

Housing, Mortgage Bailout Guarantees and the Macro Economy[☆]

Karsten Jeske^a, Dirk Krueger^b, Kurt Mitman^c

^a*Mellon Capital Management Corporation*

^b*University of Pennsylvania, CEPR and NBER*

^c*University of Pennsylvania*

Abstract

What are the macroeconomic and distributional effects of government bailout guarantees for Government Sponsored Enterprises (e.g., Fannie Mae)? A model with heterogeneous, infinitely-lived households and competitive housing and mortgage markets is constructed to evaluate this question. Households can default on their mortgages via foreclosure. The bailout guarantee is a tax-financed mortgage interest rate subsidy. Eliminating this subsidy leads to a large decline in mortgage origination and increases aggregate welfare by 0.5% in consumption equivalent variation, but has little effect on foreclosure rates and housing investment. The interest rate subsidy is a regressive policy: it hurts low-income and low-asset households.

Keywords:

Housing, Mortgage Market, Default Risk, Government-Sponsored Enterprises

JEL: E21, G11, R21

[☆]Corresponding author: Dirk Krueger, Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104. Email: dkrueger@econ.upenn.edu. Tel: +1 (215) 573 1424. Fax: +1 (215) 573 2057.

1. Introduction

The United States displays one of the highest home ownership rates in the world at close to 70%, and owner-occupied houses constitute the most important asset for most U.S. households. Part of the attractiveness of owner-occupied housing stems from a variety of subsidies the government provides to homeowners. In addition to tax-deductible mortgage interest payments and the fact that implicit income from housing capital (i.e. the imputed rental-equivalent) is not taxable, a third subsidy arises from government intervention in the mortgage market. In the US close to 50% of residential mortgages are held by so-called Government-Sponsored Enterprises (GSE's), totaling more than \$5 trillion in value. In September 2008, the US Treasury took conservatorship of Fannie Mae and Freddie Mac after huge losses following the collapse of house prices. Since then, the US government has provided about \$180 billion (see FHFA, 2012) to help GSE's remain solvent. Policy makers are currently faced with deciding the future of GSE's and the role of the government in providing insurance in the mortgage market.

What are the macroeconomic and distributional consequences of government guarantees for GSE's, and what is the optimal degree of such bailout guarantees? We model the consequences of this bailout guarantee as a tax-financed mortgage interest rate subsidy. Prior to their bailout in 2008, the GSE's could borrow at interest rates close to that on U.S. government debt, despite the fact that they were heavily exposed to aggregate house price risk (as has become abundantly clear during the recent crisis). The absence of a significant risk premium for the GSE's debt can be attributed to the then implicit government bailout guarantee these institutions enjoyed. Currently the GSE's are explicitly backed by the US government and as a consequence enjoy lower borrowing costs than private companies. To the extent that part of the interest advantage of GSE's is passed through to homeowners, there exists a mortgage interest rate subsidy from the federal government to homeowners.

The aggregate and redistributive consequences of this subsidy are evaluated by constructing a heterogeneous agent general equilibrium model with incomplete markets in the tradition of Bewley (1986), Huggett (1993) and Aiyagari (1994). This model is augmented by a housing sector and we allow households to borrow against their real estate wealth positions through collateralized mortgages. In the model households can default on their mortgages, with the consequence of losing their homes. Competitive mortgage companies price the default risk into

the mortgage interest rates they offer. The implicit (prior to 2008) or explicit (since 2008) support of GSE's is modeled as a tax-financed direct subsidy to mortgage interest rates. The stationary economy can be interpreted as a world in which the government taxes income every period and either saves the proceeds in an effort to smooth out the spending shock triggered by a potential insolvency of the GSE's, or alternatively, is able to buy insurance against that shock from the outside world via, say, a market for credit derivatives. Under this interpretation the tax revenues constitute the required funds to cover the necessary insurance premium.

In addition to addressing the applied policy question stated above, a second contribution of the paper is the theoretical characterization of mortgage interest rates in the general equilibrium model with foreclosure. First, it is shown that mortgages are priced exclusively based on leverage, with more highly levered households paying higher interest rates. This result is important because it provides a concise characterization of the mortgage price function which allows to easily deal with the continuous choice by households of mortgage contracts with endogenous interest rates. It also facilitates the efficient computation of an equilibrium in the model. Second, a minimum down payment requirement arises endogenously in our model. Finally, a partial characterization of the household decision problem is provided that delivers insights into why households might simultaneously want to save in low interest bearing risk-free bonds and borrow in mortgages that carry higher interest rates. The model provides a useful and tractable framework for future analyses of the housing and mortgage market with collateralized default, and consequently might be of independent interest.

The quantitative results can be described as follows. First, comparing allocations in stationary equilibria with and without the policy, a tax-financed interest rate subsidy of 30 basis points¹ leads to a large increase in mortgage origination, but has little effect on investment in housing assets or in the equilibrium construction of real estate. The mortgage subsidy does *not* significantly change the share of households with positive holdings of real estate, because on one hand the subsidy makes real estate ownership more attractive, but on the other hand the higher

¹Lucas and McDonald (2010) argue for a default premium of the GSE's and assumed by the government of 20 to 30 basis points. Thirty basis points is chosen, the high end of their range, but still lower than the estimates by the Congressional Budget Office (CBO, 2001) or those implied by Passmore (2005).

Several studies (e.g., Passmore, Sherlund and Burgess (2005) and Blinder, Flannery, and Kamihachi (2004)) have argued that a significant portion, if not all of the subsidy, is passed on to homeowners.

required taxes lower after-tax income and thus discourage home ownership for low-income and low-asset households. However, the subsidy *does* significantly affect the distribution of leverage in the economy by increasing both the fraction of households that have positive mortgage debt and the level of leverage, conditional on holding a mortgage. This suggests that the government subsidy of the GSE's may have contributed to the increase in mortgage debt and household leverage prior to the housing bust, which in turn may have exacerbated the economic impact of the recent decline in house prices.

Second, using a steady state utilitarian social welfare function, the welfare implications of the subsidy are significantly negative, on the order of 0.5% of consumption equivalent variation. This aggregate statistic, however, masks substantial heterogeneity across households differing in income and wealth. Low-wealth households prefer to live in a world without the subsidy since they hold little housing and mortgages, and thus do not benefit from the interest rate subsidy, but bear part of the tax bill required to finance it. In contrast, wealthy households have larger homes and mortgages, and thus the benefits accruing to them outweigh the fiscal burden of the policy. Using the same social welfare function the *optimal* interest rate subsidy is found to be small but positive, at 9.375 basis points.

1.1. Related Literature

The paper aims at making two contributions, and thus is related to two broad strands of the literature. On the substantive side, it provides a quantitative, model-based analysis of the macroeconomic and distributional consequences of mortgage interest rate subsidies through government guarantees of the GSE's. It therefore complements empirical work that evaluates the importance of the GSE's and their government guarantee for the housing and mortgage market. Frame and Wall (2002a,b) and more recently Acharya et al. (2011) provide a thorough summary of the institutional details surrounding GSE's. The empirical estimates of Lucas and McDonald (2010) that quantified the borrowing interest rate advantage of GSE's to between 20 and 30 basis points are used to motivate the policy thought experiment.

More broadly, the paper contributes to the literature that studies the positive and normative implications of government housing subsidies on equilibrium allocations. Along this dimension, it is most closely related to the pioneering study by Gervais (2002) who constructs a heterogeneous

household general equilibrium life cycle model to evaluate the effects of the other two main government housing subsidies: the tax-deductibility of mortgage interest rates and the fact that the implicit income from owner-occupied housing capital is not subject to income taxation. Our contribution, relative to his, is to introduce mortgage default (foreclosure) into a dynamic general equilibrium model, and to use it to study a third, and hitherto perhaps somewhat overlooked government housing subsidy policy

A second, model-building and theoretical contribution of the paper is to develop an equilibrium model with mortgage debt and foreclosure in which mortgage interest rates are determined by competition of financial intermediaries, and fully reflect equilibrium default probabilities. In this regard, the model can be seen as a natural extension of the literature on uncollateralized debt and equilibrium default pioneered by Chatterjee et al. (2007) and Livshits et al. (2007). More broadly, our model shares many elements with the recent model-based quantitative housing literature.² For example, Chambers, Garriga and Schlagenhauf (2009), Ríos- Rull and Sánchez-Marcos (2008) and Favilukis, Ludvigson and van Nieuwerburgh (2012) incorporate a housing and mortgage choices into a general equilibrium framework. Similarly, Gruber and Martin (2003) also study the distributional effects of the inclusion of housing wealth in a general equilibrium model, but do not address the role of government housing subsidies.

Especially relevant for the purpose of our analysis are the three recent papers by Corbae and Quintin (2011), Chatterjee and Eyigungor (2011), and Garriga and Schlagenhauf (2009) that build general equilibrium models of housing that also feature equilibrium mortgage default, in order to evaluate the effects of the drop in house prices and a change in housing supply on equilibrium foreclosure rates. Their focus is mainly to understand the underlying reasons for, and consequences of the recent foreclosure crisis³ in the U.S. whereas our purpose is to study the effects of a specific government housing market policy. Our paper is complementary to theirs in terms of focus, but also in terms of the details of how mortgages and foreclosure are modeled. In these papers mortgages are long term contracts whereas we permit households to costlessly refinance in every period. This modeling choice, in conjunction with perfect competition in the

²For a brief survey of this literature, see Fernandez-Villaverde and Krueger (2011).

³An important *empirically* oriented literature has recently studied the causes and consequences of the recent boom in foreclosures in the U.S. See e.g. Campbell et al. (2011), Gerardi et al. (2010) or Mian et al. (2011).

mortgage market allows us to obtain a sharp analytical characterization of equilibrium mortgage interest rates and default behavior in our model.⁴

The remainder of the paper is organized as follows. Section 2 introduces the model and defines equilibrium in an economy with a housing and mortgage market. Section 3 characterizes equilibria. Section 4 describes the calibration of our economy. Section 5 details the numerical results by comparing steady states in economies with and without a mortgage interest subsidy. Section 6 concludes the paper, and all proofs are relegated to the appendix.

2. The Model

The economy is populated by a continuum of measure one of infinitely lived households, a continuum of competitive banks and a continuum of housing construction companies. The analysis proceeds by immediately describing the economy recursively.

2.1. Households

Preferences: Households derive period utility $U(c, h)$ from nondurable consumption c and housing services h , which can be purchased at a price P_l (relative to the numeraire consumption good). Households discount the future with discount factor β and maximize expected utility.

Endowments: Households receive an idiosyncratic endowment of the perishable consumption good given by $y \in Y$. These endowments follow a finite state Markov chain with transition probabilities $\pi(y'|y)$ and unique invariant distribution $\Pi(y)$. Denote by $\bar{y} = \sum_{y \in Y} y\Pi(y)$ the average endowment. The terms endowment and (labor) income are used interchangeably throughout the remainder of the paper, and a law of large number is assumed to apply, so that π and Π also denote deterministic fractions of households receiving a particular income shock y .

The government levies a proportional tax τ on labor income to finance an interest rate subsidy, if such a policy is in place.

Assets: In addition to consumption and housing services the household spends income to purchase two types of assets, one-period bonds b' and perfectly divisible houses g' . The price of

⁴Krainer et al. (2009) construct a continuous time mortgage valuation model and also provide a partial analytical characterization of the interest rate and asset value of mortgages, as a function of mortgage leverage.

bonds is denoted by P_b and the price of houses by P_h . Houses are risky assets: they are subject to *idiosyncratic* house price shocks. Let $F(\delta')$ denote the continuously differentiable cumulative distribution function of the house price depreciation rate δ' tomorrow, which has support $D = [\underline{\delta}, 1]$ with $\underline{\delta} \leq 0$. A negative value of δ indicates positive house price appreciation. The realization of δ is independent across time for every household, and that a law of large number applies, so that $F(\cdot)$ is also the economy-wide distribution of house price shocks. One unit of the housing asset generates one unit of housing services. A house purchased in the current period can immediately be rented out to generate rental income in the same period as the purchase. By assumption households are prohibited from selling bonds and houses short.

Mortgages: Households can borrow against their real estate position using one-period mortgage debt.⁵ Let m' denote the size of the mortgage, and P_m the contemporaneous receipts of resources (that is, of the consumption good) for each unit of mortgage issued today and to be repaid in the subsequent period. The “price” P_m will be determined in equilibrium by competition of banks through a zero profit condition, and will in general depend on the characteristics of households as well as the size of the mortgage m' and size of the collateral g' against which the mortgage is issued. The gross mortgage interest rate is then simply given by $R_m = 1/P_m$.

Households that come into the next period with housing assets g' and a mortgage m' possess the option, after having observed the idiosyncratic house price appreciation shock δ' , of defaulting on their mortgages, at the cost of losing their entire housing collateral. There are no other costs associated with mortgage default. This assumption, together with modeling mortgages as short-term contracts, immediately implies that households will choose to default whenever the amount owed on the mortgage is greater than the value of the house after the realization of the price shock, that is, if and only if⁶ $m' > P_h(1 - \delta')g'$.

⁵Our assumption abstracts from transaction costs of refinancing a mortgage or obtaining a home equity line of credit. Long-term mortgages in addition protect households from inflation risk and prevent banks from adjusting interest rates based on changing household characteristics (such as income), providing additional insurance to households. In our real model inflation risk is absent. In principle, financial intermediaries could condition interest rates of our one-period mortgages on income (and thus adjust them in response to income shocks), but in equilibrium (shown later) they will not. We therefore think that, in the context of our model, assuming short-term debt is a defensible assumption. As demonstrated below, the payoff is a sharp analytical characterization of equilibrium mortgage interest rates and foreclosure behavior.

⁶We make the assumption that a household indifferent between defaulting or not will choose not to default.

As a consequence, the ex-ante default probability of a household at mortgage origination, that is, prior to observing the shock, is simply a function of the size of the mortgage m' and today's value of the collateral $g'P_h$. As argued below, this will imply that in equilibrium the price of a mortgage P_m today will only be a function of (m', g') , a fact we will already use when now specifying the household problem in recursive formulation.

2.2. Recursive Formulation of the Household Problem

Let a denote cash at hand, that is, after tax income plus the value from all assets brought into the period, after the current income and house price shocks (y, δ) have materialized. The individual state of a household consists of $s = (a, y)$. The cross-sectional distribution over individual states is denoted by μ . Since the analysis is restricted to stationary equilibria in which μ is constant over time, in what follows the dependence of aggregate prices and quantities on μ is left implicit.

The dynamic programming problem of a household consists of choosing consumption c , housing services h and financial and housing assets (b', g') as well as mortgages m' to solve:

$$v(a, y) = \max_{c, h, b', m', g' \geq 0} \left\{ U(c, h) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^1 v(a', y') dF(\delta') \right\} \quad s.t. \quad (1)$$

$$c + b'P_b + hP_l + g'P_h - m'P_m(g', m') = a + g'P_l \quad (2)$$

$$a'(\delta', y', m', g') = b' + \max\{0, P_h(1 - \delta')g' - m'\} + (1 - \tau)y' \quad (3)$$

Note that because of the assumption that newly purchased housing assets can immediately be rented out, rental income $g'P_l$ from newly purchased housing assets enters the current period budget constraint. Tomorrow's cash at hand a' is equal to the sum of after tax labor income $(1 - \tau)y'$, the amount of bonds b' brought into the period and the net value of real estate. If the household owes less than the realized value of her housing asset, she does not default and the net value of real estate equals $P_h(1 - \delta')g' - m'$. For bad realization of the house price shock δ' the mortgage is under water, the household defaults and is left with zero housing wealth.

2.3. The Real Estate Construction Sector

The representative firms in the perfectly competitive real estate construction sector face the linear technology $I = C_h$, where I is the output of newly build and perfectly divisible houses of a representative firm and C_h is the input of the consumption good. Note that we assume that this technology is reversible, that is, real estate companies can turn houses back into consumption goods using the same technology, although this does not happen in the equilibrium of our calibrated economies. Thus the problem of a representative firm reads as

$$\begin{aligned} \max_{I=C_h} P_h I - C_h \end{aligned} \tag{4}$$

and the equilibrium house price necessarily satisfies $P_h = 1$. In effect, ours is therefore a model with exogenous house prices normalized to $P_h = 1$, but endogenous rents (and thus endogenous house-price to rent ratios), and perfectly elastic supply of the housing asset from the real estate companies, at the exogenous house price $P_h = 1$.

2.4. The Banking Sector

Let r_b denote the risk free interest rate on one-period bonds, to be determined in general equilibrium. Competitive banks take their costs $P_b = \frac{1}{1+r_b}$ of re-financing as given. In addition, issuing mortgages is costly; let r_w be the percentage real resource cost, per unit of mortgage issued, to the bank. This cost captures screening costs, administrative costs as well as maintenance costs of the mortgage (such as preparing and mailing a quarterly mortgage balance).⁷ In addition, in order to insure mortgages against the (unmodeled) *aggregate component* of mortgage default risk, banks need to purchase insurance at a cost θ for each dollar of mortgage originated. This θ can be interpreted as a real resource cost that is transferred to an unmodeled insurance company abroad. It will enter the aggregate resource constraint of the economy.

