Bounds on the Species Scale And the Distance Conjecture

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Based on:

The Desert and the Swampland

Cody Long, Miguel Montero, C.V., Irene Valenzuela

[2112.11467][hep-th]

Moduli-dependent Species Scale

Damian van de Heisteeg, C.V., Max Wiesner, David Wu

[2212.06841][hep-th]

Bounds on Species Scale and the Distance Conjecture

Damian van de Heisteeg, C.V., Max Wiesner,

To appear.

What is the Species Scale Λ_s? [D,DG,DGL]

Suppose we have a large number N of light species.

Then it better be that EFT of gravitational sector breaks down before we get to Planck scale:

$$S_{BH} \sim 1 \quad \text{for } R_{BH} \sim l_{Pl}$$

Therefore this would not accommodate N degrees of freedom.

$(R_{BH}^{min})^{d-2} \sim N \Rightarrow R_{BH}^{min} \sim N^{\frac{1}{d-2}}$. (In Planck units)

How could this be explained?

Higher derivative corrections are important and compete at the mass scale

$\Lambda_s = \dfrac{1}{R_{BH}^{min}} \sim N^{\frac{-1}{d-2}}$ a new cutoff in Gravitational sector

In principle the species scale will depend on the vev of fields ϕ (since as we change ϕ the masses change and so the species scale could change). We thus expect

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) + \frac{1}{2}(\partial\phi)^2 + \ldots \right]$$

We expect the coefficients be of $O(1)$. Of course not all higher derivative terms will have coefficients of $O(1)$, but that should be the generic case.

In this talk we are interested in the dependence of Λ_s on ϕ.

Asymptotics of m, Λ_s

As we approach boundaries of moduli space, distance conjecture [OV] suggests we get a tower of light states:

$$m \sim \exp(-\alpha\phi) \qquad \alpha \sim O(1)$$

Assuming string emergence conjecture [LLW] one finds

$$\alpha = \frac{1}{\sqrt{d-2}} \quad strings; \qquad \alpha = \sqrt{\frac{D-2}{(D-d)(d-2)}} \quad KK \quad (D \Rightarrow d);$$

Note in both cases

$$\alpha \geq \frac{1}{\sqrt{d-2}} \Rightarrow |m'/m|^2 \gtrsim \left(\frac{1}{d-2}\right) \qquad \phi \to \infty$$

(Additional arguments for this bound [EHKQR])

Asymptotics of m, Λ_s

Similarly we expect for a tower of light states:

$$\Lambda_s \sim m^\gamma \sim \exp(-\beta\phi) \quad \beta \sim O(1)$$

one finds

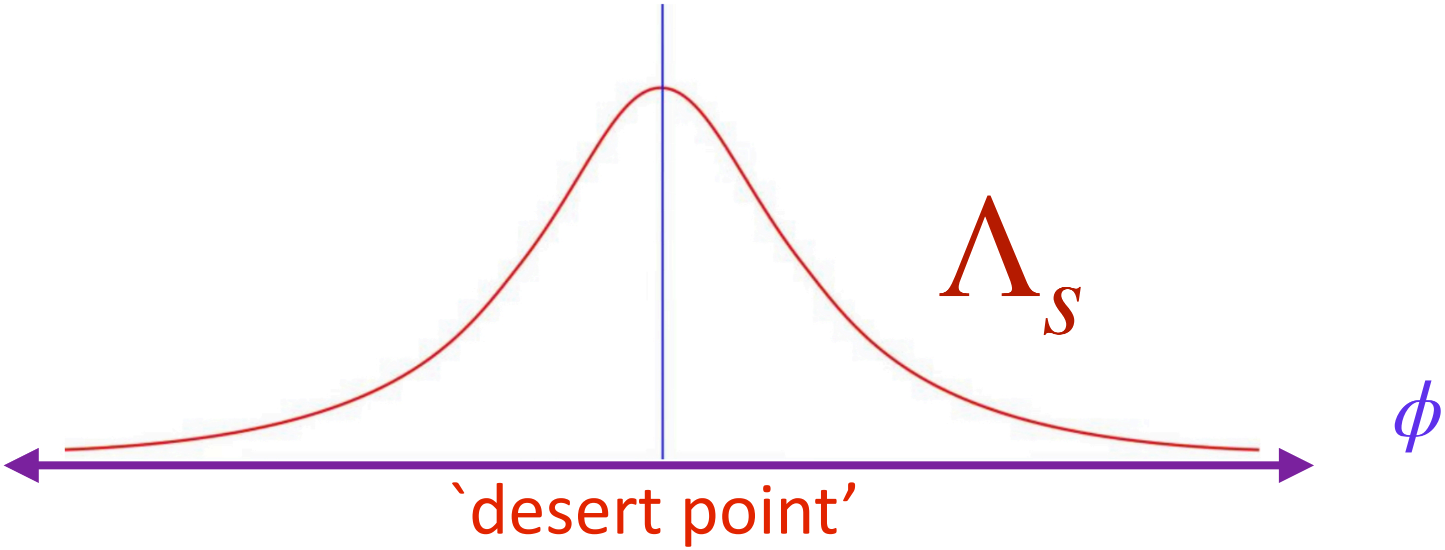
$$\beta = \frac{1}{\sqrt{d-2}} \quad strings; \quad \beta = \sqrt{\frac{D-d}{(D-2)(d-2)}} \quad KK \quad (D \Rightarrow d);$$

