An Endogenous Growth Model with a Health Sector*

Jesus Fernandez-Villaverde† Dirk Krueger†
Alexander Ludwig‡ Matthias Schön§

Preliminary and Incomplete

Abstract

We develop an overlapping generations model with endogenous growth and a health sector, in order to explain three secular facts characterizing the U.S. economy: a substantial increase in life expectancy, a rise in the share of GDP devoted to health-related expenditures as well as an increase in the relative price of medical goods. We show how to interpret these observations as the equilibrium outcome of a model in which technological progress through quality improvements is endogenously directed to the sector producing medical goods.

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†University of Pennsylvania, CEPR and NBER.
‡SAFE, Goethe University Frankfurt.
§CMR, University of Cologne.

1 Introduction

We are motivated by three salient observations about health. A strongly increasing expenditure share, longer life expectancy (and thus an aging population, given birth rates) and an increasing price of health goods (as measured by the CPI on health goods, relative to the overall CPI). We proceed by developing an overlapping generations model with endogenous growth and a health sector, in order to explain these observations as the equilibrium outcome of a model in which technological progress is endogenously directed to the sector producing medical goods. An important element of our model is an explicit notion of the relative quality of health goods. Growth in the economy is generated through quality improvements. With this element we are able to decompose the relative price changes of health goods into quality adjusted and non-adjusted price indices.

[TBC]

Related Literature.

- Jones (2013): new stuff predicting health spending share of 100%
- Strulik, Dalgaard: The economics of health demand and human aging: health capital vs. health deficits
- Schneider, Winkler (2015): Growth and welfare under endogenous lifetime
- Kuhn, Frankovic, Wrzaczek (2015): On the role of health care in general equilibrium

The remainder of this paper proceeds as follows. Section 2 presents stylized facts on life-expectancy, aggregate health spending and prices of health goods in the US. Section 3 develops a two sector endogenous growth model with a health and a consumption goods sector. Endogenous growth is modeled through quality improvements. The household sector is very stylized in that households have two periods of life and consume only in the second period. The model delivers important qualitative (and, in part, analytical) insights on the relationship between health spending, trends in life expectancies, relative prices of medical goods and their quality as well as the health spending share as a fraction of aggregate income. Next, we expand in Section **??** the household sector of the theoretical model to a multi-period quantitative model. At this stage, we also model some institutional features of a (private and public) health insurance system. Our objective is to add realistic life-cycle features of health spending, in particular, the age-increasing health expenditures. These features are important when we combine and calibrate the models in Section **??**. Specifically, we will calibrate the model so that the endogenous trends in life-expectancies, health spending and growth match the data. We will also add exogenous trends in fertility, whereby the the baby-boom baby-bust cycle will be of key relevance. Combined with the endogenous trends in life-expectancies this gives rise to an overall aging pattern in that working age population ratios will be decreasing and old-age dependency will be increasing.[1] Given the age increasing health spending predicted in the household sector, these demographic developments will add additional movement to aggregate spending patterns. Section **??** will predict spending trends, growth and life-expectancies by use of the calibrated model over the next decades. Finally, Section 5 concludes the paper.

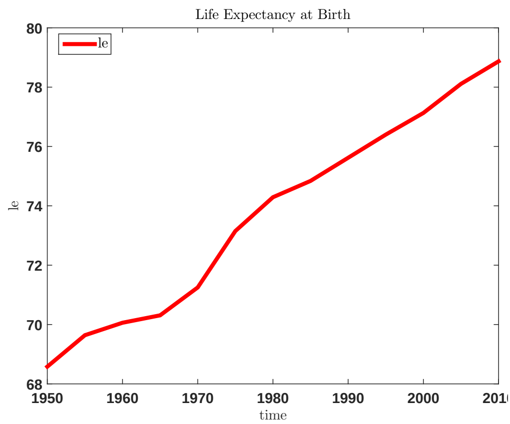
2 Stylized Facts

[TBC]

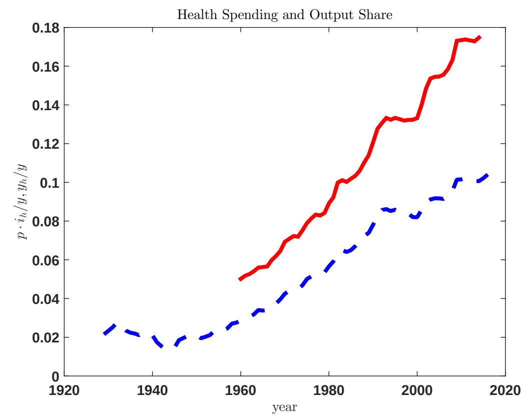
[1] Notice that these trends would also arise with constant fertility rates. Time varying fertility as observed in the data will strengthen the decrease of the working age population ratio and the corresponding increase of the old-age dependency ratio.

Figure 1: Data on Health in the US

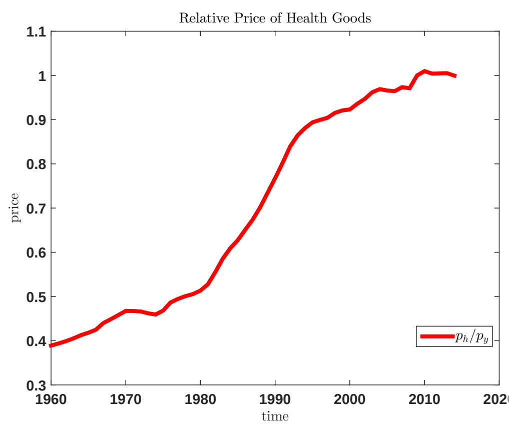
(a) Life Expectancy



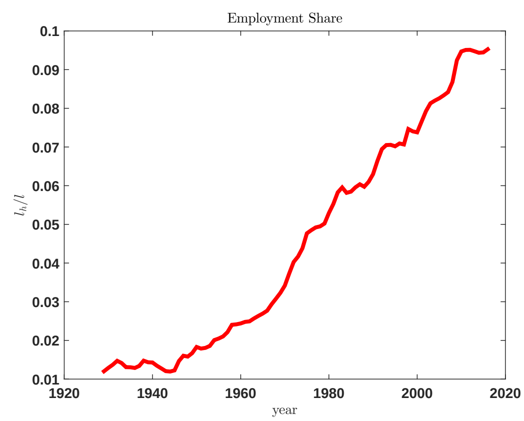
(b) Health Spending Share



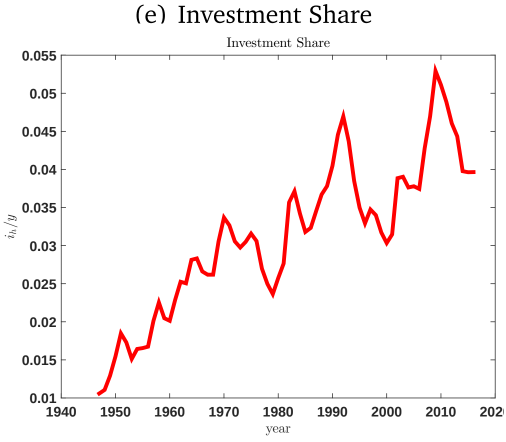
(c) Relative Price Index



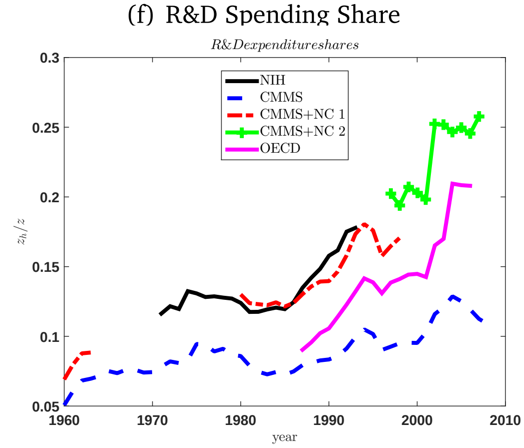
(d) Employment Share



(e) Investment Share



(f) R&D Spending Share



Notes: TBC.

3 A Simple Model

The model is populated by overlapping generations with mass 1 of identical young and n_t^o identical old households in period t. Total population is denoted by $n_t = 1 + n_t^o$. Households work, earn income, spend resources on health and save in the first period of their lives and consume in the second period of their lives. In addition, there are competitive firms in three sectors, one that produces consumption goods, one that produces health goods, and one that does research and development.

3.1 Households

Households have preferences represented by the utility function

$$\psi(i_{t+1})u(c_{t+1}^o) \tag{1}$$

where the Bernoulli utility function $u()$ is at least twice continuously differentiable with $u_c > 0$ and $u_{cc} < 0$, and satisfies the lower Inada condition, thus $\lim_{c \to 0} u_c = \infty$. Maximization is subject to the constraints

$$p_t i_t + s_t = w_t + T_t \tag{2}$$

$$c_{t+1}^o = r_{t+1} s_t. \tag{3}$$

where p_t is the relative price of health goods in terms of consumption goods (which we define formally below), c_{t+1}^o is consumption in old age, i_t is the health investment and $\psi(i_t)$ is the probability, which is increasing in health investment i_t, $\psi_i > 0$. We further assume that $\lim_{i \to \infty} \psi(i) = 1$ and $1 > \psi(i_t = 0) > 0$. T_t are transfers from accidental bequests from previous generations which households take exogenous.

We assume that the depreciation rate on capital is 1, so that the gross return on saving s_t is r_{t+1} which will equal the marginal product on capital in equilibrium. Absent borrowing constraints the budget constraints can be consolidated to the lifetime budget constraint

$$p_t i_t + \frac{c^o_{t+1}}{r_{t+1}} = w_t + T_t = x_t. \tag{4}$$

where x_t is cash-on-hand of the household. In equilibrium, transfers to generation born in period t due to accidental bequests from generation $t-1$ are given by:

$$T_t = r_t s_{t-1}(1-\psi(i_{t-1})) \tag{5}$$

Thus transfers are positive if and only if $\psi(i) < 1$ and thus households die with positive probability between young and old ages.