The government can subsidize mortgages using labor income taxes. The mortgage subsidy is modeled as an *interest rate* subsidy ϕ for each unit of mortgage issued. Thus the effective cost of the banking sector for financing one dollar of mortgage equals $(1 + r_b)(1 + r_w + \theta - \phi)$.

⁷In addition to realism the cost $r_w > 0$ insures that the interest paid on a mortgage that is repaid with probability one is still higher than the risk-free rate on bonds, which avoids an indeterminacy in the household maximization problem (since with $r_w = 0$ bonds and zero-default mortgages are perfect substitutes).

In the perfectly competitive banking sector, risk-neutral banks compete for customers *loan by loan*, as in Chatterjee et al. (2007), in the context of uncollateralized debt. Banks will only originate mortgages that yield non-negative profits in expectation. Banks, when making their origination decision, take into account the fact that a household may default on its mortgage. When a household does so, the bank seizes the housing collateral worth $(1 - \delta')g'$. However, the foreclosure technology is possibly inefficient, and therefore that the bank only recovers a fraction $\gamma \leq 1$ of the value of the collateral.

In order to define a typical banks' problem the optimal default choice of a household has to be characterized. As discussed in section 2.1 the household defaults if and only if her mortgage is under water. Thus for a household with housing assets g' and a mortgage m' there is a cutoff level for house price depreciation $\delta^*(m', g')$ at which a household is indifferent between defaulting and not defaulting on her mortgage. This cutoff is determined as $m' = (1 - \delta^*(m', g'))g'$, and thus explicitly, as

$$\delta^*(m', g') = \delta^*(\kappa') = 1 - \frac{m'}{g'} = 1 - \kappa' \quad (5)$$

where $\kappa' = \frac{m'}{g'}$ is defined as the leverage (for $g' > 0$) of a mortgage m' backed by real estate g' . Thus the household defaults for all house price depreciation realizations $\delta' > \delta^*(\kappa')$. Since the foreclosure decision of a household does not depend on bond holdings b' chosen today or current income y , in equilibrium the receipts P_m will not depend on these quantities either.

Using the characterization of the household default decision, the set of mortgage contracts that the bank will originate can be characterized. A bank will originate a mortgage if and only if:

$$m'P_m(g', m') \leq \frac{1}{(1 + r_b)(1 + r_w + \theta - \phi)} \left\{ m'F(\delta^*(\kappa')) + \gamma g' \int_{\delta^*(\kappa')}^1 (1 - \delta')dF(\delta') \right\} \quad (6)$$

$$= \frac{m'\Psi(\kappa')}{(1 + r_b)(1 + r_w + \theta - \phi)} \quad (7)$$

The left hand side $m'P_m(g', m')$ is the amount the bank pays out to a household that takes on a mortgage of size m' collateralized by g' housing assets. The term $m'\Psi(\kappa')$, the term in $\{\}$ -brackets in equation (6), denotes the receipts tomorrow for the bank from the mortgage. With

probability $F(\delta^*(\kappa'))$ the household receives a house price shock δ' that makes default suboptimal in which case she repays the full face value of the mortgage m' . For all price shocks $\delta' > \delta^*(\kappa')$ the household defaults and the bank retrieves $\gamma(1 - \delta')g'$ by foreclosing and selling off the house. The term $\Psi(\kappa')$ measures the expected revenue for the bank tomorrow for each dollar of a mortgage with leverage κ' issued today. For each dollar of mortgage issued today the costs of funds to the bank are $1 + r_b$, the direct costs of maintaining one dollar worth of mortgage are r_w , the insurance costs per dollar of mortgage are θ and the subsidy per dollar is ϕ , so that the effective discount factor of the bank is given by $\frac{1}{(1+r_b)(1+r_w+\theta-\phi)}$. Perfect competition requires that for all mortgages offered in equilibrium the inequality in (6) holds as equality.

Notice that one direct and obvious consequence of equation (6) is that $P_m(g' = 0, m') = 0$ for all $m' > 0$, that is, a mortgage $m' > 0$ without collateral, i.e. with $g' = 0$, will not generate any funds for the household today. Without collateral ($g' = 0$) and $m' > 0$ the household will default on the mortgage for sure tomorrow and foreclosure will not generate any revenues for the bank. Therefore the right-hand side of equation (6) equals zero, and thus $P_m(g' = 0, m') = 0$.

A financial intermediary that issues a mortgage of size m' with leverage $\kappa' = m'/g'$ issues bonds of value $m'P_m(g', m')(1 + r_w + \theta - \phi)$ today, transfers $m'P_m(m', g')$ to the household, uses $r_w m'P_m$ resources for mortgage origination, transfers $\theta m'P_m$ to the international insurance agency and receives a transfer $\phi m'P_m$ from the government. Tomorrow it repays the bonds (including interest) with the expected receipts $m'\Psi(\kappa')$ from the mortgage to break even.

2.5. The Government

As stated above, the government levies income taxes at a flat rate τ on households to finance the mortgage interest rate subsidy. The tax revenues of the government are given by $\tau\bar{y}$. In the baseline economy, we model the bailout guarantee provided by the government as an interest rate subsidy equal to the cost of insurance $\phi = \theta$ per unit of mortgage issued. We interpret this as the government levying income taxes to provide insurance to banks against systemic (i.e. aggregate) mortgage risk. For a loan of type (m', g') the subsidy by the government is given by

$$sub(m', g') = \theta m' P_m(g', m'; \phi = \theta) \quad (8)$$

and the total economy-wide subsidy is

$$G = \int sub(m', g') d\mu \quad (9)$$

Note that G also measures the amount of resources expended on insurance against aggregate shocks, either by the households directly (in case of no bailout policy) or by the government.

2.6. Equilibrium

We are now ready to define a stationary recursive Competitive Equilibrium for the benchmark economy. Let $S = \mathbb{R}_+ \times Y$ denote the individual state space.

Definition Given a government subsidy policy ϕ a **Stationary Recursive Competitive Equilibrium** are value and policy functions for the households, $v, c, h, b', m', g' : S \rightarrow \mathbb{R}$, policies for the real estate construction sector I, C_h , prices P_l, P_b , a mortgage pricing function $P_m : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$, a government tax rate τ and government spending G , as well as a stationary measure μ such that: (1) Given prices P_l, P_b, P_m and government policies the value function solves (1) and c, h, b', m', g' are the associated policy functions. (2) Policies I, C_h solve the maximization problem (4) of the real estate construction company. (3) Given P_b and P_m , (6) holds with equality for all m', g' . (4) The tax rate function τ satisfies $\tau = G/\bar{y}$ and government spending G satisfies (9), given the functions m', P_m . (5) The rental market clears:

$$\int g'(s) d\mu = \int h(s) d\mu. \quad (10)$$

(6) The bond market clears:

$$P_b \int b'(s) d\mu = (1 + r_w + \theta - \phi) \int P_m(g', m'; \phi) m'(s) d\mu. \quad (11)$$

(7) The goods market clears:

$$\int c(s) d\mu + C_h + (r_w + \theta - \phi) \int P_m(g', m'; \phi) m' d\mu + G = \bar{y}, \quad (12)$$

where

$$C_h = I = \int g'(s)d\mu - \int \left[\int_{\underline{\delta}}^{\delta^*(\kappa'(s))} g'(s)(1 - \delta')dF(\delta') - \gamma \int_{\delta^*(\kappa'(s))}^1 g'(s)(1 - \delta')dF(\delta') \right] d\mu \quad (13)$$

is gross investment in the housing stock. The measure μ is invariant with respect to the Markov process induced by the exogenous Markov process π and the policy functions m', g', b' .

For the policy experiments aggregate economy-wide welfare is measured via a Utilitarian social welfare function in the steady state, defined as

$$\mathcal{WEL} = \int v(s)\mu(ds) \quad (14)$$

where μ is the invariant measure over the state space for cash at hand and income, $s = (a, y)$.

3. Theoretical Results

In this section theoretical properties of our model are stated that provide insights into the forces that determine optimal household portfolio and leverage choices. In addition, the properties are useful in the computation of an equilibrium.

3.1. Mortgage Interest Rates

Recall that the implied net real interest rate from a mortgage m' with collateral g' and receipts $P_m(g', m')$ is $r_m(g', m') = 1/P_m(g', m') - 1$. From equation (6) and the fact that competition requires profits for all mortgages issued in equilibrium to be zero we immediately obtain a characterization of equilibrium mortgage interest rates:

Proposition 1. *In any steady-state equilibrium, mortgages originated with positive collateral $g' > 0$ have the following properties: (1) They are priced exclusively based on leverage $\kappa' = \frac{m'}{g'}$, that is $P_m(m', g') = P_m(\kappa')$ and $r_m(g', m') = r_m(\kappa')$; (2) $P_m(\kappa')$ is decreasing in κ' , and strictly decreasing if the household defaults with positive probability. Thus mortgage interest rates $r_m(\kappa')$ are increasing in leverage κ' ; and (3) Households that default with positive probability tomorrow receive $P_m(\kappa') < \frac{1}{(1+r_b)(1+r_w+\theta-\phi)}$ today, that is, they borrow at a risk premium $1 + r_m(\kappa') > (1 + r_b)(1 + r_w + \theta - \phi)$ that is strictly increasing in leverage κ' .*

This characterization of equilibrium mortgage interest rates now allows to obtain an endogenous upper bound for the leverage chosen by households.

3.2. Endogenous Down Payment Requirement

It is straightforward to show that it is never strictly beneficial for a household to purchase a mortgage with a leverage higher than the level that leads to subsequent default with probability one. Define the leverage that leads to certain default by $\bar{\kappa}$, which is equal to $1 - \underline{\delta}$, from equation (5). There exists a tighter *endogenous* upper bound on leverage $\kappa^* < \bar{\kappa}$ that households will never choose to exceed in equilibrium. This result also implies that it is never optimal for the household to lever up to the point in which default occurs with probability 1. Furthermore, at least with $\underline{\delta} = 0$, any mortgage chosen by households in equilibrium requires a positive down payment.

Proposition 2. *If $F(\delta)$ is \mathcal{C}^2 and log-concave with support $[\underline{\delta}, 1]$, $\underline{\delta} \leq 0$ and foreclosure is inefficient ($\gamma < 1$), there exists an endogenous borrowing limit κ^* . It is never optimal for a household to choose leverage $\kappa > \kappa^*$ at equilibrium mortgage prices P_m . Further, $\kappa^* < \bar{\kappa}$. In addition, if $\underline{\delta} = 0$, then $\kappa^* < 1$, that is, there is an endogenous minimum down payment $1 - \kappa^* > 0$.*

Log-concavity of the distribution guarantees that the resources received from a mortgage m' today, $m'P_m(\kappa')$, will be concave in m' . This fact, combined with the smoothness assumption on the distribution of the house price shocks $F(\cdot)$, guarantees that the leverage that maximizes resources today will be strictly less than the leverage that leads which to certain default. By increasing leverage beyond the level that maximizes resources today, the household receives strictly less resources today and has to repay weakly more resources tomorrow, implying that it can never be optimal for the household to take on leverage above the level that maximizes contemporaneous resources received from the mortgage. Note that the (truncated) Pareto distribution used in our quantitative analysis for $F(\delta)$ is \mathcal{C}^2 and log-concave.

3.3. Existence of a Solution to the Household Problem

It is now shown that the recursive problem of the household has a unique solution. In order to do so it is helpful to split the household problem into a static problem that optimally

allocates a given amount of resources between consumption and rental expenditures, and a dynamic consumption-saving and portfolio choice problem.⁸

As a function of the rental price P_l and total expenditures consumption c , define the indirect static utility function u as the solution to:

$$u(c; P_l) = \max_{\tilde{c}, h \geq 0} U(\tilde{c}, h) \text{ s.t.} \quad (15)$$

$$\tilde{c} + P_l h = c \quad (16)$$

Note that, in slight abuse of notation, c now denotes total expenditures $\tilde{c} + P_l h$ as opposed to just nondurable consumption (as it was defined in previous sections).

The dynamic household maximization problem can then be rewritten as:

$$v(s) = \max_{c, b', m', g' \geq 0} \left\{ u(c; P_l) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^{\infty} v(s') dF(\delta') \right\} \quad (17)$$

$$\text{s.t.} \quad c + b' P_b + g' [1 - P_l] - m' P_m(\kappa') = a \quad (18)$$

$$a'(\delta', h', m', g') = b' + \max\{0, (1 - \delta')g' - m'\} + (1 - \tau)y' \quad (19)$$

In the appendix it is shown that the recursive problem of the household has a unique solution:

Proposition 3. *Suppose that $u(\cdot; P_l)$ is unbounded from below and bounded from above. Then recursive problem of the household has a unique solution $v(a, y)$ that is strictly increasing in its first argument a .*

The fact that the utility function is bounded from above guarantees that even as the cash at hand of a household diverges, the value function will remain bounded. Therefore, since the utility function is unbounded from below, it will always be optimal to set consumption expenditures c strictly away from zero, since contemporaneous utility diverges to negative infinity as consumption goes to zero, but the continuation value is bounded for all levels of saving. Note

⁸Clearly this separation hinges crucially on the existence of frictionless housing and rental markets and the perfect substitutability of owner occupied housing and rentals in providing housing service flows.

that if the utility function U will be of CRRA form with risk aversion coefficient $\sigma > 1$ (with Cobb-Douglas aggregator between consumption and housing) as in our quantitative analysis, then the indirect utility function u satisfies the assumptions made in proposition 3.

3.4. Characterization of the Household Problem

A partial characterization of the household problem is now provided. Define as

$$P(\kappa') := 1 - P_l - \kappa' P_m(\kappa') \quad (20)$$

the net *per unit* resources required to purchase g' housing assets, partially financed by a mortgage with leverage $\kappa' = m'/g'$. Note that

$$g'P(\kappa') = (1 - P_l)g' - m'P_m(m'/g'). \quad (21)$$

Using this definition the following holds:

Proposition 4. *If $u(c; P_l)$ is differentiable in c , then for any state s for which it is optimal to choose an interior solution to the portfolio choice problem ($g', b', m' > 0$) in the current and subsequent period, the Euler equation governing for the household is given by:*

$$[P_b + P'(\kappa')]u'(c(a, y); P_l) = [1 - F(1 - \kappa')] \beta \sum_{y' \in Y} \pi(y'|y)u'(c(b' + (1 - \tau)y', y'); P_l) \quad (22)$$

Further, at any such point the optimal leverage κ' chosen by the household is determined by:

$$P(\kappa') - (1 + \kappa') P'(\kappa') = 1 - P_l \quad (23)$$

This partial characterization helps illustrate why households might simultaneously choose to borrow and save. First, note from Proposition 1 that when the default probability is zero, in the benchmark case with subsidy, households can borrow at a rate $r_b + r_w$ and save in bonds at rate r_b . If r_w were to equal zero, there would be an indeterminacy between saving and borrowing, conditional on choosing mortgages with zero default probability. Having a positive cost of issuing a mortgage, $r_w > 0$, eliminates that indeterminacy and creates a wedge between the borrowing

and saving rate, even in the absence of default. However, even when there is a wedge in the interest rates between borrowing and saving, it may be optimal for households to simultaneously save and hold mortgages (i.e. borrow).

Equation (22) emerges by inserting the first order condition for mortgages m' into the first order condition for risk-free bonds b' (and using the envelope condition). It equates the costs and benefits from a joint marginal increase in bond holdings b' and mortgages m' (holding housing g' constant). On the cost side, an extra bond costs P_b , and a simultaneous increase in mortgages by one unit brings revenue

$$P_m(\kappa') + \frac{m'}{g'} P'_m(\kappa') = -P'(\kappa') > 0 \quad (24)$$

and thus the net cost of this marginal variation is the left-hand side of equation (22), $P_b + P'(\kappa')$. In utility terms, the cost amounts to $[P_b + P'(\kappa')] u'(c)$. One can interpret $P_b + P'(\kappa')$ as the *insurance premium* of borrowing in mortgages to save at the risk free rate. On the benefit side, in states δ' in which the household does not default, she has to pay back the extra unit of the mortgage, but receives the extra bond payoff, which nets out to zero. For states δ' in which the household defaults, however, she still receives the extra unit of consumption from the bond payoff (which the household values at $u'(c')$), but does not have to repay the extra unit of the mortgage. The risk-free bonds thus provide insurance against low consumption in default states. The default probability is given by

$$1 - F(\delta^*(\kappa')) = 1 - F(1 - \kappa'). \quad (25)$$

Thus the expected benefit, in utility terms, from the joint marginal variation in b', m' is given by the right hand side of equation (22). This equation therefore shows why a household would simultaneously save at a low risk-free rate and borrow (in defaultable mortgages) at a higher rate: this strategy, together with the default option, provides insurance against low consumption in high δ' states, for which the household is willing to pay an insurance premium.

