

Uncharted Territories in Geometric Engineering

Strings and Geometry 2023 - Philadelphia

Michele Del Zotto - Uppsala University



European Research Council
Established by the European Commission

Simons Collaboration on Global Categorical Symmetries

BASED ON:

[1] 2 2 0 9 . 1 0 5 5 1 w / LIU , OEHLMANN

[2] 2 2 1 2 . 0 5 3 3 1 w / " , "

[3] 2 3 0 3 . x x x x x w / " , "

[4] 2 3 0 3 . x x x x x w / ACHARYA , HECKMAN ,
HÜBNER , TORRES

BASED ON:

[1] 2209.10551 w/ LIV, OEHLMANN

[2] 2212.05331 w/ " , "

[3] 2303.xxxxx w/ " , "

[4] 2303.xxxxx w/ ACHARYA, HECKMAN,
HÜBNER, TORRES

MOSTLY THESE TWO

BASED ON: → SEE MUYANG'S TALK

[1] 2 2 0 9 . 1 0 5 5 1 w / LIU , OEHLMANN

[2] 2 2 1 2 . 0 5 3 3 1 w / " , "

[3] 2 3 0 3 . x x x x x w / " , "

[4] 2 3 0 3 . x x x x x w / ACHARYA , HECKMAN ,
HÜBNER , TORRES

MOSTLY THESE TWO

COMMERCIALS

COMMERCIALS

- **NORDITA** CATEGORICAL ASPECTS OF SYMMETRIES PROGRAM

CONFERENCE AUG 22-25, 2023

- GCS CONFERENCE + GRADUATE SCHOOL

LES DIABLERET

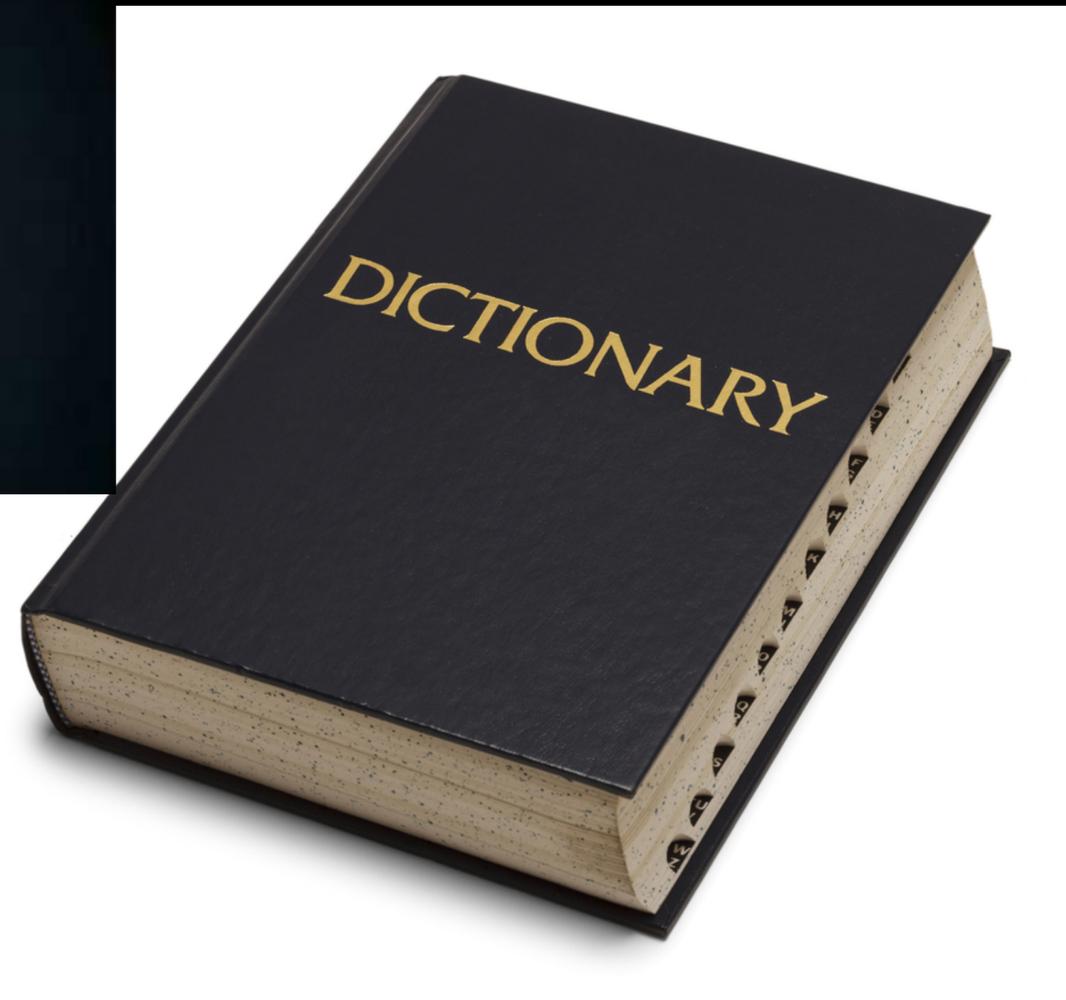
AUG 27 - SEPT 8, 2023

DETAILS ON: SCGCS.BERKELEY.EDU

GEOMETRIC ENGINEERING

[KATZ, KLEMM, VAFARIC]

[LEUNG, Vafa 97]



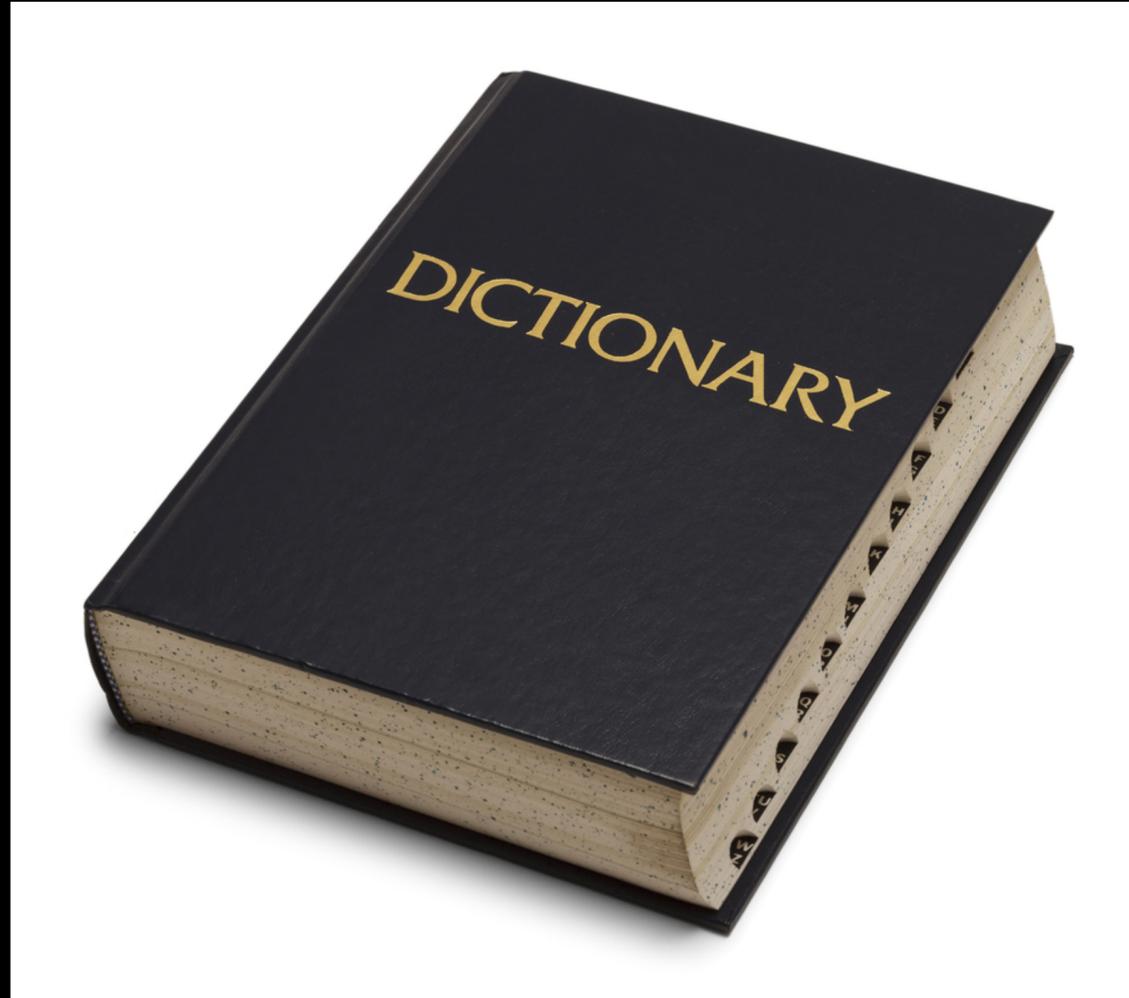
GEOMETRIC
SINGULARITIES

Volume III Supersymmetry

THE
QUANTUM
THEORY OF
FIELDS

STEVEN WEINBERG

IN TODAY'S TALK WE WILL EXPLORE TWO
OPEN GEOMETRIC ENGINEERING QUESTIONS

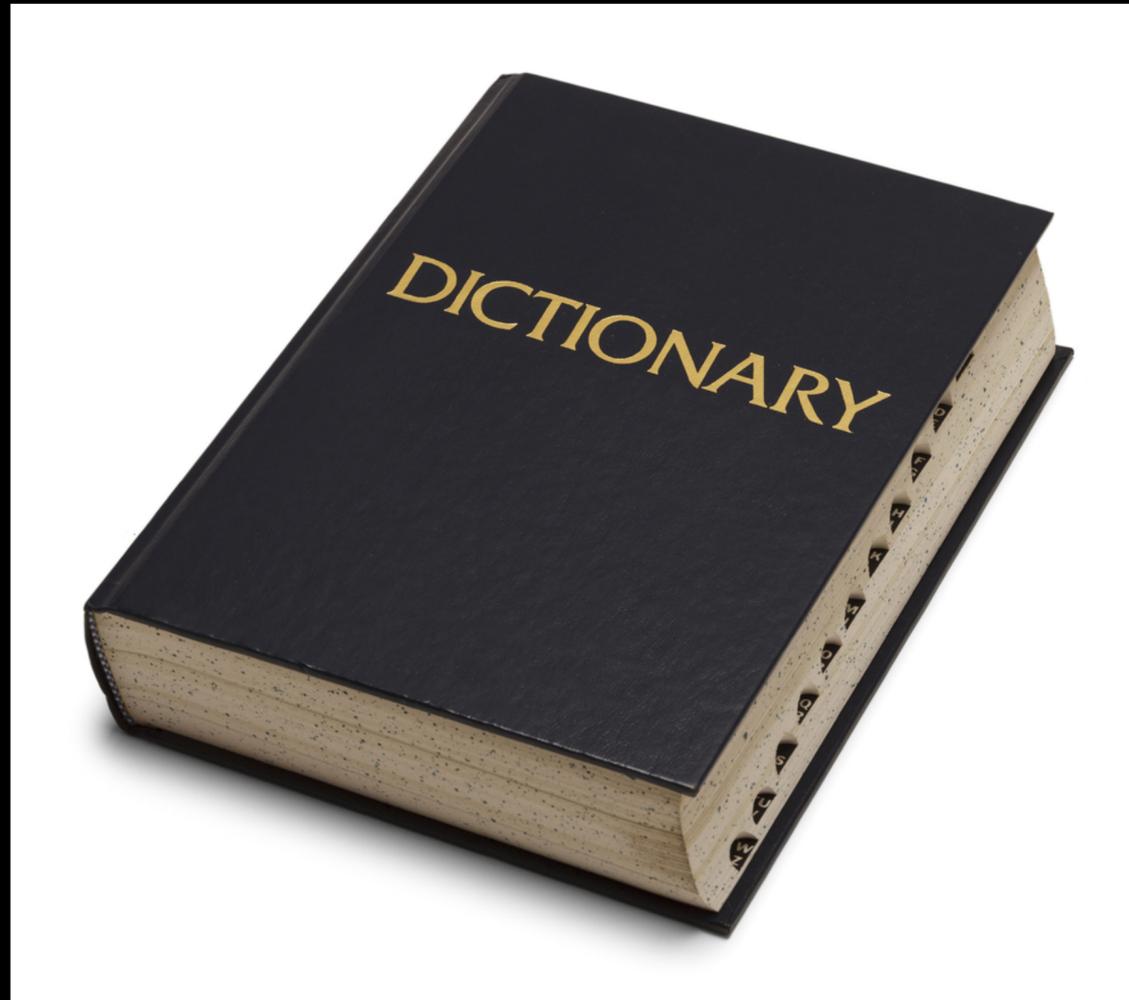


IN TODAY'S TALK WE WILL EXPLORE TWO
OPEN GEOMETRIC ENGINEERING QUESTIONS

1

GEOMETRIC
ENGINEERING
LIMIT OF
HETEROTIC
STRINGS

[1-3]

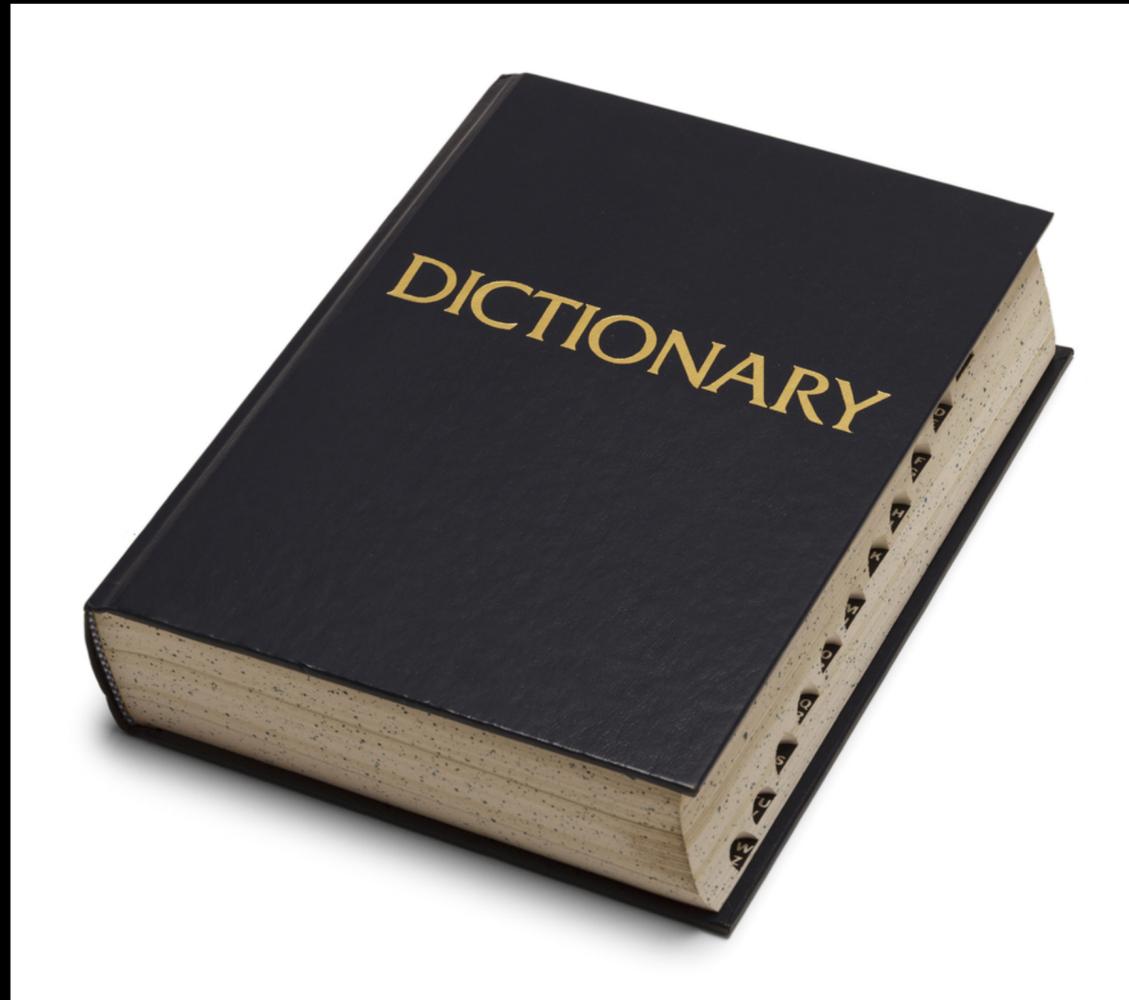


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2

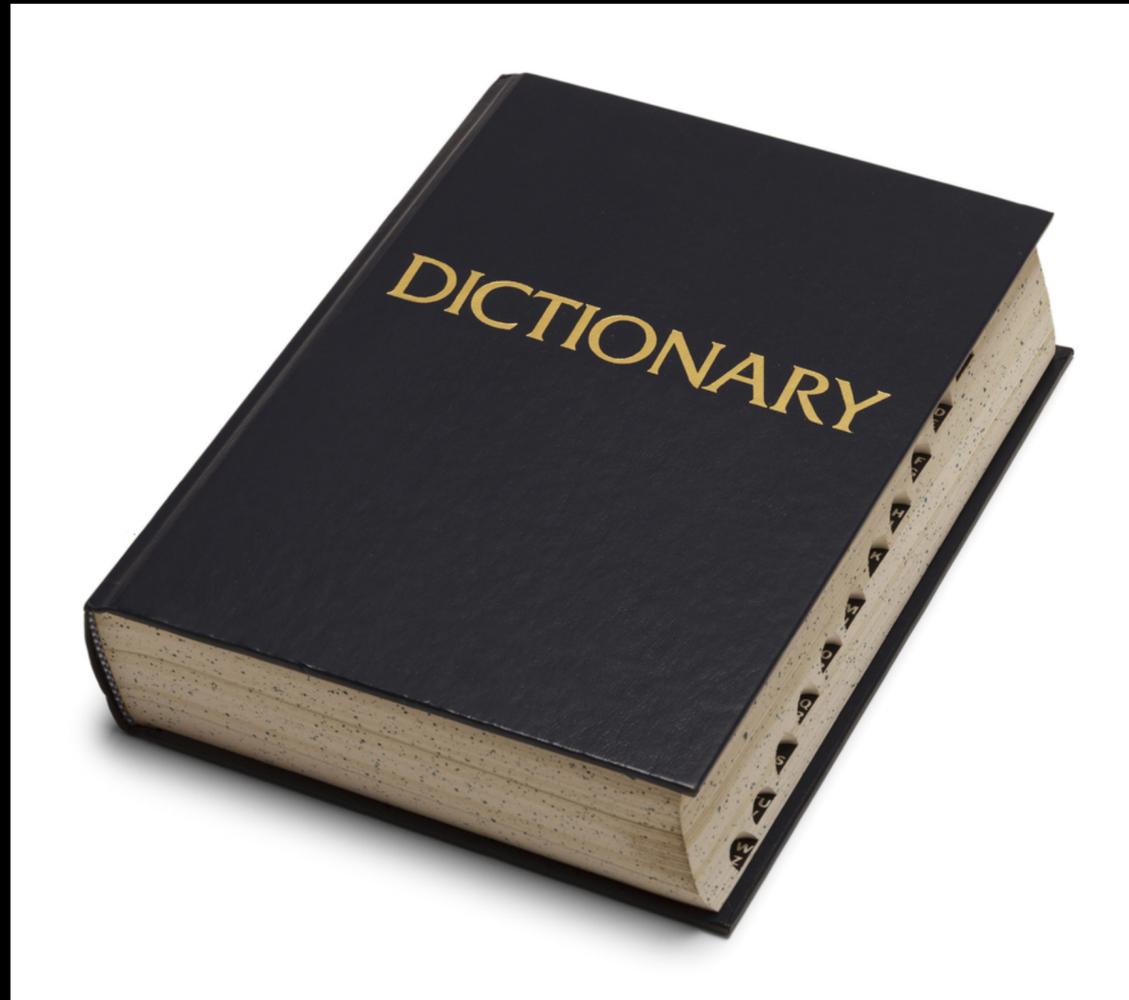
GEOMETRIC
ENGINEERING
OF M-THEORY
ON SINGULAR
 G_2 HOLONOMY
CONES

[4]

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2

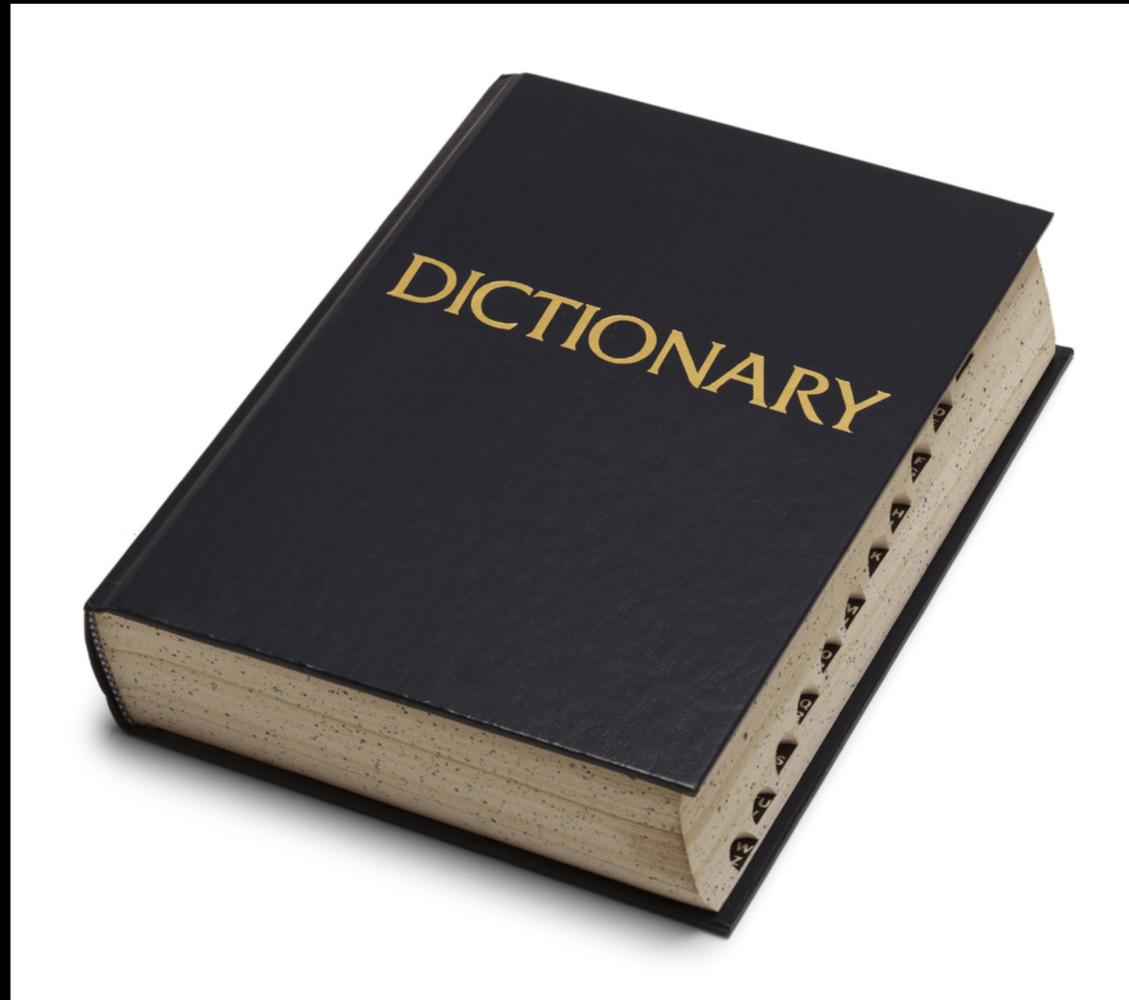
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IN BOTH CASES PROGRESS THANKS TO
RECENT ADVANCES IN SCFTS AND SYMMETRIES [4]

IN TODAY'S TALK WE WILL EXPLORE TWO
OPEN GEOMETRIC ENGINEERING QUESTIONS:

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GEOMETRIC
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LIMIT OF
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2

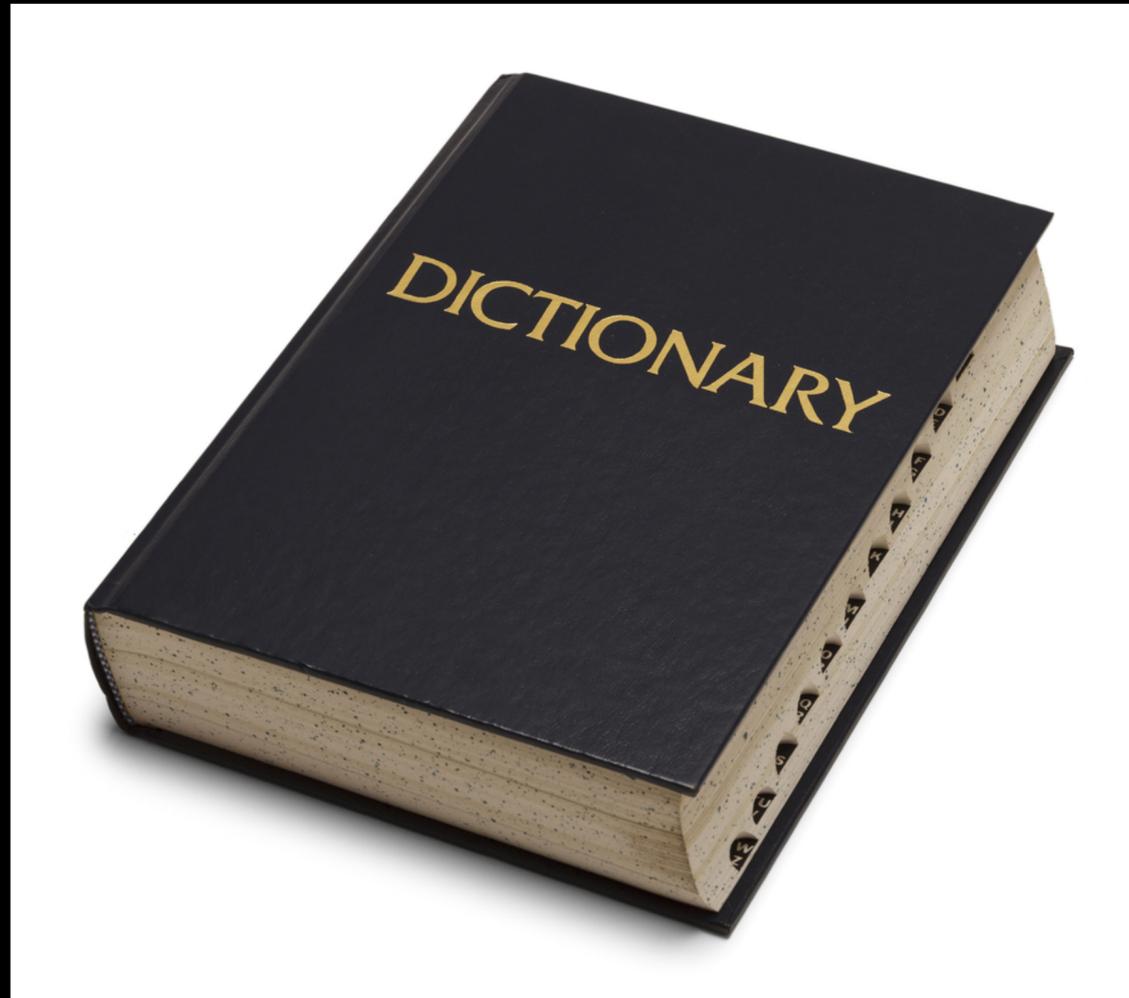
GEOMETRIC
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 G_2 HOLONOMY
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IN BOTH CASES PROGRESS THANKS TO
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NAIVELY DISTINCT BUT \exists INTERPLAY VIA DUALITY