3.2 Firms, Production and R&D

3.2.1 Final Goods Producers

Let $j \in \{f, h\}$ stand for the final and the health sector of the economy, respectively, with and p_{jt} for the price of the output of each of the two sectors. We will normalize p_{ft} to 1 and simply let p_t denote the relative price of health goods whenever it notationally more convenient and there is no room for confusion. In each sector a representative firm uses a continuum of intermediate inputs indexed by i and labor to produce sectorial output y_{jt} according to the production function:

$$y_{jt} = \left(\int_0^1 q_{jit}^{1-\alpha} y_{jit}^{\alpha} di \right) l_{jt}^{1-\alpha} \tag{6}$$

where $0 < \alpha < 1$ and y_{jit} is the quantity of intermediate input i used to produce the output good in sector j at date t and l_{jt} is the number of workers employed in sector j. The entity q_{jit} denotes the quality of intermediate input i at date t in sector j. Growth in this model results from innovations that increase the quality q_{jit} of intermediate inputs. Since the final good producer is competitive and takes factor input prices as given, she hires labor and intermediate inputs to equate marginal productivities to these input prices, taking as given their qualities q_{jit}. Let the wage rate be given by w_t and the price of one unit of intermediate good i in sector j is p_{jit}. The first order conditions read as

$$p_{jt}(1-\alpha)\left(\int_0^1 q_{jit}^{1-\alpha} y_{jit}^{\alpha} di\right) l_{jt}^{-\alpha} = w_t \tag{7}$$

for labor demand and

$$p_{jt}\alpha q_{jit}^{1-\alpha} y_{jit}^{\alpha-1} l_{jt}^{1-\alpha} = p_{jit} \tag{8}$$

for the demand for intermediate goods, given their quality q_{jit}.

3.2.2 Intermediate Goods Producers

Each intermediate good producer i is a monopolist that takes the demand function (8) as given and uses capital (which depreciates immediately after use) to produce the intermediate good according to:

$$y_{jit} = k_{jit}. \tag{9}$$

The rental rate of capital is given by r_t, so that each intermediate goods monopolist producer maximizes profits, taking as given the demand function of the final goods producer:

$$\pi_{jit} = \max_{y_{jit}} \left\{\left[p_{jt}\alpha q_{jit}^{1-\alpha} y_{jit}^{\alpha-1} l_{jt}^{1-\alpha}\right] y_{jit} - r_t y_{jit}\right\}$$

with first order condition

$$y_{jit} = \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} q_{jit} l_{jt} \tag{10}$$

and profits

$$\pi_{jit} = \frac{1-\alpha}{\alpha} r_t y_{jit} > 0 \tag{11}$$

The monopolistic price follows from using (10) in (8) as

$$p_{jit} = \frac{1}{\alpha} r_t > r_t \tag{12}$$

hence featuring the standard markup over marginal costs, r_t. It is the same across all intermediate input producers i and sectors j. Furthermore, the markup over marginal costs is also constant.

Finally, observe from (10) that $\frac{y_{jit}}{q_{jit}}$ is constant across varieties i. Likewise the ratio of profits to quality is constant, which we state for further reference using (10) in (11) to get

$$\frac{\pi_{jit}}{q_{jit}} = \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} l_{jt}. \tag{13}$$

3.2.3 Aggregation of Production Sector

Because the ratios of variety-specific intermediate outputs to quality y_{jit}/q_{jit} and profits to output (or quality) π_{jit}/y_{jit} (π_{jit}/q_{jit}) are constant across varieties i we get immediate aggregation results for each sector.

For each production sector j we can determine aggregate capital input and production as

$$k_{jt} = \int_0^1 k_{jit} di = \int_0^1 y_{jit} di = \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} q_{jt} l_{jt} \tag{14}$$

where

$$q_{jt} = \int_0^1 q_{jit} di \tag{15}$$

is an aggregate quality index of intermediate inputs in sector j. Furthermore, exploiting (10) and (14) in (6) yields as aggregate production function for sector j

$$y_{jt} = k_{jt}^{\alpha} \left(q_{jt} l_{jt} \right)^{1-\alpha}. \tag{16}$$

Using equations (7) and (14) delivers as factor prices for labor inputs and capital inputs:

$$w_t = (1-\alpha)\frac{p_{jt}y_{jt}}{l_{jt}} \tag{17a}$$

$$r_t = \alpha^2 \frac{p_{jt}y_{jt}}{k_{jt}}. \tag{17b}$$

Finally we can use (11) and (14) to determine aggregate profits in each sector j as

$$\pi_{jt} = \alpha(1-\alpha)p_{jt}y_{jt} \tag{18}$$

and thus in each sector j output exhausts factor input payments plus profits:

$$p_{jt}y_{jt} = \pi_{jt} + r_t k_{jt} + w_t l_{jt} \tag{19}$$

To summarize the aggregation result, in each of the two sectors output is produced with a Cobb-Douglas production function with capital and labor inputs in which the level of technology is given by q_{jt}. However, final goods producers cannot rent capital directly, but have to go through monopolistically competitive intermediaries. As a consequence owners of the capital (which will be the old households in equilibrium) command only a fraction α^2 of the value of output, with a fraction $\alpha(1-\alpha)$ accruing to the monopolist intermediaries.

3.2.4 Research and Development

An R&D developer that specializes in intermediate good i that spends resources of the final consumption good z_{jit} on R&D to achieve innovation. If successful in innovation, the quality of the intermediate good increases from q_{jit-1} to

$$q_{jit} = \lambda q_{jit-1} \tag{20}$$

where $\lambda > 1$ is a parameter. The successful innovator immediately becomes the monopolist, and for one period enjoys monopoly profits π_{jit} associated with technology level $q_{jit} = \lambda q_{jit-1}$. In a product line i in which innovation is not successful a randomly chosen entrepreneur becomes the monopolist and produces at quality $q_{jit} = q_{jit-1}$ with associated profits.

We assume that the probability of innovating is related to the quality reached when successfully innovating given by λq_{jit-1}. We also assume that the probability of innovating depends on the size of the economy. As formulated in Young (1998) and others this offsets scale effects. Specifically, we assume that the probability of innovation varies inversely with the size of the period t population to the effect that the probability of innovating is given by

$$\varphi\left(\frac{z_{jit}}{\lambda q_{jit-1}}\right)^{\gamma} \cdot n_t^{-1} \tag{21}$$

with $\gamma \in (0,1)$ and $\varphi > 0$. The specification implies that an increase of the scale of the economy (here measured in terms of the size of the total population), dilutes the effects of research outlays, z_{jit}. Intuitively, this captures Young (1998)'s insight that as population grows, the effectiveness of research aimed at quality improvement is reduced by being spread more thinly thus dissipating the effect on the overall rate of productivity growth. Similarly, the inverse relationship between the success probability and current quality q_{jit-1} reflects the fact that it becomes increasingly harder to innovate if already a level of quality is reached for variety i.

Thus the R&D entrepreneur spends resources z_{jit} and, if successful, collects profits π_{jit}. Hence the problem reads as

$$\max_{z_{jit}}\left\{\pi_{jit}\varphi\left(\frac{z_{jit}}{\lambda q_{jit-1}}\right)^{\gamma} n_t^{-1} - z_{jit}\right\} \tag{22}$$

with first order condition

$$\frac{\pi_{jit}}{\lambda q_{jit-1} n_t}\varphi\gamma\left(\frac{z_{jit}}{\lambda q_{jit-1}}\right)^{\gamma-1} = 1. \tag{23}$$

which yields as solution a ratio of R&D spending to potential period t technology

$\frac{z_{jit}}{\lambda q_{jit-1}}$

$$\frac{z_{jit}}{\lambda q_{jit-1}} = \left[\varphi\gamma\frac{\pi_{jit}}{\lambda q_{jit-1}n_t}\right]^{\frac{1}{1-\gamma}} \tag{24}$$

Noticing that in case of success $q_{jit} = \lambda q_{jit-1}$ we can now use equation (13) in the above to get

$$\frac{z_{jit}}{\lambda q_{jit-1}} = \left[\varphi\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}}\ell_{jt}\right]^{\frac{1}{1-\gamma}} \tag{25}$$

where $\ell_{jt} = \frac{l_{jt}}{n_t}$ is the working age population share in sector j. Using the above back in (21) we observe the share of varieties innovating is (due to the law of large numbers)

$$\mu_{jt} = \int \varphi\left(\frac{z_{jit}}{\lambda q_{jit-1}}\right)^{\gamma} di = \varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}}\ell_{jt}\right]^{\frac{\gamma}{1-\gamma}} \tag{26}$$

and is thus independent of the distribution of qualities across varieties i.

For future reference, also observe that resources spend by entrepreneur i are

$$z_{jit} = \left[\varphi\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}}\ell_{jt}\right]^{\frac{1}{1-\gamma}}\lambda q_{jit-1}$$

so that total resources devoted to R&D in sector j are equal to

$$z_{jt} = \int z_{jit}di = \left[\varphi\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}}\ell_{jt}\right]^{\frac{1}{1-\gamma}}\lambda q_{jt-1} \tag{27}$$

which are also independent of the distribution of qualities across varieties in sector j.

3.3 Equilibrium and Income and Product Accounting

3.3.1 Definition of Equilibrium

In this section we define a competitive equilibrium for our economy. We immediately proceed to defining equilibrium for the aggregate economy, thereby already exploiting the aggregation results developed in sections 3.2.3 and 3.2.4. Noticing that we can define either good as numeraire, we normalize $p_{ft} = 1$ and define all equilibrium conditions in terms of the price of health goods $p_t = \frac{p_{ht}}{p_{ft}}$.