Equation (23) then shows that conditional on wanting to “borrow to save” there is a unique optimal value κ' at which to do so. Thus the proposition predicts that the optimal policy function for leverage κ' is flat over that region of the state space for which the household finds it optimal

to “borrow to save”. Our quantitative analysis below will demonstrate that this is indeed the optimal portfolio strategy for a significant part of the state space.

3.5. Bounds on the Equilibrium Rental Price of Housing

After having partially characterized the household problem an upper bound is derived on the rental price P_l , one of the two prices to be determined in general equilibrium.⁹ For all feasible choices of the household it has to be the case that $P(\kappa') = 1 - P_l - \kappa' P_m(\kappa') \geq 0$, otherwise the household can obtain a positive cash flow today by buying a house with a mortgage; the default option on the mortgage guarantees that the cash flow from the house tomorrow is non-negative. Thus, the requirement of absence of this arbitrage opportunity in equilibrium requires $P(\kappa') \geq 0$ for all κ' , and, in particular, for $\kappa' = \bar{\kappa}$. Thus

$$P(\kappa' = \bar{\kappa}) = 1 - P_l - \bar{\kappa} P_m(\kappa' = \bar{\kappa}) \geq 0 \quad (26)$$

which implies

$$\begin{aligned} P(\bar{\kappa}) &= 1 - P_l - \bar{\kappa} P_m(\bar{\kappa}) = 1 - P_l - \left(\frac{1}{(1+r_b)(1+r_w)} \right) \gamma(1 - E(\delta')) \geq 0 \text{ or} \\ P_l &\leq 1 - \left(\frac{1}{(1+r_b)(1+r_w)} \right) \gamma(1 - E(\delta')) = \frac{r_b + r_w + r_b r_w + \gamma E(\delta') + 1 - \gamma}{(1+r_b)(1+r_w)} \quad (27) \end{aligned}$$

which places an upper bound on the equilibrium rental price.¹⁰

Appendix Appendix A.4 includes a discussion of why the riskiness of the housing asset puts also puts a lower bound on the expected return from housing, and thus a lower bound on the rental price P_l , which given in our quantitative applications by $P_l \geq \frac{r_b + E(\delta')}{1+r_b}$.

These bounds bracket the equilibrium rental price and are thus very useful for the computation of the model. Equipped with the theoretical characterization of household portfolio behavior, of the high-dimensional equilibrium mortgage interest rate function $r_m(\kappa')$ and the bounds for the equilibrium rental price P_l , we now proceed to the quantitative assessment of the aggregate and

⁹The equilibrium bond price P_b satisfies $P_b < 1/\beta$, as in Aiyagari (1994).

¹⁰If $\gamma = 1$, this condition simply states that the rental price P_l cannot be larger than the user cost of housing, inclusive of the cost r_w of originating mortgages $\frac{r_b + r_w + r_b r_w + E(\delta')}{(1+r_b)(1+r_w)}$.

distributional consequences of the government interest rate subsidy.

4. Calibration

The model is mapped to the US during the years 2000-2006, a period prior to when the implicit bailout guarantee turned explicit. Some parameters are selected exogenously; the remaining parameters are calibrated jointly in the model.

4.1. Technology and Endowments

Income process:. For a continuous state $AR(1)$ process of the form

$$\log y' = \rho \log y + (1 - \rho^2)^{0.5} \varepsilon \quad (28)$$

with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma_\varepsilon^2$, the unconditional standard deviation is σ_ε and the one-period autocorrelation (persistence) is ρ . Estimates for ρ in the literature center around values close to 1. Motivated by Storesletten et al. (2004) $\rho = 0.98$ is selected. Estimates for σ_ε range from 0.2 to 0.4 (see e.g. Aiyagari, 1994), and $\sigma_\varepsilon = 0.3$ is chosen. The $AR(1)$ process is approximated with a 5 state Markov chain using Tauchen and Hussey's (1991) procedure. Appendix B.2 gives the values for the income realizations, Markov transition matrix π and the invariant distribution Π .

Foreclosure technology:. Two recent studies, Pennington-Cross (2006) and Campbell, Giglio and Pathak (2011), estimate the default loss parameter γ . Pennington-Cross studies liquidation sales revenue from foreclosed houses and compares it to market prices constructed via the OFHEO repeat sales index. Campbell, Giglio and Pathak have access to zip code level data in Massachusetts and compare foreclosed home sales to regional prices. Pennington-Cross finds that the average loss in foreclosure is 22% as opposed to 27% in Campbell, Giglio and Pathak. Since they use data from only one state, as compared to national estimates from Pennington-Cross (and their estimates are relatively close anyhow), the lower value is chosen, hence $\gamma = 0.78$.

The depreciation process:. The house value price depreciation process is calibrated to attain realistic levels of default in the model while at the same time generating the statistical properties of *idiosyncratic* house price appreciation and depreciation rates observed in the data. Since

the analysis conducted is steady state, the aggregate component of house price fluctuations and secular aggregate house price growth are abstracted from.

According to the Mortgage Banker Association (MBA (2006)), the quarterly foreclosure rate has been about 0.4 percent in between 2000 and 2006. Abstracting from the possibility that one house may go in and out of foreclosure multiple times within one given year, this implies that on an annual basis, banks start foreclosure proceedings on about 1.6 percent of their mortgages. The ratio of mortgages in foreclosure that eventually end in liquidation was about 25 percent in 2005, according to MBA (2006). Most homeowners avoid liquidation by either selling their property, refinancing their mortgage or just paying off the arrears. Consequently, only about 0.4 percent of mortgages actually end up in liquidation, in the way our model envisions it. Given the unusually strong aggregate home price appreciation over the 2000-06 period this figure is viewed as a lower bound on the long-run foreclosure rate (that rate certainly increased strongly in subsequent years) and thus a default rate of 0.5 percent of all mortgages is targeted.

Two empirical moments of house price depreciation are targeted, the mean and the standard deviation. The mean depreciation for residential housing according to the Bureau of Economic Analysis was 1.48% between 1960 and 2002 (with standard deviation 0.05%), computed as consumption of fixed capital in the housing sector (Table 7.4.5) divided by the capital stock of residential housing. With respect to the standard deviation of idiosyncratic house price depreciation shocks, we utilize data from the Office of Federal Housing Enterprise Oversight (OFHEO). OFHEO models house prices as a diffusion process and estimates within-state and within-region annual house price volatility. The technical details can be found in the paper by Calhoun (1996). The broad range for the eight census regions is an annual volatility of 9 – 10% in the years 1998-2004. The upper bound $\sigma_\delta = 0.10$ is used to account for the fact that nationwide house price volatility is slightly higher than the within-region volatility.

Using a log-normal distribution for house price appreciation of real estate in our model, with the mean and standard deviation as stated above, does not generate a sufficiently large share of foreclosures, since, at least within the context of our model, the right tail of the distribution appears to be too thin to reproduce the empirical foreclosure target. In order to obtain more empirically realistic levels of mortgage default a generalized Pareto distribution is used (which has a fatter right tail, relative to a log-normal distribution) whose probability distribution function

is given by:

$$f(\delta) = \frac{1}{\sigma_\delta} \left(1 + \frac{k(\delta - \underline{\delta})}{\sigma_\delta} \right)^{\left(-\frac{1}{k}-1\right)} \quad (29)$$

With this parametric form for the house price depreciation distribution there are three parameters $k, \sigma_\delta, \underline{\delta}$ to pin down three moments: the mean depreciation rate, its standard deviation and the equilibrium share of mortgages in default that the model generates endogenously.¹¹

4.2. Preferences

For the period utility function, a CRRA form with a Cobb-Douglas aggregator between nondurable consumption and housing services is assumed.

$$U(\tilde{c}, h) = \frac{(\tilde{c}^\alpha h^{1-\alpha})^{1-\sigma} - 1}{1-\sigma} \quad (30)$$

Note that this functional form implies an indirect utility function of

$$u(c; P_l) = \frac{\Phi(P_l)c^{1-\sigma} - 1}{1-\sigma} \quad (31)$$

where $\Phi(P_l) = (\alpha^\alpha(1-\alpha)^{1-\alpha}P_l^{\alpha-1})^{1-\sigma}$.

The parameter α is chosen such that the share of housing in total consumption expenditures matches NIPA data, according to which this share has been fairly steady at 14.1% over the last 40 years, with a standard deviation of only about 0.5%. This yields a value of $\alpha = 0.8590$.

The time discount parameter β and the CRRA parameter σ are endogenously calibrated to match an equilibrium risk free rate of 1% and a median household leverage of 61% in the benchmark economy. Data from the 2004 Survey of Consumer Finances is used and our attention is restricted to households with heads aged 50 and younger, in order to control for strong life-cycle trends in leverage. Household leverage is then computed using data on houses owned and mortgages owed on those homes. The median leverage is calculated to be 61% from these SCF data. The resulting preference parameters are $(\beta, \sigma) = (0.919, 3.912)$.

¹¹It is understood that, strictly speaking, all parameters determine all endogenous variables jointly.

4.3. Mortgage Parameters

For the interest rate subsidy it is assumed that the pass-through of the subsidy is 100%, in order to make the case for the subsidies most favorable. The size of the subsidy is chosen to match the estimated implicit interest rate differential of 30 basis points that the GSE's enjoyed during the period of their implicit guarantee by the government, see Lucas and McDonald (2010). Finally a mortgage administration cost r_w of 10 basis points is chosen, equal one third of the mortgage subsidy. This choice corresponds to an annual cost of \$100 for servicing a \$100,000 mortgage. Tables 1 and 2 summarize our parameterization of the model.

5. Results

Before analyzing the effects of the bailout policy by comparing the equilibria with and without the policy in place it is instructive to explain household behavior in the baseline economy. In figures 1 and 2 the housing and leverage policy functions of households are plotted under both policy scenarios, as a function of cash at hand a , conditional on the lowest and highest realizations y_1 and y_5 of current labor income y . Note that by the definition of cash at hand, $a \geq y$, and thus the policy functions for $y = y_5$ starts to the right (along the x-axis) of that for $y = y_1$.

From figure 1 observe that, with the subsidy, purchases of housing assets are monotonically increasing in cash at hand. Figure 2 displays the fact that leverage is high (at close to 80%) for households with little wealth under this policy scenario. Leverage then drops quickly, as cash at hand increases, to around 61% and remains constant at that level. From the threshold for cash at hand for which a leverage of 61% is optimal onwards households no longer reduce leverage with increasing "wealth" a , but start purchasing *bonds*, as can be seen from the bond policy function for the subsidy case, displayed in figure 3. As cash at hand increases further leverage remains constant, and the holdings of bonds b' , the housing asset g' and mortgages m' rise.

This behavior is exactly what proposition 4 above predicts: households with high wealth both borrow through high-interest mortgages and save with low-interest bearing bonds. These households want to take advantage of the high return on housing, but would also like to insure consumption against adverse house price shocks. Positive bond holdings are essentially consumption insurance against bad δ' realizations that might trigger default and would thus reduce total wealth to zero, in the absence of positive bond holdings. If in addition labor income tomorrow

is low, $y' = y_1$, then a portfolio mix without positive bond holdings would lead to very low consumption realization in bad idiosyncratic states of the world (e.g. $y' = y_1$ and high δ'). In order to maintain a level of consumption above that of labor income it is therefore optimal to hold bonds as insurance. Note that equation (23) determines uniquely the constant (in cash at hand a) optimal leverage of these households, which is calculated to be 61% in the baseline model.¹²

5.1. *Effects of Removing the Subsidy on Household Behavior*

Now the bailout policy is evaluated, comparing steady state equilibria of economies with and without a tax-financed mortgage interest rates subsidy of 30 basis points. First the change in household behavior induced by the removal of the subsidy is analyzed, and then its aggregate, distributional and welfare implications are discussed.

The main economic impact on households from removing the subsidy is to make mortgages less attractive by increasing the effective interest rate. The most notable difference in household choices can thus be seen in the leverage and bond policy functions in figures 2 and 3 respectively, where each of the two panels plots the policy functions for both cases (subsidy, no subsidy) against cash at hand, for a given current income level $y \in \{y_1, y_5\}$. Observe that for households with low levels of cash at hand the change in behavior induced by the removal of the subsidy is modest: under both policy scenarios households with little wealth take on highly leveraged mortgages and hold no bonds. As households get wealthier, however, without the subsidy leverage decreases monotonically to zero. In the absence of the mortgage interest subsidy wealthy households do *not* simultaneously hold bonds and mortgages, since the wedge in the interest rate between saving and borrowing (even absent default risk) increases from 10 to 40 basis points. Thus the change in the policy induces a massive reduction in leverage for wealthier households, and thus

¹²Note that since asset poor households do not buy any bonds, households allocate a larger share of their portfolios to bonds as cash at hand increases. This behavior of households may sound counterintuitive at first, but is consistent with results from the portfolio choice literature (see e.g. Cocco et al. (2005) or Haliassos and Michaelides (2001)). These papers argue that it should be households with high cash at hand that hold a higher share of their portfolio in the safe asset since these households have high financial relative to human wealth (the present discounted value of future labor income). Consequently, these households expect to finance their current and future consumption primarily with capital income, whereas low cash-at-hand people tend to rely mostly on their labor income. Thus it is relatively more important for the high cash at hand people not to be exposed to large financial asset return risk. In fact, since idiosyncratic labor income shocks and house depreciation shocks are uncorrelated in our model, housing is not a bad asset for hedging labor income risk (of course the bond is even better in this regard, but it has a lower expected return).

a substantial reduction in mortgage debt held by these households.

As figure 1 shows, the effect on the housing choice is much smaller though. These two observations also imply that the removal subsidy causes a shift in the balance of the household portfolio away from *bonds* and towards *home equity* as seen in figure 4. As a consequence of this general shift in households' portfolio composition, the share of bonds in the net worth portfolio of the *median* household declines substantially: whereas this household holds 60.7% of its net worth in bonds with the mortgage subsidy, this share drops to 5.8% without the subsidy.

5.2. Aggregate Effects of Removing the Subsidy

How the change in household behavior translates into the main macroeconomic aggregates is summarized in table 3. Aggregate mortgages taken out by households decline sharply, by 91%. Despite this, the overall impact on aggregate investment into housing is actually slightly positive: the stock of housing properties increases by 2.38%. Household labor income net of taxes increases by 0.97%, exactly the amount required to finance the interest rate subsidy in equilibrium.

The behavioral changes induced by a change in the subsidy in turn have significant general equilibrium price effects. Since the supply of housing increases, the equilibrium rental price of housing decreases, by slightly more than one percent. The equilibrium risk-free interest rate r_b declines by a substantial 48 basis points in response to the removal of the subsidy since the demand for loans to finance house purchases collapses. Note that the effective equilibrium interest rate on borrowing, holding leverage constant, actually *decreases* by 18 basis points in the absence of the subsidy, since the 30 basis point increase due to the removal of the subsidy is more than offset by the general equilibrium. This highlights that the key margin governing household asset and portfolio choice is not so much the absolute cost of borrowing or return on saving, but the *wedge between the borrowing and saving rate*. With the subsidy, and for a mortgage with zero default risk, the difference in interest rates on saving and borrowing was 10 basis points, equal to the per dollar cost r_w of originating and maintaining the mortgage. Without the subsidy, the effective interest rate on borrowing is 40 basis points higher than (and thus almost double) than that of saving. This massive reduction in the attractiveness of mortgage borrowing is also reflected in a decline in aggregate default rates which fall from 0.51% to 0.41% per year. Note that since foreclosure is costly in terms of resources (banks only recover a fraction γ of the value

of the home), the reduction in foreclosure rates due to the removal of the subsidy will be a key factor in the welfare evaluation of the change of the government's policy.

Given that the subsidy only benefits home owners one would expect that removing it has important consequences for the distribution of home ownership, wealth and welfare. Since the only asset in positive net supply is real estate and, as was already documented, the stock of houses increases by 2.71% due to the removal of the subsidy, so does total wealth in the economy. However, median net worth falls, about 1.4%. This rising gap between average and median wealth suggests that the distribution of wealth becomes more dispersed without the subsidy, which is confirmed by a mild increase in the Gini coefficient for (net) wealth from 0.471 to 0.478. Figure 5 which displays the stationary wealth distributions with and without policy suggests that this is mainly due to a larger fraction of households at the borrowing constraint and a slightly fatter right tail of the wealth distribution in the scenario without the subsidy. Thus if wealth inequality is a direct concern of policy makers the removal of the subsidy is counterproductive along this dimension, although the effects of the subsidy policy on wealth inequality is quantitatively small.

Another potential rationale for (indirectly) subsidizing mortgage interest rates on the part of the government is to increase home ownership rates in the economy. Table 3 shows that if this is indeed the ultimate goal of the government, it is successful, according to our model. The fraction of households that own some real estate, $\mu(g' > 0)$, is slightly higher with than without the interest rate subsidy. The fraction of households that own at least as much real estate as they use for their own housing service consumption, $\mu(g' > h)$ increases more substantially, from 40% to 44% with the subsidy. Even though in our model owning real estate is not directly linked to using that same real estate as owner occupied housing¹³, the fraction of households with $g' \geq h$ is perhaps the best proxy of home ownership rates in our model, and it is negatively affected by the removal of the government subsidy.