1

GEOMETRIC
ENGINEERING
LIMIT OF
HETEROTIC
STRINGS

[1-3]



SIMPLEST

SUSY

SINGULARITIES :

DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

$$\Gamma_g \subseteq SU(2)$$

g E ADE

MACKEY

CORRESPONDENCE

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

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WELL-KNOWN BUILDING
BLOCKS IN GEOMETRIC
ENGINEERING DICTIONARIES

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WELL-KNOWN BUILDING

BLOCKS IN GEOMETRIC

ENGINEERING DICTIONARIES:

$$M / X^4_g \equiv 7D \text{ } g\text{-SYM}$$

[SEN 96]

MACKEY

CORRESPONDENCE

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WELL-KNOWN BUILDING
BLOCKS IN GEOMETRIC

MACKAY

ENGINEERING DICTIONARIES:

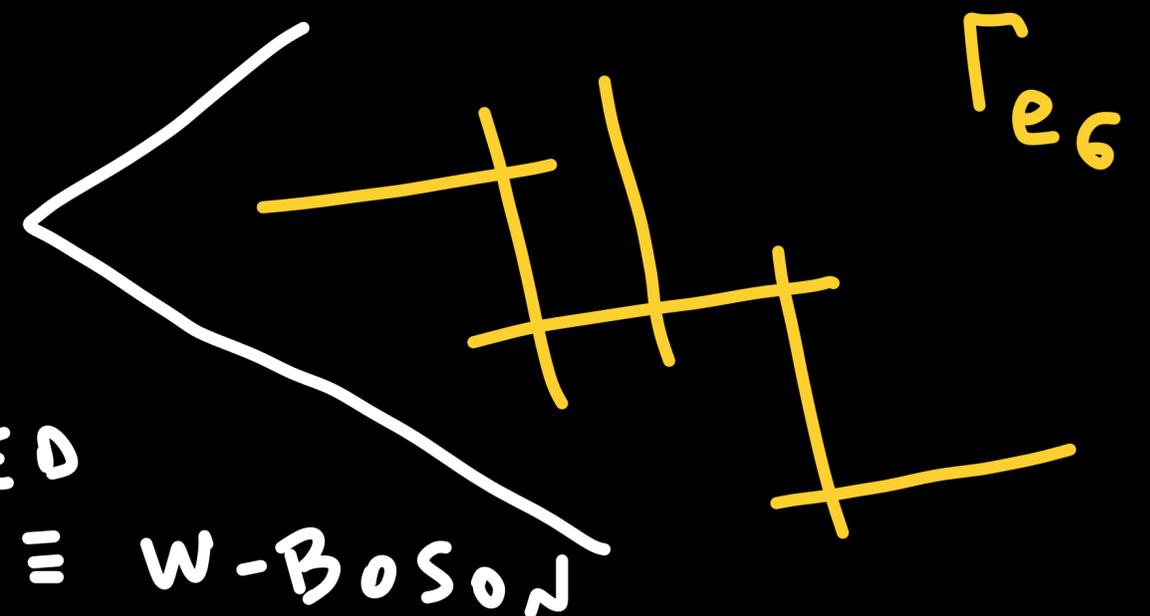
CORRESPONDENCE

$$M / X^4_g \equiv 7D \text{ } g\text{-SYM}$$

[SEN 96]

WRAPPED

$M2S \equiv W\text{-BOSON}$



SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

$$\Gamma_g \subseteq SU(2) \quad g \in ADE$$

WELL-KNOWN BUILDING
BLOCKS IN GEOMETRIC
ENGINEERING DICTIONARIES:

$$M / X^4_g \equiv 7D \quad g\text{-SYM}$$

ALL GLOBAL FORMS ARE REALIZED $M / \partial X^4_g$ SYMMETRY
TFT

ALBERTINI, DE, GARCIA ETXEBARRIA
HOSEINI (20); MORRISON,
SCHÄFER, NAMEKI, WILLET (20);
APRUZZI, BONETTI, SCHÄFER
NAMEKI, GARCIA ETXEBARRIA,
HOSEINI (21)

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

$g \in ADE$

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• $M / X^4_g \equiv 7D$ g -SYM $g \in ADE$

• $IIA / X^4_g \equiv 6D$ g -SYM $(1,1)$ [SEN 97]

SIMPLEST SUSY SINGULARITIES: DU VAL

$$x^4_g = \mathbb{C}^2 / \Gamma_g$$

• $M / x^4_g \equiv 7D$ g -SYM $g \in ADE$

• $IIA / x^4_g \equiv 6D$ g -SYM $(1,1)$ [SEN 97]

RMK: $IIA/x = D_{S^1}(M/x)$

SIMPLEST

SUSY

SINGULARITIES :

DU VAL

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- $M / X^4_g \equiv 7D$ g -SYM $g \in ADE$

- $IIA / X^4_g \equiv 6D$ g -SYM $(1,1)$ [SEN 97]

- $IIB / X^4_g \equiv 6D$ g -TYPE $(2,0)$ [WITTEN 95]

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• $IIA / X^4_g \equiv 6D$ g -SYM $(1,1)$ [SEN 97]

• $IIB / X^4_g \equiv 6D$ g -TYPE $(2,0)$ [WITTEN 95]

RMK: $D_{5'}(IIA / X^4_g) \xleftrightarrow{T} D_{5'}(IIB / X^4_g)$

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

• $M / X^4_g \equiv 7D$ g -SYM $g \in ADE$

• $IIA / X^4_g \equiv 6D$ g -SYM $(1,1)$ LITTLE STRING THEORIES

• $IIB / X^4_g \equiv 6D$ g -TYPE $(2,0)$ [SEIBERG 97]

RMK: $D_{5'}(IIA / X^4_g) \xleftrightarrow{T} D_{5'}(IIB / X^4_g)$

SIMPLEST

SUSY

SINGULARITIES :

DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

- $M / X^4_g \equiv 7D \text{ } g\text{-SYM}$

LITTLE
MEMBRANE
THEORY

[NEKRASOV,
MOORE
SHATASHVILI 98]

- $IIA / X^4_g \equiv 6D \text{ } g\text{-SYM} \text{ } (1,1)$

LITTLE
STRING

- $IIB / X^4_g \equiv 6D \text{ } g\text{-TYPE} \text{ } (2,0)$

THEORIES
[SEIBERG 97]

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WHAT ABOUT THE HETEROTIC?

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4 = \mathbb{C}^2 / \Gamma_g$$

WHAT ABOUT THE HETEROTIC?

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

NAIVELY:

- $\text{Het}_{E_8 \times E_8} / X^4_g \equiv 6D(1,0)$

WHAT ABOUT THE HETEROTIC?

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_9 = \mathbb{C}^2 / \Gamma_9$$

NAIVELY:

- $\text{Het}_{E_8 \times E_8} / X^4_9 \equiv 6D(1,0)$

- $\text{Het}_{D_{16}} / X^4_9 \equiv 6D(1,0)$

WHAT ABOUT THE HETEROTIC?

SIMPLEST

SUSY

SINGULARITIES :

DU VAL

$$X^4_9 = \mathbb{C}^2 / \Gamma_9$$



NAIVELY:

- $\text{Het}_{E_8 \times E_8} / X^4_9 \equiv 6D (1, 0)$ LSTs

- $\text{Het}_{D_{16}} / X^4_9 \equiv 6D (1, 0)$

RMK: $D_{5'} (\text{Het}_{E_8 \times E_8} / X^4_9) \xleftrightarrow{T} D_{5'} (\text{Het}_{D_{16}} / X^4_9)$

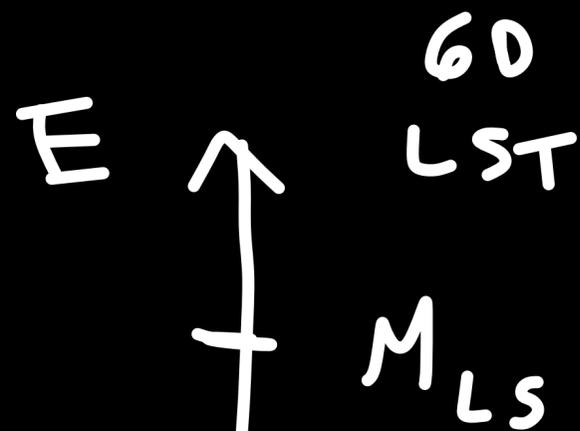
SIMPLEST

SUSY

SINGULARITIES :

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$$X^4_g = \mathbb{C}^2 / \Gamma_g$$



NAIVELY:

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BUT HETEROTIC HAS GAUGE BUNDLES!

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_9 = \mathbb{C}^2 / \Gamma_9$$

BUT HETEROTIC HAS GAUGE BUNDLES!

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4 = \mathbb{C}^2 / \Gamma_g$$

HETEROTIC HAS GAUGE BUNDLES \Rightarrow NEED
CHOICE OF FLAT CONNECTION

SIMPLEST SUSY SINGULARITIES: DU VAL

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HETEROTIC HAS GAUGE BUNDLES \Rightarrow NEED
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$$\partial X^4_g = S^3 / \Gamma_g \quad \pi_1(S^3 / \Gamma_g) \cong \Gamma_g$$

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

HETEROTIC HAS GAUGE BUNDLES \Rightarrow NEED CHOICE OF FLAT CONNECTION

$$\partial X^4_g = S^3 / \Gamma_g \quad \pi_1(S^3 / \Gamma_g) \cong \Gamma_g$$

FOR Het_H PARAMETRIZED BY

$$\xi \in \text{Hom}(\Gamma_g, H)$$

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_g = \mathbb{C}^2 / \Gamma_g$$

FOR Het_H PARAMETRIZED BY

$$\xi \in \text{Hom}(\Gamma_g, H)$$

SIMPLEST

SUSY

SINGULARITIES :

DV VAL

$$X_g^4 = \mathbb{C}^2 / \Gamma_g$$

$$\xi \in \text{Hom}(\Gamma_g, H)$$

$$\text{Het}_H / (X_g^4, \xi) \in 6D(1,0) \text{ LST}$$

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4 = \mathbb{C}^2 / \Gamma_g$$

$$\xi \in \text{Hom}(\Gamma_g, H)$$

$$\text{Het}_H / (X^4, \xi) \in \text{6D}(1,0) \text{ LST}$$

OUR TASK NOW IS TO DETERMINE THIS LST

$$\text{Het}_H / (x^4, \xi) \in \text{GD}(1,0) \text{ LST}$$

OUR TASK NOW IS TO DETERMINE THIS LST

$\text{Het}_H / (x^4, \xi) \in \text{GD}(1,0) \text{LST}$

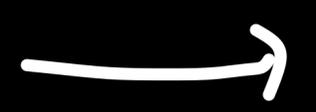
OUR TASK NOW IS TO DETERMINE THIS LST

Het_H / (x⁴, g, ξ) ∈ 6D (1,0) LST

OUR TASK NOW IS TO DETERMINE THIS LST

STRATEGY: [3]

Het_{E₈ × E₈}



HÖRVA-WITTEN PICTURE



FRACTIONALIZATION OF M₉

FROM ORBI-INSTANTONS



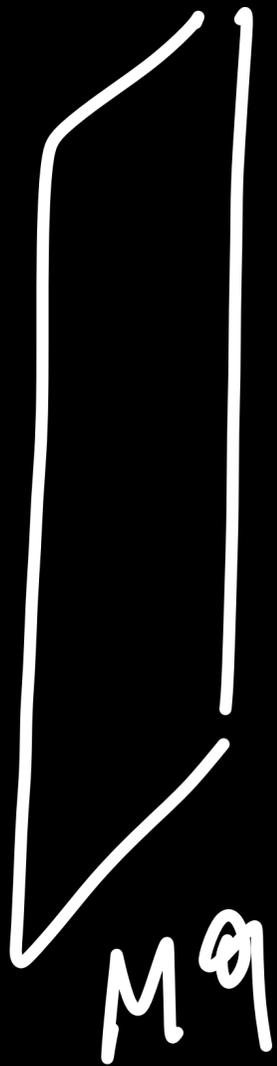
EMBED IN F-THEORY & GEOMETRIZE T-DUALITY



Het_{D₁₆}

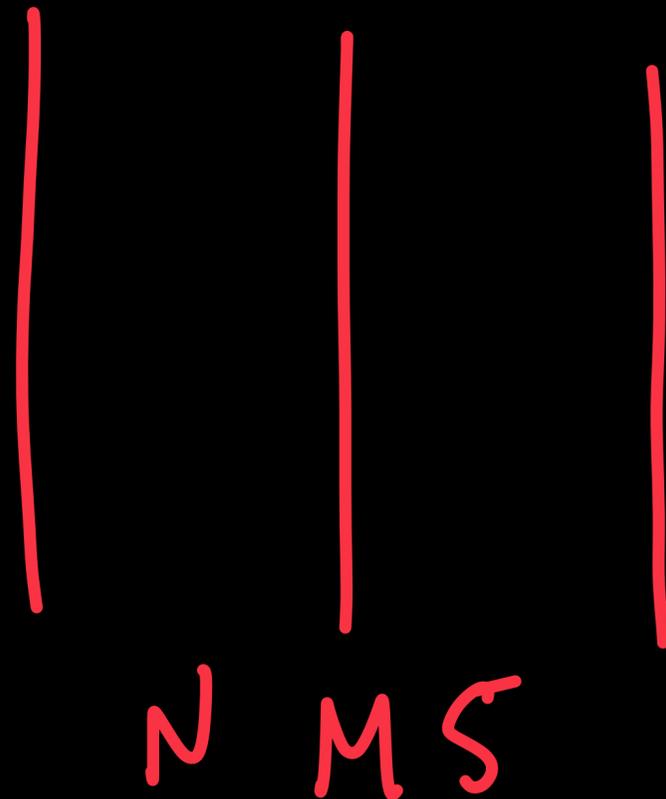
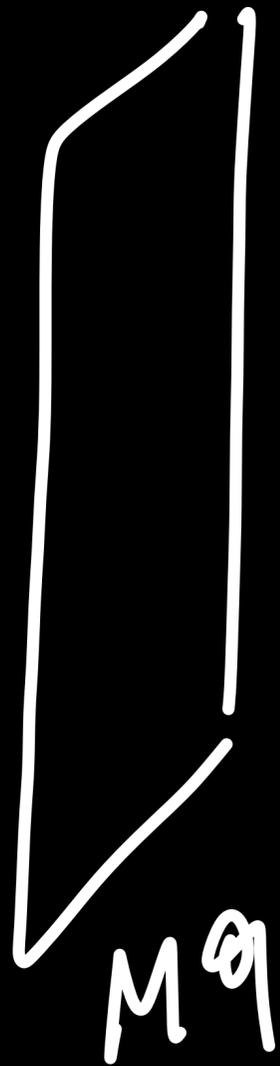
FREY, RUDELIUS ...
MEKKAREEYAI OMMOM
LAWRIE, LIU,
FAZZI, GIRI,
GIACOMELLI ...

[BHARDWAJ, DZ, HECKMAN,
MORRISON, RUDELIUS, VATAIT]



✓
HORAVA
WITTEN
95-96

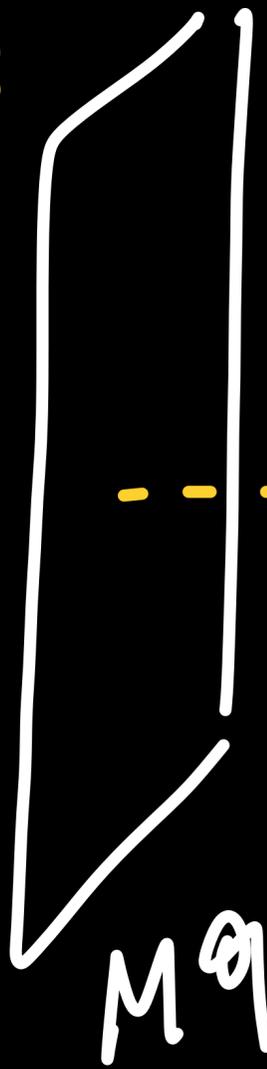
$$M / (S^1 / \mathbb{Z}_2)$$
$$= \text{Het } E_8 \times \bar{E}_8$$



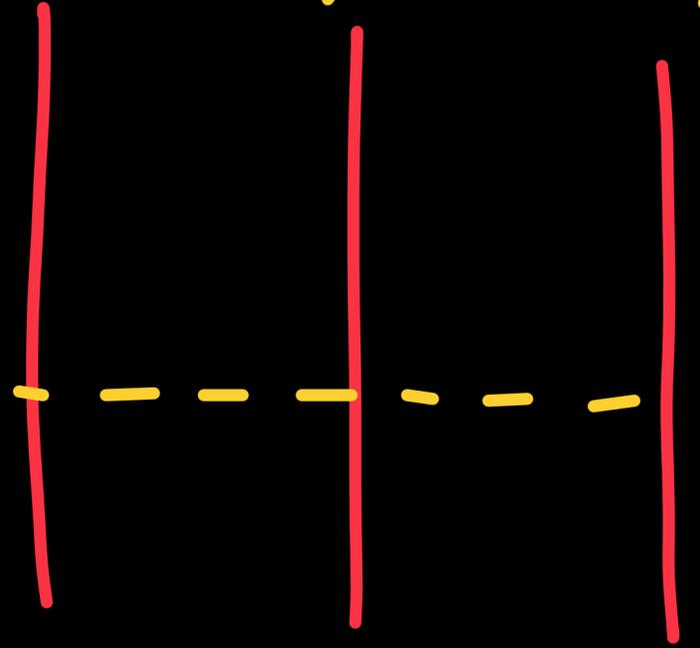
✓
HORAVA
WITTEN
95-96

$$M / (S^1 / \mathbb{Z}_2)$$
$$= \text{Het } E_8 \times E$$

$\mu_1: \Gamma \rightarrow E_8$

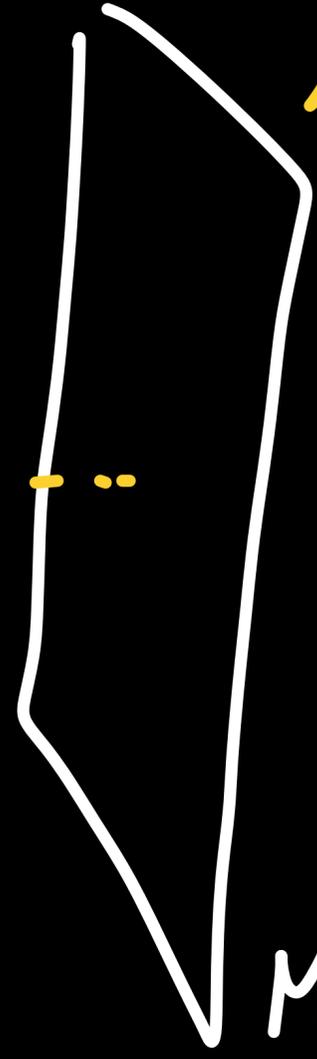


$$F_2 = C(\mu_2(\Gamma), E_8)$$



NMS

$\mu_2: \Gamma \rightarrow E_8$

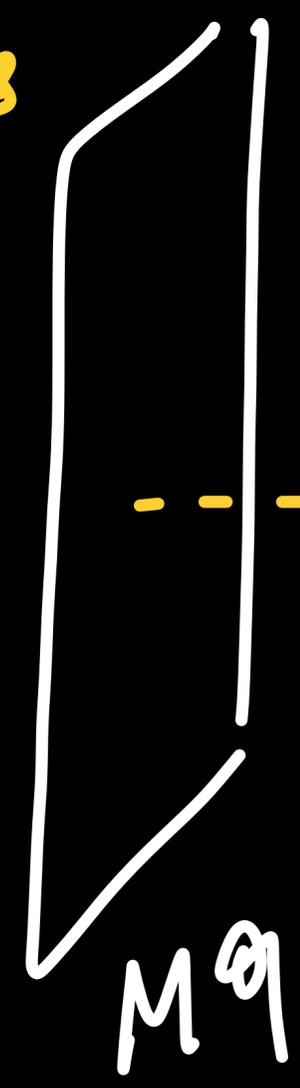


✓
 HORAVA
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 95-96

\mathbb{C}^2 / Γ

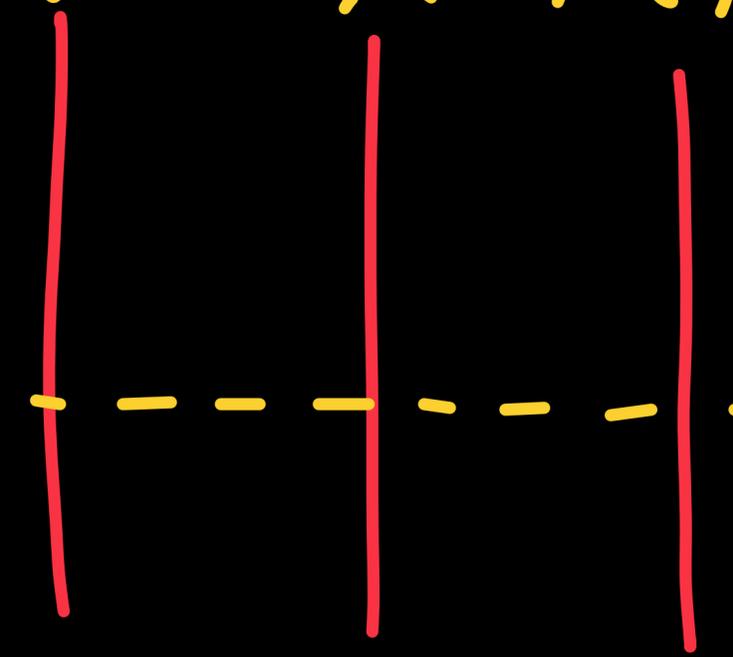
[ASPINWALL-MORRISON 98 ... LOTS OF RESULTS... FONT, GARCIA]
 [ETXEBARRIA, LÜST, MEYRHOFER 17 ... [1, 2] SYSTEMATIC]
 UNDERSTANDING

$\mu_1: \Gamma \rightarrow E_8$



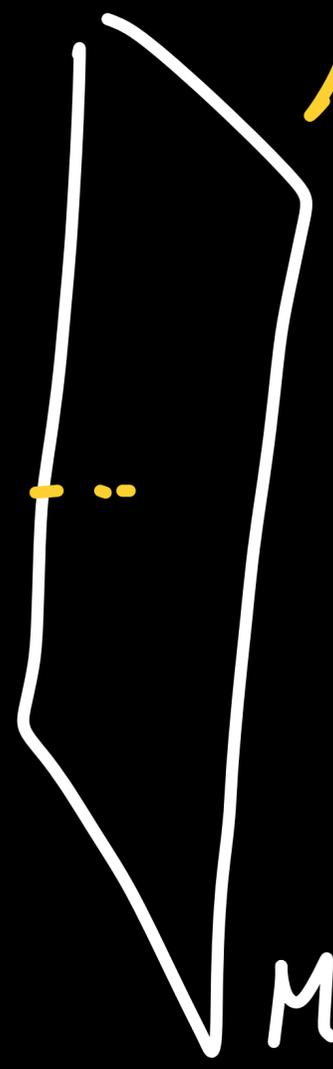
M^9

$F_2 = C(\mu_2(\Gamma), E_8)$



N MS

$\mu_2: \Gamma \rightarrow E_8$



M^9

HOŘAVA
WITTEN
95-96

\mathbb{C}^2 / Γ

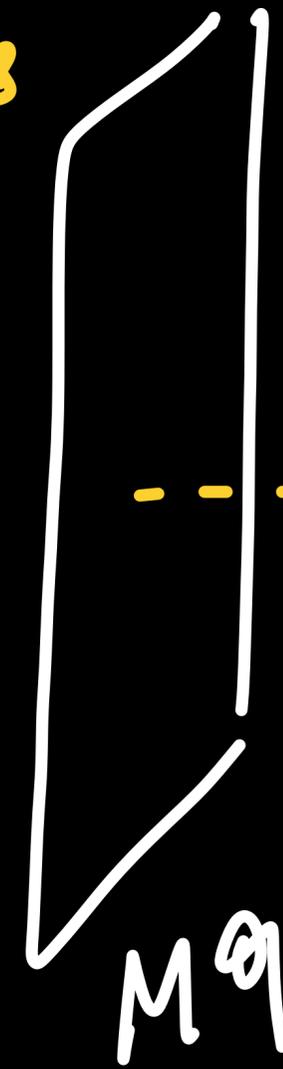


$\equiv T(M_1, \Gamma)$

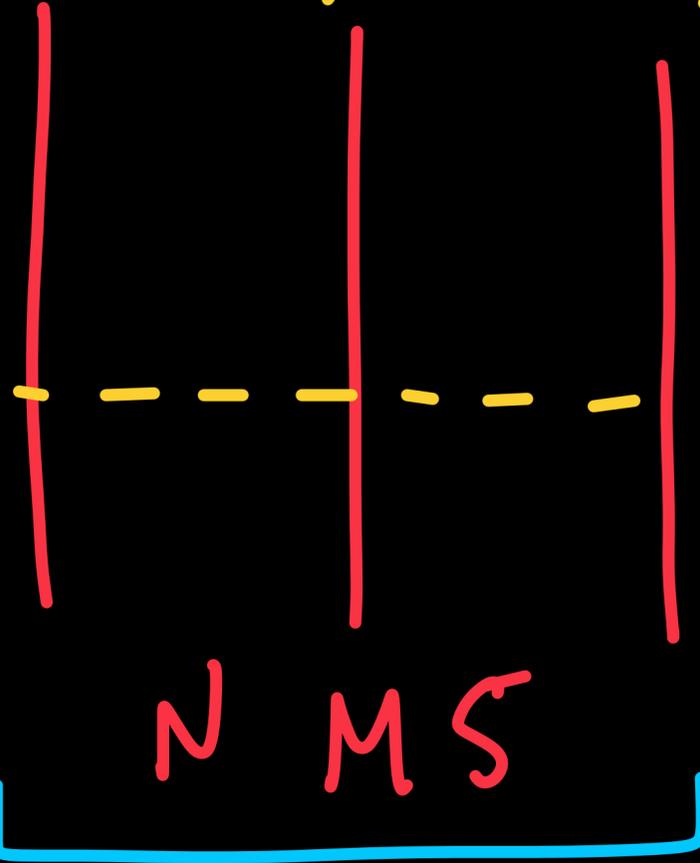
RUELICUS
FREY 17

: 1 ORBI-INSTANTON

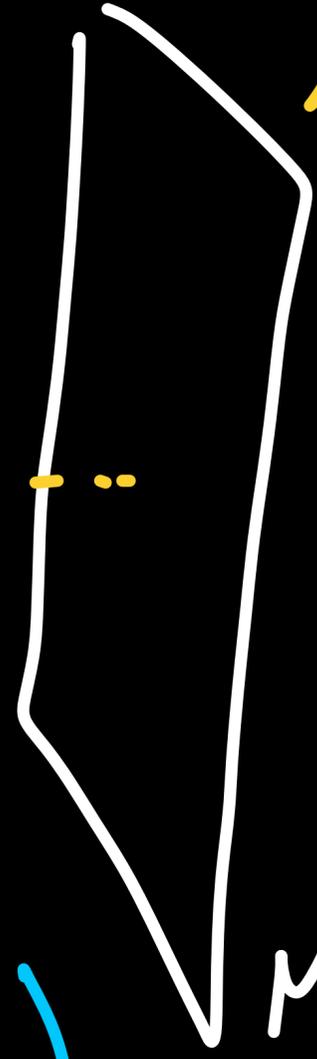
$\mu_1: \Gamma \rightarrow E_8$



$F_2 = C(\mu_2(\Gamma), E_8)$



$\mu_2: \Gamma \rightarrow E_8$



HOŘAVA
WITTEN
95-96

\mathbb{C}^2 / Γ

$\equiv T_{N-2}(g_\Gamma, g_\Pi)$

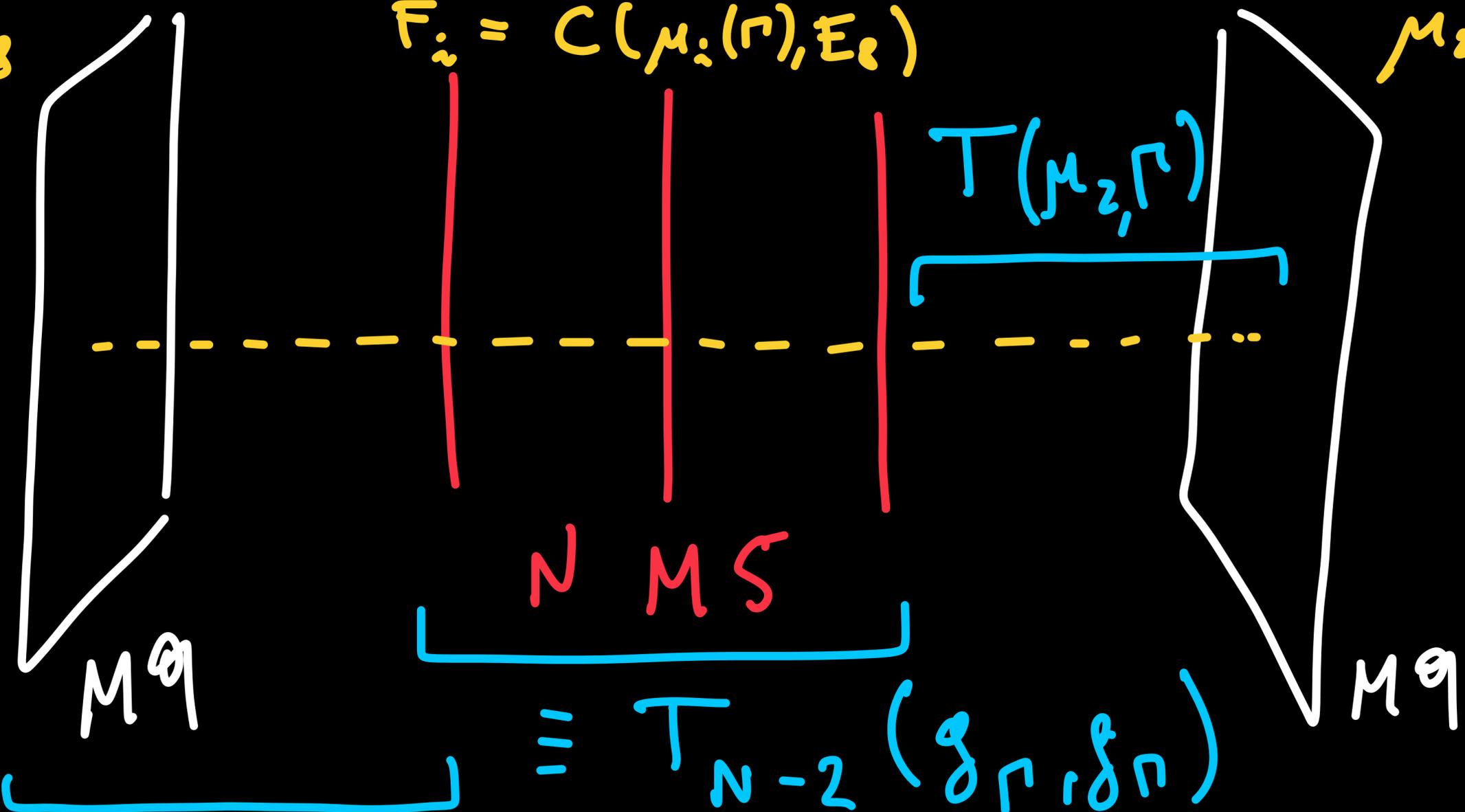
$\equiv T(M_1, \Gamma)$

RUDELIUS
FREY 17

CONFORMAL
MATTER

[02, HECKLAD,
TOMASIELLO,
VAFA 14]

