

On the Redistributive Effects of Government Bailouts in the Mortgage Market

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Abstract

In this paper we study the aggregate and distributional consequences of government bailout guarantees in the U.S. mortgage market. We construct a model with aggregate risk in which competitive financial intermediaries issue mortgages to households that can default on their debt. Default probabilities are priced into mortgage interest rates unless a government bailout guarantee makes the lenders whole even in economic downturns in which foreclosure rate surge.

1 Introduction

In this paper we study the aggregate and distributional consequences of government bailout guarantees in the U.S. mortgage market. We construct a model with aggregate risk in which competitive financial intermediaries issue mortgages to households that can default on their debt. Default probabilities are priced into mortgage interest rates unless a government bailout guarantee makes the lenders whole even in economic downturns in which foreclosure rate surge. We then use the model to evaluate the importance of this policy for aggregate housing and leverage and its cross-sectional distribution in tranquil times, as well as for the consequences of large economic crisis such as the great recession.

Discussion about the institutional details

Discussion about the implicit and now explicit bailout guarantee

Discussion about time series of household debt and leverage as well as house prices

In order to evaluate the aggregate and distributional consequences of a bailout guarantee that is perceived to be credible by market participants we construct a model with aggregate risk as well as uninsurable idiosyncratic risk, as in Krusell and Smith (1997, 1998), but augmented with a real estate sector, mortgage debt and consumer default, as in Jeske, Krueger and Mitman (2013). In that model we provide a sharp characterization of equilibrium mortgage interest rates which are shown to depend primarily (and under certain assumptions on the default consequences, exclusively) on the leverage ratio associated with the mortgage under consideration as well as on aggregate conditions determining aggregate house prices and risk free interest rates.

1.1 Related Literature

Literature on quantitative housing, with focus on aggregate risk

Literature on housing subsidies: Gervais (2001), Jeske et al. (2013)

Literature on bailouts more generally: Roch and Uhlig 2012 study the effects of bailouts in the context of international debt crises. Their mechanism that bailouts lead to cheaper credit and thus more debt in equilibrium is similar to ours, although both the context as well as the institutional details how the bailout is implemented differs. Cooper and Kempf (2011).

2 The Model

Our model features aggregate productivity risk as well as idiosyncratic income and house price risk. It can be seen as an extension of Krusell and Smith's (1997, 1998) environment to a model with risky real estate, short-term mortgages and a foreclosure option, as in the stationary model of Jeske et al. (2013).

2.1 Stochastic Structure

In order to induce business cycles the economy is subject to aggregate productivity shocks, denoted by $z \in Z = \{z_b, z_g\}$ that follow a finite Markov chain with transition matrix $\pi(z'|z)$. We interpret z_g as normal times and z_b as a severe economic crisis, such as the great depression or the great recession. The aggregate shock z affects both the aggregate component of income $A(z)$ as well as the housing technology $A_h(z)$. Note that this specification allows aggregate income and aggregate house prices to be positively correlated.

In addition households will face uninsurable idiosyncratic income and house price risk that we will spell out in greater detail below. As a consequence the endogenous aggregate state variables, from now on summarized by S , will include the cross-sectional joint income and wealth (or more precisely, cash at hand) distribution $\mu(y, a)$ across households. In what follows we will directly set up the economy recursively, thereby sidestepping the sequential formulation of the model.

2.2 Households

2.2.1 Preferences and Endowments

Households value nondurable consumption c and housing services h according to the period utility function $U(c, h)$ and discount the future at rate β . They are assumed to maximize expected utility. We normalize the price of nondurable consumption to 1 in every period and denote the rental price of one unit of real estate by $P_l(z, S)$.

In each period households are endowed with one unit of time that they supply inelastically to the labor market. Their labor productivity in the current period is denoted by $y \in Y(z)$ and follows a finite state Markov chain with transitions $\pi(y'|y, z', z)$. Both the individual states and the transition probabilities are permitted to depend on the aggregate state of the economy, in order to capture the possibility that idiosyncratic income risk is countercyclical, as argued by Storesletten, Telmer and Yaron (2004b). We assume that a law of large numbers apply, so that $\pi(y'|y, z', z)$ is not only the transition probability for each household, but also the deterministic fraction of the population making a labor productivity transition from y today to y' tomorrow, conditional on the aggregate economy transiting from z to z' . We assume that the cross-sectional distribution over individual productivity shocks $\Pi_z(y)$ only depends on the current aggregate shock and denote by

$$L(z) = \sum_y y \Pi_z(y)$$

the exogenous aggregate effective labor input in aggregate state z .

A household's labor income is then given by $w(z, S)y$, where $w(z, S)$ is the economy-wide wage per labor efficiency unit, and households pay labor income taxes according to the (potentially nonlinear) tax schedule $T(\cdot)$.

2.2.2 Assets

Households can purchase two classes of assets, real estate that carries idiosyncratic house price risk, and a full set of Arrow securities that pay off contingent on the realization of the aggregate shock z' . Their prices are denoted by $P_b(z, S, z')$ and the position chosen by the household as $b'(z')$. Real estate is perfectly divisible and carries a price $P_h(z, S)$ per unit. Houses purchased today (whose position is denoted by g') can be rented out immediately, and thus the out-of-pocket expense per unit equals $P_h(z, S) - P_l(z, S)$. Tomorrow the value of the house (per unit) is given by $P_h(z', S')(1 - \delta')$, where δ' is an idiosyncratic house price depreciation shock that is drawn from a differentiable cumulative distribution function $F(\delta')$ with domain $[\underline{\delta}, 1]$. As with the idiosyncratic labor productivity shocks an assumed law of large number assures that $F(\cdot)$ is also the

cross-sectional distribution over house price depreciation shocks.¹ By assumption households cannot sell short either Arrow securities or real estate.

2.2.3 Mortgages and Default

In order to finance the purchase of real estate households can use short-term mortgage debt. We denote by m' the amount of mortgage debt incurred today and to be repaid tomorrow, and by $P_m m'$ the amount of resources today advanced to the household for the promise to repay m' tomorrow. The mortgage pricing function $P_m(y, a, \delta, g', m', b'; z, S)$ will be determined in equilibrium and might depend on current household characteristics (y, a, δ) , the size of the mortgage m' and the housing collateral g' , the position of financial assets $b'(\cdot)$, as well as on aggregate conditions summarized by the aggregate state S . The inverse of the price, $1/P_m$ then gives the gross real mortgage interest rate.

The legal environment, which we take as given in this paper, allows households to default on their mortgage debt, with the consequence of having their real estate position foreclosed and a share ϕ of their financial assets confiscated. Thus households default on their mortgage debt tomorrow if and only if

$$P_h(z', S')(1 - \delta')g' + b'(z') - m' < (1 - \phi)b(z')$$

that is, if and only if, given the size of the mortgage and the collateral (m', g') and the financial asset position $b'(z')$, the idiosyncratic house price depreciation shock δ' is sufficiently large. Define the threshold depreciation rate δ^* at which the household is indifferent between defaulting or not by

$$P_h(z', S')(1 - \delta^*)g' + b'(z') - m' = (1 - \phi)b(z')$$

or

$$\delta^* = 1 - \frac{m' - \phi b'(z')}{P_h(z', S')g'} = \delta^*(m', g', b'(z'); z', S') \quad (1)$$

and thus the default probability in state z' tomorrow is given by $1 - F(\delta^*(m', g', b'(z'); z', S'))$. Also note that if $\phi = 0$ then households default on their mortgage debt if and only if their real estate is under water. On the other hand, if $\phi > 0$ households may refrain from defaulting even in that case (as long as they have some financial assets).