5.3. *Welfare and Distributional Implications of the Policy*

The welfare consequences of the reform are now discussed. Removing the mortgage interest rate subsidy increases aggregate steady state welfare, as measured by consumption equivalent

¹³In our model nothing links the housing stock g' a household owns to the housing services h she consumes, but it is convenient for the interpretation of our results to make that association.

variation (CEV), by a non-negligible 0.5%. That is, household consumption (of both nondurables and housing services) in the steady state with the subsidy has to be increased by this percentage in all states of the world and for all households, such that a household is indifferent *ex ante* (that is, prior to knowing what part of the distribution he will be born into) between being born into the steady state with or without the subsidy.¹⁴

Figure 6 sheds some light which households which characteristics (a, y) benefit from the subsidy. The figure plots the steady state consumption equivalent gain for households with different income realizations against cash at hand.¹⁵ This plot should be understood as a quantitative answer to the following hypothetical question: in which economy would someone with state (a, y) prefer to start her life, an economy with or without subsidy? Our results indicate that the welfare gains from the subsidy are monotonically increasing in wealth, with wealth-poor households preferring to start life in the economy without subsidy while households with high wealth benefit from the subsidy. Similarly, holding wealth fixed higher current income households view the mortgage interest rate subsidy more favorably than income-poor households.

The heterogeneity in the welfare assessment of the policy across households is due to the following factors. First, the subsidy keeps interest rates on the financial assets of wealthy households high (since the subsidy fuels a stronger mortgage demand), and second, it provides these households (which invest in bonds and leverage substantially in real estate) with a direct interest rate subsidy for this investment strategy. Poorer households, on the other hand, derive a larger share of their current resources from labor income which is subject to the income tax that finances the mortgage rate subsidy. Thus these households would prefer having the subsidy and the tax that comes with it removed. This is especially true if their wealth is so low that debt-financed investment into real estate becomes suboptimal for the household, and thus the subsidy does not apply to them.

We conclude (and view this as perhaps our most important normative finding) that masking

¹⁴Steady state welfare comparisons can be problematic since they ignore the welfare consequences of the transition path towards the new steady state (and thus the cost of additional accumulation of physical capital or the stock of housing). In our model without capital the only transitional cost stems from building up the modest extra 2.4% of the housing stock. One therefore might expect the welfare gains from removing the policy to be somewhat smaller, but still positive in the aggregate, once the transitional costs are fully accounted for.

¹⁵The same comments about ignoring the welfare effects along the transition apply, as before.

the aggregate moderate welfare gains from removing the policy is a substantial heterogeneity in the welfare assessment of this policy across the population. The disagreement between households is quantitatively sizable: the poorest member of society would pay in excess of 1% of lifetime consumption to get rid off the policy, whereas households with wealth twice the average would lose more than 1% from the same policy reform.

5.4. *The Optimal Size of the Mortgage Interest Rate Subsidy*

The previous discussion begs the question what size of a (potentially negative) subsidy is optimal, given the *utilitarian steady state social welfare function* employed above. The answer is not obvious in a model with incomplete markets and rich household heterogeneity, but is straightforward to derive computationally. A smaller (relative to the benchmark) but positive subsidy of 9 and 3/8 basis points maximizes social welfare. Table 4, column 3, displays the aggregate and distributional consequences of implementing the optimal subsidy. In order to understand why, in figure 6 consumption equivalent variation (CEV) for households with different characteristics is plotted and the optimal subsidy of 9 and 3/8 basis points, side by side with the CEV's from an elimination of the subsidy. Recall that the CEV's measure the welfare gains (or losses), relative to the benchmark case, a subsidy of 30 basis points. Notice that with the zero subsidy the CEV plots are steeper, relative to the $9\frac{3}{8}$ basis point subsidy. Low income, low CAH households benefit more from the complete removal of the subsidy, mainly because of the decrease in tax burden. However, examining the difference in household portfolio choices it becomes clear why a positive subsidy is optimal. In the baseline economy high CAH households take on large mortgages, subsidized by the government. However, as the value of the subsidy falls, these high CAH households no longer exhibit the "borrow-to-save" behavior. Essentially the lower subsidy disincentivizes wealthy households from trying to get "government financed" insurance. However, it still allows lower CAH households to engage in this strategy. The optimal subsidy imposes a small tax burden on the very low CAH, but still subsidizes "middle class" households for obtaining insurance against catastrophic house depreciation shocks. Using a utilitarian social welfare function to aggregate the welfare gains and losses then delivers the optimal subsidy rate. Of course it is important to note, in comparing the status quo, the optimal policy and the no-subsidy case, that neither policy *Pareto-dominates* another policy, and what we term "optimal"

is only socially optimal under our specific (but very commonly used in the literature) social welfare function.

6. Sensitivity Analysis

This subsection contains a discussion of two strong assumptions that have been made so far and to what extent they affect our substantive positive and normative conclusions. First, a production economy is introduced. Second, assets that allow for the diversification of house price risk are introduced.

6.1. Other Assets in Positive Supply: Introducing Capital

In the model discussed so far the only asset in *positive net supply* was risky housing. This assumption helped us to isolate the role of mortgages and foreclosure in hedging idiosyncratic house price risk. The analysis is extended to a production economy with physical capital as in Aiyagari (1994), but with risky real estate and housing services, as in the benchmark economy. Appendix Appendix C.1 discusses the details of the model and its calibration, and table 4, columns 4 and 5, summarize the results. In a nutshell, as the table shows, the introduction of physical capital leaves the results of the policy analysis qualitatively, and to a large extent quantitatively unchanged *if we re-calibrate the model* to be consistent with the same targets as was the model without capital.¹⁶ Note, though, that the risk free interest rate in the economy with capital is somewhat less sensitive to the removal of the subsidy due to the curvature in the production function.

The one quantitative exception are the welfare gains from the removal of the subsidy, which are *significantly larger* in the economy with capital, thus *reinforcing* the normative point we wish to make. The key difference to the economy without capital is a larger value of the risk aversion (prudence) parameter σ , which induces households to save more in order to absorb the

¹⁶Such re-calibration to match the same median leverage ratio and risk free interest rate requires increasing the risk aversion and prudence parameter σ from 3.9 to 7.5, and reducing the time discount factor β from 0.92 to 0.89. Under the *old calibration*, but in the model with capital, the appendix shows that since households save predominantly in riskless capital, the demand for riskless assets and mortgages completely collapses. It is our belief that this economy is not a useful laboratory to analysis the hypothetical policy reform since it results in the counterfactual absence of any meaningful mortgage market. And of course, if there are no mortgages traded in equilibrium, a policy that subsidizes these mortgages has no effect.

additional supply of assets (the physical capital stock), but also implies a larger curvature in the utility and thus value function of households. Thus a policy reform (such as the removal of the subsidy) that redistributes from rich (high income and cash at hand) to poor households constitutes larger aggregate welfare gains, under our utilitarian social welfare function.

6.2. Diversification of Idiosyncratic House Price Risk

In our model with idiosyncratic house price risk households have a strong incentive to pool that risk, something that the benchmark version of the model rules out, in the same way explicit insurance against the idiosyncratic income risk households face is ruled out, in line with the standard incomplete markets literature. Although it is empirically plausible to assume that *idiosyncratic* house price risk cannot be fully diversified through trading state-contingent claims that pay off contingent on individual house-specific price shocks, this importance assumption is now briefly explored in two ways. First, a version of the model with a housing mutual fund is analyzed. Second, sensitivity analysis is performed with respect to the variance of the house price shock, in effect assuming that a certain share of house price risk can be fully diversified.

First, consider an extended version of the model where a representative, competitively behaving mutual fund buys a portfolio of houses of positive measure, rents them out and sells the depreciated portfolio of houses tomorrow. Given that the mutual fund holds a positive measure of houses, the expected depreciation rate on its portfolio is risk-free and equal to $E(\delta)$. Households can purchase three assets, financial assets and mortgage-financed individual houses (exactly as in the benchmark model) as well as the mutual fund. Given that the mutual fund has a risk-free investment strategy, the return on the mutual fund has to equal that of risk free bonds in equilibrium. This in turn implies an equilibrium rental rate of $P_l = \frac{r_b + E(\delta)}{1 + r_b}$, equal to the user cost of housing (see appendix Appendix C.2). Note that households are still permitted to buy individual houses financed by mortgages, and might do so given the option-like mortgage cum foreclosure contracts available to them.

However, as table 4, columns 8 and 9 show, in this version essentially the entire housing stock (more than 99.9%) is held by the mutual fund, the mortgage market shuts down, and thus the removal of the interest rate subsidy has no effect on the equilibrium (since no mortgages are traded with and without the subsidy). Also note that households would be willing to pay 1.7%

of permanent consumption (as measured by the *CEV*) to be borne into the economy with the mutual fund, relative to the economy without it, signaling large welfare gains from completing markets with respect to idiosyncratic house price risk.

Second, a version of the model is computed in which the variance of the house price shocks was reduced to half its original size, $\hat{\sigma}_p^2 = 0.5\sigma_p^2 = 0.005$, assuming that the other half is perfectly diversifiable through financial assets we do not spell out explicitly. As table 4, columns 6 and 7, display, all qualitative findings remain intact under this specification, but the quantitative welfare effects shrink significantly. This is due to substantially the same reason as for the introduction of the housing mutual fund. With lower house price risk households, especially those with large housing positions and cash at hand, now tend to own real estate outright, rather than finance it with mortgages (since there is less housing risk to hedge, the foreclosure option is less attractive). As house price risk is reduced further, the economy converges to the housing mutual fund economy discussed above.

Thus, and in contrast to introducing capital and production, the assumed presence of significant undiversifiable house price risk is important, quantitatively but even qualitatively, for our positive and welfare results. In light of the substantial welfare gains that could be achieved by such diversification an investigation of the causes for why markets diversifying this risk do not exist or are imperfect seems an important area of work; see Shiller (2008) for a discussion.

7. Conclusions

The future of the GSE's and the role of the government in mortgage market remains a key question facing policy makers. We have constructed an equilibrium model of mortgage debt and foreclosures and use it to evaluate the aggregate and distributional consequences of a stylized bailout guarantee for GSE's. This guarantee leads to excessive mortgage origination, higher leverage and larger foreclosure rates in equilibrium, compared to a world without such a policy. The steady state aggregate welfare gains from abolishing the guarantee are significantly positive, and poor households would strongly benefit from such a reform.

An extension to a nonstationary model with endogenous house prices would lend itself to a quantitative evaluation of how much of the run-up in household mortgage debt and house prices in the early 2000's can be attributed to the government's involvement with the GSE's. It could

also be used to study the distributional consequences of the collapse in house prices in a world with high household leverage that is at least partially induced by this involvement. Such an analysis is deferred to future work.

8. Acknowledgments

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Table 1: Endogenously Calibrated Parameters

Parameter	Interpretation	Value	Target
σ	Risk Aversion	3.912	Median Leverage
β	Time Discount Factor	0.919	Risk-free Rate
k	Pareto shape parameter	0.7302	Foreclosure Rate
σ_δ	Pareto scale parameter	0.0078	House price volatility
$\underline{\delta}$	Pareto threshold parameter	-0.0077	Average price depreciation

Note: This table presents the parameters calibrated endogenously in the model via a moment matching procedure to match the relevant moments in US data.

Table 2: Exogenously Calibrated Parameters

Parameter	Interpretation	Value	Target
A_h	Technology Const. in Housing Constr.	1.0	none (normalized)
ρ	Income Persistence	0.98	Storesletten et al (2004)
σ_e	Income Variance	0.3	Storesletten et al (2004)
γ	Foreclosure Technology	0.78	Pennington and Cross (2004)
α	Share Parameter on Nondur. Cons.	0.8590	Exp. Share in BEA
ϕ	Implicit Interest Rate Subsidy	30 BP	Lucas and McDonald (2010)
r_w	Mortgage administration fee	10 BP	

Note: This table presents the parameters calibrated exogenously (taken from the relevant literature).

Table 3: Quantitative Results: Consequences of Removing the Subsidy

Variable	Subsidy	No Subsidy	Difference
% Sub	30bp	0bp	-100%
Sub/\bar{y}	0.97%	0%	-100%
P_l	0.0281	0.0278	-1.1%
r_b	1.0%	0.518%	-0.482%
H	5.327	5.454	2.38%
M	3.231	0.319	-91.3%
Default share	0.51%	0.41%	-19.6%
Mean Net Worth	5.327	5.454	2.38%
Median Bond Portfolio Share	60.7%	5.8%	-90.4%
Wealth Gini	0.471	0.478	1.49%
$\mu(g' > 0)$	96.74%	96.66	-0.08%
$\mu(g' > h)$	43.51%	39.77%	-8.60%
CEV	-0.6297	-0.6206	0.5%*

*Computed as consumption equivalent variation, that is

$$(EV_{no\ subs}/EV_{subs})^{1/(1-\sigma)}$$

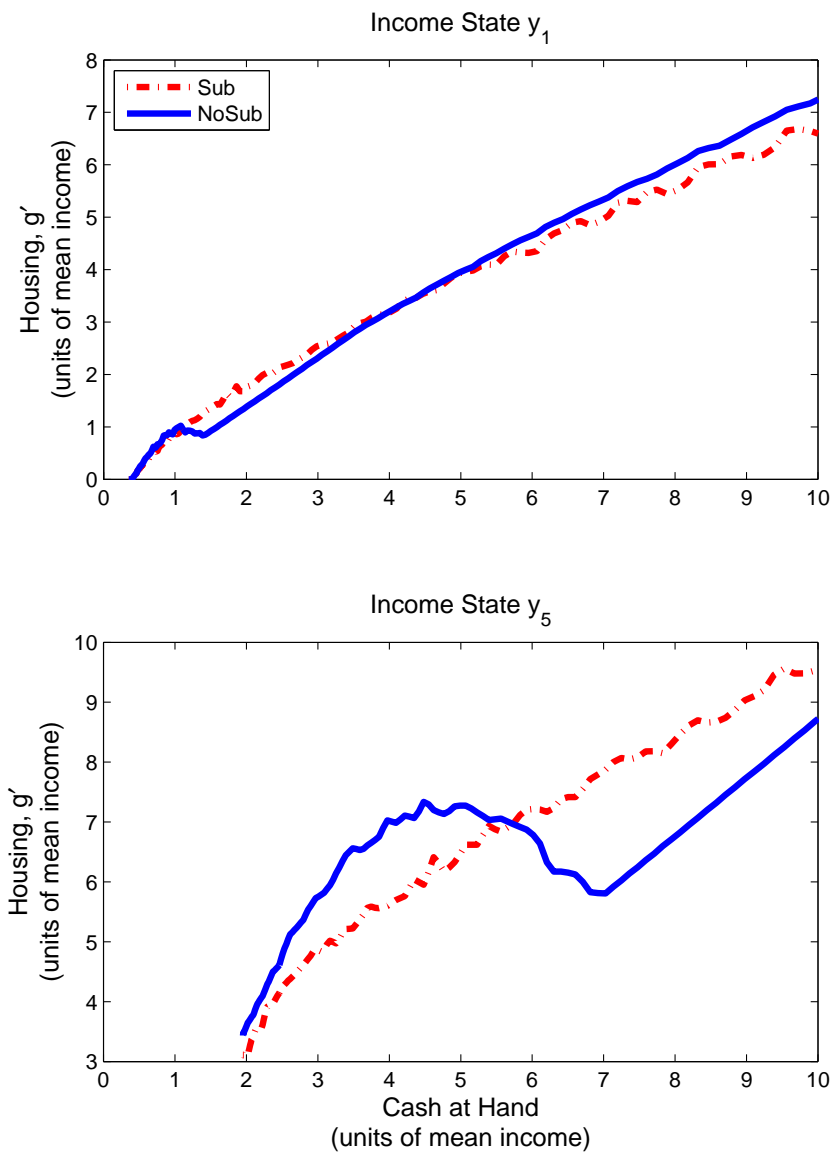
Note: This table presents relevant aggregate statistics from the baseline economy with the mortgage subsidy and compares it to the economy with the subsidy removed. The welfare comparison is presented in consumption equivalent variation.