$$\mu_1: \Gamma \rightarrow E_8$$



$$\mu_2: \Gamma \rightarrow E_8$$

HOŘAVA
WITTEN
95-96

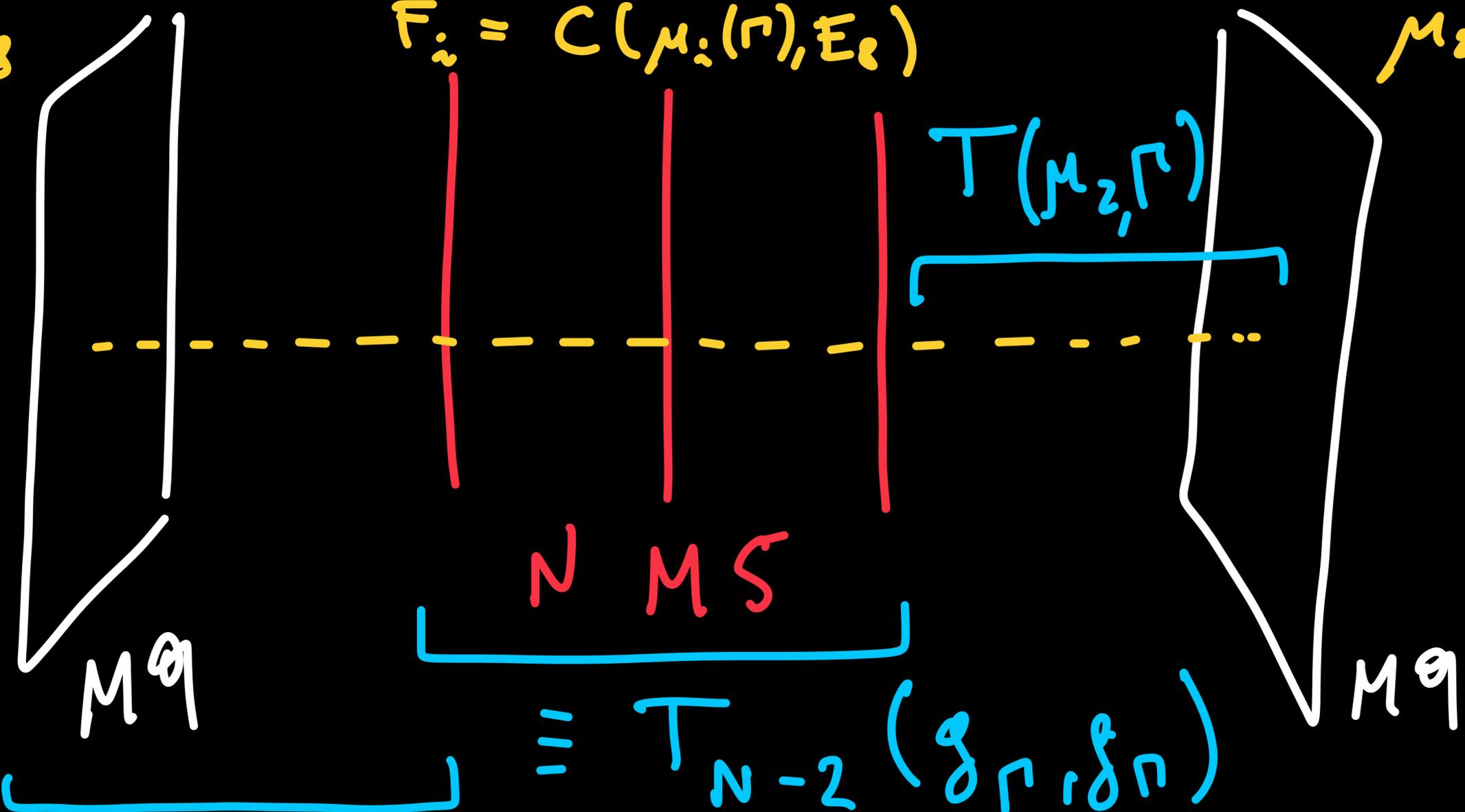
$$\mathbb{C}^2 / \Gamma$$

$$\equiv T(\mu_1, \Gamma)$$

RUELICUS
FREY 17

CONFORMAL
MATTER

$\mu_1: \Gamma \rightarrow E_8$



$F_2 = C(\mu_2(\Gamma), E_8)$

$\mu_2: \Gamma \rightarrow E_8$

HOŘAVA
WITTEN
95-96

\mathbb{C}^2 / Γ

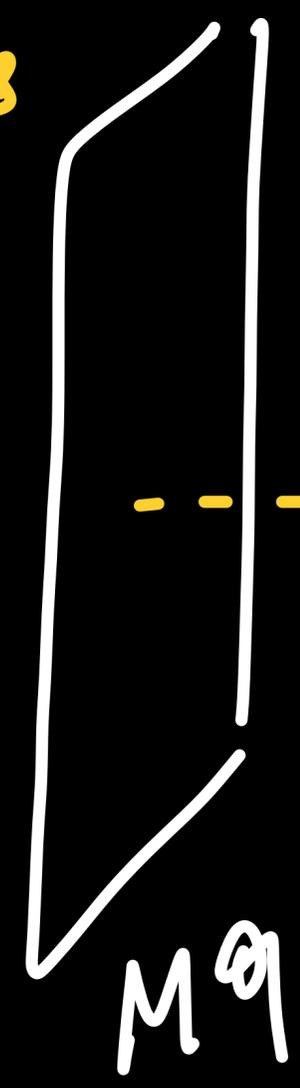
$\equiv T(\mu_1, \Gamma)$

RUDELIUS
FREY 17

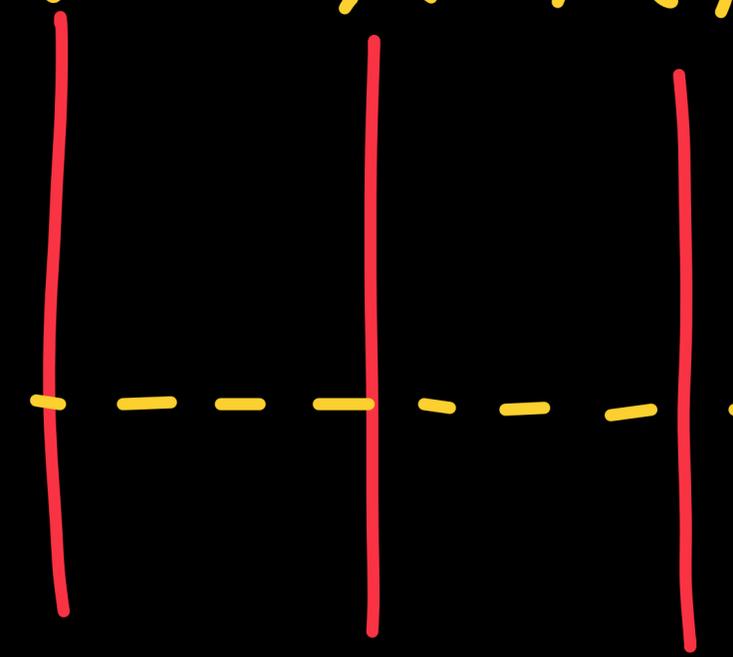
CONFORMAL
MATTER

ALL
KNOWN
6D
SCFTs!

$\mu_1: \Gamma \rightarrow E_8$

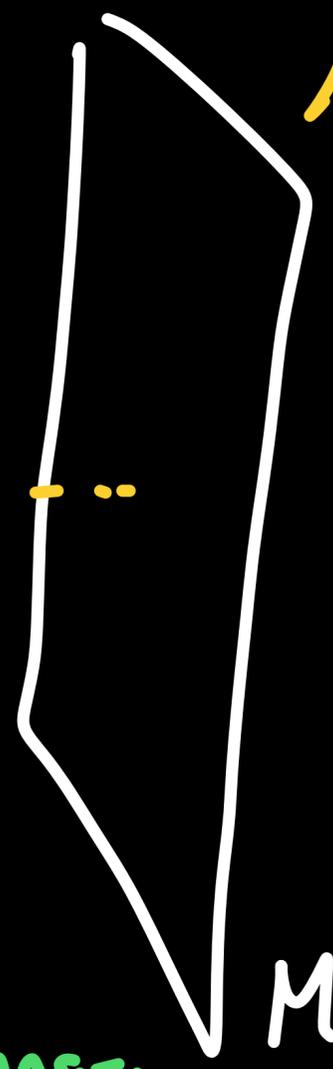


$$F_2 = C(\mu_2(\Gamma), E_8)$$



N M5

$\mu_2: \Gamma \rightarrow E_8$



✓
 HORAVA
 WITTEN
 95-96

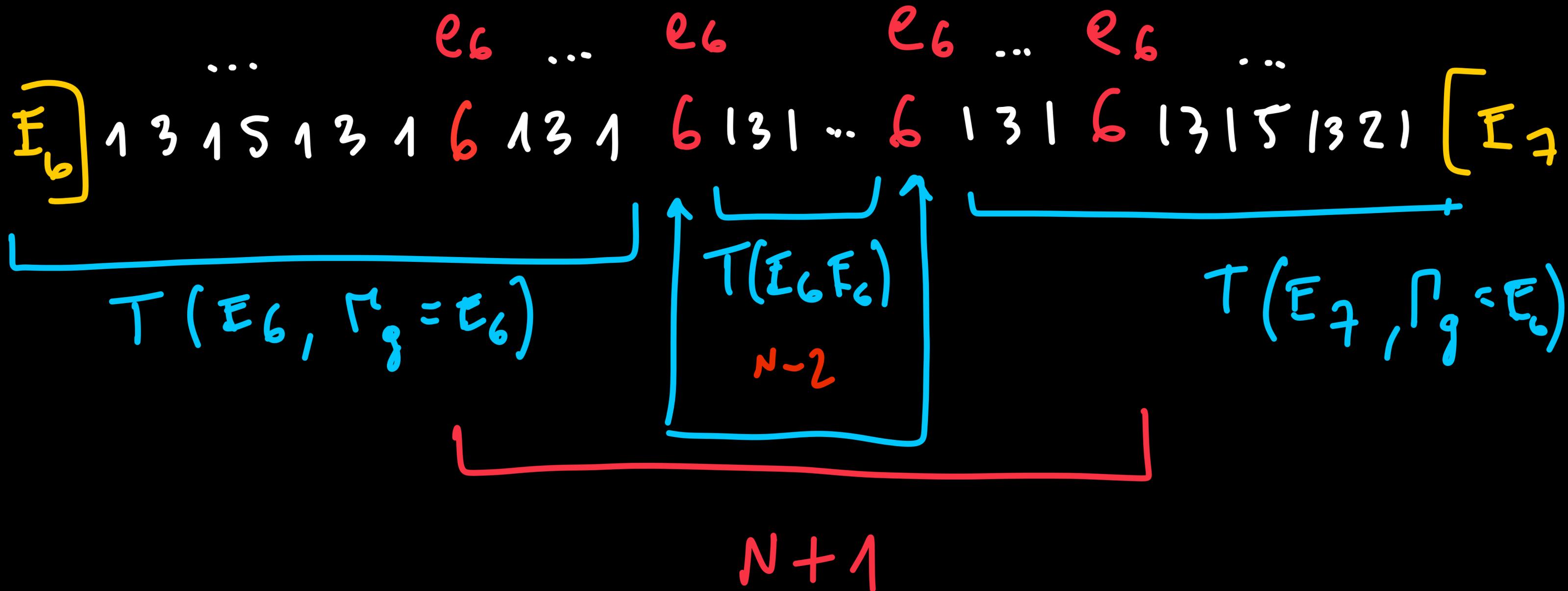
$$\mathbb{C}^2 / \Gamma$$

FUSION: HECKMAN, RUDOLPH, TOPAZIS 13
 DT, LOCKHART 18

GENERALIZED
 QUIVER!

$$\equiv T(\mu_1, \Gamma) \begin{array}{c} \textcircled{g_\Gamma} \\ \hline \end{array} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{g_\sigma} T(\mu_2, \Gamma)$$

EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$



IS T-DUAL TO [MUYANG'S TALK]

$$\begin{array}{cccccc} \text{SP}_{7+N} & \text{SO}_{22+4N} & \text{SP}_{6+3N} & \text{SU}_{8+4N} & \text{SU}_{4+2N} \\ [SO_{22}] & 1 & 4 & 1 & 2 & 2 \\ & & [SP_1] & [SO_2] & & \end{array}$$

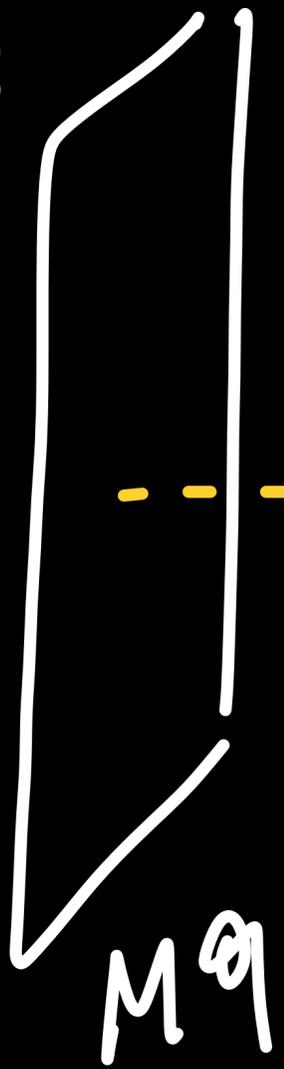
$$\hat{k}_L = 2$$

$$\hat{k}_R = 69 + 24N$$

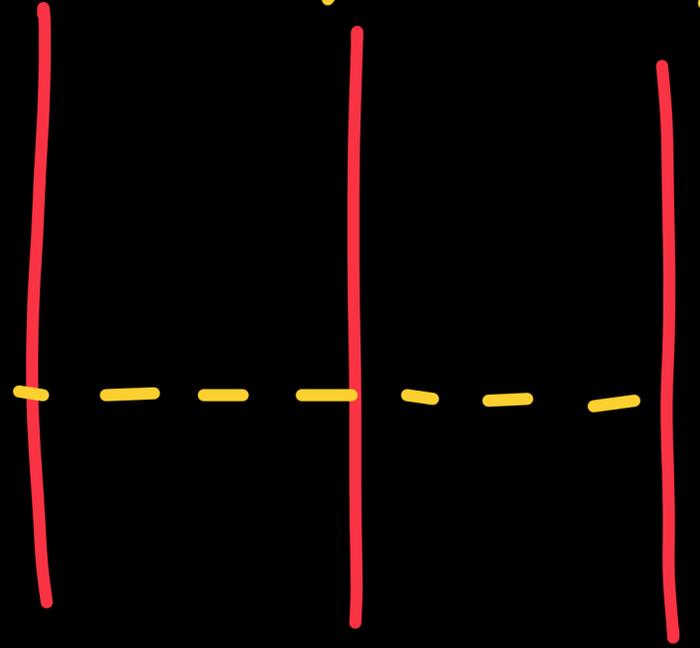
2-GROUP
STRUCTURE

[INTRILIGATOR BLUM 97]

$\mu_1: \Gamma \rightarrow E_8$

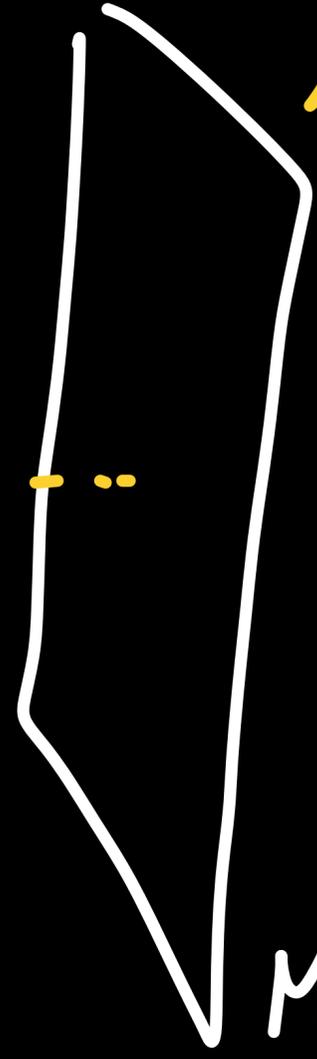


$F_2 = C(\mu_2(\Gamma), E_8)$



N M5

$\mu_2: \Gamma \rightarrow E_8$

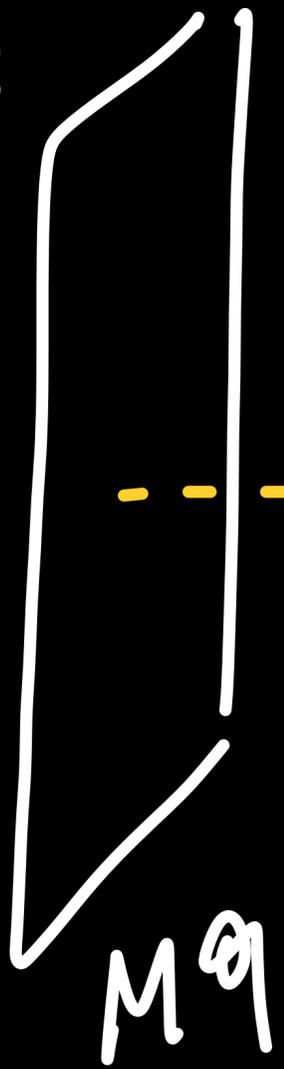


HOŘAVA
WITTEN
95-96

\mathbb{C}^2 / Γ

$$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{g_0} T(\mu_2, \Gamma)$$

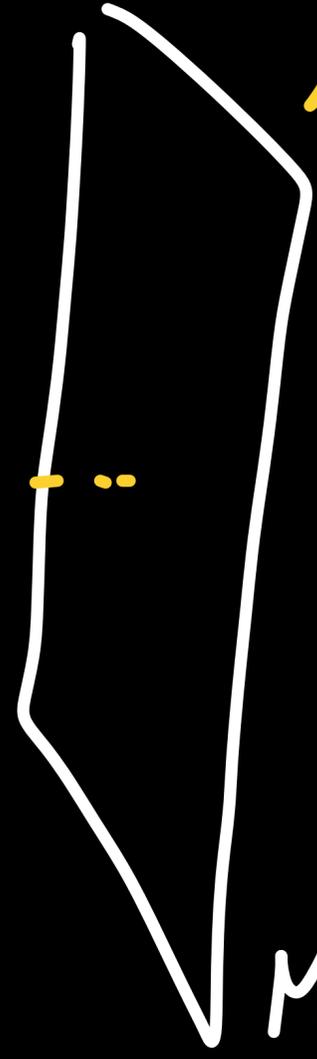
$\mu_1: \Gamma \rightarrow E_8$



$F_2 = C(\mu_2(\Gamma), E_8)$

REMOVE
NS5

$\mu_2: \Gamma \rightarrow E_8$



✓
HOŘAVA
WITTEN
95-96

\mathbb{C}^2 / Γ

M^9

M^9

$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} T_{N-2}(g_\Omega, g_\Omega) \xrightarrow{g_\Omega} T(\mu_2, \Gamma)$

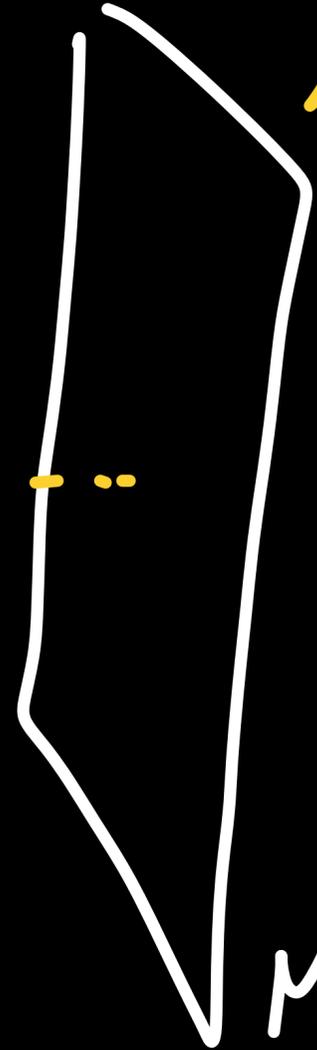
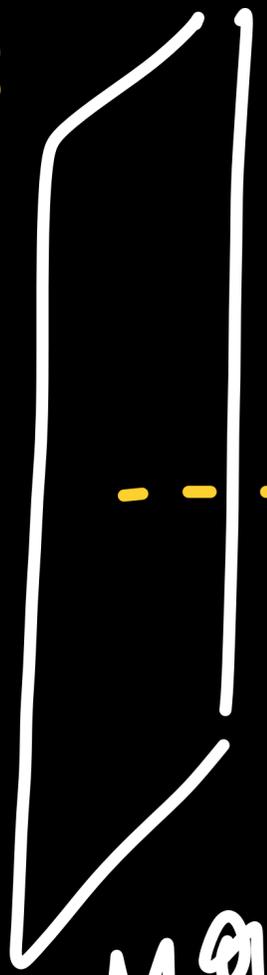
$$\mu_1: \Gamma \rightarrow E_8$$

$$F_2 = C(\mu_2(\Gamma), E_8)$$

$$\mu_2: \Gamma \rightarrow E_8$$

✓
HOŘAVA
WITTEN
95-96

$$\mathbb{C}^2 / \Gamma$$



M^4

M^4

REDUCE

REDUCE

REMOVE
NSS

≡

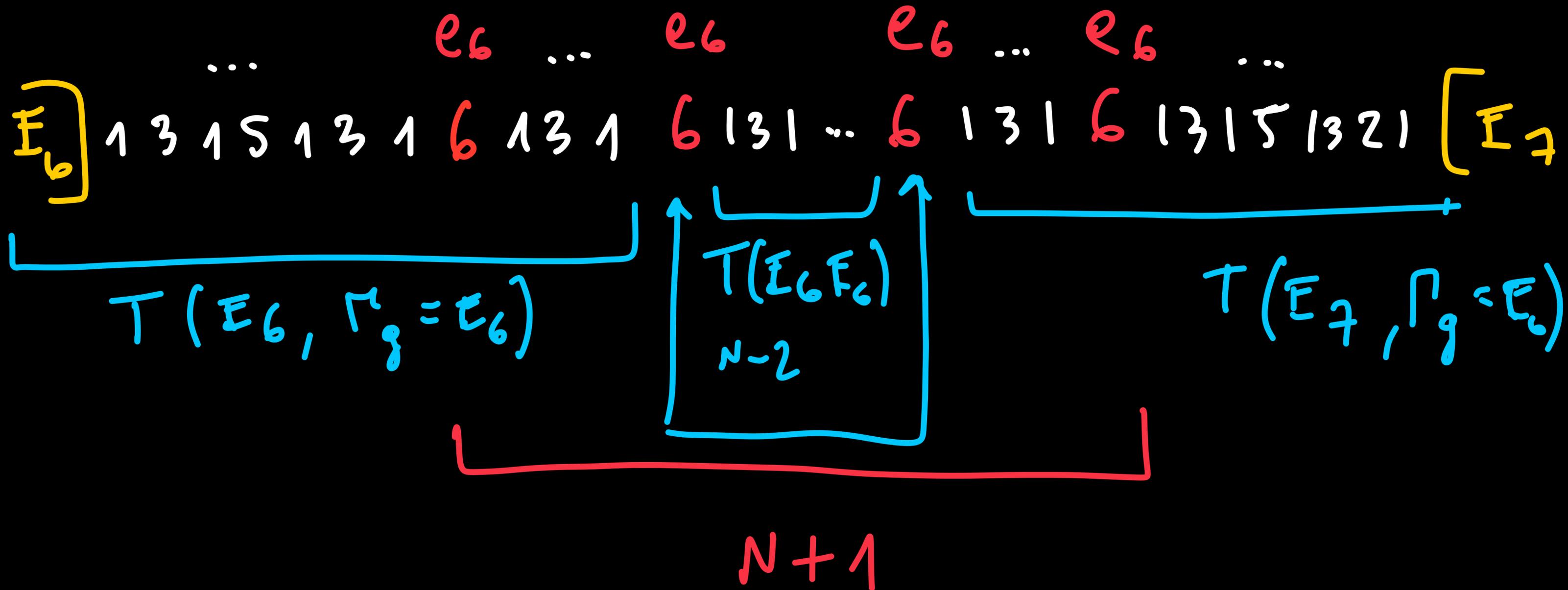
$$T_0(\mu_1, \Gamma)$$



FUSE TOGETHER

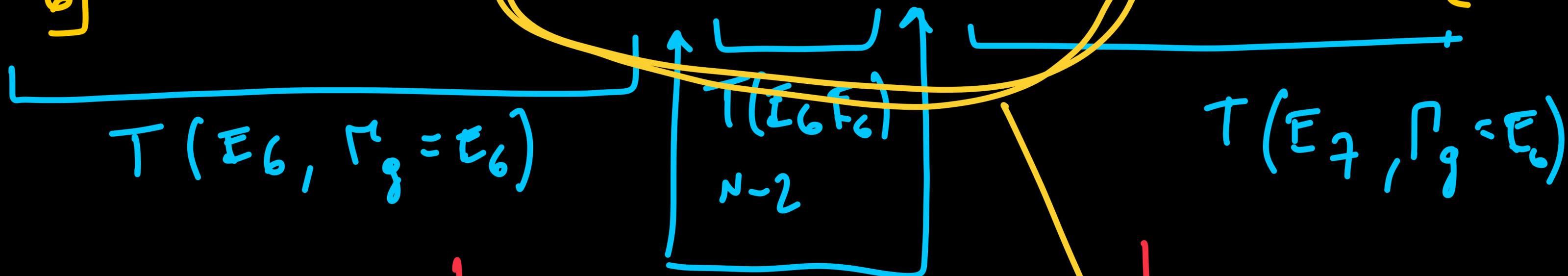
$$T_0(\mu_2, \Gamma)$$

EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$



EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$

$\left[E_6 \right]$... e_6 ... e_6 ... e_6 ... $\left[E_7 \right]$
 1 3 1 5 1 3 1 6 1 3 1 6 1 3 1 ... 6 1 3 1 6 1 3 1 5 1 3 2 1



EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$

$$\left[E_6 \right] \begin{matrix} \dots \\ 1 & 3 & 1 & 5 & 1 & 3 & 1 \end{matrix}$$

$$T_0(E_6, \Gamma_g = E_6)$$

REDUCED

$$e_6 \\ 6$$

$$\underbrace{\begin{matrix} \dots \\ 1 & 3 & 1 & 5 & 1 & 3 & 2 & 1 \end{matrix}} \left[E_7 \right]$$

$$T_0(E_7, \Gamma_g = E_6)$$

REDUCED

IN THE
 $N \rightarrow 0$ LIMIT!

EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$

$$\left[E_6 \right] \overset{\dots}{1} \overset{\dots}{3} \overset{\dots}{1} \overset{\dots}{5} \overset{\dots}{1} \overset{\dots}{3} \overset{\dots}{1} \overset{e_6}{6} \overset{\dots}{1} \overset{\dots}{3} \overset{\dots}{1} \overset{\dots}{5} \overset{\dots}{1} \overset{\dots}{3} \overset{\dots}{2} \overset{\dots}{1} \left[E_7 \right]$$

$T_0(E_6, \mu_g = e_6)$

REDUCED

$T_0(E_7, \mu_g = e_6)$

REDUCED

IS THE 6D LST FROM $\text{Het}_{E_8 \times E_8} / X_{e_6}^4, \mu_1, \mu_2$

IS T-DUAL TO $N \rightarrow 0$ LIMIT OF:

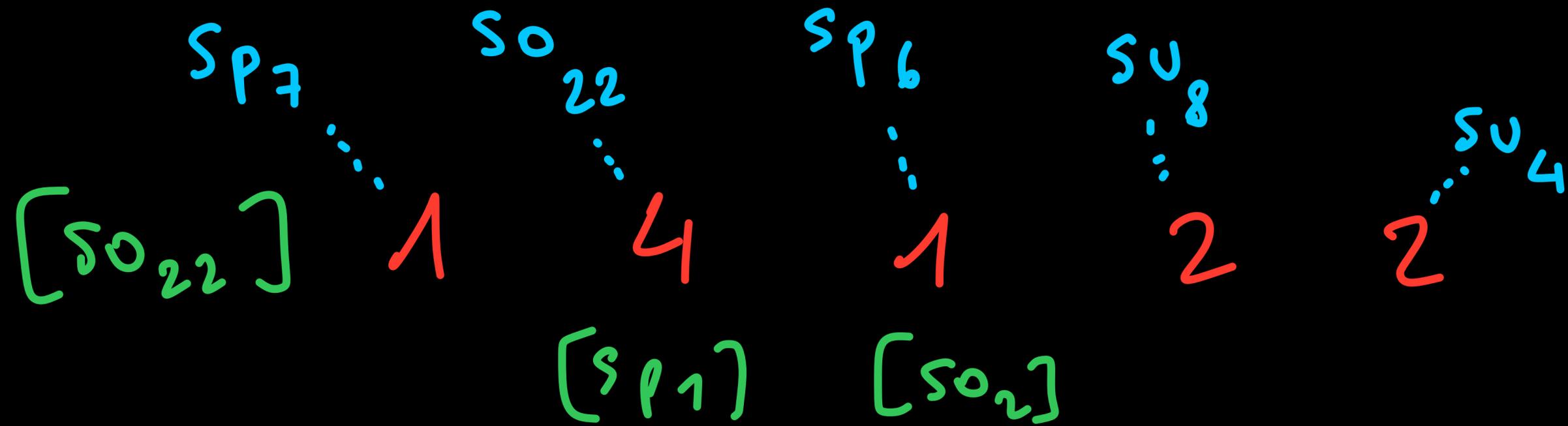
$$\begin{array}{cccccc}
 \text{SP}_{7+N} & \text{SO}_{22+4N} & \text{SP}_{6+3N} & \text{SU}_{8+4N} & \text{SU}_{4+2N} & \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \\
 [SO_{22}] & 1 & 4 & 1 & 2 & 2 \\
 & & [SP_1] & [SO_2] & &
 \end{array}$$

$$\hat{k}_L = 2 \quad \hat{k}_R = 69 + 24N$$

2-GROUP
STRUCTURE

[INTRILIGATOR BLUM 97]

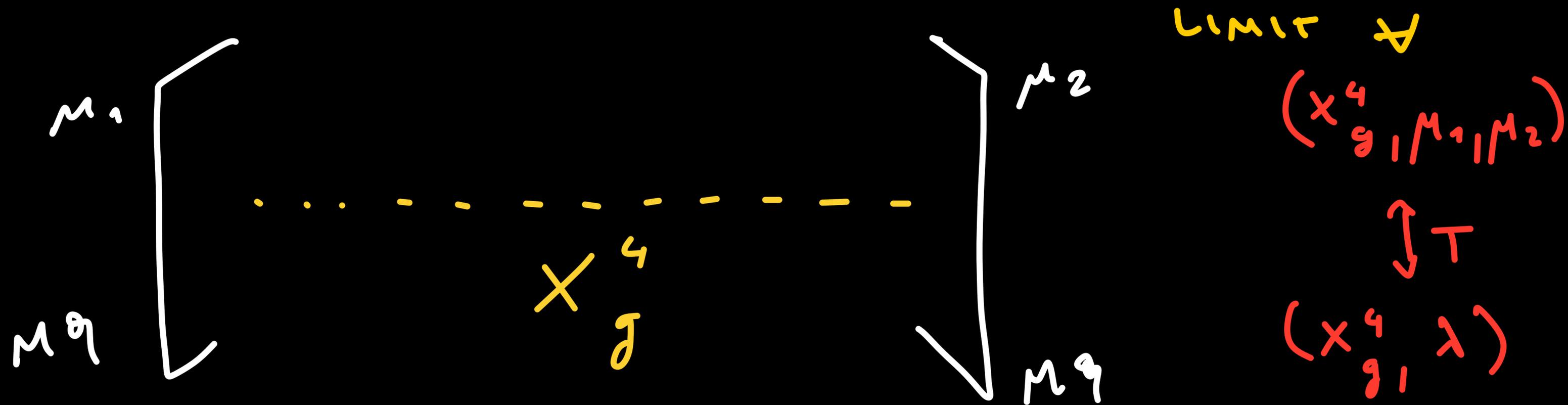
IS T-DUAL TO



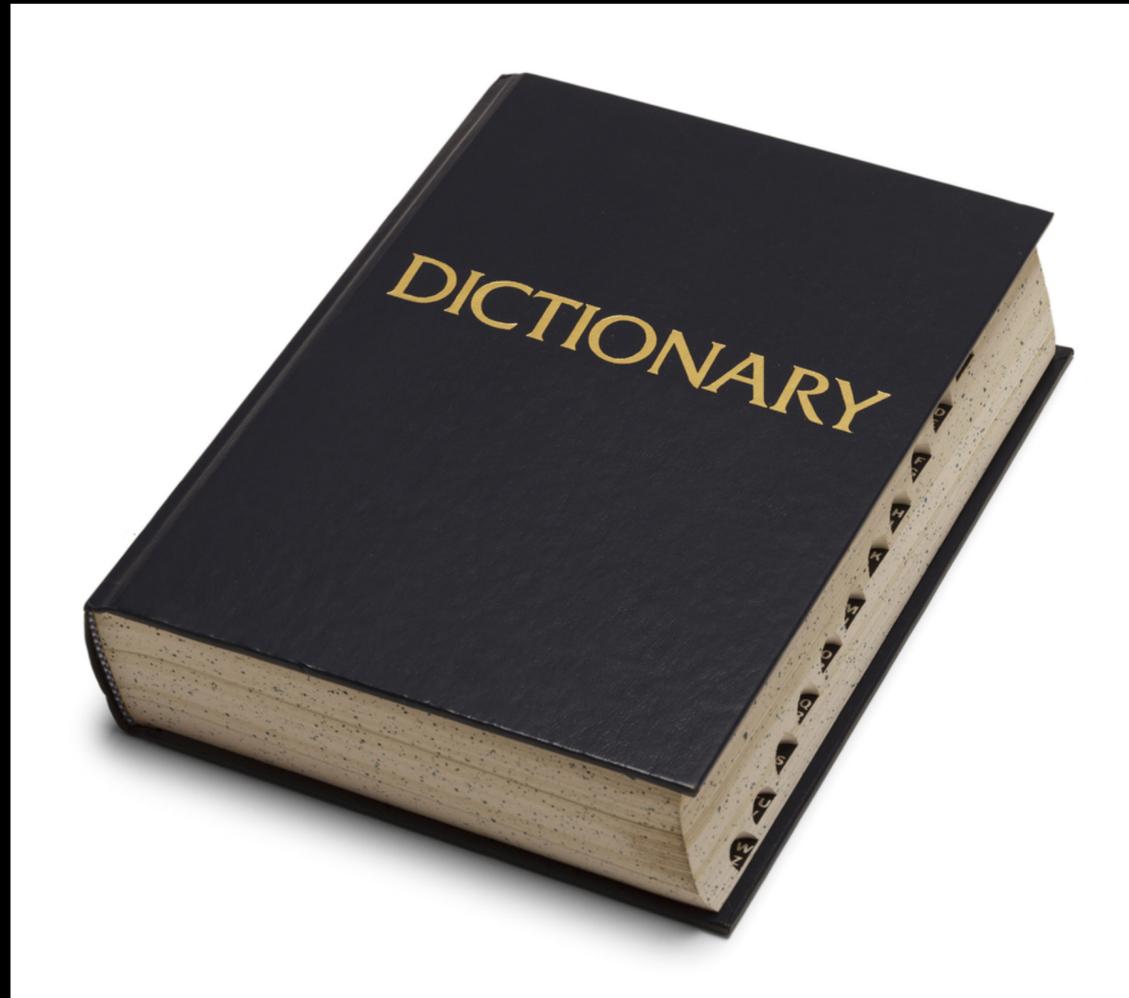
$\hat{k}_\rho = 2 \quad \hat{k}_R = 69$ 2-GROUP STRUCTURE

IS THE GD LST : $Het_{Spin(32)/\mathbb{Z}_2} / (x^4, \lambda)$

IN THE PAPER: GENERAL PRESCRIPTION TO
CAPTURE GEOMETRIC ENGINEERING



ALSO COMMENT ON TURNING ON TORSIONAL
 C_3 FLUX ON $S^3 / \Gamma_g \rightarrow$ NON SIMPLY LACED
 VERSIONS. [TRIPLES FLUXES]
 & STRINGS



2

GEOMETRIC
ENGINEERING
OF M-THEORY
ON SINGULAR
 G_2 HOLONOMY
CONES

M / X^7 : G_2 HOLONOMY \rightarrow

4D N=1
SQFT

• RICH DYNAMICS

SYMMETRY BREAKING...

CONFINEMENT...

• NON-PERTURBATIVE CORRECTIONS

EUCLIDEAN M2 BRANES ON

CALIBRATED ASSOCIATIVE 3-CYCLES

\Rightarrow GEOMETRIC ENGINEERING IS

HARDER: QUANTUM GEOMETRY

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

• $X_1^7 = \text{CONE} (S^3 \times S^3)$ [BRYANT SALAMON 89]

• $X_2^7 = \text{CONE} (CP^3)$ $ds_{X^7}^2 = dr^2 + r^2 ds_{X^6}^2$

• $X_3^7 = \text{CONE} (SU(3) / U(1) \times U(1))$

PHYSICS: [ATIYAH, MALDACENA, VAFSA 00; ACHARYA 00;
ATIYAH-WITTEN 01; ACHARYA-WITTEN 01; CVETIC,
GIBBONS, LÜ, POPE 01]

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

$$\cdot X_1^7 = \text{CONE} (S^3 \times S^3) = \mathbb{S}(S^3)$$

$$\cdot X_2^7 = \text{CONE} (CP^3) = \Lambda_-^2(S^4)$$

$$\cdot X_3^7 = \text{CONE} (S \cup (3) / U(1) \times U(1)) = \Lambda_-^2(CP^2)$$

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

$$\cdot X_1^7 = \text{CONE} (S^3 \times S^3) = \mathbb{S}(S^3)$$

$$\cdot X_2^7 = \text{CONE} (CP^3) = \Lambda_-^2(S^4)$$

$$\cdot X_3^7 = \text{CONE} (S \cup (3) / U(1) \times U(1)) = \Lambda_-^2(CP^2)$$

WE WANT SCFTS

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

• $X_{1,1}^7 = \text{CONE}(S^3 \times S^3) = \mathbb{S}(S^3)$  QUANTUM GEOMETRY

• $X_{2,2}^7 = \text{CONE}(CP^3) = \Lambda_-^2(S^4)$

• $X_{3,3}^7 = \text{CONE}(SU(3)/U(1) \times U(1)) = \Lambda_-^2(CP^2)$

WE WANT SCFTS THAT GENERALIZE MATTER FOR 4D $N=1$ G_2 EXAMPLES

VERY INTERESTING BUT \neq QUALITATIVE FEATURES

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

$X_1^7 = \text{CONE}(S^3 \times S^3) = S(S^3)$  QUANTUM GEOMETRY

$X_2^7 = \text{CONE}(CP^3) = \Lambda_-^2(S^4)$

$X_3^7 = \text{CONE}(SU(3)/U(1) \times U(1)) = \Lambda_-^2(CP^2)$

WE WANT SCFTS THAT GENERALIZE MATTER FOR 4D $N=1$ G_2 EXAMPLES

EM2

VERY INTERESTING BUT \neq QUALITATIVE FEATURES

THERE ARE 3 EXAMPLES OF LOCAL G_2 'S

$X_1^7 = \text{CONE}(S^3 \times S^3) = S(S^3)$  QUANTUM GEOMETRY

$X_2^7 = \text{CONE}(CP^3) = \Lambda_-^2(S^4)$
 $X_3^7 = \text{CONE}(SU(3)/U(1) \times U(1)) = \Lambda_-^2(CP^2)$

WE WANT SCFTS
FOCUS ON THE OTHER
TWO EXAMPLES AND
TAKE ORBIFOLDS!

EM2

VERY INTERESTING BUT \neq QUALITATIVE FEATURES

$$\bullet X_2^7 = \text{CONE}(\mathbb{C}P^3) = \Lambda_-^2(S^4)$$

$$\bullet X_3^7 = \text{CONE}\left(\text{SU}(3)/\text{U}(1) \times \overset{\uparrow}{\text{U}(1)}\right) = \Lambda_-^2(\mathbb{C}P^2)$$

WE WANT SCFTS

FOCUS ON THE OTHER

TWO EXAMPLES AND

TAKE ORBIFOLDS!

FINITE
SUBGROUPS
OF $SO(5)$
ISOMETRY

FINITE
SUBGROUPS OF
 $PSU(3)$ ISOMETRY

LOTS OF UNEXPLORED EXAMPLES! BUT BECAUSE OF G_2 HOLONOMY: GENERIC NON-ISOLATED SINGULARITIES

$$\cdot X_2^7 = \text{CONE}(\mathbb{C}P^3) = \Lambda_-^2(S^4)$$

$$\cdot X_3^7 = \text{CONE}\left(\text{SU}(3) / \text{U}(1) \times \overset{\uparrow}{\text{U}(1)}\right) = \Lambda_-^2(\mathbb{C}P^2)$$

WE WANT SCFTS

FOCUS ON THE OTHER

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 $PSU(3)$ ISOMETRY

LOTS OF UNEXPLORED EXAMPLES! BUT BECAUSE
OF G_2 HOLONOMY **GENERIC NON-ISOLATED SINGULARITIES**

IS THIS A PROBLEM?

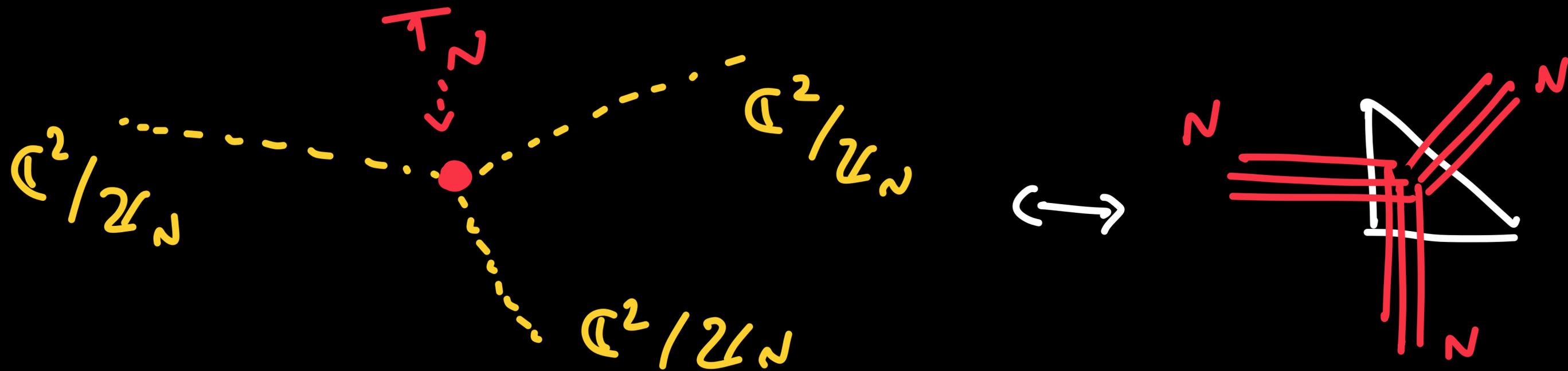
LOTS OF UNEXPLORED EXAMPLES! BUT BECAUSE OF G_2 HOLONOMY **GENERIC NON-ISOLATED SINGULARITIES**

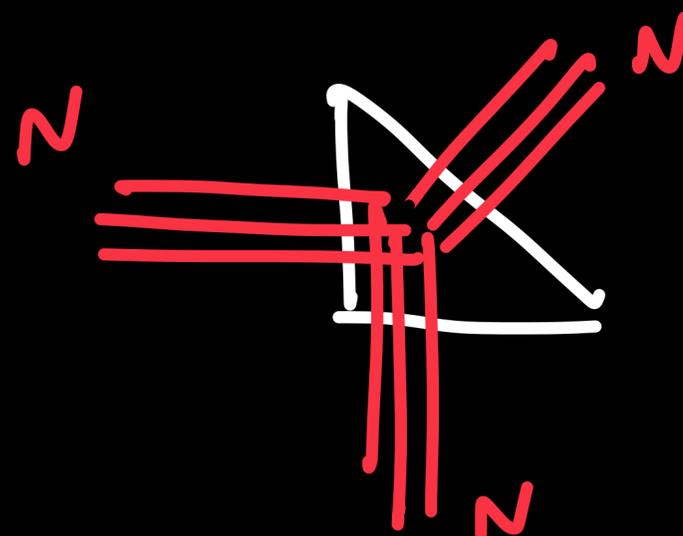
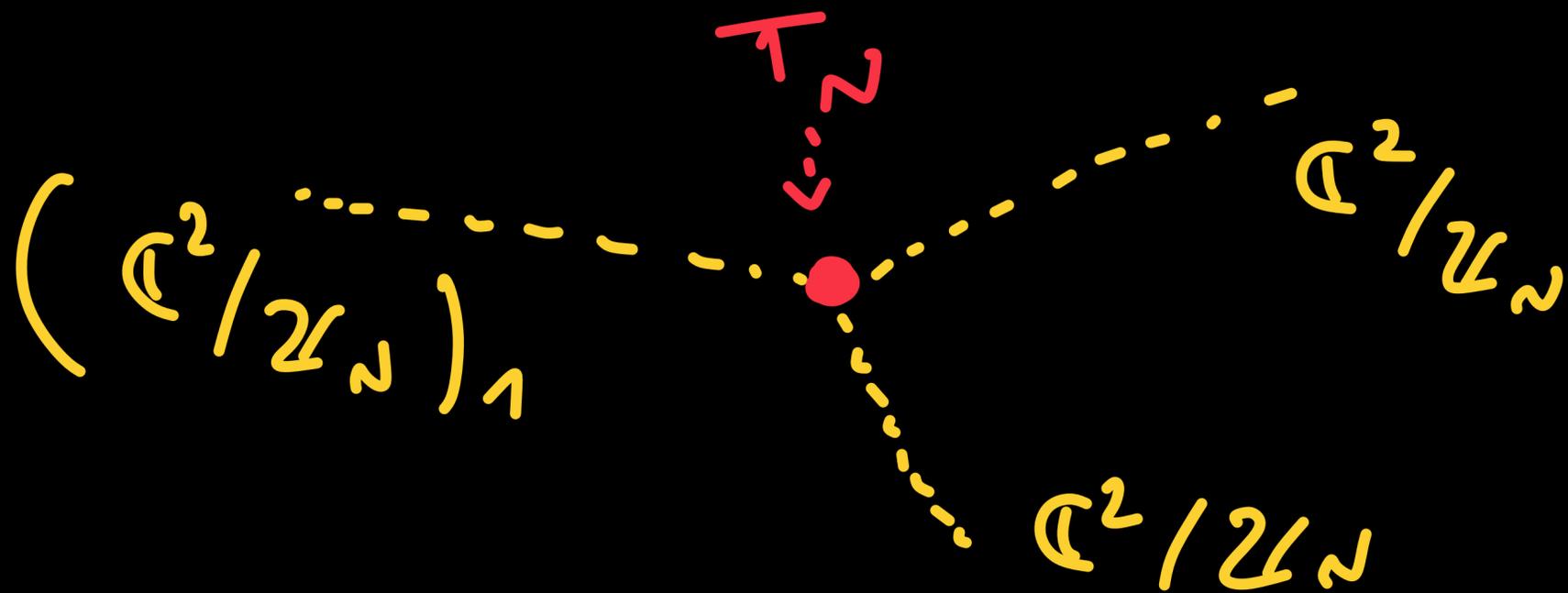
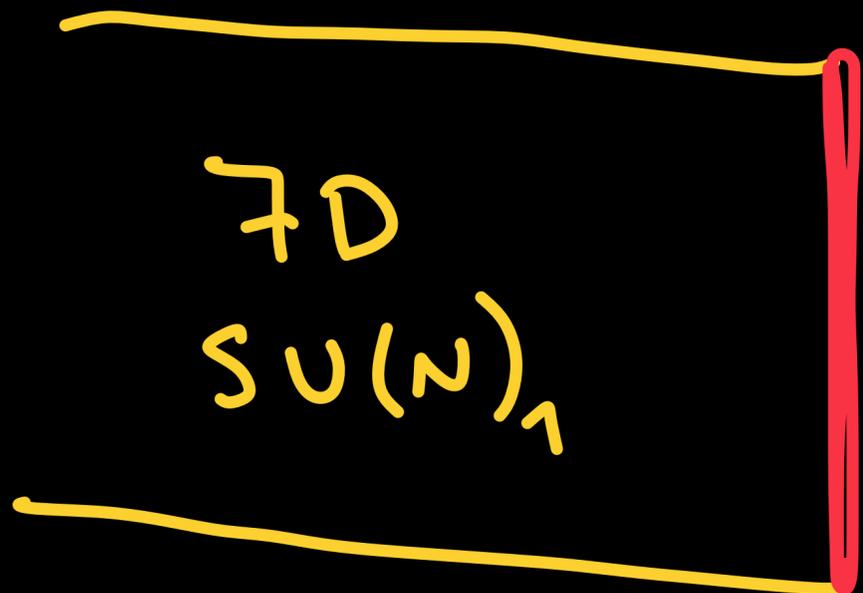
IS THIS A PROBLEM? NOT REALLY, WE ARE