Note that the current household characteristics (y, a, δ) only affect the probability of default tomorrow if they affect the distribution $F(\delta')$ of house price shocks tomorrow. Under the assumption that the distribution $F(\delta')$ does not depend on (a, y, δ) , neither will default probabilities and thus mortgage interest rates. We will already exploit this result and write $P_m(g', m', b'; z, S)$ in the recursive formulation of the household problem to which we turn next.

2.2.4 Recursive Formulation of Household Decision Problem

The individual state variables are labor productivity (labor income) y and cash at hand a . Let $s = (y, a)$ denote the individual state variables, and, as before, S as the endogenous aggregate state. The Bellman

¹Conceptually it is straightforward to make the distribution F of δ dependent on the aggregate shock z , and to introduce positive correlation between the exogenous labor productivity and house price shocks, by writing $F(\delta|y, z)$. Similarly, it is straightforward to model serial correlation in house price depreciation shocks as well.

Also note that since δ is a multiplicative shock, it constitutes a permanent shock to the individual house price of a particular household.

equation reads as

$$v(s, z, S) = \max_{\substack{c, h \geq 0 \\ b', m', g' \geq 0}} \left\{ U(c, h) + \beta \sum_{y', z'} \pi(z'|z) \pi(y'|y, z', z) \int_{\underline{\delta}}^1 v(s', z', S') dF(\delta') \right\} \quad (2)$$

s.t.

$$a = c + \sum_{z'} P_b(z, S, z') b'(z') + g' P_h(z, S) + (h - g') P_l(z, S) - m' P_m(g', m', b'; z, S) \quad (3)$$

$$a' = w(z', S') y' - T(w(z', S') y'; z', S') + \max\{(1 - \phi) b(z'), P_h(z', S') (1 - \delta') g' + b'(z') - m'\} \quad (4)$$

$$S' = \Gamma(z, S, z') \quad (5)$$

where $T(w(z, S) y; z, S)$ are the income taxes paid on household labor income $w(z, S) y$ in aggregate state (z, S) .

2.3 Real Estate Production Technology

Perfectly divisible houses are produced by competitive real estate construction companies. The representative firm in this sector has access to a linear technology that turns one unit of the final consumption good into $A_h(z)$ units of real estate, where $A_h(z)$ is total factor productivity in the real estate production sector. The firm buys C_h units of inputs to produce I_h units of houses, in order to solve the following maximization problem:

$$\begin{aligned} & \max_{I_h, C_h} P_h(z, S) I_h - C_h \\ & \text{s.t.} \\ & I_h = A_h(z) C_h \end{aligned}$$

Thus it immediately follows that the equilibrium price² for one unit of newly produced real estate equals $P_h(z, S) = P_h(z) = \frac{1}{A_h(z)}$. Effectively, this is a model with exogenous house prices driven by the exogenous process $\{A_h(z)\}$ and real estate production (residential fixed investment) I_h being determined by the demand side at the exogenous price $P_h(z) = \frac{1}{A_h(z)}$.

2.4 Final Output Production Technology

As in the real estate production sector, final output is produced by a representative, perfectly competitive firm that operates a constant returns to scale technology

$$Y = A(z) K^\alpha L^{1-\alpha}$$

where Y is the final goods output and K and L are the capital and labor inputs the firm rents from perfectly competitive factor markets at rental rates $w(z, S)$ and $r(z, S)$, respectively. For future reference, these will be given in equilibrium as

$$\begin{aligned} w(z, S) &= (1 - \alpha) \frac{Y(z, S)}{L(z)} \\ r(z, S) &= \alpha \frac{K(z, S)}{L(z)} - \delta_k \end{aligned}$$

where we have already anticipated the labor market clearing condition equating labor demand to exogenously given aggregate labor supply $L(z)$. We furthermore assume that capital depreciates at rate δ_k in every period.

Remark 1 Below we will document results for an endowment economy as well, which is a special case of our general model with $\alpha = 0$ (and $\delta_k = K = 0$). In this case $w(z, S) = A(z)$.

²This statement assumes that I_h, C_h are not constrained to be positive. If we restrict $I_h, C_h \geq 0$, then we can only conclude that $P_h(z) \leq \frac{1}{A_h(z)}$, with equality if $I_h = C_h = 0$.

2.5 Financial Intermediaries

Competitive financial intermediaries compete mortgage loan by loan for households, as in Chatterjee et al. (2007). For a loan size m' against a house of size³ $g' > 0$, to be repaid tomorrow the intermediary advances $P_m m'$ resources today and bears administrative costs $r_w m' P_m$. To finance this cost $(1 + r_w) m' P_m$ the intermediary sells a position of Arrow securities, specified below. In the next period, if the household does not default the financial intermediary collects the repayment of the loan m' . However, if the household reneges on her promise, the intermediary obtains the house and sells it for $\gamma(1 - \delta')g' P_h(z')$, where $\gamma < 1$ measures efficiency of foreclosure technology. The intermediary also extracts $\phi b'(z')$ financial assets from the household.

Thus expected revenues from a loan with characteristics (m', g') to a household with financial asset portfolio $(b'(z'))$ in aggregate state (z', S') tomorrow is given by $m' \Psi(m', g', b'(z'); z', S')$ where

$$\begin{aligned} \Psi(m', g', b'(z'); z', S') &= F(\delta^*(m', g', b'(z'); z', S')) + \phi(1 - F(\delta^*(m', g', b'(z'); z', S'))) \frac{b'(z')}{m'} \\ &\quad + \frac{\gamma P_h(z', S') g'}{m'} \int_{\delta^*(m', g', b'(z'); z', S')}^1 (1 - \delta') dF(\delta') \end{aligned} \quad (6)$$

is the expected revenue per unit of loan m' extended. Since the mortgage market is perfectly competitive loan by loan, for all contracts issued in equilibrium it has to be the case that the cost $(1 + r_w) m' P_m$ for the intermediary today equals the expected revenue tomorrow $m' \Psi(m', g', b'(z'); z', S')$, summed over all aggregate states tomorrow and appropriately discounted to today. Thus the equilibrium loan pricing function has to satisfy:

$$(1 + r_w) P_m(m', g', (b'(z'))); z, S) = \sum_{z'} P_b(z, S, z') \Psi(m', g', b'(z'); z', S')$$

or

$$P_m(m', g', (b'(z'))); z, S) = \sum_{z'} \frac{P_b(z, S, z')}{(1 + r_w)} \Psi(m', g', b'(z'); z', S') \quad (7)$$

Thus what a financial intermediary effectively does when issuing a mortgage m' against a house g' , is to sell Arrow securities in the amounts of $m' \Psi(m', g', b'(z'); z', S')$ to households and uses the proceeds $m' \sum_{z'} q(z, S, z') \Psi(m', g', b'(z'); z', S')$ to transfer $m' P_m(m', g', (b'(z'))); z, S)$ to the household that takes the mortgage and bears resource cost $r_w m' P_m(m', g', (b'(z'))); z, S)$. Tomorrow the financial intermediary takes the state-contingent revenues from the mortgages $m' \Psi(m', g', b'(z'); z', S')$ to repay their position of the Arrow securities. Thus the mortgage issuer fully hedges against the aggregate risk; by taking on a positive measure of $(m', g', (b'(z')))$ type mortgages the law of large numbers assures that the financial intermediary also fully diversifies (by pooling) idiosyncratic household default risk.