Table 4: Quantitative Results: Optimal Subsidy and Sensitivity Analysis

Variable	Baseline			With Capital		Var/2		<i>H</i> Mutual Fund	
% Sub	30bp	0bp	9.375bp	30bp	0bp	30bp	0bp	30bp	0bp
Median Leverage	61.5%	0.0%	0.0%	61.5%	0.0%	0.7%	0.0%	0.0%	0.0%
r_b	1.0%	0.518%	0.597%	1.0%	0.676%	1.043%	0.783%	1.456%	1.456%
Sub/\bar{y}	0.97%	0%	0.04%	0.96%	0%	0.2%	0%	0%	0%
P_l	0.0281	0.0278	0.0279	0.0296	0.0294	0.0281	0.0279	0.029	0.029
GDP	1.15	1.151	1.152	1.585	1.587	1.152	1.152	1.153	1.153
H/GDP	4.633	4.737	4.720	3.332	3.404	4.683	4.723	4.565	4.565
M/GDP	2.81	0.269	0.382	2.018	0.003	0.604	0.080	0	0
K/GDP	0	0	0	2.702	2.83	0	0	0	0
w	1	1	1	1	1.014	1	1	1	1
Default share	0.51%	0.41%	0.49%	0.49%	0.3%	0.28%	0.14%	0%	0%
Net Worth/GDP	4.633	4.737	5.436	6.034	6.234	5.393	5.44	5.262	5.262
Median b Share	60.7%	7.2%	8.1%	72.2%	47.8%	12.9%	1.7%	0%	0%
$\mu(g' > 0)$	96.79%	96.66%	96.68%	100%	100%	96.78%	96.66%	14.45%	14.45%
$\mu(g' > h)$	43.00%	39.77%	40.71%	45.7%	45.9%	41.20%	38.78%	0%	0%
CEV	0	0.81%	0.85%	0%	2.17%	0%	0.13%	0%	0%

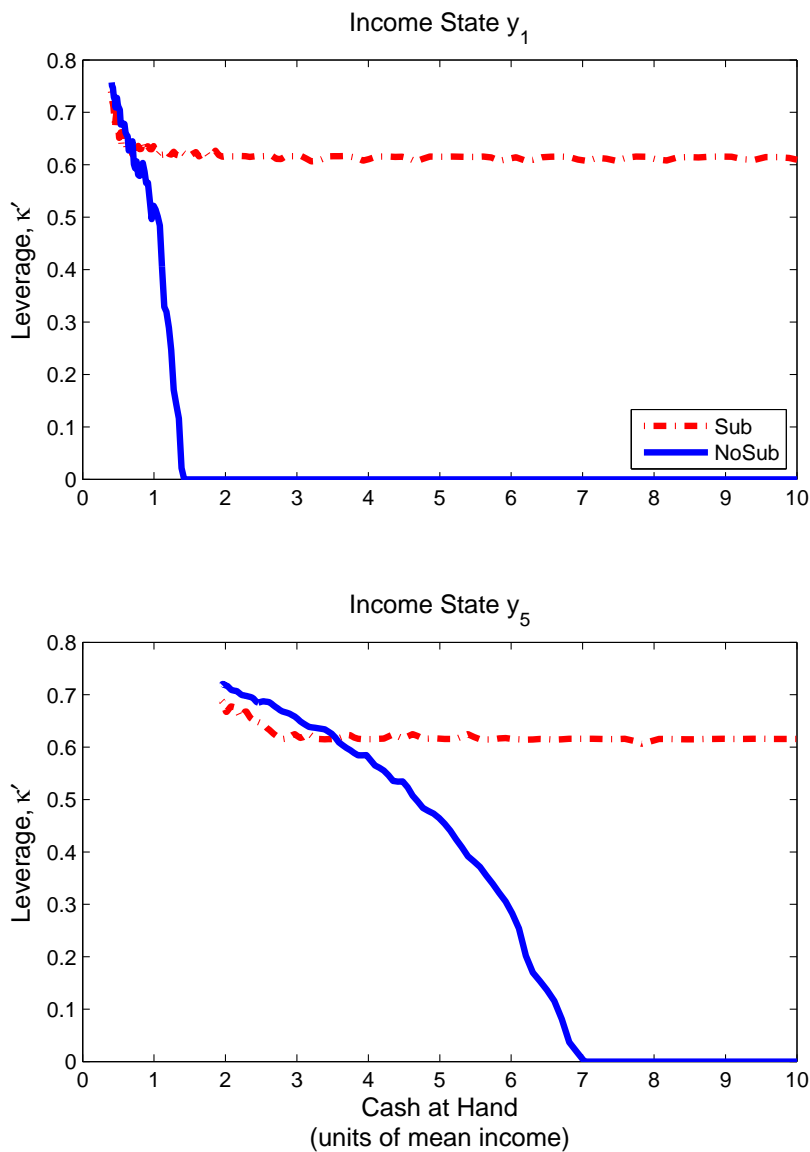
Note: This table presents relevant aggregate statistics from selected sensitivity analyses.

Figure 1: Housing Policy Function With and Without Subsidy



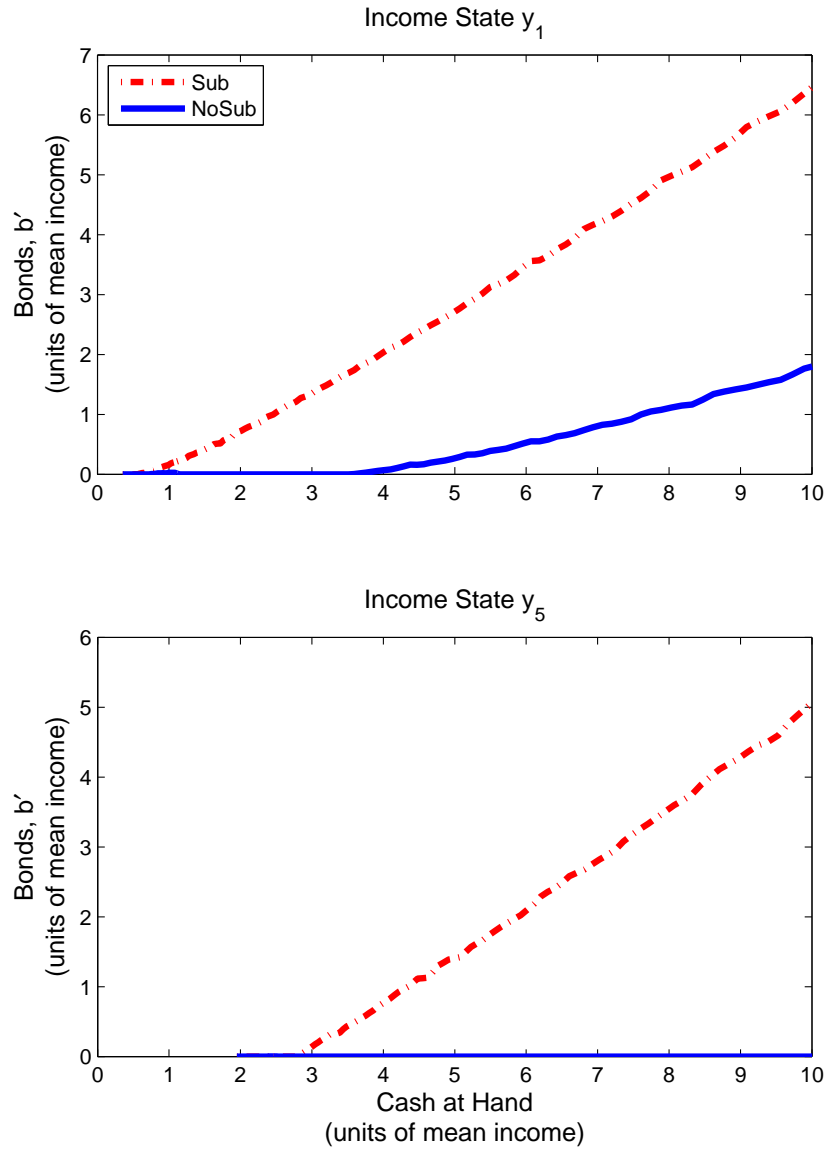
Note: This figure compares the optimal housing choice for a household in the baseline economy and the economy without the subsidy for different levels of cash at hand for the highest and lowest persistent income states.

Figure 2: Leverage Policy Function With and Without Subsidy



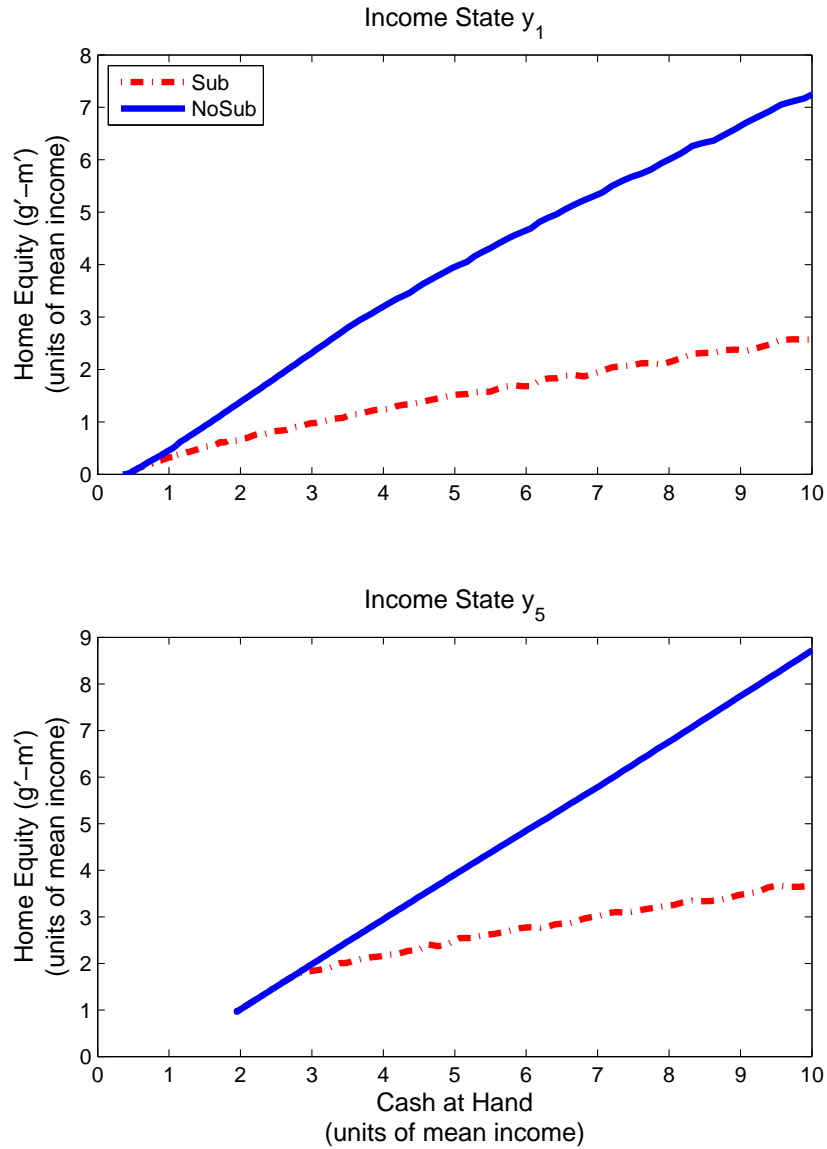
Note: This figure compares the optimal leverage choice for a household in the baseline economy and the economy without the subsidy for different levels of cash at hand for the highest and lowest persistent income states.

Figure 3: Bonds Policy Function With and Without Subsidy



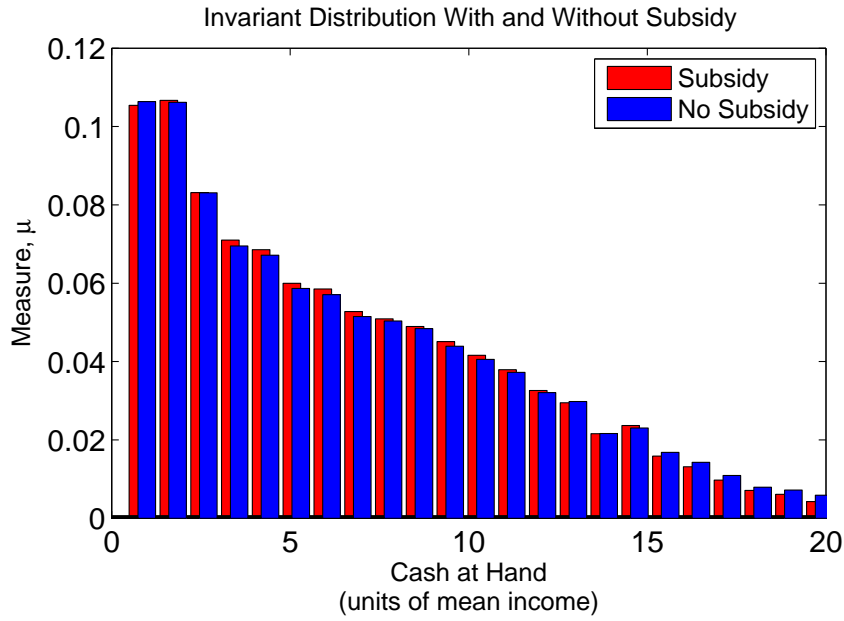
Note: This figure compares the optimal bond choice for a household in the baseline economy and the economy without the subsidy for different levels of cash at hand for the highest and lowest persistent income states.

Figure 4: Home Equity Policy Function With and Without Subsidy



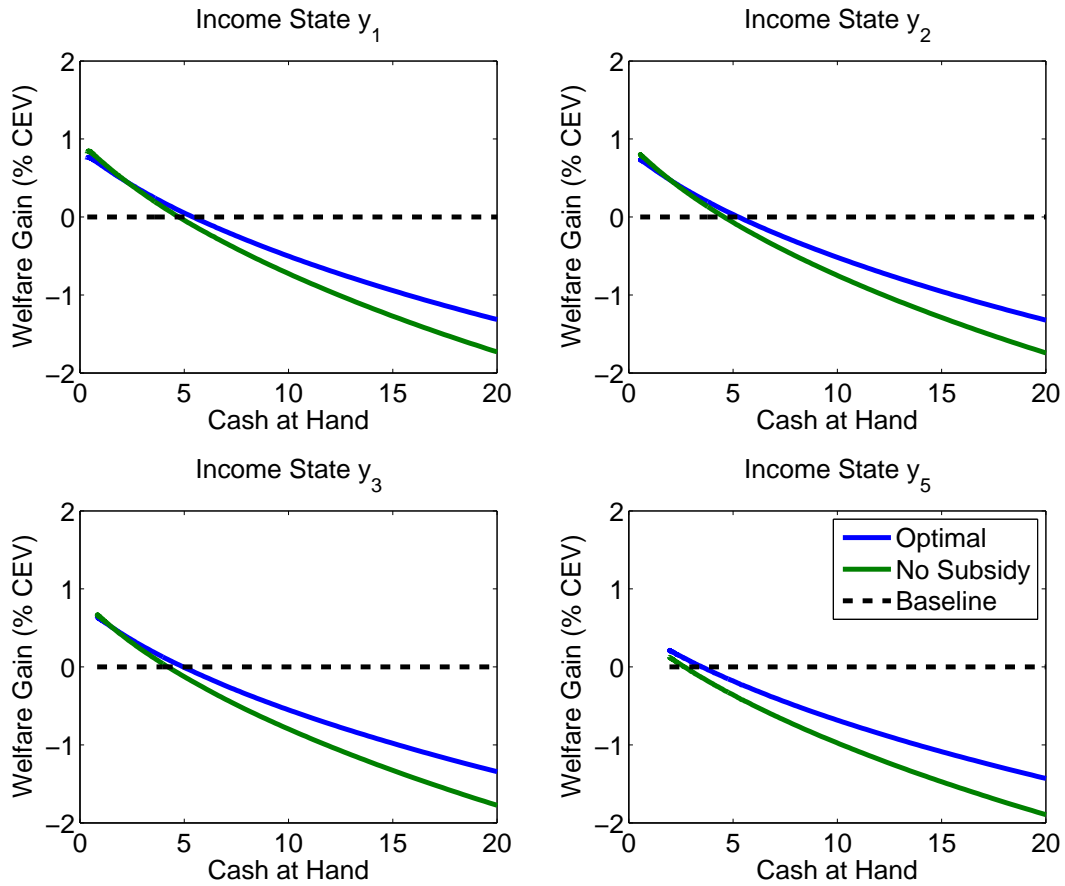
Note: This figure compares the home equity held by a household in the baseline economy and the economy without the subsidy for different levels of cash at hand for the highest and lowest persistent income states.

Figure 5: Distribution over Cash at Hand with and without Subsidy



Note: This figure compares the steady state distributions over cash at hand in the baseline economy and the economy without the subsidy.

Figure 6: Welfare Comparison



Note: This figure compares welfare in consumption equivalent variation terms between households in the baseline economy, the economy without the subsidy and the economy with optimal subsidy, for different levels of cash at hand for the four of the five persistent income states.

Appendix A. Theoretical Appendix - NOT FOR PUBLICATION

Appendix A.1. Characterization of Equilibrium Mortgage Interest Rates

In this section we will construct a proof of Proposition 1 in the main text.