Note in both cases

$$\beta \leq \frac{1}{\sqrt{d-2}} \Rightarrow |\Lambda_s'/\Lambda_s|^2 \lesssim \left(\frac{1}{d-2}\right) \qquad \phi \to \infty$$

Dependence of Λ_s on moduli

A systematic study was started in [LMVV], and the general picture we expect to get:



Difficult to study this as a function of moduli. How to find it? In a special class of theories this can be done.

Dependence of Λ_s on moduli [HVWW]

We consider $\mathcal{N} = 2$ supersymmetric theories in $d = 4$. Consider the vector multiples moduli ϕ. In this case there is a natural proposal for what $\Lambda_s(\phi)$ is:

$$\frac{1}{\Lambda_s^2} = F_1$$

Where F_1 is the effective term in the gravitation action:

$$\int d^4\theta d^4x \; F_1(\phi) \, . \, W^2 = \int d^4x \; F_1(\phi) \, . \, R^2 + \ldots$$

Expect this to be generically (but not always) a correct identification as far as vector multiplet moduli.

For CY compactification of Type II to 4d F_1 can be explicitly computed as it is related to genus one topological string partition function:

$$F_1 = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \mathrm{Tr}\left[(-1)^F F_L F_R q^{H_0} \bar{q}^{\bar{H}_0}\right]$$

There are further motivations for why this is related to the species scale. Number of light degrees of freedom should be reflected in the spectrum of Laplacian and indeed we have

$$F_1 = \frac{1}{2}\sum_{p,q} (-1)^{p+q}\left(p-\frac{3}{2}\right)\left(q-\frac{3}{2}\right) \log(\det \Delta_{(p,q)})$$

This is consistent with $F_1 \sim 1/\Lambda_s^2 \sim N$ (see also [CLS])
Also related to a gravitational ``a-function''.

We have checked that the asymptotic limits agree with that expected from the asymptotic behavior of the species scale.

Confirms the picture that Λ_s reaches maximum (`desert point') somewhere in the middle of moduli (LG point for the quintic case) and vanishes exponentially asymptotically.
We have checked this for both KK boundaries and fundamental string boundaries (K3-fibered CY3 when volume of K3 vanishes leading to heterotic strings).

Question: Are there any features which can be argued to hold both in the interior of moduli space and the asymptotic values?

To assess this, we consider integrating UV modes to obtain an EFT for the gravity sector in two steps:

Step 1-Integrate out the massive modes. As discussed before we expect a resulting gravity EFT action valid up to distance scale Λ_s^{-1} of the form:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \ldots \right) + \frac{1}{2}(\partial\phi)^2 + \ldots \right]$$

Step 2-Integrate out the short distance modes of the massless field ϕ up to the cutoff $1/\Lambda_s$.

Consistency requirement: Step 2, should not change the general structure of EFT from step 1, because the BH entropy argument for species scale comes from massive modes because ϕ is just one mode.

$$\frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} \to \frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}(2-n)\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\delta\phi$$

$$\frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_m(R)}{\Lambda_s^{m-2}(\phi)} \to \frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_m(R)}{\Lambda_s^{m-2}(\phi)}(2-m)\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\delta\phi$$

$$\frac{M_{\mathrm{pl}}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} \to \frac{M_{\mathrm{pl}}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}(2-n)\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\delta\phi$$

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$$\sim M_{\rm pl}^{2d-4}\int d^dx d^dy \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}(x)\ G(x,y)\ \frac{\mathcal{O}_m(R)}{\Lambda_s^{m-2}(\phi)}\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}(y)$$

$$\frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} \to \frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}(2-n)\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\delta\phi$$

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$$\sim M_{\rm pl}^{d-2}\int d^dx \frac{\tilde{\mathcal{O}}_{m+n}(R)}{\Lambda_s^{m+n-2}(\phi)}\left|\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\right|^2 M_{pl}^{d-2}$$