2.6 Government Policy

We model two component of government policy. First, in order to obtain realistic levels of tax rates even in the absence of a government bailout guarantee we assume that the government spends a fixed fraction g of total output as nondiscretionary government expenditures:

$$G(z, S) = gY(z, S) \quad (8)$$

where $g > 0$ is a fixed policy parameter to be calibrated. For future reference we note that

$$G(z, S) = gY(z, S) = \frac{g}{1 - \alpha} w(z, S) L(z) \quad (9)$$

Second, and at the core of this paper, we have to model a government bailout guarantee of the financial intermediaries in the real estate sector. We assume that the government bails banks out only when the

³For positive mortgages $m' > 0$ we require positive collateral $g' > 0$ by assumption. For $\phi > 0$ we need to check whether we can sustain positive uncollateralized debt; for now we rule it out by assumption.

economy turns from normal times $z = z_g$ into crisis times $z' = z_b$. In that situation, the government guarantees that mortgages, in expectation over idiosyncratic shocks, have the same payoffs for the financial intermediary in good and bad states of the world. That is, the government guarantees that if $z = z_g$, then

$$\Psi(m', g', b'(z'); z' = z_b, S') = \Psi(m', g', b'(z'); z' = z_g, S')$$

Thus the government insures the mortgage issuer both against lower house prices directly (which affects what the intermediary gets after default) and against the higher default rates due to lower house prices. The subsidy to a mortgage taker for a mortgage of type $(m', g', (b'(z')))$ is worth today

$$P_b(z, S, z') [\Psi(m', g', b'(z'); z' = z_g, S') - \Psi(m', g', b'(z'); z' = z_b, S')]$$

since the mortgage taker gets

$$\begin{aligned} P_m(m', g', (b'(z')); z_g, S) &= \Psi(m', g', b'(z'); z' = z_g, S') \sum_{z'} \frac{P_b(z, S, z')}{1 + r_w} \\ &> \sum_{z'} \frac{P_b(z, S, z')}{(1 + r_w)} \Psi(m', g', b'(z'); z', S') \end{aligned} \quad (10)$$

In the aggregate state $z' = z_b$ tomorrow (conditional on $z = z_g$, since otherwise there is no bailout tomorrow) the government has to raise taxes to cover the loss to the mortgage issuers, with the total losses equal to

$$Tr(S) = \int m' [\Psi(m', g', b'(z'); z' = z_g, S') - \Psi(m', g', b'(z'); z' = z_b, S')] d\mu. \quad (11)$$

where it is understood that m', g' and $b'(z')$ are functions of the state (s, z, S) . Note that these required tax revenues tomorrow are a deterministic function of the aggregate state S today. As a consequence, the parameters governing the tax policy function tomorrow need to be functions of the aggregate state today as well. Rather than spelling this mapping out in full generality we restrict attention to the class of tax functions used in the quantitative analysis. As in Benabou (2002), Heathcote et al. (2012) and Bacis et al. (2012), labor income taxes are given as

$$T(w(z, S)y; z, S) = w(z, S)y - (1 - \tau) (w(z, S)y)^{1-\lambda}$$

so that after-tax labor income for a household making per-tax earnings $w(z, S)y$ equals

$$(1 - \tau) (w(z, S)y)^{1-\lambda}.$$

The parameter λ governs the progressivity of the labor income tax schedule and will be treated as fixed in the analysis. The parameter τ instead determines the level of labor income taxes and will vary with the state of the economy to insure a balanced budget. Total tax revenues are given by

$$\int T(w(z)y; z, S) d\mu = \int w(z, S)y d\mu - (1 - \tau) \int (w(z, S)y)^{1-\lambda} d\mu = w(z, S)L(z) - (1 - \tau)w(z, S)^{1-\lambda}\xi(z)$$

where

$$\xi(z) = \int y^{1-\lambda} d\mu = \sum_y y^{1-\lambda} \Pi_z(y)$$

is an exogenous function of the aggregate shock z . Thus the government budget constraint for tomorrow implies, for $z = z_g$ and $z' = z_b$

$$\int T(w(z', S')y'; z', S') d\mu' = G(z', S') + Tr(S)$$

and thus, using equation (9):

$$w(z', S')L(z') - (1 - \tau)w(z', S')^{1-\lambda}\xi(z') = \frac{g}{1 - \alpha}w(z', S')L(z') + Tr(S)$$

which can be solved for the tax level in closed form as

$$\tau(z, S, z') = 1 - \frac{1 - \alpha - g}{1 - \alpha} w(z', S')^\lambda \frac{L(z')}{\xi(z')} + \frac{Tr(S)}{w(z', S')^{1-\lambda} \xi(z')}$$

where $S' = \Gamma(z, S, z')$ is determined by the aggregate law of motion. This discussion suggests that one way⁴ to deal with the fact that the tax rate tomorrow is a function of the aggregate state S (and specifically, the cross-sectional asset distribution) today is to make the current tax rate τ part of the endogenous aggregate state S today and specify as law of motion for the tax rate tomorrow⁵

$$\tau' = \tau(z, S, z') = \begin{cases} 1 - \frac{1 - \alpha - g}{1 - \alpha} w(z', S')^\lambda \frac{L(z')}{\xi(z')} & \text{if } z = z_g \text{ or } z_{-1} = z_b \\ 1 - \frac{1 - \alpha - g}{1 - \alpha} w(z', S')^\lambda \frac{L(z')}{\xi(z')} + \frac{Tr(S)}{w(z', S')^{1-\lambda} \xi(z')} & \text{if } z = z_b \text{ and } z_{-1} = z_g \end{cases} \quad (12)$$

2.7 Recursive Competitive Equilibrium

We are now ready to define a recursive competitive equilibrium for our economy. The set of endogenous aggregate state variables includes $S = (H, K, \tau, \mu)$, where H denotes the depreciated housing stock (prior to new construction).⁶

Definition 2 *A recursive competitive equilibrium with bailout guarantees are household value and policy functions $v, c, h, g', m', b'(\cdot)$, aggregate allocation functions I, C_h, I_h , pricing functions P_b, P_m, P_l, P_h, r, w and an aggregate law of motion Γ such that*

1. *[Household Optimality]: Given prices P_b, P_m, P_l, P_h, r, w and the aggregate law of motion Γ , the function v solves the household Bellman equation and $c, h, g', m', b'(\cdot)$ are the associated policy functions.*
2. *[Final Goods Production Firms' Optimality]: Wages and rental rates of capital are given by*

$$\begin{aligned} w(z, S) &= (1 - \alpha)A(z) \left(\frac{K(z, S)}{L(z)} \right)^\alpha \\ r(z, S) &= \alpha A(z) \left(\frac{K(z, S)}{L(z)} \right)^{\alpha-1} \end{aligned}$$

3. *[Real Estate Production Firms' Optimality]: The price for real estate satisfies*

$$P_h(z, S) = \frac{1}{A_h(z)}$$

and new construction equals output in the real estate production sector:

$$I_h(z, S) = A_h(z)C_h(z, S)$$

4. *[Financial Intermediaries' Optimality]: The mortgage pricing function is given by equation (10).*

5. *Market clearing*

(a) *The rental market clears*

$$\int h(s, z, S) d\mu = \int g'(s, z, S) d\mu$$

⁴Alternatively one could make the distribution μ_{-1} part of the state space (as it determines the required tax revenues $T(S_{-1})$ today), but that approach is even more burdensome.

⁵This might look circular because of $S' = \Gamma(z, S, z')$ on the right hand side of the equation, but note that S' only appears as determinant of $w(z', S')$, and the only part of S' wages depend on is K' which is determined exclusively by μ' but does not depend on τ' .

⁶Since μ is the joint distribution of labor productivity and cash at hand we cannot immediately infer the aggregate capital stock K from it, and thus we include it as an additional state variable. When computing an equilibrium we will use K as one of the state variables anyhow (as did Krusell and Smith, 1998).