Definition 1. *Given an initial population,* $1, n_t^o$*, and initial conditions* s_0, i_0, q_{f0}, q_{h0}*, a competitive equilibrium is a sequence of household allocations* $c_1^o, d_1, \{s_t, i_t, c_{t+1}^o\}_{t=1}^{\infty}$*, a sequence capital and labor inputs of goods producers* $\{k_{jt}, l_{jt}\}_{t=1}^{\infty}$*, a sequence of R&D expenditures, profits and consumption of R&D developers* $\{z_{jt}, \pi_{jt}, c_{jt}\}_{t=1}^{\infty}$*, a sequence of aggregate capital and technology* $\{k_t, q_{ft}, q_{ht}\}_{t=1}^{\infty}$*, prices* $\{p_t, w_t, r_t\}_{t=1}^{\infty}$ *and transfers* $\{T_t\}_{t=1}^{\infty}$ *and a law of motion of the old population* n_t^o *such that*

1. *Household maximization: for each* $t \geq 1$*, given prices and transfers* w_t, p_t, r_{t+1}, T_t*, the allocations* i_t, s_t, c_{t+1}^o *maximize (1) subject to (4).*

2. *Transfers* T_t *satisfy equation (5).*

3. *Factor prices satisfy equations (17a) and (17b).*

4. *Optimal R&D spending* z_{jt} *in each sector is given by (27) and consumption of R&D entrepreneurs is determined as* $c_{jt} = \pi_{jt} - z_{jt}$*.*

5. *The equilibrium innovation intensity* μ_{jt} *is given by equation (26) and technology in each sector evolves according to*

$$q_{jt} = (1 - \mu_{jt})q_{jt-1} + \mu_{jt}\lambda q_{jt-1} \tag{28}$$

6. *Markets clear: for all* $t \geq 1$

   (a) *Labor Market*

$$1 = \sum_j l_{jt} \tag{29}$$

(b) *Capital Market*

$$\sum_j k_{jt} = k_t \tag{30}$$

(c) *Asset Market*

$$k_t = s_{t-1} \tag{31}$$

(d) *Final Goods Market*

$$s_t + c_t^o n_t^o + \sum_j [c_{jt} + z_{jt}] = k_{ft}^{\alpha} \left(q_{ft} l_{ft}\right)^{1-\alpha} \tag{32}$$

(e) *Health Goods Market*

$$i_t = k_{ht}^{\alpha} \left(q_{ht} l_{ht}\right)^{1-\alpha} \tag{33}$$

7. *The population evolves according to*

$$n_t^o = \psi(i_{t-1}). \tag{34}$$

3.3.2 National Income and Product Accounting in this Economy

In order to map the equilibrium in our model to the data we now derive GDP, total income and total spending in this economy and verify that value added equals income and equals spending. We do this both to assure that the model is internally consistent as well as to make explicit the map between model variables and empirical counterparts.

Aggregate Income In each period t young non-entrepreneur households receive labor income and income from accidental bequests

$$w_t(l_{ft} + l_{ht}) + T_t$$

whereas the number $n_t^o = \psi(i_{t-1})$ of surviving old households earn capital income $r_t s_{t-1}$. Recall that transfers are given by equation (5) so that aggregate transfers write as

$$T_t = [1 - \psi(i_{t-1})] r_t s_{t-1}.$$

R&D entrepreneurs in both sectors earn total profits given by $\pi_{ft} + \pi_{ht}$ and purchase inputs $z_{ft} + z_{ht}$. Consequently their aggregate income is

$$\pi_{ft} + \pi_{ht} - (z_{ft} + z_{ht}).$$

Thus total income earned by non-entrepreneur and entrepreneur households equals

$$w_t(l_{ft} + l_{ht}) + r_t s_{t-1}[1 - \psi(i_{t-1})] + r_t s_{t-1}\psi(i_{t-1}) + \pi_{ft} + \pi_{ht} - z_{ft} - z_{ht}$$

Spending Non-entrepreneur households spend $p_t i_t$ on health goods, $s_t = k_{t+1}$ on investment goods and $c_t^o n_t^o$ on consumption goods (recall that the final output good is the numeraire and used for both consumption and investment). Monopolists spend $c_{ft} + c_{ht}$ on final consumption goods.

Thus the value of total spending equals

$$c_t^o \psi(i_{t-1}) + (s_t + p_t i_t) + (c_{ft} + c_{ht}).$$

We note that total spending equals total income since

$$c_t^o \psi(i_{t-1}) = r_t s_{t-1}\psi(i_{t-1}) \quad (35)$$

$$(s_t + p_t i_t) = w_t(l_{ft} + l_{ht}) + [1 - \psi(i_{t-1})] r_t s_{t-1} \quad (36)$$

$$c_{jt} = \pi_{jt} - z_{jt} \quad (37)$$

from the nonentrepreneur household budget constraints and the definition of consumption c_{jt} of entrepreneurs in both sectors j.

Output (Value Added) Total value added equals the value of output in the final goods sector, the health sector (the intermediate goods sector produces capital goods whose value exactly nets out with the capital inputs of the health and final goods sector), net of the inputs in the R&D sector. Thus value added is given by

$$y_{ft} + p_t y_{ht} - (z_{ft} + z_{ht}). \tag{38}$$

But we established that

$$p_{jt} y_{jt} = \pi_{jt} + r_t k_{jt} + w_t l_{jt}$$

for both sectors, and thus value added

$$y_{ft} + p_t y_{ht} - \left(z_{ft} + z_{ht}\right) = r_t k_t + w_t l_t + \sum_j [\pi_{jt} - z_{jt}]$$

equals income (which in turn, as shown in the previous section, equals spending). Therefore the basic accounting identities are satisfied in our model.

3.4 Analysis of the Decision Problems

3.4.1 Analysis of Household Problem

For given prices and transfers and resulting cash-on-hand, $\{p_t, x_t, r_{t+1}\}$, where $x_t = w_t + T_t$, the household problem boils down to a one-dimensional maximization problem choosing i_t and thus the survival probability $\psi(i_t)$. Accordingly, the maximization problem writes as

$$\begin{aligned} & \max_{c_{t+1}, i_t} \psi(i_t) u(c_{t+1}) \\ p_t i_t + s_t &= x_t := w_t + T_t \\ c_{t+1} &= r_{t+1} s_t \end{aligned}$$

Define the expenditure share of health as

$$\theta_t = \frac{p_t i_t}{x_t}$$

so that

$$
\begin{aligned}
i_t &= \frac{x_t}{p_t}\theta_t \\
c_{t+1} &= r_{t+1}x_t(1-\theta_t)
\end{aligned}
$$

and the maximization problem becomes

$$\max_{\theta_t \in [0,1]} \psi\left(\frac{x_t}{p_t}\theta_t\right) u\left(r_{t+1}x_t(1-\theta_t)\right)$$

with first-order condition

$$\frac{x_t}{p_t}\psi'\left(\frac{x_t}{p_t}\theta_t\right) u\left(r_{t+1}x_t(1-\theta_t)\right) \leq r_{t+1}x_t\psi\left(\frac{x_t}{p_t}\theta_t\right) u'\left(r_{t+1}x_t(1-\theta_t)\right),$$

with equality if $\theta_t > 0$. Notice that the corner solution $\theta_t \leq 1$ can be ignored because the utility function satisfies the lower Inada condition. Thus

$$\frac{u\left(r_{t+1}x_t(1-\theta_t)\right)}{u'\left(r_{t+1}x_t(1-\theta_t)\right)} \leq p_t r_{t+1} \frac{\psi\left(\frac{x_t}{p_t}\theta_t\right)}{\psi'\left(\frac{x_t}{p_t}\theta_t\right)} \tag{39}$$

with equality if $\theta_t > 0$.

Unfortunately equation (39) does not have a closed form solution and we cannot establish consistency with a BGP unless very specific forms of the survival function $\psi(\cdot)$ and the period utility function u are assumed.

Assumption 1. *Following* **?**) *and others, we assume that the utility function takes the form*

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + b$$

where $\sigma \geq 0$ and $b \geq 0$ are parameters.

Parameter b measures the value of life.

Assumption 2 (No Suicide)**.** *For $\sigma \geq 1$ parameter b has to be chosen sufficiently large such that $u(c) > 0$ for all relevant levels of consumption.*

Assumption 3. *The survival function satisfies the CDF of a type 2 Pareto distribution,*

$$\psi(i) = 1 - [1 + \nu + i]^{-\xi},$$

with parameters $\nu > 0$ *and* $\xi > 0$.

Note that ψ is strictly increasing in ν and ξ, and is strictly increasing in i with $\psi(i = 0) = 1 - [1 + \nu]^{-\xi} > 0$ and $\lim_{i\to\infty} \psi(i) = 1$.

With these assumptions we can rewrite (39) as

$$\frac{r_{t+1}x_t(1-\theta_t)}{1-\sigma} + b\left(r_{t+1}x_t(1-\theta_t)\right)^{\sigma} \leq \frac{p_t r_{t+1}}{\xi}[1+\nu+\frac{x_t}{p_t}\theta_t]\left([1+\nu+\frac{x_t}{p_t}\theta_t]^{\xi} - 1\right) \tag{40}$$

with equality if $\theta_t > 0$.

We can now give precise conditions under which $\theta_t = 0$ is optimal:

$$\frac{r_{t+1}x_t}{1-\sigma} + b\left(r_{t+1}x_t\right)^{\sigma} \leq \frac{p_t r_{t+1}}{\xi}[1+\nu]\left([1+\nu]^{\xi} - 1\right)$$

This is satisfied if either the economy is sufficiently poor (x_t is sufficiently small), the survival rate under zero investments is sufficiently large (ν is sufficiently large) or the price of health goods is sufficiently high (p_t is sufficiently large). If the economy is at the corner, then all production and employment takes place in the final goods sector.

Furthermore, under these functional form assumptions we can establish the following condition on the existence and the properties of the balanced growth:

Proposition 1. *Under assumptions 1, 2 and 3 a BGP with growth rate* $\lambda > 1$ *exists for* $t \to \infty$ *if and only if* $\sigma \leq 1 + \xi$. *For* $\sigma < 1 + \xi$ *health investments in the BGP are equal to zero, hence* $\theta = i = 0$. *For* $\sigma = 1 + \xi$ *the health expenditure share of young households is given by*

$$\theta^* = \frac{1}{1 + (\xi b)^{-\frac{1}{\sigma}}(pr)^{-\left(1-\frac{1}{\sigma}\right)}} = \theta^*(\xi b, pr) \in (0,1) \tag{41a}$$

Proof. See Appendix A.1 □

Thus, the expenditure share in health θ_t is strictly increasing in ξb and pr (only their products matter), and the steady state expenditure share is independent of ν (that is, only the tail of the distribution ψ matters because $i_t = \infty$ in the BGP).

We can further characterize the evolution of the health expenditure share along the transition. Now assume that $\sigma = 2$ (and thus, to permit a meaningful BGP, $\xi = 1$). Then the corner solution $\theta_t = 0$ emerges if

$$(br_{t+1}x_t - 1)\,x_t \leq p_t[1+\nu]\nu$$

that is, if the economy is sufficiently poor (x_t small), the price of health goods sufficiently high and/or the probability of survival absent any health investment is sufficiently high (ν high).