Proof. In equilibrium equation (6) becomes (by dividing both sides by mortgage size $m' > 0$):

$$\begin{aligned} P_m(g', m') &= \frac{1}{m'} \left[\left(\frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \right) \left\{ m' F(\delta^*(m', g')) + \gamma g' \int_{\delta^*(m', g')}^{\infty} (1-\delta') dF(\delta') \right\} \right] \\ &= \left(\frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \right) \left\{ F(\delta^*(\kappa')) + \frac{\gamma}{\kappa'} \int_{\delta^*(\kappa')}^{\infty} (1-\delta') dF(\delta') \right\} \\ &= P_m(\kappa') \end{aligned}$$

which proves the mortgages are priced exclusively on leverage. The second point follows from the definition of the CDF of δ' and the optimal default choice $\delta^*(\kappa')$. To prove the final point, note that for all $\delta' > \delta^*(\kappa')$ we have $\frac{\gamma}{\kappa'}(1-\delta') < 1$ and thus the term in $\{\}$ -brackets is strictly less than 1. ■

Appendix A.2. Endogenous Upper Bound on Leverage

In this section we will construct a proof of Proposition 2 in the main text.

Proof. In order to prove the statement, we will first show that there exists a leverage κ^* that maximizes the contemporaneous resources available to the household. Then we will argue that in equilibrium households will only chose values of leverage less than κ^* and that the value κ^* is strictly less than the leverage which leads to certain default.

First, consider the period budget constraint:

$$c + bP_b + hP_l + g - mP_m(g, m) = a + gP_l \quad (\text{A.1})$$

This equation can be rewritten as:

$$g[1 - P_l - \kappa P_m(g, m)] = a - c - hP_l - bP_b \quad (\text{A.2})$$

For a fixed amount of housing $g > 0$, we can calculate the value of mortgages m that will generate the maximal amount of resources available today:

$$m^* \in \arg \min_{0 \leq m \leq g(1-\underline{\delta})} [g[1 - P_l - \kappa P_m(g, m)]] \quad (\text{A.3})$$

$$\Leftrightarrow m^* \in \arg \min_{0 \leq m \leq g(1-\underline{\delta})} -mP_m(g, m) \quad (\text{A.4})$$

Note that $g(1-\underline{\delta})$, the upper bound on m , corresponds to a leverage that will result in default for sure in the second period. Borrowing above this level does not increase resources available today since an extra dollar borrowed beyond this limit will be defaulted on for sure, and thus no additional resources will be advanced today by the financial intermediary against the promise to pay nothing back tomorrow. Furthermore, this upper bound compactifies the choice space and guarantees that the pricing function is differentiable on the interior of that space. The first order

conditions for this program is:

$$-P_m(\kappa) - m \frac{\partial P_m(\kappa)}{\partial \kappa} \frac{\partial \kappa}{\partial m} + \lambda = 0 \quad (\text{A.5})$$

where $\lambda \geq 0$ is the Lagrange multiplier on the upper constraint (the lower bound trivially is not binding as long as $\mathbb{E}[\delta] < 1$ and thus $P_m > 0$). The second order condition (which is well-defined due to the differentiability of $f(\delta)$) is:

$$-\frac{1}{g} \left[2 \frac{\partial P_m(\kappa)}{\partial \kappa} + \kappa \frac{\partial^2 P_m(\kappa)}{\partial \kappa^2} \right] \quad (\text{A.6})$$

In order to simplify these conditions, consider the mortgage pricing function $P_m(\frac{m}{g})$ given by:

$$P_m(\kappa) = \frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \left\{ F(1-\kappa) + \frac{\gamma}{\kappa} \int_{1-\kappa}^{\bar{\delta}} (1-\delta) dF(\delta) \right\} \quad (\text{A.7})$$

As long as δ has a continuous pdf (guaranteed because $F \in \mathcal{C}^2$), we can apply Leibnitz's rule to obtain:

$$\begin{aligned} \frac{\partial P_m(\kappa)}{\partial \kappa} &= \frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \times \\ &\quad \left\{ -f(1-\kappa) - \frac{\gamma}{\kappa^2} \int_{1-\kappa}^{\bar{\delta}} (1-\delta) dF(\delta) + \frac{\gamma}{\kappa} (-1)(-1) (1-(1-\kappa)) f(1-\kappa) \right\} \\ &= \frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \left\{ (\gamma-1)f(1-\kappa) - \frac{\gamma}{\kappa^2} \int_{1-\kappa}^{\bar{\delta}} (1-\delta) dF(\delta) \right\} \end{aligned}$$

We can simplify the first order conditions by substituting in our prior calculations to obtain:

$$\begin{aligned} 0 &= P_m(\kappa^*) + \kappa^* \frac{1}{(1+r_b)(1+r_w+\theta-\phi)} \left\{ (\gamma-1)f(1-\kappa^*) - \frac{\gamma}{\kappa^{*2}} \int_{1-\kappa^*}^{\bar{\delta}} (1-\delta) dF(\delta) \right\} - \lambda \\ &= F(1-\kappa^*) + \frac{\gamma}{\kappa^*} \int_{1-\kappa^*}^{\bar{\delta}} (1-\delta) dF(\delta) + (\gamma-1)\kappa^* f(1-\kappa^*) - \frac{\gamma}{\kappa^*} \int_{1-\kappa^*}^{\bar{\delta}} (1-\delta) dF(\delta) - \lambda \\ &= F(1-\kappa^*) + (\gamma-1)\kappa^* f(1-\kappa^*) - \lambda \end{aligned}$$

We want to show that κ^* is an interior choice and thus that $\lambda = 0$. Consider otherwise, that is $\kappa^* = 1 - \underline{\delta}$ and $\lambda \geq 0$:

$$F(1 - \kappa^*) + (\gamma - 1)\kappa^* f(1 - \kappa^*) - \lambda = \underbrace{F(\underline{\delta})}_{=0} + \underbrace{(\gamma - 1)(1 - \underline{\delta})f(\underline{\delta})}_{\leq 0} - \underbrace{\lambda}_{\geq 0}$$

Since $f(\underline{\delta}) > 0$ (from the definition of the support of F), if foreclosure is inefficient $\gamma < 1$, then the above equation will be strictly less than 0, implying that $\kappa^* = 1 - \underline{\delta}$ cannot satisfy the first order conditions, thus guaranteeing an interior optimal choice. Note that when foreclosure is efficient ($\gamma = 1$) choosing $\kappa^* = 1 - \underline{\delta}$ maximizes the resources received today (by the same argument in the text, since this is the point at which the household defaults for sure). The multiplier λ is still zero in this case, since $\kappa^* = 1 - \underline{\delta}$ is the global maximizer of the mortgage receipts.

Thus, we can characterize interior value of leverage which satisfies the first order conditions by the implicit equation:

$$\kappa^* = \frac{F(1 - \kappa^*)}{(1 - \gamma)f(1 - \kappa^*)} \quad (\text{A.8})$$

A solution to this implicit equation exists and is unique on $(0, \bar{\kappa})$. To see this rewrite the implicit equation as:

$$\frac{1}{1 - x} = (1 - \gamma) \frac{f(x)}{F(x)} \quad (\text{A.9})$$

for $x \in (\underline{\delta}, 1)$. The right hand side is strictly increasing in x , and takes values from $[1/(1 - \underline{\delta}), \infty)$. The left hand side is decreasing and also unbounded above (both coming from log-concavity of F). Thus, from a standard fixed-point theorem in \mathbb{R} , there exists an x in that interval which satisfies the equation and it is unique.

Now that we have found an interior solution to the first order conditions, we must check the second order condition, which simplifies to:

$$-\frac{1}{g(1 + r_b)(1 + r_w + \theta - \phi)} \{(\gamma - 2)f(1 - \kappa) + (1 - \gamma)\kappa f'(1 - \kappa)\} \quad (\text{A.10})$$

If the second order conditions is strictly positive at κ^* , the first order condition will be necessary and sufficient for a minimum, and thus will characterize the leverage which yields the maximal amount of contemporaneous resources. Log-concavity of $F(\delta)$ is sufficient for the SOC to be strictly positive. To see this, first note that log-concavity implies $f'^2 < 0$. Using this fact we can show:

$$\begin{aligned} f' &< \frac{f}{F} f \\ \Rightarrow f'(1 - \kappa^*) &< \frac{f(1 - \kappa^*)}{F(1 - \kappa^*)} f(1 - \kappa^*) \\ \Rightarrow (1 - \gamma)\kappa^* f'(1 - \kappa^*) &< f(1 - \kappa^*) \end{aligned}$$

where the last inequality comes from equation (A.8). Now, this last inequality, along with the fact that $\gamma < 1$ and $f \geq 0$ implies that the second order condition at κ^* will be strictly positive. Thus, we have shown that our interior κ^* is indeed a maximum.

Now that we have shown the existence of κ^* to prove the first statement, suppose by way of contradiction that a household has chosen an affordable and conjectured optimal allocation $\{c, h, b, g, m\}$ such that $\kappa > \kappa^*$. Consider another allocation $\{c', h', b', g', m'\}$, where $h' = h$, $b' = b$, $g' = g$ and m' is such that $\frac{m'}{g'} = \kappa^*$ and $c' = c + m'P_m(\kappa^*) - mP_m(\kappa)$. Note that this new allocation is affordable, and that $m'P_m(\kappa^*) > mP_m(\kappa)$ implies that $c' > c$ (from above). Furthermore cash-at-hand in the next period will be weakly greater in all possible states under the primed allocation than the original (since $b = b'$, $g = g'$ and $m' < m$, $b + \max((1 - \delta)g - m, 0) \leq b' + \max((1 - \delta)g' - m', 0)$). Thus, from the strict monotonicity of the utility function, the primed allocation yields strictly higher period utility and weakly higher continuation value, and thus is strictly preferred to the original hypothesized optimum. ■

Appendix A.3. Existence, Uniqueness and Characterization of the Value Function

Appendix A.3.1. Definitions

First we need some definitions. Let the $M \subset \mathbb{R}_+$ mortgage choice set, $B \subset \mathbb{R}_+$ be the bond choice set, $G \subset \mathbb{R}_+$ be the housing choice set, $C \subset \mathbb{R}_+$ the consumption expenditure choice set. The state variable, cash-at-hand lies in $a \in A \subset \mathbb{R}_+$. Income resides in $y \in Y$, where Y is a finite set. We define the budget correspondence $\Gamma : A \rightarrow C \times B \times G \times M$ as:

$$\Gamma(a) = \{(c, b, g, m) \in C \times B \times G \times M : c + bP_b + g[1 - P_l] - mP_m(g, m) \leq a, \quad m \leq g\kappa^*\} \quad (\text{A.11})$$

where κ^* is the endogenous maximal leverage characterized above.

We can write down our Bellman equation as:¹⁷

$$v(a, y) = \max_{x \in \Gamma(a)} u(c; P_l) + \beta \sum_{y' \in Y} \pi(y'|y) \int v(\phi(b', g', m', \delta', y'), y') dF(\delta') \quad (\text{A.12})$$

where $x = (c, b', g', m')$ and $\phi(b', g', m', \delta', y') = b' + \max((1 - \delta')g' - m', 0) + (1 - \tau)y'$ is cash at hand tomorrow. We can now define our operator, $T : A \times Y \rightarrow A \times Y$ as:

$$Tv(a, y) = \max_{x \in \Gamma(a)} u(c; P_l) + \beta \sum_{y' \in Y} \pi(y'|y) \int v(\phi(b', g', m', \delta', y'), y') dF(\delta') \quad (\text{A.13})$$

With our definitions in hand, we proceed to discuss the properties of the budget correspondence Γ and the operator T .

Appendix A.3.2. Properties of Γ and T

We begin by establishing properties of Γ . First, it is non-empty (since $0 \in \Gamma(a)$) and monotone. Given prices that satisfy¹⁸ $1 - P_l - \kappa P_m(\kappa) > 0 \forall \kappa \in [0, \kappa^*]$, we can show that Γ is also continuous and compact valued. To prove this we want to apply Theorems 3.4 and 3.5 from

¹⁷We will show in the next section that Γ is compact valued, so that the maximum of the program will be obtained, justifying our use of the max-operator as opposed to the sup-operator.

¹⁸This is a no-arbitrage condition on the relation between the interest rate and rental rate, and thus will be true in any equilibrium. If this condition is violated the household can buy one unit of housing with leverage κ at a positive cash flow today (and due to the default option the cash flow tomorrow from this transaction is nonnegative, and strictly positive as long as the household does not default with probability 1).

Stokey and Lucas (1989). We will show that the conditions for these two theorems are met by proving two lemmas.

Lemma 1. *The graph of Γ , $G = \{(a, c, b, g, m) \in A \times C \times B \times G \times M : (c, b, g, m) \in \Gamma(a)\}$ is closed, and for any bounded set $\hat{A} \subset A$, $\Gamma(\hat{A})$ is bounded.*

Proof. To show that the graph is closed, take a sequence $\{(a_n, c_n, b_n, g_n, m_n)\}_{n=0}^{\infty}$ such that $(c_n, b_n, g_n, m_n) \in \Gamma(a_n)$ for all n that converges to (a, c, b, g, m) . Suppose that $(c, b, g, m) \notin \Gamma(a^*)$. This implies either $c + bP_b + g[1 - P_l] - mP_m(g, m) > a$ or $m > g\kappa^*$. However, since all functions involved are continuous, there exists some $N > 0$ such that one of the inequalities is also violated for $(a_N, c_N, b_N, g_N, m_N)$, which implies that $(c_N, b_N, g_N, m_N) \notin \Gamma(a_N)$, a contradiction.

To show that $\Gamma(\hat{A})$ is bounded, let $\hat{a} = \sup \hat{A}$. Since Γ is monotone, if $\Gamma(\hat{a})$ is bounded, $\Gamma(\hat{A})$ will be bounded. Now, observe that $0 \leq c \leq a$, $0 \leq b \leq a/P_b$. To prove that g is bounded above is equivalent to showing $\forall a \in \mathbb{R}_+ \exists \bar{g} \geq 0$ s.t. $\forall g > \bar{g}$, $(c, b, g, m) \notin \Gamma(a)$. We will construct such a candidate \bar{g} . We propose $\bar{g}(a)$ such that $\bar{g}(a) = a/[1 - P_l - \kappa^*P_m(\kappa^*)]$, where κ^* is the endogenous leverage limit. Now, suppose by way of contradiction that there was an allocation $(c, b, g, m) \in \Gamma(a)$ with $g > \bar{g}(a)$. This implies that:

$$c + bP_b + g[1 - P_l - \kappa P_m(g, m)] \leq a \quad (\text{A.14})$$

$$\Rightarrow c + bP_b + g[1 - P_l - \kappa^*P_m(g, m^*)] \leq a \quad (\text{A.15})$$

$$\Rightarrow g \leq \bar{g} - \frac{c + bP_b}{1 - P_l - \kappa^*P_m(g, m^*)} \quad (\text{A.16})$$

$$\Rightarrow g \leq \bar{g}, \quad (\text{A.17})$$

a contradiction. Thus, for any a , $\Gamma(a)$ is bounded, thus $\Gamma(\hat{a})$ is bounded and therefore $\Gamma(\hat{A})$ is bounded. ■

Lemma 2. *The graph of Γ , G , is convex, and for any bounded set $\hat{A} \subset A$, there exists a bounded set $\hat{X} \subset C \times B \times G \times M$ such that $\Gamma(a) \cap \hat{X} \neq \emptyset$ for all $a \in \hat{A}$.*

Proof. The second part of the lemma is trivially satisfied by letting $\hat{X} = \{(0, 0, 0, 0)\}$.