(b) The real estate market clears:

$$\int g'(s, z, S) d\mu = H + I_h(z, S)$$

(c) The market for state-contingent assets clears state by state: for all z'

$$\begin{aligned} & \int b'(s, z, S, z') d\mu \\ = & (1 + r(z', S') - \delta)K'(z, S) + \int m'(s, z, S) \Psi(m'(s, z, S), g'(s, z, S), b'(s, z, S, z'); z', S') d\mu \end{aligned}$$

(d) The goods market clears

$$\int c(s, z, S) d\mu + C_h(z, S) + I(z, S) + r_w \int m'(s, z, S) P_m(\kappa'(s, S), S) d\mu = A(z)K(z, S)^\alpha L(z)^{1-\alpha}$$

(e) No arbitrage between owning physical capital and portfolio of Arrow securities:

$$1 = \sum_{z'} P_b(z, S, z')(1 + r(z', S') - \delta)$$

6. Laws of Motion

(a) Housing stock

$$\begin{aligned} H' &= \Gamma_H(z, S, z') = \int \int_{\underline{\delta}}^{\delta^*(m', g', b'(z'); z', S')} (1 - \delta') g'(s, z, S) dF(\delta') d\mu \\ &+ \gamma \int \int_{\delta^*(m', g', b'(z'); z', S')}^1 (1 - \delta') g'(s, z, S) dF(\delta') d\mu \end{aligned}$$

(b) Capital stock

$$K' = \Gamma_K(z, S, z') = (1 - \delta_k)K + I(z, S)$$

(c) Taxes: $\tau' = \Gamma_\tau(z, S, z')$ is given by equation (12).

(d) Distribution⁷:

$$\mu' = \Gamma_\mu(z, S, z')$$

Remark 3 The corresponding equilibrium without a bailout policy is defined in exactly the same way, but with

$$\tau' = \tau(z, S, z') = 1 - \frac{1 - \alpha - g}{1 - \alpha} w(z', S')^\lambda \frac{L(z')}{\xi(z')}$$

and

$$P_m(m', g', (b'(z')); z, S) = \sum_{z'} \frac{q(z, S, z')}{(1 + r_w)} \Psi(m', g', b'(z'); z', S')$$

3 Theoretical Characterization

In order to aid the computation of an equilibrium and to derive a partial analytical characterization we now first simplify the equilibrium mortgage pricing function and then use these results to simplify the household decision problem.

⁷The explicit function Γ_μ is determined by the Markov transition function induced by the exogenous Markov chain π and the optimal household policy functions that determine a' according to equation (4).

3.1 The Mortgage Interest Rate Function

Recall that the default threshold is given, for all $g' > 0$, by

$$\delta^*(m', g', b'(z'); z', S') = 1 - \frac{m' - \phi b'(z')}{P_h(z', S')g'}.$$

Defining as leverage $\kappa' = \frac{m'}{g'}$ and as financial-to-real-wealth ratio $\theta'(z') = \frac{b'(z')}{g'}$ and recalling that $P_h(z', S') = P_h(z')$, we can rewrite this as

$$\delta^* = 1 - \frac{\kappa' - \phi\theta'(z')}{P_h(z')} = \delta^*(\kappa', \theta'(z'), z')$$

and thus a household's default probability in aggregate state z' tomorrow only depends on leverage and the financial wealth ratio chosen today.⁸

The repayment per unit of loan m' in state z' tomorrow thus can be written as

$$\begin{aligned} \Psi(m', g', b'(z'); z', S') &= F(\delta^*(\kappa', \theta'(z'), z')) + \phi(1 - F(\delta^*(\kappa', \theta'(z'), z'))) \frac{\theta'(z')}{\kappa'} \\ &\quad + \frac{\gamma P_h(z')}{\kappa'} \int_{\delta^*(\kappa', \theta'(z'), z')}^1 (1 - \delta') dF(\delta') \\ &= \Psi(\kappa', \theta'(z'), z') \end{aligned}$$

and also only depends on leverage and the financial wealth ratio. This in turn simplifies the mortgage pricing function (in the case without bailout, the expression with bailout is similar)

$$P_m(\kappa', \theta'; z, S) = \sum_{z'} \frac{q(z, S, z')}{(1 + r_w)} \Psi(\kappa', \theta'(z'), z')$$

where $\theta' = (\theta'(z'))_{z' \in Z}$ stands in for the entire state-contingent portfolio of financial (relative to real) assets chosen by the household in the current period. Of course, if there is no recourse and $\phi = 0$, then Ψ and consequently P_m is independent of financial asset choices by the household.

We summarize the properties of the mortgage pricing function P_m in the following

Proposition 4 *The equilibrium mortgage pricing function has the following properties*

1. *It depends on the aggregate state S only through the state contingent interest rates $1/q(z, S, z')$*
2. *Mortgages with higher leverage command higher interest rates: $P_m(\kappa', \theta'; z, S)$ is strictly decreasing in κ' .*
3. *Households with more financial assets pay lower interest rates: $P_m(\kappa', \theta'; z, S)$ is increasing in θ' , strictly so if $\phi > 0$.*

Proof. Should follow directly from the properties of Ψ . ■

3.2 Simplification of the Household Problem

For the purpose of the analysis and computation of the household decision problem it is helpful to divide the household maximization problem into three sub-problems, a static maximization problem that optimally allocates consumption expenditures between nondurable consumption and housing, a standard intertemporal consumption-saving problem, and a portfolio-choice problem that allocates savings between real and financial assets.

⁸Note that we could have chosen to define realized leverage as $\frac{m'}{g'P_h(z')}$, but the household cannot, via her choices, perfectly control this variable, which makes the simplification of the household problem below more burdensome.

3.2.1 Static Consumption Expenditure Problem

Since the decision of how much real estate services to consume is not directly tied to the ownership position of real estate, the decision how to allocate total consumption expenditures today is a purely static problem, given the rental price $P_l(z, S) > 0$. Thus define

$$\begin{aligned} u(ce; P_l(z, S)) &= \max_{c, h \geq 0} U(c, h) \\ &\text{s.t.} \\ c + P_l(z, S)h &= ce \end{aligned}$$

The solution to this problem is uniquely determined (under the standard concavity, monotonicity and differentiability conditions) by the two equations

$$\begin{aligned} c + P_l(z, S)h &= ce \\ \frac{U_h(c, h)}{U_c(c, h)} &= P_l(z, S) \end{aligned}$$

3.2.2 Dynamic Consumption-Saving Problem

Define as

$$x' = \sum_{z'} P_b(z, S, z')b'(z') + g' [P_h(z) - P_l(z, S)] - m' P_m(\kappa', \theta'; z, S)$$

the total net value of assets purchased today and to be brought into the next period. Then the dynamic consumption-savings problem reads as

$$v(s, z, S) = \max_{0 \leq x' \leq a} \{u(a - x'; P_l(z, S)) + \beta w(x', s, z, S)\}$$

where $w(x', s, z, S)$ is the expected lifetime utility value of saving x' units to be determined in the next subsection.