If this condition is violated there is an interior solution $\theta_t \in (0,1)$ solving the quadratic equation

$$-r_{t+1}x_t(1-\theta_t) + b\left(r_{t+1}x_t(1-\theta_t)\right)^2 \;=\; p_t r_{t+1}\left(1+\nu+\frac{x_t}{p_t}\theta_t\right)\left(\nu+\frac{x_t}{p_t}\theta_t\right) \quad (42)$$

Proposition 2. *Under assumptions 1, 2 and 3 and for $\sigma = 2 \Leftrightarrow \xi = 1$ the health expenditure share along the transition is given by*

$$\theta_t \equiv \frac{p_t i_t}{x_t} = \begin{cases} \frac{p_t(\nu+br_{t+1}x_t)}{(bp_t r_{t+1}-1)x_t} - \sqrt{\left(\frac{p_t(\nu+br_{t+1}x_t)}{(bp_t r_{t+1}-1)x_t}\right)^2 - \frac{(br_{t+1}x_t-1)x_t - p_t(1+\nu)\nu}{(bp_t r_{t+1}-1)x_t^2}p_t} \\ \qquad\qquad \textit{if } (br_{t+1}x_t-1)\,x_t > p_t[1+\nu]\nu \\ 0 \qquad\qquad \textit{otherwise.} \end{cases}$$

The health expenditure share converges to (41) as $t \to \infty$.

Proof. See Appendix A.1 □

3.4.2 Relative Prices, Qualities and the Capital Intensity

The aggregation results give rise to the relationship between relative prices and qualities in the two sectors from (16)–(17b) as

$$\frac{p_{ht}}{p_{ft}} = \left(\frac{q_{ft}}{q_{ht}}\right)^{1-\alpha}, \tag{43}$$

cf. Appendix A.2.1 for details on the derivation. The relationship is quite intuitive. If the quality (or productivity) of sector h improves relative to sector f, then relative prices of health goods are decreasing.

Using (43) in (**??**) this condition latter condition for positive health spending can be translated into one in terms of relative qualities. There will be no investment in health if

$$\frac{q_{ft}}{q_{ht}} > \left(\frac{1}{3}x_t\left(r_{t+1}bx_t - 1\right)\right)^{\frac{1}{1-\alpha}},$$

i.e., if the quality in the health sector is, ceteris paribus, too low relative to the quality in the final goods sector.

Relationship (43) also implies that along the transition the ratio of unadjusted price indices (no adjustment for quality) is

$$\frac{p_{ht}q_{ht}}{p_{ft}q_{ft}} = \left(\frac{q_{ht}}{q_{ft}}\right)^{\alpha}.$$

Therefore, if the quality (or productivity) of sector h improves relative to sector f, then relative non-quality adjusted prices of health goods are increasing.

Finally, observe from the equations for the wage rate and the interest rate, equations (17a) and (17b) that the capital intensities in the two sectors are identical. Hence, in all periods t the economy wide capital intensity satisfies $\left(\frac{k_t}{l_t}\right) = \left(\frac{k_{ht}}{l_{ht}}\right) = \left(\frac{k_{ft}}{l_{ft}}\right)$. This equality of capital intensities across the two sectors will be exploited in the computational solution of the model described in Appendix B.

3.4.3 Economic Growth

Economic growth is driven by quality improvements. In each sector a deterministic (by the law of large numbers) fraction μ_{jt} of intermediates experiences a quality improvement of size λ, and the remaining fraction $1-\mu_{jt}$ has constant quality. Thus the growth rate of the quality index q_{jt} in each sector is given by

$$g_{jt} = \frac{q_{jt}}{q_{jt-1}} = \frac{\left(1-\mu_{jt}\right) q_{jt-1} + \mu_{jt} \lambda q_{jt-1}}{q_{jt-1}} = 1 + (\lambda - 1)\mu_{jt}. \tag{44}$$

Note that given the timing assumptions whereas q_{jt-1} is predetermined in period t and thus a state variable, current technology q_{jt} can be controlled today via resources z_{jt} spent on innovation. Recall that the fraction of intermediate goods μ_{jt} that experience an innovation in each sector j is endogenous and given by equation (26). Using this in (44) gives

$$g_{jt} = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} \ell_{jt} \right]^{\frac{\gamma}{1-\gamma}}. \tag{45}$$

Hence, the growth rate of the quality index q_{jt} in sector j is a function of the relative price p_{jt}, the interest rate r_t and the (relative) market size of sector j, as measured by the working age population share employed in this sector, $\ell_{jt} = \frac{l_{jt}}{n_t}$.

3.5 Balanced Growth Path Analysis

To characterize the BGP equilibrium, notice that, in general, the dynamics of this model are determined by the dynamics of four variables. On the household side, young today households choose how much to spend on health, i_t and how much to save, s_t. Innovation firms decide how much to spend on R&D which determines q_{jt} for both sectors $j \in \{f,h\}$. Thus the state variables at the beginning of period $t+1$ are $(i_t, s_t, q_{ft}, q_{ht})$. In a balanced growth path these variables will grow at a constant rate (alongside with wages w_t, transfers b_t etc.), and with relative prices $(r_t, p_t = \frac{p_{ht}}{p_{ft}})$ being constant. Let g be the BGP gross growth rate.

Denote by

$$\begin{aligned}\tilde{x}_t &= x_t/g^t \\ x_t &= \tilde{x}_t g^t\end{aligned}$$

the growth-deflated variables. We now want to characterize a balanced growth path of this model prior to discussing the full dynamics of the model.

We have already established the properties of the household model in the balanced growth path in Proposition 1. We now turn to the production side.

3.5.1 Production Side

In the balanced growth path the growth rates of technology in both sectors are given by:

$$\begin{aligned}g_h &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{p\alpha^2}{r}\right)^{\frac{1}{1-\alpha}}\ell_h\right]^{\frac{\gamma}{1-\gamma}} \\ g_f &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}}\ell_f\right]^{\frac{\gamma}{1-\gamma}}\end{aligned}$$

Since in the long run both sectors have to grow at the same rate we have

$$\frac{\ell_f}{\ell_h} = p^{\frac{1}{1-\alpha}}$$

and exploiting the labor market clearing condition

$$\ell_f + \ell_h = \ell$$

where

$$\ell = \frac{1}{n} = \frac{1}{1+\psi(d(r,p))}$$

is the economy wide working age population ratio (we show that $\psi \equiv \psi(d(r,p))$

as part of the solution of the household problem below) we have

$$\ell_h = \frac{1}{1+p^{\frac{1}{1-\alpha}}} \frac{1}{1+\psi(d(r,p))} \quad \Leftrightarrow \quad l_h = \frac{1}{1+p^{\frac{1}{1-\alpha}}}$$
$$\ell_f = \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \frac{1}{1+\psi(d(r,p))} \quad \Leftrightarrow \quad l_f = \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}}.$$

This implies that the relative price of health goods p uniquely pins down the labor supply in both sector l_h, l_f. Using this in the expression for growth in the two sectors we find that the balanced growth path growth rate in the two sectors is given by

$$g = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{\alpha^2}{r} \right)^{\frac{1}{1-\alpha}} \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \frac{1}{1+\psi(d(r,p))} \right]^{\frac{\gamma}{1-\gamma}} \equiv g(r,p). \tag{46}$$

From the firms' optimality conditions and the production function the other growth-deflated variables in the BGP are given by:

$$\tilde{q}_f = \frac{\tilde{w} r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)} \tag{47a}$$
$$\tilde{q}_h = \frac{\tilde{w} r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha) p^{\frac{1}{1-\alpha}}} \tag{47b}$$
$$\tilde{y}_f = \frac{\tilde{w}}{(1-\alpha)} \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \tag{47c}$$
$$\tilde{y}_h = \frac{\tilde{w}}{(1-\alpha)} \frac{1}{p\left(1+p^{\frac{1}{1-\alpha}}\right)} \tag{47d}$$
$$\tilde{k}_f = \frac{\alpha^2 \tilde{w}}{(1-\alpha) r} \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \tag{47e}$$
$$\tilde{k}_h = \frac{\alpha^2 \tilde{w}}{(1-\alpha) r} \frac{1}{\left(1+p^{\frac{1}{1-\alpha}}\right)} \tag{47f}$$

which are all just functions of the BGP prices $r, p, \tilde{w}$.

3.5.2 Household Side in the BGP

The demands for investment goods and savings come from the household problem. From (5) we have that in the BGP where $i = \infty$ that $\psi(i) = 1$ by assumption 3 and therefore $T = 0$. Therefore, the demand equations in terms of de-trended variables $\tilde{i}, \tilde{s}, \tilde{w}$ are given

$$\tilde{i}(p,r,\tilde{w}) = \frac{\tilde{w}}{p}\theta(p,r) \tag{48a}$$

$$\tilde{s}(p,r,\tilde{w}) = \tilde{w}[1-\theta(p,r)] \tag{48b}$$

cf. Proposition 1.