In order to show convexity we need to establish that $-mP_m(m, g)$ is convex, which will make $c + bP_b + g[1 - P_l] - mP_m(g, m)$ convex in c, b, g, m , guaranteeing that the inequality constraint will hold for convex combinations. Consider the Hessian for $-mP_m(m, g)$:

$$\mathcal{D}^2(-mP_m(m, g)) = \begin{bmatrix} -\frac{1}{g}(2P'_m + \kappa P''_m) & \frac{\kappa}{g}(2P'_m + \kappa P''_m) \\ \frac{\kappa}{g}(2P'_m + \kappa P''_m) & -\frac{\kappa^2}{g}(2P'_m + \kappa P''_m) \end{bmatrix} \quad (\text{A.18})$$

Since the Hessian is singular, in order to show that the matrix is positive semi-definite, and hence that $-mP_m(m, g)$ is convex, we only need to show that the two diagonal elements are positive. Thus, we need to show that $2P'_m + \kappa P''_m \leq 0$. From before we know that:

$$2P'_m + \kappa P''_m = \frac{1}{(1 + r_b)(1 + r_w + \theta - \phi)} \{(\gamma - 2)f(1 - \kappa) + (1 - \gamma)\kappa f'(1 - \kappa)\} \quad (\text{A.19})$$

Focusing on:

$$(\gamma - 2)f(1 - \kappa) + (1 - \gamma)\kappa f'(1 - \kappa) \quad (\text{A.20})$$

We again employ the maintained assumption of log-concavity of $F(\delta)$ to show:

$$\frac{f(1 - \kappa^*)}{F(1 - \kappa^*)} \geq \frac{f(1 - \kappa)}{F(1 - \kappa)} \quad \forall \kappa \in [0, \kappa^*] \quad (\text{A.21})$$

$$\Rightarrow f'(1 - \kappa) < \frac{f(1 - \kappa^*)}{F(1 - \kappa^*)} f(1 - \kappa) \quad \forall \kappa \in [0, \kappa^*] \quad (\text{A.22})$$

$$\Rightarrow f'(1 - \kappa) < \frac{1}{(1 - \gamma)\kappa^*} f(1 - \kappa) \quad \forall \kappa \in [0, \kappa^*] \quad (\text{A.23})$$

$$\Rightarrow f'(1 - \kappa) < \frac{1}{(1 - \gamma)\kappa} f(1 - \kappa) \quad \forall \kappa \in [0, \kappa^*] \quad \text{since } f > 0 \quad (\text{A.24})$$

$$(1 - \gamma)\kappa f'(1 - \kappa) < f(1 - \kappa) \quad \forall \kappa \in [0, \kappa^*] \quad (\text{A.25})$$

$$(\text{A.26})$$

which guarantees that (A.19) is negative. This implies that the diagonal elements are non-negative, combined with the fact that the determinant is zero, yields that the Hessian is positive semi-definite on $\kappa \in [0, \kappa^*]$ (this follows from Debreu 1952). Now, recall our definition of $\bar{g}(a) = a/[1 - P_l - \kappa^* P_m(\kappa^*)]$ which is the maximum size house that could be purchased by a household with cash-at-hand a . Take any $g \in [0, \bar{g}(a)]$, then for any $m \in [0, g\kappa^*]$, the function $-mP_m(m, g)$ will be convex, since $\kappa \in [0, \kappa^*]$. Thus, we have established the convexity of G . ■

These two lemmas, combined with theorems 3.4 and 3.5 in Stokey and Lucas (1989), guarantee that Γ is compact valued, u.h.c. and l.h.c. Since we are analyzing the simplified recursive household problem (with the solution of the static expenditure allocation problem substituted in), we need to show some properties of the indirect utility function over consumption expenditures, $u(c, P_l)$.

Lemma 3. $u(c; P_l)$ is continuous, strictly concave and strictly increasing in c , for all $P_l > 0$.

Proof. Take $c_1, c_2 > 0$ and $c_\theta = \theta c_1 + (1 - \theta)c_2$ for $\theta \in (0, 1)$. Let $u(c_i; P_l) \equiv U(\tilde{c}_i, h_i)$ where \tilde{c}_i and h_i are the maximizers of the static problem. From the strict concavity of U , we know that

$$\begin{aligned} \theta U(\tilde{c}_1, h_1) + (1 - \theta)U(\tilde{c}_2, h_2) &< U(\theta\tilde{c}_1 + (1 - \theta)\tilde{c}_2, \theta h_1 + (1 - \theta)h_2) \\ &\leq U(\tilde{c}_\theta, h_\theta) \end{aligned}$$

where the first inequality comes from the strict concavity of U . The second inequality follows from the fact that $\theta\tilde{c}_1 + (1 - \theta)\tilde{c}_2 + P_l(\theta h_1 + (1 - \theta)h_2) = \theta c_1 + (1 - \theta)c_2 = c_\theta$, thus it is a feasible choice for the maximization for $u(c_\theta; P_l)$, and by definition of a maximum. Continuity and strict monotonicity follow from the properties of U . ■

Since we have assumed that the utility function is unbounded from below, we use a similar argument to Chatterjee et al. (2007) to establish the existence and uniqueness of the value function. Let \mathcal{V} be the set of all continuous functions $v : A \times Y \rightarrow \mathbb{R}$, such that:

$$v(a, y) \in \left[\frac{u((1 - \tau)y_{\min}; P_l)}{1 - \beta}, \frac{\bar{u}}{1 - \beta} \right] \quad (\text{A.27})$$

$$u((1 - \tau)y_{\min}; P_l) + \beta \sum_{y' \in Y} \pi(y'|y)v((1 - \tau)y', y') > u(0; P_l) + \frac{\beta \bar{u}}{1 - \beta} \quad (\text{A.28})$$

Lemma 4. \mathcal{V} is non-empty and $(\mathcal{V}, \|\cdot\|)$ is a complete metric space, where $\|\cdot\|$ is the sup-norm.

Proof. Take any constant function v_0 satisfying (A.27). v_0 is continuous and satisfies (A.28) by the assumption that u is unbounded below. In order to show that $(\mathcal{V}, \|\cdot\|)$ is complete, first note that $(\mathcal{C}, \|\cdot\|)$ is complete, where \mathcal{C} is the set of continuous functions from $A \times Y \rightarrow \mathbb{R}$. First note that $\mathcal{V} \subset \mathcal{C}$, and thus it is sufficient to show¹⁹ that \mathcal{V} is a closed subset of \mathcal{C} . So take any sequence of functions $\{v_n\}_{n=0}^{\infty} \in \mathcal{V}$ such that $v_n \rightarrow v^*$. We need to show that $v^* \in \mathcal{V}$. Suppose not, i.e. $v^* \notin \mathcal{V}$. Then either v^* is not continuous, or (A.27) or (A.28) was violated. Continuity is preserved because v_n converges to v^* uniformly given the sup-norm. If (A.27) is violated, then there must exist some N such that it is also violated for v_N , contradicting that $v_N \in \mathcal{V}$. Finally, given (A.27), (A.28) is satisfied by the assumption of unboundedness of u from below. ■

Now we need to show that for all $v \in \mathcal{V}$, $Tv \in \mathcal{V}$. Thus, we need to establish that T preserves continuity, and that (A.27) and (A.28) hold. To show that T preserves continuity, first note that $a(b', g', m', \delta', y')$ is a continuous function. Thus $v(a(\cdot), y)$ is also continuous, because the composition of continuous functions is continuous. Further, continuity is preserved by integration, and thus $\sum_{y' \in Y} \pi(y'|y) \int v(a(b', g', m', \delta', y'), y') dF(\delta')$ is also continuous. This implies that:

$$u(c; P_l) + \beta \sum_{y' \in Y} \pi(y'|y) \int v(a(b', g', m', \delta', y'), y') dF(\delta') \quad (\text{A.29})$$

is a continuous function in c, b', g', m' on $C \times B \times G \times M$. That, combined with the previously established fact that Γ is compact valued and continuous allows us to assert the continuity of Tv from the Theorem of the Maximum. To show that (A.27) is preserved, note that $u(c; P_l)$ is bounded above by \bar{u} , thus $Tv \leq \bar{u} + \beta \bar{u}/(1 - \beta) = \bar{u}/(1 - \beta)$. Further, choosing a consumption expenditure of $c = (1 - \tau)y_{\min}$ is feasible in all periods, thus $Tv \geq u((1 - \tau)y_{\min}; P_l) + \beta u((1 - \tau)y_{\min}; P_l)/(1 - \beta) = u((1 - \tau)y_{\min}; P_l)/(1 - \beta)$. This result, combined with the assumption that $u(0; P_l) = -\infty$ guarantees that Tv satisfies (A.28). Thus $T\mathcal{V} \subset \mathcal{V}$.

The final lemma necessary to prove our main result is:

Lemma 5. *The operator T is a contraction with modulus β .*

Proof. To prove this we will show that T satisfies monotonicity and discounting and then apply Blackwell's sufficient conditions (see Stokey and Lucas (1989), Theorem 3.3²⁰).

a) Monotonicity: Take $v, w \in \mathcal{V}$ such that $v(a, y) \leq w(a, y)$ for all $(a, y) \in A \times Y$. We want to show that $Tv \leq Tw$. By the definition of T and the fact that $\sum_{y' \in Y} \pi(y'|y) \int v(a', y') dF(\delta') \leq \sum_{y' \in Y} \pi(y'|y) \int w(a', y') dF(\delta')$, $Tv \leq Tw$.

¹⁹Corollary 1 on p. 52 of Stokey, Lucas and Prescott 1989, establishes that any closed subspace of a complete metric space is also complete

²⁰Note that all functions in \mathcal{V} are bounded.

b) Discounting: Take any $\gamma \in \mathbb{R}_+$, $T(v + \gamma) = Tv + \beta \sum_{y' \in Y} \pi(y'|y) \int \gamma dF(\delta') = Tv + \beta\gamma$.

Thus, from Blackwell's theorem it follows that T is a contraction mapping with modulus β .

■

Proposition 5. *Under the maintained assumptions on u and the assumption that $F(\delta)$ is \mathcal{C}^2 and log-concave, there exists a unique $v^* \in \mathcal{V}$ such that $Tv^* = v^*$. Furthermore, v^* is strictly increasing in a .*

Proof. From Lemmas 4, 5 above and the contraction mapping theorem it follows that there exists a unique $v^* \in \mathcal{V}$ such that $Tv^* = v^*$. In order to show that v^* is increasing in a we appeal to the monotonicity of Γ and the strict monotonicity of $u(c; P_L)$. Take $a, a' \in A$ such that $a < a'$. We want to show $v^*(a, y) < v^*(a', y)$.

$$v^*(a, y) = \max_{x \in \Gamma(a)} u(c; P_L) + \beta \sum_{y' \in Y} \pi(y'|y) v^*(a(b', g', m', \delta', y'), y') dF(\delta') \quad (\text{A.30})$$

$$= u(c^*; P_L) + \beta \sum_{y' \in Y} \pi(y'|y) v^*(a(b^*, g^*, m^*, \delta', y'), y') dF(\delta') \quad (\text{A.31})$$

$$< u(c^* + a' - a; P_L) + \beta \sum_{y' \in Y} \pi(y'|y) v^*(a(b^*, g^*, m^*, \delta', y'), y') dF(\delta') \quad (\text{A.32})$$

$$\leq \max_{x \in \Gamma(a')} u(c; P_L) + \beta \sum_{y' \in Y} \pi(y'|y) v^*(a(b', g', m', \delta', y'), y') dF(\delta') \quad (\text{A.33})$$

$$= v^*(a', y) \quad (\text{A.34})$$

where the third line comes from the strict monotonicity of u . The fourth line comes from the fact that $a' - a > 0$ and the fact that if $(c^*, b^*, g^*, m^*) \in \Gamma(a)$, then $(c^* + a' - a, b^*, g^*, m^*) \in \Gamma(a')$.

Appendix A.3.3. Characterization

In this section we prove Proposition 4. It is a direct consequence of Theorem 3 in Clausen and Strub (2012).

Proof. We seek to apply theorem 3 of Clausen and Strub (2012). Consider re-writing the value function as follows:

$$V(b, g, m, \delta, d, y) = \max_{x \in \Gamma(a), d' \in \{0,1\}} u(c; P_L) + \beta \sum_{y' \in Y} \pi(y'|y) V(b', g', m', \delta', d', y') dF(\delta') \quad (\text{A.35})$$

where x and Γ are defined as before, but now

$$a = b + y + (1 - d') [(1 - \delta)g - m] \quad (\text{A.36})$$

where we explicitly model the foreclosure decision with d' . Note, that there is a slight abuse of timing here, the d' denotes the choice to default in the current period, whereas d denotes the foreclosure choice in the previous period. As a result, V does not depend on d since the default decision in the previous period is not relevant for the current period's optimization.

In order to apply Theorem 3 of Clausen and Strub, we need to show that the choices satisfy what they call "one-period interior choice." The fact that the utility function is unbounded below guarantees that consumption will always be strictly interior. By assuming, as we did in proposition 4 that $g', b', m', g'', b'', m'' > 0$ guarantees that the choice of c, h, g', b', m' satisfies the "one-period interior choice" condition.

Therefore, since, as we showed above, the value function is finite and as long as the utility function is differentiable, then it will also be differentiable with respect to b, g, m , holding g', b', m', d, d' constant, and is differentiable with respect to g', b', m' , holding b, g, m, d, d' constant (based on how it is defined above). Then, we can conclude that if (g', b', m', d') is an optimal choice at b, g, m, d, δ and $g', b', m' > 0$, then V will be differentiable in g', b', m' for all δ' . Taking first order conditions and rearranging them yields the characterization in Proposition 4. ■

Appendix A.4. A Lower Bound for the Equilibrium Rental Price of Housing P_l

In the main text we established a lower bound for the equilibrium rental rate,

$$P_l \leq \frac{r_b + r_w + r_b r_w + \gamma E(\delta') + 1 - \gamma}{(1 + r_b)(1 + r_w)}.$$

We now discuss a lower bound for the rental rate; whose existence we can be sure of, but for which we have no closed-form characterization. Due to the presence of idiosyncratic house price shocks, the housing asset is an inherently risky asset. Households, however, can effectively choose the risky returns to housing by taking on different levels of leverage. Since households are risk averse, for them to purchase the housing asset, at some leverage the expected return to housing (which includes the "dividend" P_l), weighted by the stochastic discount factor of the household, must be greater than the return of the risk-free bond. This in turn implies a lower bound on the rental price P_l .

Although we do not have an analytical characterization of that lower bound, we found in our quantitative analysis that the equilibrium rental price P_l was such that the expected return of housing *at zero leverage* was always at least as high as the risk free interest rate. This implies that in our numerical analysis the following relationship holds:

$$\left(\frac{1}{1 + r_b}\right) \int_{\underline{\delta}}^1 (1 - \delta') dF(\delta') \geq 1 - P_l$$

which can be rewritten as

$$\begin{aligned} \left(\frac{1}{1 + r_b}\right) (1 - E(\delta')) &\geq 1 - P_l \text{ or} \\ P_l &\geq \frac{r_b + E(\delta')}{1 + r_b}. \end{aligned}$$

This result states that the rental price of housing was not smaller than the (expected) user cost of housing in equilibrium.

Combining these results, with $\gamma = 1$ and $r_w = 0$ we would immediately obtain that the rental price of housing P_l equals its user cost $\frac{r_b + E(\delta')}{1 + r_b}$. In fact, what happens in an equilibrium under this parameterization ($\gamma = 1, r_w = 0$) is that households purchase houses, lever up such

that they default for sure tomorrow and the houses end up in the hand of the banks. Since these are risk-neutral, and since default is fully priced into the mortgage and banks receive the full (but depreciated) value of the house, banks rather than households (which are risk averse) should (from a normative perspective) and will effectively end up owning the real estate. This equilibrium in effect replicates the equilibrium, described in section 6.2 of the paper, in the version of the model in which a housing mutual fund can diversify all idiosyncratic house price risk.

Appendix B. Computational Appendix - NOT FOR PUBLICATION

Appendix B.1. Simplification of the Household Problem

For the computation of the household problem it will be convenient to split the household maximization problem into a portfolio choice problem in which the household chooses how much to invest in bonds and houses and how much of the house to finance, and into a standard intertemporal consumption-saving problem. Define

$$x' = b'P_b + g'[1 - P_l] - m'P_m(\kappa').$$

Then the consumption-savings problem reads as

$$v(a, y) = \max_{0 \leq x' \leq a} \{u(a - x'; P_l) + \beta w(x', y)\}$$

where the value of saving x' units is given by

$$\begin{aligned} w(x', y) &= \max_{b', g', m' \geq 0} \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^{\infty} v(a', y') dF(\delta') \\ &\text{s.t.} \\ x' &= b'P_b + g'[1 - P_l] - m'P_m(\kappa') \\ a' &= b' + \max\{0, (1 - \delta')g' - m'\} + (1 - \tau)y' \end{aligned}$$

This last problem can be conveniently expressed as a choice problem of just bond b' and leverage $\kappa' = m'/g'$. Note that if $g' = 0$ the household cannot borrow and thus $\kappa' = 0$. First, we can write the last two equations in terms of κ' instead of m' :

$$\begin{aligned} x' &= b'P_b + g' \{1 - P_l - \kappa'P_m(\kappa')\} \\ a' &= b' + g' \max\{0, (1 - \delta') - \kappa'\} + (1 - \tau)y' \end{aligned}$$

Now we solve the first equation for g' to obtain

$$\begin{aligned} g' &= \frac{x' - b'P_b}{1 - P_l - \kappa'P_m(\kappa')} \\ a' &= b' + \frac{x' - b'P_b}{1 - P_l - \kappa'P_m(\kappa')} \max\{0, (1 - \delta') - \kappa'\} + (1 - \tau)y' \end{aligned}$$

and thus the portfolio choice problem boils down to a choice of b' and κ' :

$$w(x', y) = \max_{\substack{0 \leq b' \leq x'/P_b \\ 0 \leq \kappa'}} \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^{\infty} v \left\{ b' + \frac{x' - b'P_b}{1 - P_l - \kappa'P_m(\kappa')} \max\{0, (1 - \delta') - \kappa'\} + (1 - \tau)y', y' \right\} dF(\delta')$$

Appendix B.2. Labor Income Process

From the Tauchen procedure we obtain (after de-logging) the five labor productivity shock realizations $y \in \{0.3586, 0.5626, 0.8449, 1.2689, 1.9909\}$ and the following transition matrix:

$$\pi = \begin{bmatrix} 0.7629 & 0.2249 & 0.0121 & 0.0001 & 0.0000 \\ 0.2074 & 0.5566 & 0.2207 & 0.0152 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0152 & 0.2207 & 0.5566 & 0.2074 \\ 0.0000 & 0.0001 & 0.0121 & 0.2249 & 0.7629 \end{bmatrix}$$

which in turn implies the stationary distribution $\Pi = (0.1907, 0.2066, 0.2053, 0.2066, 0.1907)$ and an average labor productivity of one.