3.2.3 Portfolio Allocation Problem

Given a total amount of saving x' the household decides how to optimally allocates it between financial assets b' , real housing assets h' and the extent of mortgage debt financing. This choice solves

$$w(x', s, z, S) = \max_{b'(z'), g', m', \kappa', \theta'(z') \geq 0} \sum_{y', z'} \pi(z'|z) \pi(y'|y, z', z) \int_{\delta}^1 v(s', z', S') dF(\delta') \quad (13)$$

s.t.

$$x' = \sum_{z'} P_b(z, S, z')b'(z') + g' [P_h(z) - P_l(z, S)] - m' P_m(\kappa', \theta'; z, S) \quad (14)$$

$$a' = w(z', S')y' - T(w(z', S')y'; z', S') + \max\{(1 - \phi)b(z'), P_h(z')(1 - \delta')g' + b'(z') - m'\} \quad (15)$$

where

$$s' = (a', y'), \kappa' = \frac{m'}{g'}, \theta'(z') = \frac{b'(z')}{g'}$$

$$S' = \Gamma(z, S, z')$$

Now we note that this maximization problem can be written exclusively as a choice problem of leverage κ' and the financial wealth ratios $\theta'(z')$. To see this, we first rewrite (14) and (15) as

$$\begin{aligned} x' &= g' \left[\sum_{z'} P_b(z, S, z')\theta'(z') + [P_h(z) - P_l(z, S)] - \kappa' P_m(\kappa', \theta'; z, S) \right] \\ a' &= w(z', S')y' - T(w(z', S')y'; z', S') + g' \max\{(1 - \phi)\theta'(z'), P_h(z')(1 - \delta') + \theta'(z') - \kappa'\} \end{aligned}$$

We can solve the first equation for g' and plug it into the second equation to obtain

$$a'(\kappa', \theta'(\cdot); x', y', z', S') = w(z', S')y' - T(w(z', S')y'; z', S') \\ + \frac{x' \max\{(1-\phi)\theta'(z'), P_h(z')(1-\delta') + \theta'(z') - \kappa'\}}{\sum_{z'} P_b(z, S, \hat{z}')\theta'(\hat{z}') + [P_h(z) - P_l(z, S)] - \kappa' P_m(\kappa', \theta'; z, S)}$$

Some care has to be taken for the case $g' = 0$ since then κ' is not well-defined. Thus the maximization problem (13) can conveniently be expressed as

$$w(x', s, z, S) = \max_{\kappa', \theta'(z') \geq 0} \sum_{y', z'} \pi(z'|z)\pi(y'|y, z', z) \int_{\underline{\delta}}^1 v((a'(\kappa', \theta'(\cdot)); x', y', z', S'), y'), z', S') dF(\delta') \\ \text{with } a'(\kappa', \theta'(\cdot); x', y', z', S') \text{ given by (??)}$$

Remark 5 *This discussion has implicitly assumed that $g' > 0$ and thus $\kappa', \theta'(z')$ are well-defined. It can easily be amended to incorporate the $g' = 0$ case. As a matter of convention, let $\kappa' = \infty$ in this case. First we note that if $g' = 0$ it is optimal to set $m' = 0$ since $P_m = 0$ for all mortgage contracts without collateral. Therefore in this case (14) and (15) read as*

$$x' = \sum_{z'} P_b(z, S, z')b'(z') \\ a' = w(z', S')y' - T(w(z', S')y'; z', S') + b'(z')$$

Defining, for this case,

$$\theta'(z') = \frac{b'(z')}{\sum_{z'} P_b(z, S, \hat{z}')b'(\hat{z}')} = \frac{b'(z')}{x'}$$

as financial portfolio shares, and noting that they have a different interpretation as above, we can write

$$a'(\kappa', \theta'(\cdot); x', y', z', S') = w(z', S')y' - T(w(z', S')y'; z', S') + x'\theta'(z')$$

and thus

$$a'(\kappa', \theta'(\cdot); x', y', z', S') \\ = w(z', S')y' - T(w(z', S')y'; z', S') \\ + \begin{cases} x'\theta'(z') & \text{if } \kappa' = \infty \\ \frac{x' \max\{(1-\phi)\theta'(z'), P_h(z')(1-\delta') + \theta'(z') - \kappa'\}}{\sum_{z'} P_b(z, S, \hat{z}')\theta'(\hat{z}') + [P_h(z) - P_l(z, S)] - \kappa' P_m(\kappa', \theta'; z, S)} & \text{else} \end{cases} \quad (16)$$

Therefore the portfolio allocation problem can compactly be written as

$$w(x', s, z, S) = \max_{\substack{\theta'(z') \geq 0 \\ \kappa' \in [0, \infty]}} \sum_{y', z'} \pi(z'|z)\pi(y'|y, z', z) \int_{\underline{\delta}}^1 v((a'(\kappa', \theta'(\cdot)); x', y', z', S'), y'), z', S') dF(\delta') \\ \text{with } a'(\kappa', \theta'(\cdot); x', y', z', S') \text{ given by (16)}$$

Proposition 6 *The solution to the household problem can be obtained by first solving the static problem in section 3.2.1, and then, taking the indirect utility function from the static problem $u(ce; P_l(z, S))$ as given, iterating on the solutions of the dynamic consumption-savings problem (which takes the function w as given and delivers the function v) and the portfolio allocation problem (which takes the function v as given and delivers w).*

Proof. Shown by the preceding discussion. ■

4 Calibration

In this section we calibrate our economy. In this version of the paper we consider the endowment economy version of our model and thus follow closely the calibration strategy of Jeske, Krueger and Mitman (2013), who consider an stationary endowment economy with a real estate sector very similar to the one modeled here.

4.1 Aggregate Risk

We let the aggregate shock z take two values, $Z = \{z_b, z_g\}$ and interpret z_g as normal (or *good*) times, and z_b as bad times, i.e. as a severe economic crisis such as the great depression or the great recession. In normal times productivity in both sectors is normalized to $A(z_g) = A_h(z_g) = 1$, and in bad times we set

$$A(z_b) = 0.92 \text{ and } A_h(z_b) = 1/0.75.$$

These choices imply that output falls by 8% relative to trend, and that house prices drop by 25%, as was the case between 2007 and 2009 for the U.S. according to the Case-Shiller house price index.

The transition matrix π is chosen such that the economy, in the long run, spends 15% of the time in severe recessions, $\Pi(z_b) = 0.15$ and that the expected length of a severe recession (conditional on falling into one) is 7.5 years. These choices are motivated by the occurrence of two severe recessions in the last 100 years (the great depression and the great recession) as well as the long time it took for the economy to return back to trend after the great depression and the great recession (for the latter it has not happened yet, six years after its onset). These two targets imply a Markov transition matrix for the aggregate shock given by:

$$\pi = \begin{pmatrix} 0.87 & 0.13 \\ 0.02 & 0.98 \end{pmatrix}.$$

4.2 Idiosyncratic Risk

In order to conserve on state variables we specify the idiosyncratic part of the household labor income process as a simple AR(1) process of the form (see Aiyagari (1994) and many others since):

$$\begin{aligned} \log y' &= \rho \log y + (1 - \rho^2)^{0.5} \varepsilon \\ E(\varepsilon) &= 0, E(\varepsilon^2) = \sigma_\varepsilon^2 \end{aligned}$$

Estimates for the parameters $(\rho, \sigma_\varepsilon)$ vary across studies, but our choices of $\rho = 0.98$ and $\sigma_\varepsilon = 0.30$ are well within the range the literature (see e.g. Storesletten, Telmer and Yaron, 2004a) reports. In the benchmark calibration we assume that the size σ_ε of the innovations is constant over time (and thus over the cycle), thus ruling out countercyclical cross-sectional variance of income risk. We discretize this continuous AR(1) process into a five state Markov chain using Tauchen and Hussey's (1991) procedure.