3.5.3 BGP Equilibrium

Proposition 3. *In the BGP, the level of qualities in the two sectors $\tilde{q}_h$ and $\tilde{q}_f$ is not determined. The economy scales in absolute qualities.*

Proof of Proposition 3. Observe from Appendix A.2.1 that

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} q_{jt} \tag{49}$$

hence, for given relative prices p_{jt} scaling qualities by some factor $\lambda > 0$ scales wages by factor λ. From (**??**) we observe that this scales the demands from the household sector by factor λ. From the equality of capital intensities in the two sectors, cf. Section 3.4.2, we observe that we can compute the capital intensity directly from the solution of the household model. We therefore have in any period t (also outside the BGP)

$$\frac{\tilde{k}_t}{l_t} = \tilde{k}_t = \tilde{s}_{t-1}(\cdot)/g.$$

because $l_t = 1$. By the equality of capital intensities in both sectors we therefore have

$$\frac{\tilde{k}_{ht}}{l_{ht}} = \tilde{k}_{ht} = \tilde{s}_{t-1}(\cdot)/g.$$

Using results from Section 3.5.1 we can therefore compute the BGP capital stock in the health sector as

$$\begin{aligned}\tilde{k}_h(p,r,\tilde{w}) &= \tilde{s}(p,r,\tilde{w})/g(r,p)l_h \\ &= \tilde{s}(p,r,\tilde{w})/g(r,p)\frac{1}{1+p^{\frac{1}{1-\alpha}}},\end{aligned} \tag{50}$$

where $\tilde{s}(p,r,\tilde{w})$ follows from the household side, cf. Section 3.5.2. From this we observe that any scaling by the demands of households by factor λ will also scale capital in the economy by factor λ. Finally, observe from the homogeneity of aggregate production in the two sectors, cf. equation (16), that scaling both inputs k_{jt}, q_{jt} by factor λ under constant relative prices (and therefore constant labor) leads to scaling of output by factor λ. □

In consequence, we can normalize the absolute quality in the final goods sector to some arbitrary constant in the BGP, denoted $\bar{\tilde{q}}_f$. From equation (49) and recalling that we normalize $p_f = 1$ we can therefore express the level of wages in the BGP as a function of the interest rate r:

$$\tilde{w}(r) = (1-\alpha)\left(\frac{\alpha^2}{r}\right)^{\frac{\alpha}{1-\alpha}}\bar{\tilde{q}}_f \tag{51}$$

Therefore, demands in the household sector in equation (**??**) are also functions only of (p,r), denoted $\tilde{s}(p,r), \tilde{i}(p,r)$. Then, combining equations (50) and (47f) gives the equilibrium condition for the capital market,

$$\tilde{s}(p,r)/g(p,r) = \frac{\alpha^2\tilde{w}(r)}{(1-\alpha)r} \tag{52}$$

and from (47d) we observe that investment demand for health goods satisfies

$$\tilde{i}(p,r) = \frac{\tilde{w}(r)}{(1-\alpha)} \frac{1}{p\left(1+p^{\frac{1}{1-\alpha}}\right)} \tag{53}$$

where the respective demand equations $\tilde{s}(p,r), \tilde{i}(p,r)$ are given in (**??**). These two equations uniquely determine the equilibrium (noticing that the equilibrium in the final goods market is implied by Walras' law).

We describe a (slightly modified) fixed-point solution algorithm to determine market clearing prices and interest rates in Section B.1 of the Appendix.

3.6 Dynamics

[TBC]

Characterization of the dynamics of the economy

1. On the household side we have a representative household per generation, so no question that in period t savings and health investment of the current old generation s_{t-1}, h_{t-1} fully characterizes the aggregate state of the household sector.

2. On the firm side the cross-sectional distribution over q_{jit-1} is not a state variable. The law of motion for q_{jt-1} is given by

$$q_{jt} = (1-\mu_{jt})q_{jt-1} + \mu_{jt}\lambda q_{jt-1} \tag{54}$$

   The significance of this result is that we do not keep track of the cross sectional quality distribution in each sector, but rather only of the two aggregate indices q_{ft}, q_{ht}.

   Conjecture 1. *There exists a competitive equilibrium in which aggregate allocations do not depend on the initial cross sectional distributions.*

   Proof. See appendix □

3. Under the previous result the state variables of the system are $(q_{t-1}, q_{ht-1}, s_{t-1}, h_{t-1})$.

4 Quantitative Evaluation

We now evaluate the quantitative implications of the model.

4.1 Calibration

We choose initial conditions such that the economy is poor in the first period and it will not be optimal to invest in health, $\theta_1 = 0$. As investment will take place in the final goods sector during the initial period(s), the economy starts to grow. At some point in time $t_k > 1$, there will be a kickoff in that it will be optimal to invest in health $\theta_t > 0$. The parametrization

4.2 Simulation Output

New sequences to be compared directly to the data.

- Life expectancy: $1 + \psi(i_t)$

- Health spending share: According to **?**), R&D expenses are not included in the measures of national health expenditures as these expenditures are treated intermediate purchases under the definitions of national income accounting; that is, the value of that research is deemed to be recouped through product sales. For this reason, our relevant measure for the health expenditure share in GDP is

$$\frac{\theta_t x_t}{p_t y_{ht} + y_{ft} - (z_{ht} + z_{ft})} = \frac{p_t i_t}{p_t y_{ht} + y_{ft} - (z_{ht} + z_{ft})}$$

- Relative price index: the model price for health goods (with appropriate quality adjustment) is $p_t = \left(\frac{q_{ft}}{q_{ht}}\right)^{1-\alpha}$, without quality adjustment it is $p_t \frac{q_{ht}}{q_{ft}} = \left(\frac{q_{ht}}{q_{ft}}\right)^{\alpha}$

- The employment share in the health sector is equal to employment in the health sector l_{ht} because the total workforce sums to one.

Table 1: Parameters

Parameter	Value
Households	
Value of Life b	100
Inter-temporal elasticity of substitution $1/\sigma$	1/2
Pareto tail parameter, survival function ξ	1
Minimum survival probability parameter at $i = 0$, ν	0.1
Firms	
Capital elasticity α	0.33
Capital elasticity α	0.33
Growth factor λ	5
Innovation probability: curvature γ	0.5
Innovation probability: scaling ϕ	0.2
Initial Conditions	
Quality in health sector q_{h0}	0.001
Quality in final goods sector q_{f0}	1
Initial capital stock k_1	0.1
Initial health investment i_0	0

Notes: TBC.

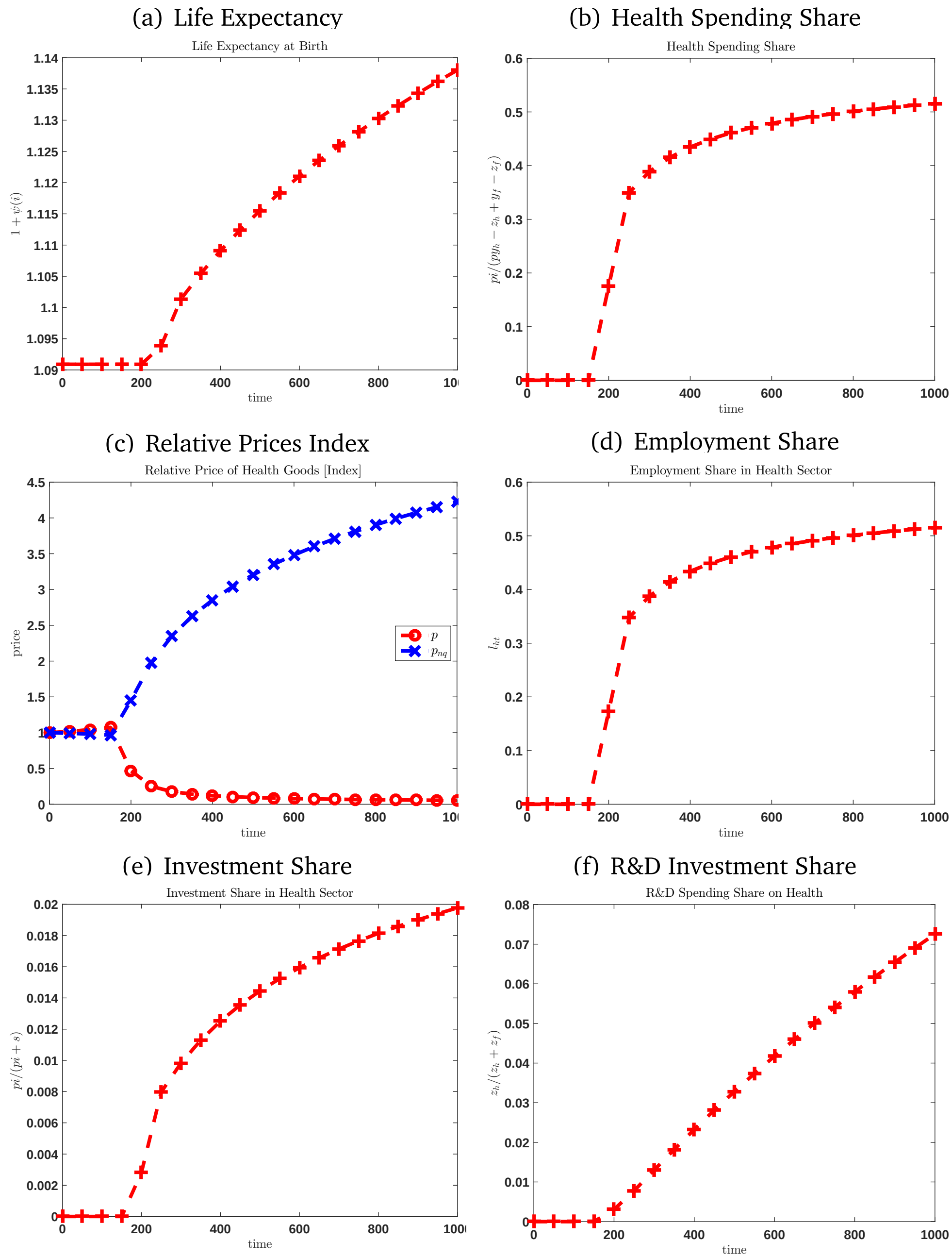
- Investment share: **TBC: ask Leon for data sources**. The investment share is given by $\frac{p_t i_t}{p_t i_t + s_t}$
- Output share: this should be the net output share, i.e., output net of expenditures on R&D. Accordingly, we will have

$$\frac{p_t y_{ht} - z_{ht}}{p_t y_{ht} + y_{ft} - (z_{ht} + z_{ft})}$$

- R&D spending share:

$$\frac{z_{ht}}{z_{ht} + z_{ft}}$$

Figure 2: Simple Model: Detrended Variables in Transition

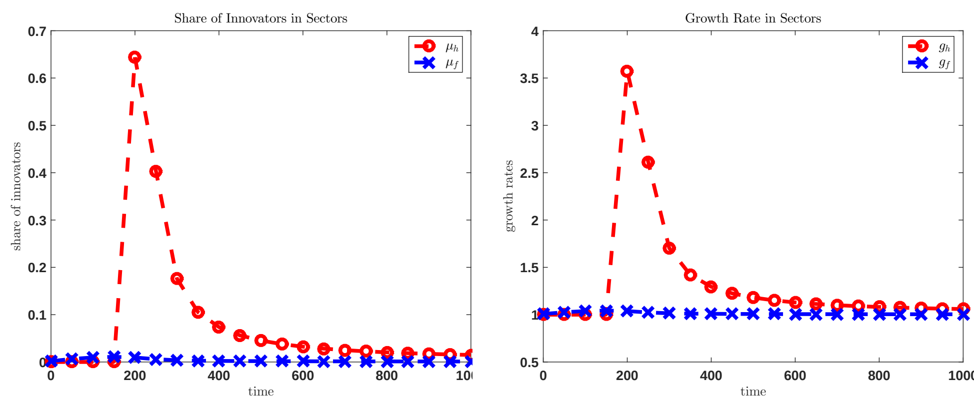


Notes: Illustration of Simple Model.