Appendix C. Sensitivity Analysis Appendix - NOT FOR PUBLICATION

Appendix C.1. The Economy with Capital

Appendix C.1.1. Details of the Model

The economy with capital is identical to the Aiyagari (1994) economy, augmented by risky housing assets and housing services, as in the benchmark economy. Instead of labor income, households now receive idiosyncratic labor endowments $y \in Y$ in each period. These endowments follow a finite state Markov chain with transition probabilities $\pi(y'|y)$ and unique invariant distribution $\Pi(y)$, as does labor income in the benchmark model. A household's labor income in a given period is then given by wy , where w is the economy-wide wage. We denote by $\bar{y} = \sum_{y \in Y} y\Pi(y)$ the average labor endowment and normalize it to 1. We will use the terms labor endowment and labor income interchangeably throughout the remainder of this appendix, and as before assume that a law of large number applies, so that π and Π also denote deterministic fractions of households receiving a particular labor income shock y .

In addition to consumption and housing services the household spends income to purchase three types of assets, one-period pure discount bonds b' , perfectly divisible houses g' (as before) and physical capital \tilde{k}' . Capital pays a net return $r - \delta_k$ where r is the rental rate on capital and δ_k its depreciation rate. The recursive problem of the households now reads as

$$v(a, y) = \max_{c, h, b', m', g', \tilde{k}' \geq 0} \left\{ U(c, h) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^1 v(a', y') dF(\delta') \right\} \quad (C.1)$$

subject to

$$c + b'P_b + hP_l + g'P_h + \tilde{k}' - m'P_m(g', m') = a + g'P_l \quad (C.2)$$

$$a'(\delta', y', m', g') = (1 + r - \delta_k)\tilde{k}' + b' + \max\{0, P_h(1 - \delta')g' - m'\} + (1 - \tau)wy' \quad (C.3)$$

On the production side, a representative firm rents labor and capital and produces the final good which can be used for consumption and physical capital investment purposes. The production technology is given by

$$Y = AK^\alpha L^{1-\alpha}$$

where K is the physical capital stock, L is the labor input used by the representative firm and Y is aggregate output. The parameter $\alpha \in [0, 1)$ measures the capital share. Note that $\alpha = 0$ corresponds to our endowment economy. The real estate construction sector and the financial intermediary sector works as before. The same is true for the government who levies income taxes at a flat rate τ on households to finance the mortgage interest rate subsidy. The tax revenues of the government are now given by τw , recalling that aggregate labor supply is normalized to 1. For a loan of type (m', g') the subsidy by the government is given by

$$sub(m', g') = \theta m' P_m(g', m'; \phi = \theta)$$

and the total economy-wide subsidy is

$$G = \int sub(m', g') d\mu \tag{C.4}$$

The income tax rate then has to satisfy

$$\tau = \frac{G}{w}. \tag{C.5}$$

Appendix C.1.2. Definition of Competitive Equilibrium

We are now ready to define a stationary recursive Competitive Equilibrium for the benchmark economy. Let $S = \mathbb{R}_+ \times Y$ denote the individual state space.

Definition Given a government subsidy policy ϕ a **Stationary Recursive Competitive Equilibrium** are value and policy functions for the households, $v, c, h, b', m', g' : S \rightarrow \mathbb{R}$, policies for the production sector K, L , for the real estate construction sector I, C_h , prices P_l, P_b , wages and rental rates w, r , a mortgage pricing function $P_m : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$, a government tax rate τ and government spending G , as well as a stationary measure μ such that

1. (Household Maximization) Given prices P_l, P_b, P_m and government policies the value function solves (C.1) and c, h, b', m', g' are the associated policy functions.
2. Production Firm Maximization

$$\begin{aligned} w &= (1 - \alpha)A \left(\frac{K}{L}\right)^\alpha \\ r &= \alpha A \left(\frac{K}{L}\right)^{\alpha-1} \end{aligned}$$

3. (Real Estate Construction Company Maximization) Policies I, C_h solve (4).
4. (Loan-by-Loan Competition) Given P_b and P_m , (6) holds with equality for all m', g' .
5. (Government Budget Balance) The tax rate function τ satisfies (C.5) and government spending G satisfies (C.4), given the functions m', P_m .

6. (Market Clearing in Rental Market)

$$\int g'(s)d\mu = \int h(s)d\mu$$

7. (Market Clearing in the Bond Market)

$$P_b \int b'(s)d\mu = (1 + r_w + \theta - \phi) \int P_m(g', m'; \phi)m'(s)d\mu$$

8. (Market Clearing in the Capital Market)

$$\int \tilde{k}'(s)d\mu = K$$

9. (Market Clearing in the Labor Market)

$$\int yd\mu = L$$

10. (Market Clearing in the Goods Market)

$$\int c(s)d\mu + C_h + \delta_k K + (r_w + \theta - \phi) \int P_m(g', m'; \phi)m'd\mu + G = AK^\alpha L^{1-\alpha}$$

where

$$C_h = I = \int g'(s)d\mu - \int \left[\int_{\underline{\delta}}^{\delta^*(\kappa'(s))} g'(s)(1 - \delta')dF(\delta') - \gamma \int_{\delta^*(\kappa'(s))}^1 g'(s)(1 - \delta')dF(\delta') \right] d\mu$$

is gross investment in the housing stock.

11. (Invariance of Distribution μ). The measure μ is invariant with respect to the Markov process induced by the exogenous Markov process π and the policy functions m', g', b' .

When we derive the welfare consequences of removing the mortgage interest subsidy, as before we measure aggregate economy-wide welfare via a Utilitarian social welfare function in the steady state, defined as

$$\mathcal{WEL} = \int v(s)\mu(ds)$$

where μ is the invariant measure over the state space for cash at hand and income, $s = (a, y)$.

Appendix C.1.3. Theoretical Results

Simplification of the Household Problem. First, we note that bonds and capital are both risk-free assets, and for either to be demanded in positive but finite amounts, it needs to be the case that both assets command the same returns. Thus in any equilibrium where both assets are traded

$$P_b = \frac{1}{1 + r - \delta_k}.$$

Define a new variable $k' = \tilde{k}' + P_b b'$. Then the household problem can be written as

$$v(a, y) = \max_{c, h, m', g', k' \geq 0} \left\{ U(c, h) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^1 v(a', y') dF(\delta') \right\}$$

subject to

$$c + k' + hP_l + g'P_h - m'P_m(g', m') = a + g'P_l \quad (\text{C.6})$$

$$a'(\delta', y', m', g') = (1 + r - \delta_k)k' + \max\{0, P_h(1 - \delta')g' - m'\} + (1 - \tau)wy' \quad (\text{C.7})$$

As before the household problem can be separated into a static and dynamic problem, with end result:

$$\begin{aligned} v(s) &= \max_{c, b', m', g' \geq 0} \left\{ u(c; P_l) + \beta \sum_{y'} \pi(y'|y) \int_{\underline{\delta}}^1 v(s') dF(\delta') \right\} \\ a &= c + k' + g'[1 - P_l] - m'P_m(k') \\ a'(\delta', h', m', g') &= (1 + r - \delta_k)k' + \max\{0, (1 - \delta')g' - m'\} + (1 - \tau)wy' \end{aligned}$$

and the same properties of the household problem go through as before go through.

Simplification of the Recursive Competitive Equilibrium. Trivially $L = 1$ from the labor market clearing condition, and thus

$$\begin{aligned} w &= (1 - \alpha)AK^\alpha \\ r &= \alpha AK^{\alpha-1} \end{aligned}$$

Adding the bond and capital market clearing conditions yields

$$P_b \int b'(s) d\mu + \int \tilde{k}'(s) d\mu = K + (1 + r_w + \theta - \phi) \int P_m(g', m'; \phi) m'(s) d\mu$$

and thus

$$\int k'(s) d\mu = K + (1 + r_w + \theta - \phi) \int P_m(g', m'; \phi) m'(s) d\mu$$

Consequently the only changes the shift from the endowment to the production economy entails is that now the risk-free interest rate in the economy is tied to the marginal product of labor, and that the market clearing conditions for risk-free assets now equates household demand for these assets to the *sum* of real assets (the physical capital stock) and financial assets (mortgages), rather than just financial assets, as in the endowment economy.

Appendix C.1.4. Calibration

Our objective is to calibrate the steady state equilibrium of the production economy with a 30bp subsidy to the same targets as we did for the endowment economy. We retain the same parameters for the idiosyncratic income process (since the common wage w just adds an aggregate constant to the log-income equation) as well as for the idiosyncratic house price depreciation process. Parameters governing the foreclosure technology and the government subsidy remain the same as well.

Variable	Baseline		With Capital		With Capital (Recalibrated)	
σ	3.912		3.912		7.512	
β	0.919		0.919		0.893	
% Sub	30bp	0bp	30bp	0bp	30bp	0bp
Median Leverage	61.5%	0.0%	0.0%	0.0%	61.5%	0.0%
r_b	1.0%	0.518%	2.841%	2.840%	1.0%	0.676%
Sub/\bar{y}	0.97%	0%	0.006%	0%	0.96%	0%
P_l	0.0281	0.0278	0.0450	0.0450	0.0296	0.0294
GDP	1.150	1.151	1.430	1.430	1.585	1.587
H/GDP	4.633	4.737	2.491	2.491	3.332	3.404
M/GDP	2.810	0.269	0.012	0.0001	2.018	0.003
K/GDP	0.0	0.0	2.353	2.355	2.702	2.830
w	1	1	0.93	0.93	1	1.014
Default share	0.51%	0.41%	0.28%	0.15%	0.49%	0.30%
Mean Net Worth/GDP	4.633	4.737	4.844	4.846	6.034	6.234
Median Bond Share	60.7%	7.2%	48.8%	48.6%	72.2%	47.8%
$\mu(g' > 0)$	96.79%	96.66%	97.6%	97.6%	100%	100%
$\mu(g' > h)$	43.00%	39.77%	47.6%	48.2%	45.7%	45.9%
CEV^*	0	0.81%	0%	0.04%	0%	2.17%

The production side of the economy is characterized by the three parameters A, α, δ_k . We normalize $A = 0.9232$ such that $w = 1$ in the benchmark equilibrium which facilitates comparisons with the endowment economy and choose a capital share of $\alpha = 0.3$ and a depreciation rate of $\delta_k = 9\%$. With these choices and a target capital-to-output ratio of $K/Y = 3$ (achieved by judicious choice of the preference parameters below) the model delivers the same risk-free interest rate of 1% as in the benchmark endowment economy.

The time discount parameter β and the CRRA parameter σ are endogenously calibrated to match an equilibrium risk free rate of 1% (equivalently, given the other parameters discussed above, a capital-output ratio of 3) and a median household leverage of 61% in the benchmark economy. In order to simultaneously match these targets we find that we need a much higher coefficient of relative risk aversion and a much lower time discount rate. The calibrated parameters are $(\beta, \sigma) = (0.893, 7.512)$. In order to understand the intuition, increasing the value of σ has a two-fold effect - first, it increases precautionary saving demand, helping to match a capital-output of 3 (rather than zero, as in the endowment economy). However, the higher contemporaneous risk-aversion also makes households more willing to "borrow-to-save" because they value the increased insurance. Lowering the time discount factor then allows us to then match the risk-free interest rate. For comparison we also report results obtained for the production economy under the original, endowment economy, preference specification with $(\beta, \sigma) = (0.919, 3.912)$.

Appendix C.1.5. Quantitative Results with Physical Capital

The results of our experiment in the model with physical capital are presented in Table Appendix C.1.5. The main text contains a shortened version of this table in which only results from the re-calibrated economy with capital are presented.

From the table we observe that the introduction of physical capital leaves the results of the policy analysis qualitatively, and to a large extent quantitatively unchanged *if we re-calibrate the model* to be consistent with the same targets as was the model without capital. Under the *old calibration*, but in the model with capital, households save predominantly in riskless capital, the demand for riskless assets and mortgages collapses and the equilibrium interest rate rises significantly (from 1% in the model without capital to 2.8% in the model with capital). Median leverage, one of the calibration targets falls from 61% in the benchmark model to zero in the model with capital (both in the presence of the subsidy). Since the mortgage market almost completely collapses the removal of the subsidy has essentially no effect on allocations and welfare. We don't think that the economy with capital, under the no-capital calibration, is a useful laboratory to analysis the hypothetical policy reform since it results in the counterfactual absence of any meaningful mortgage market. And of course, if there are no mortgages traded in equilibrium, a policy that subsidizes these mortgages has no effect.

Therefore we would argue that, in order to assess the sensitivity of our benchmark results to the inclusion of capital, one should re-calibrate the capital economy to be consistent with the same targets as the no-capital economy, and especially a risk-free interest rate of 1% as well as a median leverage of 61%. As discussed above, in order to induce households to borrow more (and saving households to save more) in the model we have to make the precautionary savings motive more potent by increasing the risk aversion and prudence parameter σ from 3.9 to 7.5. In addition, the required discount factor β falls from 0.92 to 0.89. The changes in these parameters resurrect the quantitative importance of the borrow to save motive and results in median leverage equal to the empirical target of 61% as well as a risk free rate of 1%, as in the economy without capital.

With this parameterization, both in terms of aggregate allocations, and specifically the equilibrium allocation of mortgages, houses and financial assets, the tax rate to finance the subsidy, and the rental price, as well as in terms of the welfare consequences, the endowment economy and the re-calibrated economy with capital display qualitatively, and to a large part quantitatively, the same consequences of a removal of subsidy. The one quantitative exception are the welfare gains from the removal of the subsidy, which are significantly *larger* in the economy with capital, thus reinforcing the normative point we wish to make. The key difference to the economy without capital is a larger value of the risk aversion (prudence) parameter σ , which induces larger curvature in the utility and thus value function of households. Thus a policy reform (such as the removal of the subsidy) that redistributes from rich (high income and cash at hand) to poor households constitutes larger aggregate welfare gains, under our utilitarian social welfare function.

Appendix C.2. The Economy with Housing Mutual Fund

Now, instead of introducing physical, suppose there exists a housing mutual fund that can purchase an entire portfolio (with positive measure) of housing assets and thus exploit the law of large numbers to perfectly diversify the idiosyncratic house price risk. Since the mutual fund is a risk-free asset it has to earn the same return r_b as risk-free bonds.

For each dollar invested, the mutual fund buys one unit of housing (recall that the price of housing was normalized to 1), rents it out immediately at price P_t and tomorrow sell the non-depreciated part $(1 - E(\delta))$, again at price 1. Note the fact that the mutual fund can perfectly diversify idiosyncratic depreciation risk explains why a deterministic fraction $1 - E(\delta)$ of the

fund's housing stock depreciates. Implicit in this discussion is that the mutual fund cannot default on part of its portfolio.

Appendix C.2.1. Equilibrium Rental Price

Thus the gross return of the fund's investment strategy tomorrow is

$$(1 + r_b)P_l + 1 - E(\delta)$$

which has to be equal to the return on bonds and capital, or

$$(1 + r_b)P_l + 1 - E(\delta) = 1 + r_b$$

and thus in the mutual fund economy the rental price is given by the user cost of housing:

$$P_l = \frac{r_b + E(\delta)}{1 + r_b} \tag{C.8}$$

At this rental price the housing mutual fund is a perfect substitute for the risk-free bond and thus the mutual fund is a redundant asset *from the perspective of the household*. Thus the existence of the mutual fund does not alter the household decision problem²¹, relative to the benchmark economy, but it pins down the equilibrium rental price P_l in equation (C.8).

Appendix C.2.2. Asset Market Clearing Condition

The housing position of the housing mutual fund is then given by

$$M = \int h(s)d\mu - \int g'(s)d\mu$$

and the asset market clearing condition that determines the still endogenous risk-free interest rate r_b now reads as

$$P_b \int b'(s)d\mu = M + (1 + r_w + \theta - \phi) \int P_m(g', m'; \phi)m'(s)d\mu.$$

Note that in this economy households can still buy individual houses carrying idiosyncratic risk using mortgages, and might opt to do so given the option-like mortgage cum foreclosure contract. But if in fact $\int g'(s)d\mu \equiv 0$, this economy collapses to one in which the entire housing stock is owned by the housing mutual fund, and individual households have neither houses nor mortgages in their asset portfolio. Table 4 shows that this is indeed what happens in the benchmark economy to a good first approximation. As a consequence this economy cannot reproduce the empirically observed median level or distribution of mortgage leverage in the economy, and the removal of the mortgage subsidy is inconsequential for allocations and welfare.

²¹The risk-free asset position is now a composite of risk-free bonds and risk-free housing mutual funds, both earning the same return.

References for Online Appendix

G. Debreu. 1952. "Definite and semidefinite quadratic forms." *Econometrica* 20(2):295-300.