For the process governing house price depreciation distribution we assume a generalized Pareto distribution

$$f(\delta) = \frac{1}{\sigma_\delta} \left(1 + \frac{k(\delta - \underline{\delta})}{\sigma_\delta} \right)^{(-\frac{1}{k} - 1)}$$

with the three parameters $k, \sigma_\delta, \underline{\delta}$. We pin down these parameters to match, in the steady state version of the model, three targets in the data: a mean annual house depreciation rate of $E(\delta) = 1.48\%$, a volatility of individual house prices of $\sigma(\delta) = 10\%$ and the observation that 0.5% of mortgages are in default.

4.3 Preferences

We assume that the utility function takes the form

$$u(c, h) = \frac{(c^\theta h^{1-\theta})^{1-\sigma} - 1}{1 - \sigma}$$

where θ governs the expenditure share on nondurable consumption and is set to $\theta = 0.86$. The remaining preference parameters (σ, β) are chosen such that the steady state of model has risk-free rate of $r_b = 100$ basis points, and a median leverage of 61% (the median leverage in the 2004 SCF of households 50 and younger). This yields $\beta = 0.92$ and $\sigma = 3.9$.

4.4 Real Estate Market

We set the recovery rate on foreclosed properties to $\gamma = 0.78$, following Pennington-Cross (2004) who reports an average loss in value of a property due to foreclosure of 22%. Finally we set the servicing cost to $r_w = 10$ basis points. Recall that this cost creates a wedge between risk free saving and risk-free mortgage borrowing (i.e. obtaining a mortgage with leverage that leads to certain repayment), and thus makes the household portfolio problem have a determinate solution.

4.5 Government Policy

In the current version of the paper we assume that $g = \lambda = 0$, that is, the government does not spend resources and implements a flat labor income tax schedule. For future versions of the paper we will choose $g = 17\%$ and use the estimate of Heathcote, Storesletten and Violante (2012) of $\lambda = 0.26$ as the estimate for the progressivity of the income tax code.⁹ The level of taxes τ is then determined endogenously through the budget constraint of the government and will vary endogenously over the cycle. Finally, in our benchmark exercises we consider the case of $\phi = 0$ where financial intermediaries cannot seize any financial assets of defaulting households.

5 Results

5.1 Consequences of the Bailout Guarantee on Aggregates and Distributions

Table I shows macroeconomic aggregates under both policy choices (bailout vs. no bailout). These are averages across a long simulation for each economy. First note that the bailout guarantee is quite costly for the tax payer: income taxes rise by about 0.7 percentage points.

Variable	Bail	No Bail	% Δ
τ	0.7%	0%	70bp
M	2.93	2.75	6.6%
$\bar{\kappa}$	0.54	0.43	26%
H	5.39	5.62	-4.3%
r_b	1.24%	1.07%	17bp
P_l	0.031	0.028	7.4%
% Default	.54%	.34%	59%

We observe a strong increase in average leverage $\bar{\kappa}$ (which is especially accentuated during normal times, that is, in periods with $z = z_g$), and a significant rise in foreclosure rates, especially in times of a crisis (due to higher average leverage coming into the crisis). Using a utilitarian social welfare function to aggregate lifetime utilities of the heterogeneous households in the model we find that a bailout guarantee lowers societal welfare. However, households with different characteristics experience very heterogeneous welfare consequences from the bailout policy.

Add two plots here, one that shows the leverage distribution in both economies, and one that displays CEV's against cash at hand, with the CAH distributions plotted in the background. To see results more clearly the second plot will likely require subplots by current income state y .

⁹Note that since labor supply is exogenous the progressivity of the labor income tax code is mainly important for the redistributive consequences of the government bailout guarantee.

5.2 Simulating the Great Recession

After having discussed the long-run average impact of the policy in the context of our model, we now use it to describe a great recession. In order to do so, in figures x-z we plot time series generated by the model under the assumption that the aggregate state of the economy has been good for a long time (so that aggregates and distributions have settled down), and that then, in period 5, the economy experiences a great recession that lasts for eight years, prior to the aggregate state turning good again. That is, we display the implications of the model for a sequence of aggregate shock realizations given by

$$\begin{aligned} z_t &= z_g \text{ for } t = -T, \dots, 4 \\ z_t &= z_b \text{ for } t \in \{5, \dots, 12\} \\ z_t &= z_g \text{ for } t \geq 13. \end{aligned}$$

Figure ?? displays the corresponding time path for the aggregate price of housing, $P_h(z)$ which is, by assumption, simply a function of the aggregate state of the economy z , as is aggregate total factor productivity $A(z)$. As described in the calibration section, a great recession is characterized by a fall in house prices of 25% and a simultaneous fall in aggregate incomes by 8%.

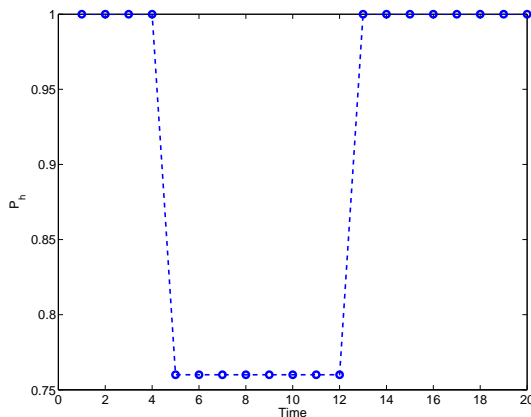


Figure 1: Evolution of Aggregate House Prices

In figures 2 and 3 we plot the aggregate time series for the foreclosure rate and aggregate leverage, defined as M/H , both for the economy without and the economy with a government bailout guarantee. We call the latter the benchmark economy (since it is this economy that we calibrated). The foreclosure rate plot displays the rates in absolute terms, the leverage plots displays percentage deviations of M'/H' from its pre-crisis level.

From figure 2 we observe that a) foreclosure rates are higher under the bailout guarantee, b) that they spike upwards in the first period of the great recession under both policy scenarios and c) that the spike is much larger for the bailout economy. As explained above, the first observation is the consequence of higher average leverage in the bailout economy even in normal times. A decline in the aggregate component of house prices by 25% in the great recession triggers additional defaults in the first period of the great recession in both economies, given the pre-recession leverage distribution in the economy (recall that, ex ante, a great recession was perceived by households as a possible, but fairly unlikely event). However, as was seen in the previous section, the bailout economy has significantly more households with mortgage leverage in excess of 75%, and thus many more households for which the decline in aggregate house prices, coupled with their own idiosyncratic house price shock leads to their real estate being under water. Consequently the foreclosure rate in the bailout economy shoots up to 9% in the first period of a great recession; in contrast, the no-bailout economy sees only slightly elevated mortgage default rates, in the order of 1.2%.

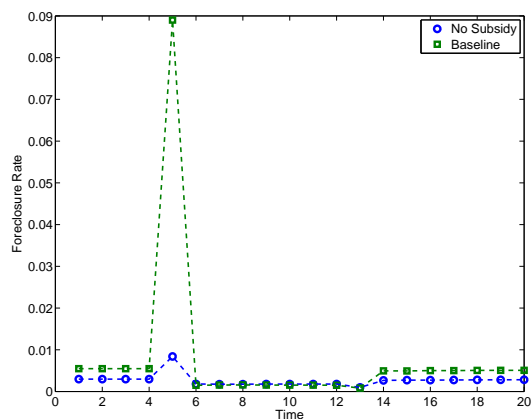


Figure 2: Evolution of Foreclosure Rates

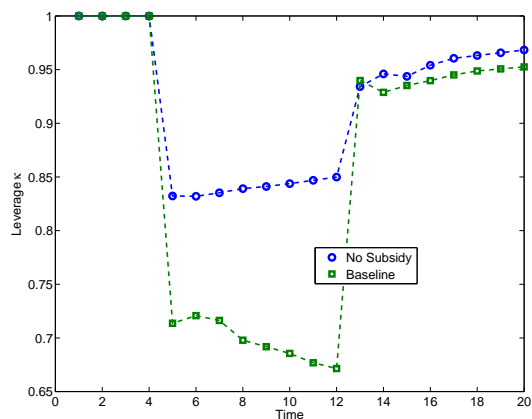


Figure 3: Evolution of Aggregate Leverage

As the great recession continues to unfold, foreclosure rates fall below their pre-recession levels, despite the fact that house prices remain depressed. The key force responsible for this finding, which holds true for both economies, is massive de-leveraging of households during the great recession. This is displayed in figure 3 which shows that average leverage in the economy falls by about 17% in the no-subsidy economy, and close to 30% in the bailout economy (in which of course households entered the great recession with much higher leverage).

