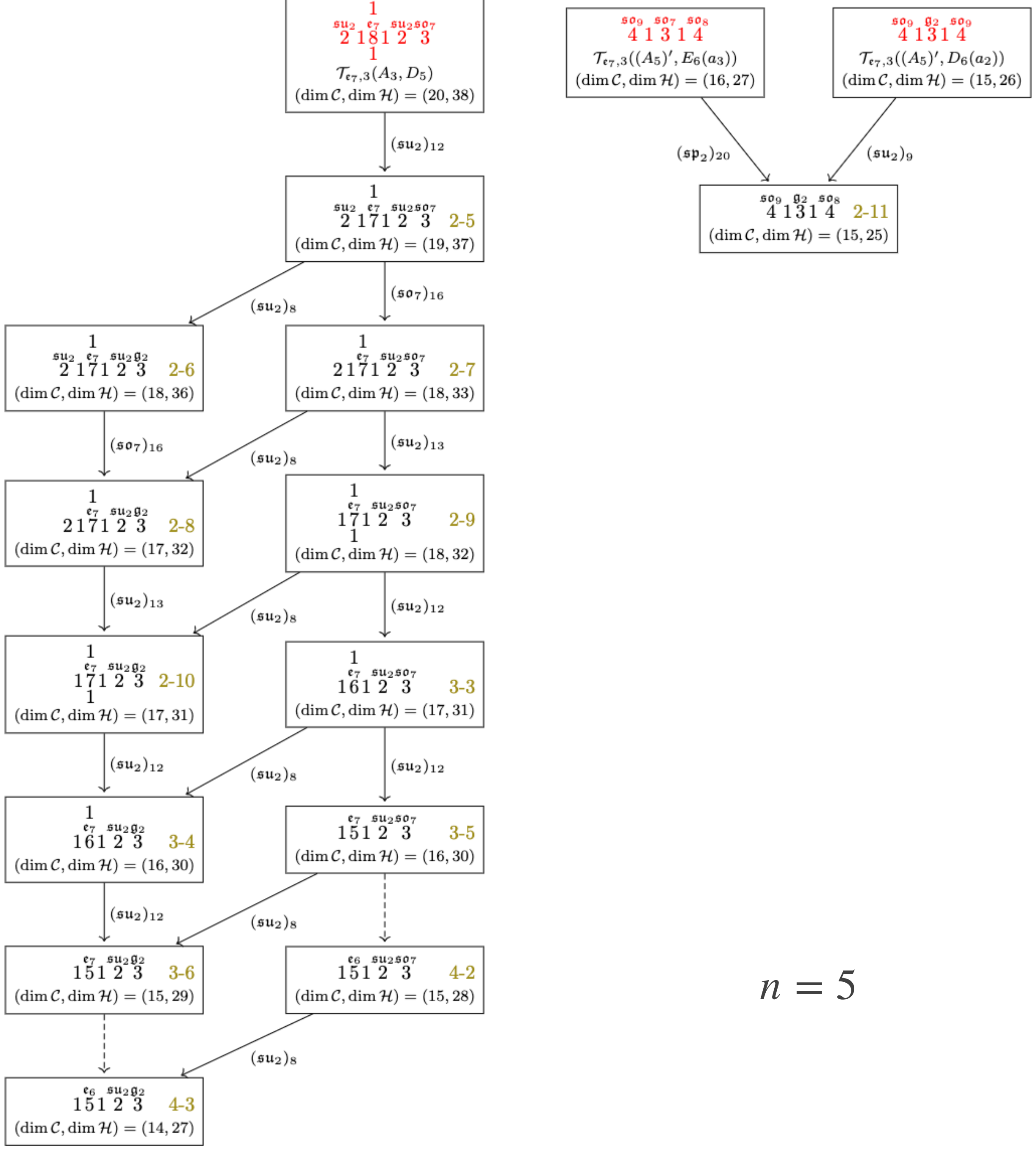


A 6d reveal

[Distler, Elliot, MJK, Lawrie]