Figure 3: Simple Model: Innovation & Growth

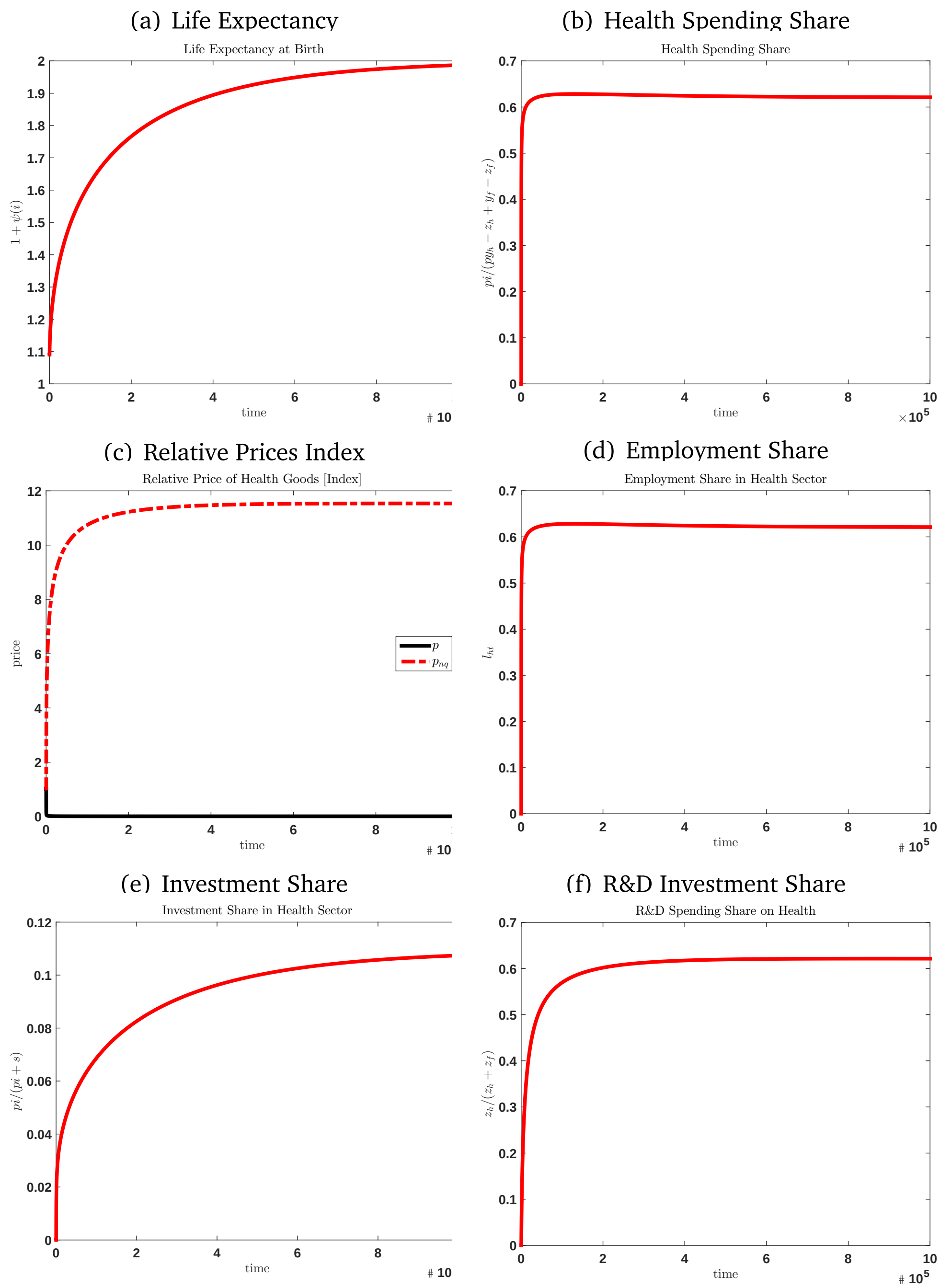
(a) Share of Innovators

(b) Growth Rates



Notes: Illustration of Simple Model.

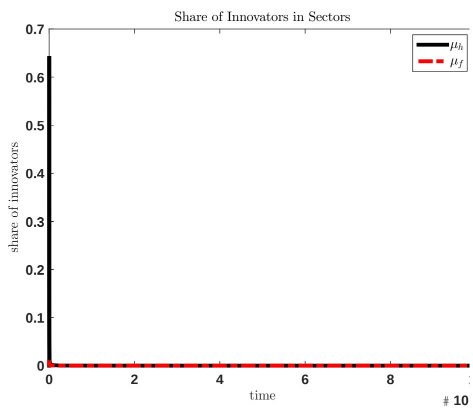
Figure 4: Simple Model: Detrended Variables in Transition: All Periods



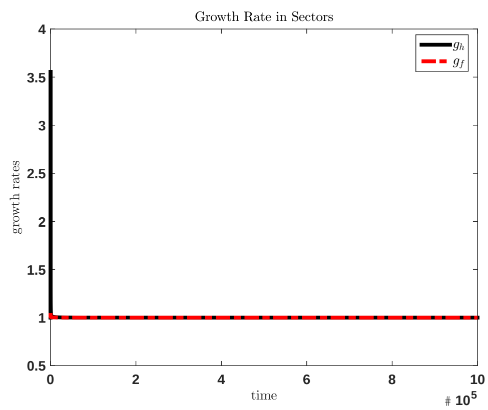
Notes: Illustration of Simple Model.

Figure 5: Simple Model: Innovation & Growth: All Periods

(a) Share of Innovators



(b) Growth Rates



Notes: Illustration of Simple Model.

5 Conclusion

[TBC]

A Analytical Appendix

A.1 Proofs

Proof of Proposition 1. In a balanced growth path r and p are constant and i_t as well as $w_t, T_t, i_t, s_t, c_{t+1}$ grow at constant rates. To derive parameter restrictions that have to be satisfied for the existence of a blanced growth path, divide the first order condition (40) by $(x_t)^{1+\xi}$ to get

$$\frac{r_{t+1}(1-\theta_t)}{(1-\sigma)(x_t)^{\xi}}+b\left(r_{t+1}(1-\theta_t)\right)^{\sigma}(x_t)^{\sigma-(1+\xi)}=\frac{p_t r_{t+1}}{\xi}\left(\frac{1+\nu}{x_t}+\frac{\theta_t}{p_t}\right)\left(\left(\frac{1+\nu}{x_t}+\frac{\theta_t}{p_t}\right)^{\xi}-\frac{1}{(x_t)^{\xi}}\right)$$

As $x_t \to \infty$, this expression can be rewritten for $\xi > 0$ and p, r, θ constant as

$$b\left(r(1-\theta)\right)^{\sigma}(x_t)^{\sigma-(1+\xi)}=\frac{pr}{\xi}\left(\frac{\theta}{p}\right)^{1+\xi}.$$

Let us consider the following three cases:

- Case 1: $\sigma > 1+\xi$. Then the LHS diverges to ∞ and the RHS converges to a finite number, a contradiction.
- Case 2: $\sigma < 1+\xi$. Then the LHS converges to 0 and thus in the BGP we get from the RHS that $\theta = 0$, that is health investments converge to zero and are equal to zero in the BGP.
- Case 3: $\sigma = 1+\xi$. Then in the BGP

  $$\xi b p^{\xi} r^{\sigma-1}(1-\theta)^{\sigma}=(\theta)^{1+\xi}$$

  and under the restriction $\xi = \sigma - 1$ we get (41).

□

Proof of Proposition 2. We analyze the quadratic equation (42) and argue that is has a unique plausible root in (0,1). Factoring out and simplifying (and suppressing time indices) yields

$$
\begin{aligned}
\left[br(1-\theta)-\frac{1}{x}\right]p(1-\theta) &= \left(\frac{p(1+\nu)}{x}+\theta\right)\left(\frac{p\nu}{x}+\theta\right) \\
bpr(1-\theta)^2-\frac{p}{x}(1-\theta) &= \frac{p^2(1+\nu)\nu}{x^2}+\frac{p(1+2\nu)}{x}\theta+\theta^2 \\
bpr(1-2\theta+\theta^2)-\frac{p}{x}+\frac{p}{x}\theta &= \frac{p^2(1+\nu)\nu}{x^2}+\frac{p(1+2\nu)}{x}\theta+\theta^2 \\
\theta^2-\frac{2p\nu+2bprx}{(bpr-1)x}\theta-\frac{p^2(1+\nu)\nu+px-bprx^2}{(bpr-1)x^2} &= 0 \\
\theta^2-\frac{2p(\nu+brx)}{(bpr-1)x}\theta-\frac{p^2(1+\nu)\nu+(1-brx)px}{(bpr-1)x^2} &= 0
\end{aligned}
$$

and thus

$$
\begin{aligned}
\theta_{1,2} &= \frac{\frac{2p(\nu+brx)}{(bpr-1)x}}{2}\pm\sqrt{\left(\frac{2p(\nu+brx)}{2(bpr-1)x}\right)^2+\frac{p^2(1+\nu)\nu+(1-brx)px}{(bpr-1)x^2}} \\
&= \frac{p(\nu+brx)}{(bpr-1)x}\pm\sqrt{\left(\frac{p(\nu+brx)}{(bpr-1)x}\right)^2+\frac{p^2(1+\nu)\nu+(1-brx)px}{(bpr-1)x^2}} \\
&= \frac{p\left(\frac{\nu}{x}+br\right)}{(bpr-1)}\pm\sqrt{\left(\frac{p\left(\frac{\nu}{x}+br\right)}{(bpr-1)}\right)^2+\frac{p^2(1+\nu)\frac{\nu}{x^2}+\left(\frac{1}{x}-br\right)p}{(bpr-1)}}
\end{aligned}
$$