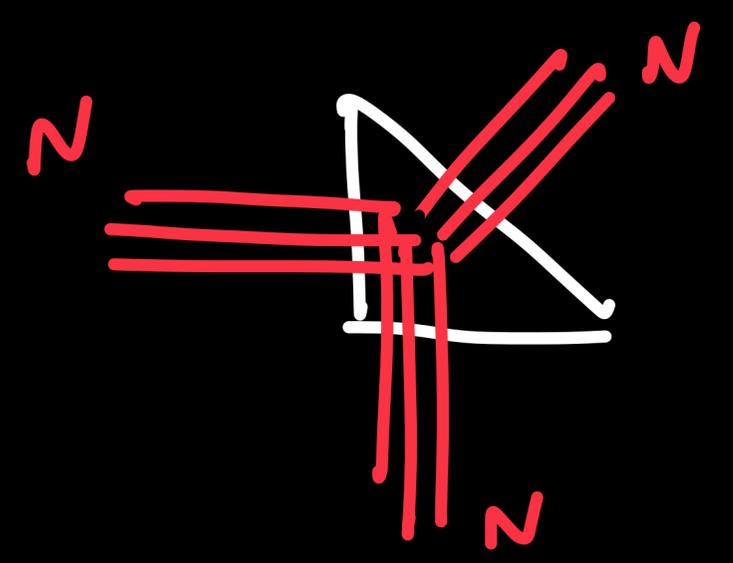
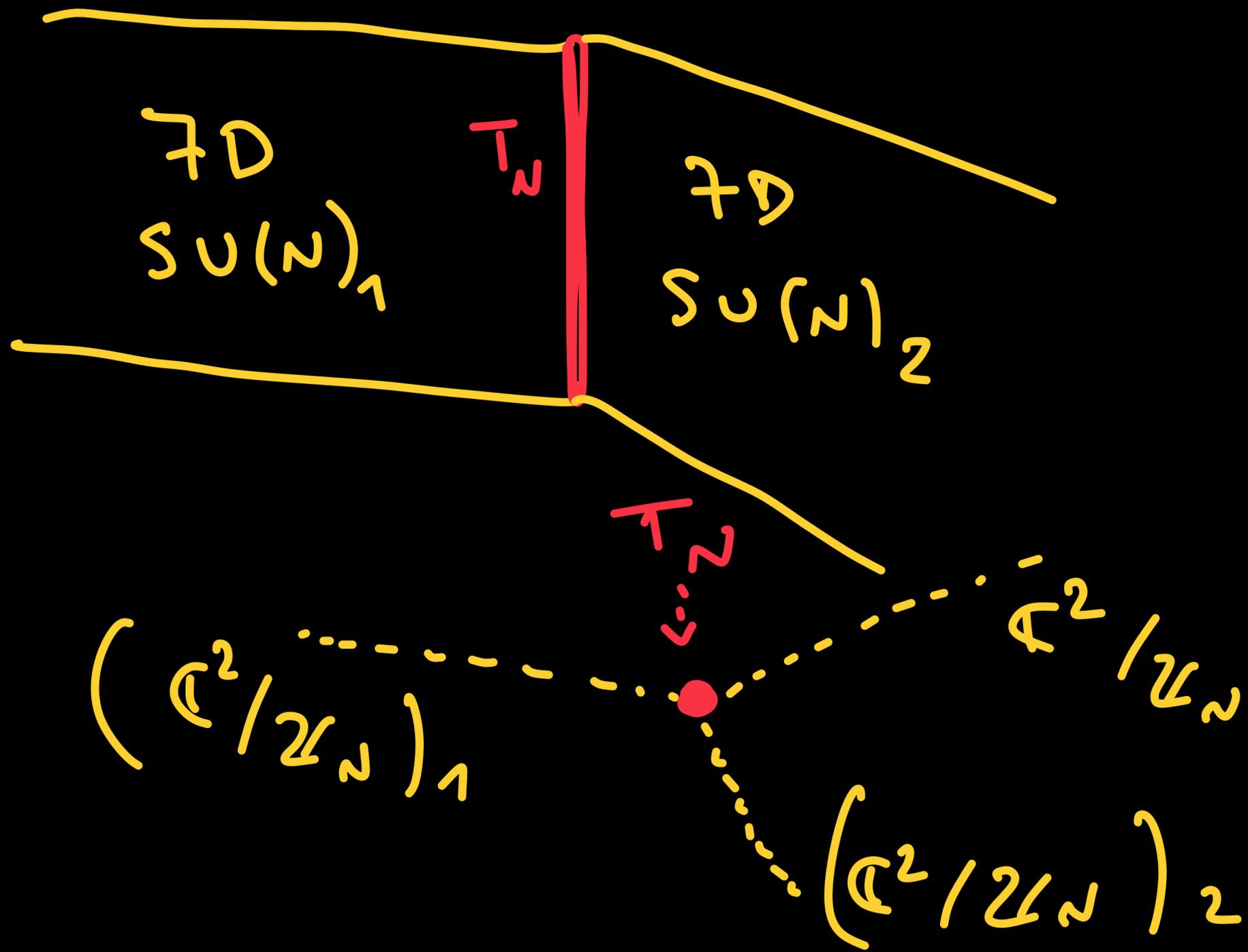
QUITE FAMILIAR WITH THESE:

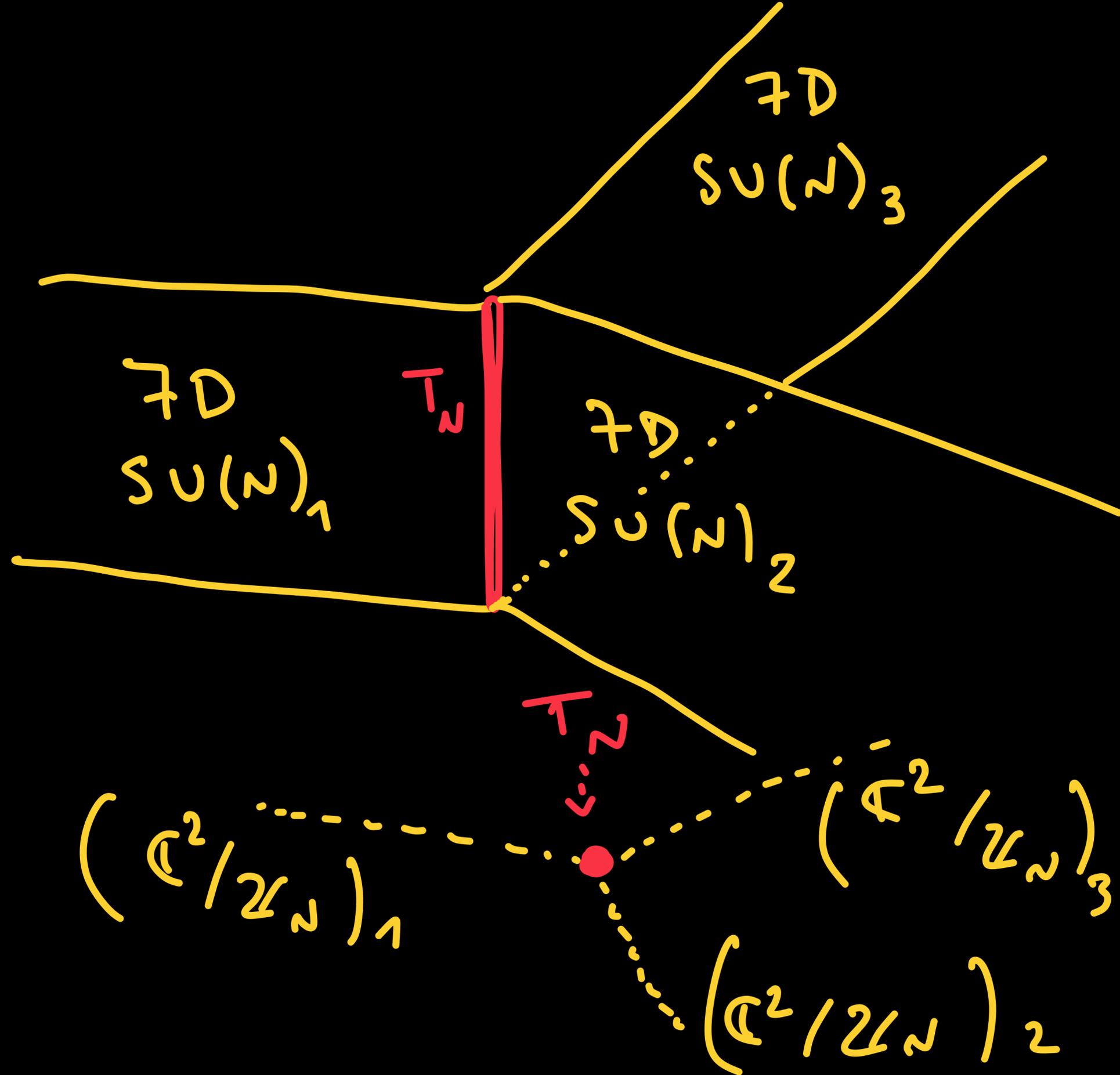
[BENINI, BENVENUTI,
TACHIKAWA 09]

$$M / \mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N \equiv T_N \quad \begin{matrix} \text{SD} \\ \text{SCFT} \end{matrix}$$

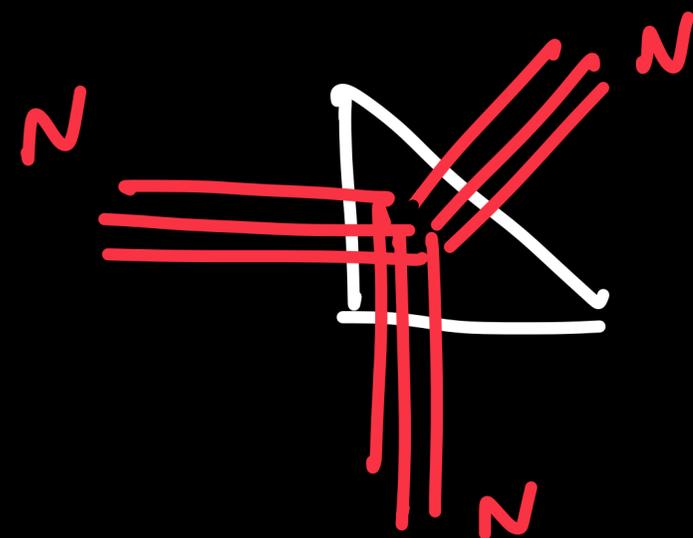


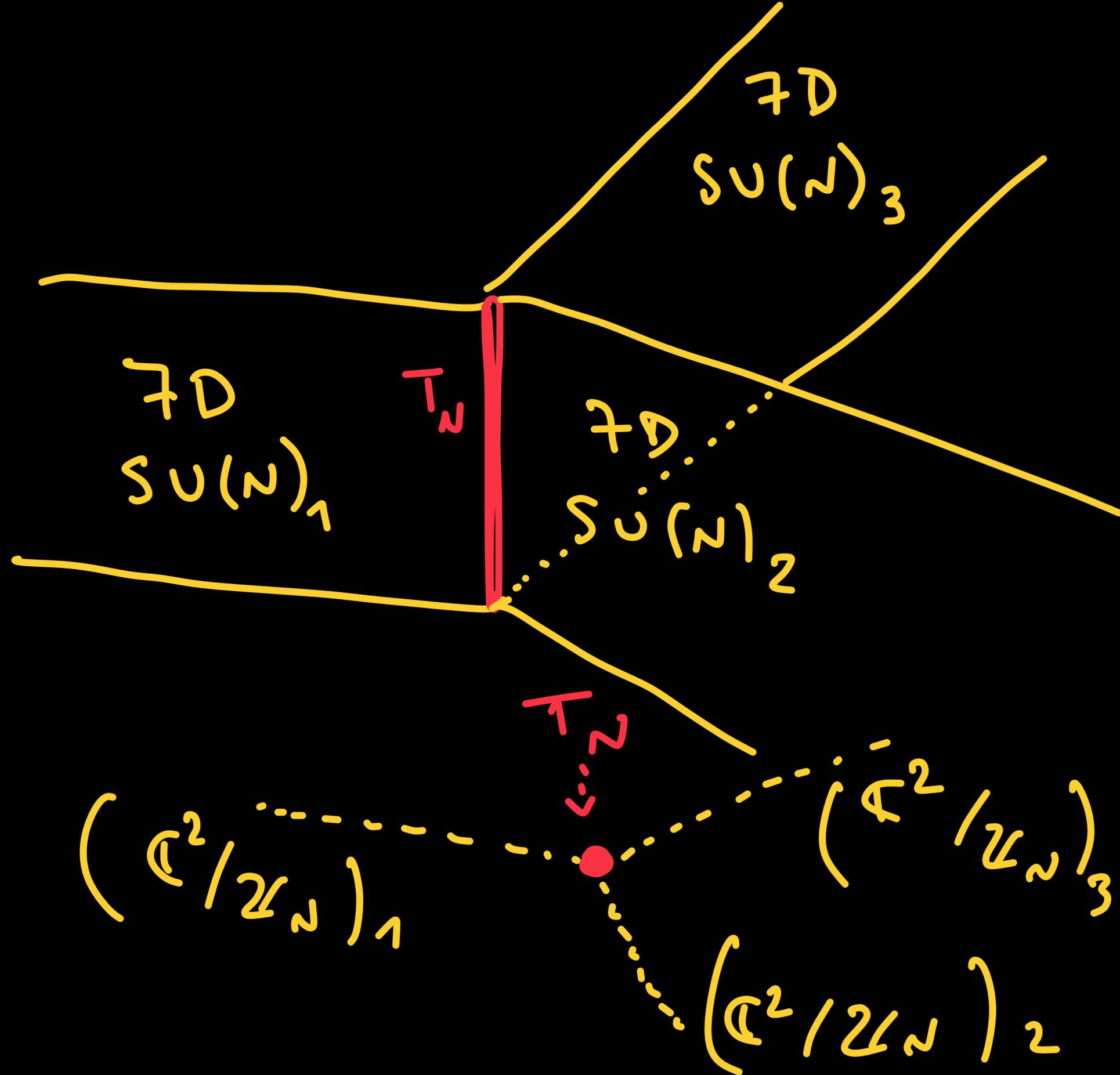






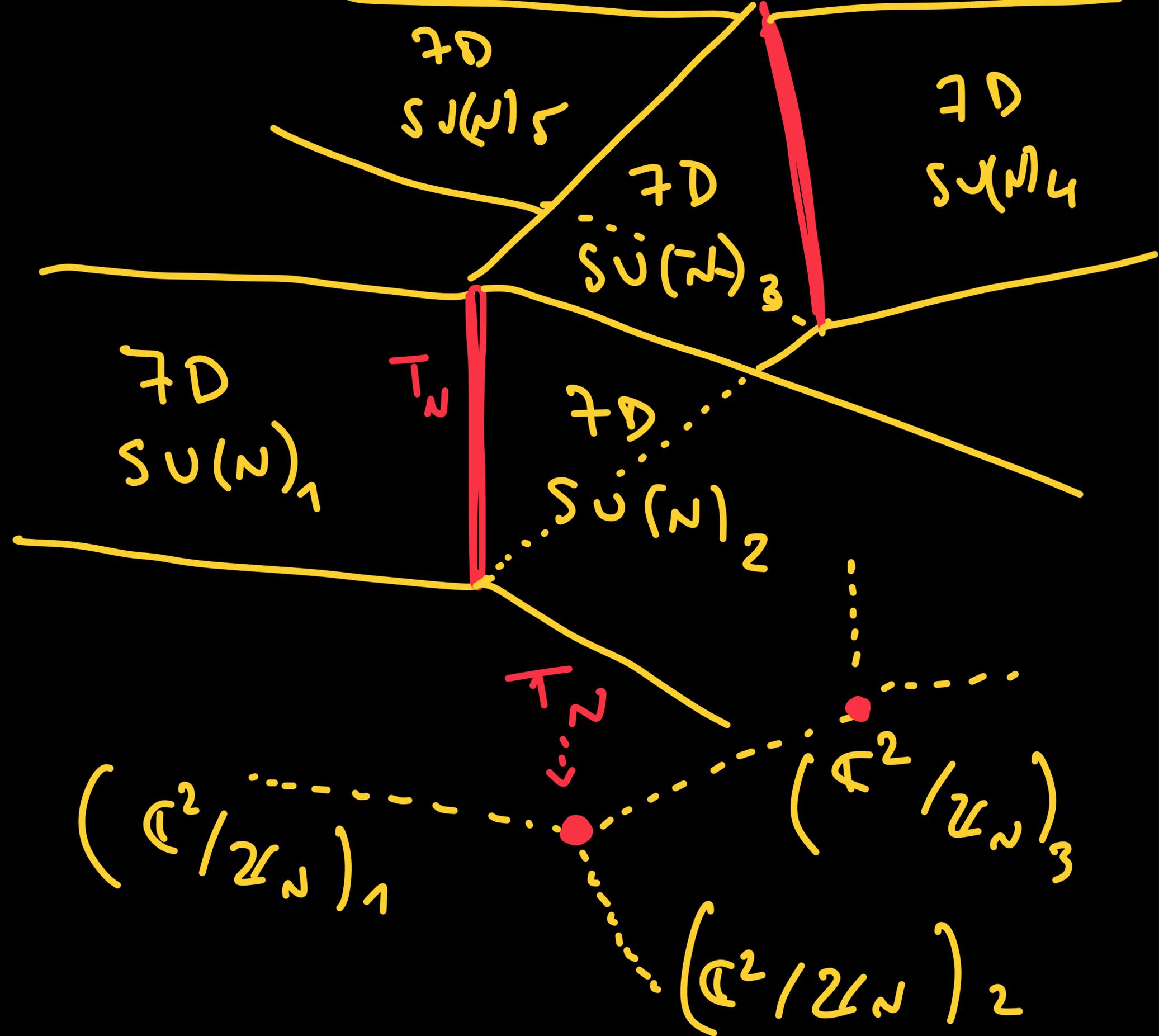
THE 5D T_N THEORY IS A CONFORMAL INTERFACE BETWEEN 7D THEORIES





THE 5D T_N THEORY IS A CONFORMAL INTERFACE BETWEEN 7D THEORIES

THE 7D DOFS DECOUPLE AND ENCODE GLOBAL SYMMETRY



GAUGING \equiv [HAYASHI,
 GLUEING JEFFERSON,
 INTERFACES KIK,
 OHMORI,
 VATA]

THE 5D T_N 19
 THEORY [APRUZZI,
 CONFORMAL SCHÄFER
 NASEBI
 WAKA]
 INTERFACE BETWEEN
 7D THEORIES

THE 7D DOFS
 DECOUPLE AND
 ENCODE
 GLOBAL SYMMETRY

FOR G_2 -ORBIFOLDS OF X^7_2 AND X^7_3
FIXED LOCUS CAN HAVE:

- CODIMENSION 4 \longleftrightarrow 7D THEORY
 M / X^4_2

FOR G_2 -ORBIFOLDS OF X^7_2 AND X^7_3
FIXED LOCUS CAN HAVE:

• CODIMENSION 4 \longleftrightarrow 7D THEORY
 M / X^4_2

• CODIMENSION 6 \longleftrightarrow 5D THEORY
 M / X^6_n
e.g. $\mathbb{C}^3 / \mathbb{Z}_n \times \mathbb{Z}_n$

FOR G_2 -ORBIFOLDS OF X^7_2 AND X^7_3
FIXED LOCUS CAN HAVE:

• CODIMENSION 4 \longleftrightarrow

7D
THEORY

M / X^4

• CODIMENSION 6 \longleftrightarrow

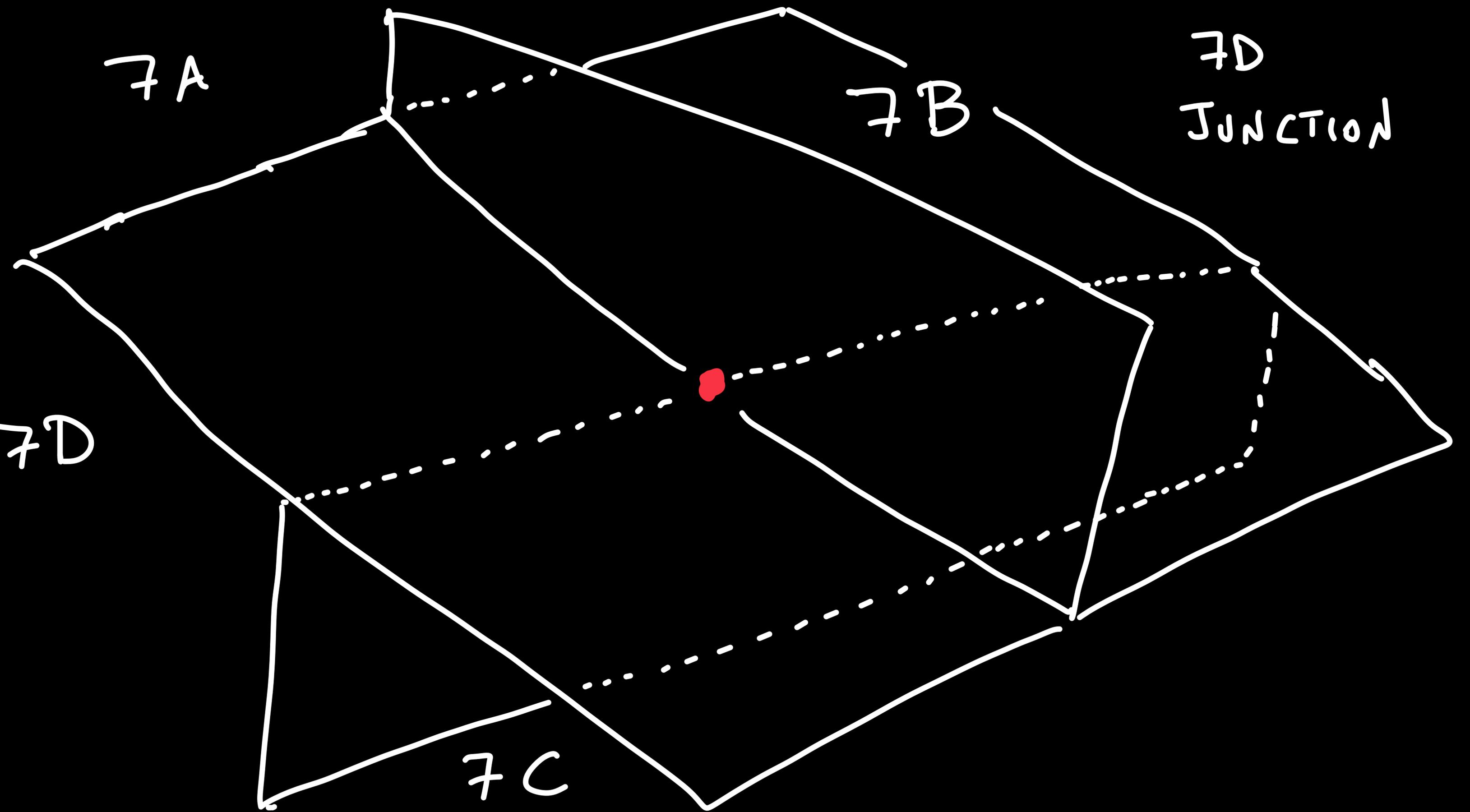
5D
THEORY

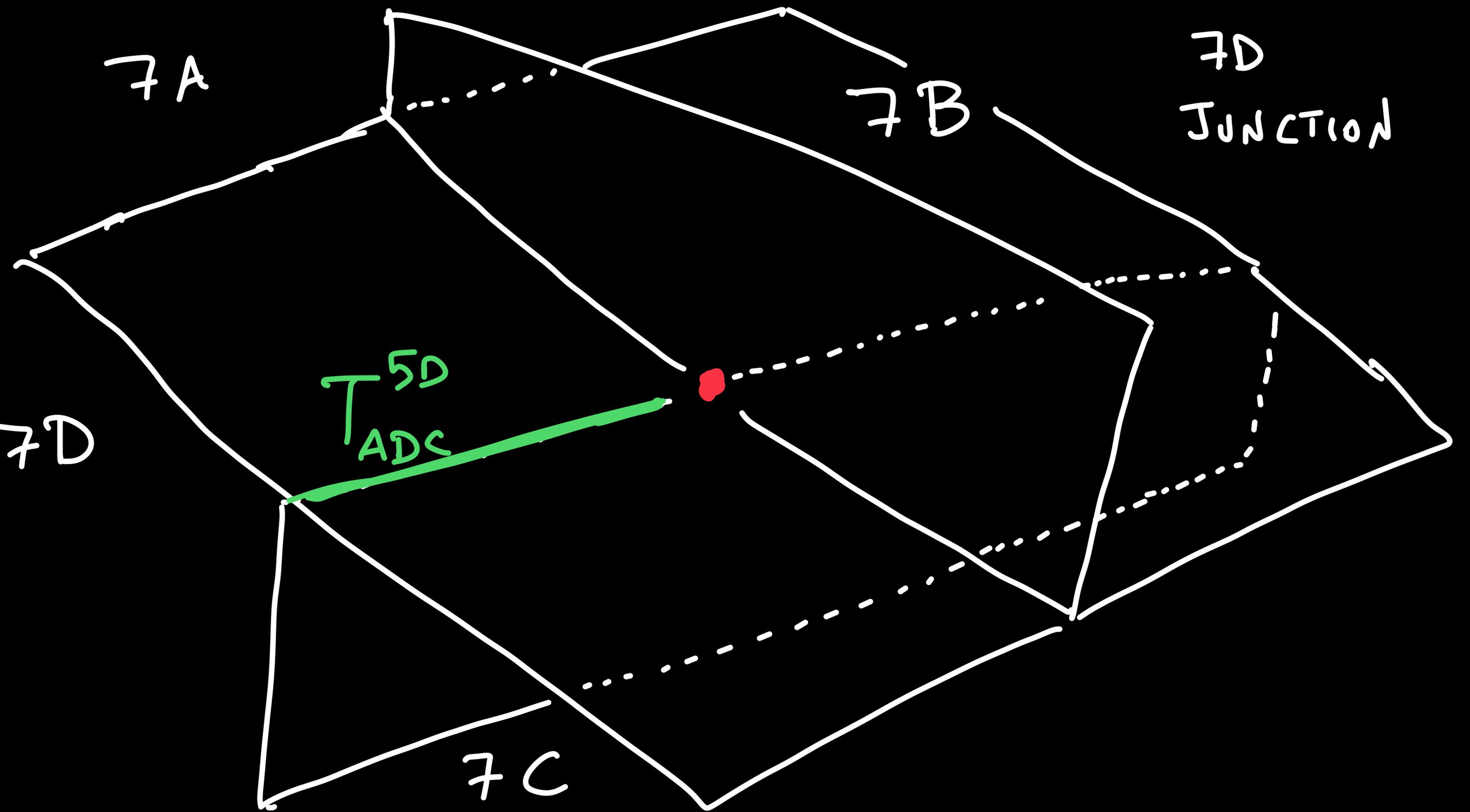
M / X^6

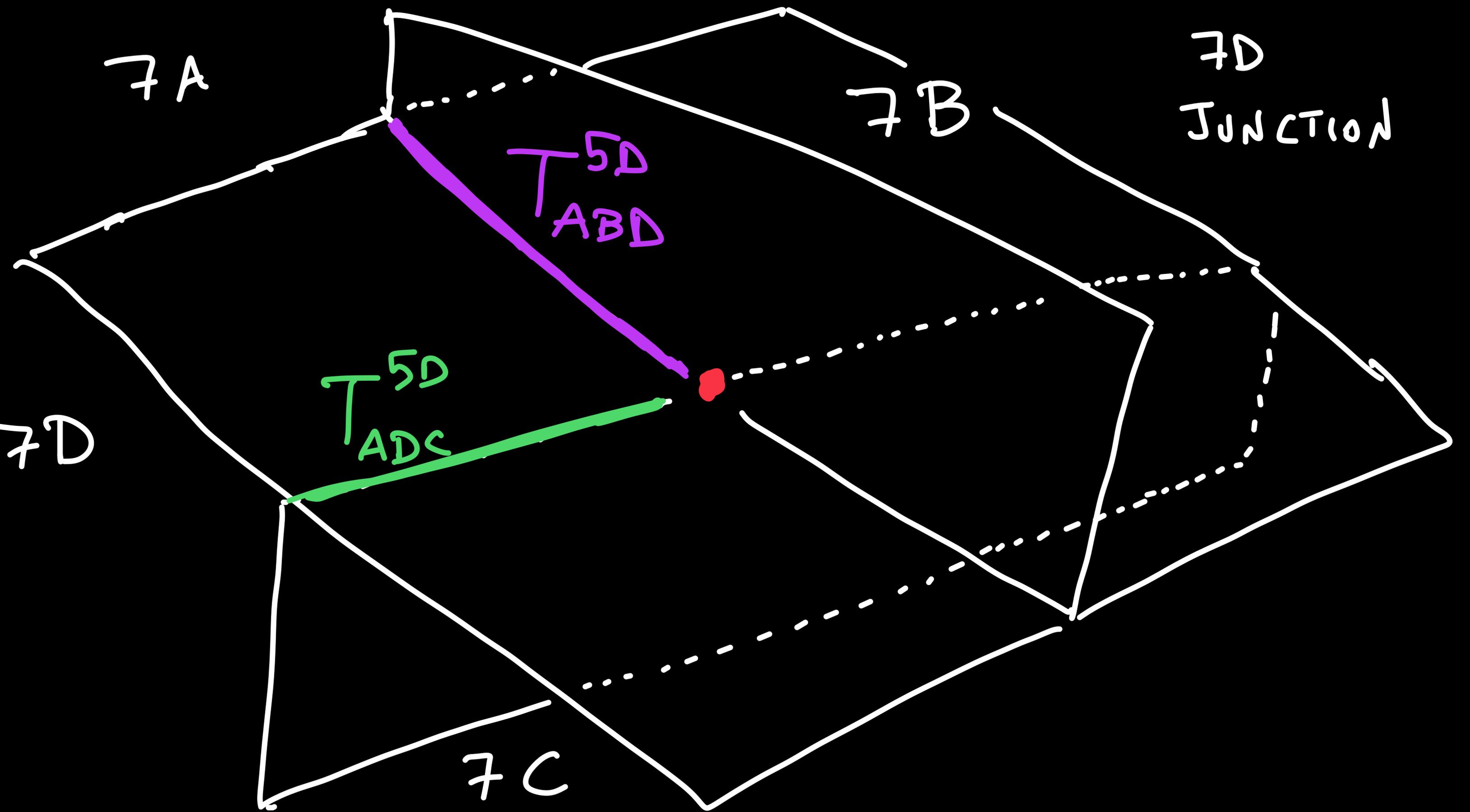
4D THEORY ARISES AS EDGE
MODE OF INTRICATED NESTED

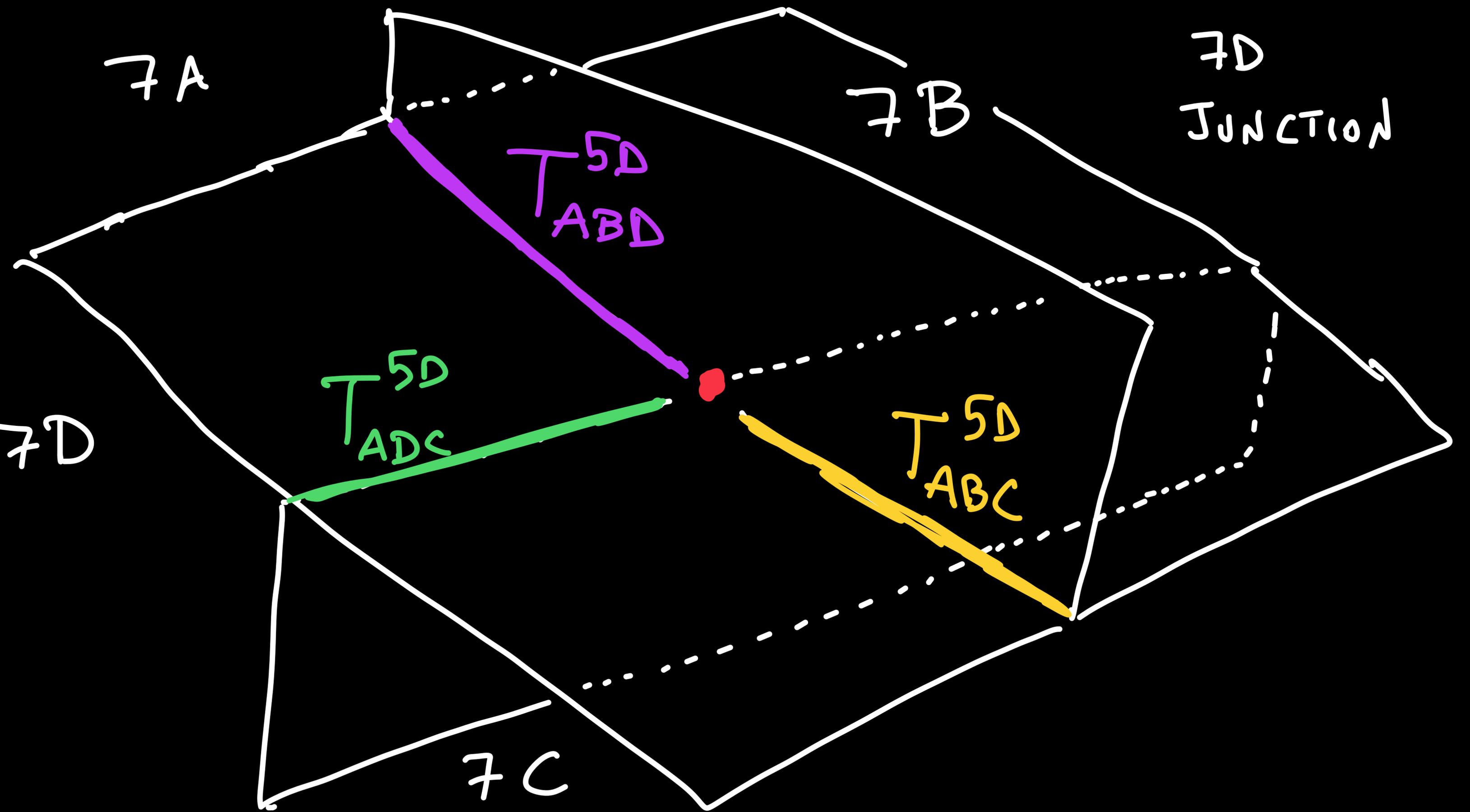
5D/7D INTERFACES

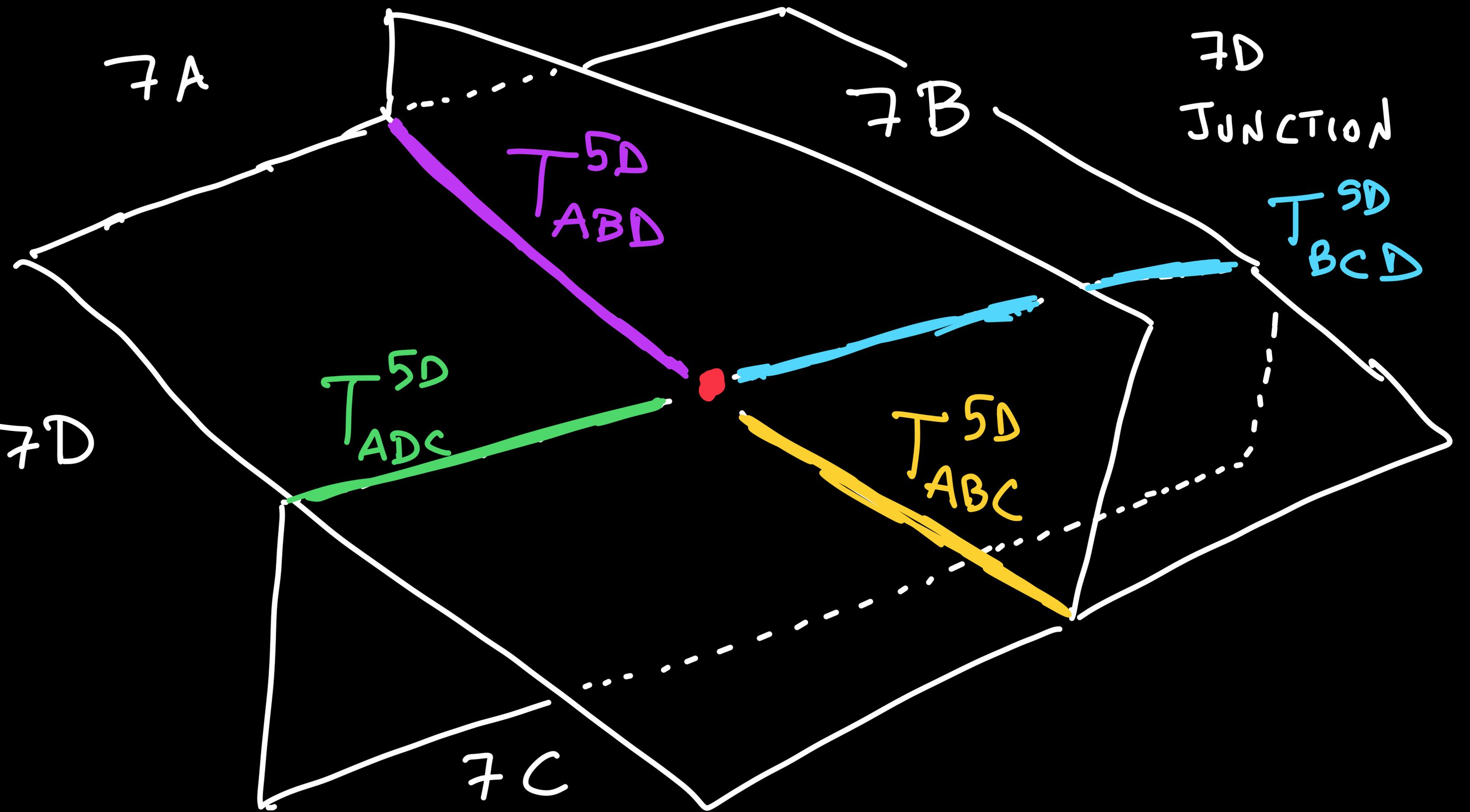
e.g. $\mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N$

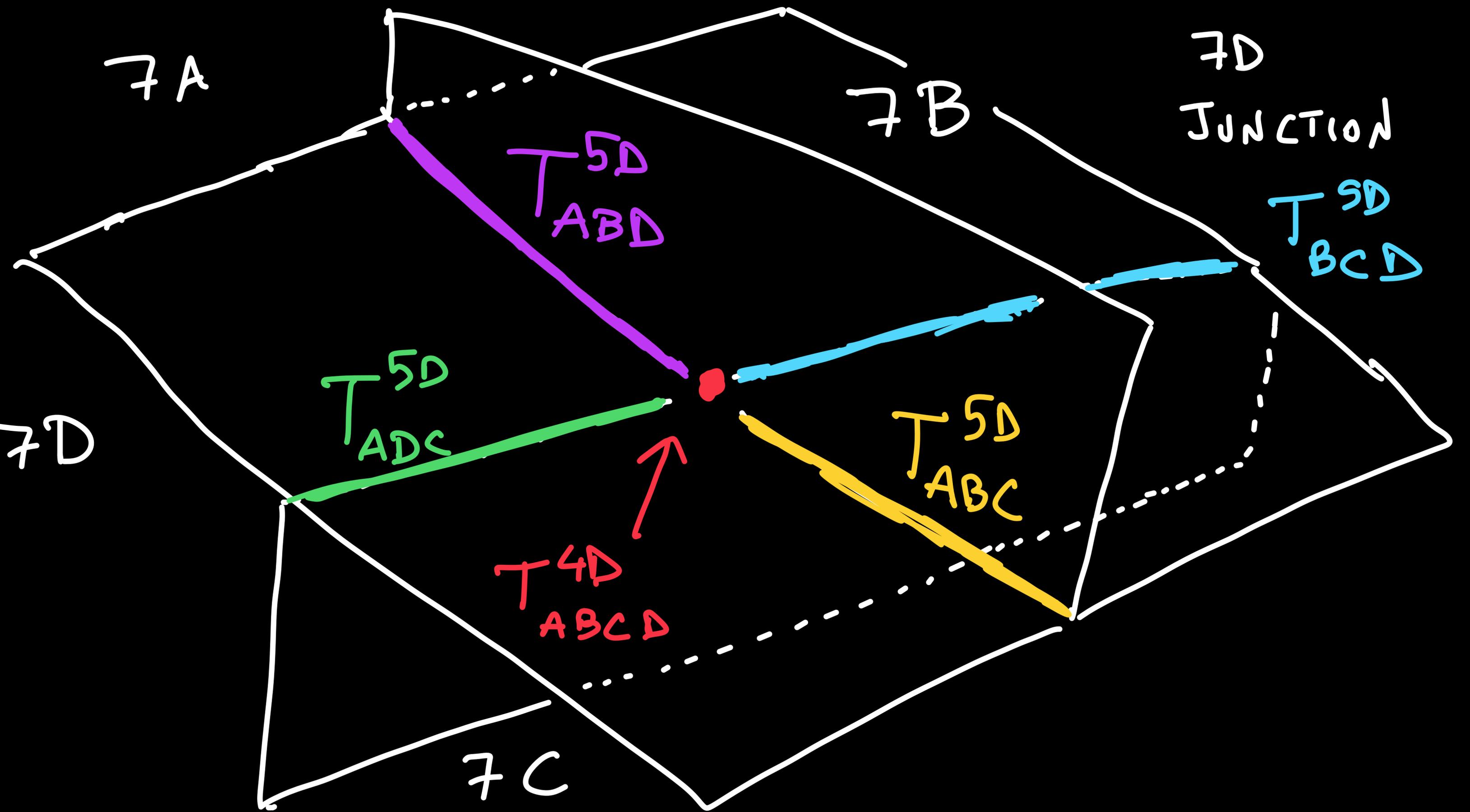












CRUCIAL QUALITATIVE DIFFERENCE:

CODIMENSION 6 : 5D SCFTs, THAT DO NOT
DECOUPLE IN THE IR

WE OBTAIN

4D-5D SYSTEMS

CRUCIAL QUALITATIVE DIFFERENCE:

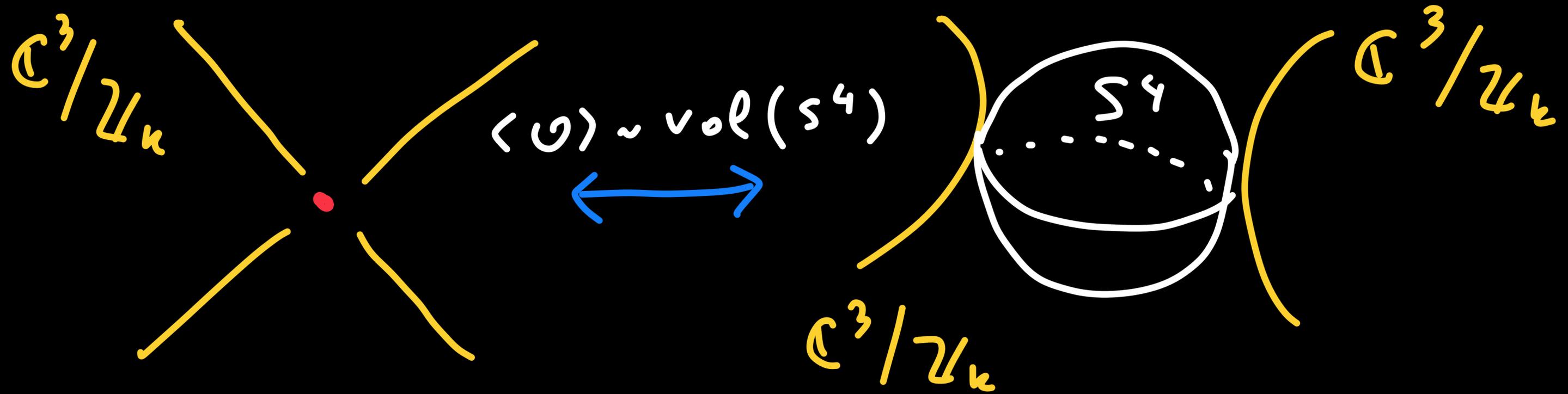
CODIMENSION 6 : 5D SCFTS, THAT DO NOT
DECOUPLE IN THE IR

THIS ALLOWS WE OBTAIN **4D-5D SYSTEMS**

QUALITATIVELY NEW FEATURES:

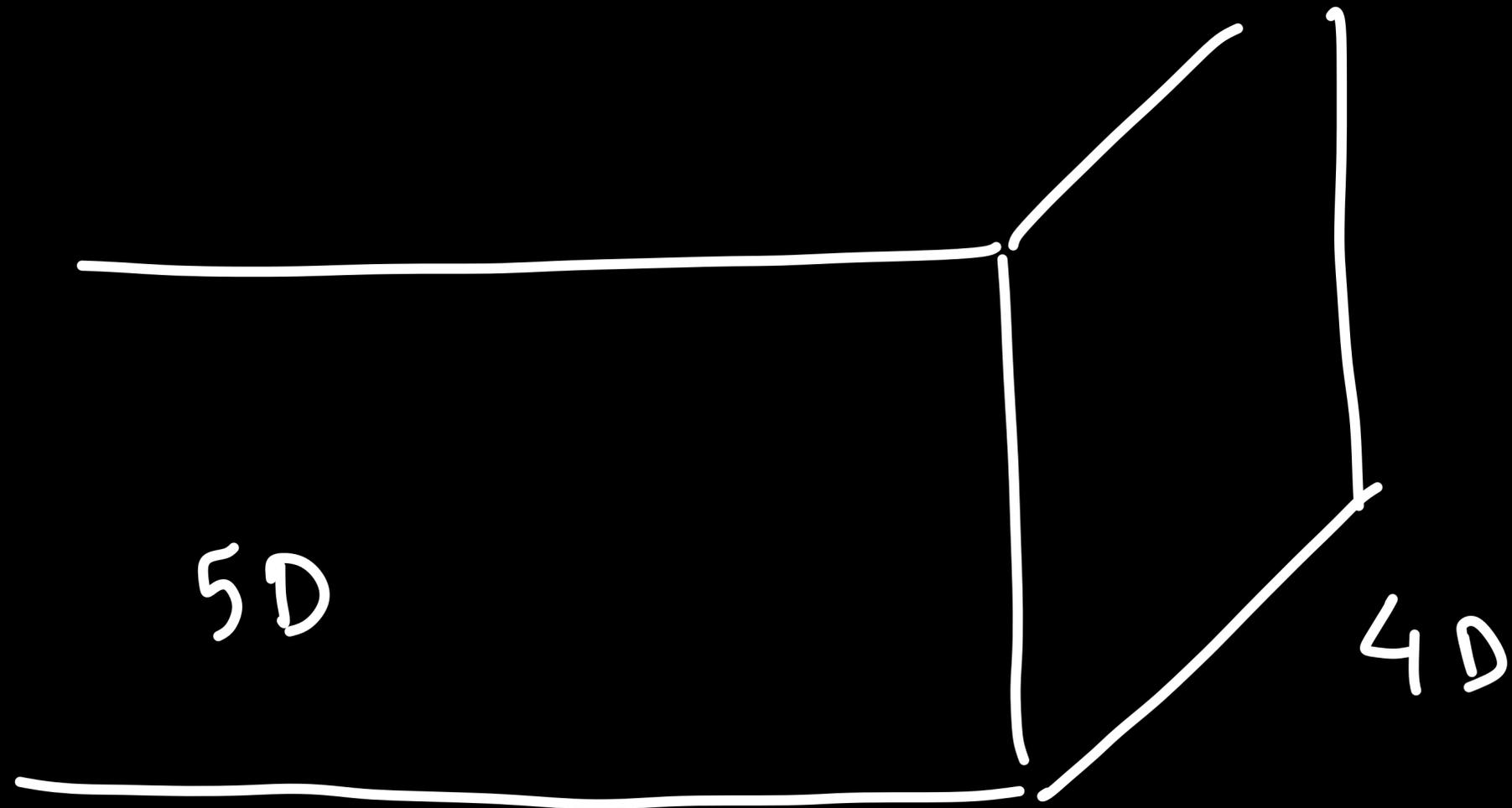
- SPONTANEOUS SYMMETRY BREAKING
- HIGHER SYMMETRY INHERITANCE
- 4D QUASI-SCFTS: NECESSARILY INTERACTING!

CRUCIAL QUALITATIVE DIFFERENCE:



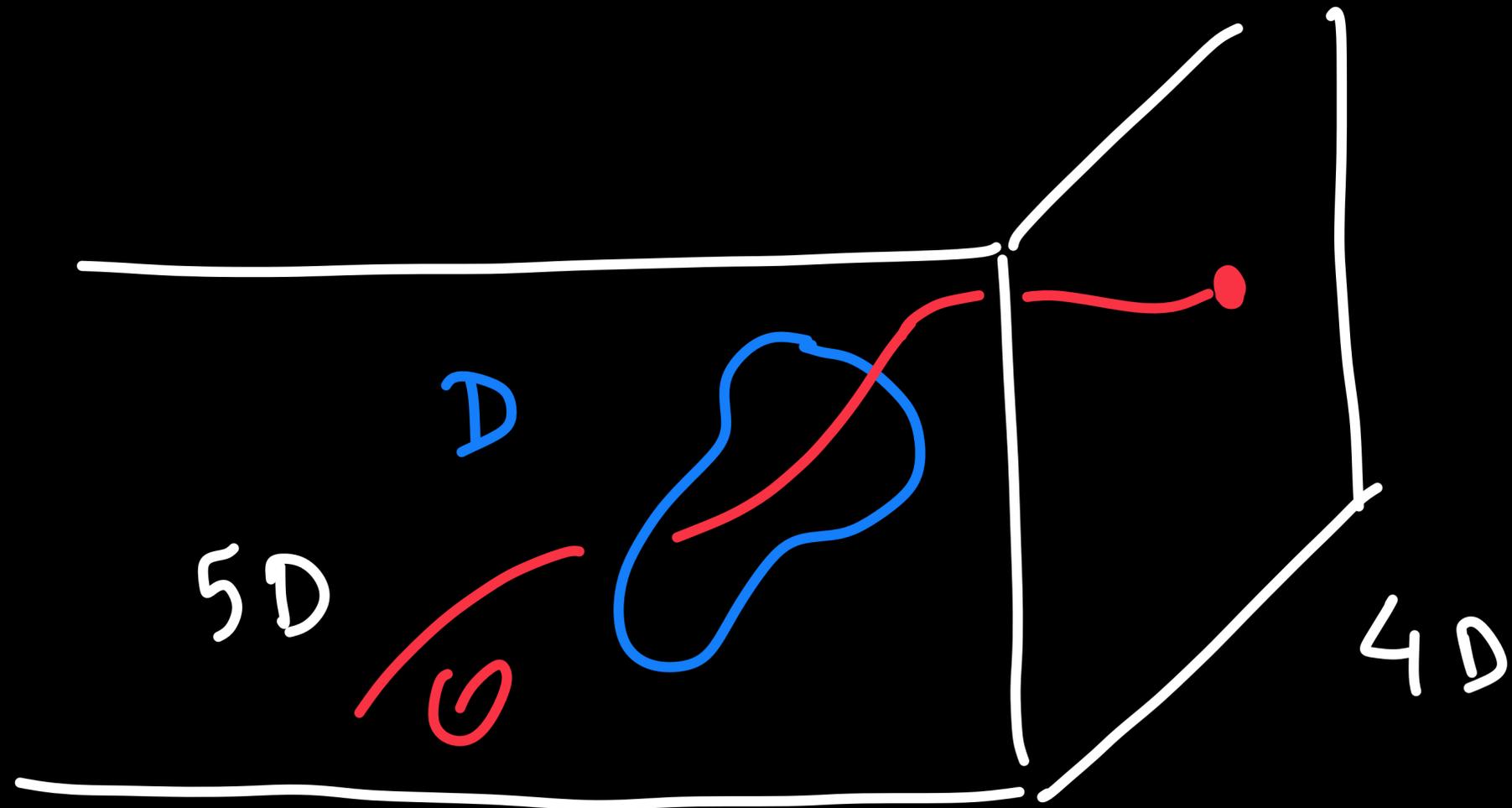
- SPONTANEOUS SYMMETRY BREAKING

CRUCIAL QUALITATIVE DIFFERENCE:



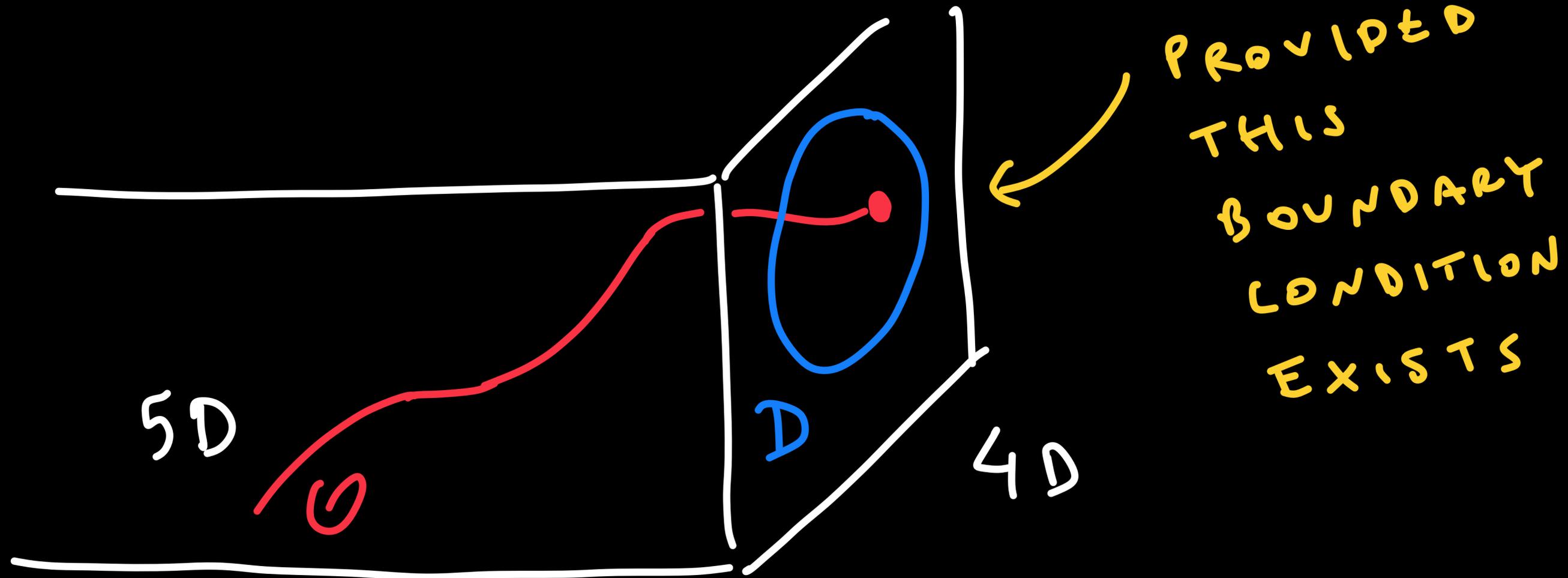
- HIGHER SYMMETRY INHERITANCE

CRUCIAL QUALITATIVE DIFFERENCE:



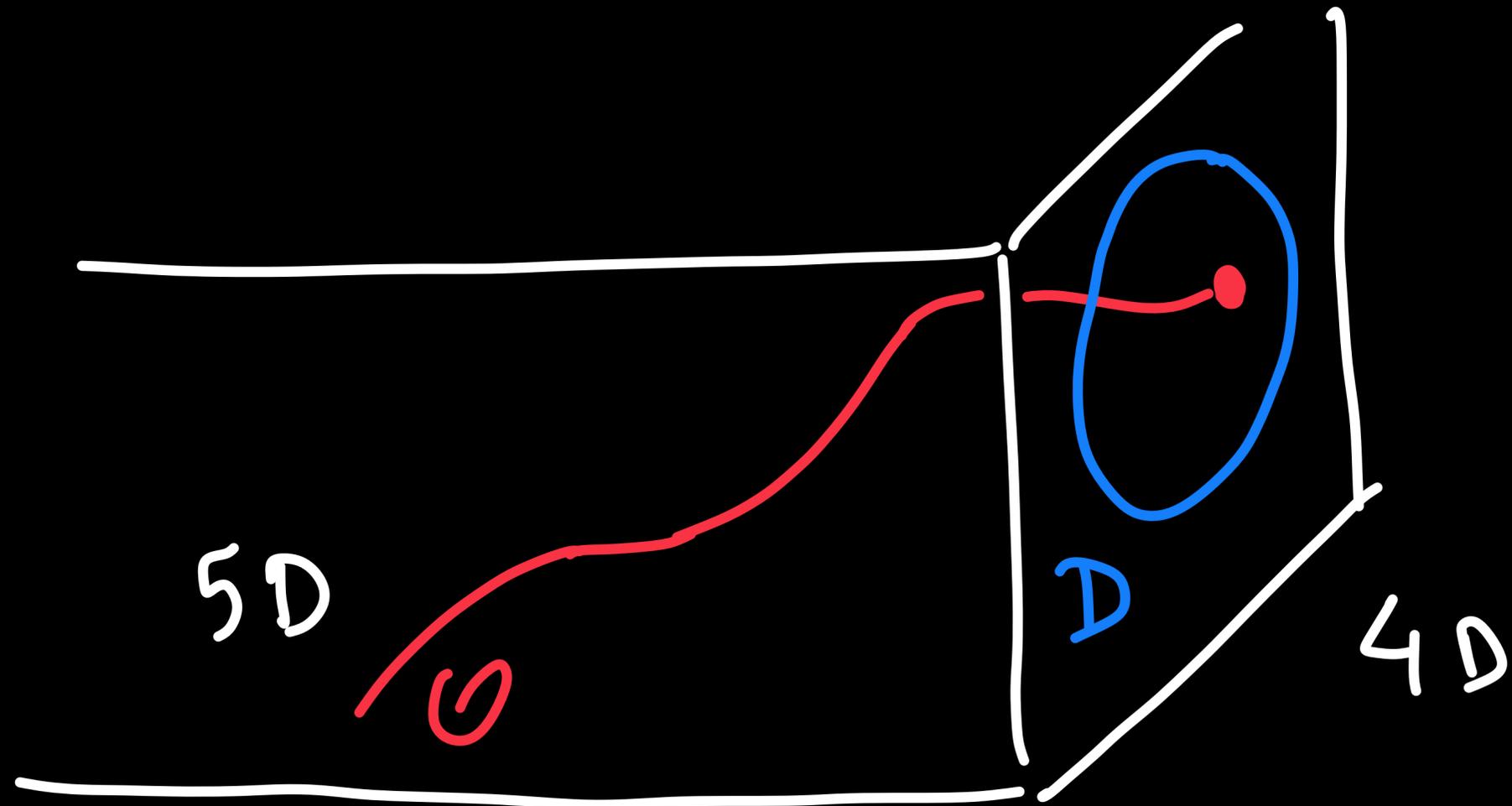
- HIGHER SYMMETRY INHERITANCE
5D 1-FORM
SYMMETRY

CRUCIAL QUALITATIVE DIFFERENCE:



- HIGHER SYMMETRY INHERITANCE
5D 1-FORM SYMMETRY → 4D 0-FORM SYMMETRY

CRUCIAL QUALITATIVE DIFFERENCE:



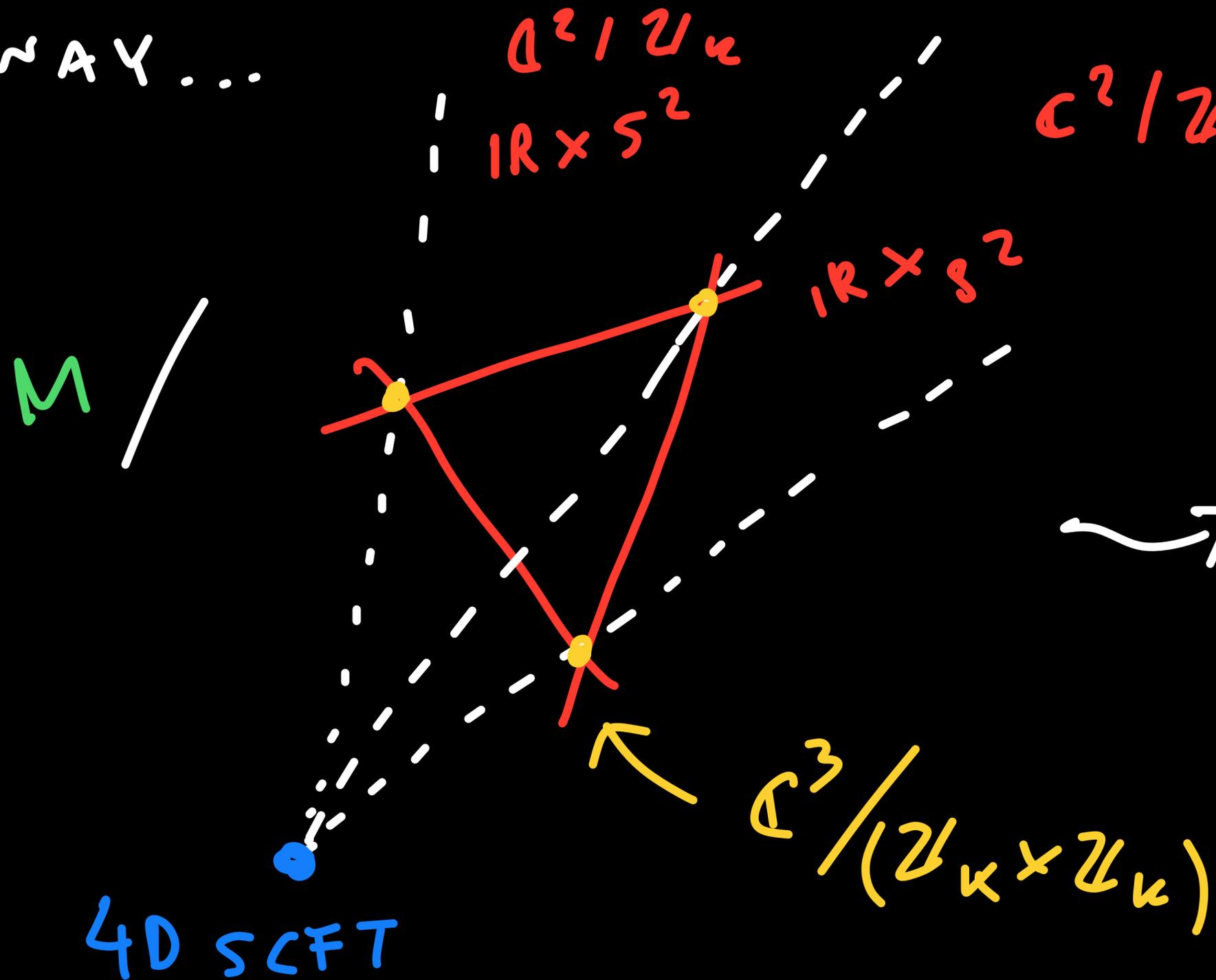
EXPLICITLY
DETERMINED
FROM
DEFECT GROUP

$$ID^{(n)} = \bigoplus_{\substack{p-k+1 \\ = n}} \frac{H_k(x, \partial x)}{H_k(x)}$$

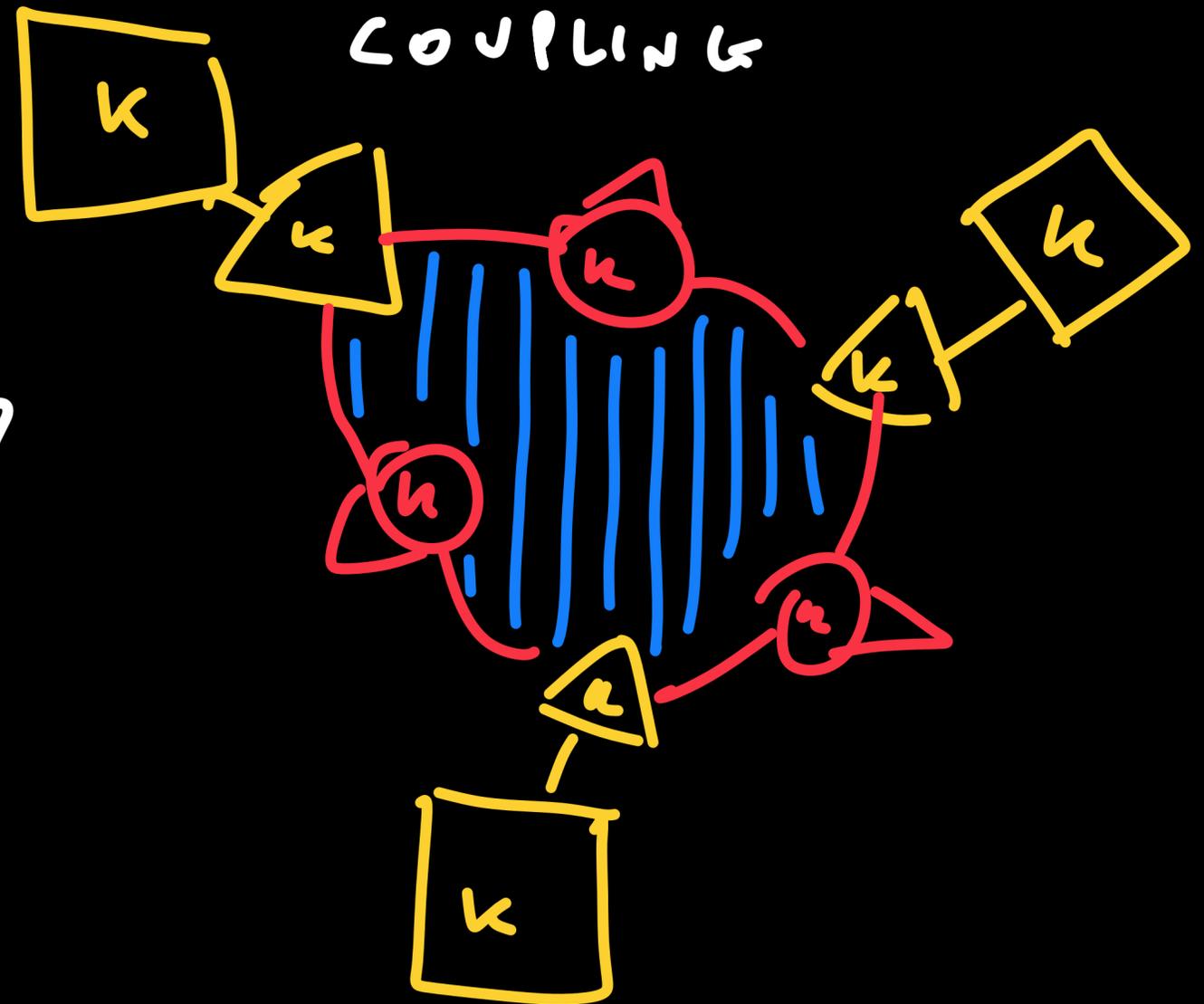
P-BRANES

- HIGHER SYMMETRY INHERITANCE
 5D 1-FORM SYMMETRY \rightarrow 4D 0-FORM SYMMETRY

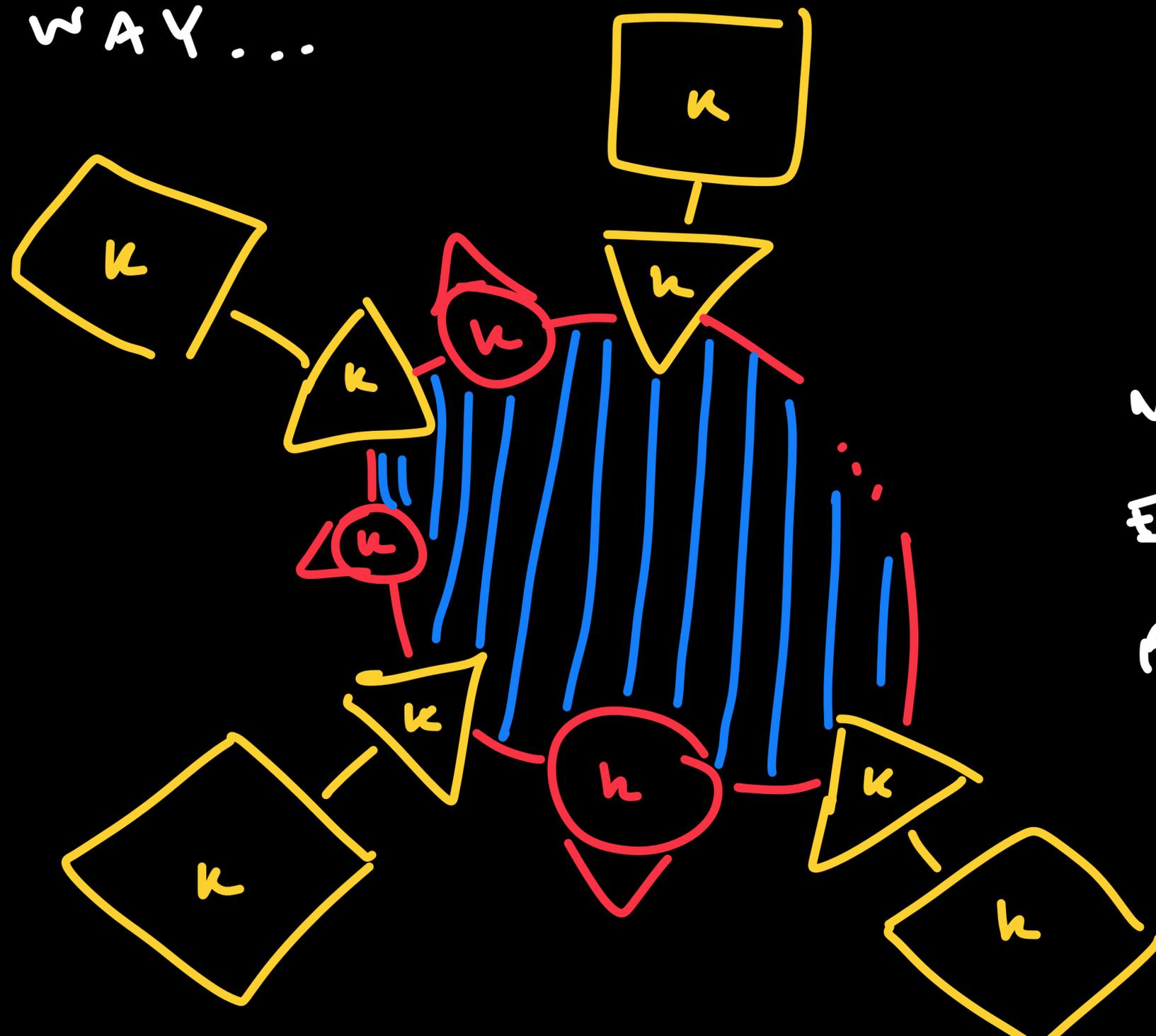
MANY NEW EXAMPLES OF 4D QUASI SCFT
 ARE GEOMETRICALLY ENGINEERED IN THIS
 WAY...



5D GAUGING w/
 POSITION DEPENDENT
 COUPLING



MANY NEW EXAMPLES OF 4D QUASI SCFT
ARE GEOMETRICALLY ENGINEERED IN THIS
WAY...



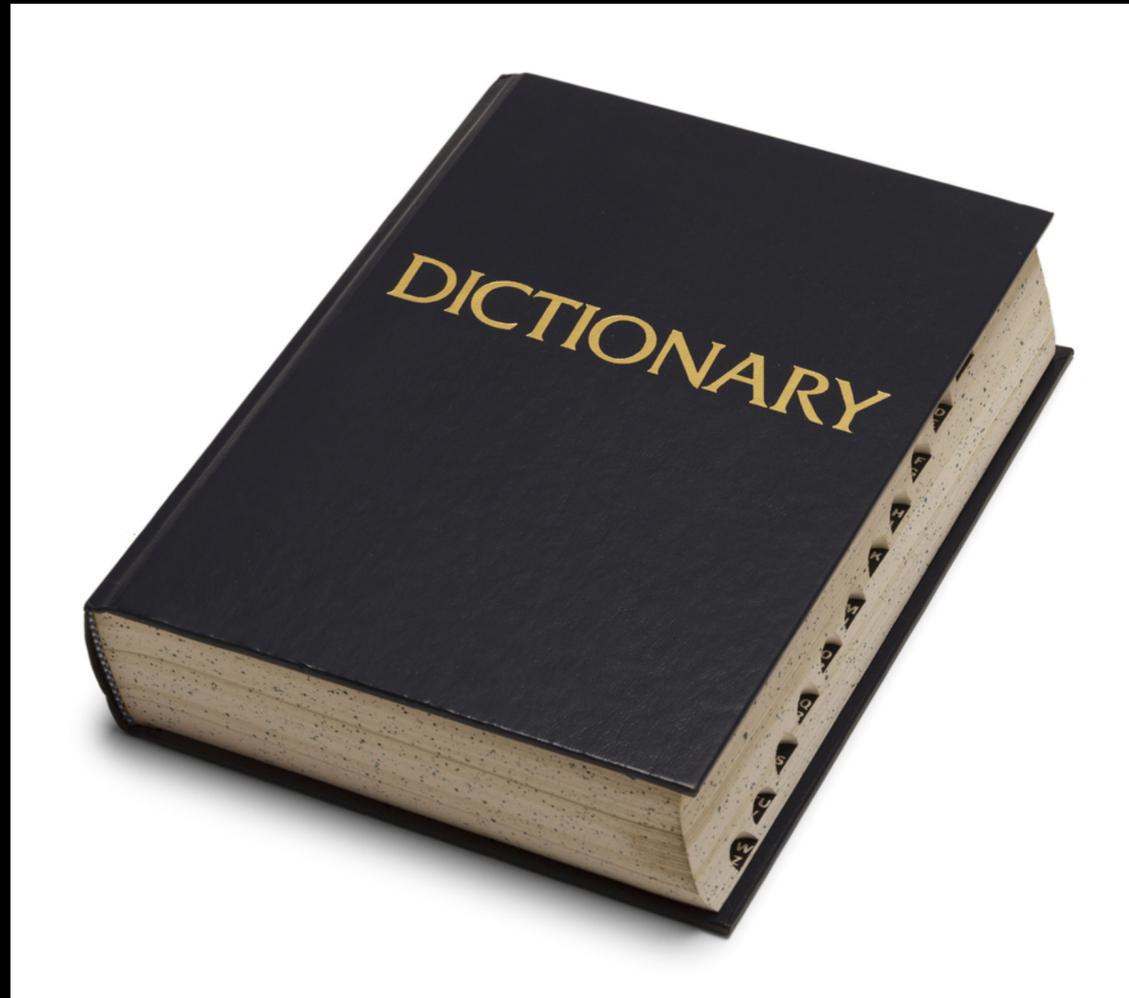
5D GAUGING w/
POSITION DEPENDENT
COUPLING

MANY MORE DETAILED
EXAMPLES IN THE
PAPER [4]

1

GEOMETRIC
ENGINEERING
LIMIT OF
HETEROTIC
STRINGS

[1-3]



2

GEOMETRIC
ENGINEERING
OF M-THEORY
ON SINGULAR
 G_2 HOLONOMY
CONES

[4]

NAIVELY DISTINCT BUT \exists INTERPLAY VIA DUALITY

IN THE FIRST PART OF THE SEMINAR :

$$\text{Het}_H / \mathbb{C}^2 / \Gamma_g$$

BUT HOW ABOUT

$$\text{Het}_H / \mathbb{C}^3 / \Gamma \quad \Gamma \subseteq \text{SU}(3) \quad ?$$

IN THE FIRST PART OF THE SEMINAR:

$$\text{Het}_H / \mathbb{C}^2 / \Gamma_g$$

BUT HOW ABOUT

$$\text{Het}_H / \mathbb{C}^3 / \Gamma \quad \Gamma \subseteq \text{SU}(3) \quad ?$$

T-DUALITY STILL HOLDS:

$$D_{\delta'} \text{Het}_{E_8 \times E_8} / \mathbb{C}^3 / \Gamma \quad \xleftrightarrow{T} \quad D_{\delta'} \text{Het}_{D_{16}} / \mathbb{C}^3 / \Gamma$$

\Rightarrow 4D LSTs [GIDEON, KUTASOV], BELOW LS SCALE
3D DUALITY
[CLOUSTET, SCHÄFER-NANEK, WNK]

IN THE FIRST PART OF THE SEMINAR :

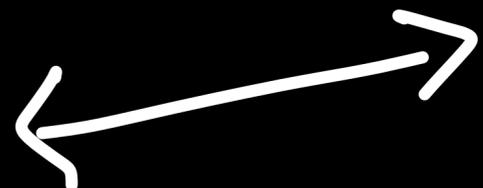
$$\text{Het}_H / \mathbb{C}^2 / \Gamma_g$$

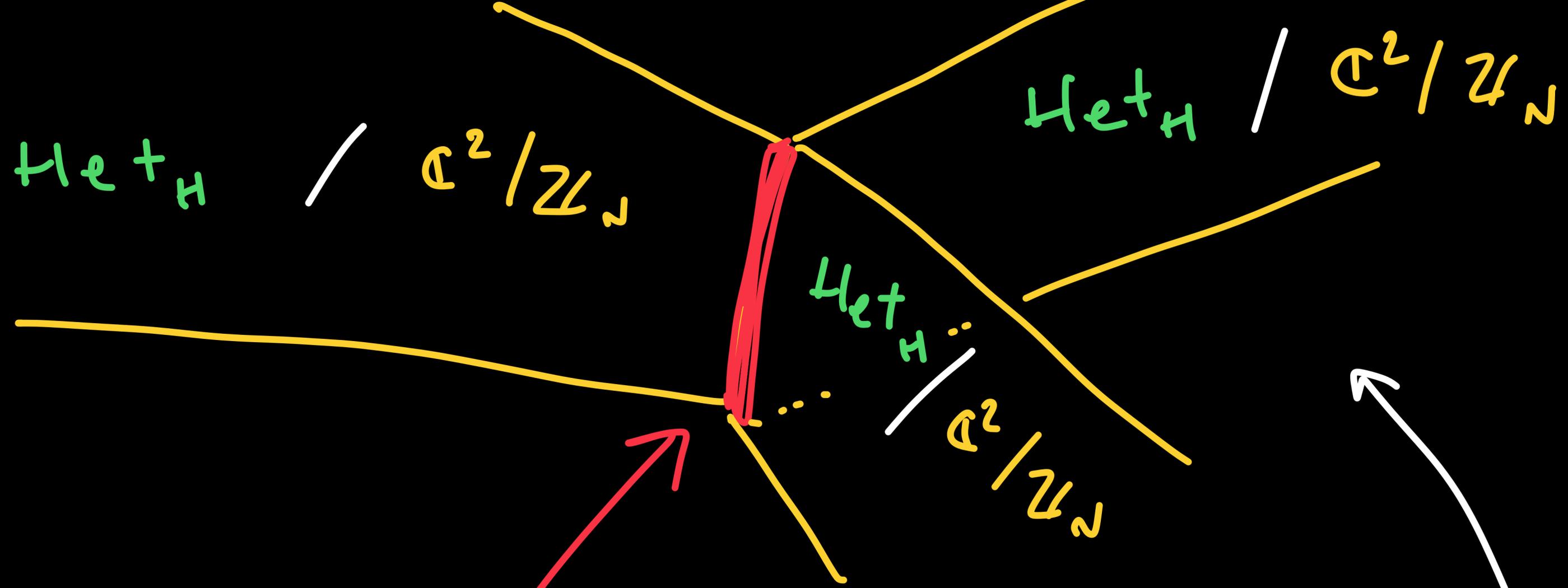
BUT HOW ABOUT

$$\text{Het}_H / \mathbb{C}^3 / \Gamma \quad \Gamma \subseteq \text{SU}(3) \quad ?$$

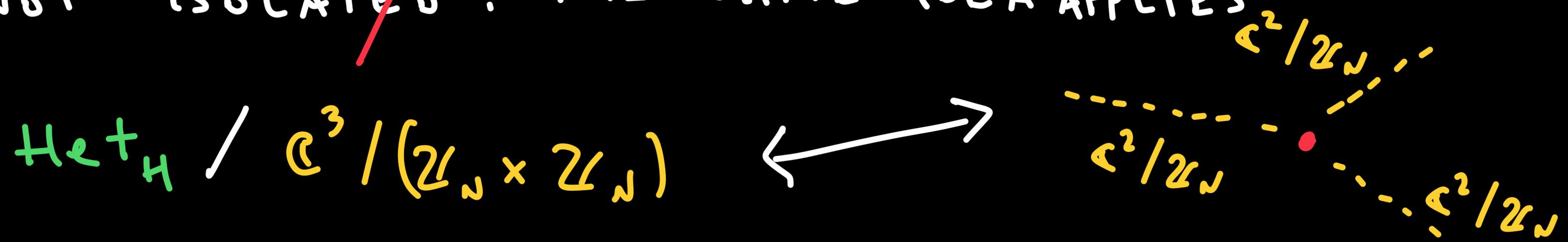
WHENEVER THE SINGULARITIES ARE NOT ISOLATED : THE SAME IDEA APPLIES

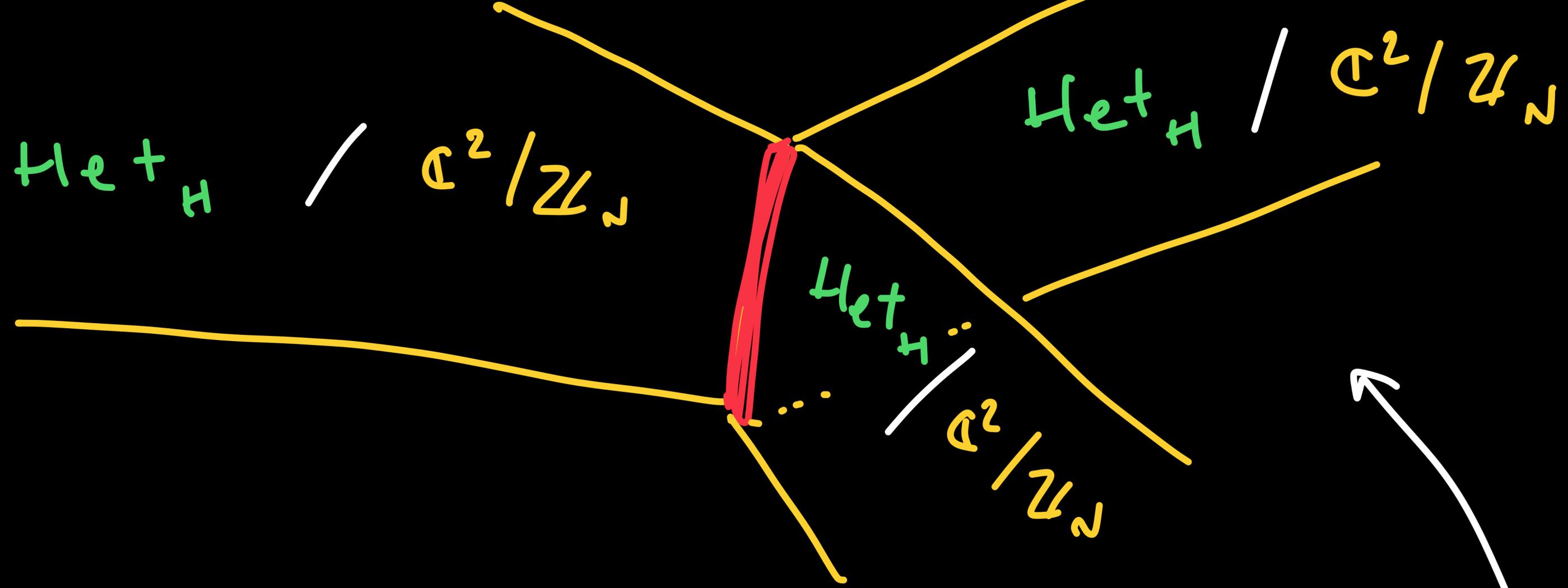
$$\text{Het}_H / \mathbb{C}^3 / (\mathbb{Z}_N \times \mathbb{Z}_N)$$