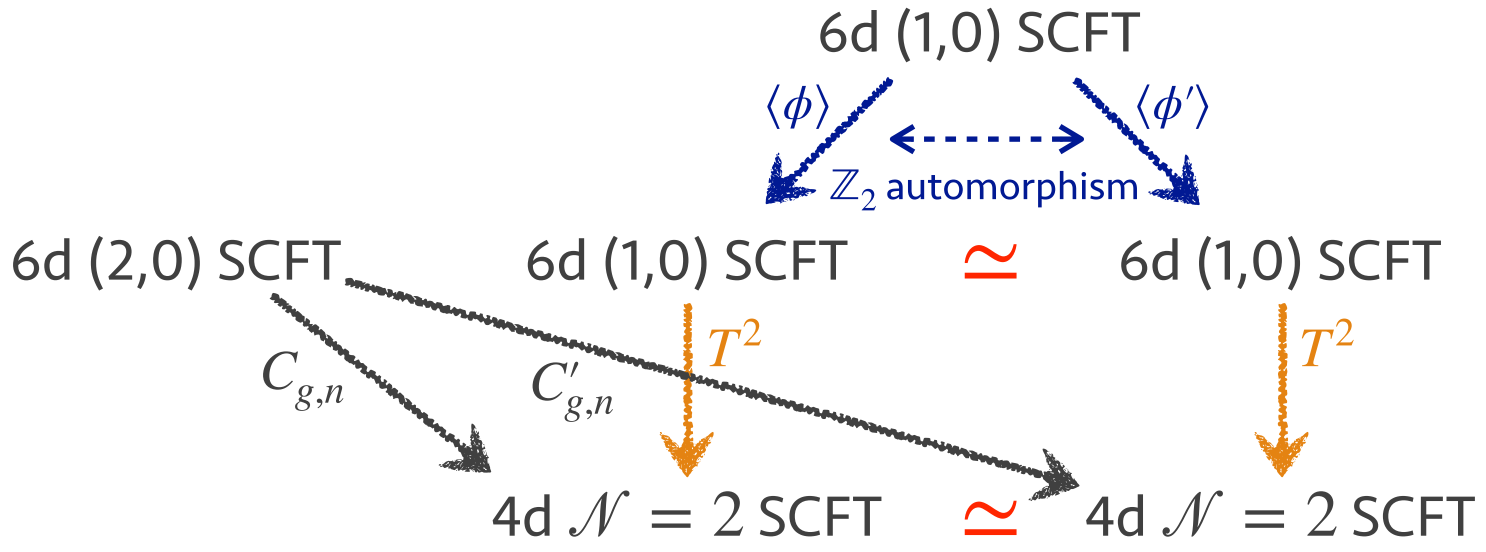




A 6d reveal

[Distler, Elliot, MJK, Lawrie]

- ➡ We exhaustively worked out for type $(\mathfrak{e}_7, \mathfrak{e}_7)$ theories.
- ➡ We showed that this methodology works for numerous type ADE theories, and expect to work more in general class S.
- ➡ General Methodology:

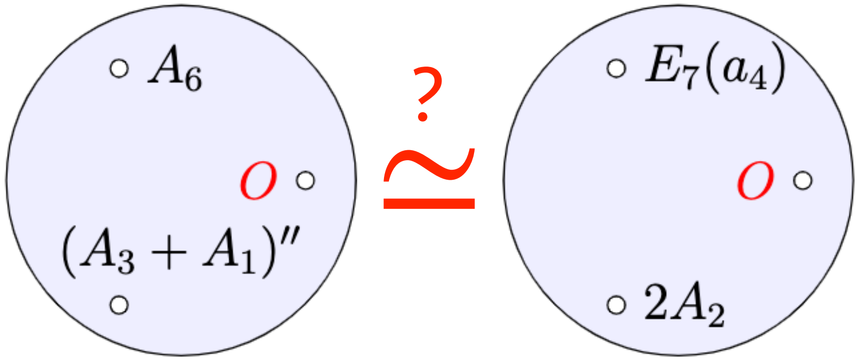


Infinitely-many pairs of 4d SCFTs with distinct constructions are shown to describe the same physics!

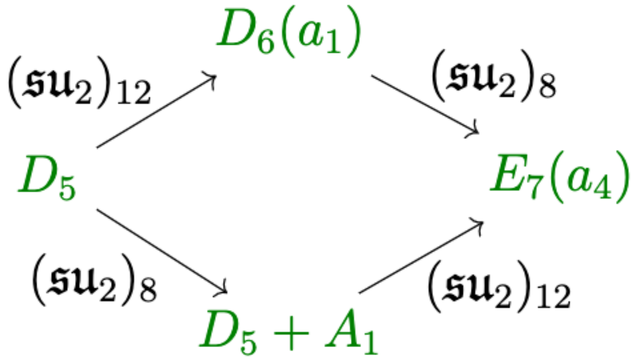
Future directions

➡ There exists class S theories which has the same "conventional" invariants but does not have 6d (1,0) prescriptions.

➡ For example:



where O is chosen from the four punctures related by RG flow



$$I_{Schur} = 1 + 37\tau^2 + 853\tau^4 + 15305\tau^6 + 233552\tau^8 + 3168458\tau^{10} + O(\tau^{11})$$

➡ The 6d perspective nevertheless gives us a hint to look for a $\mathbb{Z}_2$ automorphism between such pairs of theories.

Thank you for listening!