Note that as x goes to infinity

$$\begin{aligned}
\theta_{1,2} &= \frac{pbr}{bpr-1} \pm \sqrt{\left(\frac{pbr}{bpr-1}\right)^2 - \frac{bpr}{(bpr-1)}} \\
&= \frac{pbr}{bpr-1} \pm \sqrt{\frac{(bpr)^2 - bpr\,(bpr-1)}{(bpr-1)^2}} \\
&= \frac{pbr}{bpr-1} \pm \sqrt{\frac{bpr}{(bpr-1)^2}} \\
&= \frac{pbr \pm (bpr)^{0.5}}{bpr-1} = \frac{(bpr)^{0.5}\left((bpr)^{0.5} \pm 1\right)}{((bpr)^{0.5}-1)((bpr)^{0.5}+1)} \\
&= \frac{\left((bpr)^{0.5} \pm 1\right)}{((bpr)^{0.5}-1)(1+(bpr)^{-0.5})}
\end{aligned}$$

So if we take the smaller root, then in the limit

$$\theta_1 = \frac{\left((bpr)^{0.5}-1\right)}{((bpr)^{0.5}-1)(1+(bpr)^{-0.5})} = \frac{1}{1+(bpr)^{-0.5}} = \theta^*$$

whereas the larger root yields

$$\theta_2 = \frac{\left((bpr)^{0.5}+1\right)}{((bpr)^{0.5}-1)(1+(bpr)^{-0.5})} = \frac{(bpr)^{0.5}+1}{(bpr)^{0.5}-1} * \frac{1}{1+(bpr)^{-0.5}} > \theta^*$$

So, the smaller root is the relevant one, but of course only if that smaller root satisfies $\theta \geq 0$, otherwise there is a corner solution at $\theta = 0$.

Throughout, we also have to assume that the value of life is positive. Thus we need that

$$\begin{aligned}
& u(c_{t+1}) > 0 \\
\Leftrightarrow\quad & \left(\frac{1}{1-\sigma} c_{t+1}^{1-\sigma} + b\right) > 0 \\
\Leftrightarrow\quad & ((1-\theta_t)x_t r_{t+1})^{1-\sigma} < -b(1-\sigma)
\end{aligned}$$

where the sign change in the last line above is due to $\sigma > 1$. Setting $\sigma = 2$ gives

$$((1-\theta_t)x_t r_{t+1})^{-1} < b \iff b(1-\theta_t)x_t r_{t+1} > 1$$

which, evaluated at $\theta_t = 0$, gives the condition $bx_t r_{t+1} > 0$.

With that assumption, and the assumption that the corner solution is **not** optimal

$$(brx-1)x > p[1+\nu]\nu$$

the smaller root $\theta_1 \in (0,1)$ is guaranteed to be positive and smaller than 1. Note that for the second assumption, the first one $brx \geq 1$ is necessary, but not sufficient.

□

A.2 Additional Derivations

A.2.1 Relative Prices and Quantities

Using (16) in (17b) we get

$$r_t = \alpha^2 p_{jt} \left(\frac{k_{jt}}{l_{lj}}\right)^{\alpha-1} q_{jt}^{1-\alpha}$$

from which

$$\frac{k_{jt}}{l_{lj}} = \left(\frac{\alpha^2 p_{jt}}{r_t}\right)^{\frac{1}{1-\alpha}} q_{jt}.$$

Next, use (16) and the above in (17a) to get

$$
\begin{aligned}
w_t &= (1-\alpha)\, p_{jt} \left(\frac{k_{jt}}{l_{jt}}\right)^{\alpha} q_{jt}^{1-\alpha} \\
&= (1-\alpha)\, p_{jt} \left(\left(\frac{\alpha^2 p_{jt}}{r_t}\right)^{\frac{1}{1-\alpha}} q_{jt}\right)^{\alpha} q_{jt}^{1-\alpha} \\
&= (1-\alpha)\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} q_{jt}
\end{aligned}
$$

Equating this across the two sectors gives (43).

A.2.2 Balanced Growth Analysis of the Production Side

The key equations from the production side are the production functions in each sector j, the marginal pricing conditions, the resources devoted to R&D, profits and the equation specifying the growth of output in each sector j:

$$
\begin{aligned}
y_{jt} &= k_{jt}^{\alpha}\left(q_{jt} l_{jt}\right)^{1-\alpha} \\
\frac{y_{jt}}{l_{jt}} &= \frac{w_t}{(1-\alpha)\, p_{jt}} \\
\frac{k_{jt}}{y_{jt}} &= \frac{\alpha^2 p_{jt}}{r_t} \\
\frac{y_{jt}}{q_{jt}} &= \left(\frac{\alpha^2 p_{jt}}{r_t}\right)^{\frac{\alpha}{1-\alpha}} l_{jt} \\
z_{jt} &= \left[\varphi\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{1}{1-\gamma}} \lambda \frac{q_{jt}}{g_{jt}} \\
\pi_{jt} &= p_{jt}\alpha(1-\alpha)\, y_{jt} \\
g_{jt} &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{\gamma}{1-\gamma}}.
\end{aligned}
$$

In the balanced growth path these equations become, for the $h$ealth sector and the $f$inal output sector become, normalizing $p_{ft} = 1$:

$$
\begin{aligned}
\tilde{y}_h &= \tilde{k}_h^{\alpha} \left(\tilde{q}_h l_h\right)^{1-\alpha} \\
\tilde{y}_f &= \tilde{k}_f^{\alpha} \left(\tilde{q}_f l_f\right)^{1-\alpha} \\
\frac{\tilde{y}_h}{l_h} &= \frac{\tilde{w}}{(1-\alpha)p} \\
\frac{\tilde{y}_f}{l_f} &= \frac{\tilde{w}}{(1-\alpha)} \\
\frac{\tilde{k}_h}{\tilde{y}_h} &= \frac{\alpha^2 p}{r} \\
\frac{\tilde{k}_f}{\tilde{y}_f} &= \frac{\alpha^2}{r} \\
\frac{\tilde{y}_h}{\tilde{q}_h} &= \left(\frac{\alpha^2 p}{r}\right)^{\frac{\alpha}{1-\alpha}} l_h \\
\frac{\tilde{y}_f}{\tilde{q}_f} &= \left(\frac{\alpha^2}{r}\right)^{\frac{\alpha}{1-\alpha}} l_f \\
\tilde{z}_h &= \left[\varphi\gamma\left(\frac{p\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} \ell_h\right]^{\frac{1}{1-\gamma}} \frac{\lambda}{g}\tilde{q}_h \\
\tilde{z}_f &= \left[\varphi\gamma\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} \ell_f\right]^{\frac{1}{1-\gamma}} \frac{\lambda}{g}\tilde{q}_f \\
\tilde{\pi}_h &= p\alpha(1-\alpha)\,\tilde{y}_h \\
\tilde{\pi}_f &= \alpha(1-\alpha)\,\tilde{y}_f \\
g &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{p\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} \ell_h\right]^{\frac{\gamma}{1-\gamma}} \\
&= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} \ell_f\right]^{\frac{\gamma}{1-\gamma}}
\end{aligned}
$$

From the last equation we get that

$$\frac{\ell_f}{\ell_h} = p^{\frac{1}{1-\alpha}}.$$

Next, exploit the labor market clearing condition

$$\ell_f + \ell_h = \frac{1}{n} = \frac{1}{1+\psi(d(r,p))}$$

to get

$$\begin{aligned} \ell_h &= \frac{1}{1+p^{\frac{1}{1-\alpha}}}\frac{1}{1+\psi(d(r,p))} \quad \Longleftrightarrow \quad l_h = \frac{1}{1+p^{\frac{1}{1-\alpha}}} \\ \ell_f &= \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}}\frac{1}{1+\psi(d(r,p))} \quad \Longleftrightarrow \quad l_f = \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}}. \end{aligned}$$

This implies a growth rate in the economy in the balanced growth path of

$$g = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}}\frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}}\frac{1}{1+\psi(d(r,p))}\right]^{\frac{\gamma}{1-\gamma}} \equiv g(r,p). \quad (55)$$

Furthermore, we get the equations

$$\begin{aligned} \tilde{q}_f &= \frac{\tilde{w}r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)} \\ \tilde{q}_h &= \frac{\tilde{w}r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)p^{\frac{1}{1-\alpha}}} \\ \tilde{y}_f &= \frac{\tilde{w}}{(1-\alpha)}\frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \\ \tilde{y}_h &= \frac{\tilde{w}}{(1-\alpha)}\frac{1}{p\left(1+p^{\frac{1}{1-\alpha}}\right)} \\ \tilde{k}_f &= \frac{\alpha^2\tilde{w}}{(1-\alpha)r}\frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \\ \tilde{k}_h &= \frac{\alpha^2\tilde{w}}{(1-\alpha)r}\frac{1}{\left(1+p^{\frac{1}{1-\alpha}}\right)} \end{aligned}$$

which are all just functions of the BGP prices r, w, p. Furthermore, for any BGP, scaling all variables but labor by a constant will produce another BGP, and thus there is a continuum of BGP's indexed by the level of $\tilde{q}$. This might become relevant in the analysis of the full dynamics.

B Computational Appendix

B.1 The Simple Model in the BGP

Although we could solve the market clearing conditions of the simple model as a rootfinding problem, it is instructive to write it as a fixed-point problem because this will provide the basis for the solution of the problem outside the BGP. We will loop over a vector of prices such that the solution of the household problem can be computed. The key step will be to update these prices as well as to update quality indices in the two sectors.