Household reduce their mortgage debt, relative to normal times for two related reasons. First, since the aggregate shock is highly positively autocorrelated, the expected aggregate house price P_h tomorrow is lower if today's aggregate state is bad. Thus, for a *given* leverage, mortgage interest rates are higher today if $z = z_b$ relative to normal times. Note that this is *especially* true for the bailout economy since the guarantee makes mortgage interest rate cheaper in normal times, but not if the recession has already commenced (recall that the bailout guarantee is only operational if the aggregate state switches from z_g to z_b (but not if the economy is already stuck in the great recession and a bailout has already happened).

Second, given mean reversion in the aggregate component of house prices, expected house price *growth* is higher in bad times than in good times, which (for fixed interest rates and rental rates) makes investing in housing more attractive than investing in financial assets (the opposite side of the mortgage market). The

risk-free interest rate has to rise to bring about equilibrium in the financial market, which makes mortgage debt even less attractive for those inclined to borrow.¹⁰ The end result is a shift of household savings into real assets, and away from financial assets, and a corresponding reduction in household debt. As explained above, this shift is quantitatively more potent for the bailout economy.

The shift of assets away from financial assets and into real assets is also apparent from figure 4 which plots the evolution of the value of the aggregate housing stock $P_h H'$ over time. It shows that this value falls significantly by 13 – 18% (depending on the economy), but it falls less than the corresponding unit price P_h of housing.

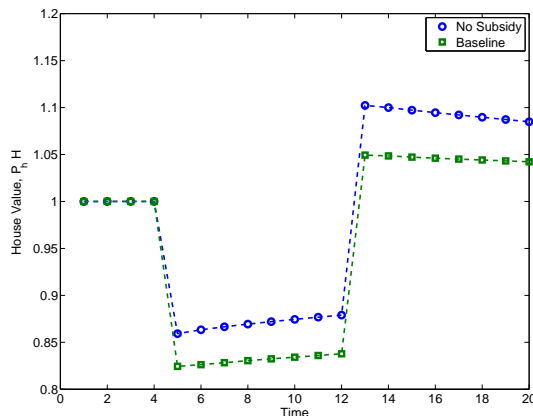


Figure 4: Evolution of the Value of the Aggregate Housing Stock

6 Sensitivity Analysis

6.1 Recourse

The previous results were derived under the assumption of no recourse, $\phi = 0$. One somewhat problematic empirical implication of that version of the model was that households default on their mortgages if and only if their mortgages are under water. As a consequence, in the benchmark economy a decline of house prices by 25% and the equilibrium distribution of leverage prior to the crisis leads to an explosion of the foreclosure rate at the onset of the crisis.

In this section we consider a version of the model in which financial intermediaries can fully seize part of the financial assets of those households that default on their mortgages, that is, we now study equilibria with $\phi > 0$. In that case some households with under-water mortgages might not default, and we conjecture that the aggregate foreclosure rate will move more sluggishly in the event of a great recession.¹¹ [TBC]

7 Conclusion

Fantastic paper

Next is the more explicit study of government fiscal and asset purchase policy in the great recession, beyond the simple bailout guarantee studied here. Also need to extend to allow for other public subsidies of the housing market, most notably interest rate deductibility of mortgage interest payments.

¹⁰We should document this by plotting the time series $r_b = (\sum_{z'} q(z'))^{-1} - 1$ for both economies as well.

¹¹It seems to me that the case $\phi = 1$ is uninteresting. In that case we should not observe any household holding simultaneously mortgage debt and financial assets. If that is true, then in equilibrium we again have all households with under-water mortgages defaulting, and thus this case does not help at all rectifying this problem.

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8 Theoretical Appendix

8.1 Alternative Tax Function

Alternatively one could specify taxes as

$$T(w(z, S)y; z, S) = \tau w(z, S)y - dw(z, S)L(z)$$

and thus as a flat tax τ with lump-sum transfer¹² d . Given this tax function we have

$$\int T(w(z) y; z, S) d\mu = (\tau - d) w(z, S) L(z)$$

and thus

$$\tau' = \tau(z, S, z') = \begin{cases} \frac{g+d(1-\alpha)}{1-\alpha} & \text{if } z = z_g \text{ or } z_{-1} = z_b \\ \frac{g+d(1-\alpha)}{1-\alpha} + \frac{Tr(S)}{w(z', S')L(z)} & \text{if } z = z_b \text{ and } z_{-1} = z_g \end{cases}$$

8.2 Mortgage Interest Rate Deductibility

Mortgage interest tax deductibility can be modeled in the following way: first, define the interest on a mortgage of size m' , net of administrative costs, as

$$r_m(\kappa', \theta', S) = \frac{1}{P_m(\kappa', \theta', S)(1 + r_w)} - 1$$

and thus total interest payments are given as $m'r_m(\kappa', \theta', S)$. As input into the tax function we define as taxable income

$$yt' = y'A(z') - \max\{m'r_m(\kappa', \theta', S), \chi A(z')\}$$

where $\chi A(z')$ is the maximal mortgage interest rate that can be deducted from income taxes. Taxes paid tomorrow are then given as $T(yt', z, z')$ and we can use whatever tax function T we like. The budget constraint of the household continues to read as in (??) and (14), but with $T(yt', z, z')$ replacing $T(y'A(z'), z, z')$. The problem is that aggregate taxes paid tomorrow,

$$\int T(yt', z, z') d\mu = \sum_{y'} \int \pi(y'|y) T(y'A(z') - \max\{m'(s, S)r_m(\kappa'(s, S), \theta'(s, S), S), \chi A(z')\}) d\mu$$

are a function of the distribution of cash at hand today (since the distribution of mortgages m' taken out today is), so of course in principle we would need to know today's cash at hand distribution μ to calculate tax receipts tomorrow. In contrast to the case with $\chi = 0$ we cannot solve for the tax rate that is needed in closed form as in (??), but have to add a period government budget constraint that would need to be satisfied. This might make the computation fairly tricky and induce further approximation error since we are approximating the wealth distribution μ with a small number of its moments. Thus I would conclude that while conceptually it is straightforward to include mortgage interest tax deductibility, it might create additional problems in the implementation. Evidently, our previous analysis is a special case with $\chi = 0$.

The corresponding mortgage pricing formula is

$$P_m(\kappa', S) = \begin{cases} \frac{\sum_{z'} \frac{q(S, z') \Psi(\kappa'(z'))}{1+r_w}}{\Psi(\kappa'(z_g)) \sum_{z'} \frac{q(S, z')}{1+r_w}} & \text{if } z = z_b \\ \Psi(\kappa'(z_g)) \sum_{z'} \frac{q(S, z')}{1+r_w} & \text{if } z = z_g \end{cases}$$

Note that in either case taxes for tomorrow are known today and are a function of the distribution μ today (and thus taxes today are functions of the distribution μ_{-1} yesterday). It seems that this requires to make the level of taxes τ today a state variable, with the law of motion $\tau' = \tau'(S, z')$ given by (??).