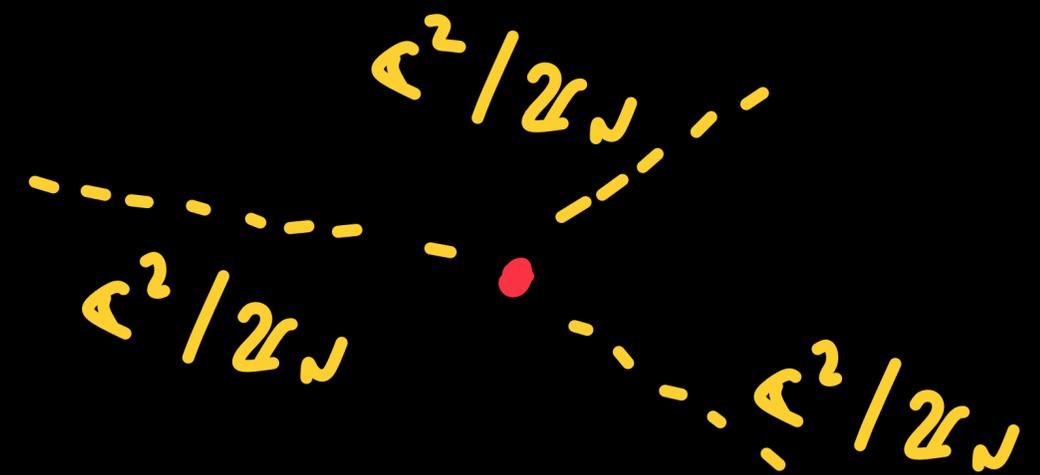
WHENEVER THE SINGULARITIES ARE NOT ISOLATED: THE SAME IDEA APPLIES

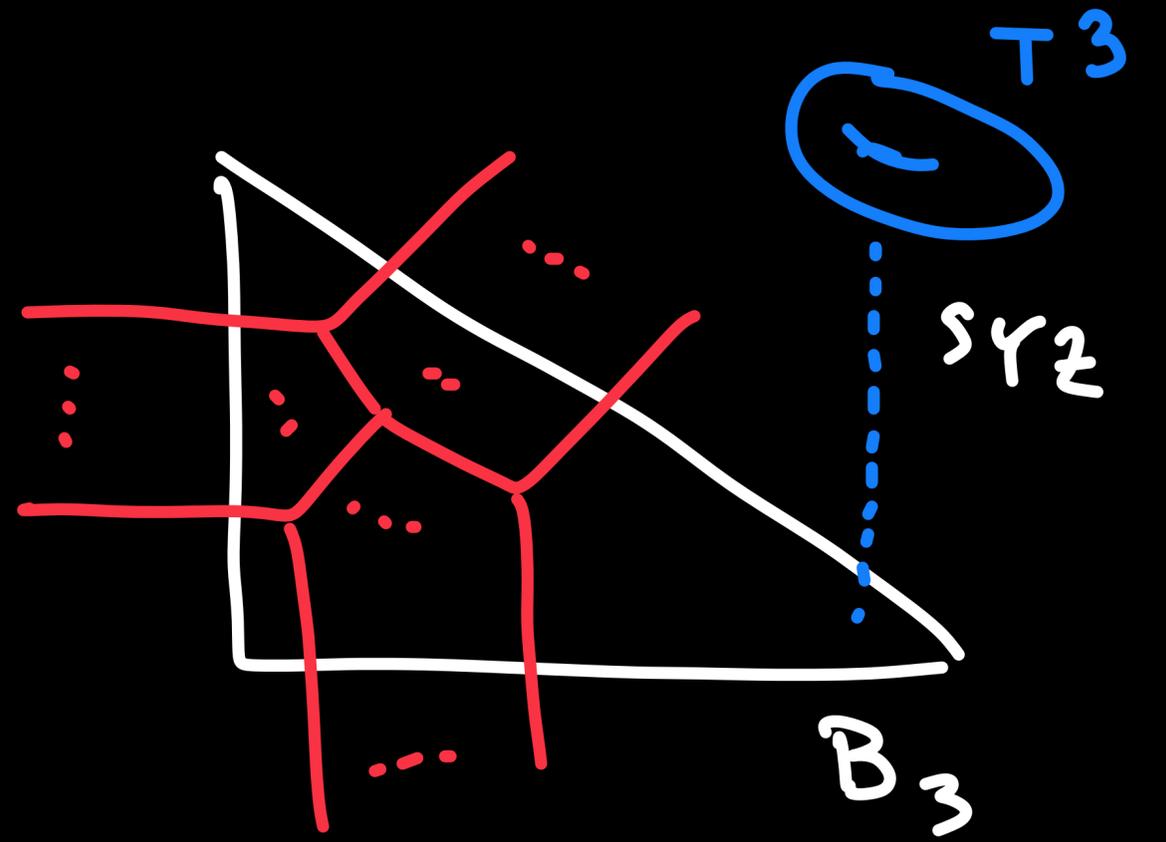




WE CAN ALSO DUALIZE
 [AGANAGIC, VAFA, KLENN]

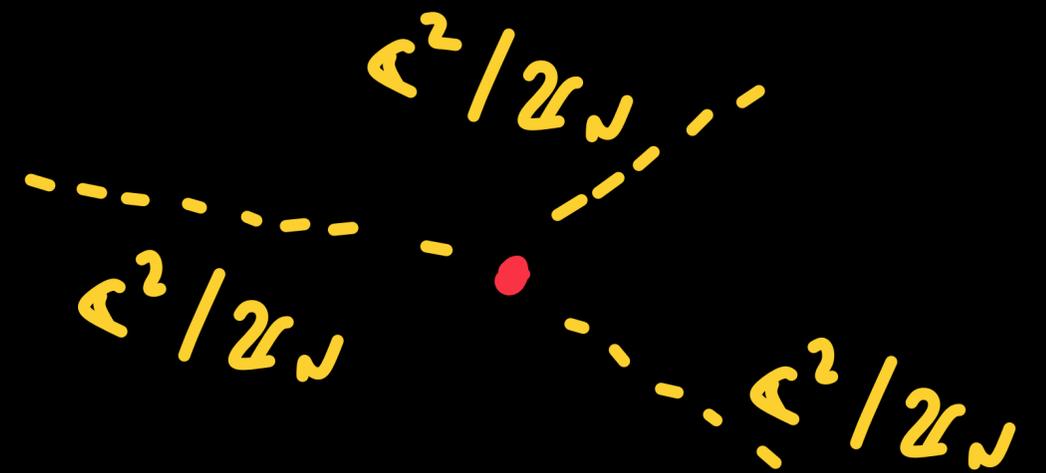
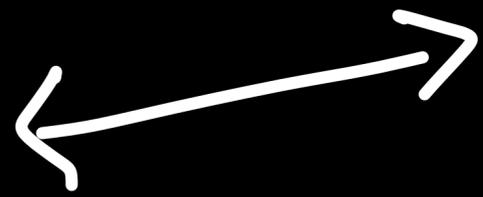
$\text{Het}_H / \mathbb{C}^3 / (\mathbb{Z}_N \times \mathbb{Z}_N)$





WE CAN ALSO DUALIZE
 [AGANAGIC, VAFA, KLEIN]

$$Het_H / \mathbb{C}^3 / (\mathbb{Z}_N \times \mathbb{Z}_N)$$



$$M / G_2 \xrightarrow{k_3}$$

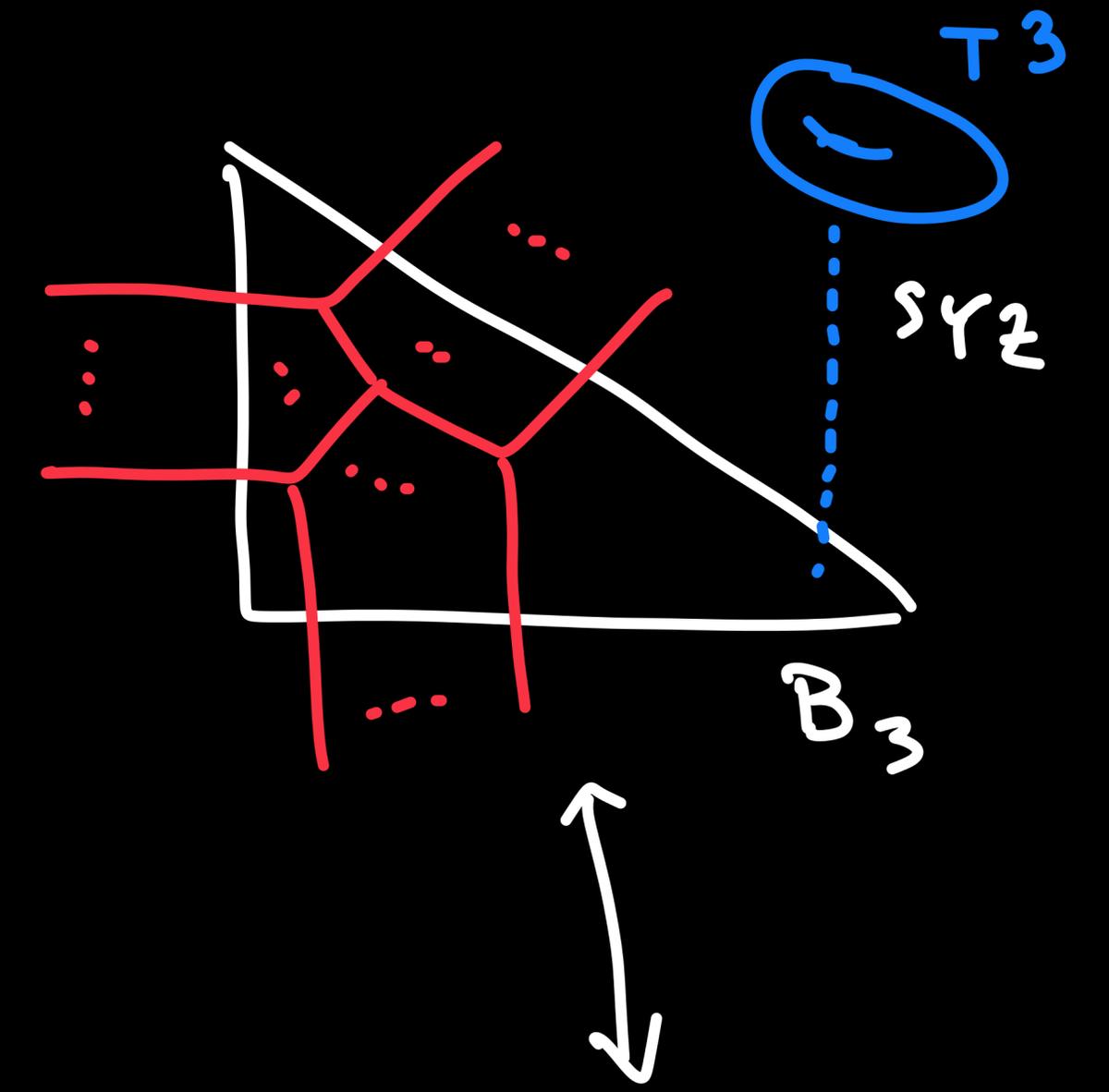
$$\downarrow$$

$$B_3$$

... LOCAL MODELS OF "TORIC" G_2 'S?

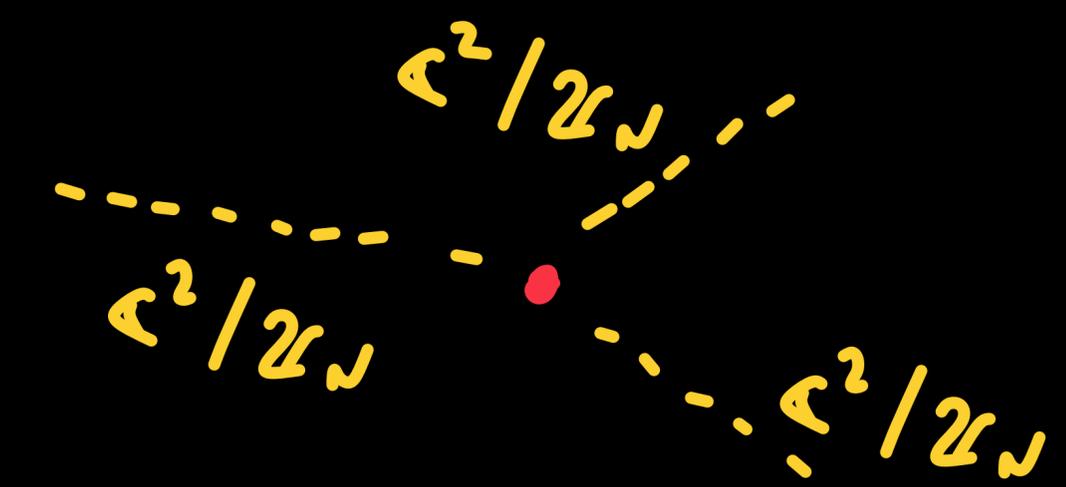
FIBERWISE

$$\text{Het}_F / T^3 \equiv M / k_3$$



WE CAN ALSO DUALIZE [AGANAGIC, VAFA, KLENN]

$$\text{Het}_H / \mathbb{C}^3 / (\mathbb{Z}_N \times \mathbb{Z}_N)$$



IN SUMMARY: MANY UNCHARTED CORNERS
OF GEOMETRIC ENGINEERING LANDSCAPE!

SCFT
METHODS



INTERPLAY

GENERALIZED
SYMMETRIES

ADVANCES THE
EXPLORATION

IN SUMMARY: MANY UNCHARTED CORNERS
OF GEOMETRIC ENGINEERING LANDSCAPE!

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EXPLORATION

THANK YOU FOR YOUR ATTENTION!

BACK UP

GENERALIZATION OF SYMMETRIES

MANTRA: GAIOTTO KAPUSTIN SEIBERG WILLET 14
KAPUSTIN THORNGREN 13

SYMMETRIES IN QFT ARE ENCODED BY
SUBSECTOR OF (QUASI) TOPOLOGICAL
OPERATORS WITH SUPPORTS OF VARIOUS
CODIMENSIONS

GENERALIZATION OF SYMMETRIES

MANTRA #2:

FUSION \leftrightarrow GENERALIZES GROUP LAW

CAN INVOLVE TOPOLOGICAL
OPERATORS OF DIFFERENT
CODIMENSION

GENERALIZATION OF SYMMETRIES

ANOMALY
 TFT_{D+1} | QFT_D

RG

CONSTRAINTS
ON DYNAMICS

ANOMALY
 TFT_{D+1} | QFT_D

GENERALIZATION OF SYMMETRIES

ANOMALY
TFT_{D+1}

SYMMETRY
TFT_{D+1}

QFT_D

APRUZZI, BONETTI, GARCÍA ETXEBARRIA;
HOSSKINI, SCHÄFFER-JAMEKI 21

RG

CONSTRAINTS
ON DYNAMICS

ANOMALY
TFT_{D+1}

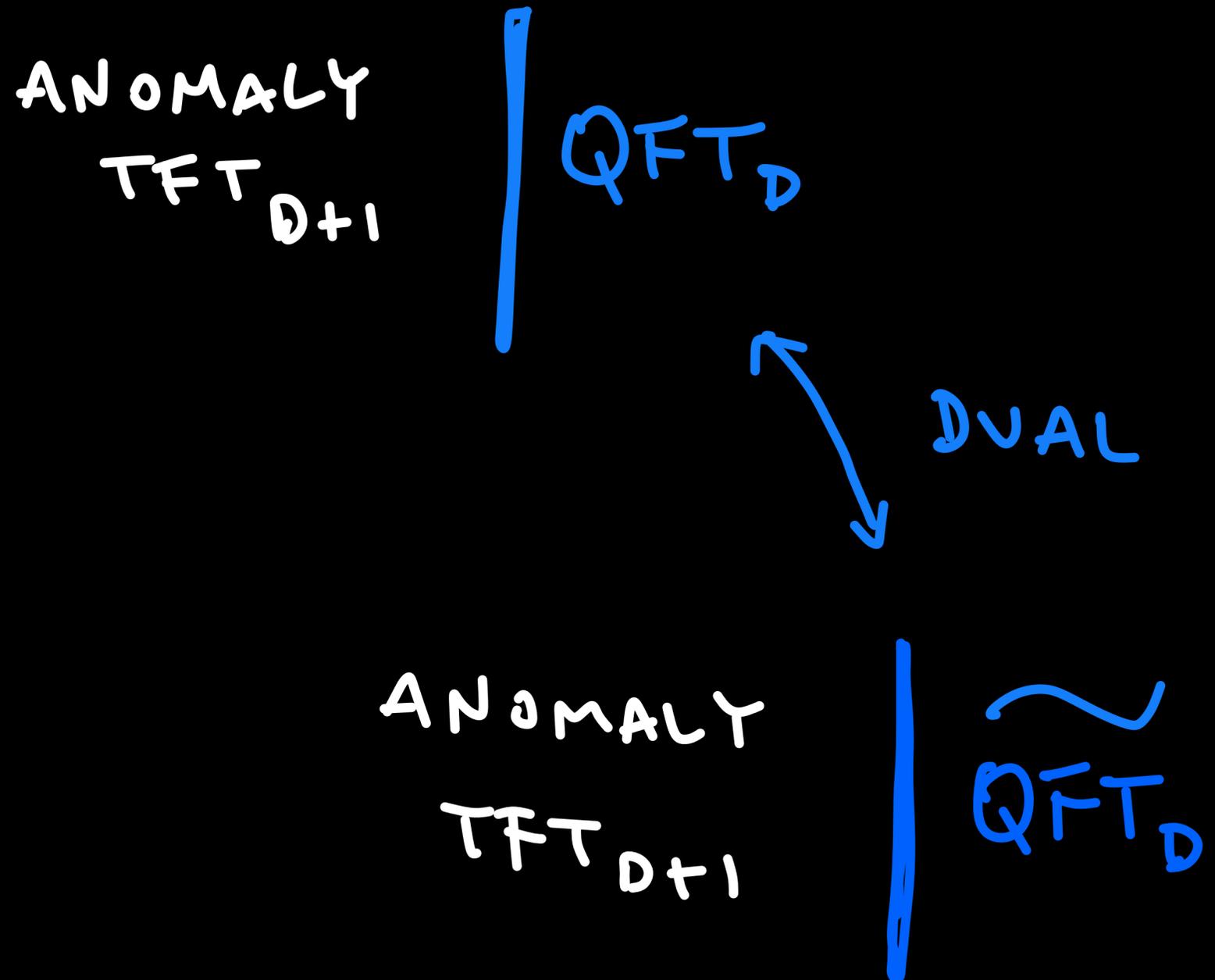
SYMMETRY
TFT_{D+1}

QFT_D

GAIOTTO, KULP 20 ;

DZ, GARCÍA ETXEBARRIA 22

GENERALIZATION OF SYMMETRIES



CONSTRAINTS
ON DYNAMICS

GENERALIZATION OF SYMMETRIES

ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

QFT_D

DUAL



CONSTRAINTS
ON DYNAMICS

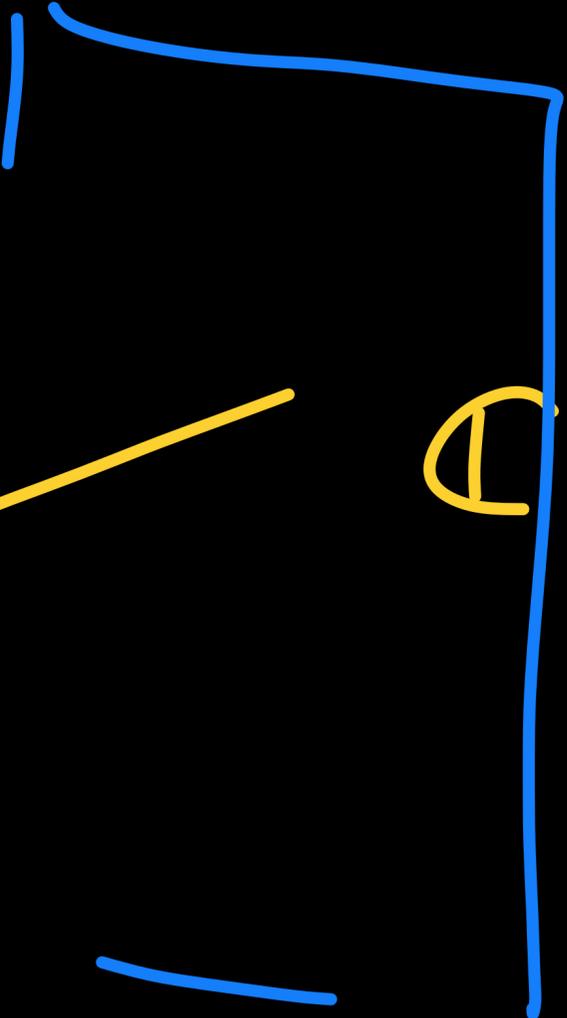
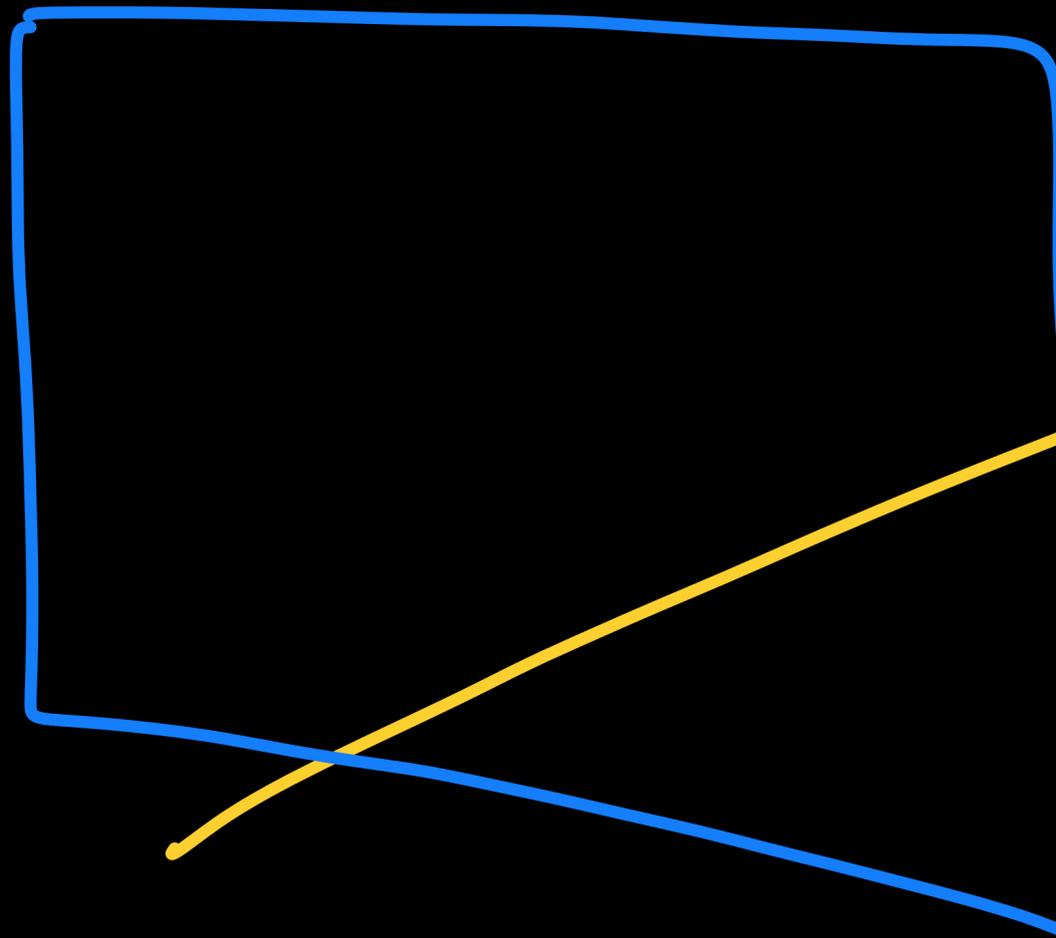
ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

\sim
 QFT_D

[DZ, OHNORI '20 ; OHNORI, TACHIKAWA '21]

$$F = \text{Pic} \subseteq H_2(K3)$$

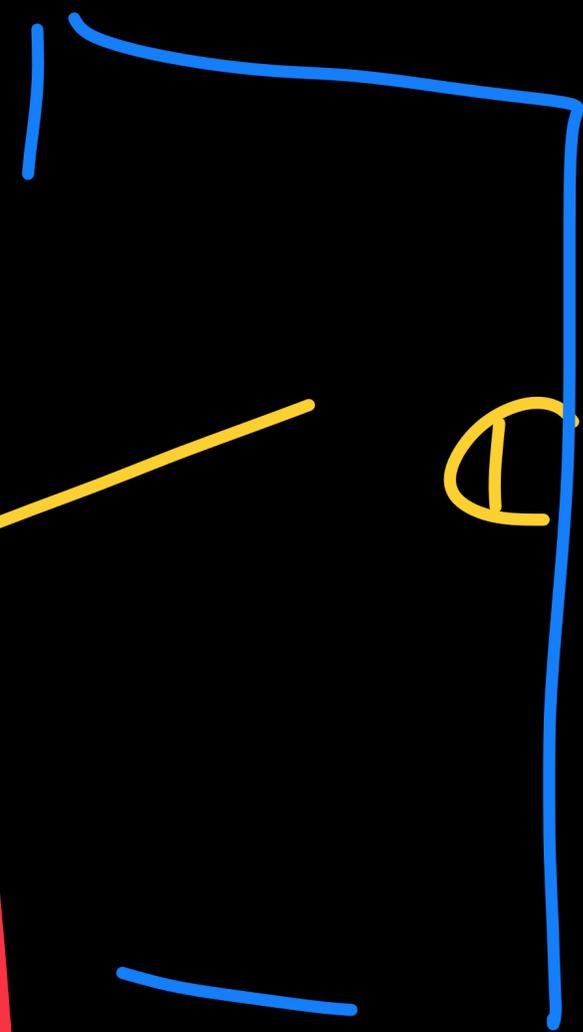
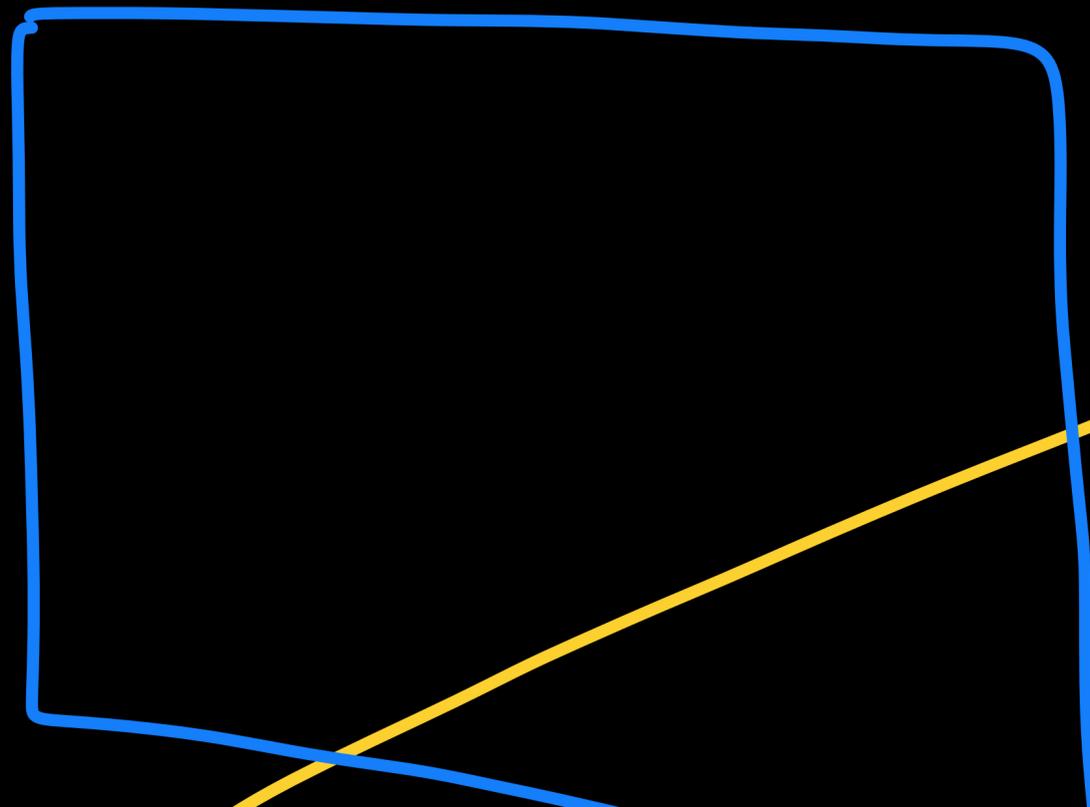


$K3_F$

LST

$K3_F$

$$\tilde{F} = \tilde{\text{Pic}} \subseteq H_2(K3)$$



$K3 \sim H_2$