Throughout, we work with a detrended version of the model. To implement the iteration we work on the following set of equations (that hold both in the transition as well as in the BGP):

$$
\begin{aligned}
\frac{p_{ht}}{p_{ft}} &= p_t = \left(\frac{\tilde{q}_{ft}}{\tilde{q}_{ht}}\right)^{1-\alpha} \\
\tilde{y}_{jt} &= \tilde{k}_{jt}^{\alpha}(\tilde{q}_{jt} l_{jt})^{1-\alpha} \\
\tilde{w}_t &= (1-\alpha)\frac{p_{jt}\tilde{y}_{jt}}{l_{jt}} \\
r_t &= \alpha^2 \frac{p_{jt}\tilde{y}_{jt}}{\tilde{k}_{jt}} \\
\tilde{\pi}_{jt} &= \alpha(1-\alpha)p_{jt}\tilde{y}_{jt} \\
p_{jt}\tilde{y}_{jt} &= \tilde{\pi}_{jt} + r_t\tilde{k}_{jt} + \tilde{w}_t l_{jt} \\
\tilde{k}_t &= \sum_j \tilde{k}_{jt} = \tilde{s}_{t-1}^{hh}(\cdot)/g \\
\tilde{i}_t &= \tilde{y}_{ht} = \tilde{i}_t^{hh}(\cdot) \\
\tilde{T}_t^{hh} &= (1-\psi(d_0))r_t\tilde{s}_{t-1}^{hh}(\cdot)/g.
\end{aligned}
$$

In addition to these equations, we use the following condition for the BGP growth rate that only holds in the BGP of the model:

$$
g_j = g = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}} \frac{1}{1+\psi(d_0)} \right]^{\frac{\gamma}{1-\gamma}}
$$

We also normalize the BGP quality in the final good sector to some constant, $\bar{\tilde{q}}_f$. Wages are then given by

$$
\tilde{w} = (1-\alpha)\left(\frac{\alpha^2}{r}\right)^{\frac{\alpha}{1-\alpha}} \bar{\tilde{q}}_f.
$$

The iteration in the BGP is accordingly as follows:

1. Start with an initial guess of $v^0 = [r^0, p^0, \tilde{T}^0]$

2. In iteration m, for given $v^m = [r^m, p^m, \tilde{T}^m]$ compute

(a) Additional objects given to households: Compute $\tilde{w}, g$ as follows:

$$\tilde{w} = (1-\alpha)\left(\frac{\alpha^2}{r}\right)^{\frac{\alpha}{1-\alpha}} \bar{\tilde{q}}_f$$
$$g = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{\alpha^2}{r^m}\right)^{\frac{1}{1-\alpha}} \frac{p^{m\frac{1}{1-\alpha}}}{1+p^{m\frac{1}{1-\alpha}}} \frac{1}{1+\psi(d_0)}\right]^{\frac{\gamma}{1-\gamma}}$$

(b) Solution of household model: Given $[r^m, p^m, \tilde{T}^m, \tilde{w}^m, g^m]$ solve the household model, cf. Proposition 1, giving $\tilde{s}^{hh}(r,p,\tilde{T}), \tilde{i}^{hh}(r,p,\tilde{T})$.

(c) Aggregation: Compute labor supply in both sectors

$$l_h = \frac{1}{1+p^{\frac{1}{1-\alpha}}}$$
$$l_f = 1 - l_h$$

and, given the household decisions, the economy wide capital intensity

$$\frac{\tilde{k}}{l} = \tilde{k} = \tilde{s}(r,p,\tilde{T})/g$$

cf. Sections 3.4.2 and 3.5.3.

Next, compute the capital stocks and output in the two sectors as

$$\tilde{k}_h = \frac{\tilde{k}}{l} l_h$$
$$\tilde{k}_f = \frac{\tilde{k}}{l} l_f$$
$$\tilde{y}_h = \tilde{i}^{hh}$$
$$\tilde{y}_f = \tilde{k}_f^{\alpha}\left(\tilde{q}_f l_f\right)^{1-\alpha}$$
$$\tilde{\pi}_h = \alpha(1-\alpha) p \tilde{y}_h.$$

(d) Updating: Update $r, p, \tilde{T}$ using

$$\check{p}^m = (\tilde{y}_h)^{-1}\left(\tilde{\pi}_h + r^m\tilde{k}_h + \tilde{w}^m l_h\right)$$
$$\check{r}^m = \alpha^2\frac{\tilde{y}_f}{k_f}$$
$$\check{\tilde{T}}^m = (1-\psi(d_0))\check{r}^m\tilde{s}^{hh}(\cdot)/g$$

and collect the updated variables as $\check{v}^m = [\check{r}^m, \check{p}^m, \check{\tilde{T}}^m]$.

(e) Define $d^m = \|v^m - \check{v}^m\|_\infty$. If $d^m < \epsilon$ stop, else compute $v^{m+1} = (1-\lambda)v^m + \lambda\check{v}^m$ for some dampening factor $\lambda \in (0,1)$ and proceed with step 2.

Observe that as a byproduct of the algorithm we get

$$\tilde{q}_h = \frac{\bar{\tilde{q}}_f}{p^{\frac{1}{1-\alpha}}}.$$

B.2 The Simple Model in the Transition

The iteration in the transition is very similar to the BGP solution with some modifications. Recall that all variables are detrended with the growth rate in the BGP, denoted as g.

As additional equation we use equation (49) which relates qualities to wages. From this relationship we get:

$$\tilde{q}_{ft} = \left((1-\alpha)\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}\right)^{-1}\tilde{w}_t$$

Hence, we can always map from wages to qualities and, once we include the time vector of wages as an additional outer loop variable, we therefore do not need to also store the quality in the health sector.

To update qualities along the transition, we also need the (trend-adjusted) growth rates:

$$\tilde{g}_{jt} = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} \ell_{jt} \right]^{\frac{\gamma}{1-\gamma}} / g.$$

Recall that g is the previously computed growth rate in the BGP.

Also notice that we take as given the initial asset holdings $\tilde{s}^{hh}_{-1}$ which we need to update the transfers from accidental bequests to the first generation. In our factual implementation we assume that $\tilde{s}^{hh}_{-1} = \tilde{s}^{hh}_{0}$. In our quantitative model we will replace this assumption with assuming no growth in the initial equilibrium.

1. Start with an initial guess of $v^0 = [\vec{r}^{\,0}, \vec{p}^{\,0}, \vec{\tilde{T}}^{0}, \vec{\tilde{w}}^{0}]$ and a length of the transition, T

2. In iteration m, for given V^m compute

   (a) Additional objects:

      i. Quality in final goods sector:

$$\tilde{q}_{ft} = \left((1-\alpha) \left(\frac{\alpha^2}{r_t} \right)^{\frac{\alpha}{1-\alpha}} \right)^{-1} \tilde{w}_t$$

      ii. Quality in health sector:

$$\tilde{q}_{ht} = \frac{\tilde{q}_{ft}}{p_t^{\frac{1}{1-\alpha}}}$$

   (b) Solve the household model. Compute the solution iterating backwards in time from $t = \mathrm{T}, \ldots, 0$ and set $\tilde{s}^{hh}_{-1} = \tilde{s}^{hh}_{0}$. Also, if $i_t < 0$, set $i_t = 0$ (in these periods, the health sector is not operative).

   (c) Aggregation: Compute the economy wide capital intensity

$$\frac{\tilde{k}_t}{l_t} = \tilde{k}_t = \tilde{s}_{t-1}(r, p, \tilde{T})/g$$

and

i. If $i_t > 0$ then compute the capital intensity in the health sector as well as the output to labor ratio in the sector in all periods t as

$$\frac{\tilde{y}_{ht}}{l_{ht}} = \left(\frac{\tilde{k}_t}{l_t}\right)^{\alpha} q_{ht}^{1-\alpha}$$
$$y_{ht} = i_t$$
$$l_{ht} = \frac{\tilde{y}_{ht}}{\frac{\tilde{y}_{ht}}{l_{ht}}}.$$

If $l_{ht} > 1$ then set $l_{ht} = 1$. Compute

$$k_{ht} = \frac{\tilde{k}_t}{l_t} \cdot l_{ht} = \tilde{k}_t \cdot l_{ht}$$

ii. If $i_t = 0$, then set $y_{ht} = l_{ht} = k_{ht} = 0$.

iii. Aggregate all other (relevant) variables:

$$l_{ft} = 1 - l_{ht}$$
$$k_{ft} = k_t \cdot l_{ft}$$
$$y_{ft} = k_{ft}^{\alpha}(q_{ft} l_{ft})^{1-\alpha}$$

(d) Compute labor shares (labor relative to total population) in the two sectors

$$\ell_{jt} = \frac{l_{jt}}{n_t}$$

where $n_t = 1 + \psi(d_t)$ and trend adjusted growth rates:

$$\tilde{g}_{jt} = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} \ell_{jt} \right]^{\frac{\gamma}{1-\gamma}} /g.$$

(e) Update $r, \tilde{T}$ as:

$$\check{r}^m_t = \alpha^2 \frac{\tilde{y}_{ft}}{k_{ft}}$$
$$\check{\tilde{T}}^m_t = (1-\psi(d_t))\check{r}^m_t \tilde{s}^{hh}_{t-1}(\cdot)/g$$

(f) Update p, w by backward shooting on qualities[2], that is for $t = T-1, T-2, \ldots, 0$ compute, initializing the backward iteration with the BGP qualities $\tilde{q}_{fT}, \tilde{q}_{hT}$:

$$\tilde{q}_{ft} = \frac{\tilde{q}_{ft+1}}{\tilde{g}_{ft+1}}$$
$$\tilde{q}_{ht} = \frac{\tilde{q}_{ht+1}}{\tilde{g}_{ht+1}}$$
$$\check{p}^m_t = \left(\frac{q_{ft}}{q_{ht}}\right)^{1-\alpha}$$
$$\check{\tilde{w}}_t = (1-\alpha)\left(\frac{\alpha^2}{\check{r}^m_t}\right)^{\frac{\alpha}{1-\alpha}} \tilde{q}_{ft}$$

(g) Collect updated variables as $\check{v}^m = [\vec{\check{r}}^m, \vec{\check{p}}^m, \vec{\check{\tilde{T}}}^m, \vec{\check{\tilde{w}}}^m]$. Define $D^m = \|v^m - \check{v}^m\|_\infty$. If $D^m < \epsilon$ stop, else compute $v^{m+1} = (1-\lambda)v^m + \lambda\check{v}^m$ for some dampening factor $\lambda \in (0,1)$ and proceed with step 2.

[2]Unlike in the BGP computation, where the relative price was updated using the resource constraint of the health sector, we here employ the relationship between prices and qualities to update prices. Reason is that updating the relative (shadow) with the resource constraint is not possible if the health sector is not operative (case $i_t = 0$). Observe that along the transition, for $i_t = 0$ both the quality in the health sector and the relative price of health goods are fictive (i.e., shadow) prices.

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