¹²It would be harder to make d a deduction so that taxes are the maximum of the above and zero, so that

$$T(w(z, S)y; z, S) = \max\{0, (1 - \tau)w(z, S)y - dw(z, S)\}$$

9 Computational Appendix

The computation strategy follows the framework developed in Krusell and Smith (1997), which was further adapted by Storesletten, Telmer and Yaron (2007) and Gomes and Michaelides (2008). In the current period, there are three "deep" prices that the household needs to know: $q(S, z_g), q(S, z_b), P_l(S)$ (knowing the two values of q is sufficient for determining P_m since $\Psi(\kappa', z')$ depends only on the exogenous aggregate state). These are the three prices that will clear the bond and housing markets. If we have transitioned into the bad state from the good state, the households also need to know the tax, τ , used to finance the bailout. This is where things differ slightly from the standard Krusell & Smith setup. Unlike with capital, H , does not directly determine any prices in the subsequent period. However, we need to know it for the aggregate law of motion of the housing stock, and can use it as a statistic of μ in order to forecast the evolution of the prices $q(S, z_g), q(S, z_b), P_l(S)$. We will use H as that statistic (it's also something that we'll be computing anyway along the simulated path). Krusell & Smith note that the bond pricing function exhibits high serial correlation. Gomes and Michaelides suggest that to overcome this issue to include the bond price as an aggregate state for forecasting the next period's bond prices. Further, our tax function depends on the contemporaneous and one lagged realization of z , so we will need to forecast tomorrow's τ as a function of today's and tomorrow's z values. Since foreclosure will (we assume) be highly dependent on changes in z , our forecast law of motion for the housing stock H' will also be allowed to be state contingent.

The algorithm is as follows:

1. We approximate the true aggregate state ($S = (z, H, \mu)$) by $\hat{S} = (z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau)$.
2. We conjecture laws of motion for \hat{S} :

$$\log H'(H, z, z') = a_0(z, z') + a_1(z, z') \log H \quad (17)$$

$$P'_l(H, z, z', P_l) = b_0(z, z') + b_1(z, z') \log H + b_2(z, z') \log P_l \quad (18)$$

$$\log q'_{zz}(H, z, z', q_{zz}) = c_0(z, z') + c_1(z, z') \log H + c_2(z, z') \log q_{zz} \quad (19)$$

$$\log q'_{z\bar{z}}(H, z, z', q_{z\bar{z}}) = d_0(z, z') + d_1(z, z') \log H + d_2(z, z') \log q_{z\bar{z}} \quad (20)$$

$$\tau(H, z, z') = e_0(z, z') + e_1(z, z') \log H \quad (21)$$

We solve an approximated version of the household problem (\hat{P}):

$$v(s, \hat{S}) = \max_{c, h, b', m', g' \geq 0} \left\{ U(c, h) + \beta \sum_{y'} \sum_{z'} \pi(y'|y) \pi(z'|z) \int_{\underline{\delta}}^1 v(s', \hat{S}') dF(\delta') \right\}$$

subject to

$$c + \sum_{z'} q_{zz'} b'(z') + (h - g') P_l + g' P_h(z) - m' P_m(g', m', \hat{S}) = a$$

$$a'(\delta', y', m', g', \hat{S}') = b'(z') + \max\{0, P_h(z')(1 - \delta')g' - m'\} + (1 - \tau')y'A(z')$$

$$\hat{S}' = \hat{\Gamma}(z, \hat{S}, z') \text{ given by the forecasts (2)-(6)}$$

The solution to \hat{P} , will generate policy functions as a function of the approximate aggregate states. Not also that in this formulation we are taking the "market clearing" prices as given in \hat{S} , but they need not necessarily clear markets for the values on the grid.

3. We initialize the economy by assuming that it is in the good state z_g (and thus $\tau = 0$), and set the distribution over cash at hand to our steady state distribution $\mu = \mu^{SS}$, and set H equal to the steady state H . This pins down H, z, τ , and then the algorithm proceeds as follows:

(a) Calculate the aggregate demand functions for each $P_l, q_{zz}, q_{z\bar{z}}$ on the grid:

$$\begin{aligned}
B'_{z'}(P_l, q_{zz}, q_{z\bar{z}}; H, z, \tau) &= \int b'_{z'}(s, z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau) d\mu \\
G'(P_l, q_{zz}, q_{z\bar{z}}; H, z, \tau) &= \int g'(s, z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau) d\mu \\
M'_{z'}(P_l, q_{zz}, q_{z\bar{z}}; H, z, \tau) &= \int m'(s, z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau) \Psi(\kappa'(s, \hat{S}), z') d\mu \\
H(P_l, q_{zz}, q_{z\bar{z}}; H, z, \tau) &= \int h(s, z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau) d\mu
\end{aligned}$$

Using the market clearing conditions, we can compute the vector valued excess demand function (suppressing the dependence on H, z, τ):

$$\Xi(P_l, q_{zz}, q_{z\bar{z}}) = \begin{bmatrix} H(P_l, q_{zz}, q_{z\bar{z}}) - G'(P_l, q_{zz}, q_{z\bar{z}}) \\ B'_z(P_l, q_{zz}, q_{z\bar{z}}) - M'_z(P_l, q_{zz}, q_{z\bar{z}}) \\ B'_{\bar{z}}(P_l, q_{zz}, q_{z\bar{z}}) - M'_{\bar{z}}(P_l, q_{zz}, q_{z\bar{z}}) \end{bmatrix}$$

Note that the second and third lines are for the two asset securities, where B'_z is the aggregate demand of bonds by households that pay off tomorrow if z is unchanged and M'_z is the aggregate supply issued the financial intermediaries to cover their mortgage positions.

- (b) We then have Ξ defined over the cube in $P_l \times q_{zz} \times q_{z\bar{z}}$ space. We can then interpolate that function using B-splines, and then solve to the value $(P_l^*, q_{zz}^*, q_{z\bar{z}}^*)$ that yields market clearing ($\Xi = (0, 0, 0)$).
- (c) We have then calculated our initial approximate state $\hat{S}_0 = (z, H, P_l^*, q_{zz}^*, q_{z\bar{z}}^*, \tau)$.
- (d) Next, we draw z' .

i. Then we update H' using the policy rules and the law of motion:

$$H' = \int \left\{ \int_{\underline{\delta}}^{\delta^*(\kappa'(s, \hat{S}_0), z')} (1 - \delta') g'(s, \hat{S}_0) dF(\delta') + \gamma \int_{\delta^*(\kappa'(s, \hat{S}_0), z')}^1 (1 - \delta') g'(s, \hat{S}_0) dF(\delta') \right\} d\mu_0$$

- ii. Then we calculate τ (0, unless we've switched from z_g to z_b).
- iii. Finally, we update the distribution, μ' using:

$$a'(\delta', y', m', g', \hat{S}') = b'(z') + \max\{0, P_h(z')(1 - \delta')g' - m'\} + (1 - \tau')y'A(z')$$

iv. We now have our new (z', H', τ') , so we return to (a) and solve for the market clearing prices.

4. We now have generated a sequence of $\left\{ \hat{S}_t(z, H, P_l, q_{zz}, q_{z\bar{z}}, \tau) \right\}_{t=1}^N$. We discard the first 500 values, and then we run the regressions in (2)-(6) to evaluate how close our forecast rules were to actual path of \hat{S} . If the coefficients from the regressions are the same as the conjectured ones, STOP, otherwise, update the coefficients for the forecasts and then return to 2., where we resolve the approximated household problem (P') and then repeat.