$\tilde{\mathcal{O}}_{m+n}(R)$ smeared version of operators in box of size Λ_s^{-1}

$$\sim M_{\rm pl}^{d-2}\int d^dx \frac{\tilde{\mathcal{O}}_{m+n}(R)}{\Lambda_s^{m+n-2}(\phi)}$$ expected form

$$\frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} \rightarrow \frac{M_{\rm pl}^{d-2}}{2}\frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}(2-n)\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\delta\phi$$

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$$\sim M_{\rm pl}^{2d-4}\int d^dx d^dy \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)}\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}(x)\ G(x,y)\ \frac{\mathcal{O}_m(R)}{\Lambda_s^{m-2}(\phi)}\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}(y)$$

$$\sim M_{\rm pl}^{d-2}\int d^dx \frac{\tilde{\mathcal{O}}_{m+n}(R)}{\Lambda_s^{m+n-2}(\phi)}\left|\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\right|^2 M_{pl}^{d-2} \leq O(1)$$

$\tilde{\mathcal{O}}_{m+n}(R)$ smeared version of operators in box of size Λ_s^{-1}

$$\sim M_{\rm pl}^{d-2}\int d^dx \frac{\tilde{\mathcal{O}}_{m+n}(R)}{\Lambda_s^{m+n-2}(\phi)}$$ expected form

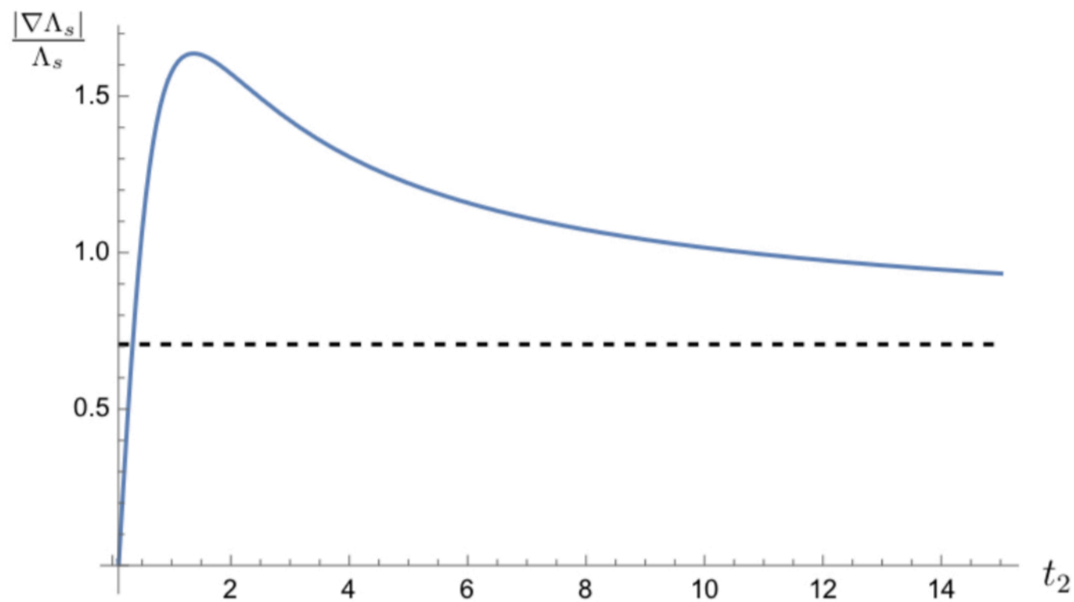
One may be tempted based on expected asymptotic value, to fix the $O(1)$ constant for all points:

$$\left|\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\right|^2 M_{pl}^{d-2} \leq \frac{1}{d-2}$$

One may be tempted based on expected asymptotic value, to fix the $O(1)$ constant for all points:

$$\left|\frac{\Lambda_s'(\phi)}{\Lambda_s(\phi)}\right|^2 M_{pl}^{d-2} \leq \frac{1}{d-2}$$

Using $\Lambda_s^2 = \dfrac{1}{F_1}$, we find the typical curve:



$$\frac{1}{\sqrt{d-2}} = \frac{1}{\sqrt{2}}$$

Application to Distance Conjecture

If we have any tower of light particles with typical mass scale for the tower m we expect the species scale to be related to it by a power law:

$$m \sim \Lambda_s^{\beta}$$

This implies

$$\left|\frac{m'}{m}\right|^2 \leq \frac{O(1)}{M_{pl}^{d-2}}$$

Which implies that the exponential nature of distance conjecture is the fastest it can be!

What is needed is also the asymptotic inequality in the other direction to lead to expected exponential behavior.

Conclusion

We have seen that the species scale which can depend on massless moduli varies over the moduli but its slope cannot be too large, and at the fastest rate it can change exponentially, as is expected at boundaries. Black hole entropy and its relation to species scale is a key in deriving this bound.

The same is true for the mass of towers as we approach the boundaries, leading at most to exponential falling off of the mass of the asymptotic tower.